Pradeep's FUNDAMENTAL PHYSICS Vol. I

Strictly according to new syllabus & examination pattern of CBSE, ISCE and state boards of Punjab, Haryana, H.P., J & K, Odisha, Karnataka, Kerala, Nagaland, Manipur, Rajasthan, Jharkhand, Bihar, U.P., Uttarakhand, M.P., Chhatisgarh, Assam, Gujarat, West Bengal, A.P. etc.
for **class XII of +2 stage.**

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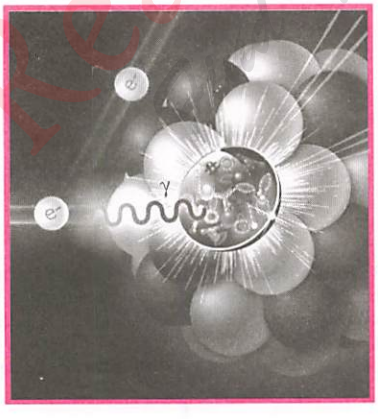

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Contents

Unit – 1 [Electrostatics]

Chapters		Pages
1.	**Electrostatic Charges and Fields**	**1/1—1/134**
1.1.	Introduction	1/1
1.2.	Electric Charge	1/2
1.3.	Two Kinds of Charges	1/2
1.4.	Conductors, Insulators and Dielectrics	1/4
1.5.	Gold Leaf Electroscope (GLE)	1/5
1.6.	Origin of Electric Charge in Electrostatics	1/5
*1.7.	Charging by Induction	1/6
1.8.	Quantization of Electric Charge	1/8
1.9.	Additivity of Charge	1/9
1.10.	Conservation of Charge	1/10
1.11.	Comparison of Charge and Mass	1/10
1.12.	Coulomb's Law	1/11
1.13.	Coulomb's Law in Vector Form	1/12
1.14.	Units of Charge	1/14
1.15.	Dielectric Constant or Relative Electrical Permittivity	1/14
1.16.	Forces between Multiple Charges : Principle of Superposition	1/15
1.17.	Continuous Charge Distribution	1/16
1.18.	Force Due to Continuous Distribution of Charges	1/17
1.19.	Concept of Electric Field	1/18
1.20.	Electric Field Intensity	1/18
1.21.	Electric Field Intensity due to a Point Charge	1/19
1.22.	Electric Field Intensity due to a Group of Charges	1/20
1.23.	Electric Field Intensity due to Continuous Charge Distribution	1/21
1.24.	Rectangular Components of Electric Intensity due to a Point Charge	1/21
1.25.	Physical Significance of Electric Field	1/22
1.26.	Electric Field Lines	1/22
1.27.	Properties of Electric Field Lines	1/23
1.28.	Electric Dipole	1/24
1.29.	Dipole Moment	1/25
1.30.	Dipole Field	1/26
1.31.	Field Intensity on Axial Line of Electric Dipole	1/26
1.32.	Field Intensity on Equatorial Line of Electric Dipole	1/27
1.33.	Electric Field Intensity at any Point due to a Short Electric Dipole	1/28

** Not included in CBSE syllabus. This article has been given only for the preparation of competitive examinations.*

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1.34. Electric Field Intensity at any Point on the axis of a Uniformly Charged Ring 1/29
1.35. Electric Dipole in a Uniform Two Dimensional Electric Field 1/30
1.36. Potential Energy of Dipole in a Uniform Electric Field 1/32
1.37. Area Vector 1/33
1.38. Electric Flux 1/33
1.39. Gauss's Theorem or Gauss's Law in Electrostatics 1/34
1.40. Simple Verification of Gauss's Theorem 1/37
1.41. Deduction of Coulomb's Law from Gauss's Theorem 1/38
1.42. Field due to an Infinitely Long Straight Uniformly Charged Wire 1/38
1.43. Electric Field Intensity due to a Uniformly Charged Spherical Shell 1/40
1.44. Electric Field Intensity due to a Non-conducting Charged Solid Sphere 1/41
1.45. Electric Field Intensity due to a Thin Infinite Plane Sheet of Charge 1/42
1.46. Electric Field Intensity due to Two Thin Infinite Plane Parallel Sheets of Charge 1/43
Solved Examples 1/44
Revision At A Glance 1/60
Multiple Choice Questions 1/62
Assertion-Reason Type Questions 1/65
Conceptual Problems 1/69
Very Short Answer Questions [*1 Mark*] 1/75
Short Answer Questions [*2 Marks*] 1/78
Short Answer Questions [*3 Marks*] 1/80
Long Answer Questions [*5 or more Marks*] 1/81
Case-Based Very Short/Short Questions 1/82
Case-Based MCQs and Assertion-Reason Questions 1/83
Problems For Practice 1/85
***NCERT** Exercises with Solutions* 1/97
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 1/102
Competition Focus [*NEET/JEE Special*] 1/116

2. Electrostatic Potential and Capacitance **2/1—2/140**

2.1. Introduction 2/1
2.2. Electrostatic Potential Energy 2/1
2.3. Electrostatic Potential 2/2
2.4. Electrostatic Forces are conservative 2/3
2.5. Electrostatic Potential due to a Point Charge 2/4
2.6. Potential at a Point due to Group of Electric Charges 2/5
2.7. Electrostatic Potential at a Point due to an Electric Dipole 2/6
2.8. Equipotential Surfaces 2/8
2.9. Relation Between Electric Intensity and Electric Potential 2/9
2.10. Expression for Electrostatic Potential Energy of a System of Charges 2/10
2.11. Potential Energy of Charges in an External Electric Field 2/12
2.12. Electrostatics of Conductors 2/13

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2.13. Electrostatic Shielding 2/15
2.14. Electrical Capacitance 2/16
2.15. Capacity of an Isolated Spherical Conductor 2/17
2.16. Capacitor and Its Principle 2/17
2.17. Parallel Plate Capacitor 2/18
2.18. Relative Permittivity or Dielectric Constant 2/19
2.19. Spherical Capacitor 2/20
2.20. Grouping of Capacitors/Condensers 2/20
2.21. Energy Stored in a Capacitor 2/22
2.22. Total Energy Stored in a Combination of Capacitors 2/24
2.23. Energy Density of a Parallel Plate Capacitor 2/24
2.24. Common Potential 2/25
2.25. Loss of Energy on Sharing Charges 2/25
2.26. Non-polar and Polar Dielectrics 2/26
2.27. Dielectric Polarization 2/27
2.28. Dielectric Strength 2/29
2.29. Capacitance of a Parallel Plate Capacitor with a Conducting Slab 2/29
2.30. Capacitance of a Parallel Plate Capacitor with a Dielectric Slab 2/30
2.31. Action of Sharp Points 2/32
2.32. Lightning Conductor 2/32
2.33. Van de Graaff Electrostatic Generator 2/32
Solved Examples 2/33
Revision At A Glance 2/55
Multiple Choice Questions 2/58
Assertion-Reason Type Questions 2/61
Conceptual Problems 2/64
Very Short Answer Questions [*1 Mark*] 2/71
Short Answer Questions [*2 Marks*] 2/74
Short Answer Questions [*3 Marks*] 2/82
Long Answer Questions [*5 or more Marks*] 2/83
Case-Based Very Short/Short Questions 2/84
Case-Based MCQs and Assertion-Reason Questions 2/86
Problems For Practice 2/88
***NCERT** Exercises with Solutions* 2/107
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 2/109
Competition Focus [*NEET/JEE Special*] 2/119

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Unit – 2 [Current Electricity]

3. Current Electricity 3/1—3/214

3.1. Introduction 3/1
3.2. Current Carriers 3/1
3.3. Electromotive Force 3/2
3.4. Difference Between Electromotive Force and Potential Difference 3/2
3.5. Electric Current 3/3
3.6. Electric Current in Conductors 3/5
3.7. Drift Velocity 3/5
3.8. Mobility 3/6
3.9. Relation Between Current and Drift Velocity 3/7
3.10. Ohm's Law 3/8
3.11. Electrical Resistance 3/9
3.12. Electrical Resistivity or Specific Resistance 3/9
3.13. Current Density, Conductance and Electrical Conductivity 3/12
3.14. Classification of Materials in Terms of Conductivity 3/14
3.15. Effect of Temperature on Resistance 3/14
3.16. Variation of Electrical Resistivity with Temperature 3/16
3.17. Non-Ohmic Devices or Limitations of Ohm's Law 3/17
3.18. Thermistors 3/17
3.19. Super-Conductivity 3/18
*3.20. Meissner Effect 3/19
*3.21. Colour Code for Carbon Resistors 3/19
*3.22. Resistances in Series 3/20
*3.23. Resistances in Parallel 3/21
3.24. EMF, Internal Resistance and Terminal Potential Difference of a Cell 3/22
3.25. Distinction Between E.M.F. and Terminal Potential Difference of a Cell 3/24
3.26. Grouping of Two Unidentical Cells in Series and Parallel 3/24
3.27. Grouping of Number of Identical Cells 3/26
3.28. Heating Effect of Current ; Joule's Law 3/29
3.29. Cause of Heating Effect of Current 3/29
3.30. Heat Produced by Electric Current 3/29
3.31. Electric Power 3/30
3.32. Electric Energy 3/30
3.33. Some Aspects of Heating Effects of Current 3/31
3.34. Maximum Power Transfer Theorem 3/32
3.35. Kirchhoff's Rules 3/32
3.36. Wheatstone Bridge Principle 3/34
3.37. Temperature Measurement Using Wheatstone Bridge 3/35
3.38. Slide wire Bridge or Meter Bridge 3/36
*3.39. Potentiometer and its Principle of Working 3/37

** Not included in CBSE syllabus. These articles have been given only for the preparation of competitive examinations.*

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*3.40. Determination of Potential Difference Using Potentiometer 3/38
*3.41. Comparison of EMFs of Two Cells Using Potentiometer 3/39
*3.42. Determination of Internal Resistance of a Cell by Potentiometer Method 3/40
*3.43. Sensitiveness of Potentiometer 3/40
*3.44. Difference Between Potentiometer and Voltmeter 3/41
Solved Examples 3/41
Revision At A Glance 3/74
Multiple Choice Questions 3/77
Assertion-Reason Type Questions 3/82
Conceptual Problems 3/89
Very Short Answer Questions [*1 Mark*] 3/102
Short Answer Questions [*2 Marks*] 3/110
Short Answer Questions [*3 Marks*] 3/122
Long Answer Questions [*5 or more Marks*] 3/124
Case-Based Very Short/Short Questions 3/125
Case-Based MCQs and Assertion-Reason Questions 3/127
Problems For Practice 3/130
***NCERT** Exercises with Solutions* 3/162
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 3/165
Competition Focus [*NEET/JEE Special*] 3/175

Unit – 3 [Moving Charges and Magnetism]

4. Moving Charges and Magnetism 4/1—4/192
4.1. Introduction 4/1
4.2. Oersted's Experiment 4/1
4.3. Magnetic Field 4/2
4.4. Biot-Savart's Law 4/4
4.5. Similarities and Dis-Similarities Between the Biot-Savart's Law for the Magnetic Field and Coulomb's Law for Electrostatic Field 4/7
4.6. Magnetic Field due to a Straight Wire Carrying Current 4/7
4.7. Magnetic Field at the Centre of the Circular Coil Carrying Current 4/9
4.8. Magnetic Field at a Point on the Axis of a Circular Coil Carrying Current 4/11
4.9. Magnetic Field due to Various Current Carrying Configurations 4/13
4.10. Ampere's Circuital Law 4/15
4.11. Applications of Ampere's Circuital Law 4/17
4.12. Magnetic Field due to Infinite Long Straight Wire Carrying Current 4/18
4.13. Magnetic Field due to Current Through a Very Long Circular Cylinder 4/18
4.14. The Solenoid 4/20
4.15. The Toroid 4/22
4.16. Comparison Between Ampere's Circuital Law and Biot Savart's Law 4/23
4.17. Motion of a Charged Particle in a Uniform Electric Field 4/24
4.18. Motion of a Charged Particle in a Uniform Magnetic Field 4/25
4.19. Lorentz Force 4/27

** Not included in CBSE syllabus. These articles have been given only for the preparation of competitive examinations.*

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4.20. Velocity Filter 4/28
4.21. Cyclotron 4/28
4.22. Force on a Current Carrying Conductor Placed in a Magnetic Field 4/31
4.23. Force Between Two Parallel Linear Conductors Carrying Current 4/33
4.24. Experimental Demonstration of the Magnetic Force Between Parallel Currents 4/34
4.25. Comparison of Electrical and Magnetic Forces 4/36
4.26. Torque on a Current Carrying Coil in a Magnetic Field 4/37
4.27. Moving Coil Galvanometer 4/39
4.28. Weston Type Galvanometer 4/41
4.29. Shunt 4/42
4.30. Ammeter 4/43
4.31. Voltmeter 4/43
Solved Examples 4/45
Revision At A Glance 4/66
Multiple Choice Questions 4/70
Assertion-Reason Type Questions 4/74
Conceptual Problems 4/83
Very Short Answer Questions [*1 Mark*] 4/93
Short Answer Questions [*2 Marks*] 4/98
Short Answer Questions [*3 Marks*] 4/109
Long Answer Questions [*5 or more Marks*] 4/112
Case-Based Very Short/Short Questions 4/112
Case-Based MCQs and Assertion-Reason Questions 4/114
Problems For Practice 4/116
***NCERT** Exercises with Solutions* 4/136
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 4/139
Competition Focus [*NEET/JEE Special*] 4/150

5. Magnetism And Matter 5/1—5/116

5.1. Introduction 5/1
5.2. The Bar Magnet 5/1
5.3. Some Basics of Magnetism 5/1
5.4. The Magnetic Field Lines 5/3
5.5. Properties of Magnetic Field Lines 5/5
5.6. Magnetic Dipole 5/5
5.7. Magnetic Field Strength at a Point due to Bar Magnet 5/6
5.8. Current Loop as a Magnetic Dipole 5/8
5.9. Bar Magnet as an Equivalent Solenoid 5/9
5.10. Torque on a Bar Magnet in a Magnetic Field 5/10
5.11. Determination of Magnitude of Magnetic Field Using a Compass Needle 5/11
5.12. Potential Energy of a Magnetic Dipole in a Magnetic Field 5/11
5.13. The Electrostatic Analogue of Magnetism 5/13
5.14. Gauss's Law for Magnetism 5/14

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5.15. Magnetic Field of Earth 5/15
5.16. Cause of Earth's Magnetism 5/15
5.17. General Features of Earth's Magnetic Field 5/15
5.18. Magnetic Elements 5/16
5.19. Variation of Earth's Magnetic Field 5/19
5.20. Neutral Points 5/20
5.21. The Tangent Law in Magnetism 5/21
5.22. Tangent Galvanometer 5/22
5.23. Bohr Magneton or Magnetic Dipole Moment of an Atom due to Revolving Electron 5/24
5.24. Some Important Terms Used in Magnetism 5/25
5.25. Relation Between Magnetic Permeability and Magnetic Susceptibility 5/28
5.26. Classification of Magnetic Materials 5/29
5.27. Curie Law in Magnetism 5/36
5.28. Hysterisis Curve 5/37
5.29. Energy Dissipation Due to Hysteresis 5/38
5.30. Uses of Ferromagnetics 5/39
Solved Examples 5/41
Revision At A Glance 5/54
Multiple Choice Questions 5/57
Assertion-Reason Type Questions 5/58
Conceptual Problems 5/62
Very Short Answer Questions [*1 Mark*] 5/67
Short Answer Questions [*2 Marks*] 5/70
Short Answer Questions [*3 Marks*] 5/73
Long Answer Questions [*5 or more Marks*] 5/74
Case-Based Very Short/Short Questions 5/75
Case-Based MCQs and Assertion-Reason Questions 5/77
Problems For Practice 5/79
***NCERT** Exercises with Solutions* 5/89
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 5/92
Competition Focus [*NEET/JEE Special*] 5/101

Unit – 4 [Electromagnetic Induction And Alternating Currents]

6. Electromagnetic Induction 6/1—6/102

6.1. Introduction 6/1
6.2. Magnetic Flux 6/1
6.3. The Experiments of Faraday and Henry 6/4
6.4. Cause of Induced E.M.F. 6/5
6.5. Faraday's Laws of Electromagnetic Induction 6/6
6.6. Lenz's Law 6/7
6.7. Lenz's Law and Energy Conservation 6/8
6.8. Fleming's Right Hand Rule 6/8
6.9. Various Methods of Producing Induced E.M.F. 6/9

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6.10. Motional Electromotive Force 6/10
6.11. Power Dissipated in a Conductor Moved in Magnetic Field 6/11
6.12. Eddy Currents or Foucault Currents 6/12
6.13. Applications of Eddy Currents 6/13
6.14. Self Induction 6/14
6.15. Self Inductance of a Long Solenoid 6/17
6.16. Grouping of Coils 6/17
6.17. Mutual Induction 6/18
6.18. Mutual Inductance of Two Long Coaxial Solenoids 6/20
6.19. A.C. Generator or A.C. Dynamo 6/22
6.20. Multi Phase A.C. Generator 6/25
*6.21. D.C. Generator or D.C. Dynamo 6/26
*6.22. D.C. Motor 6/27
*6.23. Motor Starter 6/29
Solved Examples 6/30
Revision At A Glance 6/37
Multiple Choice Questions 6/39
Assertion-Reason Type Questions 6/43
Conceptual Problems 6/47
Very Short Answer Questions [*1 Mark*] 6/52
Short Answer Questions [*2 Marks*] 6/55
Short Answer Questions [*3 Marks*] 6/59
Long Answer Questions [*5 or more Marks*] 6/59
Case-Based Very Short/Short Questions 6/60
Case-Based MCQs and Assertion-Reason Questions 6/62
Problems For Practice 6/64
***NCERT** Exercises with Solutions* 6/70
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 6/73
Competition Focus [NEET/JEE Special] 6/85

7. Alternating Currents 7/1—7/132

7.1. Introduction : The Alternating Current 7/1
7.2. Transient Currents 7/2
7.3. Growth and Decay of Current in an Inductor 7/3
7.4. Charging and Discharging of a Capacitor Through Resistor 7/5
7.5. Mean Value or Average Value of Alternating Current 7/8
7.6. Mean Value or Average Value of Alternating E.M.F. 7/9
7.7. Root Mean Square Value of Alternating Current 7/10
7.8. Root Mean Square Value of Alternating E.M.F. 7/11
7.9. A.C. Circuit Containing Resistance Only 7/12
7.10. A.C. Circuit Containing Inductance Only 7/13
7.11. A.C. Circuit Containing Capacitance Only 7/15

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7.12. A.C. Circuit Containing Resistance, Inductance and Capacitance in Series (RLC Circuit) 7/17
7.13. Impedance Triangle 7/20
7.14. A.C. Circuit Containing Resistance and Inductance 7/21
7.15. A.C. Circuit Containing Resistance and Capacitance 7/21
7.16. Energy Stored in an Inductor 7/23
7.17. LC Oscillations 7/24
7.18. Analogy Between Mechanical and Electrical Quantities 7/26
7.19. Electric Resonance 7/27
7.20. Q Factor of Resonance Circuit or Sharpness of Resonance 7/29
7.21. Parallel Resonance Circuit 7/30
7.22. Average Power Associated with Resistance or Non-inductive Circuit 7/31
7.23. Average Power Associated with an Inductor 7/32
7.24. Average Power Associated with a Capacitor 7/33
7.25. Average Power in RLC Circuit or Inductive Circuit 7/34
7.26. Power Factor of An A.C. Circuit 7/36
7.27. Wattless Current or Idle Current 7/37
7.28. Real Resistors, Inductors and Capacitors 7/38
7.29. Advantages and Drawbacks of A.C. Over D.C. 7/38
7.30. Transformer 7/38
7.31. Energy Losses in a Transformer 7/40
7.32. Uses of Transformer 7/41
7.33. Choke Coil 7/42
Solved Examples 7/43
Revision At A Glance 7/59
Multiple Choice Questions 7/61
Assertion-Reason Type Questions 7/64
Conceptual Problems 7/68
Very Short Answer Questions [*1 Mark*] 7/75
Short Answer Questions [*2 Marks*] 7/77
Short Answer Questions [*3 Marks*] 7/80
Long Answer Questions [*5 or more Marks*] 7/81
Case-Based Very Short/Short Questions 7/82
Case-Based MCQs and Assertion-Reason Questions 7/84
Problems For Practice 7/86
***NCERT** Exercises with Solutions* 7/102
***NCERT Exemplar Problems** with Answers, Hints & Solutions* 7/104
Competition Focus [*NEET/JEE Special*] 7/114

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USEFUL INFORMATION

A. DERIVED SI UNITS

Physical quantity	Name	Symbol
frequency	hertz	Hz
energy	Joule	J
force	newton	N
power	watt	W
pressure	pascal	Pa
electric charge	coulomb	C
electric current	ampere	A
electric potential	volt	V
electric resistance	ohm	Ω
capacitance	farad	F
magnetic flux	weber	Wb
inductance	henry	H
magnetic induction	tesla	T

B. FUNDAMENTAL CONSTANTS

Constant	Symbol	Value [with estimated error]
Speed of light	c	$3 \times 10^{8} ms^{-1}$
Gravitational constant	G	$6{\cdot}67 \times 10^{-11} Nm^{2}kg^{-2}$
Acceleration of free fall	g	$9{\cdot}81 ms^{-2}$
Atmospheric pressure at sea level	atm	$1{\cdot}01 \times 10^{5} Nm^{-2}$

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Fundamental Constants (Contd.)

Constant	Symbol	Value [with estimated error]
Absolute zero	K	–273·15°C
Boltzmann's constant	k	$1{\cdot}38\times10^{-23}JK^{-1}$
Avogadro's number	N	$6{\cdot}023\times10^{23}mol^{-1}$
Universal gas constant	R	$8{\cdot}31\ J\ mol^{-1}k^{-1}$
Stefan's constant	σ	$5{\cdot}67\times10^{-8}Wm^{-2}K^{-4}$
Permittivity of free space	ϵ_0	$8{\cdot}85\times10^{-12}C^2N^{-1}m^{-2}$
Permeability of free space	μ_0	$4\pi\times10^{-7}TmA^{-1}(WbA^{-1}m^{-1})$
Electronic charge	e	$1{\cdot}60\times10^{-19}C$
Electron rest mass	m_e	$9{\cdot}1\times10^{-31}kg$
Specific charge	e/m	$1{\cdot}76\times10^{11}Ckg^{-1}$
Proton rest mass	m_p	$1{\cdot}6726\times10^{-27}kg$
Neutron rest mass	m_n	$1{\cdot}6749\times10^{-27}kg$
Atomic mass unit	u	$1{\cdot}67\times10^{-27}kg = 931MeV$
Planck's constant	h	$6{\cdot}63\times10^{-34}Js$
Electronic Radius	r_e	$2{\cdot}8\times10^{-15}m$
Rydberg constant	R_H	$1{\cdot}09678\times10^{7}m^{-1}$
Bohr Magneton	μ_B	$9{\cdot}27\times10^{-24}Am^2$

C. CONVERSION FACTORS

1 mile	= 1·61 km
1 radian	= 57·30° = 57° 18′
1 amu	= $1{\cdot}66\times10^{-27}$kg = 931MeV
1 electron volt [eV]	= $1{\cdot}6\times10^{-19}$J
$\log_e X$	= $2{\cdot}3026\log_{10}x$
$\pi = 3{\cdot}1416$, π^2	= 9·8696
1 emu of charge	= 3×10^{10}esu of charge
1 emu of current	= 3×10^{10}esu of current
1 emu of current	= 10 ampere
1 emu of potential	= 10^{-8}volt
1 esu of potential	= 300 volt

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UNIT-1

ELECTROSTATICS

CHAPTER 1.

ELECTROSTATIC CHARGES AND FIELDS

CHAPTER 2.

ELECTROSTATIC POTENTIAL AND CAPACITANCE

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1

ELECTROSTATIC CHARGES AND FIELDS

1.1. INTRODUCTION

Many of us have the experience of seeing a spark or hearing a crackle, when we take off our synthetic shirts or nylon sweaters, particularly in dry weather. Sometimes, we feel the sensation of an electric shock while opening the door of our car or by holding the iron bar of a bus, after sliding from our seat. The reason for these experiences is discharge of electric charges through our body, which were accumulated due to rubbing of insulating surfaces. Another common example of electric discharge is the lightning that we see in the sky during thunderstorms.

We know that when a glass rod is rubbed with a piece of silk, the rod acquires the property of attracting light objects like bits of paper, straw, pith balls, dry leaves or even dust particles towards it. The glass rod is said to be *electrified* or *charged.* Similarly, a plastic comb gets electrified on passing through dry hair.

The metallic bodies of cars and trucks also get charged because of friction between them and the air rushing past them. This charge being large can produce even a spark. Such a spark can be dangerous in case of a petrol tanker. That is why petrol tankers often have a metal chain dragging along the ground. The charge produced leaks to the ground through this chain. Now a days, the tyres of cars and trucks are made by adding some carbon compound to the rubber. This facilitates the charge built up on the body of the vehicle to leak to the ground.

We learn from above that **electric charges are produced due to friction between two insulating bodies, which are rubbed against each other.** The charges on insulating bodies cannot move on their own. That is why they are called ***static charges.***

> ***The branch of Physics, which deals with the study of charges at rest (i.e., static charges), the forces between the static charges, fields and potentials due to these charges is called Electrostatics or Static Electricity or Frictional Electricity.***

Historically, this phenomenon was discovered around 600 BC by a Greek philosopher *'Thales of Miletus'*. The name electricity was taken from Greek word 'Elecktron'.

Note that most of the early experiments on electrostatics work best on a dry day, because excessive moisture provides a pathway for charge to leak off a charged object.

1.2. ELECTRIC CHARGE

Electric charge is a characteristic that accompanies fundamental particles, wherever they exist.

According to William Gilbert, ***charge is something possessed by material objects that makes it possible for them to exert electrical force and to respond to electrical force.***

The three most common elementary particles are electrons, protons and neutrons having masses

$$m_e = 9{\cdot}10940 \times 10^{-31} \text{ kg} ; m_p = 1{\cdot}67262 \times 10^{-27} \text{ kg} \quad \text{and} \quad m_n = 1{\cdot}67493 \times 10^{-27} \text{ kg}$$

Because of their mass, these particles attract one another by gravitational forces. Thus, an electron attracts another electron at a distance of 1 cm, with a gravitational force

$$F = \frac{Gm_1m_2}{r^2} = \frac{6{\cdot}67 \times 10^{-11} \times (9{\cdot}1 \times 10^{-31})^2}{(10^{-2})^2} = 5{\cdot}5 \times 10^{-67} \text{ N}$$

However, an electron *repels* another electron at a distance of 1 cm with a force = $2{\cdot}3 \times 10^{-24}$ *N*. This force is called ***electric force.*** We observe that electric force is very large compared to the gravitational force. The electrons must have some additional property (other than their mass), which is responsible for the electric force between them. ***This additional property of electron, which gives rise to electric force between two electrons is called electric charge.*** **Just as masses are responsible for the gravitational force, charges are responsible for the electric force.**

Two protons placed at a distance of 1 cm also repel each other with the same force = $2{\cdot}3 \times 10^{-24}$ *N*. It shows that protons also have charge which in magnitude, must be equal to charge on electron. Two neutrons placed at a distance of 1 cm attract each other with a force = $1{\cdot}9 \times 10^{-60}$ *N*, which is the gravitational force due to their masses. However, electric force between two neutrons is zero. Therefore, neutrons have no charge, though they have mass.

1.3. TWO KINDS OF CHARGES

Let us look at some simple facts, which were established from years of efforts and careful experiments :

(*i*) A glass rod rubbed with a piece of silk brought close to a suspended glass rod rubbed with silk repels the latter as shown in Fig. 1.1 (*a*).

(*ii*) The two pieces of silk cloth with which the glass rods were rubbed also repel each other. However, each glass rod attracts the silk piece with which it was rubbed.

(*iii*) Two ebonite/amber rods rubbed with cat's fur repel eachother as shown in Fig. 1.1(*b*). However, each ebonite rod attracts the cat's fur with which it was rubbed.

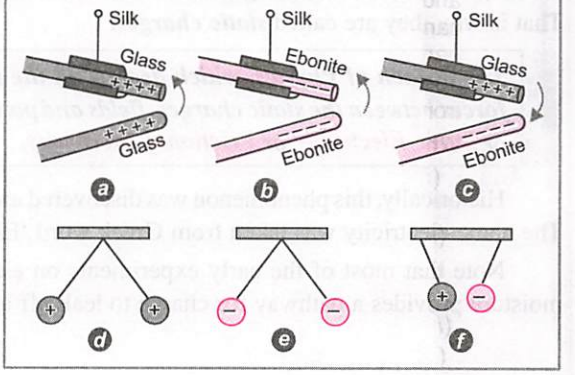
(*iv*) An ebonite rod rubbed with cat's fur attracts a glass rod rubbed with a piece of silk as shown in Fig. 1.1(*c*). However, the charged glass rod repels the cat's fur.

(*v*) When a glass rod rubbed with a piece of silk is made to touch two small pith balls (or polystyrene balls) suspended by silk/nylon threads, the balls repel eachother, as shown in Fig. 1.1(*d*).

(*vi*) Similarly, when an ebonite/amber rod rubbed with cat's fur is made to touch two small pith balls suspended by silk/nylon threads, the balls repel eachother, as shown in Fig. 1.1(*e*).

(*vii*) However, a pith ball (or polystyrene ball) touched with charged glass rod attracts another pith ball touched with charged ebonite rod, as shown in Fig. 1.1(*f*).

FIGURE 1.1



A careful analysis of these facts observed leads us to the following conclusions :

(*i*) The bodies like glass rods, ebonite/amber rods, silk, fur etc. acquire electric charge on rubbing.

(*ii*) The pith balls or polystyrene balls acquire electric charge by *actual contact* with a charged body.

(*iii*) The charge developed on glass rod rubbed with silk is different from the charge developed on ebonite rod rubbed with fur. This is because a charged glass rod repels another charged glass rod, but attracts a charged ebonite rod. Hence, there are two kinds of charges. When a glass rod is rubbed with silk, the rod acquires one kind of charge and the silk acquires second kind of charge. This is true for any pair of objects that are rubbed to be electrified.

Further, like charges repel each other and unlike charges attract each other.

(*iv*) When an electrified glass rod is brought in contact with the silk piece, with which it was rubbed, they no longer attract eachother. They also do not attract other light objects as they did on being electrified. It means the charges acquired on rubbing are lost when the two oppositely charged bodies are brought in contact with eachother. It leads us to the conclusion that ***charges acquired by the objects on rubbing against eachother must be equal and opposite.*** That is why they neutralise or nullify eachother's effect. *Du Fay was the first to show two kinds of charges.* An *American scientist Benjamin Franklin* named the two kinds of charges as positive and negative. By convention, ***charge acquired by glass rod or cat's fur is called positive. And the charge acquired by ebonite/amber rod or silk cloth is called negative.***

Hence, there are two kinds of charges. *The property which differentiates the two kinds of charges is called the polarity of charge.*

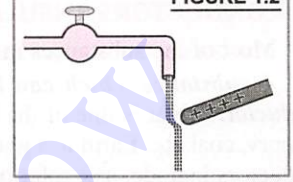
If a body possesses an electric charge, it is said to be charged or electrified. *When it has no charge, it is said to be neutral.*

Further, ***note that only rubbed area of non conducting body gets charged, and this charge does not move to other parts of the body. The charge is static on rubbed portion only.***

DO YOU KNOW ?

- If an electrically charged rod is brought near normal flow of water from a tap, the flow gets slightly diverted towards the rod as shown in Fig. 1.2. This is due to force of attraction exerted by the charged rod.

FIGURE 1.2



Curiosity Questions

Q. 1. What role does electrostatics play in a xerox copying machine ?

Ans. A xerox copying machine is one of the many industrial applications of the forces of attraction and repulsion between charged bodies. Particles of black powder, called **toner**, stick to a tiny carrier bead of the machine on account of electrostatic forces. The negatively charged toner particles are attracted from carrier bead to a rotating drum, where a positively charged image of document being copied has formed. A charged sheet of paper then attracts the toner particles from the drum to itself. They are then heat fused in place to produce the photo copy.

Q. 2. What are some other applications of electrostatics ?

Ans. (*i*) Electric capacitors or condensers, which have variety of applications are based on the principle of electrostatics.

(*ii*) High electric potential sources like van-de-graaf generator are based on principle of electrostatics.

(*iii*) Pollution is controlled by electrostatic precipitators of fly ash.

(*iv*) Spray paintings and powdered coatings are based on the principle of electrostatics.

(*v*) Power factor correctors, which reduce energy losses are based on the principle of electrostatics.

(*vi*) Electrostatic shielding protects sensitive instruments from external electric fields.
(*vii*) Natural phenomena like lightning and thunder storms are explained on the basis of electrostatics.
(*viii*) Electrostatics plays a great role in the reproduction of flowers.

Q. 3. When your friend chews a winter green life saver in a dark room, you see a faint flash of blue light from his mouth. How ?

Ans. This display of light is often called sparking. It occurs due to electric discharge of the electrostatic charges produced in chewing the winter green life saver.

1.4. CONDUCTORS, INSULATORS AND DIELECTRICS

Most of the substances in nature are divided into two categories, namely, *conductors* and *insulators*.

A substance which can be used to conduct electric charge from one end to the other is called a conductor. Silver is one of the best conductors. Other examples of conductors are copper, iron, aluminium, mercury, coal etc. Earth is a good conductor. Human body is also a good conductor of electricity. The liquid conductors include, salt solutions, acids, alkalis etc.

In metallic conductors, there are very large number of free electrons which act as carriers of charge. Infact, in a metal, the outer (valence) electrons part away from their atoms and are free to roam about in the body of the metal, but they cannot leave the metal under normal circumstances. The free electrons form a kind of *electron gas,* they collide with one another ; and also with the ions ; and move randomly in different directions. In an ex- ternal electric field, the free electrons drift against the direction of the field. The residual atoms made up of nuclei and the bound electrons remain held in their fixed positions. They constitute the *bound charges* in the conductor as they cannot move. In electrolytic conductors, the charge carriers are both, the positive and negative ions.

The* insulators *are the materials which cannot conduct electricity, i.e., they are poor conductors of electricity. Common examples of insulators are glass, rubber, plastic, ebonite, mica, wax, paper, wood etc. They are called insulators, because they prevent charge from going to places where it is not desired. Such substances possess *almost no free electrons.*

Infact, in an insulator, each electron is *attached or bound* to a particular atom and is *not free* to move in the body of the insulator. As each electron stays near its '*parent*' nucleus or within its atom or molecule, and cannot move far away from it, therefore, *an insulator does not possess freely movable charges.* Hence it fails to conduct electricity.

Insulators are also called Dielectrics. Obviously, dielectrics cannot conduct electricity. However, when an external electric field is applied on them, induced charges appear on the surface of the dielectric. *Hence we may define* ***dielectrics as the insulating materials which transmit electric effects without conducting.***

When some charge is transferred to a conductor, it gets distributed readily over the entire surface of the conductor. In contrast, if some charge is put on an insulator, it stays at the same place.

A nylon or plastic comb gets electrified on combing dry hair or on rubbing. But a metallic rod does not get electrified on rubbing. This is because charges on metal leak through our body to the ground as both are good conductors of electricity.

Retain in Memory

When a charged body is brought in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through our body. This process of sharing charges with the earth is called **grounding** or **earthing.** Earthing near the mains supply of a building is done by burying the neutral (earth) metal wire deep into earth and there, it is connected to a thick metal plate. The electric wiring in our houses has three wires ; live, neutral and earth. The first two carry electric current from the power station and the third (earth) is connected to the buried metal plate. Metallic bodies of appliances like T.V., refrigerator, electric iron etc. are connected to earth wire. If a live wire were to touch the metallic body, charge flows to earth without damaging the appliance and without causing injury to us.

1.5. GOLD LEAF ELECTROSCOPE (GLE)

A gold leaf electroscope (GLE) is an instrument which is used for detecting the presence of electric charge and its polarity (*i.e.*, ± sign of charge). The instrument can also be used for measuring potential difference.

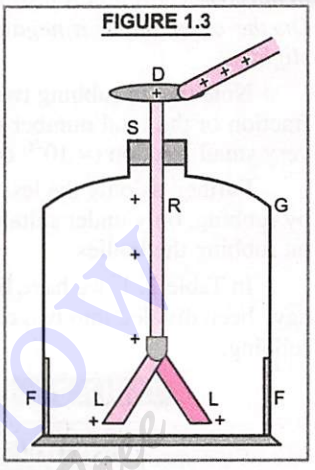


FIGURE 1.3

The essential parts of a gold leaf electroscope are shown in Fig. 1.3. LL are two extremely thin gold foils attached to lower end of a metal rod R fitted in a glass jar G through an insulating stopper S of cork or rubber etc. D is a metal disc at the free end of the metal rod. The sensitivity of the instrument is increased by pasting two tin foils F, F on the inner side of glass jar opposite to the gold leaves.

As the gold leaves are extremely thin conducting foils which have low mass per unit area and are flexible, therefore, they respond very quickly to small electrostatic forces. Thin aluminium foils can also serve the same purpose. When a charged rod is touched with the metal disc D, the same charge is transferred to the gold leaves through the metal rod. The leaves repel eachother and diverge as shown in Fig. 1.3. By measuring the divergence of leaves, the amount of charge on the body can be estimated.

Nature of electric charge on the body can be determined using a gold leaf electroscope, which is already charged.

When charge on the charged body is of opposite sign to that of the charge on GLE, then on touching the body with the disc of GLE, divergence of leaves would decrease.

When charge on the charged body is of same sign as that of charge on GLE, then on touching the charged body with the disc of GLE, the divergence of leaves would increase.

1.6. ORIGIN OF ELECTRIC CHARGE IN ELECTROSTATICS

It is known that all matter is made up of atoms and/or molecules, the basic unit being an atom. We also know that every atom consists of a central core called the *atomic nucleus,* around which negatively charged electrons revolve in circular orbits. Every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus. *Thus, even though normally, the materials are electrically neutral, they do contain charges, but their charges are exactly balanced.*

The vast amount of charge in an object is usually hidden as the object contains equal amounts of positive charge and negative charge. With such an equality or balance of charge, the object is said to be electrically neutral, i.e., it contains no net charge.

If the positive and negative charges are not in balance, then there is a net charge. ***Thus, an object is charged if it has a charge imbalance or some net charge.*** Hence, to electrify or charge a neutral body, we need to add or remove one kind of charge. When we say that a body is charged, we always refer to ***excess charge or deficit charge.***

In solids, some of the electrons are less tightly bound in the atom. These are the charges, which are transferred from one body to the other.

When we rub two insulating substances against each other, we provide energy to overcome friction between them. This energy is used in removing electrons from one substance and transferring them to the other. The transfer takes place from the material in which electrons are held less tightly (with lower work function) to the material in which electrons are held more tightly (with higher work function), *i.e.*, ***electrons are transferred from the material whose work function is lower to the material whose work function is higher.*** Consequently, the material which loses electrons acquires a positive charge and material which gains electrons acquires an equal negative charge. For example, when we rub a glass rod with silk, electrons are transferred from glass rod to silk. The glass rod becomes positively charged and silk acquires an equal negative charge. Thus, *charging by rubbing is due to actual transfer of electrons.*

The cause of charging is actual transfer of electrons from one material to the other. The insulating material with lower work function loses electrons and becomes positively charged and vice-versa.

Further, as an electron has a mass, howsoever small it may be, therefore *there does occur some change in mass on charging. A positively charged body has lost some electrons and hence its mass reduces slightly. On the other hand, a negatively charged body has gained some electrons and hence its mass increases slightly.*

Note that in rubbing two insulating bodies, the number of electrons that are transferred, is a very small fraction of the total number of electrons in the material bodies. Hence, the charge acquired by friction is a very small fraction ($\approx 10^{-6}$ coulomb) of the total positive and negative charge content of the bodies.

Further, as only the less tightly bound electrons in a material body can be transferred from it to another by rubbing, only under suitable conditions, we have to stick to certain pairs of materials to observe charging on rubbing the bodies.

In Table 1. 1, we have listed the pairs of objects which get charged on rubbing against each other. They have been divided into two classes, one acquiring positive charge and the other acquiring negative charge on rubbing.

TABLE 1.1. Objects acquiring two kinds of charges on rubbing

POSITIVE CHARGE	NEGATIVE CHARGE
1. Glass rod	**1.** Silk cloth
2. Fur or woolen cloth	**2.** Ebonite, Amber, Rubber rod
3. Woolen coat	**3.** Plastic seat
4. Woolen carpet	**4.** Rubber shoes
5. Nylon or Acetate	**5.** Cloth
6. Dry hair	**6.** Comb

Obviously, any two charged objects, in the same column repel each other and any two charged objects from different columns attract each other.

Further, different substances have been arranged in a series, called **Triboelectric Series.** When any two of the substances in series are rubbed together, the one occurring earlier in the series acquires positive charge and the other occurring later in series acquires negative charge. Some of the substances in triboelectric series are : 1. Fur 2. Flannel 3. Wool 4. Glass 5. Paper 6. Cotton 7. Silk 8. Wood 9. Metals 10. Rubber 11. Orlon 12. Polyethylene 13. Teflon 14. Ebonite, etc.

Thus glass acquires a positive charge when rubbed with silk and it acquires a negative charge when rubbed with fur.

1.7. CHARGING BY INDUCTION

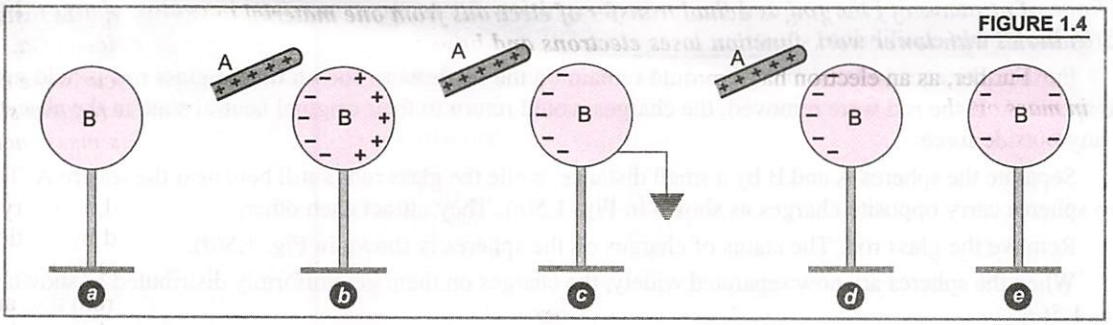
We know that a body can be charged by putting it in contact with another charged body either directly or by means of a conductor. For example, when a charged ebonite rod is in contact with a pith ball or connected to it by a copper wire, it transfers some of its negative charge to the pith ball. ***This is charging by conduction, i.e., charging by actual contact.***

When a charged body *A* is brought near an uncharged metallic body *B*, then some charge of opposite sign appears on the near face of body *B* and an equal charge of same sign appears on the rear face of body *B*. The total charge on body *B* is zero. In this process, body *A* does not lose any charge as it is not in direct contact with body *B*. This phenomenon of **charging body *B* without actual contact is called charging by induction.**

Thus, phenomenon of charging an uncharged conducting body, by bringing a charged body near it, without making a direct contact between the two bodies is called charging by induction.

The steps involved in charging a metallic sphere by induction are shown in Fig. 1.4.

FIGURE 1.4



(*a*) To begin with, a metallic sphere *B* on an insulating stand is uncharged, Fig. 1.4(*a*).

(*b*) When a positively charged glass rod *A* is brought near the uncharged metallic sphere, free electrons of the sphere are attracted and start piling up at the near end. This end therefore, becomes negatively charged and the farther end of the sphere becomes positively charged to the same extent, due to deficit of electrons as the sphere on the whole remains neutral, Fig. 1.4(*b*). The redistribution of charge is almost instantaneous and stops as soon as net force on free electrons in the metallic sphere becomes zero.

(*c*) When the sphere is grounded, *i.e.*, it is connected to earth by a conducting wire, electrons flow from the ground to the sphere and neutralise the positive charge on the farther end of the sphere. The negative charge at the near end of the sphere remains bound there due to attractive force of glass rod, Fig. 1.4(*c*).

(*d*) When the sphere is disconnected from the ground, the negative charge continues to be held on the near end, Fig. 1.4(*d*).

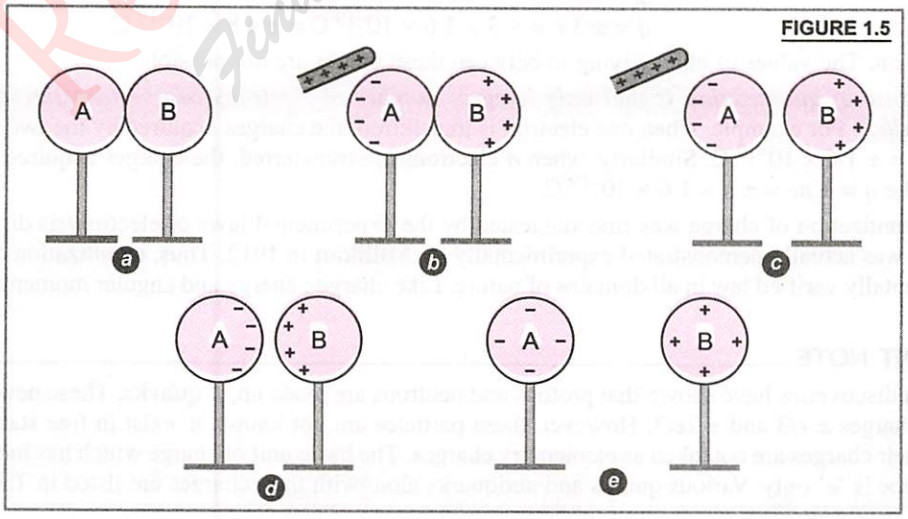
(*e*) When the glass rod is removed, the negative charge spreads uniformly over the sphere *B*, Fig. 1.4(*e*).

Similar steps are involved when a negatively charged rod is used for charging the sphere positively by induction.

Let us now understand **how we charge two spheres by induction.** Fig. 1.5(*a*) shows two metal spheres A and B supported on insulating stands, held in contact with eachother.

Let a positively charged glass rod be brought near the sphere A. Free electrons in both the spheres are attracted towards the rod. Therefore, left surface of left sphere A has an excess of negative charge and right surface of right sphere B has an excess of positive charge. *Note that all the electrons in the spheres have not accumulated on the left surface of sphere A.* This is because as negative charge starts building up at the left face of A, further electrons are repelled by these. *An equilibrium is reached almost instantly under the action*

FIGURE 1.5



of force of attraction of the rod and the force of repulsion due to the accumulated negative charges. Fig. 1.5(*b*) shows this equilibrium situation.

Further, the accumulated charges would remain on the surfaces as shown till the glass rod is held near the sphere A. If the rod were removed, the charges would return to their original neutral state in the absence of any outside force.

Separate the spheres A and B by a small distance, while the glass rod is still held near the sphere A. The two spheres carry opposite charges as shown in Fig. 1.5(*c*). They attract each other.

Remove the glass rod. The status of charges on the spheres is shown in Fig. 1.5(*d*).

When the spheres are now separated widely, the charges on them get uniformly distributed as shown in Fig. 1.5(*e*).

This is how two metal spheres get oppositely charged by induction. ***Note that in this process of electric induction, the positively charged glass rod does not lose any charge. This is contrary to charging by conduction, i.e., charging by actual contact where the charged glass rod loses some charge.***

1.8. QUANTIZATION OF ELECTRIC CHARGE

The quantization of electric charge is the property by virtue of which, the charge on a body is integral multiple of a basic unit of charge of an electron/proton, represented by e.

Thus, charge q of a body is always given by $q = ne$ ...(1)

where n is any integer, positive or negative. The basic unit of charge is the charge that an electron or proton carries. By convention, charge on an electron is taken to be negative. Therefore, charge on an electron is written as $(-e)$ and charge on a proton is $(+e)$.

The value of the basic unit of charge or elementary charge is $e = 1{\cdot}6 \times 10^{-19}$ coulomb ...(2)

It is one of the important constants of nature.

If a body contains n_1 electrons and n_2 protons, the total amount of charge on the body is

$$q = n_2 (e) + n_1 (-e) = (n_2 - n_1)\, e$$

As n_1, n_2 are integers, their difference must also be an integer. Thus, the charge on anybody is always an integral multiple of e, and can be increased or decreased also in steps of e.

Thus, any charged body or charged particle can possess charge equal to $\pm 1\,e, \pm 2\,e, \pm 3\,e$ and so on, *i.e.*, the possible values of charge are

$$q = \pm 1\, e = \pm 1 \times 1{\cdot}6 \times 10^{-19}\ \text{C} = \pm 1{\cdot}6 \times 10^{-19}\ \text{C}$$
$$q = \pm 2\, e = \pm 2 \times 1{\cdot}6 \times 10^{-19}\ \text{C} = \pm 3{\cdot}2 \times 10^{-19}\ \text{C}$$
$$q = \pm 3\, e = \pm 3 \times 1{\cdot}6 \times 10^{-19}\ \text{C} = \pm 4{\cdot}8 \times 10^{-19}\ \text{C}$$

and so on. The values of charge lying in between these values are not possible.

The cause of quantization is that only integral number of electrons can be transferred from one body to another. For example, when one electron is transferred, the charges acquired by the two bodies will be $q = \pm 1e = \pm 1{\cdot}6 \times 10^{-19}$ C. Similarly, when n electrons are transferred, the charges acquired by the two bodies will be $q = \pm ne = \pm n \times 1{\cdot}6 \times 10^{-19}$ C.

The quantization of charge was first suggested by the experimental laws of electrolysis discovered by **Faraday.** It was actually demonstrated experimentally by **Millikan** in 1912. Thus, quantization of charge is an experimentally verified law in all domains of nature. Like charge ; energy and angular momentum are also quantized.

IMPORTANT NOTE

Recent discoveries have shown that protons and neutrons are made up of **quarks.** These new particles have charges $\pm e/3$ and $\pm 2e/3$. However, these particles are not known to exist in free state. That is why their charges are not taken as elementary charges. The basic unit of charge which has independent existence is 'e' only. Various quarks and antiquarks alongwith their charges are listed in Table 1.2.

TABLE 1.2. Various quarks and antiquarks alongwith their charges

S. No.	QUARK	ANTI-QUARK
1.	up (u) = + 2e/3	anti-up ($\bar{u}$) = −2 e/3
2.	down (d) = − e/3	anti-down ($\bar{d}$) = e/3
3.	charm (c) = $+\frac{2e}{3}$	anti charm ($\bar{c}$) = $-\frac{2e}{3}$
4.	strange (s) = $-\frac{e}{3}$	anti strange ($\bar{s}$) = $\frac{e}{3}$
5.	top (t) = $+\frac{2e}{3}$	anti-top ($\bar{t}$) = $-\frac{2e}{3}$
6.	bottom (b) = $-\frac{e}{3}$	anti-bottom ($\bar{b}$) = $\frac{e}{3}$

Retain in Memory

Quantization of charge is meaningful only at the microscopic level, where the charges involved are of the order of a few tens or hundreds of e, *i.e.*, they can be counted. Such charges appear in discrete lumps and quantization of charge cannot be ignored.

However, at the *macroscopic level*, we deal with charges of a few microcoulomb. A charge of magnitude 1 μC contains electrons whose number $n = \frac{q}{e} = \frac{1\times10^{-6}\,\text{C}}{1{\cdot}6\times10^{-19}\,\text{C}} \approx 10^{13}$, which is very large. At this scale, the fact that charge can increase or decrease only in units of e is not visible. The grainy nature of charge is lost and it appears to be continuous.

Sample Problem 1 **Is a charge of $5{\cdot}8 \times 10^{-18}$ C possible ?**

Sol. From $q = n\,e$, $n = \frac{q}{e} = \frac{5{\cdot}8\times10^{-18}}{1{\cdot}6\times10^{-19}} = \mathbf{36{\cdot}25}$

As n is not an integer, this value of charge is **not possible.**

Sample Problem 2 **What is the charge on a body from which one million electrons are removed ?**

Sol. Here, $n = 10^6$, $e = 1{\cdot}6 \times 10^{-19}$ C

As electrons are removed, charge acquired is positive

$$q = ne = 10^6 \times 1{\cdot}6 \times 10^{-19}\,\text{C} = \mathbf{1{\cdot}6 \times 10^{-13}\,C}$$

1.9. ADDITIVITY OF CHARGE

***Additivity of charge* is a property by virtue of which total charge of a system is obtained simply by adding algebraically all the charges present anywhere on the system.**

It means charges are scalars like the mass of a body and are added by simple laws of Mathematics. If a system contains n charges $q_1, q_2, q_3, ..., q_n$, then the total charge of the system is

$$q = q_1 + q_2 + q_3 + ... + q_n.$$

Charge has magnitude only, but no direction, similar to the mass. However, mass of a body is always positive, but charge can be either positive or negative. Therefore, ***proper signs have to be used while adding the charges in a system.*** For example, if a system contains charge $+q$, $-2q$, $+3q$ and $+5q$, then the total charge of the system is $= +q - 2q + 3q + 5q = +7q$

1.10. CONSERVATION OF CHARGE

Conservation of charge ***is the property by virtue of which total charge of an isolated system* always remains constant or conserved.***

Within an isolated system consisting of many charged bodies, charges may get redistributed due to interactions among the bodies, but the total charge of the system shall remain the same.

For example, when we rub two insulating bodies, what one body gains in charge, the other body loses the same amount of charge. Thus, ***it is not possible to create or destroy net charge carried by any isolated system.*** However, charge carrying particles may be created or destroyed in a process. For example, a neutron turns into a proton and an electron. The proton and electron thus created have equal and opposite charges. The total charge is zero before and after the creation. Thus, ***charges can be created or destroyed in equal and unlike pairs only.***

Following examples illustrate the property of conservation of charge.

(*i*) In the phenomenon of **pair production,** a γ ray photon materialises into an electron and a positron having total charge $-e + e = 0$, which is the initial charge on a photon.

$$\gamma = e^- + e^- \qquad \text{(pair production)}$$

(*ii*) In **annihilation of matter,** an electron and a positron annihilate eachother to produce two γ-ray photons with zero charge. Charge is thus conserved.

$$e^- + e^+ = \gamma + \gamma \qquad \text{(annihilation)}$$

(*iii*) In all **radioactive decays,** charge number is always conserved.

For example, in radioactive decay of U-238, the nucleus is transformed into Th-234 with the emission of an alpha particle, which is a helium nucleus, *i.e.*,

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} \qquad \text{(radioactive decay)}$$

(*iv*) In all **nuclear transformations,** charge number is always conserved. For example :

$$_{7}N^{14} + {}_{2}He^{4} \longrightarrow {}_{8}O^{17} + {}_{1}H^{1}$$

IMPORTANT NOTE

Note that in applying the conservation of charge principle, we must add the charges algebraically, with regard to their signs.

1.11. COMPARISON OF CHARGE AND MASS

We are familiar with role of mass in gravitation, and we have just studied some features of electric charge. We can compare the two as shown in Table 1.3.

TABLE 1.3. Comparison of charge and mass

CHARGE	MASS
1. Electric charge on a body may be positive, negative or zero.	1. Mass of a body is a positive quantity.
2. Charge carried by a body does not depend upon velocity of the body.	2. Mass of a body increases with its velocity as $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$, where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is rest mass of the body.
3. Charge is quantized.	3. The quantization of mass is yet to be established.

*An isolated system is that which is free from external influence *i.e.*, no charged matter is allowed to enter or leave the volume of space occupied by the isolated system.

CHARGE	MASS
4. Electric charge is always conserved.	4. Mass is not conserved as it can be changed into energy and vice-versa.
5. Force between charges can be attractive or repulsive, according as charges are unlike or like charges.	5. The gravitational force between two masses is always attractive.
6. The force between two charges follows inverse square law.	6. The force between two masses also follows inverse square law.
7. Charge cannot exist without mass.	7. Mass can exist without charge.
8. Unit of charge is a derived unit (1 C = 1 As).	8. Unit of mass is a fundamental unit.
9. An accelerated charge emits radiation.	9. Accelerated mass emits no radiation.

1.12. COULOMB'S LAW

Coulomb performed several experiments to measure the extent of force between two point charged bodies.*

According to **Coulomb's law,**

The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

The force acts always along the line joining the two charges.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r.

According to Coulomb's law, $F \propto \frac{|q_1||q_2|}{r^2}$ or $F = \frac{k|q_1||q_2|}{r^2}$ ...(1)

where k is **electrostatic force constant.**

The value of electrostatic force constant k depends on the nature of medium separating the charges, and on the system of units.

When the charges are situated in free space (air/vacuum), then in cgs system, $k = 1$.

In SI, $k = 9 \times 10^9 \text{ N } m^2 \text{ C}^{-2}$

We write, $k = \frac{1}{4\pi\epsilon_0}$ ...(2)

where ϵ_0 is called ***absolute electrical permittivity of the free space.***

From (1), the magnitude of force is $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$ ...(3)

Units, Dimensions and Value of ϵ_0

From (3), $\epsilon_0 = \frac{1}{4\pi F}\frac{q_1 q_2}{r^2}$

As SI unit of charge is coulomb (C), therefore, Units of $\epsilon_0 = \frac{1}{\text{N}}\frac{\text{C.C}}{\text{m}^2} = \text{C}^2\text{N}^{-1}\text{m}^{-2}$

Dimensions of $\epsilon_0 = \frac{(\text{AT})(\text{AT})}{(\text{MLT}^{-2})(\text{L}^2)} = [\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$

*When linear sizes of charged bodies are much smaller than the distance separating them, the size may be ignored. The charged bodies are then treated as point charged bodies.

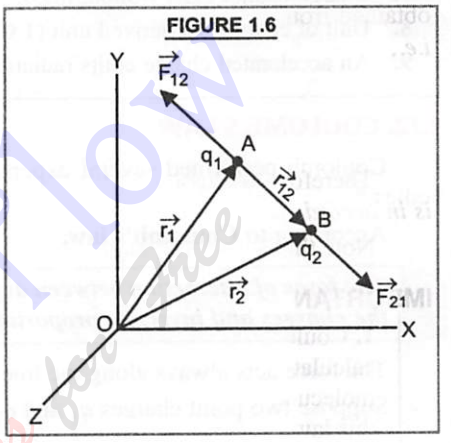
From (2), $k = \dfrac{1}{4\pi\epsilon_0}$ or $\epsilon_0 = \dfrac{1}{4\pi k}$ ...(4)

$\therefore$ $\epsilon_0 = \dfrac{1}{4\times 3{\cdot}14\times 9\times 10^9}$ $\epsilon_0 = 8{\cdot}85 \times 10^{-12}\ C^2N^{-1}\ m^{-2}$ ...(5)

1.13. COULOMB'S LAW IN VECTOR FORM

As stated already, according to Coulomb's law, the force of interaction F between any two point charges q_1 and q_2 is directly proportional to the product of the charges, and inversely proportional to the square of the distance (r) between them. *i.e.*, $F \propto \dfrac{|q_1||q_2|}{r^2}$

$$F = \frac{k|q_1||q_2|}{r^2}$$



where k is electrostatic force constant.

As force is a vector, it is better to write Coulomb's law in the vector notation. In Fig. 1.6, let

$\vec{r}_1 = \overrightarrow{OA}$ = position vector of charge q_1

$\vec{r}_2 = \overrightarrow{OB}$ = position vector of charge q_2

$\therefore$ The vector leading from q_1 to q_2 is $\overrightarrow{AB} = \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ ...(6)

In the same way, vector leading from q_2 to q_1 is $\overrightarrow{BA} = \vec{r}_{21} = \vec{r}_1 - \vec{r}_2$ ...(7)

The magnitude of $\vec{r}_{12}$ is r_{12} and magnitude of $\vec{r}_{21}$ is r_{21}.

As the direction of a vector is specified by a unit vector along the vector, we define

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}} \text{ and } \hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}}$$

If $\vec{F}_{12}$ = force on q_1 due to q_2, and $\vec{F}_{21}$ = force on q_2 due to q_1, then as is clear from Fig. 1.6,

Coulomb's force law between two point charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$ in vacuum is expressed as

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{AB^2}, \text{ along AB}$$

or $$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\times\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^3}\times\vec{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$ ...(8)

It should be clearly understood, that eqn. (8) is valid for any sign of q_1 and q_2, whether positive or negative.

If q_1, q_2 are of same sign (either both positive or both negative) ; $q_1 q_2 > 0$; $\vec{F}_{21}$ is along $\vec{r}_{12}$, which denotes repulsion for like charges ; Fig. 1.7(a).

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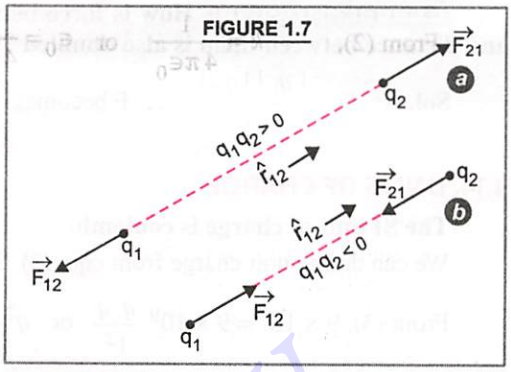
If q_1 and q_2 are of opposite sign (*i.e.*, one is positive and other is negative), $q_1\, q_2 < 0$; $\vec{F}_{21}$ is along $-\hat{r}_{12}$, which denotes attraction between unlike charges, Fig. 1.7(*b*).

Thus, eqn. (8) takes care of both the cases of like and unlike charges correctly.

The force $\vec{F}_{12}$, on charge q_1 due to charge q_2 is obtained from eqn. (8), by simply interchanging 1 and 2, *i.e.*,

FIGURE 1.7



$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\times\hat{r}_{21} = -\vec{F}_{21} \qquad \text{...(9)}$$

Therefore, force on q_1 due to q_2 is equal and opposite to the force on q_2 due to q_1. Thus, *Coulomb's law is in accordance with Newton's third law of motion.*

Note that eqns. (8) and (9) give us the forces between two charges q_1 and q_2 in vacuum only.

IMPORTANT NOTES

1. Coulomb's law is universal. The force which binds the electrons to the nucleus to form an atom is calculated from this law. The same law applies to van der waal forces which unite the atoms to form molecules. The three states of matter viz. solids, liquids and gases are also explained on the basis of this law.

2. Coulomb's law has been verified over distances ranging from nuclear dimensions ($\simeq 10^{-15}$ m) to macroscopic distances ($\simeq 10^{18}$ m). Further, the law is applicable only to point charges. This is the limitation of Coulomb's Law.

3. Coulomb's law of electrostatic force between two charges corresponds to Newton's law of gravitational force between two masses, *i.e.*, $F = \frac{G m_1 m_2}{r^2}$, where G is universal gravitational constant $= 6{\cdot}67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$. This value is much smaller compared to the value of electrostatic force constant, $k = 9 \times 10^9\ \text{Nm}^2/\text{C}^2$. That is why electrostatic forces are far more stronger than the gravitational forces. This is evident from the fact that a charged glass rod attracts a piece of paper against the gravitational pull of earth on the paper.

Further, whereas electrostatic force may be attractive or repulsive depending on the sign of charges, gravitational force is always attractive.

4. Characteristics of Electrostatic force or Coulomb's force

(*i*) *The Coulomb force is a* **central force** as it acts along a line joining the two point charges.

(*ii*) *The Coulomb force is a* **conservative force** as the work done in taking a charge from one point to another in the electric field of another charge is independent of the path followed.

(*iii*) Electrostatic force between two charges is **spherically symmetric.**

(*iv*) Electrostatic force between two charges **obeys inverse square law.**

(*v*) Electrostatic force between two charges is attractive for unlike charges and it is repulsive for like charges.

(*vi*) Electrostatic force between two charges is **not affected by the presence or absence of any other charge.**

(*vii*) Coulomb's law governing electrostatic force between two charges corresponds to Newton's Law of gravitation.

(*viii*) Coulomb's law is in accordance with Newton's third law of motion *i.e.,* **forces of action and reaction are equal and opposite.**

Sample Problem **How is force between two charges affected when each charge is doubled and distance between them is also doubled ?**

Sol. As $F \propto \frac{|q_1||q_2|}{r^2}$ $\therefore$ F becomes $\frac{(2)\ (2)}{(2)^2}$ time = 1 time, *i.e.*, **force remains the same.**

1.14. UNITS OF CHARGE

The SI unit of charge is coulomb.

We can define unit charge from eqn. (3). Suppose $q_1 = q_2 = q$; $r = 1$ m and $F = 9 \times 10^9$ N

From (3), $9 \times 10^9 = 9 \times 10^9 \frac{q\ q}{1^2}$ or $q^2 = 1$ or $q = \pm 1$ (coulomb), Hence,

Unit charge in SI (i.e. one coulomb) is that much charge which when placed in vacuum at a distance of one metre from an equal and similar charge would repel it with a force of 9×10^9 newton.

The cgs unit of charge is **1 electrostatic unit (e.s.u.) of charge** or **stat coulomb.**

It is also called one **franklin** (Fr), in honour of an American scientist Franklin for his contributions to the study of electrostatics.

As charge on an electron is $4{\cdot}8 \times 10^{-10}$ stat coulomb, therefore,

$1{\cdot}6 \times 10^{-19}$ coulomb $= 4{\cdot}8 \times 10^{-10}$ stat coulomb or 1 coulomb $= \frac{4{\cdot}8 \times 10^{-10}}{1{\cdot}6 \times 10^{-19}}$ stat coulomb

$$\boxed{1 \text{ coulomb} = 3 \times 10^9 \text{ stat coulomb}} \quad \text{...(10)}$$

Yet another unit of charge is **electromagnetic unit (e.m.u.) of charge,** where

$$\boxed{1 \text{ e.m.u. of charge} = 3 \times 10^{10} \text{ e.s.u. of charge (stat coulomb)} = 10 \text{ coulomb.}}$$

1.15. DIELECTRIC CONSTANT OR RELATIVE ELECTRICAL PERMITTIVITY

When the charges are situated in a medium other than free space (vacuum or air), the force between them is given by

$$F_m = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2} \quad \text{...(11)}$$

where ϵ is called ***absolute electrical permittivity of the intervening medium.***

The force between the same two charges held the same distance apart in vacuum is

$$F_0 = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \quad \text{...(12)}$$

Dividing (12) by (11), we get

$$\boxed{\frac{F_0}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } K} \quad \text{...(13)}$$

where ϵ_r is called **relative electrical permittivity** of the medium. It is also called **dielectric constant** of the medium and is denoted by K.

From (13), we may define

Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the absolute electrical permittivity of free space. Also,

Dielectric constant of a medium may be defined as the ratio of force of interaction between two point charges separated by a certain distance in air/vacuum to the force of attraction/ repulsion between the same two point charges, held the same distance apart in the medium.

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The value of K depends only on the nature of medium.

For example, for vacuum, K = 1·00000 ; for air, K = 1·006 ; for hydrogen, K= 1·00026 ; for glass, K = 3 to 4 ; for mica, K = 3 to 6 ; for water, K = 81 and so on.

From (13), $\epsilon = \epsilon_0 K$

Using it in (11), $$F_m = \frac{1}{4\pi\epsilon_0 K}\frac{q_1 q_2}{r^2} = \frac{F_0}{K}$$

Thus, force between two given charges held a given distance apart in water (K = 81) is only 1/81 of the force between them in air/vacuum.

Sample Problem **Two equal like charges in air repel eachother with a force F. By what percentage should each charge be reduced so that the force between them in a medium of dielectric constant 2 reduces by 28% ?**

Sol. Let $q_1 = q_2 = q$; and r be the distance between them.

$$\therefore \quad F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \qquad ...(i)$$

Let the charge be reduced to $q_1' = q_2' = q'$

$$\therefore \quad F' = \frac{K q' q'}{4\pi\epsilon_0 r^2} = \frac{2q'^2}{4\pi\epsilon_0 r^2}$$

Now, $$F' = (100 - 28)\ \% = \frac{72}{100}$$

$$\therefore \quad \frac{F'}{F} = \frac{2q'^2}{q^2} = \frac{72}{100}$$

$$\left(\frac{q'}{q}\right)^2 = \frac{36}{100}\ ;\ \frac{q'}{q} = \frac{6}{10}$$

Percentage reduction in charge

$$\frac{(q - q')}{q} \times 100 = \frac{(q - 6q/10)}{q} \times 100 = \mathbf{40\%}$$

1.16. FORCES BETWEEN MULTIPLE CHARGES : PRINCIPLE OF SUPERPOSITION

The mutual electric force between two charges is given by Coulomb's law. However, when we have to calculate the force on a charge due to several stationary charges, we use the principle of superposition in addition to Coulomb's law.

According to **superposition principle,** ***total force on any charge due to a number of other charges at rest is the vector sum of all the forces on that charge due to other charges, taken one at a time. The forces due to individual charges are unaffected by the presence or absence of other charges.***

Suppose point charges q_1, q_2, q_3 .... q_n are situated at points with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\vec{r}_n$ respectively w.r.t. the origin O of the rectangular co-ordinate system XYZ, Fig. 1.8.

In general, total force $\vec{F}_0$ on a test charge q_0 at position $\vec{r}_0$ due to all the n discrete charges can be written as

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} .... + \vec{F}_{0n} \qquad ...(14)$$

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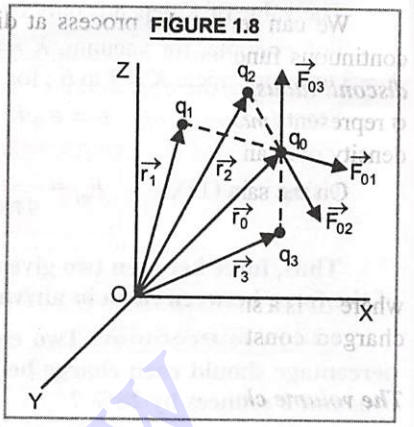
The component forces are shown in Fig. 1.8.

Here, $\vec{F}_{01}$ = force on q_0 due to q_1, $\vec{F}_{02}$ = force on q_0 due to q_2 and so on, $\vec{F}_{0n}$ = force on q_0 due to q_n.

FIGURE 1.8



According to Coulomb's law, $\vec{F}_{01} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{r_{10}^2}\hat{r}_{10}$

This is the force on q_0 due to q_1, even though other charges are present.

Similarly, force on q_0 due to q_2, even when other charges are present is given by

$$\vec{F}_{02} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_2}{r_{20}^2}\hat{r}_{20} \quad \text{and so on}$$

$$\vec{F}_{0n} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_n}{r_{n0}^2}\hat{r}_{n0}$$

Putting these values in eqn. (14), we get total force on charge q_0 as

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0 q_1}{r_{10}^2}\hat{r}_{10} + \frac{q_0 q_2}{r_{20}^2}\hat{r}_{20} + \frac{q_0 q_3}{r_{30}^2}\hat{r}_{30} + \ldots + \frac{q_0 q_n}{r_{n0}^2}\hat{r}_{n0}\right]$$

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{q_0 q_i}{r_{i0}^2}\hat{r}_{i0}$$

$$\vec{F}_0 = \frac{q_0}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{q_i}{r_{i0}^3}\vec{r}_{i0}$$

$$\vec{F}_0 = \frac{q_0}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{q_i}{|\vec{r}_0 - \vec{r}_i|^3}(\vec{r}_0 - \vec{r}_i) \quad \ldots(15)$$

The direction of net force $\vec{F}_0$ on charge q_0 due to n discrete charges can be determined using *polygon law of vectors.*

Retain in Memory

We can use superposition principle for computing (*i*) net force (*ii*) net field (*iii*) net flux (*iv*) net potential as well as net potential energy at the observation point P due to any configuration of charges.

1.17. CONTINUOUS CHARGE DISTRIBUTION

As charge can exist only as integral multiple of basic charge (e), therefore, charge distribution is always discrete, on account of atomicity of charge. However, it is impractical to work in terms of discrete charges always. For example, on the surface of a charged conductor, we cannot specify the charge distribution in terms of the locations of the microscopic charged constituents. However, we can consider a small area element ΔS on the surface of the conductor. This area element is very small on the macroscopic scale, but big enough to include a very large number of electrons. If ΔQ is the amount of charge on this element, we define **surface charge density** (σ) at the area element by $\sigma = \frac{\Delta Q}{\Delta S}$

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We can repeat the process at different points on the surface of the conductor and thus arrive at a continuous function σ, called the surface charge density. ***At the microscopic level, charge distribution is discontinuous,*** as there are discrete charges separated by intervening space, where there is no charge. Therefore, σ represents *macroscopic surface charge density* which is a smoothed out average of the microscopic charge density over an area element ΔS, which is small macroscopically but large microscopically.

On the same basis, when charge is distributed along a line, straight or curved, we define

linear charge density, $\lambda = \frac{\Delta Q}{\Delta l}$

where Δl is a small line element of wire on the macroscopic scale that includes a large number of microscopic charged constituents and ΔQ is the charge contained in that line element. The units of λ are *C/m*.

The volume charge density is defined in a similar manner as $\rho = \frac{\Delta Q}{\Delta V}$,

where ΔQ is the charge included in the macroscopically small volume element ΔV that includes a large number of microscopic charged constituents. The units of ρ are C/m^3.

Note that the notion of continuous charge distribution is similar to the continuous mass distribution in mechanics. For example, when we talk of density of a liquid, we are referring to its macroscopic density treating it as a continuous fluid and ignoring its discrete molecular constitution.

1.18. FORCE DUE TO CONTINUOUS DISTRIBUTION OF CHARGES

The force due to a continuous charge distribution can be obtained in almost the same way as for a system of discrete charges.

Suppose a continuous charge distribution in space has a volume charge density ρ. With respect to any suitably chosen origin O, let the position vector of any point in the charge distribution be $\vec{r}_i$. The volume charge density ρ is a function of $\vec{r}_i$, *i.e.*, it may vary from point to point. Divide the charge distribution into small volume elements of size ΔV. Therefore, charge in this volume element is $\Delta Q = \rho \cdot \Delta V$

Consider any general point P inside or outside the charge distribution with position vector $\overrightarrow{OP} = \vec{r}_0$. Using Coulomb's law, force due to charge element ΔQ on a small test charge q_0 at P is

$$d\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_0(\Delta Q)}{r'^2}\hat{r}' = \frac{q_0}{4\pi\epsilon_0}\frac{\rho\Delta V}{r'^2}\hat{r}'$$

where r' is the distance between the charge element and point P ; and $\hat{r}'$ is unit vector directed from charge element to the point P. As is clear from Fig. 1.9(*a*), $\vec{r}' = (\vec{r}_0 - \vec{r}_i)$

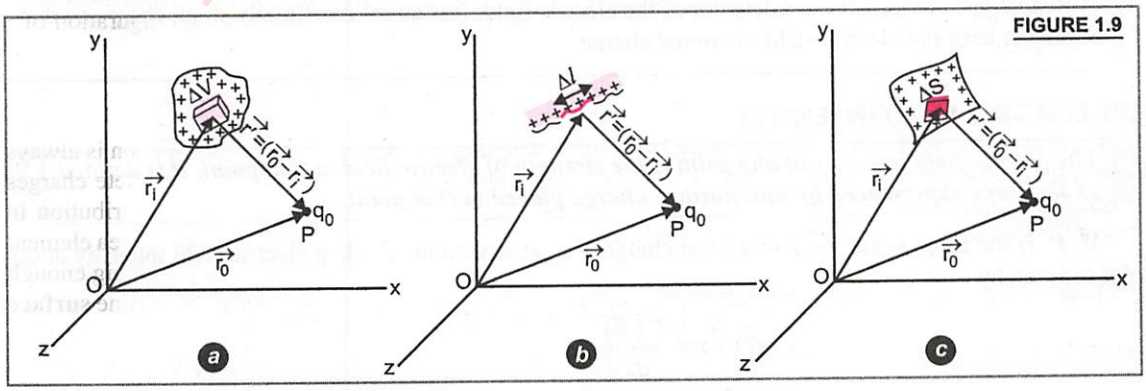


FIGURE 1.9

By the superposition principle, total force due to entire volume charge distribution is obtained by summing over the forces due to different volume elements.

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \sum_{\text{all}\,\Delta V} \frac{\rho\,\Delta V}{r'^2}\,\hat{r}' \qquad \text{...(16)}$$

When $\Delta V \to 0$, the sum becomes an integral and total force can be written as

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_V \frac{\rho\,dV}{r'^2}\,\hat{r}' \qquad \text{...(17)}$$

Proceeding as above, we can write total force due to *continuous line distribution of charge* as shown in Fig. 1.9(*b*) as

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_L \frac{\lambda\,dl}{r'^2}\,\hat{r}' \qquad \text{...(18)}$$

And total force due to *continuous surface distribution of charge* as shown in Fig. 1.9(*c*) as

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_S \frac{\sigma\,ds}{r'^2}\,\hat{r}' \qquad \text{...(19)}$$

1.19. CONCEPT OF ELECTRIC FIELD

When an electric charge q_0 is held in the vicinity of another charge Q, then the charge q_0 experiences a force of attraction or repulsion. We say that this force is due to interaction of electric fields of two charges Q and q_0. Thus, electric field of charge Q may be conceived as the *space property* by virtue of which, the presence of charge Q modifies the space around Q leading to the development of a force on any charge q_0 held in this space. We may define,

> *Electric field* ***due to a given charge as the field that permeates the space around the charge, in which electrostatic force of attraction or repulsion due to the charge can be experienced by any other charge.***

If a test charge experiences no force at a point, the electric field at that point (due to some other charge some where) must be zero.

The concept of field was introduced first of all by **Faraday.**

The charge Q, which is producing the electric field is called a **source charge** and the charge q_0 which tests the effect of source charge is called a **test charge.** The source charge can be a single charge or a group of discrete charges or any continuous distribution of charge.

IMPORTANT NOTE

> Note that test charge acts as a detector of the electric field. It is an infinitesimally small charge so that it affects least the electric field of source charge.

1.20. ELECTRIC FIELD INTENSITY

> *The electric field intensity* ***at any point is the strength of electric field at that point. It is defined as the force experienced by unit positive charge placed at that point.***

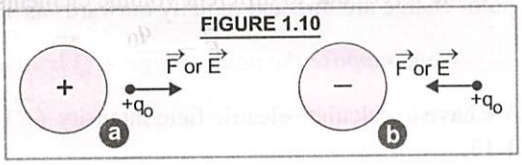
If $\vec{F}$ is the force acting on a small test charge + q_0 at any point $\vec{r}$, then electric field intensity at this point is given by

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q_0}$$

The SI unit of electric field intensity is newton per coulomb (N/C)

Electric field intensity is obviously **a vector.** The direction of $\vec{E}$ is the same as the direction of $\vec{F}$, *i.e.*, $\vec{E}$ is along the direction in which the test charge + q_0 would tend to move if free to do so.

FIGURE 1.10



For a positive source charge, the electric field will be directed radially outwards from the charge, Fig. 1.10(*a*). If the source charge is negative, the electric field vector at each point is radially inwards, as shown in Fig. 1.10(*b*).

As the test charge + q_0 may have its own electric field, it may modify the electric field of the source charge. Therefore, to minimise this effect and ultimately remove it, we rewrite electric field intensity at $\vec{r}$ as

$$\vec{E}(\vec{r}) = \underset{q_0 \to 0}{\text{Limit}} \frac{\vec{F}(\vec{r})}{q_0} \qquad \text{...(20)}$$

As the magnitude of test charge q_0 decreases, electric field intensity $\vec{E}(\vec{r})$ at a point is defined more and more accurately. On account of discrete nature of charge, the minimum possible value of test charge q_0 is $1{\cdot}6 \times 10^{-19}$ coulomb (which is the unit charge). It cannot be zero.

However, on the macroscopic scale, it is as good as taking the limit $q_0 \to 0$. *If the small test charge q_0 is positive, the measured value of electric intensity will be somewhat less than the actual value of electric intensity. However, if the small test charge q_0 is negative, the measured value of electric intensity will be somewhat more than the actual value of electric intensity.*

Values of some electric fields are given in Table 1.4.

TABLE 1.4. Values of some electric fields

Field Location	Value of E (N/C) or (V/m)
1. At the surface of uranium nucleus.	3×10^{21}
2. Within a hydrogen atom.	5×10^{11}
3. Electric breakdown of dry air.	3×10^{6}
4. Near a charged comb.	10^{3}
5. In a copper wire of house hold circuits.	10^{-2}

Sample Problem **Calculate force on an electron in a uniform electric field of 5×10^4 *N/C* due north.**

Sol. Here, $q_0 = 1\text{ e} = -1{\cdot}6 \times 10^{-19}$ C, E = 5×10^4 N/C, due north

$\therefore$ $F = q_0 E = -1{\cdot}6 \times 10^{-19} \times 5 \times 10^4$ N $= -8 \times 10^{-15}$ N due north = **8×10^{-15} N due south.**

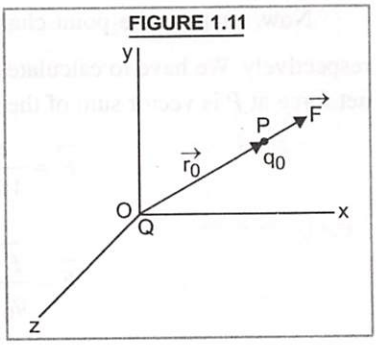
1.21. ELECTRIC FIELD INTENSITY DUE TO A POINT CHARGE

Suppose we have to calculate electric field intensity at any point P due to a point charge Q at O, where $OP = r_0$. Imagine a small positive test charge q_0 at P, Fig. 1.11. According to Coulomb's law, force on charge q_0 at P is

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Q\,q_0}{r_0^2} \hat{r}_0$$

where $\hat{r}_0$ is unit vector directed from Q towards q_0.

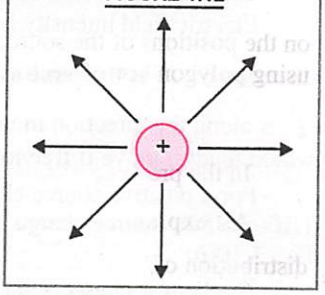
FIGURE 1.11



As $\vec{E} = \dfrac{\vec{F}}{q_0}$ $\therefore$ $\vec{E} = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{r_0^2} \hat{r}_0$ ...(21)

In general, the electric field vectors at several points around a positive point charge are directed radially outwards as shown in Fig. 1.12.

FIGURE 1.12



Now, suppose the point charge + Q is situated at A, where $\overrightarrow{OA} = \vec{r}_i$.

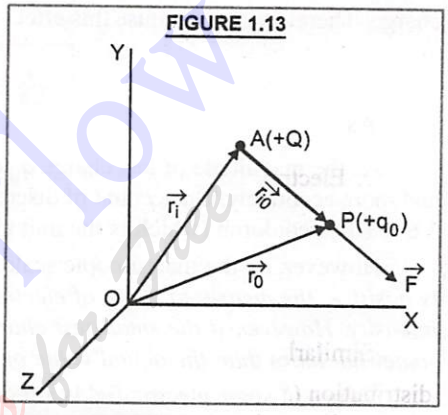
We have to calculate electric field intensity $(\vec{E})$ at P, where $\overrightarrow{OP} = \vec{r}_0$, Fig. 1.13.

According to Coulomb's law, force on a small test charge $(+q_0)$ at P is

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q q_0}{r_{i0}^2}\hat{r}_{i0} = \frac{1}{4\pi\epsilon_0}\frac{Q q_0}{r_{i0}^3}\vec{r}_{i0}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q q_0}{|\vec{r}_0 - \vec{r}_i|^3}(\vec{r}_0 - \vec{r}_i),$$

FIGURE 1.13



where $\vec{r}_0 - \vec{r}_i = \vec{r}_{i0} = \overrightarrow{AP}$

As $\vec{E} = \dfrac{\vec{F}}{q_0}$

$$\therefore \quad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{|\vec{r}_0 - \vec{r}_i|^3}(\vec{r}_0 - \vec{r}_i) \qquad \text{...(22)}$$

Obviously, it is along AP produced.

1.22. ELECTRIC FIELD INTENSITY DUE TO A GROUP OF CHARGES

The electric field intensity at any point due to a group of point charges is equal to the vector sum of the electric field intensities due to individual charges at the same point. For this, we first calculate the electric field intensity at the given point P due to individual charges, as if other charges were not there and then add them vectorially, *i.e.*, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots \vec{E}_n$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{Q_i}{r_i^2}\hat{r}_i \qquad \text{...(23)}$$

Here, r_i is the distance of the point P from the ith charge Q_i and $\hat{r}_i$ is unit vector directed from Q_i to the point P.

Now, suppose the point charges $Q_1, Q_2, Q_3, \ldots Q_n$ are located in vacuum at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_n$ respectively. We have to calculate electric field intensity $\vec{E}$ at any point P whose position vector is $\vec{r}_0$. As net force at P is vector sum of the forces due to individual charges, therefore, we can write

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{Q_i}{|\vec{r}_0 - \vec{r}_i|^3}(\vec{r}_0 - \vec{r}_i)$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{Q_i}{|\vec{r}_0 - \vec{r}_i|^3}(\vec{r}_0 - \vec{r}_i)$$

Note that $\vec{E}$ is a vector quantity that varies from one point to another point in space. Its value depends on the positions of the source charges. The direction of $\vec{E}$ due to a group of discrete charges is determined using polygon law of vectors.

1.23. ELECTRIC FIELD INTENSITY DUE TO CONTINUOUS CHARGE DISTRIBUTION

In the previous chapter, we have studied three kinds of continuous charge distributions in Article 1.17.

As explained already, total force on a small test charge + q_0 at $P(\vec{r_0})$ due to linear charge distribution of density $\lambda(\vec{r_i})$ is

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_L \frac{\lambda(\vec{r_i})\, dl}{|\vec{r_0} - \vec{r_i}|^3} (\vec{r_0} - \vec{r_i})$$

As $\vec{E} = \vec{F}/q_0$

$\therefore$ Electric field intensity at location $\vec{r_0}$ due to linear charge distribution $\lambda(\vec{r_i})$ is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_L \frac{\lambda(\vec{r_i})\, dl}{|\vec{r_0} - \vec{r_i}|^3} (\vec{r_0} - \vec{r_i}) \quad \text{...(24)}$$

Similarly, we can write expressions for electric field intensity at position r_0 due to surface charge distribution (S) and volume charge distribution (V) at $\vec{r_i}$ as

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma(\vec{r_i})\, dS}{|\vec{r_0} - \vec{r_i}|^3} (\vec{r_0} - \vec{r_i}) \quad \text{...(25)}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r_i})\, dV}{|\vec{r_0} - \vec{r_i}|^3} (\vec{r_0} - \vec{r_i}) \quad \text{...(26)}$$

1.24. RECTANGULAR COMPONENTS OF ELECTRIC INTENSITY DUE TO A POINT CHARGE

Suppose a point charge + Q is held at O, the origin of co-ordinate system shown in Fig. 1.14. We have to calculate rectangular components of electric field intensity $(\vec{E})$ at any point $P(x, y, z)$, where

$$\overrightarrow{OP} = \vec{r} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$$

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2} \quad \text{...(27)}$$

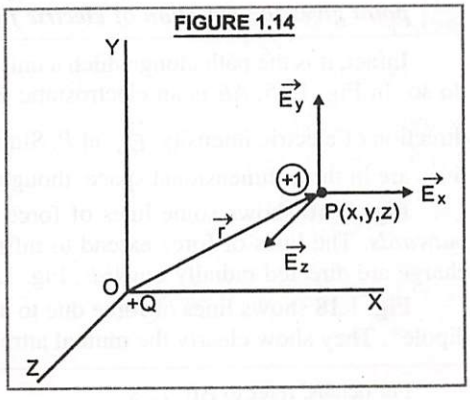
Force on unit + charge at P *i.e.*

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \quad \text{...(28)}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^3} \vec{r} = \frac{Q(\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \quad \text{...(29)}$$

If $\vec{E}_x, \vec{E}_y, \vec{E}_z$ are the components of $\vec{E}$ along the three co-ordinate axes, then

FIGURE 1.14



$$\vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z \quad \text{or} \quad \vec{E} = \hat{i}E_x + \hat{j}E_y + \hat{k}E_z \qquad ...(30)$$

Comparing (29) and (30), we get

$$E_x = \frac{Qx}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \qquad ...(31)$$

$$E_y = \frac{Qy}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \qquad ...(32)$$

$$E_z = \frac{Qz}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \qquad ...(33)$$

We find from (28) that as seen from O, the position of point charge + Q, the electric field is **spherically symmetric**, *i.e.*, at $x = y = z$; $E_x = E_y = E_z$, *i.e.*, at equal distances from the charge, field intensity is equal. Further, electric intensity at any point varies inversely as the square of the distance of the point from the charge.

1.25. PHYSICAL SIGNIFICANCE OF ELECTRIC FIELD

In Physics, the term 'field' generally refers to a quantity that is defined at every point in space. The electric field at a point in space around a system of charges gives us the force a unit positive test charge would experience, when placed at that point, without disturbing the system. The field is independent of the test charge that we place at a point to determine the electric field. This field varies from point to point. As force is a vector quantity, therefore, electric field is a vector field.

From the knowledge of electric intensity $\vec{E}$ at any point $\vec{r}$, we can readily calculate the magnitude and direction of force experienced by any charge q_0 held at that point, i.e.,

$$\vec{F}(\vec{r}) = q_0 \vec{E}(\vec{r})$$

This is the physical significance of electric field.

1.26. ELECTRIC FIELD LINES

Electric field lines map pictorially, the electric field around a configuration of electric charges. The concept was developed by Faraday and is a non mathematical way of visualizing electric fields around charged configurations. We may define,

An electric field line as a path, straight or curved in electric field, such that tangent to it at any point gives the direction of electric field intensity at that point.

Infact, it is the path along which a unit positive charge actually moves in the electrostatic field, if free to do so. In Fig. 1.15, AB is an electrostatic line of force. The tangent to the line at any point P gives us the direction of electric intensity $\vec{E}_P$ at P. Similarly, tangent to AB at Q gives us the direction of $\vec{E}_Q$. The field lines are in three dimensional space, though Fig. 1.15 shows them only in a plane.

Fig. 1. 16 shows some lines of force due to single *positive* point charge. These are directed radially *outwards*. The lines of force extend to infinity. On the contrary, lines of force due to single *negative* point charge are directed radially *inwards*, Fig. 1.17.

Fig. 1.18 shows lines of force due to a pair of equal and opposite charges, $(q, -q)$, forming an electric dipole*. They show clearly the mutual attraction between the opposite charges.

For details, refer to Art. 1.28

FIGURE 1.15

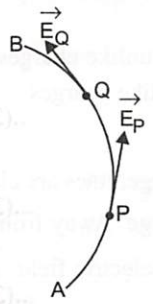


FIGURE 1.16

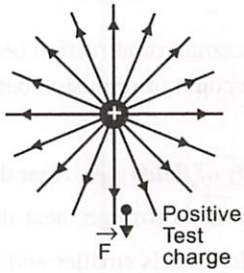


FIGURE 1.17

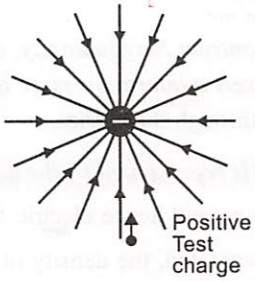
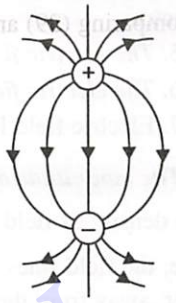


FIGURE 1.18



The lines of force due to two equal positive point charges (q, q) are shown in Fig. 1.19.

As is clear from Fig. 1.19, the field lines due to two charges ($+q_1$ and $+q_2$) exert lateral pressure on each other, which is responsible for repulsion between the like charges. Further, in Fig. 1.19, point P represents the neutral point, where net electric field intensity is zero.

When the charges are *equal, P* lies at the *centre* of the line joining the charges. However, when the charges are unequal, the neutral point P is *closer to the smaller charge* as shown in Fig. 1.20. Fig. 1.21 shows field lines for a section of an infinitely large sheet of positive charge.

FIGURE 1.19

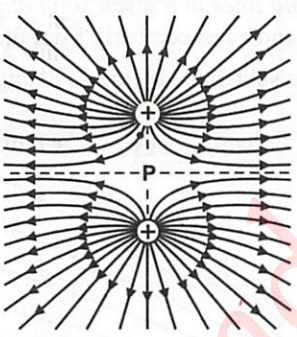


FIGURE 1.20

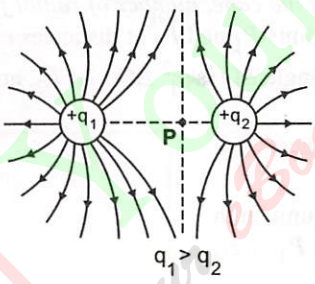
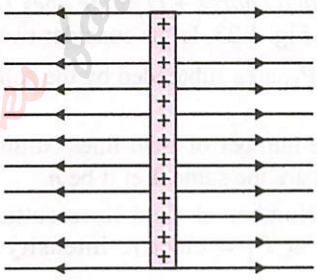


FIGURE 1.21



1.27. PROPERTIES OF ELECTRIC FIELD LINES

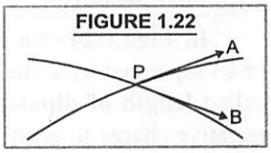
1. *Electric field lines are* **continuous curves.** They start from a positively charged body and end at a negatively charged body. No electric lines of force exist inside the charged body. Thus, ***electrostatic field lines are continuous but do not form closed loops.***

If there is a single charge, the electric field lines may start or end at infinity. For example, due to a single positive charge, field lines would start from the charge and end at infinity. Due to a single negative charge, field lines would start from infinity and end at the negative charge.

We shall study in unit 3 that ***magnetic lines of force are continuous (or endless) closed loops as against the electric lines of force which are continuous but not closed loops.***

2. *Tangent to the electric field line at any point gives the* **direction** *of electric field intensity at that point.*

FIGURE 1.22



3. *No two electric field lines of force can intersect eachother.* This is because at the point of intersection P, we can draw two tangents PA and PB to the two lines of force, Fig. 1.22.

This would mean two directions of electric field intensity at the same point, which is not possible. Hence no two electric lines of force can cross each other.

4. *The electric field lines are always normal to the surface of a conductor*, both while starting or ending on the conductor. Therefore, there is no component of electric field intensity parallel to the surface of the conductor.

5. *The electric field lines contract longitudinally*, on account of attraction between unlike charges.

6. *The electric field lines exert a lateral pressure* on account of repulsion between like charges.

7. Electric field lines pass through dielectrics.

The magnitude of the field is represented by the density of field lines. Near the charge, lines are closer, so the density of field lines is more and hence electric field $\vec{E}$ is stronger near the charge. Away from the charge, the field lines are well separated, the density of field lines is smaller and hence electric field $\vec{E}$ is weaker, away from the charge. We may, therefore, define

> ***Electric field intensity** at a point is equal to number of field lines crossing normally a unit area around that point.*

As field lines are three dimensional and all figures are drawn on the plane of the paper (in two dimensions), therefore, to estimate the density of field lines, we have to consider the number of field lines per unit cross-sectional area perpendicular to the lines. We know that in three dimensions, the solid angle $\Delta\Omega$ subtended by a small perpendicular plane area ΔS at a distance r from apex can be written as $\Delta\Omega = \dfrac{\Delta S}{r^2}$.

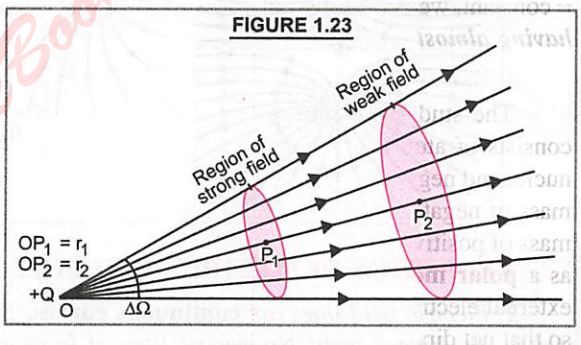
With a charge + Q at the apex O of the cone, number of radial field lines in a given solid angle is the same. In Fig. 1.23, let us consider two points P_1 and P_2 at distances r_1 and r_2 respectively from the charge.

At P_1, area subtended by the solid angle $\Delta\Omega$ is $r_1^2.\Delta\Omega$. At P_2, area subtended by the solid angle $\Delta\Omega$ is $r_2^2.\Delta\Omega$.

The number of field lines cutting these area elements are the same. Let it be n.

∴ Number of field lines cutting unit area element at P_1 = electric intensity at P_1, *i.e.*, $E_1 = \dfrac{n}{r_1^2.\Delta\Omega}$, and number of field lines cutting unit area element at P_2 = electric intensity at P_2, *i.e.*, $E_2 = \dfrac{n}{r_2^2.\Delta\Omega}$

FIGURE 1.23



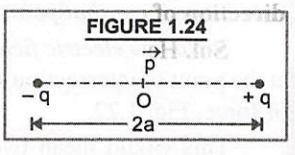
Clearly ; $\dfrac{E_1}{E_2} = \dfrac{r_2^2}{r_1^2}$, *i.e.*, $E \propto \dfrac{1}{r^2}$ *i.e., electric field intensity is more near the charge and vice-versa.*

1.28. ELECTRIC DIPOLE

> ***An electric dipole** consists of a pair of equal and opposite point charges separated by some small distance.*

In Fig. 1.24, we have shown two equal and opposite point charges ($\pm q$) separated by a small distance '2*a*'. It represents an electric dipole. 2 *a* is called **length of dipole.** It can be taken as a vector whose direction is from negative charge to positive charge of dipole.

FIGURE 1.24



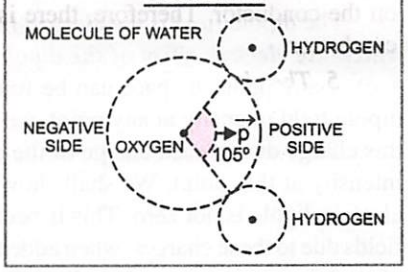
The middle point of locations of $-q$ and $+q$ is called the **centre** of the dipole. It is represented by *O*.

The total charge of the electric dipole $= -q + q = 0$.

This does not mean that the field of the electric dipole is zero.

Atoms or molecules of ammonia, water, alcohol, carbon dioxide, HCl etc. are some of the examples of electric dipoles, because in their cases, the centres of positive and negative charge distributions are separated by some small distance. Fig. 1.25 shows a molecule of water (H_2O) with three nuclei represented by dots. The electric dipole moment $\vec{p}$ points from the (negative) oxygen side to the (positive) hydrogen side of the molecule as explained in Art. 1.29.

FIGURE 1.25



1.29. DIPOLE MOMENT

Dipole moment ($\vec{p}$) ***is a measure of the strength of electric dipole. It is a vector quantity whose magnitude is equal to product of the magnitude of either charge and the distance between them.***

i.e., $\vec{p} = q(2\vec{a})$ or $|\vec{p}| = q(2a)$ ...(34)

By convention, the direction of $\vec{p}$ ***is from negative charge to positive charge.***

The SI unit of dipole moment is **coulomb-metre** (C–m).

The **dimensional formula** of dipole moment is $M^0 (AT) L = [M^0 L^1 A^1 T^1]$

If charge q gets larger, and the distance $2a$ gets smaller and smaller, keeping the product $|\vec{p}| = q \times 2a$ = constant, we get what is called an **ideal dipole** or **point dipole.** Thus, ***an ideal dipole is the smallest dipole having almost no size.***

Physical significance of electric dipoles

The study of electric dipoles is important for electrical phenomena in matter. Matter, as we know, consists of atoms or molecules which are electrically neutral. In a molecule, there are positively charged nuclei and negatively charged electrons. If the centre of mass of positive charges coincides with the centre of mass of negative charges, the molecule behaves as a **non-polar molecule.** On the contrary, if the centre of mass of positive charges does not coincide with the centre of mass of negative charges, the molecule behaves as a **polar molecule** and it possesses some intrinsic or permanent dipole moment. In the absence of an external electric field, the dipole moments of different molecules in a piece of matter are randomly oriented, so that net dipole moment of the piece is zero. When an external electric field is applied, the polar molecules tend to align themselves along the field and some net dipole moment develops. The piece of matter is said to have been polarised.

When non-polar molecules are subjected to the action of an external electric field, the centres of mass of positive and negative charges in the molecule get displaced in opposite directions. Thus, the external field induces some dipole moment in the molecule in the direction of the field. The induced dipole moments of different molecules in the sample add up vectorially to produce some net total dipole moment.

Sample Problem **Charges ± 20 nC are separated by 5 mm. Calculate the magnitude and direction of dipole moment.**

Sol. Here $q_1 = q_2 = q = \pm 20\ nC = \pm 20 \times 10^{-9}$ C

$2a = 5$ mm $= 5 \times 10^{-3}$ m

$p = q(2a) = 20 \times 10^{-9} \times 5 \times 10^{-3} = \mathbf{10^{-10}}$ **C.m**

The direction of $\vec{p}$ is **from negative charge to positive charge.**

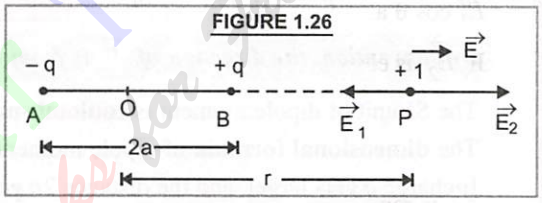
1.30. DIPOLE FIELD

The dipole field is the electric field produced by an electric dipole. ***It is the space around the dipole in which the electric effect of the dipole can be experienced.*** The electric field of the pair of charges ($-q$ and $+q$) at any point in space can be found from Coulomb's law and the superposition principle. To calculate dipole field intensity at any point, we imagine a unit positive charge held at that point. We calculate force on this charge due to each charge of the dipole and take vector sum of the two forces. This gives us dipole field intensity at that point. We shall show that though the total charge of electric dipole is zero, the field of the electric dipole is not zero. This is because the charges q and $-q$ are separated by some distance. The electric fields due to these charges, when added, donot cancel out. However, at distances much larger than the separation of two charges, *i.e.*, $r >> 2a$, the fields due to q and $-q$ largely cancel out. Therefore, at large distances from the dipole, the dipole field falls off more rapidly $\left(\text{like } E \propto \frac{1}{r^3}\right)$ than like $E \propto \frac{1}{r^2}$ for a point charge.

1.31. FIELD INTENSITY ON AXIAL LINE OF ELECTRIC DIPOLE

Consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a small distance $2a$. We have to calculate electric intensity $\vec{E}$ at a point P on the axial line of the dipole, and at a distance $OP = r$ from the centre O of the dipole, Fig. 1.26.

FIGURE 1.26



If $\vec{E_1}$ is the electric intensity at P due to charge $-q$, at A, then

$$|\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}$$

It is along PA

Suppose $\vec{E_2}$ is the electric intensity at P due to charge $+q$ at B, then

$$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}$$

It is along BP produced.

As $\vec{E_1}$ and $\vec{E_2}$ are collinear vectors acting in opposite directions and $|\vec{E_2}| > |\vec{E_1}|$ therefore, the resultant intensity $\vec{E}$ at P will be difference of two, acting along BP produced.

$$|\vec{E}| = |\vec{E_2}| - |\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right] = \frac{q}{4\pi\epsilon_0}\frac{4ar}{(r^2-a^2)^2} = \frac{q\times 2a\times 2r}{4\pi\epsilon_0(r^2-a^2)^2}$$

But $q \times 2a = |\vec{p}|$, the dipole moment $\therefore$

$$|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0}\cdot\frac{2r}{(r^2-a^2)^2} \quad \text{...(35)}$$

If dipole is short, $2a << r$,

$$|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0}\frac{2r}{r^4} = \frac{2|\vec{p}|}{4\pi\epsilon_0 r^3} \quad \text{...(36)}$$

The direction of $\vec{E}$ is along BP produced, *i.e.*, along the direction of $\vec{p}$.

Clearly, $|\vec{E}| \propto \frac{1}{r^3}$

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1.32. FIELD INTENSITY ON EQUATORIAL LINE OF ELECTRIC DIPOLE

Consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a small distance $AB = 2a$ with centre at O and dipole moment, $p = q\,(2\,a)$, Fig. 1.27.

We have to find electric intensity $\vec{E}$ at a point P on the equatorial line of the dipole, where $OP = r$.

If $\vec{E_1}$ is electric intensity at P due to charge $-q$ at A, then

$$|\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{AP^2}$$

But $AP^2 = OP^2 + OA^2 = r^2 + a^2$

$$\therefore \quad |\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \qquad ...(37)$$

$\vec{E_1}$ is represented by $\overrightarrow{PC}$.

Let $\angle PBA = \angle PAB = \theta$.

$\vec{E_1}$ has two rectangular components :

$E_1 \cos\theta$ along $PR \,||\, BA$ and $E_1 \sin\theta$ along $PE \perp BA$

If $\vec{E_2}$ is electric intensity at P, due to charge $+q$ at B, then

$$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)} \qquad ...(38)$$

$\vec{E_2}$ is represented by $\overrightarrow{PD}$ (along BPD). $\vec{E_2}$ has two rectangular components,

$E_2 \cos\theta$ along $PR \,||\, BA$ and $E_2 \sin\theta$ along PF (opposite to PE)

From (32) and (33), as $|\vec{E_1}| = |\vec{E_2}|$ $\therefore$ $E_1 \sin\theta$ along PE and $E_2 \sin\theta$ along PF cancel out

$\therefore$ Resultant intensity at P is given by $|\vec{E}| = E_1 \cos\theta + E_2 \cos\theta$

$$|\vec{E}| = 2E_1 \cos\theta \qquad (\because |\vec{E_1}| = |\vec{E_2}|)$$

$$|\vec{E}| = \frac{2}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)}\cos\theta = \frac{2}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)}\left(\frac{OA}{AP}\right) = \frac{2}{4\pi\epsilon_0}\frac{q}{(r^2+a^2)}\frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{q\times 2a}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}$$

But $q \times 2a = |\vec{p}|$, the dipole moment $\therefore$

$$\boxed{|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}} \qquad ...(39)$$

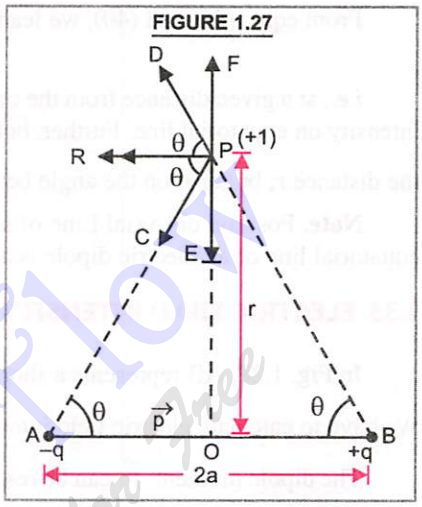
The direction of $\vec{E}$ is along $\overrightarrow{PR} \,||\, \overrightarrow{BA}$ (*i.e.*, opposite to $\vec{p}$).

In vector form, we can rewrite (34) as

$$\boxed{\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}}$$

Obviously, $\vec{E}$ **is in a direction opposite to the direction of** $\vec{p}$ **.**

FIGURE 1.27



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If the dipole is short, $2a << r$ $\therefore$ $$|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0 r^3} \quad ...(40)$$

Clearly, $$|\vec{E}| \propto \frac{1}{r^3}$$

From eqns. (36) and (40), we learn that $$\frac{E_{axial}}{E_{equatorial}} = 2$$

i.e., at a given distance from the centre of dipole, electric field intensity on axial line is twice the electric intensity on equatorial line. Further, both the magnitude and the direction of dipole field depend not only on the distance r, but also on the angle between the position vector $\vec{r}$ and dipole moment $\vec{p}$.

Note. Position on Axial Line of an electric dipole is also known as ***End on position.*** And position on equatorial line of an electric dipole is also known as ***Broadside on position.***

1.33. ELECTRIC FIELD INTENSITY AT ANY POINT DUE TO A SHORT ELECTRIC DIPOLE

In Fig. 1.28, AB represents a short electric dipole of moment $\vec{p}$ along $\overrightarrow{AB}$. O is the centre of dipole. We have to calculate electric field intensity $\vec{E}$ at any point K, where $OK = r$, $\angle BOK = \theta$.

The dipole moment $\vec{p}$ can be resolved into two rectangular components :

($p \cos\theta$) along $A_1 B_1$ and ($p \sin\theta$) along $A_2 B_2 \perp A_1 B_1$.

Field intensity at K on the axial line of $A_1 B_1$,

$$|\vec{E_1}| = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}$$

Let it be represented by $\overrightarrow{KL}$ along OK.

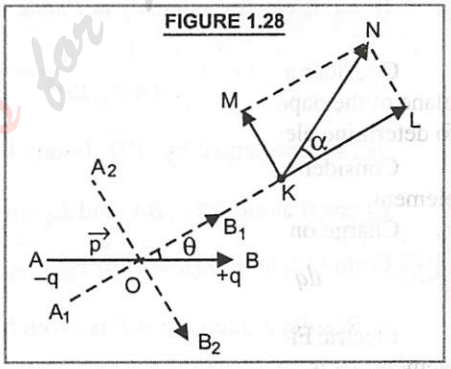
Field intensity at K on equatorial line of $A_2 B_2$,

$$|\vec{E_2}| = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

Let it be represented by $\overrightarrow{KM} \parallel B_2 A_2$ and $\perp \overrightarrow{KL}$. Complete the rectangle $KLNM$. Join $\overrightarrow{KN}$.

According to | | gm law, $\overrightarrow{KN}$ represents resultant intensity ($\vec{E}$) at K due to the short dipole.

FIGURE 1.28



As $$KN = \sqrt{KL^2 + KM^2}$$

$\therefore$ $$|\vec{E}| = \sqrt{E_1^2 + E_2^2} = \sqrt{\left(\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\epsilon_0 r^3}\right)^2} = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$|\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + (\cos^2\theta + \sin^2\theta)} = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

i.e., $$|\vec{E}| = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\epsilon_0 r^3} \quad ...(41)$$

Let $\angle LKN = \alpha$

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In ΔKLN, $\tan\alpha = \frac{LN}{KL} = \frac{KM}{KL} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \cdot \frac{4\pi\epsilon_0 r^3}{2p\cos\theta}$, or $\boxed{\tan\alpha = \frac{1}{2}\tan\theta}$ $\therefore$ α can be calculated.

Particular cases :

1. **When the point K lies on axial line of dipole.**

$\theta = 0°$, $\cos\theta = \cos 0° = 1$

$$\therefore \quad |\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2 0° + 1} = \frac{2p}{4\pi\epsilon_0 r^3} \quad \text{and} \quad \tan\alpha = \frac{1}{2}\tan 0° = 0, \quad \therefore \quad \alpha = 0°$$

i.e., resultant intensity is along the axial line.

2. **When the point K lies on equatorial line of dipole.**

$\theta = 90°$, $\cos\theta = \cos 90° = 0$

$$\therefore \quad E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2 90° + 1} = \frac{p}{4\pi\epsilon_0 r^3} \quad \text{and} \quad \tan\alpha = \frac{1}{2}\tan 90° = \infty, \quad \alpha = 90°$$

i.e., direction of resultant field intensity is perpendicular to the equatorial line (and hence antiparallel to axial line of dipole).

We have already proved these results in Arts 1.31. and 1.32.

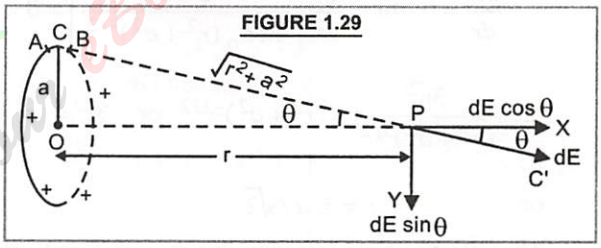
1.34. ELECTRIC FIELD INTENSITY AT ANY POINT ON THE AXIS OF A UNIFORMLY CHARGED RING

Consider a circular loop of wire of negligible thickness, radius a and centre O held perpendicular to the plane of the paper. Let the loop carry a total charge $+q$ distributed uniformly over its circumference. We have to determine electric field intensity at any point P on the axis of the loop, where $OP = r$, Fig. 1.29.

Consider a small element AB of the loop. Let length of element $AB = dl$ and C be the centre of the element.

Charge on the element AB is

$$dq = \frac{q}{2\pi a}dl \qquad \text{...(42)}$$



Electric Field intensity at P due to the charge element AB is

$$|\vec{dE}| = \frac{1}{4\pi\epsilon_0}\frac{dq}{CP^2}$$

along PC' at $\angle\theta$ with the axis OX.

$$|\vec{dE}| = \frac{1}{4\pi\epsilon_0}\frac{dq}{(r^2+a^2)} \qquad \text{...(43)}$$

$\vec{dE}$ can be resolved into two rectangular components.

$dE\cos\theta$ along PX, the axis of the loop, and $dE\sin\theta$ along PY, perpendicular to the axis.

For a pair of diametrically opposite elements of the loop, components of electric field intensity perpendicular to the axis will cancel, whereas the components along the axis of the loop will add. As the loop can be considered to be made up of a large number of pairs of diametrically opposite elements, therefore,

$\sum dE\sin\theta = 0$

Hence the resultant electric field intensity $\vec{E}$ at P is $|\vec{E}| = \sum dE\cos\theta$

In ΔOPC, $\cos\theta = \frac{OP}{CP} = \frac{r}{(r^2+a^2)^{1/2}}$ $\therefore$ $|\vec{E}| = \sum \frac{1}{4\pi\epsilon_0}\frac{dq}{(r^2+a^2)}\frac{r}{(r^2+a^2)^{1/2}}$

Using (42), we get $|\vec{E}| = \sum \frac{1}{4\pi\epsilon_0}\left(\frac{q\,dl}{2\pi a}\right)\frac{r}{(r^2+a^2)^{3/2}}$

$$= \frac{q\,r}{4\pi\epsilon_0\, 2\pi a\,(r^2+a^2)^{3/2}} \sum_{\text{whole loop}} dl = \frac{q\,r\,(2\pi a)}{4\pi\epsilon_0\, 2\pi a\,(r^2+a^2)^{3/2}}$$

$$|\vec{E}| = \frac{q\,r}{4\pi\epsilon_0\,(r^2+a^2)^{3/2}} \quad ...(44)$$

The direction of $\vec{E}$ is along PX, the axis of the loop.

Special Cases. (*i*) **When *P* lies at the centre of the loop.**

$r = 0$, therefore, from (44), $\vec{E} = 0$

(*ii*) **When $r >> a$** (*i.e.* P lies far off from the loop), neglecting a^2 in comparison to r^2 in (44), we get

$$|\vec{E}| = \frac{q\,r}{4\pi\epsilon_0\, r^3} = \frac{q}{4\pi\epsilon_0\, r^2}, \text{ along } PX$$

This is the expression for $\vec{E}$ at a distance r from a point charge q. ***Hence a circular loop of charge behaves as a point charge when the observation point (P) is at very large distance from the loop, compared to the radius of the loop.***

(*iii*) Electric field intensity due to a uniformly charged ring will be *maximum.* When

$$\frac{dE}{dr} = 0 \quad \text{or} \quad \frac{d}{dr}\left[\frac{q\times r}{4\pi\epsilon_0\,(r^2+a^2)^{3/2}}\right] = 0 \quad \text{or} \quad \frac{q}{4\pi\epsilon_0}\left[\frac{-3r^2}{(r^2+a^2)^{5/2}} + (r^2+a^2)^{-3/2}\right] = 0$$

$$\text{or} \quad \frac{3r^2}{(r^2+a^2)^{5/2}} = (r^2+a^2)^{-3/2} \quad \text{or} \quad 3r^2 = (r^2+a^2)$$

or $r = \pm\, a/\sqrt{2}.$

Thus electric field intensity due to a uniformly charged ring will be maximum at a distance $r = \frac{a}{\sqrt{2}}$ from its centre on either side on the axis of the ring. The variation of electric field intensity due to a charged ring with the distance r from the centre of the ring is shown in Fig. 1.30.

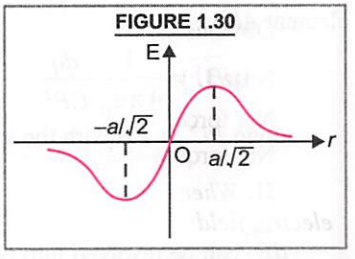


FIGURE 1.30

1.35. ELECTRIC DIPOLE IN A UNIFORM TWO DIMENSIONAL ELECTRIC FIELD

Consider an electric dipole consisting of two equal and opposite point charges $-q$ at A and $+q$ at B separated by a small distance $AB = 2a$, having dipole moment $|\vec{p}| = q \times 2a$, Fig. 1.31.

Let this dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.

Force on charge $+q$ at $A = q\vec{E}$, along the direction of $\vec{E}$.

Force on charge $-q$ at $B = q\vec{E}$, in a direction opposite to $\vec{E}$.

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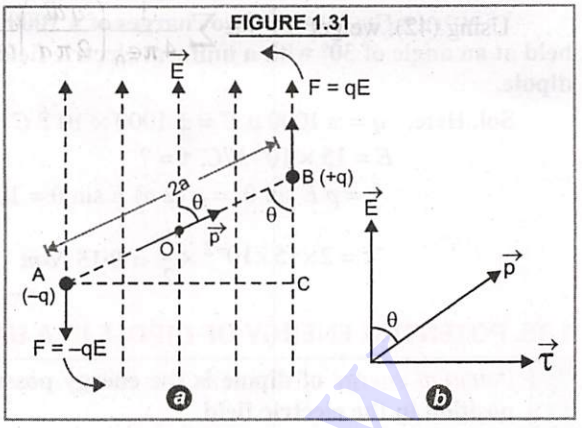
Since $\vec{E}$ is uniform, therefore, net force on the dipole is $(qE - qE) = 0$. However, as the forces are equal, unlike and parallel, acting at different points, therefore, they form a **couple** which rotates the dipole in the **anticlockwise direction,** as shown in Fig. 1.31. Thus, the couple tends to align the dipole axis along the direction of field $\vec{E}$.

FIGURE 1.31



Draw $AC \perp \vec{E}$

$\therefore$ perpendicular distance between the forces = arm of couple = AC

As torque = moment of the couple

$\tau = force \times arm\ of\ couple.$

$\tau = F \times AC = F \times AB \sin\theta = F \times 2a \sin\theta = (qE)\, 2a \sin\theta = (q \times 2a)\, E \sin\theta$

As $q \times 2a = p$, therefore, $\boxed{\tau = pE \sin\theta}$ ...(45)

$\therefore$ In vector form, we can rewrite this eqn. as $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$ ...(46)

The direction of $\vec{\tau}$ is given by *right hand screw rule* and is perpendicular to $\vec{p}$ and $\vec{E}$, *i.e.*, perpendicular to the plane of the paper and outwards.

Special cases. As stated already, this torque tries to align the electric dipole in the direction of the field. When $\vec{p}$ is along $\vec{E}$, $\theta = 0°$, $\tau = pE \sin 0° = 0$.

The dipole is in **stable equilibrium.**

However, when dipole is held in a direction *opposite* to $\vec{E}$, the torque would turn the dipole through 180°. As such, the dipole will be in an **unstable equilibrium.**

The torque will be maximum, when $\theta = 90°$.

$$\boxed{\text{Max. value of torque} = pE \sin 90° = pE}$$

Obviously, units of τ are N-*m* and its dimensional formula is $[M^1 L^2 T^{-2}]$.

Note I. *When electric dipole is in a uniform electric field (along* $\vec{p}$*),* Fig. 1.32(*a*)

Net force = $qE - qE = 0$

Net torque = $pE \sin\theta = 0$, as $\theta = 0°$

II. *When electric dipole is in a non-uniform electric field*

(a) *Field is increasing along* $\vec{p}$, Fig. 1.32(*b*)

Net force on dipole is along $\vec{E}$

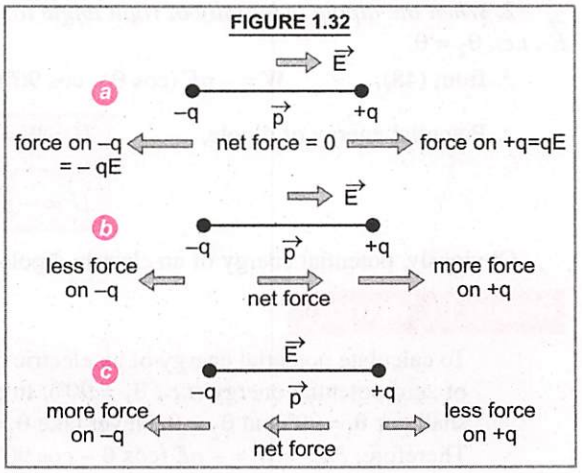
Net torque = $pE \sin\theta = 0$, as $\theta = 0°$, *i.e.*, $\vec{p}$ is along $\vec{E}$

(b) *Field is decreasing along* $\vec{p}$, Fig. 1.32(*c*)

Net force on dipole is opposite to $\vec{p}$

Net torque = $pE \sin\theta = 0$, as $\theta = 0°$, *i.e.* $\vec{p}$ is along $\vec{E}$.

FIGURE 1.32



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Sample Problem **Two charges of $\pm 1000\ \mu C$ are separated by 2 mm. The dipole so formed is held at an angle of 30° with a uniform electric field of $15 \times 10^4\ N/C$. Calculate the torque acting on the dipole.**

Sol. Here, $q = \pm 1000\ \mu C = \pm 1000 \times 10^{-6}\ C = \pm 10^{-3}\ C$, $2a = 2\ mm = 2 \times 10^{-3}\ m$, $\theta = 30°$

$E = 15 \times 10^4\ N/C$, $\tau = ?$

$\tau = pE \sin\theta = q(2a) E \sin\theta = 10^{-3} \times 2 \times 10^{-3} \times 15 \times 10^4 \sin 30°$

$$\tau = 2 \times 15 \times 10^{-2} \times \frac{1}{2} = \mathbf{0{\cdot}15\ N\text{-}m}$$

1.36. POTENTIAL ENERGY OF DIPOLE IN A UNIFORM ELECTRIC FIELD

Potential energy **of dipole is the energy possessed by the dipole by virtue of its particular position in the electric field.**

Suppose an electric dipole of moment $\vec{p}$ is oriented at an $\angle\theta$ with the direction of uniform external electric field $\vec{E}$, Fig. 1.31. We know, the torque $\vec{\tau}$ acting on the dipole is $\tau = pE \sin\theta$

It tries to rotate the dipole to align it with $\vec{E}$.

Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

∴ Total work done in rotating the dipole from orientation θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta = pE\,[-\cos\theta]_{\theta_1}^{\theta_2} = -pE\,[\cos\theta_2 - \cos\theta_1] \quad \text{...(47)}$$

Potential energy of dipole, $U = W = -pE(\cos\theta_2 - \cos\theta_1)$ ...(48)

Particular cases.

1. *When the dipole is initially aligned along the electric field, i.e.,* $\theta_1 = 0°$, and we have to set it at angle θ with $\vec{E}$, *i.e.,* $\theta_2 = \theta$.

From (48), $W = -pE(\cos\theta - \cos 0°)$

$W = -pE(\cos\theta - 1)$ ...(49)

This work done is stored in the dipole in the form of potential energy.

2. *When the dipole is initially at right angle to* $\vec{E}$, *i.e.,* $\theta_1 = 90°$, and we have to set it at angle θ with $\vec{E}$, *i.e.,* $\theta_2 = \theta$

∴ from (48), $W = -pE(\cos\theta - \cos 90°) = -pE\cos\theta$

∴ Potential energy of dipole, $\boxed{U = W = -pE\cos\theta}$ ...(50)

$$\boxed{U = -\vec{p}.\vec{E}} \quad \text{...(51)}$$

Obviously, potential energy of an electric dipole is a **scalar quantity.** It is measured in **joule.**

Retain in Memory

To calculate potential energy of an electric dipole at any angle θ, we must start from the position of zero potential energy (*i.e.,* $\theta_1 = 90°$). In equation (48), *P.E.* $= W = -pE(\cos\theta_2 - \cos\theta_1)$, we shall put $\theta_1 = 90°$ and $\theta_2 = \theta$. Never take $\theta_1 = 0°$, as it is not the position of zero potential energy. Therefore, *P.E.* $= W = -pE(\cos\theta - \cos 90°) = -pE\cos\theta$.

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Sample Problem **A dipole consists of an electron and a proton separated by a distance of 5×10^{-9} m. The dipole is aligned in a uniform electric field of $1{\cdot}44 \times 10^4$ N/C. Calculate potential energy of dipole to hold it at 60° with the direction of electric field.**

Sol. Here $q = 1{\cdot}6 \times 10^{-19}$ C ; $2a = 5 \times 10^{-9}$ m.

$\theta_1 = 0°$, $E = 1{\cdot}44 \times 10^4$ N/C, $PE = ?$, $\theta_2 = 60°$

$$P.E. = -pE(\cos\theta_2 - \cos\theta_1) = -q \times 2a\,E(\cos\theta_2 - \cos\theta_1)$$

$$= -1{\cdot}6 \times 10^{-19} \times 5 \times 10^{-9} \times 1{\cdot}44 \times 10^4 (\cos 60° - \cos 0°)$$

$$= -8 \times 1{\cdot}44 \left(\frac{1}{2} - 1\right) \times 10^{-24}\,\text{J}$$

$$P.E. = \mathbf{5{\cdot}76 \times 10^{-24}\ J}$$

1.37. AREA VECTOR

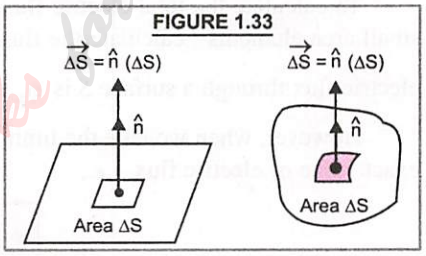
Area is a scalar quantity. But in some of the problems, it is convenient to treat it as a vector. The question is how to associate some direction with the area of a curved surface. Let us divide the given closed area into a large number of very small area elements. Each small area element may be treated as planar. As normal to the plane specifies the orientation of the plane, therefore, the direction of a planar area vector is along its normal. But a normal can point in two directions, inwards or outwards. ***By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward drawn normal.***

Thus, an area element vector $\Delta\vec{S}$ at a point on a closed surface can be written as

$$\Delta\vec{S} = \hat{n}\,(\Delta S) \qquad \text{...(52)}$$

where ΔS is magnitude of the area element and $\hat{n}$ is a unit vector in the *direction of outward drawn normal* at that point, as shown in Fig. 1.33.

FIGURE 1.33

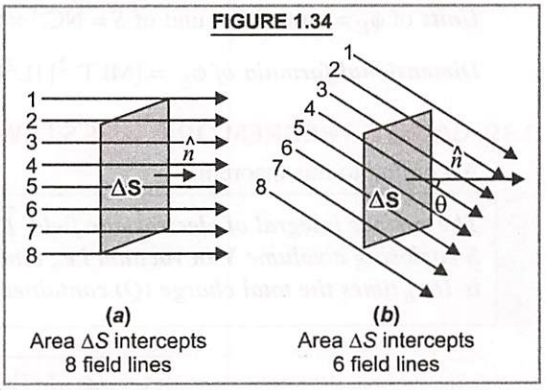


1.38. ELECTRIC FLUX

***Electric flux** over an area in an electric field is a measure of the number of electric field lines crossing this area.*

We know that the number of electric field lines crossing a unit area placed normal to the field at a point is a measure of strength of electric field $\vec{E}$ at that point. If we place a small planar element of area ΔS normal to $\vec{E}$ at this point, [Fig. 1.34(*a*)], ***the number of electric field lines crossing this area element is proportional to E*** (ΔS). Note that it is not proper to say that number of field lines crossing the area is equal to E (ΔS). The number of field lines is after all, a matter of how many field lines we choose to draw. *What is physically significant is the relative number of field lines crossing a given area at different points.*

FIGURE 1.34



If we tilt the area element by angle θ [or we tilt $\vec{E}$ *w.r.t.* area element by angle θ, Fig. 1.34(*b*)], the number of field lines crossing the area will be smaller. As projection of area element normal to E is

$\Delta S \cos\theta$ (or component of $\vec{E}$ normal to area element is $E \cos\theta$), therefore, number of electric field lines crossing area ΔS is proportional to $E(\Delta S \cos\theta)$. This is shown in Fig. 1.34(*b*).

Hence electric flux $\Delta\phi$ through an area element $\Delta\vec{S}$ in an electric field $\vec{E}$ is defined as

$$\Delta\phi = \vec{E}.\Delta\vec{S} = E\,(\Delta S)\cos\theta \quad \text{...(53)}$$

which is proportional to the number of field lines cutting the area element.

Here, θ is smaller angle between $\vec{E}$ and $\Delta\vec{S}$. For a closed surface, θ is the angle between $\vec{E}$ and outward normal to the area element.

Now, $\Delta\phi = \vec{E}.\Delta\vec{S} = E\,\Delta S\cos\theta = E\,(\Delta S\cos\theta)$, *i.e.*, E times the projection of area normal to $\vec{E}$.

Also, $\Delta\phi = \vec{E}.\Delta\vec{S} = E\,\Delta S\cos\theta = (E\cos\theta)\,\Delta S$, *i.e.*, component of $\vec{E}$ along normal to the area element times the magnitude of area element. When $\vec{E}$ is normal to area element, $\theta = 0°$, electric flux is maximum. When $\vec{E}$ is along the area element, $\theta = 90°$, electric flux is zero. When $\theta > 90°$, $\cos\theta$ is negative. Therefore, electric flux is negative.

To calculate the total electric flux through any given surface, we have to divide the given surface into small area elements ; calculate the flux at each element and add them up with proper sign. Therefore, total electric flux through a surface S is $\phi_E \approx \Sigma\,\vec{E}.\Delta\vec{S}$

However, when we take the limit $\Delta S \to 0$, the summation can be written as integral and we obtain the exact value of electric flux, *i.e.*,

$$\phi_E = \oint \vec{E}.\vec{ds} = \oint E\,ds\cos\theta$$

The circle on the integral sign indicates that the surface of integration is a closed surface.

Note that electric flux over a closed surface S can be positive ; negative or zero, according as $\theta < 90°$; $\theta > 90°$ and $\theta = 90°$.

Electric flux is a scalar quantity.

Units of ϕ_E = unit of E × unit of $S = NC^{-1} \times m^2 = Nm^2\,C^{-1}$.

Dimensional formula *of* $\phi_E = [MLT^{-2}]\,[L^2]\,[AT]^{-1} = [M^1\,L^3\,T^{-3}\,A^{-1}]$

1.39. GAUSS'S THEOREM OR GAUSS'S LAW IN ELECTROSTATICS

According to this theorem,

The surface integral of electrostatic field $\vec{E}$ produced by any sources over any closed surface S enclosing a volume V in vacuum i.e., total electric flux over the closed surface S in vacuum, is $1/\epsilon_0$ times the total charge (Q) contained inside S, i.e.,

$$\phi_E = \oint_S \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0} \quad \text{...(54)}$$

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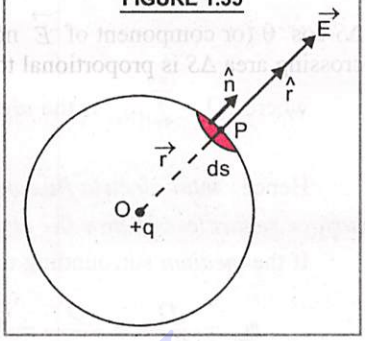
The charges inside S may be point charges or even continuous charge distributions. *There is no contribution to total electric flux from the charges outside S. Further, the location of Q inside S does not affect the value of surface integral.*

The surface chosen to calculate the surface integral is called *Gaussian surface**. While selecting such a surface, we shall avoid charges on S itself.

Proof of Gauss's theorem (*for spherically symmetric surfaces only*)

Suppose an isolated positive point charge q is situated at the centre O of a sphere of radius r, Fig. 1.35.

FIGURE 1.35



According to Coulomb's law, electric field intensity at any point P on the surface of the sphere is

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2} \qquad ...(55)$$

where $\hat{r}$ is unit vector directed from O to P.

Consider a small area element ds of the sphere around P. Let it be represented by the vector $\vec{ds} = \hat{n}\,ds$ where $\hat{n}$ is unit vector along outdrawn normal to the area element.

$\therefore$ Electric flux over the area element $d\phi_E = \vec{E}.\vec{ds} = \left(\frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\right).(\hat{n}\,ds)$

$$\vec{E}.\vec{ds} = \frac{q}{4\pi\epsilon_0}\frac{ds}{r^2}\hat{r}.\hat{n}$$

As normal to a sphere at every point is along the radius vector at that point, therefore, $\hat{r}.\hat{n} = 1$

$$\vec{E}.\vec{ds} = \frac{q}{4\pi\epsilon_0}\frac{ds}{r^2}$$

Integrating over the closed surface area of the sphere, we get total normal electric flux over the entire spherical surface, $\phi_E = \oint_S \vec{E}.\vec{ds} = \frac{q}{4\pi\epsilon_0 r^2}\oint_S ds = \frac{q}{4\pi\epsilon_0 r^2} \times$ total area of spherical surface

$$= \frac{q}{4\pi\epsilon_0 r^2}(4\pi r^2) = \frac{q}{\epsilon_0}$$

Hence, $\boxed{\phi_E = \oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}}$, which proves Gauss's theorem.

If there are point charges, $q_1, q_2, q_3 \ldots\ldots q_n$ lying inside the surface, each will contribute to the electric flux, independent of the others (*superposition principle*),

$$\therefore\ \phi_E = \phi_{E_1} + \phi_{E_2} + \phi_{E_3} + \ldots\ldots \phi_{E_n} = \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} + \ldots \frac{q_n}{\epsilon_0} = \frac{1}{\epsilon_0}(q_1 + q_2 + q_3 + \ldots + q_n) = \frac{1}{\epsilon_0}\sum_{i=1}^{i=n} q_i$$

*Gaussian surface around a point charge or a continuous charge distribution is an imaginary closed surface at every point of which electric intensity is same. As Gaussian surface is an imaginary geometric surface, it may lie in empty space or it may be embedded in a solid body.

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$$\therefore \quad \boxed{\phi_E = \frac{Q}{\epsilon_0}}$$

where $Q = \sum_{i=1}^{i=n} q_i$ is the *algebraic sum of all the charges inside the closed surface.*

Hence, ***total electric flux over a closed surface in vacuum is*** $\frac{1}{\epsilon_0}$ ***time the total charge within the surface regardless of how the charges may be distributed.***

If the *medium* surrounding the charge has a dielectric constant K, then

$$\phi_E = \frac{Q}{K\epsilon_0} = \frac{Q}{\epsilon_r \epsilon_0} = \frac{Q}{\epsilon} \qquad \text{where} \quad K = \epsilon_r = \epsilon/\epsilon_0 \qquad \therefore \quad \boxed{\phi_E = \frac{Q}{\epsilon}}$$

If there is *no net charge* within the closed surface *i.e.*, when $Q = 0$, $\therefore \phi_E = 0$

It means that the ***total electric flux through a closed surface is zero if no charge is enclosed by the surface*** or **when algebraic sum of all the charges enclosed by the surface is zero.**

It can be shown that charges situated outside the closed surface make no contribution to total electric flux over the surface.

Retain in Memory

1. Gauss's theorem holds good for any closed surface, regardless of its shape or size.
2. The surface that we choose for the application of Gauss's law is called the Gaussian surface. However, we usually choose a spherical Gaussian surface, because this choice has three simplifying features : (*i*) The dot product $\vec{E} \cdot \vec{ds} = E\, ds \cos 0° = E\,(ds)$, because at all points on spherical Gaussian surface, $\theta = 0°$. (*ii*) Magnitude of $\vec{E}$ is constant at all points on the spherical Gaussian surface. Therefore, E can be brought out of the integral sign. (*iii*) The remaining integral is merely the surface area of the sphere ($= 4\pi r^2$), which is obtained without actually doing the integration.
3. Take care to see that the Gaussian surface chosen does not pass through any discrete charge. This is because electric field is not well defined at the location of the charge.
4. In the situation when the surface is so chosen that there are some charges inside and some outside the electric field $\vec{E}$ (whose flux is calculated) is due to all the charges, both inside and outside the closed surface. However, for electric flux, the term (q) represents only the total charge inside the closed surface.
5. Gauss's theorem is used most commonly for symmetric charge configurations.
6. Gauss's theorem is based on inverse square dependence of $\vec{E}$ on distance $\left(i.e.,\ E \propto \frac{1}{r^2}\right)$.
7. From Gauss's theorem, we can calculate the *number of electric lines of force that radiate outwards from one coulomb of positive charge in vacuum.* As

$$\phi_E = \frac{q}{\epsilon_0}, \text{ therefore, when } q = 1 \text{ coulomb, } \phi_E = \frac{1}{8{\cdot}85\times10^{-12}} = 1{\cdot}13\times10^{11}$$

Sample Problem 1 **The total electric flux emanating from a closed surface enclosing an alpha particle (e = electronic charge) is** **(Kerala CEE 2011)**

Sol. Charge of α particle, $q = 2e$

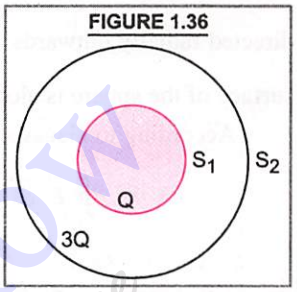
According to Gauss's theorem,

electric flux emanating from closed surface surrounding the alpha particle is $\phi = \frac{q}{\epsilon_0} = \frac{2e}{\epsilon_0}$

Sample Problem 2 **S_1 and S_2 are two hollow concentric spheres enclosing charges Q and $3Q$ respectively as shown in Fig. 1.36. What is the ratio of electric flux through S_1 and S_2? What would be electric flux through S_1? If air inside S_1 is replaced by a medium of dielectric constant 3?**

Sol. Electric flux through S_1, $\phi_1 = \frac{Q}{\epsilon_0}$

FIGURE 1.36



Electric flux through S_2, $\phi_2 = \frac{Q+3Q}{\epsilon_0} = \frac{4Q}{\epsilon_0}$ $\therefore$ $\frac{\phi_1}{\phi_2} = \frac{1}{4}$

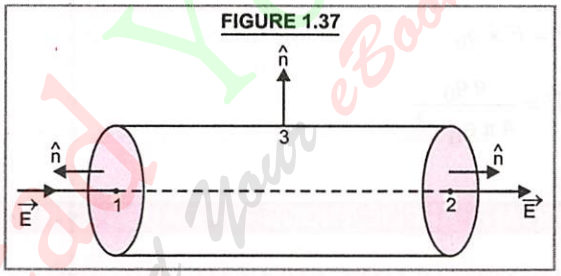
When air inside S_1 is replaced by a medium of $\epsilon_r = 3$,

then electric flux through $S_1 = \phi_1' = \frac{Q}{\epsilon} = \frac{Q}{\epsilon_0 \epsilon_r} = \frac{Q}{3\epsilon_0}$

1.40. SIMPLE VERIFICATION OF GAUSS'S THEOREM

To verify Gauss's theorem, let us calculate electric flux through a closed cylindrical surface containing no charge. Let the cylinder be held in an external uniform electric field of intensity $\vec{E}$ along the axis of the cylinder, as shown in Fig. 1.37.

FIGURE 1.37



Suppose ϕ_1 and ϕ_2 represent the electric flux through the surfaces 1 and 2 of area s_1 and s_2 of the cylinder and ϕ_3 is the electric flux through the curved surface 3 of the cylinder.

Now, outward normal to surface 1 is opposite to $\vec{E}$ and outward normal to surface 2 is along $\vec{E}$.

$\therefore$ $\phi_1 = -E s_1$ and $\phi_2 = E s_2$

As $s_1 = s_2 = s$ = area of circular cross section of the cylinder, therefore,

$$\phi_1 + \phi_2 = -E s_1 + E s_2 = -E s + E s = 0.$$

Also, normal to surface 3 at every point is perpendicular to $\vec{E}$.

$\therefore$ $\phi_3 = E s_3 \cos 90° = 0$

Hence, $\phi_1 + \phi_2 + \phi_3 = 0$, *i.e.*, total electric flux over the closed cylindrical surface containing no charge is zero. This verifies Gauss's theorem.

Sample Problem **A uniform electric field exists in space. Find the electric flux of the field through curved surface area of the cylinder with its axis parallel to the field.**

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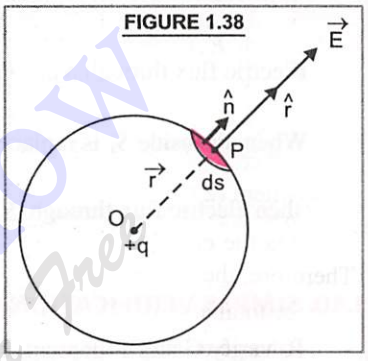
Sol. As normal to curved surface area of the cylinder is at 90° to electric field,

$$\therefore \qquad \phi = \int_S \vec{E}.\vec{ds} = \int_S E\,ds\cos 90° = \textbf{zero}$$

1.41. DEDUCTION OF COULOMB'S LAW FROM GAUSS'S THEOREM

Consider an isolated positive point charge q at O. Imagine a sphere of radius r with centre O, Fig. 1.38. The magnitude of electric field intensity $\vec{E}$ at every point on the surface of the sphere is the same and it is directed radially outwards. Further, the direction of vector $\vec{ds}$ representing a small area element ds on the surface of the sphere is along $\vec{E}$ only, *i.e.*, $\theta = 0°$



According to Gauss's theorem,

$$\oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0} \quad \text{or} \quad \oint_S E\,ds\cos 0° = \frac{q}{\epsilon_0}$$

or $$E\oint_S ds = \frac{q}{\epsilon_0} \quad \text{or} \quad E\,(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

This is the electric field intensity at any point P distant r from an isolated point charge q at the centre of the sphere. If another point charge q_0 were placed at P, then force on q_0 would be

$$F = E \times q_0$$

$$F = \frac{q\,q_0}{4\pi\epsilon_0 r^2} \qquad \text{...(56)}$$

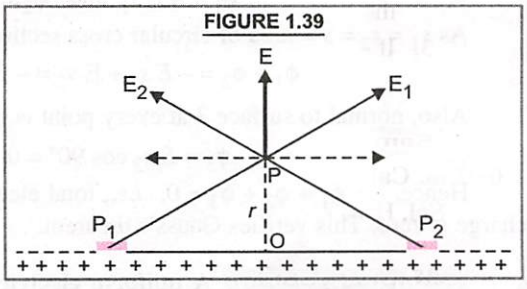
This is Coulomb's law.

IMPORTANT APPLICATIONS OF GAUSS'S THEOREM

1.42. FIELD DUE TO AN INFINITELY LONG STRAIGHT UNIFORMLY CHARGED WIRE

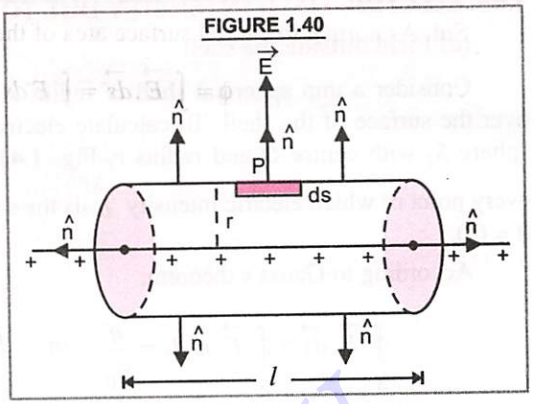
Consider an infinitely long thin wire with uniform linear charge density λ. To calculate field due to this wire at any point P, we consider a pair of line elements P_1 and P_2 of the wire at equal distances on either side of an arbitrary origin O.

Electric fields $\vec{E_1}$ and $\vec{E_2}$ at point P due to the two line elements are shown in Fig. 1.39. Their components normal to the radius vector $OP = r$ cancel out, being equal and opposite. However, the components along OP add. Therefore, resultant electric field is radial. This is true for any such pair of line elements. Therefore, total field at any point P is **radial.** As the wire is infinite, electric field does depend upon distance of point from the wire, but not on the position of P. Thus, electric field at every point in the plane cutting the wire normally is radial ; and its magnitude depends only on the radial distance r.



Let us consider a right circular closed cylinder of radius r and length l with the infinitely long line of charge as its axis, Fig. 1.40. The magnitude of electric intensity $\vec{E}$ at every point on the curved surface of the cylinder is the same, because all such points are at the same distance from the line charge.

FIGURE 1.40



Also, $\vec{E}$ and unit vector $\hat{n}$ along outward normal to curved surface are in the same direction, so that $\theta = 0°$.

$\therefore$ Electric flux over the curved surface of the cylinder,

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\, ds = \oint_S E\,(1)\cos 0°\, ds$$

$$= E \oint_S ds = E\,(2\pi\, rl)$$

where $(2\pi\, rl)$ is area of the curved surface of the cylinder.

On the ends of the cylinder, angle between electric field intensity $\vec{E}$ and outward normal $\hat{n}$ is 90°. Therefore, these ends make no contribution to electric flux of the cylinder.

$\therefore$ total electric flux over the whole cylinder, $\phi_E = E\,(2\pi\, rl)$

Charge enclosed in the cylinder = linear charge density × length

$$q = \lambda\, l$$

According to Gauss's theorem, $\phi_E = \dfrac{q}{\epsilon_0}$ $\quad\therefore\quad$ $E\,(2\pi\, rl) = \dfrac{\lambda l}{\epsilon_0}$

$$E = \frac{\lambda}{2\pi\epsilon_0\, r} \qquad \text{...(57)}$$

Clearly, $E \propto \dfrac{1}{r}$

If $\lambda > 0$, the direction of electric field at every point is **radially outwards.**

If $\lambda < 0$, the direction of electric field at every point is **radially inwards.**

Retain in Memory

1. In the above derivation, electric field $\vec{E}$ is due to charge on the entire wire of infinite length. However, the charge enclosed by the cylindrical Gaussian surface is only $\lambda \times l$.
2. The assumption that the wire is infinitely long is crucial. $\vec{E}$ is taken normal to the curved part of the cylindrical Gaussian surface only on account of this assumption.
3. If the wire is not infinitely long, we have to take into consideration the end effects.

Sample Problem **An infinite line charge produces a field of 9×10^4 N C^{-1} at a distance of 0·02 m. Calculate the linear charge density.**

Sol. Here, $E = 9 \times 10^4$ N C^{-1}, $r = 0{\cdot}02$ m, $\lambda = ?$

As $\quad E = \dfrac{\lambda}{2\pi\epsilon_0\, r} = \dfrac{2\lambda}{4\pi\epsilon_0\, r}$ $\quad\therefore\quad$ $\lambda = \dfrac{E \times 4\pi\epsilon_0\, r}{2} = 9\times10^4 \times \dfrac{1}{9\times10^9} \times \dfrac{0{\cdot}02}{2} = \mathbf{10^{-7}\ Cm^{-1}}$

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1.43. ELECTRIC FIELD INTENSITY DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

(a) Field outside the shell

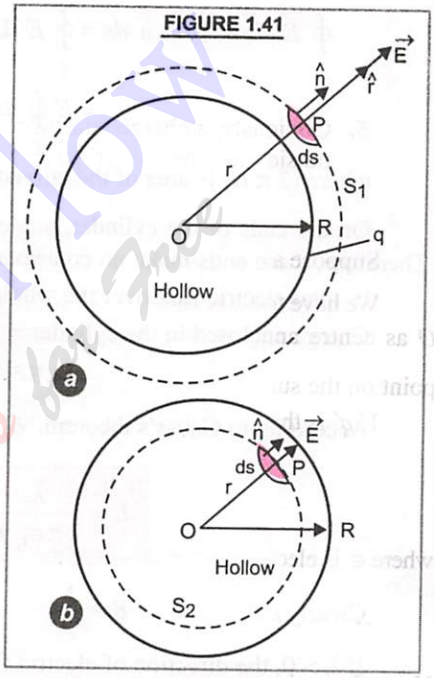
Consider a thin spherical shell of radius R with centre O. Let a charge $+q$ be distributed uniformly over the surface of the shell. To calculate electric field intensity at any point P, where $OP = r$, imagine a sphere S_1 with centre O and radius r, Fig. 1.41(*a*). The surface of this sphere is a Gaussian surface at every point of which electric intensity $\vec{E}$ is the same, directed radially outwards (as is unit vector $\hat{n}$, so that $\theta = 0°$).

According to Gauss's theorem,

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\, ds = \frac{q}{\epsilon_0} \quad \text{or} \quad E \oint_S ds = \frac{q}{\epsilon_0}$$

$$\therefore \; E \,.\, (4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} \qquad \text{...(58)}$$



This is exactly the field produced by a charge q placed at the centre O. Hence we conclude ***that for points outside the spherical shell, the field due to uniformly charged shell is as if the entire charge of the shell is concentrated at the centre of the shell.***

At a point on the surface of the shell

$r = R$, from (58), $E = q/4\pi\epsilon_0 R^2$ = Maximum

If σ is surface density of charge on the shell, then

$$q = 4\pi R^2 \,.\, \sigma$$

$$\therefore \; E = \frac{4\pi R^2 \sigma}{4\pi\epsilon_0 R^2} = \frac{\sigma}{\epsilon_0} = \text{constant}, \; i.e., \; E = \frac{\sigma}{\epsilon_0} \qquad \text{...(59)}$$

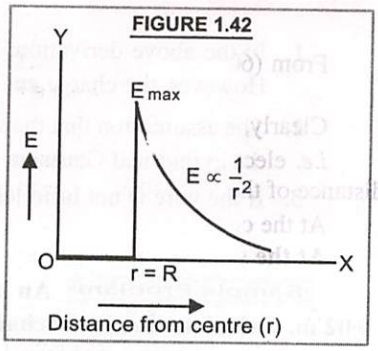
(b) Field inside the shell

In Fig. 1.41(*b*), the point P where we have to find electric intensity is inside the shell. The Gaussian surface is the surface of a sphere S_2 passing through P and with centre at O. The radius of sphere S_2 is $r < R$.

The electric flux through the Gaussian surface, as calculated above is $E \times 4\pi r^2$. As charge inside a spherical shell is zero, the Gaussian surface encloses no charge. The Gauss's theorem gives

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0} = 0 \quad \therefore \quad E = 0 \text{ for } r < R.$$

Hence, ***the field due to a uniformly charged spherical shell is zero at all points inside the shell.***

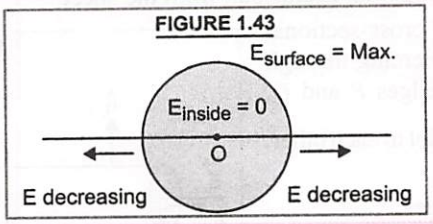


The variation of electric field intensity E with distance from the centre of a uniformly charged spherical shell is shown in Fig. 1.42.

Retain in Memory

1. When a uniform spherical shell carries charge Q, electric field at any point inside the shell is zero, Fig. 1.43.
2. The shell behaves as if the entire charge is concentrated at its centre.
3. Electric intensity at any point on the surface of the shell is maximum.
4. Outside the shell, E varies inversely as the square of distance from the centre.

FIGURE 1.43



5. Obviously, a charged particle inside the spherical shell will not experience any force. However, outside the shell, charged particle will be attracted or repelled.

1.44. ELECTRIC FIELD INTENSITY DUE TO A NON-CONDUCTING CHARGED SOLID SPHERE

Suppose a non conducting solid sphere of radius R and centre O has uniform volume density of charge ρ.

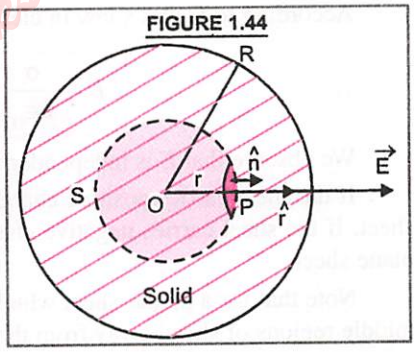
We have to calculate electric field intensity $\vec{E}$ at any point P inside the solid sphere, where $OP = r$. With O as centre and r as radius, imagine a sphere S, which acts as a Gaussian surface, Fig. 1.44. At every point on the surface of S, magnitude of $\vec{E}$ is same, directed radially outwards.

If q' is the charge enclosed by the sphere S, then according to Gauss's law

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\, ds = E\oint_S ds = \frac{q'}{\epsilon}$$

where ϵ is electrical permittivity of the material of the insulating sphere.

FIGURE 1.44



$$\therefore \quad E(4\pi r^2) = \frac{q'}{\epsilon} \quad \text{or} \quad \boxed{E = \frac{q'}{4\pi\epsilon r^2}} \quad \text{...(60)}$$

Now, charge inside S *i.e.* q' = volume of $S \times$ volume density of charge

$$q' = \frac{4}{3}\pi r^3 \times \rho$$

From (60), $$E = \frac{4}{3}\frac{\pi r^3 \rho}{4\pi\epsilon r^2} = \frac{r\rho}{3\epsilon}$$, *i.e.*, $$\boxed{E = \frac{r\rho}{3\epsilon}} \quad \text{...(61)}$$

Clearly, $E \propto r$

i.e. electric intensity at any point inside a non-conducting charged solid sphere varies directly as the distance of the point from the centre of the sphere.

At the **centre** of the sphere, $r = 0$, $\therefore \quad E = 0$

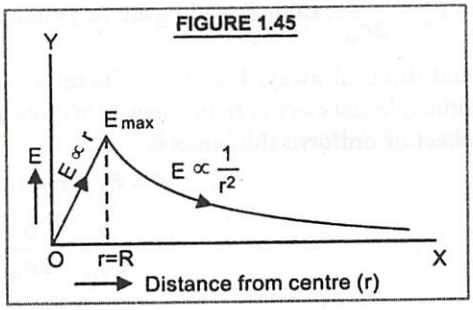
At the surface of the sphere, $r = R$

$$\therefore \quad E = \frac{R\rho}{3\epsilon} \Rightarrow \textbf{maximum.}$$

We have already proved that outside the sphere, $E \propto 1/r^2$.

All these results are plotted in Fig. 1.45, which represents the variation of electric field intensity E with distance (r) from the centre of a non conducting uniformly charged solid sphere.

FIGURE 1.45



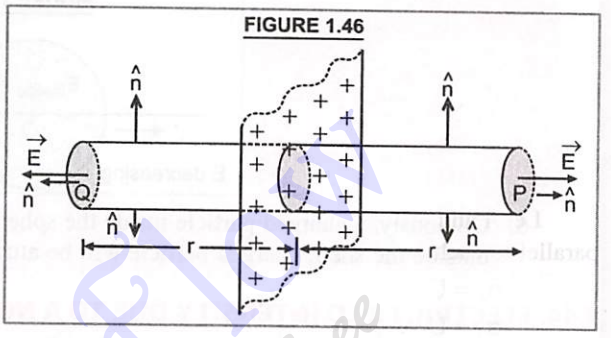
1.45. ELECTRIC FIELD INTENSITY DUE TO A THIN INFINITE PLANE SHEET OF CHARGE

Consider a thin, *infinite plane sheet* of charge. Let σ be the surface density of charge on the sheet (*i.e.* charge/area). We have to calculate electric field intensity $\vec{E}$ at any point P, (at a perpendicular distance) r from the sheet.

From symmetry, we find that $\vec{E}$ on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.

Let us imagine a cylinder of cross-sectional area ds around P and length 2 r, piercing through the sheet. At the two cylindrical edges P and Q, $\vec{E}$ and outward normal $\hat{n}$ are parallel to each other, Fig. 1.46.

FIGURE 1.46



∴ electric flux over these edges

$$= 2\vec{E}.\hat{n}\, ds = 2E\, ds$$

On the curved surface of the cylinder, $\vec{E}$ and outward normal $\hat{n}$ are ⊥ to each other. Therefore, no contribution to electric flux is made by the curved surface of the cylinder.

∴ Total electric flux over the entire surface of the cylinder, $\phi_E = 2\,E\,ds$

As area of the sheet intercepted by the cylinder is ds, therefore.

Total charge enclosed by the cylinder = $\sigma\, ds$

According to Gauss's law in electrostatics, $\phi_E = 2\,E\,ds = \frac{q}{\epsilon_0} = \frac{\sigma\, ds}{\epsilon_0}$

∴ $$E = \frac{\sigma}{2\epsilon_0} \qquad \text{...(62)}$$

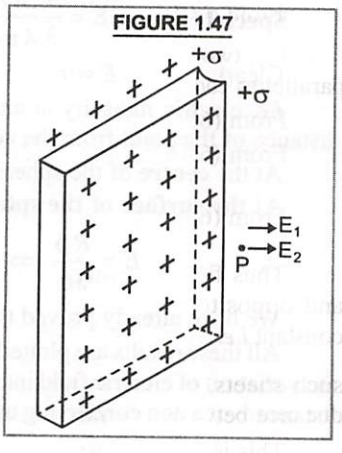
We observe that E is independent of r, the distance of the point from the plane charged sheet.

If the sheet carries positive charge, $\sigma > 0$. The electric field is uniform, normal and outwards from the sheet. If the sheet carries negative charge, $\sigma < 0$. The field is uniform and along the inward normal to the plane sheet.

Note that for a planar sheet which is large, but not infinite, eqn. (62) is only approximately true in the middle regions of sheet, away from the ends ; and at distances not far away from the sheet.

If the infinite plane sheet has uniform thickness, the surface density of charge σ is uniform and *same* on both the surfaces of the sheet.

FIGURE 1.47



The electric field intensity at any point P due to each surface is $E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$. Both E_1 and E_2 are perpendicular to the plane of the sheet and directed away, Fig. 1.47. Therefore, according to superposition principle, net electric field intensity at P due to an **infinite charged plane sheet of uniform thickness is**

$$E = E_1 + E_2$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \quad \therefore \quad E = \frac{\sigma}{\epsilon_0}$$

Note that the sheet is supposed to be of infinite size so that the '*edge effect*' due to distortion of electric field at the edges (of a finite sheet) can be ignored.

Sample Problem **A charge of $17{\cdot}7 \times 10^{-4}$ C is distributed over a large sheet of area 400 m^2. Calculate the electric field intensity at a distance of 10 cm from it.**

Sol. Here, $q = 17{\cdot}7 \times 10^{-4}$ C, $A = 400$ m^2, $E = ?$, $r = 10$ cm $= 10^{-1}$ m

In case of a large plane sheet, distance of the point ($= r$) does not matter.

$$E = \frac{\sigma}{2\epsilon_0} = \frac{q/A}{2\epsilon_0} = \frac{q}{2\epsilon_0 A} = \frac{17{\cdot}7\times10^{-4}}{2\times(8{\cdot}85\times10^{-12})\times400} = \mathbf{2{\cdot}5 \times 10^5\ N/C}$$

1.46. ELECTRIC FIELD INTENSITY DUE TO TWO THIN INFINITE PLANE PARALLEL SHEETS OF CHARGE

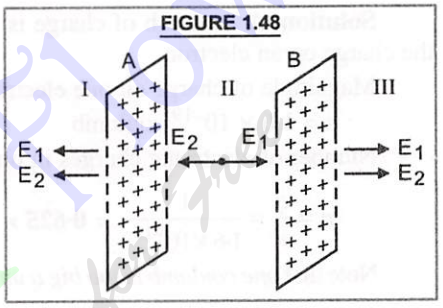
Let A and B be two thin infinite plane charged sheets held parallel to each other, Fig. 1.48. Suppose

σ_1 = Uniform surface density of charge on A

σ_2 = Uniform surface density of charge on B

If E_1, E_2 are electric field intensities at a point due to charged sheets A and B respectively, then

FIGURE 1.48



$$E_1 = \frac{\sigma_1}{2\epsilon_0} \quad \text{and} \quad E_2 = \frac{\sigma_2}{2\epsilon_0}.$$

The arrangement shows three regions I, II and III. We apply superposition principle to calculate the net field intensity in the three regions. As a matter of convention, a field pointing from left to right is taken as positive and the one pointing from right to left is taken as negative.

We assume that $\sigma_1 > \sigma_2 > 0$

$\therefore$ **In region I,** $E_I = -E_1 - E_2 \quad \therefore \quad E_I = \dfrac{-\sigma_1}{2\epsilon_0} - \dfrac{\sigma_2}{2\epsilon_0} = \dfrac{-1}{2\epsilon_0}(\sigma_1 + \sigma_2)$ ...(63)

Similarly, **in region II,** $E_{II} = E_1 - E_2 = \dfrac{\sigma_1}{2\epsilon_0} - \dfrac{\sigma_2}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$ ...(64)

and, **in region III,** $E_{III} = E_1 + E_2 = \dfrac{\sigma_1}{2\epsilon_0} + \dfrac{\sigma_2}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$ ...(65)

Special cases: Suppose $\sigma_1 = \sigma$, and $\sigma_2 = -\sigma$

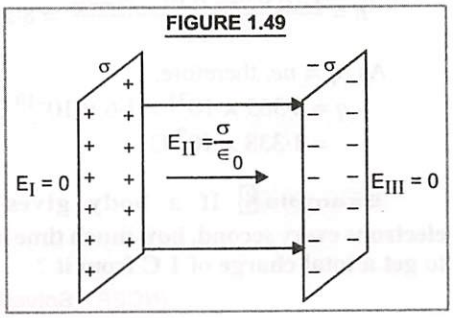
i.e., two thin infinite plane sheets with equal and opposite uniform surface densities of charge are held parallel to each other.

From (63), $E_I = 0$

From (65), $E_{III} = 0$

From (64), $E_{II} = \dfrac{2\sigma}{2\epsilon_0} = \dfrac{\sigma}{\epsilon_0} =$ constant.

FIGURE 1.49



Thus field intensity inbetween such sheets having equal and opposite uniform surface densities of charge becomes constant *i.e.*, a uniform electric field is produced inbetween two such sheets, Fig. 1.49. Also, $\vec{E}$ does not depend upon the distance between the thin sheets.

This is how uniform electric fields are produced in practice.

SOLVED EXAMPLES

TYPE I. QUANTIZATION OF CHARGE

Formula used. $q = \pm ne$

Units used. q and e are in coulomb, n is a number

Standard value. $e = 1{\cdot}6 \times 10^{-19}$ coulomb

Example 1 **Which is bigger, a coulomb or charge on an electron ? How many electronic charges form one coulomb of charge ?**

(Pb. Board 2011)

Solution. A coulomb of charge is bigger than the charge on an electron.

Magnitude of charge on one electron,

$e = 1{\cdot}6 \times 10^{-19}$ coulomb

Number of electronic charges in one coulomb,

$$n = \frac{q}{e} = \frac{1}{1{\cdot}6 \times 10^{-19}} = \mathbf{0{\cdot}625 \times 10^{19}}$$

Note that *one coulomb is too big a unit of charge.*

Example 2 **How much positive and negative charge is there in a cup of water ?**

(NCERT Solved Example)

Solution. Suppose the cup contains 250 cc of water (H_2O).

Mass of 250 cm^3 of water = 250 g.

Molecular weight of water = 2 + 16 = 18

Number of molecules in 18 g of water

$$= 6{\cdot}023 \times 10^{23}$$

Number of molecules in 250 g of water

$$= \frac{6{\cdot}023 \times 10^{23} \times 250}{18}$$

As each molecule of water contains 10 electrons, therefore, total number of electrons,

$$n = \frac{10 \times 6{\cdot}023 \times 10^{23} \times 250}{18} = 8{\cdot}365 \times 10^{25}$$

As $q = ne$, therefore,

$$q = 8{\cdot}365 \times 10^{25} \times 1{\cdot}6 \times 10^{-19}\ \text{C}$$

$$= \mathbf{1{\cdot}338 \times 10^{7}\ C}$$

Example 3 **If a body gives out 10^9 electrons every second, how much time is required to get a total charge of 1 C from it ?**

(NCERT Solved Example)

Solution. Here, $n = 10^9$ electrons/sec

Charge given/sec,

$$q = ne = 10^9 \times 1{\cdot}6 \times 10^{-19}\ \text{C}$$

$$= 1{\cdot}6 \times 10^{-10}\ \text{C}$$

Total charge, Q = 1 C

$$\therefore \text{Time required} = \frac{Q}{q} = \frac{1}{1{\cdot}6 \times 10^{-10}}\ \text{sec}$$

$$= 6{\cdot}25 \times 10^9\ s = \frac{6{\cdot}25 \times 10^9}{3600 \times 24 \times 365}\ \text{year}$$

$$= \mathbf{198{\cdot}18\ year}$$

Example 4 **Two bodies A and B carry charges – 3·00 μC and – 0·44 μC. How many electrons should be transferred from A to B so that they acquire equal charges?**

Solution. Here, $q_1 = -3{\cdot}00\ \mu C$

and $q_2 = -0{\cdot}44\ \mu C$

Let n electrons be transferred from A to B, when A and B would carry same charge.

$\therefore$ Charge on A = Charge on B

$$-3{\cdot}00 + ne = -0{\cdot}44 - ne$$

$$2ne = 3{\cdot}00 - 0{\cdot}44 = 2{\cdot}56\ (\mu C)$$

$$n = \frac{2{\cdot}56}{2e}$$

Taking $e = 1{\cdot}6 \times 10^{-19}\ \text{C} = 1{\cdot}6 \times 10^{-13}\ \mu C$

$$n = \frac{2{\cdot}56}{2 \times 1{\cdot}6 \times 10^{-13}} = 0{\cdot}8 \times 10^{13}$$

$$= \mathbf{8 \times 10^{12}}$$

TYPE II. COULOMB'S LAW

Formulae used. $F_0 = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$;

$$F_m = \frac{1}{4\pi\epsilon} \frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{|q_1||q_2|}{r^2}$$

Units used. q_1, q_2 are in coulomb, F in newton and r in metre.

Standard Values. $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{Nm}^2\ \text{C}^{-2}$

$$\epsilon_0 = 8{\cdot}85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

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Example 5 **A free pith ball P of 10 g carries a positive charge of 5×10^{-8} C. What must be the nature and magnitude of charge that should be given to another pith ball Q fixed 7 cm below the former ball, so that upper ball is stationary ?**

(Hr. Board 2001)

Solution. Here, $m = 10 \text{ g} = 10 \times 10^{-3}$ kg,

$q_1 = 5 \times 10^{-8}$ C, $r = 7 \text{ cm} = 7 \times 10^{-2}$ m

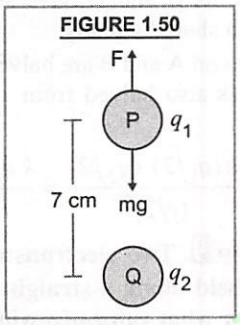
Second ball Q must carry positive charge so that force of repulsion balances the weight of ball P.

When ball remains stationary, Fig. 1.50,

$$F = mg$$

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = mg$$

FIGURE 1.50



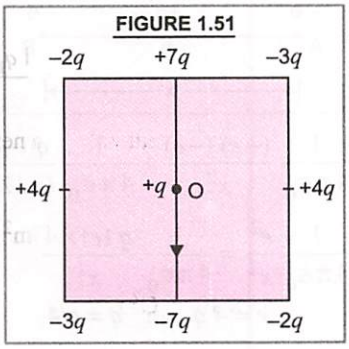
$$\frac{9\times10^9 \times 5\times10^{-8} \times q_2}{(7\times10^{-2})^2} = 10 \times 10^{-3} \times 9{\cdot}8$$

$$q_2 = \frac{10\times10^{-3}\times9{\cdot}8\times(7\times10^{-2})^2}{9\times10^9\times5\times10^{-8}}$$

$$= \mathbf{1{\cdot}067 \times 10^{-6}\ C}$$

Example 6 **A particle carrying charge $+q$ is held at the centre of a square of each side one metre. It is surrounded by eight charges arranged on the square as shown in Fig. 1.51. If $q = 2$ μC, what is the net force on the particle?**

FIGURE 1.51



Solution. As is clear from Fig. 1.51, forces on the particle at O due to $(-2q, -2q)$; $(-3q, -3q)$ and $(+4q, +4q)$ are equal and opposite. They cancel out in pairs. However, forces due to $+7q$ and $-7q$ add up. Therefore, net force on the particle at O is

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{(7q)(q) + 7q(q)}{(1/2)^2}$$

$$= \frac{9\times10^9 \times 14\,(2\times10^{-6})^2}{1/4}$$

$$= 36 \times 14 \times 4 \times 10^{-3} \text{ N}$$

$$= \mathbf{2{\cdot}016\ N}$$

Example 7 **Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses, both have inverse square dependence on the distance between the charges/masses (*a*) compare the strength of these forces by determining the ratio of their magnitudes (*i*) for an electron and a proton (*ii*) for two protons (*b*) estimate the accelerations for electron and proton due to electrical force of their mutual attraction when they are 1 Å apart.**

(NCERT Solved Example)

Solution. (*a*) (*i*) For an electron and proton

$$|F_e| = \frac{1}{4\pi\epsilon_0}\frac{e\times e}{r^2};$$

$$|F_g| = \frac{G m_e . m_p}{r^2}$$

$$\frac{|F_e|}{|F_g|} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{G m_e m_p}$$

$$= \frac{9\times10^9\,(1{\cdot}6\times10^{-19})^2}{6{\cdot}67\times10^{-11}\times9\times10^{-31}\times1{\cdot}66\times10^{-27}}$$

$$= \mathbf{2{\cdot}3 \times 10^{39}}$$

(*ii*) Similarly, for two protons,

$$\frac{|F_e|}{|F_g|} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{G.m_p m_p}$$

$$= \frac{9\times10^9\,(1{\cdot}6\times10^{-19})^2}{6{\cdot}67\times10^{-11}\times(1{\cdot}66\times10^{-27})^2}$$

$$= \mathbf{1{\cdot}3 \times 10^{36}}$$

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(*b*) Force of mutual attraction between an electron and a proton,

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{9\times10^9\,(1{\cdot}6\times10^{-19})^2}{(10^{-10})^2}$$

$$= 2{\cdot}3\times10^{-8}\ \text{N}$$

$$\text{Acceleration of electron} = \frac{F}{m_e} = \frac{2{\cdot}3\times10^{-8}}{9\times10^{-31}}$$

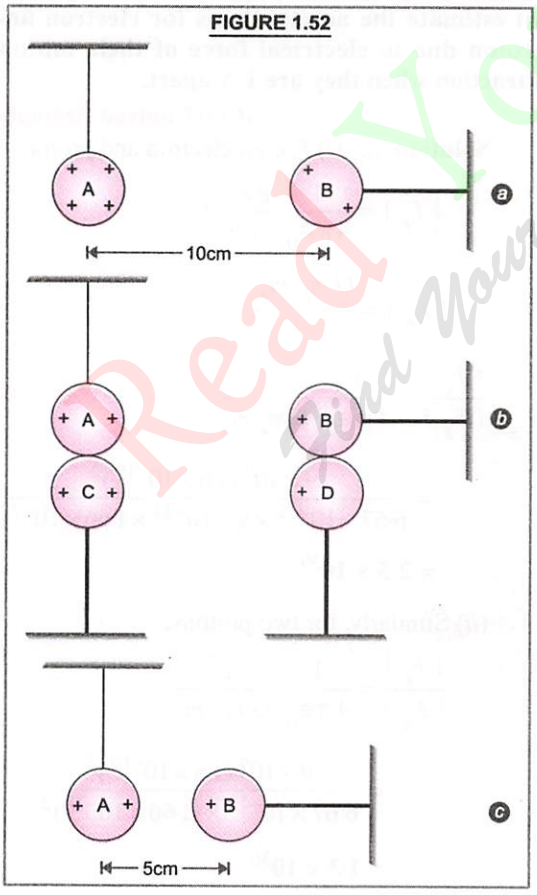
$$= 2{\cdot}5\times10^{22}\ \text{m/s}^2$$

$$\text{Acceleration of proton} = \frac{F}{m_p} = \frac{2{\cdot}3\times10^{-8}}{1{\cdot}66\times10^{-27}}$$

$$= 1{\cdot}3\times10^{19}\ \text{m/s}^2$$

Example 8 **A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B carried by an insulating handle is brought close to A such that the distance between their centres is 10 cm as shown in Fig. 1.52(*a*). The resulting repulsion of A is noted (for example, by shining a beam of light and measuring the deflection of its shadow on a calibrated screen).**

FIGURE 1.52



Spheres A and B are touched by uncharged spheres C and D respectively, as shown in Fig. 1,52(*b*). C and D are then removed and B is brought closer to A to a distance of 5·0 cm between their centres, as shown in Fig. 1.52(*c*). What is the expected repulsion of A on the basis of Coulomb's Law ? Spheres A and C and spheres B and D have identical sizes. Ignore the sizes of A and B in comparison to separation between their centres.

(NCERT Solved Example)

Solution. Let the original repulsive force between A and B be

$$F = \frac{k\,q_1\,q_2}{r^2}$$

As A and C have same size, charges are shared equally. Again, as B and D have same size, their charges are also shared equally.

As charges on A and B are halved, and distance between them is also halved from 10 cm to 5 cm, therefore,

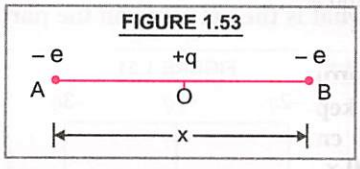
$$F' = \frac{k\,(q_1/2)\,(q_2/2)}{(r/2)^2} = \frac{k\,q_1\,q_2}{r^2} = F$$

Example 9 **Two electrons and a positive charge q are held along a straight line. At what position and for what value of q will the system be in equilibrium ? Check whether it is stable, unstable or neutral equilibrium.**

Solution. Let two electrons of charges $-e$ each be held at A and B. The third charge $+q$ must be placed at the centre O of AB. The forces on $+q$, due to two electrons being equal and opposite, cancel eachother and it is in equilibrium.

For the charge $(-e)$ at A to be in equilibrium, Fig. 1.53, force on *charge at A* due to $-e$ charge at B + force on charge at A due to $+q$ charge at O = Zero

FIGURE 1.53



$$\frac{1}{4\pi\epsilon_0}\frac{(-e)\,(-e)}{x^2} + \frac{1}{4\pi\epsilon_0}\frac{q\,(-e)}{(x/2)^2} = 0$$

$$\text{or}\quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q\,(e)\times4}{x^2}$$

$$\text{or}\quad e = 4\,q \quad \text{or} \quad q = e/4$$

If charge at O is moved slightly towards A, it would not return to O on its own and shall continue to move towards A. Hence ***equilibrium is unstable.***

Example 10 **Two point charges 4 μC and 1 μC are separated by a distance of 2 m in air. Find the point on the line joining the charges at which net electric field of the system is zero.**

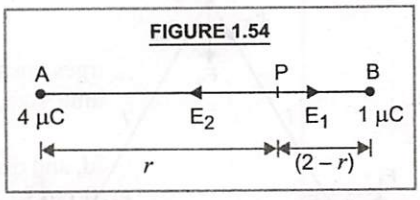
[CBSE 2017(C)]

Solution. Here, $q_1 = 4\ \mu C = 4 \times 10^{-6}$ C

$q_2 = 1\ \mu C = 10^{-6}$ C

$d = 2$ m

FIGURE 1.54



Let electric field of the system be zero at P, where $AP = r$, Fig. 1.54.

At P, $E_1 = E_2$

$$\therefore \quad \frac{q_1 \times 1}{4\pi\epsilon_0 r^2} = \frac{q_2 \times 1}{4\pi\epsilon_0 (2-r)^2}$$

$$\frac{4}{r^2} = \frac{1}{(2-r)^2} \quad \text{or} \quad \frac{2}{r} = \frac{1}{2-r}$$

$$r = 4 - 2r$$

$$3r = 4$$

$$r = \mathbf{4/3\ m}$$

TYPE III. SUPERPOSITION PRINCIPLE

Formula used. $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ...$

Units used. Force is in newton, when charges are in coulomb and distance is in metre.

Example 11 **Ten positively charged particles are kept fixed on x-axis at points $x = 10$ cm, 20 cm, 30 cm, ..., 100 cm. The first particle has charge $1{\cdot}0 \times 10^{-8}$ C, second 8×10^{-8} C, third 27×10^{-8} C, and so on. Tenth particle will have charge 1000×10^{-8} C. Find the magnitude of electric force acting on a 1C charge placed at the origin.**

Solution. By superposition principle, force on charge 1 C placed at origin

$$F_0 = F_{01} + F_{02} + ... + F_{010}$$

$$= \frac{q_0}{4\pi\epsilon_0}\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + ... + \frac{q_{10}}{r_n^2}\right]$$

$$= 9\times10^9 \times 1\left[\frac{1{\cdot}0\times10^{-8}}{(0{\cdot}10)^2} + \frac{8\times10^{-8}}{(0{\cdot}20)^2} + \frac{27\times10^{-8}}{(0{\cdot}30)^2} + ... \frac{1000\times10^{-8}}{(1{\cdot}00)^2}\right]$$

$$= 9 \times 10^9 \times 10^{-6}\ [1 + 2 + 3 + ... + 10]$$

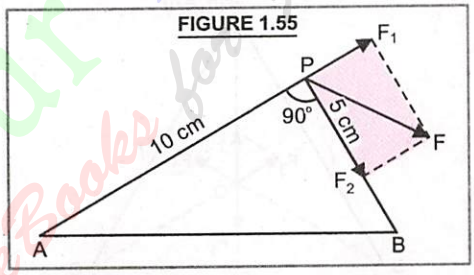
$$= 9 \times 10^3 \times 55 = \mathbf{4{\cdot}95 \times 10^5\ N}$$

Example 12 **Find the magnitude of the resultant force on a charge of 1 μC held at P due to two charges of $+ 2 \times 10^{-8}$ C and $- 10^{-8}$ C at A and B respectively.**

Given $AP = 10$ cm and $BP = 5$ cm.

$\angle APB = 90°$, Fig. 1.55.

FIGURE 1.55



Solution. Here, F = ?,

Charge at P, $q = 1\ \mu C = 10^{-6}$ C

Charge at A, $q_1 = + 2 \times 10^{-8}$ C

Charge at B, $q_2 = - 10^{-8}$ C

AP = 10 *cm* = 0·1 *m*, BP = 5 *cm* = 0·05 m

$\angle APB = 90°$

Force at P due to q_1 charge at A,

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q}{AP^2}, \text{ along } AP \text{ produced}$$

$$= \frac{9\times10^9 \times 2\times10^{-8} \times 10^{-6}}{(0{\cdot}1)^2} = 18 \times 10^{-3}\ \text{N}$$

Force at P due to q_2 charge at B,

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2\, q}{BP^2}, \text{ along } PB$$

$$= \frac{9\times10^9 \times 10^{-8} \times 10^{-6}}{(0{\cdot}05)^2} = 36 \times 10^{-3}\ \text{N}$$

As angle between $\vec{F}_1$ and $\vec{F}_2$ is 90°,

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$\therefore$ Resultant force, $F = \sqrt{F_1^2 + F_2^2}$

$= \sqrt{(18\times10^{-3})^2 + (36\times10^{-3})^2}$

$= 18 \times 10^{-3} \times 2{\cdot}236$

$= \mathbf{4{\cdot}0 \times 10^{-2}\ N}$

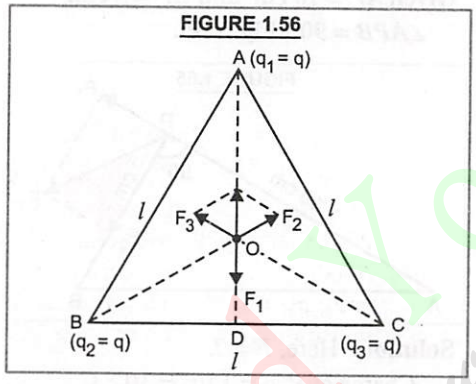
Example 13 **Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle ?**

(NCERT Solved Example)

Solution. As shown in Fig. 1.56, draw AD $\perp$ BC.

$\therefore$ $\quad AD = AB \cos 30° = \dfrac{l\sqrt{3}}{2}$

FIGURE 1.56



Distance AO of the centroid O from A

$$= \frac{2}{3}AD = \frac{2}{3}\frac{l\sqrt{3}}{2} = \frac{l}{\sqrt{3}}$$

$\therefore$ Force on Q at O due to charge $q_1 = q$ at A

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{Qq}{(l/\sqrt{3})^2} = \frac{3Qq}{4\pi\epsilon_0 l^2},\ \text{along AO}$$

Similarly, force on Q due to charge $q_2 = q$ at B

$$F_2 = \frac{3Qq}{4\pi\epsilon_0 l^2}\ \text{along BO}$$

and force on Q due to charge $q_3 = q$ at C

$$F_3 = \frac{3Qq}{4\pi\epsilon_0 l^2},\ \text{along CO}$$

Angle between forces F_2 and $F_3 = 120°$

By parallelogram law, resultant of F_2 and

$$F_3 = \frac{3Qq}{4\pi\epsilon_0 l^2}\ \text{along OA}$$

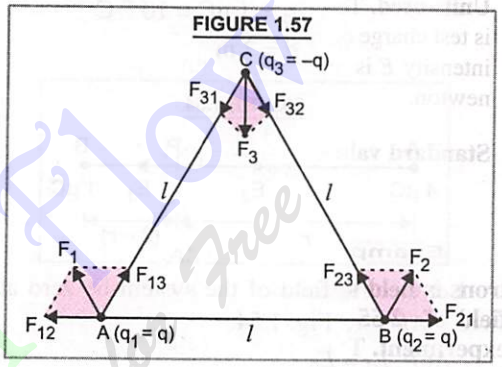
$\therefore$ Total force on $Q = \dfrac{3Qq}{4\pi\epsilon_0 l^2} - \dfrac{3Qq}{4\pi\epsilon_0 l^2} = 0$

Example 14 **Consider the charges q, q and $-q$ placed at the vertices of an equilateral triangle of each side l. What is the force on each charge ?**

(NCERT Solved Example)

Solution. As is clear from Fig. 1.57, force on $q_1 = q$ at A

FIGURE 1.57



$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} = F\,\hat{r_1}$$

where $F = \dfrac{qq}{4\pi\epsilon_0 l^2}$

and $\hat{r_1}$ = unit vector along BC

Force on $(q_2 = q)$ at B,

$$\vec{F_2} = \vec{F_{21}} + \vec{F_{23}} = F\,\hat{r_2},$$

where $\hat{r_2}$ = unit vector along AC

Force on $q_3 = -q$ at C,

$$\vec{F_3} = \vec{F_{31}} + \vec{F_{32}}$$

$$= [\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°}]\,\hat{n}$$

$$= \sqrt{3}\,F\,\hat{n}$$

where $\hat{n}$ = unit vector along the direction bisecting $\angle$BCA.

We can show that

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

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TYPE IV. ELECTRIC FIELD AND RELATION BETWEEN ELECTRIC INTENSITY AND FORCE

Formulae used.

1. $|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ **2.** $\vec{F} = q_0\vec{E}$

3. For calculating field due to any number of charges and continuous charge distributions, principle of super-position is applied.

Units used. The source charge q is in coulomb, so is test charge q_0. The distance r is in metre. Electric intensity E is in newton/coulomb and force F is in newton.

Standard value. $\frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{Nm}^2\ \text{C}^{-2}$

Example 15 **An oil drop of 12 excess electrons is held stationary under a constant electric field of $2{\cdot}55 \times 10^4$ NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1·26 g cm^{-3}. Estimate the radius of the drop.**

($g = 9{\cdot}81$ ms^{-2} ; $e = 1{\cdot}60 \times 10^{-19}$ C)

Solution. Here, $n = 12$,

$E = 2{\cdot}55 \times 10^4$ NC^{-1},

$\rho = 1{\cdot}26$ gm cm^{-3} $= 1{\cdot}26 \times 10^3$ kg m^{-3} ;

$r = ?$

As the droplet is stationary,

$\therefore$ weight of droplet = force due to electric field

$$\frac{4}{3}\pi r^3 \rho g = E ne \quad ;$$

$$r^3 = \frac{3Ene}{4\pi\rho g}$$

$$= \frac{3\times2{\cdot}55\times10^4\times12\times1{\cdot}6\times10^{-19}}{4\times3{\cdot}14\times1{\cdot}26\times10^3\times9{\cdot}81}$$

$$= 0{\cdot}94 \times 10^{-18}$$

$$\therefore \quad r = (0{\cdot}94 \times 10^{-18})^{1/3} = \mathbf{9{\cdot}81 \times 10^{-7}\ m}$$

Example 16 **A small sphere of mass 1 g carries a charge of + 6 μC. The sphere is suspended by a string in an electric field of 400 NC^{-1} acting downwards. Calculate tension in the string. What will be the tension if charge on the sphere were – 6 μC ?**

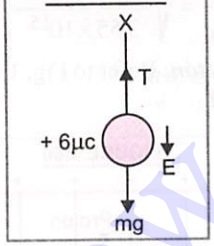
Solution. Here, $m = 1\ g = 10^{-3}$ kg,

$q = +6\ \mu\text{C} = +6 \times 10^{-6}$ C

$E = 400$ NC^{-1}, downwards.

As is clear from Fig. 1.58,

FIGURE 1.58



$$T = F_e + mg = qE + mg$$
$$= 6 \times 10^{-6} \times 400 + 10^{-3} \times 9{\cdot}8$$
$$= 122 \times 10^{-4}\ \text{N} = 1{\cdot}22 \times 10^{-2}\ \text{N}$$

However, when $q = -6\ \mu$C, force due to electric field is upwards.

$\therefore T + F_e = mg$

or $\quad T = mg - F_e = 9{\cdot}8 \times 10^{-3} - 24 \times 10^{-4}$

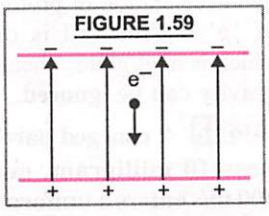
$$= \mathbf{74 \times 10^{-4}\ N}$$

Example 17 **An electron falls through a distance of 1·5 cm in a uniform electric field of value 2×10^4 N/C. When the direction of electric field is reversed, a proton falls through the same distance. Compare the time of fall in each case. Contrast the situation with that of 'free fall under gravity'.** **[CBSE 2018 (C)]**

(NCERT Solved Example)

Solution. (*a*) ***For electron,*** Refer to Fig. 1.59

FIGURE 1.59



$y_1 = 1{\cdot}5$ cm $= 1{\cdot}5 \times 10^{-2}$ m, $E_1 = 2 \times 10^4$ N/C

$q_0 = (-)\ 1{\cdot}6 \times 10^{-19}$ coulomb, $m_1 = 9 \times 10^{-31}$ kg.

$$\text{Acceleration, } a_1 = \frac{F_1}{m_1} = \frac{q_0 E_1}{m_1}$$

$$= \frac{1{\cdot}6\times10^{-19}\times2\times10^4}{9\times10^{-31}}$$

$$= 3{\cdot}55 \times 10^{15}\ \text{m/s}^2$$

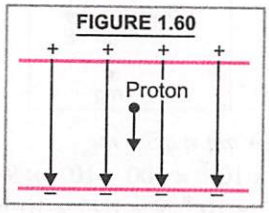
From $y_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2$

$$y_1 = 0 + \frac{1}{2} a_1 t_1^2$$

or $t_1 = \sqrt{2\frac{y_1}{a_1}} = \sqrt{\frac{2\times1{\cdot}5\times10^{-2}}{3{\cdot}55\times10^{15}}} = \mathbf{2{\cdot}9 \times 10^{-9}}$ **sec.**

(b) ***For proton,*** Refer to Fig. 1.60, when electric field is reversed.

FIGURE 1.60



Charge $q_0 = +1{\cdot}6 \times 10^{-19}$ *C,*

$m_2 = 1{\cdot}67 \times 10^{-27}$ kg.

$\therefore$ Acceleration, $a_2 = \frac{F_2}{m_2} = \frac{q_0 E_2}{m_2}$

$$a_2 = \frac{1{\cdot}6\times10^{-19}\times2\times10^4}{1{\cdot}67\times10^{-27}} = 1{\cdot}92 \times 10^{12} \text{ m/s}^2$$

Similarly, $t_2 = \sqrt{\frac{2\,y_2}{a_2}} = \sqrt{\frac{2\times1{\cdot}5\times10^{-2}}{1{\cdot}92\times10^{12}}}$

$= 1{\cdot}25 \times 10^{-7}$ s

and $\frac{t_1}{t_2} = \frac{2{\cdot}9\times10^{-9}}{1{\cdot}25\times10^{-7}} = \mathbf{2{\cdot}3 \times 10^{-2}}$

We observe that acceleration of electron $\approx 10^{15}$ m/s^2 and acceleration of proton $\approx 10^{12}$ m/s^2. The value of 'g' in free fall is only 9·8 m/s^2 $\approx 10^1$ m/s^2, which is negligible. Therefore, **effect of acc. due to gravity can be ignored.**

Example 18 **A charged particle of charge 2 μC and mass 10 milligram, moving with a velocity of 1000 m/s enters a uniform electric field of strength 10^3 N/C directed perpendicular to its direction of motion. Find the velocity and displacement of the particle after 10 s.**

(CBSE Sample Paper 2011)

Solution. Here, $q = 2\ \mu C = 2 \times 10^{-6}$ C,

$m = 10$ mg $= 10^{-5}$ kg

$v_x = 1000$ m/s, $E_y = 10^3$ N/C

$$v_y = u_y + a_y t = 0 + \frac{q E_y t}{m}$$

$$= \frac{2\times10^{-6}\times10^3\times10}{10^{-5}} = 2000 \text{ m/s}$$

As v_x remains constant, therefore, net velocity after 10 s

$$= \sqrt{v_x^2 + v_y^2} = \sqrt{(1000)^2 + (2000)^2}$$

$$= \mathbf{1000\sqrt{5}\ m/s}$$

Displacement along x-axis after 10 s

$x = v_x t = 1000 \times 10 = 10000$ m

Displacement along y-axis after 10 s

$$y = u_y \times t + \frac{1}{2}\frac{q E_y}{m} t^2$$

$$= 0 + \frac{1}{2} \times \frac{2\times10^{-6}\times10^3}{10\times10^{-6}} (10^2) = 10000 \text{ m}$$

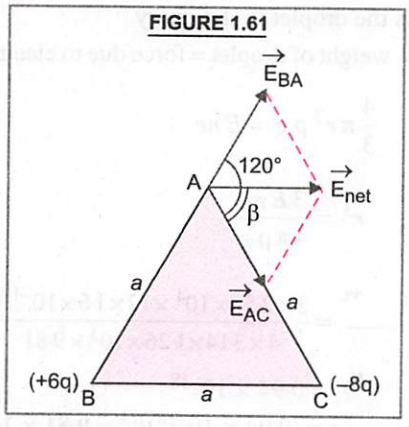
Net displacement $= \sqrt{x^2 + y^2}$

$$= \sqrt{(10000)^2 + (10000)^2} = \mathbf{10000\sqrt{2}\ m}$$

Example 19 **Two point charges + 6 q and − 8 q are placed at the vertices B and C of an equilateral triangle ABC of side a. Obtain an expression for magnitude and direction of resultant electric field at the vertex A due to these two charges.** **[CBSE 2014 (C)]**

Solution. As is clear from Fig. 1.61.

FIGURE 1.61



$$E_{BA} = \frac{1}{4\pi\epsilon_0}\frac{6q\times1}{a^2} = 6E$$

$$E_{AC} = \frac{1}{4\pi\epsilon_0}\frac{8q}{a^2} = 8E$$

where $E = \frac{q}{4\pi\epsilon_0 a^2}$

Magnitude of resultant field is

$$E_{net} = \sqrt{E_{BA}^2 + E_{AC}^2 + 2E_{BA}E_{AC}\cos 120^\circ}$$

$$= \sqrt{(6E)^2 + (8E)^2 + 2\times 6E\times 8E\left(-\frac{1}{2}\right)}$$

$$E_{net} = E\sqrt{52} = \frac{q\sqrt{52}}{4\pi\epsilon_0 a^2}$$

Let the resultant field make an angle β with AC.

$$\tan\beta = \frac{E_{BA}\sin 120^\circ}{E_{AC} + E_{BA}\cos 120^\circ}$$

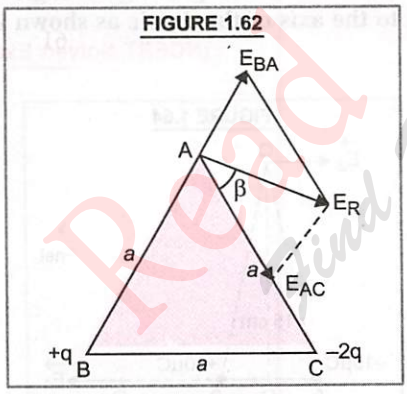
$$= \frac{6E\sqrt{3}/2}{8E + 6E\left(-\frac{1}{2}\right)} = \frac{3\sqrt{3}}{5}$$

$$\beta = \tan^{-1}\left(\frac{3\sqrt{3}}{5}\right)$$

Example 20 **Two point charges + q and – 2 q are placed at the vertices B and C of an equilateral triangle ABC of side a. Obtain the expression for magnitude and direction of resultant electric field at the vertex A due to these two charges.** **[CBSE 2014 (C)]**

Solution. In Fig. 1.62, imagine a charge of + 1 C at A.

FIGURE 1.62



$$E_{BA} = \frac{1}{4\pi\epsilon_0}\frac{q\times 1}{a^2} = E, \text{ along } BAE$$

$$\text{and } E_{AC} = \frac{-2q}{4\pi\epsilon_0 a^2} = -2E, \text{ along } AC$$

$$\theta = 120^\circ$$

Resultant electric field at A

$$E_R = \sqrt{E_{BA}^2 + E_{AC}^2 + 2E_{BA}.E_{AC}\cos 120^\circ}$$

$$= \sqrt{E^2 + (-2E)^2 + 2E(2E)\left(-\frac{1}{2}\right)}$$

$$E_R = E\sqrt{3} = \frac{q\sqrt{3}}{4\pi\epsilon_0 a^2}$$

If β is angle which E_R makes with AC, then

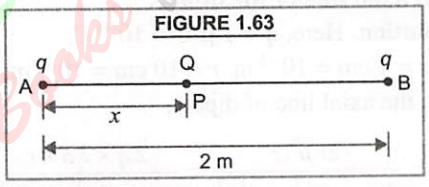
$$\tan\beta = \frac{E_{BA}\sin\theta}{E_{AC} + E_{BA}\cos\theta} = \frac{E\sin 120^\circ}{2E + E\cos 120^\circ}$$

$$= \frac{E\sqrt{3}/2}{E\left(2 - \frac{1}{2}\right)} = \frac{1}{\sqrt{3}} \quad \therefore \beta = 30^\circ$$

Example 21 **Two identical point charges, q each, are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q.** **(CBSE 2019)**

Solution. In Fig. 1.63, two identical point charges, q each are placed at A, B, where AB = 2 m. Let a third point charge Q be placed at P on the line AB such that $AP = x$. Therefore, $BP = (2 - x)$.

FIGURE 1.63



The charge Q will be in equilibrium when it is repelled/attracted equally by the two charges at A and B. Therefore, it must be at the centre of AB. it may be positive or negative.

Again, charge q at B will be in equilibrium, when

$$\frac{kQq}{1^2} + \frac{kqq}{2^2} = 0 \quad \text{or} \quad \frac{Q}{1} + \frac{q}{4} = 0 \quad \text{or } Q = -q/4$$

TYPE V. ELECTRIC DIPOLE, DIPOLE MOMENT, DIPOLE FIELD

Formulae used.

1. Dipole moment $|\vec{p}| = q\times 2a$, where $2a$ = distance between two point charges of dipole.

2. Dipole field intensity on axial line of dipole

$$|\vec{E}_1| = \frac{2|\vec{p}|r}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

where r is distance of the point from the centre of the dipole.

3. Dipole field intensity on equatorial line of dipole

$$|\vec{E}_2| = \frac{|\vec{p}|}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}}$$

4. Dipole field intensity at any point due to a short dipole is $|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$,

and $\tan\alpha = \frac{1}{2}\tan\theta$,

where θ is the angle which the line joining the point to centre of dipole makes with the axis of dipole.

And α is the angle at which $\vec{E}$ is inclined to the line joining the point to the centre of dipole.

Units used. q in coulomb, $2a$ in metre, $|\vec{p}|$ in coulomb metre (C-m), r in metre, E_1, E_2 are in newton/coulomb

Example 22 **Two charges each of 1 μ C but opposite in sign are 1 cm apart. Calculate electric field at a point distant 10 cm from the mid point on axial line of the dipole.**

Solution. Here, $q = 1\ \mu C = 10^{-6}$ C,

$2a = 1$ cm $= 10^{-2}$ m, $r = 10$ cm $= 10^{-1}$ m

On the axial line of dipole,

$$E = \frac{2|\vec{p}|r}{4\pi\epsilon_0 (r^2 - a^2)^2} = \frac{2q \times 2a \times r}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

$$= \frac{9\times10^9 \times 2\times10^{-6} \times 10^{-2} \times 10^{-1}}{(10^{-2} - 0{\cdot}25\times10^{-4})^2}$$

$$= \frac{18}{10^{-4}(1 - 0{\cdot}0025)^2}$$

$$E = \frac{18\times10^4}{0{\cdot}9975\times0{\cdot}9975} = \mathbf{18 \times 10^4\ N/C}$$

Example 23 **Two charges + 20 μ C and − 20 μ C are held 1 cm apart. Calculate the electric field at a point on the equatorial line at a distance of 50 cm from the centre of the dipole.**

Solution. Here, $q = \pm 20\ \mu C = \pm 20 \times 10^{-6}$ C,

$2a = 1$ cm $= 10^{-2}$ m, $r = 50$ cm $= \frac{1}{2}$ m

As $2a << r$, therefore, intensity on equatorial line of short dipole is

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3} = \frac{q\times2a}{4\pi\epsilon_0 r^3}$$

$$= \frac{9\times10^9 \times 20\times10^{-6} \times 10^{-2}}{(1/2)^3}$$

$$E = \mathbf{1{\cdot}44 \times 10^4\ N/C}$$

Example 24 **What is the magnitude of electric intensity due to a dipole of moment 2×10^{-8} C-m at a point distant 1 m from the centre of dipole, when line joining the point to the centre of dipole makes an angle of 60° with dipole axis ?**

Solution. Here, $p = 2 \times 10^{-8}$ C-m, $r = 1$ m,

$\theta = 60°$, $E = ?$

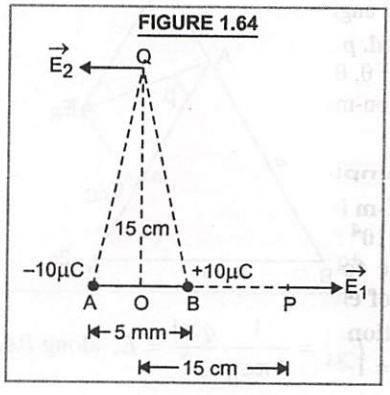
$$E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

$$= \frac{2\times10^{-8} \times 9\times10^9}{1^3}\sqrt{3(\cos 60°)^2 + 1}$$

$$E = \mathbf{238{\cdot}1\ N/C}$$

Example 25 **Two charges ± 10 μ C are placed 5·0 mm apart. Determine the electric field at (*a*) a point P on the axis of dipole 15 cm away from its centre on the side of the positive charge, as shown in Fig. 1.64 and at (*b*) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole as shown in Fig. 1.64.** (NCERT Solved Example)

FIGURE 1.64



Solution. Here, $q = \pm 10\ \mu C = \pm 10^{-5}$ C

$2a = 5{\cdot}0$ mm $= 5 \times 10^{-3}$ m

$r = OP = 15$ cm $= 15 \times 10^{-2}$ m

$|\vec{p}| = q \times 2a = 10^{-5} \times 5 \times 10^{-3}$

$= \mathbf{5 \times 10^{-8}\ C\text{-}m}$

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(*a*) As P lies on axial line of dipole,

$$\therefore \quad E_1 = \frac{2|\vec{p}|r}{4\pi\epsilon_0 (r^2 - a^2)^2}, \text{ along } BP$$

As $a << r$, $\therefore$

$$E_1 = \frac{2|\vec{p}|}{4\pi\epsilon_0 r^3} = \frac{2\times5\times10^{-8}\times9\times10^9}{(15\times10^{-2})^3}$$

$$= \mathbf{2{\cdot}67 \times 10^5\ N/C, \text{ along } BP}$$

(*b*) As Q lies on equatorial line of dipole,

$$\therefore \quad E_2 = \frac{|\vec{p}|}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}} = \frac{|\vec{p}|}{4\pi\epsilon_0 r^3}$$

$(\because\ a << r)$

$$= \frac{1}{2}E_1 = \frac{1}{2}\times2{\cdot}67\times10^5 = \mathbf{1{\cdot}33 \times 10^5\ N/C}$$

$\vec{E_2}$ **is along a line parallel to** BA as shown in Fig. 1.64.

TYPE VI. TORQUE ON DIPOLE AND POTENTIAL ENERGY OF DIPOLE IN ELECTRIC FIELD

Formulae used. 1. Torque, $\tau = pE \sin\theta$, where θ is angle between $\vec{p}$ and $\vec{E}$

2. Potential energy of dipole,

$$U = -pE(\cos\theta_2 - \cos\theta_1)$$

where θ_1 is initial angle between $\vec{p}$ and $\vec{E}$, and θ_2 is final angle between $\vec{p}$ and $\vec{E}$.

Units used. p in coulomb-metre, E in newton/coulomb ; θ, θ_1, θ_2 in degrees ; τ in newton-metre and U in joule.

Example 26 **An electric dipole of moment 5×10^{-8} C-m is aligned in a uniform electric field of $1{\cdot}44 \times 10^4$ N/C. Calculate potential energy of the dipole to hold the dipole at 60° with the direction of electric field.**

Solution. Here, $p = 5 \times 10^{-8}$ *C-m*, $\theta_1 = 0°$,

$E = 1{\cdot}44 \times 10^4$ N/C, $\theta_2 = 60°$, *P.E.* = ?

$$P.E. = -pE(\cos\theta_2 - \cos\theta_1)$$

$$= -5\times10^{-8}\times1{\cdot}44\times10^4(\cos60° - \cos0°)$$

$$P.E. = -7{\cdot}2\times10^{-4}(0{\cdot}5 - 1)$$

$$= \mathbf{3{\cdot}6 \times 10^{-4}\ J}$$

Example 27 **An electric dipole of dipole moment 4×10^{-5} Cm is placed in a uniform electric field of 10^{-3} N/C making an angle of 30° with the direction of the field. Determine the torque exerted by the electric field on the dipole.**

(Hr. Board 2002)

Solution. Here, $p = 4 \times 10^{-5}$ C.m,

$E = 10^{-3}$ N/C, $\theta = 30°$

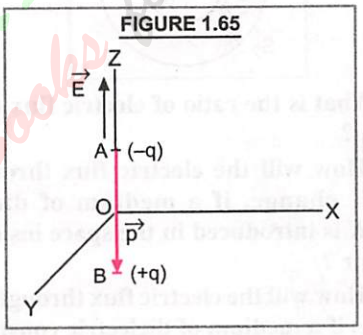
$$\tau = pE\sin\theta = (4\times10^{-5})\ 10^{-3}\sin30°$$

$$= \mathbf{2 \times 10^{-8}\ N\text{-}m}$$

Example 28 **In a certain region of space, electric field is along the Z-direction throughout. The magnitude of electric field is, however, not constant, but increases uniformly along the positive Z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C-m in the negative Z-direction ?**

Solution. Consider an electric dipole with $-q$ charge at A and $+q$ charge at B, placed along Z-axis, so that its dipole moment is in negative Z direction, , *i.e.*, $p_z = -10^{-7}$ C-m, Fig. 1.65.

FIGURE 1.65



The electric field is along + direction of Z-axis, such that

$$\frac{dE}{dz} = 10^5\ NC^{-1}\ m^{-1}$$

From, $F = q\,dE = q \times \frac{dE}{dz} \times dz$

$$= (q \times dz) \times \frac{dE}{dz} = p\frac{dE}{dz}$$

$$\therefore \quad F = -10^{-7} \times 10^5 = \mathbf{-10^{-2}\ N}$$

i.e., the force on the dipole is along negative direction of Z-axis.

Now, $\theta = 180°$.

$\therefore$ torque on dipole $\tau = pE\sin180° = \mathbf{0}$

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TYPE VII. ELECTRIC FLUX AND GAUSS'S THEOREM IN ELECTROSTATICS

Formulae used.

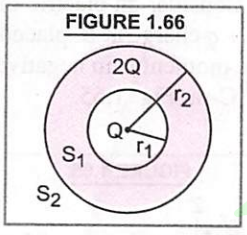
1. $\phi_E = \oint_S \vec{E}.\vec{ds} = E\,s\cos\theta$, where θ is angle which normal to the area makes with $\vec{E}$.

2. $\phi_E = \oint \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0}$, where Q is total charge (*i.e.* algebraic sum of all the charges) inside the closed surface.

Units used. E is in NC^{-1}; s in m^2; ϕ_E in $NC^{-1}\,m^2$.

Example 29 **S_1 and S_2 are two concentric spheres enclosing charges Q and $2Q$ respectively as shown in Fig.1.66.**

FIGURE 1.66



(*i*) What is the ratio of electric flux through S_1 and S_2 ?

(*ii*) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant K is introduced in the space inside S_1 in place of air ?

(*iii*) How will the electric flux through sphere S_1 change, if a medium of dielectric constant K is introduced in the space inside S_2 in place of air ? (CBSE 2014)

Solution. (*i*) From Gauss's theorem

electric flux through S_1 is $\phi_1 = \frac{Q}{\epsilon_0}$

electric flux through S_2 is $\phi_2 = \frac{2Q+Q}{\epsilon_0} = \frac{3Q}{\epsilon_0}$

$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{1}{3}$

(*ii*) When a medium of dielectric constant K is introduced in the space inside S_1, then

$$\phi_1' = \oint \vec{E'}.\vec{ds} = \oint \frac{\vec{E}}{K}.\vec{ds} = \frac{1}{K}\frac{Q}{\epsilon_0}$$

(*iii*) On introducing dielectric medium inside S_2, electric flux through S_1 **will not change.**

Example 30 **Consider a uniform electric field $\vec{E} = 3\times10^3\,\hat{i}\ NC^{-1}$. Calculate the flux of this field through a square surface of area 10 cm square**

(*i*) when its plane is parallel to Y-Z plane.

(*ii*) when the normal to its plane makes an angle of 60° with X-axis. (CBSE 2013)

Solution. Here, $E = 3\times10^3\,\hat{i}$ N/C;

$A = (0{\cdot}1\ m)^2 = 10^{-2}\ m^2$

(*i*) As normal to area is in the direction of electric field, therefore, $\theta = 0°$

$\phi = EA\cos\theta = 3\times10^3\times10^{-2}\cos 0°$

$= 30\ Nm^2\ C^{-1}$

(*ii*) In this case, $\theta = 60°$

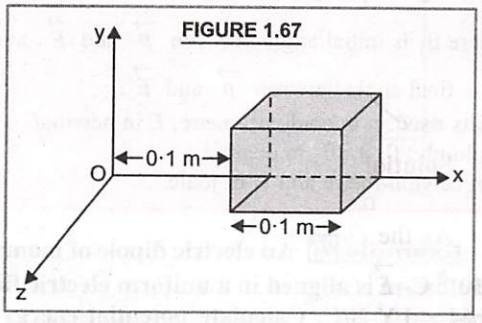
$\therefore\ \phi' = EA\cos\theta = 3\times10^3\times10^{-2}\cos 60°$

$= 15\ Nm^2\ C^{-1}$

Example 31 **The electric field components due to a charge inside the cube of side 0·1 m are $E_x = \alpha x$, where $\alpha = 500$ (N/C) m^{-1}, $E_y = 0$, $E_z = 0$. Calculate the flux through the cube and the charge inside the cube.** [CBSE Sample Paper 2019-20, CBSE (OD) 2008, 2018 (C)]

Solution. As is clear from Fig.1.67, flux is only along X-axis.

FIGURE 1.67



Through the left face

$\phi_1 = E_x \cdot A\cos 180°$

$= 500\times0{\cdot}1\times10^{-2}\,(-1) = -0{\cdot}5$

Through the right face

$\phi_2 = E_x \cdot A\cos 0°$

$= 500\,(0{\cdot}2)\times10^{-2}\times1 = 1{\cdot}0$

$\therefore$ Net flux through the cube

$\phi = \phi_1 + \phi_2 = -0{\cdot}5 + 1 = \mathbf{0{\cdot}5\ Nm^2\ C^{-1}}$

Charge inside the cube $= \epsilon_0\,\phi$

$= 8{\cdot}85\times10^{-12}\times0{\cdot}5 = \mathbf{4{\cdot}425\times10^{-12}\ C.}$

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Example 32 **A uniformly charged conducting sphere of 2·4 m diameter has a surface charge density of 180·0 μC/m² (*i*) Find the charge on the sphere. (*ii*) What is the total flux leaving the surface of the sphere ?** (CBSE 2009)

Solution. Here, $R = \frac{2{\cdot}4}{2} = 1{\cdot}2$ m

$\sigma = 180{\cdot}0\ \mu C/m^2 = 180 \times 10^{-6}\ Cm^{-2}$

(*i*) Charge on the sphere $q = 4\pi R^2 \sigma$

$= 4 \times 3{\cdot}14 \times (1{\cdot}2)^2 \times 180 \times 10^{-6}$

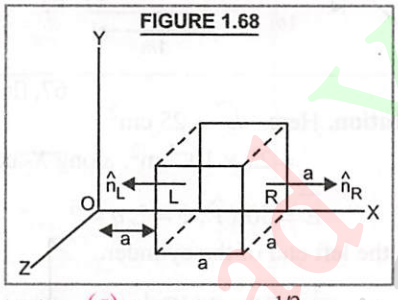
$= \mathbf{3{\cdot}53 \times 10^{-3}\ C}$

(*ii*) Electric flux, $\phi_E = \frac{q}{\epsilon_0} = \frac{3{\cdot}53 \times 10^{-3}}{8{\cdot}85 \times 10^{-12}}$

$= \mathbf{3{\cdot}9 \times 10^8\ Nm^2\ C^{-1}}$

Example 33 **The electric field components in Fig. 1.68 are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$ in which $\alpha = 800$ N/C–m$^{1/2}$. Consider the cube shown in Fig. 1.67. Calculate (*a*) the flux ϕ_E through the cube, and (*b*) the charge within the cube. Assume that $a = 0{\cdot}1$ m.** [CBSE 2018 (C)] (NCERT Solved Example)

FIGURE 1.68



Solution. (*a*) Here, $E_x = \alpha x^{1/2}$, $E_y = 0$, $E_z = 0$

$\alpha = 800$ N/C–m$^{1/2}$; $a = 0{\cdot}1$ m.

As the electric field has only x component, therefore, $\vec{E}.\Delta\vec{S} = \phi_E = 0$ for each of four faces of cube ⊥ to Y-axis and Z-axis. Flux is there only for left face L and right face R of the cube shown in Fig. 1.68.

At the left face, $x = a$

$\therefore \quad E_L = \alpha a^{1/2}$

$\phi_L = \vec{E}_L.\Delta\vec{S} = \alpha a^{1/2}(a^2)\cos 180° = -\alpha a^{5/2}$

At the right face, $x = a + a = 2a$

$\therefore \quad E_R = \alpha (2a)^{1/2}$

$\phi_R = \vec{E}_R.\Delta\vec{S} = \alpha (2a)^{1/2}(a^2)\cos 0°$

$= \alpha a^{5/2}\sqrt{2}$

$\therefore$ Net flux through the cube $= \phi_R + \phi_L$

$= \alpha a^{5/2}\sqrt{2} - \alpha a^{5/2}$

$\phi = \alpha a^{5/2}(\sqrt{2} - 1) = 800\ (0{\cdot}1)^{5/2}(\sqrt{2} - 1)$

$= \mathbf{1{\cdot}05\ N\ m^2\ C^{-1}}$

(*b*) By Gauss's theorem,

$q = \epsilon_0 \phi = 8{\cdot}85 \times 10^{-12} \times 1{\cdot}05$

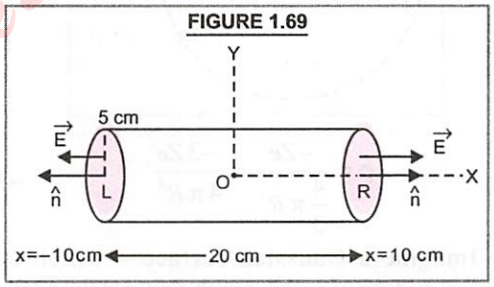
$= \mathbf{9{\cdot}27 \times 10^{-12}\ C}$

Example 34 **An electric field is uniform, and in the positive x-direction for positive x ; and uniform with the same magnitude, but in the negative x-direction for negative x. It is given that $\vec{E} = 200\,\hat{i}$ N/C for $x > 0$ and $\vec{E} = -200\,\hat{i}$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one face is at $x = +10$ cm and the other is at $x = -10$ cm. (*a*) What is the net outward flux through each flat face ? (*b*) What is the flux through the side of the cylinder ? (*c*) What is the net outward flux through the cylinder ? (*d*) What is the net charge inside the cylinder ?** (NCERT Solved Example)

Solution. (*a*) Here, $l = 20$ cm, $r = 5$ cm $= 0{\cdot}05$ m.

As is clear from Fig. 1.69, on the left face L, $\vec{E}$ and $\Delta\vec{S}$ are parallel.

FIGURE 1.69



$\therefore \quad \phi_L = \vec{E}.\Delta\vec{S} = -200\,\hat{i}.\Delta\vec{S}$

$= -200\ [\pi (0{\cdot}05)^2] \cos 0°$

$= \mathbf{-1{\cdot}57\ N\ m^2\ C^{-1}}$

Again, on right face, $\vec{E}$ and $\Delta\vec{S}$ are parallel.

$\therefore \quad \phi_R = \vec{E}.\Delta\vec{S} = +200\,\hat{i}.\Delta\vec{S}$

$= 200\ [\pi (0{\cdot}05)^2] \cos 0°$

$= \mathbf{1{\cdot}57\ N\ m^2\ C^{-1}}$

(*b*) For any point on the curved surface of the cylinder, $\vec{E} \perp \Delta\vec{S} \quad \therefore \quad \phi = \vec{E}.\Delta\vec{S} = 0$

(c) Net outward flux through the cylinder,

$\phi = 1{\cdot}57 + 1{\cdot}57 + 0$

$= 3{\cdot}14 \text{ N m}^2 \text{ C}^{-1}$

(d) By Gauss's Theorem,

$q = \epsilon_0 \phi = 8{\cdot}85 \times 10^{-12} \times 3{\cdot}14$

$= 2{\cdot}78 \times 10^{-11} \text{ C}$

Example 35 **An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge upto a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus ?**

(NCERT Solved Example)

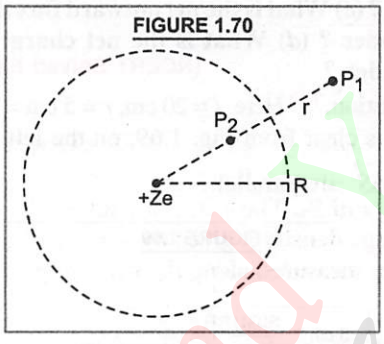
Solution. The charge distribution for this model of atom is shown in Fig. 1.70.

Charge on nucleus = $+ Ze$

As atom is neutral, total negative charge = $- Ze$

$\therefore$ Negative charge density, $\rho = \dfrac{\text{Charge}}{\text{Volume}}$

FIGURE 1.70



$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3} = \frac{-3Ze}{4\pi R^3} \quad ...(i)$$

Imagine a Gaussian surface – which is a spherical surface of radius r with centre at the nucleus (not shown).

According to Gauss's theorem in electrostatics.

$$\phi = E(r) \times 4\pi r^2 = \frac{q}{\epsilon_0} \quad ...(ii)$$

(i) When point P_1 is outside,

i.e., $r > R$, $q = Ze - Ze = 0$ $\therefore$ $E(r) = 0$.

(ii) When point P_2 is inside, i.e., $r < R$, Charge enclosed by Gaussian surface.

$$q' = Ze + \frac{4\pi r^3}{3}\rho = Ze - Ze\frac{r^3}{R^3} \quad ...\text{using } (i)$$

From (ii), $E(r) = \dfrac{q'}{4\pi\epsilon_0 r^2} = \dfrac{Ze - Ze\, r^3/R^3}{4\pi\epsilon_0 r^2}$

$$= \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

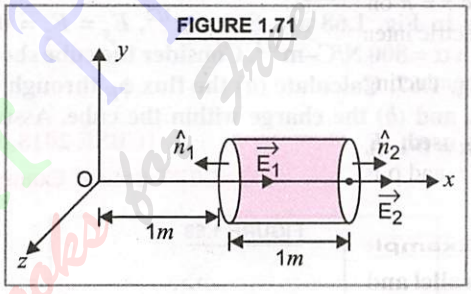
Example 36 **A hollow cylindrical box of length 1 m and area of cross section 25 cm^2 is placed in a three dimensional co-ordinate system as shown in Fig. 1.71. The electric field in the region is given by $\vec{E} = 50x\,\hat{i}$, where E is in NC^{-1}, and x is in metre. Find**

(i) Net flux through the cylinder

(ii) Charge enclosed by the cylinder.

(CBSE 2013)

FIGURE 1.71



Solution. Here, $\vec{ds} = 25 \text{ cm}^2$

$= 25 \times 10^{-4} \text{ m}^2$, along X-axis

$\vec{E} = 50x\,\hat{i}$, $\phi = ?$, $q = ?$

On the left end of the cylinder,

$\vec{E}_1 = 50 \times 1 \text{ NC}^{-1}$, along X-axis

$\therefore$ $\phi_1 = E_1\, ds \cos 180°$

$= 50 \times 25 \times 10^{-4}\,(-1)$

$= -0{\cdot}125 \text{ NC}^{-1} \text{ m}^2$

On the right end of the cy'inder,

$\vec{E}_2 = 50 \times 2 = 100 \text{ NC}^{-1}$ along X-axis

$\therefore$ $\phi_2 = E_2\, ds \cos 0° = 180 \times 25 \times 10^{-4}\,(1)$

$= 0{\cdot}250 \text{ NC}^{-1} \text{ m}^2$

Net flux through the cylinder

$\phi = \phi_1 + \phi_2 = -0{\cdot}125 + 0{\cdot}250$

$= \mathbf{0{\cdot}125 \text{ NC}^{-1} \text{ m}^2}$

$q = \epsilon_0 \phi = 8{\cdot}85 \times 10^{-12} \times 0{\cdot}125$

$= \mathbf{1{\cdot}106 \times 10^{-12} \text{ C}}$

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TYPE VIII. APPLICATIONS OF GAUSS'S THEOREM

Formulae used. 1. Electric intensity due to a line charge, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$

2. Electric intensity due to thin infinite plane sheet of charge, $E = \sigma/2\epsilon_0$

When sheet of charge is thick, $E = \sigma/\epsilon_0$

3. Electric intensity between two thin infinite plane sheets carrying equal and opposite charge densities $E = \sigma/\epsilon_0$

4. Electric intensity at a point on the surface or outside a charged spherical shell. $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$, where $r = R$ on the surface

5. Electric intensity at a point inside a charged solid non conducting sphere, $E = \dfrac{r\rho}{3\epsilon}$

Units used. E is in NC^{-1} ; λ is in C m^{-1}, σ is in C m^{-2} and ρ is in C m^{-3}

Example 37 **Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 19×10^{-22} Cm^{-2}. What is E (*a*) to the left of the plates (*b*) to the right of the plates (*c*) between the plates ?**

Here, $\sigma = 19 \times 10^{-22}$ Cm^{-2}

Solution. (*a*) On the left, the fields of the two plates are equal and opposite, So E = **Zero.**

(*b*) On the right, the fields of the two plates are equal and opposite, So E = **Zero.**

(*c*) Between the plates, the fields due to both plates are in same direction, so the resultant field is

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{19\times10^{-22}}{8{\cdot}85\times10^{-12}}$$

$$= 2{\cdot}14 \times 10^{-10}\ NC^{-1}$$

Example 38 **A plastic rod of length 2·2 m and radius 3·6 mm carries a negative charge of $3{\cdot}8 \times 10^{-7}$ C spread uniformly over its surface. What is the electric field near the mid-point of the rod, at a point on its surface ?**

Solution. Here, $l = 2{\cdot}2$ m,

$r = 3{\cdot}6$ mm $= 3{\cdot}6 \times 10^{-3}$ m,

$q = -3{\cdot}8 \times 10^{-7}$ C, E = ?

Linear charge density, $\lambda = \dfrac{q}{l} = \dfrac{-3{\cdot}8\times10^{-7}}{2{\cdot}2}$

$$= -1{\cdot}73 \times 10^{-7}\ Cm^{-1}$$

As $E = \dfrac{\lambda}{2\pi\epsilon_0 r} = \dfrac{2\lambda}{4\pi\epsilon_0 r}$

$$\therefore \quad E = \frac{2(-1{\cdot}73\times10^{-7})\times9\times10^{9}}{3{\cdot}6\times10^{-3}}$$

$$= -8{\cdot}6 \times 10^{5}\ N/C$$

Example 39 **A large plane sheet of charge having surface charge density 5×10^{-16} cm^{-2} lies in *XY* plane. Find electric flux through a circular area of radius 1 cm. Given normal to the circular area makes an angle of 60° with *Z*-axis.**

Solution. Here, $\sigma = 5 \times 10^{-16}$ cm^{-2}, ϕ = ?,

$r = 1$ cm $= 10^{-2}$ m, $\theta = 60°$

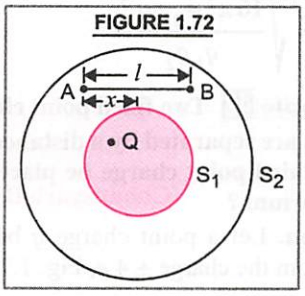
$$\phi = E(\Delta S)\cos\theta = \left(\frac{\sigma}{2\epsilon_0}\right)\pi r^2 \cos 60°$$

$$= \frac{5\times10^{-16}\times3{\cdot}14(10^{-2})^2\times1/2}{2\times8{\cdot}85\times10^{-12}}$$

$$= 4{\cdot}44 \times 10^{-9}\ Nm^2\ C^{-1}$$

Example 40 **In Fig. 1.72, calculate the total flux of the electrostatic field through the spheres S_1 and S_2. The wire *AB* shown here has a linear charge density λ given by $\lambda = kx$, where x is the distance measured along the wire from end *A*.**

FIGURE 1.72



Solution. Charge on an element of length dx of wire *AB*

$$dq = \lambda\, dx = kx\, dx$$

Total charge on wire *AB*,

$$q = \int dq = \int_0^l kx\, dx = k\left(\frac{x^2}{2}\right)_0^l = \frac{1}{2}kl^2$$

Total flux through S_1 $= \phi_1 = \dfrac{Q}{\epsilon_0}$

Total flux through $S_2 = \phi_2 = \dfrac{Q+q}{\epsilon_0}$

$$= \frac{Q+\frac{1}{2}kl^2}{\epsilon_0}$$

TYPE IX. TYPICAL EXAMPLES

Example 41 **A particle of mass m carrying a charge $-q_1$ is moving around a charge $+q_2$ along a circular path of radius r. Prove that period of revolution of charge $-q_1$ about $+q_2$ is given by**

$$T = \sqrt{\frac{16\pi^3 \epsilon_0 mr^3}{q_1 q_2}}$$

Solution. Here, force of attraction between charges = centripetal force

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \frac{mv^2}{r}$$

So $$v = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{mr}}$$

Time period of revolution

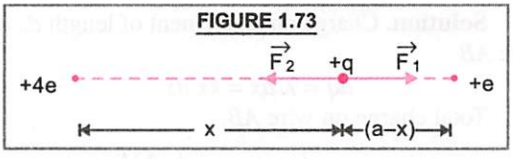
$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{4\pi\epsilon_0 mr}{q_1 q_2}}$$

$$T = \sqrt{\frac{16\pi^3\epsilon_0 mr^3}{q_1 q_2}}$$

Example 42 **Two fixed point charges $+4e$ and $+e$ units are separated by a distance a. Where should the third point charge be placed for it to be in equilibrium ?** (Chhatisgarh Board 2012)

Solution. Let a point charge q be held at a distance x from the charge $+4e$, Fig. 1.73.

FIGURE 1.73



$\therefore$ Distance of q from charge $+e = (a-x)$

Force on this charge exerted by the charge $+4e$ is $F_1 = \dfrac{q(4e)}{4\pi\epsilon_0 x^2}$, *directed away from* $(4e)$

Force on this charge exerted by the charge $+e$

$F_2 = \dfrac{q(e)}{4\pi\epsilon_0 (a-x)^2}$, *directed away from* (e)

For the charge q to be in equilibrium $F_1 = F_2$

i.e. $$\frac{q(4e)}{4\pi\epsilon_0 x^2} = \frac{q(e)}{4\pi\epsilon_0 (a-x)^2}$$

or $$\frac{4}{x^2} = \frac{1}{(a-x)^2} \quad \text{or} \quad \frac{2}{x} = \frac{1}{a-x}$$

or $$x = 2a - 2x$$

$\therefore$ $$3x = 2a \quad \text{or} \quad x = 2a/3$$

Hence the charge q should be held at a distance $2a/3$ from charge $(+4e)$.

Example 43 **An infinite number of charges each equal to 4 μC are placed along X-axis at $x = 1$ m, $x = 2$ m, $x = 4$ m, $x = 8$ m and so on. Find the total force on a charge of 1C placed at the origin.**

Solution. Here, $q = 4\ \mu C = 4 \times 10^{-6}$ C, $q_0 = 1C$

From the principle of superposition, total force acting on 1C charge at the origin due to all the given charges is

$$F = \frac{qq_0}{4\pi\epsilon_0}\left[\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+....\right]$$

$$= 9\times10^9 \times 4\times10^{-6}\times 1\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+.....\right]$$

As sum of infinite geometric progression

$$= \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\therefore F = 9\times10^9\times4\times10^{-6}\times\frac{4}{3} = 4{\cdot}8\times10^4\ \text{N}$$

Example 44 **A pendulum bob of mass 80 milligram and carrying a charge of 2×10^{-8} C is at rest in a horizontal uniform electric field of 2×10^4 Vm^{-1}. Find the tension in the thread of the pendulum and the angle it makes with the vertical.**

Solution. Here, $m = 80$ mg $= 80\times10^{-6}$ kg,

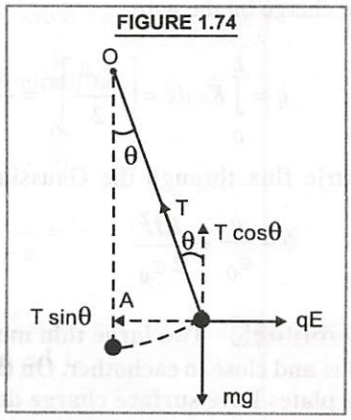
$q = 2\times10^{-8}$ C, $E = 2\times10^4$ V/m

Let T be the tension in the string and θ be the angle it makes with the vertical, Fig. 1.74.

In equilibrium, $T\sin\theta = qE$ ...(*i*)

and $T\cos\theta = mg$

FIGURE 1.74



Dividing, we get

$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{2\times10^{-8}\times2\times10^{4}}{80\times10^{-6}\times9{\cdot}8} = 0{\cdot}5102$$

$\therefore \quad \theta = 27°$

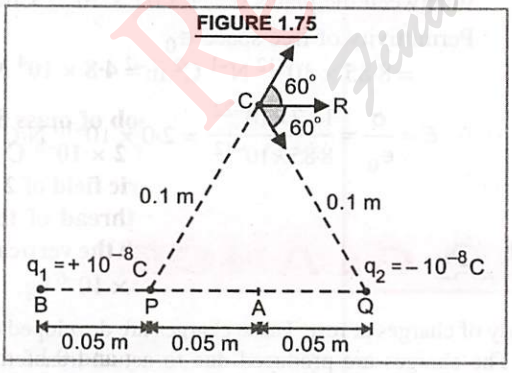
From (*i*),

$$T = \frac{qE}{\sin\theta} = \frac{2\times10^{-8}\times2\times10^{4}}{\sin 27°}$$

$$= \mathbf{8{\cdot}8\times10^{-4}\ N}$$

Example 45 **Two point charges of 10^{-8} C and -10^{-8} C are placed 0·1 m apart. Calculate electric field intensity at A, B and C shown in Fig. 1.75.** (NCERT Solved Example)

FIGURE 1.75



Solution. The charges $+10^{-8}$ C and -10^{-8} C are held at P and Q respectively, PQ = 0·1 m, Fig. 1.75.

Also, $PA = PB = 0{\cdot}05$ m

$QA = 0{\cdot}05$ m and $CP = CQ = 0{\cdot}1$ m

At A, Field intensity due to charge q_1

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1\times1}{AP^2}\ ;\ \text{along } PA$$

$$= \frac{9\times10^{9}\times10^{-8}}{(0{\cdot}05)^2} = 36000\ \text{N/C}$$

Field intensity due to charge q_2

$$= \frac{9\times10^{9}\times10^{-8}}{(0{\cdot}05)^2}\ ;\ \text{along } AQ = 36000\ \text{N/C}$$

$\therefore$ Net field intensity at A = 36000 + 36000

= 72000 N/C = $\mathbf{7{\cdot}2\times10^{4}}$ **N/C along** $\boldsymbol{AQ}$

At B, Field intensity due to charge q_1

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1\times1}{PB^2}\ :\ \text{along } PB \text{ produced}$$

$$= \frac{9\times10^{9}\times10^{-8}}{(0{\cdot}05)^2} = 36000\ \text{N/C}$$

Field intensity due to charge q_2

$$= \frac{1}{4\pi\epsilon_0}\frac{q_2\times1}{BQ^2},\quad \text{along } BQ$$

$$= \frac{9\times10^{9}\times10^{-8}}{(0{\cdot}15)^2} = 4000\ \text{N/C}$$

$\therefore$ Net field intensity at B = 36000 – 4000

= 32000 N/C

= $\mathbf{3{\cdot}2\times10^{4}}$ **N/C, along** $\boldsymbol{PB}$ **produced**

At C, Field intensity due to charge q_1,

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{PC^2},\ \text{along } PC.$$

$$E_1 = \frac{9\times10^{9}\times10^{-8}}{(0{\cdot}1)^2} = 9\times10^{3}\ \text{N/C}$$

Field intensity due to charge q_2,

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{QC^2},\ \text{along } CQ.$$

$$E_2 = \frac{9\times10^{9}\times10^{-8}}{(0{\cdot}1)^2} = 9\times10^{3}\ \text{N/C}.$$

Both, E_1 and E_2 are equal in magnitude, acting at 60° to CR parallel to BA.

$\therefore$ Resultant intensity at R

$$= E_1\cos60° + E_2\cos60° = 2E_1\cos60°$$

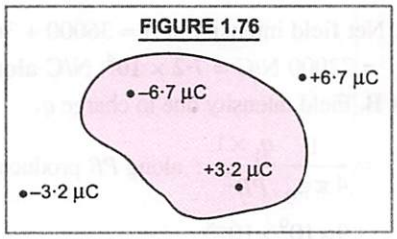
$$= 2\times9\times10^{3}\times\frac{1}{2}$$

$$= \mathbf{9\times10^{3}\ N/C}$$

It is represented by CR parallel to BA. The components of $\vec{E}_1$ and $\vec{E}_2$ in directions $\perp$ to CR cancel out.

Example 46 **Fig. 1.76 shows a closed surface surrounding some electric charges (*a*) what is the net electric flux through the surface ? (*b*) Is the electric flux directed inward or outward from the surface ?**

FIGURE 1.76



Solution. (*a*) Here, total charge inside the surface :

$$q = -6{\cdot}7\ \mu C + 3{\cdot}2\ \mu C = -3{\cdot}5\ \mu C$$

$$\text{As}\quad \phi_E = \frac{q}{\epsilon_0} = \frac{-3{\cdot}5\times10^{-6}}{8{\cdot}85\times10^{-12}}$$

$$= -3{\cdot}95 \times 10^5\ \text{Nm}^2\ \text{C}^{-1}$$

(*b*) As ϕ_e is negative, it is directed **Inwards.**

Example 47 **A wire AB of length L has linear charge density $\lambda = Kx$, where x is measured from the end A of the wire. This wire is enclosed by a Gaussian hollow surface. Find the expression for electric flux through the surface.**

[CBSE 2017 (C)]

Solution. Here, $\lambda = \dfrac{dq}{dx} = Kx$; $\phi = ?$

$$dq = Kx\,dx$$

Total charge on the wire

$$q = \int_0^L Kx\,dx = \left[\frac{Kx^2}{2}\right]_0^L = \frac{KL^2}{2}.$$

Electric flux through the Gaussian hollow surface is $\phi = \dfrac{q}{\epsilon_0} = \dfrac{KL^2}{2\epsilon_0}$

Example 48 **Two large thin metal plates are parallel and close to eachother. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17{\cdot}7 \times 10^{-22}$ C/m^2. What is electric field intensity E (*a*) in the outer region of the first plate and between the plates ?**

(CBSE Sample Paper 2022-23)

Solution. Here, surface charge density of plate

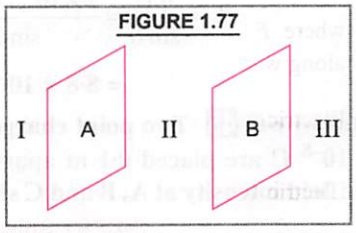
$$A = +\,17{\cdot}7 \times 10^{-22}\ \text{C/m}^2$$

Surface charge density of plate

$$B = -\,17{\cdot}7 \times 10^{-22}\ \text{C/m}^2$$

In the outer region of plate A, electric field intensity $E = 0$

FIGURE 1.77



Inbetween the plates $\sigma = +\,17{\cdot}7 \times 10^{-22}$ C/m^2

Permittivity of free space ϵ_0

$$= 8{\cdot}85 \times 10^{-12}\ \text{N}^{-1}\ \text{C}^2\ \text{m}^{-2}$$

$$\therefore\ E = \frac{\sigma}{\epsilon_0} = \frac{17{\cdot}7\times10^{-22}}{8{\cdot}85\times10^{-12}} = 2{\cdot}0 \times 10^{-10}\ \text{N/C}$$

REVISION AT A GLANCE

1. Electrostatics stands for ***static electricity*** involving study of charges at rest. These charges are developed on insulators when they are rubbed against each other. The charges are produced due to actual transfer of electrons.
2. Two important properties of charge are ***Quantization and Conservation.*** The quantization means that charge can exist only as integral multiple of electronic charge. ($-e$), *i.e.*, $q = \pm ne$, where n is an integer.

   The conservation of charge means that on an isolated system, total electric charge always remains constant. Obviously, the charges can be created only in equal and unlike pairs.

3. Like charges repel and unlike charges attract each other with a force F governed by ***Coulomb's law***

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{12} \text{ (when } q_1 q_2 > 0\text{), } i.e., \text{ charges are like charges}$$

and $$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0 r^3}\vec{r}_{21} \text{ (when } q_1 q_2 < 0\text{), } i.e., \text{ charges are unlike charges.}$$

where q_1, q_2 are the charges held r distance apart in vacuum ; $\vec{F}_{21}$ is force on q_2 by q_1 and $\vec{r}_{12}$ is vector pointing q_1 to q_2. $\frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{N m}^2\ \text{C}^{-2}$, where ϵ_0 is absolute electrical permittivity of free space.

When the same charges are separated by the same distance in a medium of relative permittivity ϵ_r or K, the force between the charges becomes : $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0 K}\frac{q_1 q_2}{r^3}\vec{r}_{12}$

4. To find total force on a given charge q_0 due to any number of charges q_1, q_2.....q_n. we use ***superposition principle***, according to which $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ...\vec{F}_{0n}$

i.e. vector sum of forces due to individual charges would give us the total force.

5. ***Electric Field***

Electric field intensity at any position (r) is defined as $\vec{E}(r) = \underset{q_0 \to 0}{Limit}\ \frac{\vec{F}}{q_0}$

where $\vec{F}$ is total force experienced by a small positive test charge q_0. The direction of $\vec{E}$ is the direction along which the positive test charge would try to move, if free to do so.

Electric field intensity at a point distant r from a point charge q in air is $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

Electric field intensity due to a group of charges is obtained by applying superposition principle. The SI unit of $\vec{E}$ *is* NC^{-1}.

6. ***An electric line of force is a path***, straight or curved, such that tangent to the path at any point gives us the direction of electric field intensity at that point. No two electric lines of force can intersect eachother.

7. Two equal and opposite charges separated by a small distance are said to form an **electric dipole.** Many of the atoms/molecules are electric dipoles.

Every dipole is associated with a ***dipole moment*** $\vec{p}$ whose magnitude is equal to the product of the magnitude of either charge (q) and the distance $2\vec{a}$ between the charges, *i.e.*, $\vec{p} = q\times(2\vec{a})$

The direction of $\vec{p}$ is from negative charge to positive charge. It is measured in coulomb-metre. Every dipole has an electric field around it.

Field intensity at any point on *axial line* of electric dipole is, $|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0}\frac{2r}{(r^2-a^2)^2}$, along $\vec{p}$ where r is distance of the point from the centre of the dipole.

If the point lies on *equatorial line* of dipole at the same distance r, then $|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0}\frac{1}{(r^2+a^2)^{3/2}}$, opposite to $\vec{p}$.

8. When an electric dipole of moment $\vec{p}$ is held at an angle θ with the direction of a uniform electric field E, a ***torque*** $\vec{\tau}$ acts on the dipole, given by : $\vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$, in magnitude.
This torque tries to align the dipole in the direction of the field, reducing angle θ to zero.
To orient the dipole at any other angle with $\vec{E}$, work has to be done on the *dipole,* which is stored as P.E. of dipole given by : $P.E. = -pE\cos\theta = -\vec{p}.\vec{E}$

9. ***Electric Flux*** over an area in an electric field represents the total number of electric field lines crossing this area. It is represented by ϕ_E
Electric flux is a scalar quantity, and is measured in $Nm^2\ C^{-1}$
According to ***Gauss's theorem*** in electrostatics, total normal electric flux over a closed surface S in vacuum is $1/\epsilon_0$ time the total charge (Q) contained inside the surface.

$$Q_E = \oint_S \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0}$$

Coulomb's law of force can be obtained from Gauss's theorem. This theorem can be used for calculating electric field intensity due to highly symmetric charge distributions.

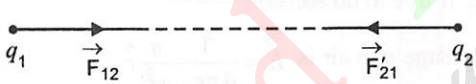
OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. According to Coulomb's law, which is the correct relation for the following figure ?

FIGURE 1(Q).1



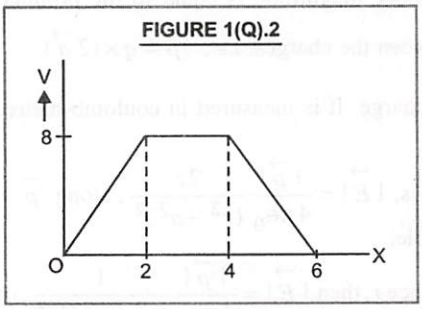
(a) $q_1 q_2 > 0$ (b) $q_1 q_2 < 0$

(c) $q_1 q_2 = 0$ (d) $1 > \frac{q_1}{q_2} > 0$

(CBSE Sample Paper 2022-23)

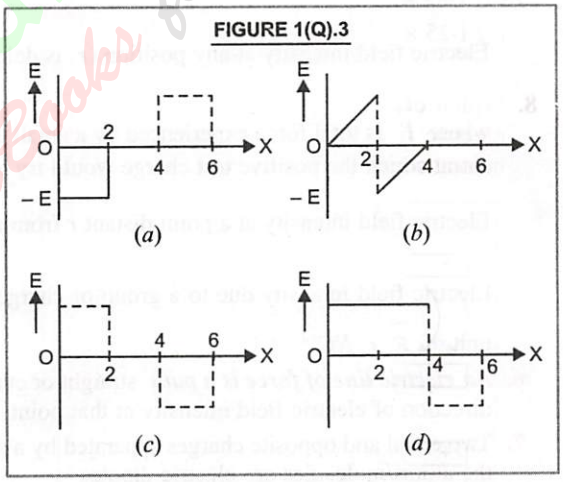
2. The electric potential V as a function of distance X is shown in Fig. 1(Q).2.

FIGURE 1(Q).2



The graph of the magnitude of electric field (E) as a function of X is

FIGURE 1(Q).3



(CBSE Sample Paper 2022-23)

3. An object has a charge of 1 C and gains $5{\cdot}0 \times 10^{18}$ electrons. The net charge on the object becomes

(a) – 0·80 C (b) + 0·80 C

(c) + 1·80 C (d) + 0·20 C

(CBSE 2022)

4. A negatively charged object X is repelled by another charged object Y. However, an object Z is attracted to object Y. Which of the following is the most possibility for object Z ?

(a) positively charged only

(b) negatively charged only

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(*c*) neutral or positively charged

(*d*) neutral or negatively charged (CBSE 2022)

5. In a chamber, three spheres carry charges + 3 *e*, + 5 *e* and – 3 *e*. They came in contact for a moment and got separated. Which one of the following are possible values for the final charges on the spheres ?

(*a*) + 5 *e*, – 4 *e*, + 5 *e*

(*b*) + 6 *e*, + 6 *e*, – 7 *e*

(*c*) – 4 *e*, + 3·5 *e*, + 5·5 *e*

(*d*) + 5 *e*, – 8 *e*, + 7 *e* (CBSE 2022)

6. The electric potential on the axis of an electric dipole at a distance *r* from its centre is *V*. Then the potential at a point at the same distance on its equatorial line will be

(*a*) 2 *V* (*b*) – *V*

(*c*) *V*/2 (*d*) Zero

(CBSE Sample Paper 2022-23)

7. A potential difference of 200 V is maintained across a conductor of resistance 100 Ω. The number of electrons passing through it in 1 second is

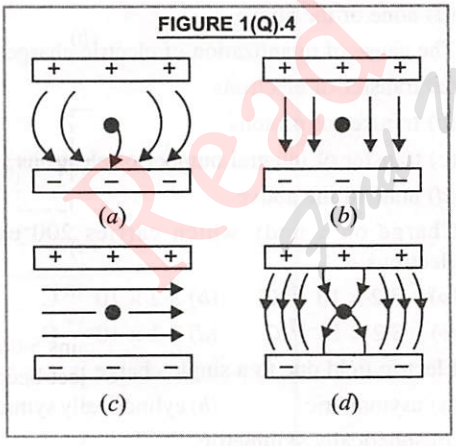
(*a*) $1{\cdot}25 \times 10^{19}$ (*b*) $2{\cdot}5 \times 10^{18}$

(*c*) $1{\cdot}25 \times 10^{18}$ (*d*) $2{\cdot}5 \times 10^{16}$

(CBSE 2022)

8. Which of the diagrams correctly represents the electric field between two charged plates if a neutral conductor is placed inbetween the plates ?

FIGURE 1(Q).4



(CBSE 2022)

9. The magnitude of electric field due to a point charge 2 *q* at a distance *r* is *E*. Then the magnitude of electric field due to a uniformly charged thin spherical shell of radius *R* with total charge *q* at a distance $\frac{r}{2}(r >> R)$ will be

(*a*) 2 *E* (*b*) *E*

(*c*) *E*/2 (*d*) *E*/4 (CBSE 2022)

10. A square sheet of side '*a*' is lying parallel to *XY* plane at $z = a$. The electric field in the region is $\vec{E} = cz^2 \hat{k}$. The electric flux through the sheet is

(*a*) $\frac{1}{2}ca^3$ (*b*) Zero

(*c*) $a^4 c$ (*d*) $\frac{1}{2}a^4 c$ (CBSE 2022)

11. An electric dipole placed in a non-uniform electric field will experience

(*a*) only a force (*b*) only a torque

(*c*) both, force and torque

(*d*) neither force nor torque (CBSE 2022)

12. Let N_1 be the number of electric field lines going out of an imaginary cube of side *a*, that enclosed an isolated point charge 2 *q* and N_2 be the corresponding number for an imaginary sphere of radius *a* that encloses an isolated point charge 3 *q*. Then N_1/N_2 is

(*a*) $\frac{1}{\pi}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{9}{4}$ (*d*) π (CBSE 2022)

13. Which statement is true for Gauss Law ?

(*a*) All the charges whether inside or outside the Gaussian surface contribute to electric flux

(*b*) Electric flux depends upon the geometry of the Gaussian surface

(*c*) Gauss's theorem can be applied to non-uniform electric field

(*d*) The electric field over the Gaussian surface remains continuous and uniform at every point

14. In the process of charging, a positively charged body

(*a*) loses mass (*b*) gains mass

(*c*) neither of the two

(*d*) depends on conditions

15. Electric flux of an electric field $\vec{E}$ through an area $\vec{dA}$ is given by

(*a*) $\frac{\vec{E}}{\vec{dA}}$ (*b*) $\vec{E} \cdot \vec{dA}$

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(c) $\frac{\vec{E}\cdot d\vec{A}}{\epsilon_0}$ (d) $\frac{\vec{E}\times d\vec{A}}{\epsilon_0}$ (CBSE 2022)

16. A charge Q is placed at the centre of the line joining two charges q and q. The system of three charges will be in equilibrium if Q is

(a) $-q/3$ (b) $-q/2$

(c) $\frac{q}{4}$ (d) $-q/4$ (CBSE 2022)

17. Two point charges + 16 q and − 4 q are located at $x = 0$ and $x = L$. The location of the point on x-axis at which resultant electric field due to these charges is zero is

(a) L (b) 6 L

(c) 4 L (d) 2 L (CBSE 2022)

18. An electric dipole of dipole moment 4×10^{-5} Cm kept in a uniform electric field of 10^{-3} N/C experiences a torque of 2×10^{-8} N-m. The angle which the dipole makes with the electric field is

(a) 30° (b) 45°

(c) 60° (d) 90° (CBSE 2020)

19. Which of the following is a vector quantity ?

(a) Electric charge (b) Electric flux

(c) Electric field (d) Electric potential

20. Electric field lines contract longitudinally. This is because of

(a) attraction between unlike charges

(b) repulsion between like charges

(c) both (a) and (b)

(d) neither (a) nor (b)

21. Charge on a conducting sphere resides

(a) inside the sphere

(b) at the centre of the sphere

(c) outside the sphere

(d) on the outer surface of the sphere

22. Four charges + 5 Q, − 3 Q, + 2 Q, − 4 Q are enclosed inside some surface. What will be the outgoing flux through the surface

(a) 14 V-m (b) Zero

(c) 4 V-m (d) − 4 V-m

23. Charge on a body is integral multiple of ± e where e is electronic charge. This is because of

(a) Conservation of charge

(b) Quantisation of charge

(c) Conservation of mass

(d) Quantisation of charge

24. SI unit of surface integral of electric field is

(a) $Nm^2 C^{-2}$ (b) $Nm^{-2} C^2$

(c) $Nm^2 C^{-1}$ (d) $Nm C^{-2}$

25. The relation between electric intensity (E) and electric flux ϕ is

(a) $\phi = \oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$ (b) $\phi = \frac{q^2}{\epsilon_0}$

(c) $\phi = \frac{\sigma}{\epsilon_0}$ (d) none of these

26. What does $q_1 + q_2 = 0$ signify ?

(a) Electric potential (b) Electric intensity

(c) Electric dipole (d) None of these

27. 1 stat-Coulomb = ... Coulomb

(a) 3×10^9 (b) 3×10^{-9}

(c) $\frac{1}{3}\times 10^9$ (d) $\frac{1}{3}\times 10^{-9}$

28. When a plastic comb is passed through dry hair, the charge acquired by the comb is

(a) always negative

(b) always positive

(c) sometimes negative

(d) none of the above

29. The cause of quantization of electric charge is

(a) transfer of electrons

(b) transfer of protons

(c) transfer of integral number of electrons

(d) none of the above

30. Charge on a body which carries 200 excess electrons is

(a) $-3{\cdot}2 \times 10^{-18}$ C (b) $3{\cdot}2 \times 10^{-18}$ C

(c) $-3{\cdot}2 \times 10^{-17}$ C (d) $3{\cdot}2 \times 10^{-17}$ C

31. Electric field due to a single charge is

(a) asymmetric (b) cylindrically symmetric

(c) spherically symmetric

(d) none of the above

32. At a given distance from the centre of short electric dipole, field intensity on axial line is k times the field intensity on equatorial line, where k =

(a) 2 (b) 3

(c) 4 (d) 1

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33. Electric field due to an electric dipole is

(*a*) spherically symmetric

(*b*) cylindrically symmetric

(*c*) asymmetric

(*d*) none of the above

34. When an electric dipole is held at an angle in a uniform electric field, the net force F and torque τ on the dipole are

(*a*) $F = 0, \tau = 0$ (*b*) $F \neq 0, \tau \neq 0$

(*c*) $F = 0, \tau \neq 0$ (*d*) $F \neq 0, \tau = 0$

35. The dimensional formula of electric flux is

(*a*) $[M^1L^2T^{-2}A^{-1}]$ (*b*) $[M^{-1}L^3T^{-3}A]$

(*c*) $[M^1L^3T^{-3}A^{-1}]$ (*d*) $[M^1L^{-3}T^{-3}A^{-1}]$

36. A closed surface in vacuum encloses charges $-q$ and $+3q$. Another charge $-2q$ lies outside the surface. Total electric flux over the surface is

(*a*) zero (*b*) $2q/\epsilon_0$

(*c*) $-\frac{3q}{\epsilon_0}$ (*d*) $4q/\epsilon_0$

37. The number of electric lines of force radiating from a closed surface in vacuum is $1{\cdot}13 \times 10^{11}$. The charge enclosed by the surface is

(*a*) 1 C (*b*) 1 μ C

(*c*) 0·1 C (*d*) 0·1 μ C

38. Two charges $3{\cdot} \times 10^{-5}$ C and 5×10^4 C are placed at a distance 10 cm from each other. Find the value of electrostatic force acting between them.

(*a*) $13{\cdot}5 \times 10^{11}$ N (*b*) 40×10^{11} N

(*c*) 180×10^9 N (*d*) $13{\cdot}5 \times 10^{10}$ N

39. What is the S.I. unit of electric flux ?

(*a*) $\frac{N}{C} \times m^2$ (*b*) $N \times m^2$

(*c*) $\frac{N}{m^2} \times C$ (*d*) $\frac{N^2}{m^2} \times C^2$

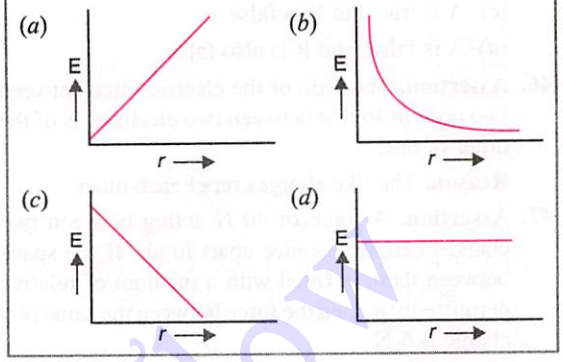
40. What is the value of minimum force acting between two charges each of charge e placed at 1 m apart from each other ?

(*a*) Ke^2 (*b*) Ke

(*c*) $\frac{Ke}{4}$ (*d*) $\frac{Ke^2}{2}$

41. For a point charge, the graph between electric field versus distance is given by :

FIGURE 1(Q).5



42. Four charges $+8Q - 3Q + 5Q$ and $-10Q$ are kept inside a closed surface. What will be the outgoing flux through the surface.

(*a*) 26 V – m (*b*) 0 V – m

(*c*) 10 V – m (*d*) 8 V – m

43. On charging a neutral balloon, its size–

(*a*) Increases

(*b*) Decreases

(*c*) Remains same

(*d*) No relation between charge and size

44. Electric field lines contract lengthwise. It shows

(*a*) Repulsion between same charges

(*b*) Attraction between opposite charges

(*c*) No relation between force & contraction

(*d*) Electric field lines do not move on straight path

45. A charge q is placed at the point of interrection of body diagonals of a cube. The electric flux passing through any one of its face is

(*a*) $\frac{q}{6\epsilon_0}$ (*b*) $\frac{3q}{\epsilon_0}$

(*c*) $\frac{6q}{\epsilon_0}$ (*d*) $\frac{q}{3\epsilon_0}$

(CBSE Sample Paper 2020)

II. Assertion-Reason Type Questions

Direction. For question numbers 46 to 53, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

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(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

46. Assertion. The ratio of the electric force between two protons to that between two electrons is of the order of one.

Reason. The like charges repel each other.

47. Assertion. A force of 40 N acting between two charges certain distance apart in air. If the space between them is filled with a medium of relative permittivity 8, then the force between the same two charge is 5 N.

Reason. The coulomb's force between two charges q_1 and q_2 placed distance r apart in medium of relative permittivity $\in_r$ is

$$F = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2}$$

48. Assertion. The coulomb force is the dominating force in the universe.

Reason. The coulomb force is weaker than the gravitational force.

49. Assertion. Two identical charges repel each other with a force equal to 10 mg wt, when they are 0·6 m apart in air. The value of each charge is 2 μC and $g = 10\ ms^{-2}$.

Reason. Like charges repel each other.

50. Assertion. If sum of two charges q_1 and q_2 is Q, then the force between q_1 and $q_2$will be maximum if the ratio Q/q_1 is 2.

Reason. The coulomb force between the two charges is

$$F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$$

51. Assertion. When a conducting medium is placed between two charges then the electric force between them will become infinite.

Reason. The addition of conducting medium between two charges reduces the distance to zero.

52. Assertion. The number of electric lines of force emanating from 1 μC charge in vacuum is $1{\cdot}13 \times 10^8$.

Reason. It follows from Gauss's Law in electrostatics.

53. Assertion. The number of electrons in one coulomb of charge is $6{\cdot}25 \times 10^{-18}$.

Reason. This follows from the relation $q = ne^2$, where symbols have usual meaning.

ANSWERS

I. Multiple Choice Questions

1. (*b*)	**2.** (*a*)	**3.** (*d*)	**4.** (*c*)	**5.** (*b,c*)	**6.** (*d*)	**7.** (*a*)	**8.** (*d*)	**9.** (*a*)	**10.** (*c*)
11. (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*a*)	**15.** (*b*)	**16.** (*d*)	**17.** (*a*)	**18.** (*a*)	**19.** (*c*)	**20.** (*a*)
21. (*d*)	**22.** (*b*)	**23.** (*b*)	**24.** (*c*)	**25.** (*a*)	**26.** (*c*)	**27.** (*d*)	**28.** (*a*)	**29.** (*c*)	**30.** (*c*)
31. (*c*)	**32.** (*a*)	**33.** (*b*)	**34.** (*c*)	**35.** (*c*)	**36.** (*b*)	**37.** (*a*)	**38.** (*a*)	**39.** (*a*)	**40.** (*a*)
41. (*b*)	**42.** (*b*)	**43.** (*a*)	**44.** (*b*)	**45.** (*a*)					

II. Assertion-Reason Type Questions

46. (*b*)	**47.** (*a*)	**48.** (*d*)	**49.** (*b*)	**50.** (*a*)	**51.** (*d*)	**52.** (*d*)	**53.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. In the Fig. 1(Q).1 shown, $\vec{F}_{12}$ and $\vec{F}_{21}$ are the forces of attraction. Therefore, charges q_1, q_2 must be of opposite sign, *i.e.*, $q_1\, q_2 < 0$.

2. We find that from $x = 0$ to $x = 2$, $\frac{dV}{dx} = \frac{8}{2} = 4$.

As $E = -\frac{dV}{dx}$, so, E is negative constant.

From $x = 2$ to 4, V = constant. Therefore, $E = 0$ and so on. **Choice (*a*) is correct.**

3. $Q_{total} = Q_{object} + ne$
$= 1{\cdot}0 + 5 \times 10^{18} (-1{\cdot}6 \times 10^{-19})$
$Q_{total} = 1{\cdot}0 + (-0{\cdot}8) = \mathbf{0{\cdot}20\ c}$

4. As X is repelled by Y, therefore, charge on Y must be negative. As Z is attracted by Y, Z must carry no charge or positive charge. **Choice (*c*) is correct.**

5. Here, total charge $= Q_1 + Q_2 + Q_3$
$= 3e + 5e - 3e = 5e$
As all the three spheres came in contact simultaneously and got separated, the charge must be conserved

i.e., $Q_1' + Q_2' + Q_3' = 5e$
Now, $6e + 6e - 7e = 5e$
As $-4e + 3{\cdot}5e + 5{\cdot}5e = 5e$
Thus, choice (*b*) and (*c*) are correct.

6. From the knowledge of theory, electric potential at any point on the equatorial line of electric dipole is **zero.**

7. $I = \dfrac{V}{R} = \dfrac{ne}{t}$

$\therefore\ n = \dfrac{I \times t}{e} = \dfrac{V}{R} \times \dfrac{t}{e} = \dfrac{200 \times 1}{100 \times 1{\cdot}6 \times 10^{-19}}$
$= \mathbf{1{\cdot}25 \times 10^{19}}$

8. Electric field lines always start from positive charge and end at negative charge. Therefore **option (*d*) is the correct choice.**

9. Electric field due to a uniformly charged thin spherical shell at a distance r is

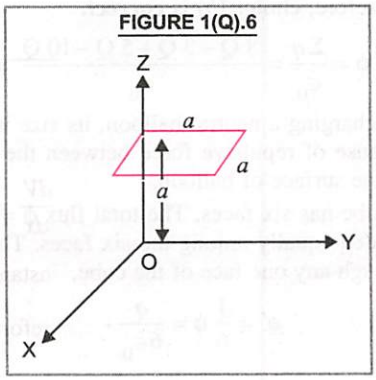
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{2q}{4\pi\epsilon_0 r^2}$$

When charge becomes q and distance is $r/2$, then

$$E' = \frac{q}{4\pi\epsilon_0 (r/2)^2} = \frac{4q}{4\pi\epsilon_0 r^2} = \mathbf{2E}$$

10. Here $z = a$ [Refer to Fig. 1(Q).6]

FIGURE 1(Q).6



$$\phi = \vec{E} \cdot \vec{A} = (cz^2 \hat{k})\, a^2 \cdot \hat{k} = cz^2 a^2$$

At, $z = a$

$$\phi = ca^2 a^2 = \mathbf{ca^4}$$

11. In a non-uniform electric field, an electric dipole will experience both, force and torque. Thus **option (*c*) is true.**

12. According to Gauss's theorem,

$$N_1 = \frac{2q}{\epsilon_0} \qquad N_2 = \frac{3q}{\epsilon_0}$$

$$\frac{N_1}{N_2} = \frac{2q}{\epsilon_0} \times \frac{\epsilon_0}{3q} = \frac{2}{3}$$

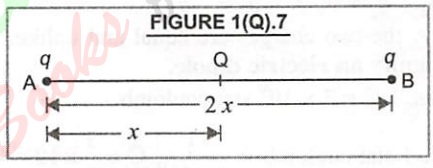
13. For Gauss's Law, the electric field over the Gaussian surface remains continuous and uniform at every point. **Option (*d*) is true.**

14. A body gets positively charged when it loses some electrons. It involves loss of mass. **Choice (*a*) is correct.**

15. By definition, $d\phi = \vec{E} \cdot \vec{dA}$

16. As is clear from Fig. 1(Q).7, charge q at B will be in equilibrium if net force at B is zero.

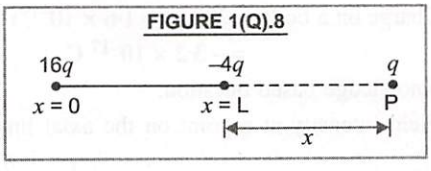
FIGURE 1(Q).7



$$\frac{q \times q}{(2x)^2} = -\frac{Qq}{x^2} \quad \text{or} \quad Q = -\frac{q}{4}$$

17. Let the resultant electric field due to the three charges be zero at P, at a distance x from charge $-4q$, Fig. 1(Q).8.

FIGURE 1(Q).8



$$\therefore\ \frac{kq \times 16}{(x+L)^2} = \frac{k(4q)}{x^2}$$

$$\frac{4}{x+L} = \frac{2}{x} \quad \text{or} \quad 4x = 2x + 2L$$
$$\boldsymbol{x = L}$$

18. Here, $p = 4 \times 10^{-5}$ cm, $E = 10^{-3}$ N/C
$\tau = 2 \times 10^{-8}$ Nm, $\theta = ?$
As $\tau = pE \sin\theta$
$2 \times 10^{-8} = 4 \times 10^{-5} \times 10^{-3} \sin\theta$

$$\sin\theta = \frac{1}{2} \quad \therefore \quad \theta = 30°$$

19. Out of the given quantities, **electric field is a vector.**

20. Electric field lines contract longitudinally **because of attraction between unlike charges.**

21. On a charged conducting sphere, **charge resides on the outer surface of the sphere.**

22. Electric flux $= \frac{\Sigma q}{\epsilon_0} = \frac{5Q - 3Q + 2Q - 4Q}{\epsilon_0}$

$= \text{Zero}$

23. This property of electric charge is due to **quantisation of charge.**

24. Surface integral of electric field $= \oint_S E\,ds\cos\theta$.

SI unit of surface integral of electric field is

$= (N/C)\,(m^2) \times 1$

$= Nm^2\,C^{-1}$

25. According to Gauss's theorem in electrostatics,

$$\phi = \oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

26. $q_1 + q_2 = 0 \quad \therefore \quad q_2 = -q_1$

i.e. the two charges are equal and unlike. **They signify an electric dipole.**

27. As, $1\text{ C} = 3 \times 10^9$ stat coulomb

$$\therefore \text{ 1 stat coulomb} = \frac{1}{3\times10^9}\text{ C} = \frac{1}{3}\times10^{-9}\text{ C}$$

28. When a plastic comb is passed through dry hair, the electrons pass from hair to comb. Due to it, the comb acquire **negative charge.**

29. The cause of quantization of electric charge is due to transfer of integral number of electrons.

30. Charge on a body $= 200 \times (-1{\cdot}6 \times 10^{-19})\text{ C}$

$= -3{\cdot}2 \times 10^{-17}\text{ C}$

31. Knowledge based question.

32. Field intensity at a point on the axial line of a short dipole at distance r, $E_A = \frac{1}{4\pi\epsilon_0}\times\frac{2p}{r^3}$

Field intensity at a point on the equitorial line at distance r, $E_E = \frac{1}{4\pi\epsilon_0}\times\frac{p}{r^3}$

$$\therefore \quad \frac{E_A}{E_E} = 2 \quad \text{or} \quad E_A = 2\,E_E$$

33. Knowledge based question.

34. When an electric dipole is held at an angle in a uniform electric field, there will be equal and opposite forces on the positive and negative charges of the dipole. Due to it, the resultant force on the dipole will be zero. But these forces will form a torque to rotate the dipole to align in the direction of electric field.

35. Electric flux, $\phi = \frac{q}{\epsilon_0} = \frac{q}{q^2/4\pi F r^2} = \frac{4\pi F r^2}{q}$

$$\text{Dimensions of } \phi = \frac{(MLT^{-2})\times L^2}{AT} = [ML^3T^{-3}A^{-1}]$$

36. Electric flux over a closed surface, $\phi = \frac{Q}{\epsilon_0}$

where Q is the total charge enclosed by the closed surface. Here $Q = -q + 3q = 2q$

$$\therefore \quad \phi = 2q/\epsilon_0$$

37. No. of electric lines of force radiating from a closed surface in vacuum = electric flux linked with the closed surface, $\phi = \frac{Q}{\epsilon_0} = 1{\cdot}13 \times 10^{11}$

$$\therefore \quad Q = 1{\cdot}13 \times 10^{11} \times \epsilon_0$$

$$= 1{\cdot}13 \times 10^{11} \times (8{\cdot}85 \times 10^{-12}) \approx 1\text{ C}$$

38. Here, $q_1 = 3 \times 10^{-5}$ C, $q_2 = 5 \times 10^4$ C,

$r = 10\text{ cm} = 10^{-1}\text{ m}$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{9\times10^9 \times (3\times10^{-5}) \times (5\times10^4)}{(10^{-1})^2}$$

$$= 135 \times 10^{10}\text{ N} = 13{\cdot}5 \times 10^{11}\text{ N}$$

39. Electric flux, $\phi = E \times ds$

SI unit of electric flux $= (N/C) \times m^2$

40. Force $F = K\frac{q_1 q_2}{r^2} = K\frac{e\times e}{1^2} = K\,e^2$

41. From the knowledge of theory, $E \propto \frac{1}{r}$.

Therefore, **choice (*b*) is correct.**

42. As, $\phi = \frac{\Sigma q}{\epsilon_0} = \frac{8\,Q - 3\,Q + 5\,Q - 10\,Q}{\epsilon_0} = \text{Zero}$

43. On charging a neutral balloon, its size increases because of repulsive force between the charges on the surface of balloon.

45. A cube has six faces. The total flux $\phi = q/\epsilon_0$ is divided equally among the six faces. Therefore, through any one face of the cube,

$$\phi' = \frac{1}{6}\phi = \frac{q}{6\epsilon_0}.$$

II. Assertion-Reason Type Questions

46. The magnitude of charge on proton is same as that on electron. According to coulomb's law

$$F = \frac{K q_1 q_2}{r^2}. \text{ As } q_1 = q_2 = e$$

(for protons and electrons)

$$\therefore \quad \frac{F_{\text{protons}}}{F_{\text{electrons}}} = \frac{K \times e \times e/r^2}{K \times e \times e/r^2} = 1$$

Thus **Assertion is true.** Here **Reason is also true but can not explain the Assertion.**

47. Here, **Reason is true,**

$$F_{\text{air}} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} = 40 \text{ N} \qquad ...(i)$$

and $$F_{\text{med}} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0 \times 8} \times \frac{q_1 q_2}{r^2} = \frac{1}{8}$$

$$F_{\text{air}} = \frac{1}{8} \times 40 = 5 \text{ N}$$

Thus **Assertion is true** and **Reason is the correct explanation of Assertion.**

48. Every mass in the universe is governed by gravitational force of attraction. Hence gravitational force is dominating force but it is a weakest force as compared to other forces in nature. So, both **Assertion and Reason are false.**

49.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

or $$(10 \times 10^{-3} \times 10) = \frac{(9 \times 10^9) \times q \times q}{(0{\cdot}6)^2}$$

or $$q^2 = \frac{10^{-1} \times (0{\cdot}6)^2}{9 \times 10^9} = 4 \times 10^{-12}$$

$$\therefore \quad q = 2 \times 10^{-6} \text{ C} = 2 \ \mu\text{C}$$

Thus **Assertion is true.** Here **Reason is true but can not explain the Assertion.**

50. Given, $q_1 + q_2 = Q$ or $q_2 = (Q - q)$

Coulomb force, $$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 (Q - q_1)}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 Q - q_1^2}{r^2}$$

For F to be maximum, $\frac{dF}{dq_1} = 0$

$$\therefore \quad \frac{dF}{dq_1} = \frac{1}{4\pi\epsilon_0} [Q - 2q_1] = 0$$

or $Q - 2q_1 = 0$ or $Q = 2q_1 \quad \therefore \ Q/q_1 = 2$

Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

51. When a conducting medium is inserted between two charges then $\epsilon_r = \infty$.

$$F = \frac{1}{4\pi\epsilon_0 \epsilon_r} \times \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0 \times \infty} \times \frac{q_1 q_2}{r^2} = 0.$$

This both **Assertion and Reason are false.**

52. Both the Assertion and Reason are false.

53. In 1 C charge, number of electrons = $6{\cdot}25 \times 10^{18}$. The relation given in Reason $q = ne^2$ is also false.

CONCEPTUAL PROBLEMS

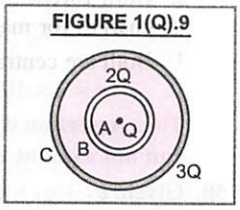
I. Electric charge and its properties

1. The electric charge of any body is actually a surplus or deficit of electrons. Why not protons ?

Sol. Electrons are loosely bound to atoms and can be readily exchanged during rubbing. Protons are firmly bound inside the nucleus. They cannot be easily detached. This is the reason why electric charge of any body is just a surplus or deficit of electrons.

2. A charge Q is kept in the inner cavity and a charge 2 Q is given to the inner shell. A charge 3 Q is given to the outermost shell, as shown in Fig. 1(Q).9. Find the charges at the surfaces A, B and C.

FIGURE 1(Q).9



Sol. Due to induction, $-Q$ charge is induced on the inner surface A of inner shell and $+Q$ charge is induced on the outer surface B of inner shell. Thus, charge on surface $A = -Q$.

Now charge on surface $B = 2Q + Q = 3Q$

Due to induction, $-3Q$ charge is induced on the inner surface of outer shell C and $+3Q$ charge is induced on the outer surface of outer shell C. Thus, total charge on surface $C = 3Q + 3Q = 6Q$.

3. A glass rod rubbed with silk is brought close to two uncharged metallic spheres in contact with each other, inducing charges on them as shown in Fig. 1(Q).10(*a*).

Describe what happens when

(i) the spheres are slightly separated and

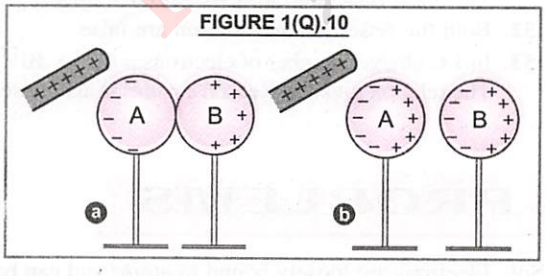
(ii) the glass rod is subsequently removed and finally

(iii) the spheres are separated far apart.

Sol. A glass rod rubbed with silk acquires positive charge. When this rod is brought close to two uncharged metallic spheres A and B in contact, negative charge is induced on the left side of sphere A and an equal positive charge is induced on the right side of sphere B, Fig. 1(Q).10(*a*).

(*i*) When the spheres are slightly separated, the induced charges are shown in Fig. 1(Q).10(*b*).

FIGURE 1(Q).10



(*ii*) As the glass rod is removed subsequently and separation is too small, the distribution of charges on A and B remains the same as shown in Fig. 1(Q).10(*b*).

(*iii*) When A and B are placed at a large distance, the positive and negative charges on each flow and neutralise, so that no charge is left on A or B.

4. (a) A comb run through one's hair attracts small bits of paper. Why ? What happens if the hair are wet or if it is a rainy day ?

(b) Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary ?

(c) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why ?

(d) A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why ?

(NCERT Solved Example)

Sol. (*a*) This is because the comb gets charged by friction. If the hair are wet, or if it is a rainy day, friction between the hair and the comb reduces. The comb does not get charged, and it will not attract small bits of paper.

(*b*) The special rubber tyres of air crafts are made slightly conducting so that electricity generated on account of friction between the tyres and the runway goes to earth.

(*c*) When a vehicle moves, its body gets charged on account of friction due to air. The tyres also accumulate the charge on account of friction between the tyres and the road. The metallic ropes from the vehicle touching the ground enable the accumulated charges to flow to earth. This would otherwise be hazardous to the inflammable materials.

(*d*) When a bird perches on single bare high power line, nothing happens to him as no current flows through his body because the circuit is not complete. In case of a person standing on ground, the circuit is completed with the ground through his body. Current flows through his body resulting in a fatal shock.

5. An insulating rod carries some net charge, and a copper sphere is neutral. The rod and the sphere do not touch. Can there be force of attraction/repulsion between the two?

Sol. When the charged rod is brought near the neutral sphere, the sphere gets oppositely charged by induction. Therefore, the sphere and the rod will

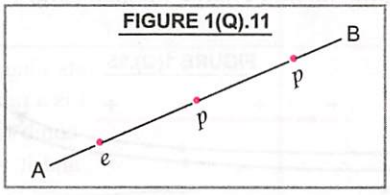
attract each other. However, force of repulsion between the two is not possible.

6. Neutral metal objects, especially in industry, are often coated with electrically charged paint or powder particles. How do these particles stick on the metal objects?

Sol. The electrically charged paint/powder particles charge the neutral metal surface oppositely by the process of induction. The particles stick on the metal objects on account of force of attraction between unlike charges.

7. Figure 1(Q).11 shows two protons (symbol *p*) and one electron (symbol *e*) on a straight line *AB*. What is the direction of net electrostatic force on the central proton?

FIGURE 1(Q).11



Sol. In Fig. 1(Q).11, the central proton is repelled by the other proton, along *BA* ; and it is attracted by electron again along *BA*. Therefore, net electrostatic force on central proton is along *BA*.

8. Give some points of dissimilarity and similarity between electrostatic forces and gravitational forces.

Sol. (*a*) **Dissimilarity**

1. Electrostatic forces are between two charges and gravitational forces are between two masses.
2. Electrostatic forces may be attractive or repulsive. But gravitational forces are always attractive.
3. Electrostatic force between two protons is 10^{36} times stronger than gravitational force between them.
4. Electrostatic forces operate over distances which are not large. Gravitational forces operate over very large distances.
5. Electrostatic force depends on the medium between the two charges. Gravitational force does not depend upon the medium between the two masses.

(*b*) **Similarity**

1. Both the forces obey inverse square law.
2. Both forces are proportional to product of charges (or masses) of interacting particles.
3. Both are central forces.
4. Both are conservative forces.

II. Coulomb's Law

9. If two objects repel one another, you know both carry either positive charge or negative charge. How would you determine whether these charges are positive or negative?

Sol. To determine the polarity of charge on the two objects, we bring one of the objects near a positively charged glass rod. If the object is repelled away from the rod, it must be positively charged (as like charges repel). However, if the object is attracted towards the glass rod, it must be negatively charged (as unlike charges attract).

10. Is coulomb's law in electrostatics valid in all situations ?

Sol. Coulomb's law is not applicable in all situations. It is appliable under the following conditions :

1. The electric charges must be at rest.
2. The electric charges must be point charges, *i.e.*, size of charges must be smaller than separation between the charges.
3. The separation between the charges must be greater than nuclear size (10^{-15} m) because if distance is $< 10^{-15}$ m, strong nuclear forces dominate over the electrostatic forces.

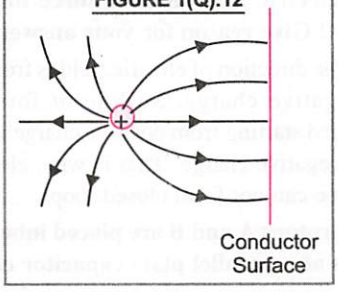
III. Electric field

11. A point charge + *Q* is placed in the vicinity of a conducting surface. Draw the electric field lines between the surface and the charge.

[CBSE 2017 (C)]

Sol. The electric field lines are always normal to the surface of a conductor, both while starting or ending on the conductor. This is shown in Fig. 1(Q).12.

FIGURE 1(Q).12

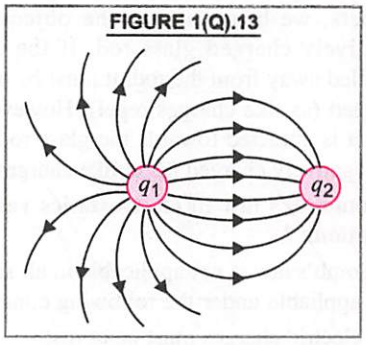


12. Draw the pattern of electric field lines when a point charge $-Q$ is kept near an uncharged conducting plate. (CBSE 2019)

Sol. An explained in the above question, the pattern of electric field lines will remain the same as shown in Fig. 1(Q).12 except that all arrows showing the direction are reversed as the charge is negative.

13. Fig. 1(Q).13 shows the electric field lines for two point charges separated by a distance. What are the signs of q_1 and q_2? Can you determine the ratio q_1/q_2?

FIGURE 1(Q).13



Sol. As the electric field lines emanate from q_1 and some of them end at q_2, therefore q_1 must be a positive charge and q_2 must be a negative charge.

As the number of lines leaving q_1 is twice the number of field lines terminating on q_2, therefore, magnitude of q_1 must be twice the magnitude of q_2, *i.e.*, $q_1/q_2 = 2$.

14. In defining electric field due to a point charge, the test charge has to be vanishingly small. How this condition can be justified, when we know that charge less than that of electron or a proton is not possible.

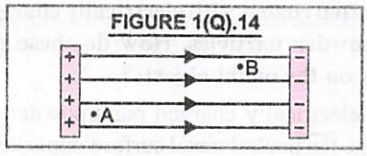
Sol. This is true that charge less than electron or proton is not possible but in macroscopic situations, source charge is much larger than charge on electron or proton, so the limit $q_0 \to 0$ for the test charge is justified.

15. Can electric field lines of force form closed loops ? Give reason for your answer.

Sol. No. The direction of electric field is from positive to negative charge. So line of force can be regarded starting from positive charge and ending on a negative charge. That is why, electric lines of force can not form closed loops.

16. Two protons A and B are placed inbetween the plates of a parallel plate capacitor charged to a potential difference V as shown in Fig. 1(Q).14. The forces on the two protons are identical. Is this statement true ?

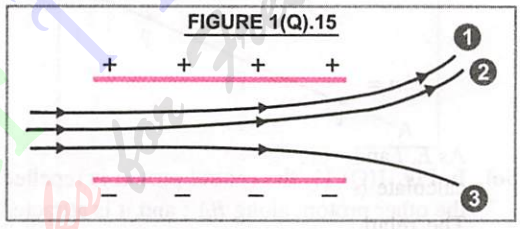
FIGURE 1(Q).14



Sol. *True.* This is because field between two parallel plates is uniform (E), charge on each proton is same (q). Therefore, force on each proton $F = qE$ is same.

17. Fig. 1(Q).15 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio ?

FIGURE 1(Q).15



(*a*) Suppose two particles have identical curved trajectories. Which of the following are necessarily true ?

(*i*) they have same charge (*ii*), they have same mass (*iii*) the charges have the same sign, (*iv*) they have the same *e/m* ratio.

(*b*) You are given the initial velocity v of a beam particle and the length of the capacitor l. What other measurement would enable one to find *e/m* ?

Sol. The charge on particles 1 and 2 must be negative and charge on particle 3 must be positive. We shall show that charge to mass ratio (*e/m*) is highest in case of particle 3.

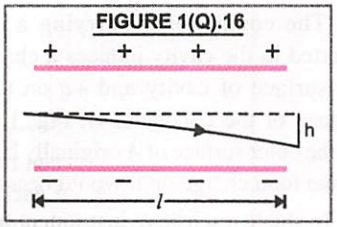
If we assume that the three charged particles have entered the electric field with the same velocity, then the deflection is proportional to *e/m i.e.*, charge to mass ratio of the particle.

As deflection of particle 3 is maximum, the (*e/m*) of this particle must be the highest.

(*a*) When two particles have identical curved trajectories, their charges must have the same sign and the particles must have the same *e/m* ratio.

(*b*) If *h* is the vertical displacement of particle at the end of capacitor plates as shown in Fig. 1(Q).16, then, from

FIGURE 1(Q).16



$$s = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{l}{v}\right)^2 = \frac{1}{2}\left(\frac{e}{m}\right)\frac{El^2}{v^2}$$

$$e/m = \frac{2v^2h}{El^2}$$

As *E*, *l* and *v* are known, by measuring *h*, we can calculate (*e*/*m*).

The relation shows that for given E, *l* and *v* ; deflection (*h*) ∝ (*e*/*m*). This is what we have stated above.

IV. Electric dipole

18. A point charge placed at any point on the axis of an electric dipole at some large distance experiences a force *F*. What will be the force acting on the point charge when its distance from the dipole is doubled.

Sol. As electric field on axial line varies as

$$E \propto \frac{1}{r^3} \text{ or } \frac{F}{q} \propto \frac{1}{r^3} \text{ or } F \propto \frac{1}{r^3}$$

So, when distance is doubled, force reduces to *F*/8.

V. Electric Flux and Gauss's Theorem

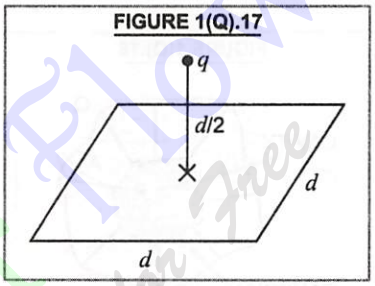
19. Two charges of magnitude – 2 *Q* and + *Q* are located at points (*a*, 0) and (4 *a*, 0) respectively. What is the electric flux due to charges through a sphere of radius '3 *a*' with its centre at the origin. (CBSE 2013)

Sol. Here, only the charge – 2 *Q* is enclosed by the sphere of radius 3 *a* so, By Gauss theorem

$$\phi_E = \frac{-2Q}{\epsilon_0}$$

20. (*a*) Define electric flux. Is it a scalar or a vector ? A point charge *q* is at a distance *d*/2 directly above the centre of a square of side *d*, as shown in Fig. 1(Q).17. Use Gauss's Law to obtain the expression for the electric flux through the square.

FIGURE 1(Q).17



(*b*) If the point charge is now moved to a distance *d* from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected. (CBSE 2018)

Sol. (*a*) Electric flux over an area in an electric field is a measure of the number of electric field lines crossing this area. It is a **scalar** quantity.

According to Gauss's theorem, electric flux passing through six faces of the cube is

$$\phi = \frac{q}{\epsilon_0}$$

The given square is one face of the cube. Therefore, electric flux through the square is

$$\phi' = \frac{\phi}{6} = \frac{q}{6\epsilon_0}$$

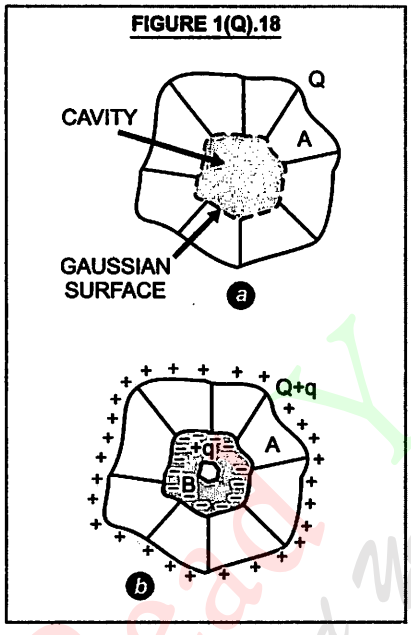
(*b*) When point charge is moved to a distance *d* from the centre of the square and the side of the square is doubled there will be no change in electric flux through the square. It will remain the same, *i.e.*, $\phi' = \frac{q}{6\epsilon_0}$

Q. 21. (a) A conductor A with a cavity as shown in Fig. 1(Q).18 (*a*) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor.

(b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is (Q+q), [Fig. 1(Q).18 (*b*)].

(c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.

FIGURE 1(Q).18



Sol. (*a*) We know, the net field inside a charged conductor is zero, *i.e.*, $\vec{E} = 0$, inside. Let us choose a Gaussian surface lying wholly within the conductor and enclosing the cavity, Fig. 1(Q).18(*a*). According to Guass's law,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0} = 0 \qquad (\because \vec{E} = 0, \text{inside})$$

$\therefore q = 0$, *i.e.*, charge inside the cavity is zero. Hence the entire charge Q on the conductor must appear on the outer surface of the conductor.

(*b*) The conductor B carrying a charge $+q$ inserted in the cavity induces a charge $-q$ on the surface of cavity and $+q$ on the outside surface of the conductor A, Fig. 1(Q). 18(*b*). As the outer surface of A originally had a charge Q, the total charge on it would become $(Q+q)$.

(*c*) To shield a sensitive instrument from strong electrostatic fields, the instrument must be put in a metallic cover.

Q. 22. (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e. where $\vec{E} = 0$) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Sol. (*a*) To prove this, suppose the equilibrium of test charge is *stable*. When the test charge is displaced from the null point in any direction, it must experience a *restoring force towards the null point.* This indicates that there is net inward flux of electric field through a closed surface around the null point. But, according to Gauss's law, the electric flux through a surface not enclosing any charge must be zero. Hence the equilibrium cannot be stable.

(*b*) The middle point of the line joining two like charges is a null point. When we displace a test charge slightly along the line, a restoring force tends to bring the test charge back to the centre. But when we displace the test charge normal to the line, the net force on the test charge takes it further away from the null point. Hence the equilibrium cannot be stable.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Electric charge and its properties

1. What is the cause of charging ? (Jharkhand Board 2011)

Ans. The cause of charging is actual transfer of electrons from one body to the other.

2. An isolated conducting sphere is given a positive charge. Does its mass increase, decrease or remain the same ?

Ans. Its mass decreases slightly as it loses some electrons.

3. What is meant by quantization of charge ? (Bihar Board 2011)

Ans. Charge on any body or particle can be integral multiple of charge on an electron ($-e$), *i.e.*,

$q = \pm ne$, where $n = 1, 2, 3, ...$

4. What do you mean by conservation of electric charge ?

Ans. Conservation of electric charge means that the total charge on an isolated system remains unchanged with time.

5. Is the total charge of the universe conserved ?

Ans. Yes, charge conservation is a global phenomenon.

6. Name any two basic properties of electric charge. (Bihar Board 2012)

Ans. (*i*) Quantization of charge (*ii*) Conservation of charge.

7. What is the value of charge on an electron ? Is a charge less than this value possible ? (Manipur Board 2011)

Ans. $e = -1{\cdot}6 \times 10^{-19}$ C. No, charge less than this value does not exist.

8. Give two points of distinction between charge and mass.

Ans. Charge can be positive, negative or zero. But mass is a positive quantity. Further, electric charge is always conserved. But mass is not conserved as it can be changed into energy and vice-versa.

9. Can ever photons have a charge ? If not why ?

Ans. No, photons can never have a charge. This is because charge cannot exist without rest mass.

10. Which of the following quantities depends on state of rest or motion of a body : (*a*) mass (*b*) length (*c*) time (*d*) charge (*e*) charge density and (*f*) phase ?

Ans. Mass, length, time and charge density depend on state of rest or motion of a body. However, charge and phase are invariant.

11. Is the force acting between two point charges q_1 and q_2 kept at some distance in air, attractive or repulsive when :
(*i*) $q_1q_2 > 0$ (*ii*) $q_1q_2 < 0$ (CBSE 2007, 2003)

Ans. (*i*) When $q_1q_2 > 0$, the force is repulsive.
(*ii*) When $q_1q_2 < 0$, the force is attractive.

II. Coulomb's Law

12. Does Coulomb's law of electric force obey Newton's third law of motion ?

Ans. Yes, it obeys. Forces exerted by two charges on eachother are always equal and opposite.

13. Is the electric force between two electrons greater than the gravitational force between them ? If so, by what factor ?

Ans. Yes, electric force between two electrons is greater than gravitational force between them by a factor of 10^{42}.

14. Electrostatic forces are much stronger than gravitational forces. Give one example.

Ans. A charged glass rod can lift a piece of paper against the gravitational pull of earth on this piece.

15. What is the dimensional formula for ϵ_0 ?

Ans. $[M^{-1} L^{-3} T^4 A^2]$.

16. Write down the value of absolute permittivity of free space. (Pb. Board 1996)

Ans. Absolute permittivity of free space $\epsilon_0 = 8{\cdot}85 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$.

17. What is the relevance of large value of K (= 81) for water ? (Chhatisgarh Board 2011)

Ans. It makes water a great solvent. This is because binding force of attraction between oppositely charged ions of the substance in water becomes 1/81 of the force between these ions in air.

18. Force of attraction between two point charges placed at a distance d is F. What distance apart should they be kept in the same medium so that force between them is F/3 ? (CBSE 2011)

Ans. As $F \propto 1/r^2$ $\therefore$ New distance = $\sqrt{3}\,r$.

19. In coulomb's law, on what factors does the value of electrostatic force constant K depend ? (Pb. Board 2011)

Ans. The value of K depends on nature of medium separating the charges and on the system of units.

III. Superposition Principle

20. When two charges q_1 and q_2 are kept at some distance apart, force acting between these charges is F. If a third charge q_3 is placed quite close to q_2, what will happen to the force between q_1 and q_2 ?

Ans. As force b7etween any two charges doesn't depend upon the presence of any other charge, so force between F_1 and F_2 will remain F only.

21. Does the coulomb force that one charge exerts on another, change if other charges are brought nearby ?

Ans. No, the coulomb force due to one charge on another charge is not changed.

IV. Eletric field

22. A point charge q is placed at the origin. How does the electric field due to the charge vary with distance r from the origin ?

Ans. Electric field varies inversely as square of distance from the point charge.

23. Write the dimensional formula of electric field.

Ans. $[MLT^{-3}A^{-1}]$

24. A proton is placed in a uniform electric field directed along the positive x-axis. In which direction will it tend to move ? (CBSE 2011)

Ans. As proton is positively charged, it will tend to move along $+x$ axis, *i.e.*, along the direction of electric field.

25. Name any four vector fields.

Ans. Electric field, magnetic field, gravitational field and flow field (of a liquid) are vector fields.

26. What is the use of the concept of electric field intensity ?

Ans. It enables us to find force on any charge in the field as $\vec{F} = q\vec{E}$

27. Name the physical quantity whose SI unit is NC^{-1} ? (Pb. Board 2011)

Ans. Electric field intensity.

28. Why is electric field intensity inside a charged conductor zero ?

Ans. This is because there are no electric lines of force inside the body of a conductor.

29. Two point charges of $+3\ \mu C$ each are 100 cm apart. At what point on the line joining the charges will the electric intensity be zero ?

Ans. At the centre of the two charges.

30. What is nature of symmetry of field due to a point charge ?

Ans. The field due to point charge has spherical symmetry.

31. When is an electric line of force straight ?

Ans. Electric line of force is straight in the field of a single charge.

32. When is an electric line of force curved ?

Ans. Electric line of force is curved in the field of more than one point charges.

V. Electric dipole

33. Define electric dipole moment. Write its SI unit. (Jharkhand Board 2011 ; Raj. Board 2011 ; CBSE 2011)

Ans. Electric dipole moment is equal to product of magnitude of either charge and distance between them. Its SI unit is C-m.

34. What is an ideal dipole ?

Ans. For a given dipole moment $|\vec{p}| = q(2a)$, when the charge q gets larger and larger, while the separation $(2a)$ between two charges gets smaller and smaller, we obtain an ideal dipole. Thus, an ideal dipole has almost no size.

35. In which orientation, a dipole placed in a uniform field is in (*i*) stable (*ii*) unstable equilibrium ? (CBSE 2010)

Ans. If θ is angle between $\vec{p}$ and $\vec{E}$, then $\theta = 0°$ for stable equilibrium, $\theta = 180°$ for unstable equilibrium.

36. How does a torque affect the dipole in an electric field ?

Ans. Torque tries to align the dipole along the field.

37. Which rule gives you the direction of torque ?

Ans. The direction of torque is given by right hand screw rule.

38. What happens when an electric dipole is held in a non uniform electric field ?

Ans. It experiences some net force and some net torque.

39. At what points, dipole field intensity is parallel to the line joining the charges ?

Ans. At any point on axial line or equatorial line of dipole.

40. When does an electric dipole placed in a non-uniform electric field experience a zero torque but non-zero force.

Ans. When the dipole is placed parallel to the non-uniform electric field.

41. Will an electric dipole have translational motion when placed in a non-uniform electric field ? Give reason for your answer.

Ans. Yes, in a non-uniform electric field, an electric dipole experiences unequal forces at its ends. The two forces, add up to give a resultant force, which gives a translatory motion to the dipole.

VI. Electric Flux and Gauss's Theorem

42. Name the principle which is mathematical equivalent of coulomb's law and super-position principle.

Ans. Gauss's theorem of electrostatics.

43. If the radius of the Gaussian surface enclosing a charge q is halved, how does the electric flux through the Gaussian surface change ? **(CBSE 2008)**

Ans. As $\phi = \frac{q}{\epsilon_0}$, so flux does not depend upon the radius of Gaussian surface, it will remain unchanged.

44. A charge q is placed at the centre of a cube of side l what is the electric flux passing through two opposite faces of the cube ? **(CBSE 2012)**

Ans. Flux through each face $= \frac{q}{6\epsilon_0}$

$\therefore$ flux thorugh two opposite faces

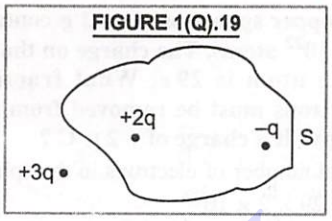
$$= \frac{q}{6\epsilon_0} + \frac{q}{6\epsilon_0} = \frac{2q}{6\epsilon_0} = \frac{q}{3\epsilon_0}$$

45. Two concentric spherical shells of radii R and $2R$ are given charges Q_1 and Q_2 respectively. The surface charge densities of the outer surfaces are equal. Determine the ratio $Q_1 : Q_2$. **(CBSE 2013)**

Ans. Here, $\frac{Q_1}{Q_2} = \frac{4\pi R^2 \sigma}{4\pi (2R)^2 \sigma} = 1:4$

46. Fig. 1(Q).19 shows three point charges $+2q$, $-q$ and $+3q$. What is the electric flux due to this configuration through the surface S ? **(CBSE 2010)**

FIGURE 1(Q).19



Ans. $\phi_E = \frac{2q - q}{\epsilon_0} = \frac{q}{\epsilon_0}$

47. What is the relation between electric intensity and electric flux ?

Ans. The surface integral of electric field intensity over a closed surface in free space (*i.e.*, electric flux) is $1/\epsilon_0$ times the total charge q enclosed by the surface, *i.e.*, $\phi = \oint \vec{E}.\vec{ds} = q/\epsilon_0$

48. What is the number of electric lines of force that radiate outwards from one coulomb of charge in vacuum ?

Ans. From $\phi = \frac{q}{\epsilon_0}$, when $q = 1$ C,

$$\phi = \frac{1}{8{\cdot}85 \times 10^{-12}} = 1{\cdot}13 \times 10^{11}$$

49. What is the SI unit of surface integral of electric field ?

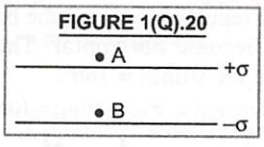
Ans. $N\,m^2\,C^{-1}$ or volt-metre, as the surface integral of electric field represents electric flux.

50. An electric dipole of dipole moment 20×10^{-6} Cm is enclosed by a closed surface. What is the net electric flux coming out of the surface ?

Ans. Net electric flux coming out of closed surface is zero, because net charge on electric dipole is zero.

51. Two plane sheets of charge densities $+\sigma$ and $-\sigma$ are kept in air as shown in Fig. 1(Q).20. What are the electric field intensities at points A and B ? **(Kerala Board 2012)**

FIGURE 1(Q).20



Ans. At A, $E = 0$, and At B, $E = \frac{\sigma}{\epsilon_0}$

SHORT ANSWER QUESTIONS — Carrying 2 marks

I. Electric charge and its properties

1. A copper sphere of mass 2 g contains nearly 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a charge of + 2 μ C ?

Ans. Total number of electrons in the sphere

$= 29 \times 2 \times 10^{22}$

No. of electrons removed

$$= \frac{q}{e} = \frac{2 \times 10^{-6}}{1{\cdot}6 \times 10^{-19}} = 1{\cdot}25 \times 10^{13}$$

Fraction of electrons removed

$$= \frac{1{\cdot}25 \times 10^{13}}{29 \times 2 \times 10^{22}} = \mathbf{2{\cdot}16 \times 10^{-11}}$$

2. Give four properties of electric charges.

Ans. (*i*) Like charges repel and unlike charges attract each other.

(*ii*) Charge is quantized.

(*iii*) Charge is conserved.

(*iv*) Charge on a body is not affected by its motion.

II. Coulomb's Law

3. Dielectric constant of a medium is unity. What will be its permittivity ?

Ans. We know that dielectric constant of a medium is

$$K = \in_r = \frac{\in}{\in_0}$$

$$\therefore \quad \in = K \in_0 = 1 \times 8{\cdot}854 \times 10^{-12}$$
$$= \mathbf{8{\cdot}854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}}$$

4. Two small balls having equal positive charge q coulomb are suspended by two insulating strings of equal length l metre from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity. What is the angle between the two strings and the tension in each string ?

Ans. In a satellite, there is condition of weightlessness. Therefore, $mg = 0$. On account of electrostatic force of repulsion between the balls, the strings would become horizontal. Therefore, angle between the strings = **180°.**

Also, tension in each string = force of repulsion

$$T = \frac{1}{4\pi\in_0} \frac{q^2}{(2l)^2}\ \text{N}$$

5. An attractive force of 5 N is acting between two charges of + 2 μ C and – 2 μ C placed at some distance. If the charges are mutually touched and placed again at the same distance, what will be the new force, between them ?

Ans. On touching, charges neutralise. Therefore,

$F = 0$

6. Two point charges of + 2 μ C and + 6 μ C repel each other with a force of 12 N. If each is given an additional charge of – 4 μ C, what will be the new force ?

Ans. $q_1 = +2\ \mu C,\ q_2 = +6\ \mu C,\ F = 12$ N

$q_1' = +2-4 = -2\ \mu C\ ;\ q_2' = +6-4 = 2\ \mu C,$

$F' = ?$

$$\frac{F'}{F} = \frac{(q_1')(q_2')}{q_1 q_2} = \frac{(-2)\,(2)}{(2)\,(6)} = -\frac{1}{3}$$

$$F' = \frac{-F}{3} = \frac{-12}{3} = \mathbf{-4\ N\ (attractive)}$$

7. What is the importance of expressing coulomb's law in vector form ?

Ans. As in vector form $\hat{r}_{12} = -\hat{r}_{21} \ \therefore\ \vec{F}_{12} = -\vec{F}_{21}$

It shows two charges exert equal and opposite forces on each other. So Newton's third law is obeyed.

As Coulombian force acts along $\vec{F}_{12}$ and $\vec{F}_{21}$ *i.e.*, along the line joining the centres of two charges, so they are central forces.

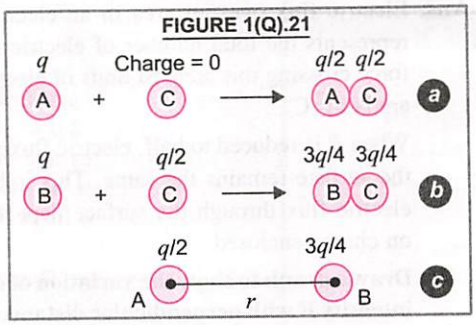
III. Superposition Principle

8. Two identical metallic spheres A and B, each carrying a charge q repel each other with a force F. A third metallic uncharged sphere C of the same size is made to touch the spheres A and B alternately and then removed away. What is the force of repulsion between A and B ?

Ans. Let r be the distance between two spheres A and B. Then force of repulsion between spheres A and B is

$$F = \frac{1}{4\pi\in_0} \frac{q \times q}{r^2} \qquad ...(1)$$

When uncharged sphere C of same size is placed in contact with sphere A, Fig. 1(Q).21(*a*), each sphere acquires a charge



$$= \frac{q+0}{2} = \frac{q}{2}$$

When the sphere C is placed in contact with sphere B, Fig. 1(Q).21(b), the charge is redistributed equally between B and C. Thus, charge on B or C is

$$= \frac{1}{2}\left[q + \frac{q}{2}\right] = \frac{3q}{4}$$

Refer to Fig. 1(Q).21(c), Net force of repulsion between spheres A and B is

$$F' = \frac{1}{4\pi\epsilon_0}\frac{(q/2)(3q/4)}{r^2} = \frac{3}{8}F$$

IV. Electric field

9. Why no two electric lines of force can intersect each other ? (Hr. Board 2007)

Ans. This is because at the point of intersection, we can draw two tangents to the two lines of force. This would mean two directions of electric field intensity at the point of intersection, which is not possible.

10. Give two properties of electric field lines.

Ans. **1.** Electric field lines are continuous curves. They start from a positively charged body and end at a negatively charged body. They are continuous but do not form closed loops as there are no lines of force inside the charged body.

2. Tangent to the electric field line at any point gives the direction of electric field intensity at that point.

11. A charged particle is free to move in an electric field. Will it always move along an electric line of force ?

Ans. If charged particle was initially at rest, it will move along the direction of electric field. If initial velocity of the charged particle makes a certain angle with electric field, then the charged particle will not move along the line of force.

V. Electric dipole

12. What does $(q_1 + q_2) = 0$ signify ?

Ans. $q_1 + q_2 = 0$, $q_1 = -q_2$. Such a system of point charges is called an electric dipole. It is a pair of equal and opposite point charges separated by a small distance.

13. An electric dipole is placed at rest in a uniform electric field, and released. How will it move ?

Ans. A torque will develop and align the electric dipole in the direction of the electric field, if the dipole is not aligned already. The dipole shall not move as net force on the dipole is zero.

14. Define the term electric dipole moment. Is it scalar or vector ?

Ans. Electric dipole moment (p) is the product of either charge ($\pm q$) and the distance ($2a$) between the charges, *i.e.*, $p = q(2a)$. It is a vector quantity, directed from $-q$ to $+q$.

15. What is the direction of field intensity at a point (*i*) on axial line of dipole and (*ii*) on equatorial line of dipole ?

Ans. **For a point on the axial line of dipole,** the direction of electric field intensity $\vec{E}$ is along a line parallel to the axis of dipole directed along the direction of dipole moment $\vec{p}$.

For a point on the equatorial line of dipole, the direction of electric field $\vec{E}$ is along a line parallel to the axis of dipole directed opposite to the direction of dipole moment $\vec{p}$.

16. What is the nature of symmetry of the electric field due to (*i*) point charge and (*ii*) electric dipole ?

Ans. The electric field due to a point charge has **spherical symmetry** with point charge at the centre. It is so because, at equal distances from the point charge, field intensity is equal.

The electric field due to a dipole has a **cylindrical symmetry.** The axis of the cylinder passes through the dipole axis. It is so because, the electric field due to dipole will be same at every point on the surface of a right circular cylinder with electric dipole as the axis.

17. When an electric dipole is suspended in a uniform electric field, then under what conditions the dipole is in (*i*) stable equilibrium and (*ii*) unstable equilibrium.

Ans. The dipole in an electric field will be in stable equilibrium if the following conditions are satisfied :

(*i*) The resultant force on dipole is zero, *i.e.*, there is no translatory motion of dipole. (*ii*) The torque on dipole is zero, *i.e.*, there is no rotatory motion of dipole. (*iii*) The potential energy of dipole is minimum.

It will be so when dipole is aligned along the direction of electric field.

The dipole will be in unstable equilibrium if (*i*) the resultant force on dipole is zero. (*ii*) the torque on the dipole is zero. (*iii*) the potential energy of dipole is maximum.

It will be so when dipole is aligned opposite to the direction of electric field.

18. Show that when an electric dipole is placed in a uniform electric field $\vec{E}$, potential energy U is given by $U = -\vec{p}\cdot\vec{E}$.

Ans. Refer to Art. 1.36.

19. An electric dipole is held at an angle θ in a uniform electric field *E*. Will there be any (*i*) net translating force (*ii*) torque acting on it ? Explain.

Ans. (*i*) There will be no net translating force.

(*ii*) Torque on the diple ; $\tau = pE \sin\theta$. It will align the dipole along the field. Once the dipole is aligned, $\tau = 0$. For details, refer to Art. 1.35.

20. Charge q_1 is inside the Gaussian surface ; charge q_2 just outside the surface. Does the electric flux through the surface depend on q_1 ? Does it depend on q_2 ? Explain.

Ans. Electric flux through the surface depends on q_1 (as q_1 is inside the Gaussian surface). But it does not depend on q_2, it being outside Gaussian surface. q_2 does not contribute to electric flux.

21. Using Gauss's law, derive an expression for the electric field intensity at any point near a uniformly charged thin wire of charge/length = λ C/m. (CBSE 2008)

Ans. Refer to Art. 1.35.

22. Define electric flux. Write its SI unit. A charge *q* is enclosed by a spherical surface of radius *R*. If the radius is reduced to half, how would the electric flux through the surface change ? [CBSE 2009, 2018 (C)]

Ans. Electric flux over an area in an electric field represents the total number of electric lines of force crossing this area. SI units of electric flux are N m^2 C^{-1}.

When *R* is reduced to half, electric flux through the surface remains the same. This is because electric flux through the surface depends only on charge enclosed.

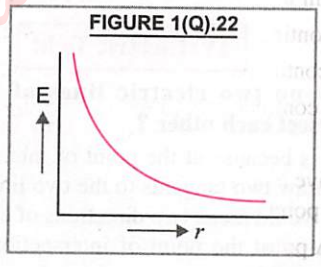
23. Draw a graph to show the variation of electric intensity *E* with perpendicular distance *r* from an infinite line of charge density λ C/m. Find work done in bringing a charge *q* from perpendicular distance r_1 to r_2 ($r_2 > r_1$). (CBSE 2018)

Ans. As is known, electric intensity *E* at a perpendicular distance *r* from an infinite line of charge density λ C/m is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

The variation of *E* with *r* is as shown in Fig. 1(Q).22. Work done in bringing charge *q* from distance r_1 to r_2 is

FIGURE 1(Q).22



$$dW = \Sigma F\, dr = qE\, dr = q\frac{\lambda}{2\pi\epsilon_0 r}dr$$

$$W = \int_{r_1}^{r_2} \frac{\lambda q}{2\pi\epsilon_0 r}\, dr$$

$$W = \frac{\lambda q}{2\pi\epsilon_0}[\log r]_{r_1}^{r_2} = \frac{\lambda q}{2\pi\epsilon_0}\log\left(\frac{r_2}{r_1}\right)$$

SHORT ANSWER QUESTIONS — Carrying 3 marks

1. What do you understand by (*a*) quantisation of charge and (*b*) conservation of charge. Explain each with illustrations. (HP Board 2013) **[Arts. 1.8 and 1.10]**

2. Explain Coulomb's law in vector form and discuss the importance of this study. **[Art. 1.13]**

3. Define *SI* unit of charge, electromagnetic unit of charge and electrostatic unit of charge. State the relation between them. **[Art 1.14]**

4. What is meant by Dielectric constant and absolute electrical permittivity of medium. How

are they related ? State *SI* units of dielectric constant and absolute electrical permittivity. [Art. 1.15]

5. Explain super position principle. Apply it to obtain total force on a point charge due to n discrete charges. [Art. 1.16]
6. Establish relation between electric field strength and force. [Art. 1.20]
7. Explain what is meant by an electric line of force ? Give its two important properties ? (Raj. Board 2011) [Arts. 1.26 & 1.27]
8. Give two properties of electric lines of force. Sketch them for an isolated positive point charge and an electric dipole. (Kerala Board 2012) [Art. 1.26]
9. State and prove Gauss's law in electrostatics. (Bihar Board 2012, Jharkhand Board 2011, J & K Board 2011) [Art. 1.39]
10. State Gauss's Theorem in electrostatics and deduce coulomb's law from Gauss's theorem. (Pb. Board 2011) [Art. 1.41]

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. How will you show that there are two kinds of charges ? Explain with proper illustrations. [Art. 1.3]
2. What do you understand by the phenomenon of charging by induction. Give some illustrations to support your answer. [Art. 1.7]
3. What is meant by continuous charge distribution ? Apply superposition principle to obtain total force on a point charge due to
   (*i*) continuous line distribution of charge
   (*ii*) continuous surface distribution of charge and
   (*iii*) continuous volume distribution of charge. [Art. 1.17 & 1.18]
4. Derive an expression for electric field intensity at a point due to
   (*i*) A point charge [Art. 1.21]
   (*ii*) A group of charges [Art. 1.22]
   (*iii*) Continuous charge distribution [Art. 1. 23]
5. Derive general expression for rectangular components of electric intensity due to point charge in space. [Art. 1. 24]
6. Derive an expression for dipole field intensity at any point on (*i*) axial line of dipole (CBSE 2017, Raj. Board 2012) [Art. 1. 31]
   (*ii*) equatorial line of dipole (CBSE 2019, 2013, MP Board 2011) [Art. 1. 32]
   (*iii*) at any point [Art. 1. 33]
   (*iv*) axis of uniformly charged ring [Art. 1. 34]
7. Derive an expression for force and torque acting on an electric dipole in a uniform two dimensional electric field. In which situation, torque on the dipole is (*i*) maximum and (*ii*) minimum. (CBSE 2017, HP Board 2013, WB Board 2012) [Art. 1.35]
8. Derive an expression for potential energy of an electric dipole in a uniform electric field. In which situation, the potential energy of dipole is (*i*) maximum and (*ii*) minimum ? (Raj. Board 2011) [Art. 1.36]
9. Using Gauss's theorem, derive an expression for electric field intensity at a point due to
   (*i*) a line of charge. [Art. 1.42]
   (*ii*) a uniformly charged thin spherical shell. (CBSE 2011, 2013, CBSE (F) 2017, Jharkhand Board 2012, Kerala Board 2012, Manipur Board 2011, Pb. Board 2011) [Art. 1.43]
   (*iii*) a charged solid sphere. (Chhatisgarh Board 2011) [Art. 1.44]
   (*iv*) an infinite plane sheet of charge. (CBSE 2017, 2016, Uttarakhand Board 2012, Manipur Board 2012, Raj. Board 2011) [Art. 1.45]
   (*v*) two parallel sheets of charge with charge densities, σ and $-\sigma$. (CBSE 2017, 2009) [Art. 1.46]
10. State Gauss's Law. Using this law, obtain the expression for the electric field due to an infinitely long straight conductor of linear charge density λ. [CBSE Sample Paper 2019-20, CBSE 2018, 2017 (C)] [Arts. 1.39 and 1.42]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. The vast amount of charge in an object is usually hidden as the object contains equal amounts of positive charge and negative charge. When a glass rod is rubbed with a piece of silk, some electrons from glass are transferred to silk piece. Therefore, glass rod acquires positive charge and silk piece acquires an equal negative charge. The charge acquired by a body is an integral multiple of basic unit of charge of an electron/proton represented by (e), *i.e.*, $q = ne$. This property is called **quantisation of charge.** Further, total charge on an isolated system always remains constant or conserved. This property is called **conservation of charge.** We can compare the role of charge to the role of mass in gravitation.

Read the above paragraph carefully and answer the following very short and short answer questions :

1. Which of the following is not an insulator ? glass, ebonite, paper and copper ?
2. How is the vast amount of charge in every object hidden ?
3. What are the two basic properties of electric charge ?
4. What is the charge acquired by a body from which one million electrons have been removed ?

CASE 2. According to Gauss's theorem in electrostatics, total normal electric flux over a closed surface S in vacuum is $1/\epsilon_0$ times the total charge Q contained inside S, *i.e.*,

$$\phi_E = \oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

The charges enclosed may be point charges or even continuous charge distributions. Further, location of Q inside the closed surface does not affect the electric flux.

An imaginary closed surface at every point of which electric intensity is same is called **Gaussian surface.** It may be an empty space or it may be embedded in a solid body.

If the medium surrounding the charges has a dielectric constant K, then

$$\phi_E = \frac{Q}{K\epsilon_0}$$

Read the above passage carefully and answer the following very short answer and short answer questions.

5. A cube of side of 0·1 m encloses charges of 3 μC and – 5 μC. What is electric flux from any one face of the cube ?
6. In the above question, a charge of + 2 μC is held outside the cube. What is the electric flux from any one surface of the cube ?
7. Point charges of 1 μC, 2 μC, 3 μC and 4 μC are held outside a conducting sphere and another charge of 10 μC is held inside the sphere. What is the total electric flux from the sphere ?
8. Algebraic sum of point charges present inside a hollow sphere is zero. A point charge of – 10 μC is now held outside the hollow sphere. The total electric flux from the sphere will be.

CASE 3. From the knowledge of electric intensity $(\vec{E})$ at any point $(\vec{r})$, we can calculate the magnitude and direction of force experienced by any charge q_0 held at that point

$$\vec{f}(\vec{r}) = q_0 \vec{E}(\vec{r})$$

This is the physical significance of electric field.

As against the electric lines of force which are continuous but not closed loops, magnetic lines of force are continuous, endless closed loops.

An electric dipole consists of a pair of equal and opposite point charges separated by some small distance. The **dipole moment** $(\vec{p})$ is a measure of strength of electric dipole $\vec{p} = q(2\vec{r})$. The direction of $(\vec{p})$ is from negative charge to positive charge. On axial line of electric dipole

$$|\vec{E}| = \frac{|\vec{p}|2r}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

The direction of $\vec{E}$ is opposite to direction of $\vec{p}$. On equatorial line of electric dipole,

$$|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}}$$

Read the above passage carefully and answer the following very short and short questions.

9. A charge q_0 is held in an electric field of intensity $\vec{E}$. What is the force on the charge ?
10. The algebraic sum of charges on an electric dipole is zero. Why does it have an electric field ?
11. Electric field $\vec{E}$ on axial line of electric dipole is $\vec{E} \propto r^n$. What is the value of n ?
12. Give the magnitude and direction of electric intensity due to an electric dipole at any point on equatorial line of electric dipole.

ANSWERS

1. Copper is not an insulator.
2. The vast amount of charge in an object is usually hidden as the object contains equal amounts of positive charge and negative charge. The object is said to be electrically neutral, *i.e.*, it contains no charge.
3. Two basic properties of electric charge are :
   (*i*) Quantisation of charge
   (*ii*) Conservation of charge.
4. $Q = ne = 10^6 \times 1{\cdot}6 \times 10^{-19}$ C $= \mathbf{1{\cdot}6 \times 10^{-13}}$ C
   As electrons have been removed, charge on the body is positive.
5. A cube has six faces. According to Gauss's theorem in electrostatics, from one face of the cube,
$$\phi_E = \frac{1}{6}\frac{\Sigma q}{\epsilon_0} = \frac{1}{6}\frac{(3-5)\mu C}{\epsilon_0}$$
$$= \frac{-2\times10^{-6}}{6\epsilon_0} = -\frac{1\times10^{-6}}{3\hat{1}_0}$$
6. Any charge present outside the cube does not affect the electric flux from the cube.
7. As charges outside the sphere do not affect the electric flux, therefore,
$$\phi_E = \frac{q}{\epsilon_0} = \frac{10\times10^{-6}}{\epsilon_0} = \frac{10^{-5}}{\epsilon_0}$$
8. As algebraic sum of charges inside a hollow sphere is zero, $\phi_E = 0$. Further, presence of a charge of $-10\ \mu$C outside the hollow sphere does not affect the electric flux through the sphere. Therefore, $\phi_E = 0$.
9. $\vec{F} = q_0 \vec{E}$
10. The electric dipole has an electric field because charges of the dipole do not lie at the same point.
11. Electric field on axial line of electric dipole is proportional to $\frac{1}{r^3}(=r^{-3})$. Therefore, $n = -3$.
12. On equatorial line of electric dipole,
$$|\vec{E}| = \frac{\vec{p}}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}}$$
$\vec{E}$ is in a direction opposite to the direction of $\vec{p}$.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. According to Coulomb's law, the force of interaction between any two point charges is directly proportional to the product of charges and inversely proportional to the square of distance between them. The force acts always along the line joining the two charges.
$$F = \frac{k|q_1||q_2|}{r^2}$$
Here, k is electrostatic force constant.
In CGS system, $k = 1$. In SI, $k = 9 \times 10^9$ Nm^2 C^{-2}
$$k = \frac{1}{4\pi\epsilon_0}, \text{ where}$$
ϵ_0 is called absolute electric permittivity of free space.

Based on the above paragraph, answer questions no. 1 to 4 :

1. Units of ϵ_0 are
   (*a*) C^2 Nm^{-2}
   (*b*) C^{-2} Nm^2
   (*c*) C^2 N^{-1} m^{-2}
   (*d*) C^{-2} Nm^2
2. Value of absolute electrical permittivity of free space is

(*a*) $8{\cdot}85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

(*b*) $8{\cdot}85 \times 10^{12}\ C^2\ N^{-1}\ m^{-2}$

(*c*) $9 \times 10^{9}\ C^2\ N^{-1}\ m^{-2}$

(*d*) $9 \times 10^{-9}\ C^2\ N^{-1}\ m^{-2}$

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false and R is also false.

3. Assertion. The value of electrostatic force constant depends on nature of medium separating the charges and also on the system of units.

Reason. In SI, $k = 9 \times 10^9\ Nm^2\ C^{-2}$. In CGS system $k = 1$.

4. Assertion. Coulomb's law corresponds to Newton's law of gravitation.

Reason. According to Newton's law of gravitation

$$F = \frac{G m_1 m_2}{r^2}$$

The symbols have standard meanings.

CASE 2. An electric line of force is a path, straight or curved, such that tangent to the path at any point gives us electric field intensity at that point. No two electric lines of force can intersect each other.

Two equal and opposite charges separated by a small distance are said to form an **electric dipole.** Its dipole moment is $\vec{p} = q(2\vec{a})$. Direction of $\vec{p}$ is from negative charge to positive charge, $\vec{p}$ is measured in Coulomb-meter.

Based on the above paragraph, answer questions no. 5 to 8 :

5. Can two electric lines of force intersect each other ?

(*a*) No, never (*b*) Yes, always

(*c*) sometimes (*d*) cannot predict

6. Dipole moment $\vec{p}$ is equal to

(*a*) $q\ (2\ a)$ (*b*) $q/2\ a$

(*c*) $\frac{2a}{q}$ (*d*) none of these

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is correct explanation of A.

(*b*) Both A and R are true but R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

7. Assertion. Electric dipole moment is a scalar.

Reason. $\vec{p} = q(2a)$.

8. Assertion. No two electric lines of force can intersect each other.

Reason. Because they lie far apart.

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*a*) **4.** (*a*) **5.** (*a*) **6.** (*a*) **7.** (*d*) **8.** (*c*)

HINTS/EXPLANATIONS For Difficult Questions

1. From $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$

$$\epsilon_0 = \frac{q_1 q_2}{4\pi F r^2} = \frac{C^2}{Nm^2}$$

$$= C^2\ N^{-1}\ m^{-2}.$$

2. $\epsilon_0 = 8{\cdot}85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

3. Both, the Assertion and Reason are true, and Reason is correct explanation of the Assertion.

4. Both, the assertion and Reason are true and Reason is correct explanation of the Assertion.

7. As $\vec{p} = q \times (2\vec{a})$. Hence **Reason is false.** Here $2\vec{a}$ is also a vector, therefore Electric dipole moment is a vector. Thus **Assertion is also false.**

8. Assertion is true, but the Reason is false.

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PROBLEMS FOR PRACTICE

TYPE I. QUANTIZATION OF CHARGE

1. A person combs his hair on a dry day. The comb causes 10^{22} electrons to leave the person's hair and stick to the comb. Calculate the charge the comb carries. **[Ans. $-1{\cdot}6 \times 10^3$ C]**
2. Estimate the number of free electrons in 36 g of water and the negative charge possessed by them. Given : Avogadro's number = $6{\cdot}023 \times 10^{23}$ and molecular weight of water = 18.
[Ans. $12{\cdot}046 \times 10^{24}$, $-1{\cdot}93 \times 10^6$ C]
3. What is the total charge on 75·0 kg of electrons ?
[Ans. $-1{\cdot}33 \times 10^{13}$ C]
4. How many mega coulombs of positive (or negative) charge are in 1·00 mole of neutral molecular hydrogen gas (H_2) ? **[Ans. 0·1927 MC]**
5. Calculate the total positive or negative charge on a 3·11 *g* copper penny. Given, Avogadro's number = $6{\cdot}023 \times 10^{23}$ per gram mole ; for copper, atomic charge = 29 and atomic mass = 63·5.
[Ans. $\pm 1{\cdot}37 \times 10^5$ C]

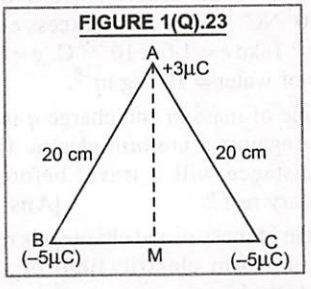
TYPE II. COULOMB'S LAW

6. A charge of magnitude Q is divided into two parts q and $(Q - q)$ such that the two parts exert maximum force on each other. Calculate the ratio Q/q. **[Ans. 2]**
7. Two identical metal spheres A and B have equal and similar charges. They repel each other with a force 103 N, when they are placed 10 cm apart in a medium of dielectric constant 7. Determine the charge on each sphere. **[Ans. $28{\cdot}3 \times 10^{-6}$ C]**
8. Calculate the Coulomb force between two alpha particles separated by a distance of $3{\cdot}2 \times 10^{-15}$ m in air. **[Ans. 90 N, repulsive)]**
9. What equal charges would have to be placed on earth and moon to neutralize their gravitational attraction ? Given, mass of earth = 10^{25} kg, mass of moon = 10^{23} kg. **[Ans. $8{\cdot}6 \times 10^{13}$ C]**
10. The electrostatic force of repulsion between two positively charged ions carrying equal charges is $3{\cdot}7 \times 10^{-9}$ N, when they are separated by a distance of 5 Å. How many electrons are missing from each ion ? **[Ans. 2]**
11. Two small spheres each of mass 'm' kg and charge q coulomb are suspended from a point by insulating threads each of l metre length, but of negligible mass. If θ is the angle which each string makes with the vertical when equilibrium has been reached, show that
$q^2 = 4\, mg\, l^2 \sin^2\theta \tan\theta\, (4\pi \in_0)$
12. Two particles, each having a mass of 5 g and charge 10^{-7} C stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient. **[Ans. $\mu = 0{\cdot}18$]**
13. Two point charges $q_2 = 3 \times 10^{-6}$ C and $q_1 = 5 \times 10^{-6}$ C are located at (3, 5, 1) and (1, 3, 2) m. Find $\vec{F}_{12}$ and $\vec{F}_{21}$ using vector form of Coulomb's law. Also, find their magnitude.
[Ans. $\vec{F}_{12} = -\vec{F}_{21} = -(5\times10^{-3})(2\hat{i} + 2\hat{j} - \hat{k})$ N
$= 1{\cdot}5 \times 10^{-2}$ N, in magnitude]
14. Two small charged spheres contain charges + q_1 and + q_2 respectively. A charge dq is removed from sphere carrying charge q_1 and is transferred to the other. Find charge on each sphere for maximum electric force between them.
$\left[\text{Ans. } \frac{1}{2}(q_1 + q_2)\right]$

TYPE III. SUPERPOSITION PRINCIPLE

15. Three point charges of + 3 μC, – 5 μC and – 5 μC are kept on the vertices of an equilateral triangle of side 20 cm as shown in Fig. 1(Q).23. What should be the sign and magnitude of charge to be placed at the mid point (M) of side BC of triangle ABC so that charge A remains in equilibrium ?

FIGURE 1(Q).23



[Ans. 6·49 μC, positive]

16. Four equal point charges each 16 μC are placed on four corners of a square of side 0·4 m. Calculate force on any one of the changes.

[Ans. 27·56 N]

17. Two opposite corners of a square carry Q charge each and the other two opposite corners of the same square carry q charge each. If the resultant force on q is zero, how are Q and q related ?

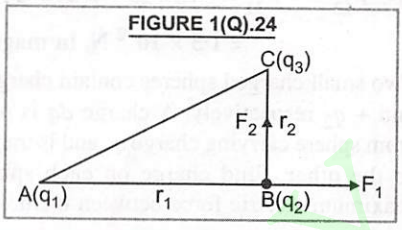
[Ans. $q = -2\sqrt{2}\,Q$]

18. Equal charges each of 20 μ C are placed at $x = 0$, 2, 4, 8, 16 cm on X-axis. Find the force experienced by the charge at $x = 2$ cm.

[Ans. $1{\cdot}2 \times 10^3$ N]

19. Charges $q_1 = 1{\cdot}5\ mC$, $q_2 = 0{\cdot}2\ mC$ and $q_3 = -0{\cdot}5\ mC$, are placed at points A, B, C respectively as shown in Fig. 1(Q).24. If $r_1 = 1{\cdot}2\ m$ and $r_2 = 0{\cdot}6\ m$, calculate magnitude of resultant force on q_2. **[Ans. $3{\cdot}1 \times 10^3$ N]**

FIGURE 1(Q).24



TYPE IV. ELECTRIC FIELD AND RELATION BETWEEN ELECTRIC INTENSITY AND FORCE

20. A small ball of paper has mass 9×10^{-5} kg and carries a charge of 5 μ C. When it is held over another charged ball of paper at a distance of 2 cm above it, the two balls stay in equilibrium. What is the charge on the second ball ?

[Ans. $7{\cdot}84 \times 10^{-12}$ C]

21. A water droplet of radius 1 micron in Millikan oil drop apparatus is first held stationary under the influence of an electric field of intensity $5{\cdot}1 \times 10^4$ NC^{-1}. How many excess electrons does it carry ? Take $e = 1{\cdot}6 \times 10^{-19}$ C, $g = 9{\cdot}8$ ms^{-2} and density of water = 10^3 kg m^{-3}. **[Ans. 5]**

22. A particle of mass m and charge q is thrown at a speed u against a uniform electric field E. How much distance will it travel before coming to momentary rest ? **[Ans. $m\,u^2/2\,qE$]**

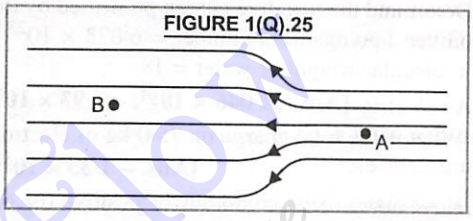
23. A particle of mass m and charge q is released from rest in uniform electric field of intensity E. Calculate the kinetic energy it attains after moving a distance x between the plates. **[Ans. qEx]**

24. Two electric charges + q and + 4 q are placed at a distance $6a$ apart on a horizontal plane. Find the position of the point on the line joining the two charges where the electric field is zero.

[Ans. 2 a from + q]

25. In Fig. 1(Q).25, the electric field lines on the left have twice the separation of those on the right. (*a*) If the magnitude of the field at A is 40 N/C, what force acts on a proton at A ? (*b*) What is the magnitude of the field at B ?

FIGURE 1(Q).25



[Ans. (*a*) $6{\cdot}4 \times 10^{-18}$ N (*b*) 20 N/C]

26. Eight identical point charges of q coulomb each are placed at the corners of a cube of each side 0·1 m. Calculate electric field at the centre of the cube. Calculate the field at the centre when one of the corner charges is removed.

[Ans. Zero ; ($1{\cdot}2 \times 10^{12}\,q$) NC^{-1} towards the corner without charge]

27. A charge of 4×10^{-9} C is distributed uniformly over the circumference of a conducting ring of radius 0·3 m. Calculate the field intensity at a point on the axis of the ring at 0·4 m from its centre, and also at the centre. **[Ans. 115·2 N/C ; zero]**

TYPE V. ELECTRIC DIPOLE, DIPOLE MOMENT, DIPOLE FIELD

28. Calculate the magnitude of the force, due to an electric dipole of dipole moment $3{\cdot}6 \times 10^{-29}$ C-m, on an electron 25 nm from the centre of the dipole, along the dipole axis. Assume that this distance is large relative to the dipole's charge separation.

[Ans. $6{\cdot}6 \times 10^{-15}$ N]

29. Charge $q_1 = +\,6{\cdot}0$ n C is on y-axis at $y = +\,3$ cm and charge $q_2 = -\,6{\cdot}0$ n C is on y-axis at $y = -\,3$ cm. Calculate force on a test charge $q_0 =$ 2 n C placed on X-axis at $x = 4$ cm. **[Ans. $-\,51{\cdot}8\ \hat{j}$ μN]**

30. Two charges + q and − q are located at points A (0, 0, − 2) and B (0, 0, 2) respectively. How much work will be done in moving a test charge from point P (4, 0, 0) to Q (− 5, 0, 0) ?

(CBSE 2009) **[Ans. Zero]**

31. Two point charges each of 5 μC but opposite in sign are placed 4 cm apart. Calculate the electric

field intensity at a point distant 4 cm from the mid point on the axial line of dipole.
(Pb. Board 2002) **[Ans. 10^8 NC^{-1}]**

TYPE VI. TORQUE ON DIPOLE AND POTENTIAL ENERGY OF DIPOLE IN ELECTRIC FIELD

32. An electric dipole of dipole moment 4×10^{-5} C.m is placed in a uniform electric field of 10^3 NC^{-1} making an angle of 30° with the direction of field. Determine the torque exerted by the electric field on the dipole. (Hr. Board 2001)
[Ans. 2×10^{-8} Nm]

33. An electric dipole of length 10 cm having charges $\pm 6 \times 10^{-3}$ C, placed at 30° with respect to a uniform electric field experiences a torque of magnitude $6\sqrt{3}$ N-m. Calculate (i) magnitude of electric field (ii) the potential energy of dipole.
[Ans. (i) $2\sqrt{3} \times 10^4$ NC^{-1} (ii) – 18 J]

34. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has a charge ± 8 nC. (CBSE 2014) **[Ans. – 4 J]**

TYPE VII. ELECTRIC FLUX AND GAUSS'S THEOREM IN ELECTROSTATICS

35. Given a uniform electric field
$\vec{E} = 5 \times 10^3 \hat{i}\ NC^{-1}$.
Find the flux of this field through a square of 10 cm on a side, whose plane is parallel to Y-Z plane. What would be the flux through the same square if plane makes an angle of 30° with X-axis ? (CBSE 2014) **[Ans. 50 Nm^2 C^{-1} ; 25 Nm^2 C^{-1}]**

36. A uniform electric field $\vec{E} = E_x \hat{i}\ N/C$ for $x > 0$ and $\vec{E} = -E_x \hat{i}\ N/C$ for $x < 0$ exists. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along X-axis. Find out the net outward flux. What is the net charge within the cylinder ? [CBSE 2008 (C)]
[Ans. $2\pi r^2 E_x$ (10^{-4}) Nm^2 C^{-1}. $2\pi r^2 E_x \in_0$ (10^{-4}) C]

37. If the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$ NC^{-1}, calculate the electric flux through a surface of area 100 m^2 lying in X-Y plane.
[Ans. 300 Nm^2 C^{-1}]

38. A spherical Gaussian surface encloses a charge of $8{\cdot}85 \times 10^{-8}$ C (i) Calculate the electric flux passing through the surface (ii) If the radius of Gaussian surface is doubled, how would the flux change ?
(CBSE 2007, 2001)
[Ans. 10^4 Nm^2 C^{-1}, No change]

39. A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric field of 25 Vm^{-1}, such that normal to the surface makes an angle of 60° with the direction of electric field. Find the flux of electric field through the rectangular surface. **[Ans. $1{\cdot}8 \times 10^{-1}$ Nm^2 C^{-1}]**

40. If the electric field is given by $(6\hat{i} + 4\hat{j} + 4\hat{k})$, calculate the electric flux through a surface of area 20 units lying in $Y - Z$ plane. **[Ans. 120 units]**

41. The electric field in a certain region of space is $(5\hat{i} + 4\hat{j} - 4\hat{k}) \times 10^5$ N/C. Calculate electric flux due to this field over an area of $(2\hat{i} - \hat{j}) \times 10^{-2}$ m^2. **[Ans. 6×10^3 NC^{-1} m^2]**

42. A charge of 2×10^{-9} C is placed on a corner of a cube of side 1 m. Find the electric flux passing through this cube. **[Ans. 28·25 NC^{-1} m^2]**

43. Five thousand lines of force enter a certain volume of space and three thousand lines emerge from it. What is the total charge in coulomb within this volume? **[Ans. $-1{\cdot}77 \times 10^{-8}$ C]**

44. A positive charge of 17·7 μC is placed at the centre of a hollow sphere of radius 0·5 m. Calculate the flux density through the surface of the sphere.
(CBSE 2003) **[Ans. $6{\cdot}4 \times 10^5$ NC^{-1}]**

TYPE VIII. APPLICATIONS OF GAUSS'S THEOREM

45. An infinite line charge produces a field of 9×10^4 NC^{-1} at a distance of 4 cm. Calculate the linear charge density. (Hr. Board 2001)
[Ans. 2×10^{-7} Cm^{-1}]

46. An electron is revolving around a long line charge having charge density 2×10^{-8} Cm^{-1}. Find the kinetic energy of the electron, assuming that it is independent of the radius of electron's orbit.
[Ans. $2{\cdot}88 \times 10^{-17}$ J]

47. A large plane sheet of charge having surface charge density 5×10^{-6} Cm^{-2} lies in XY plane. Find the electric flux through a circular area of radius 0·1 m, if the normal to the circular area makes an angle of 60° with the Z-axis.
[Ans. $4{\cdot}44 \times 10^3$ Nm^2 C^{-1}]

48. A particle of mass 9×10^{-5} g is kept over a large horizontal sheet of charge density 5×10^{-5} C m^{-2}. What charge should be given to the particle so that it does not fall on release?

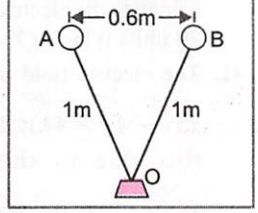
[Ans. $3{\cdot}12 \times 10^{-13}$ C]

TYPE IX. TYPICAL PROBLEMS

49. In a cartesian co-ordinate system, charge $q_1 = -2 \times 10^{-4}$ C is at $x = 0$, $y = 0$, charge $q_2 = 1 \times 10^{-3}$ C is at $x = 10$ m and $y = 0$ and charge $q_3 = -1 \times 10^{-4}$ C is at $x = 0$, $y = -5$ cm. Find the magnitude of resultantant force on q_1. **[Ans. 19·4 N]**

50. Two identical helium filled balloons A and B fastened to a weight of 5 gram by threads float in equilibrium as shown in Fig. 1(Q).26. Calculate the charge on each balloon, assuming that they carry equal charges.

FIGURE 1(Q).26



[Ans. $\pm 5{\cdot}6 \times 10^{-7}$ C]

51. Identify X in the following nuclear reactions (in the first reaction, n represents a neutron) :

(*a*) ${}_1H^1 + {}_4Be^9 \longrightarrow X + {}_0n^1$;

(*b*) ${}_6C^{12} + {}_1H^1 \longrightarrow X$;

(*c*) ${}_7N^{15} + {}_1H^1 \longrightarrow {}_2He^4 + X$.

[Ans. (*a*) ${}_5B^9$ (*b*) ${}_7N^{13}$ (*c*) ${}_6C^{12}$]

52. Four particles, each having a charge q are placed on the four corners A, B, C, D of a regular pentagon $ABCDE$. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon. **[Ans. $q/4\pi\epsilon_0 a^2$, along OE]**

53. A copper ball of density 8·6 g/cc and 1 cm in diameter is immersed in oil of density 0·8 g/cc. What is the charge on the ball, if it remains just suspended in oil in electric field of intensity 3600 V/m acting in the upward direction ?

[Ans. $1{\cdot}1 \times 10^{-5}$ C]

54. Two charges of $-4\ \mu$C and $+4\ \mu$C are placed at the points A (1, 0, 4) and B (2, −1, 5) located in an electric field $\vec{E} = 0{\cdot}20\,\hat{i}$ V/cm. Calculate the torque acting on the dipole.

[Ans. $1{\cdot}13 \times 10^{-4}$ N-m]

55. An infinite number of charges each equal to q, are placed along X-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$, ... and so on.

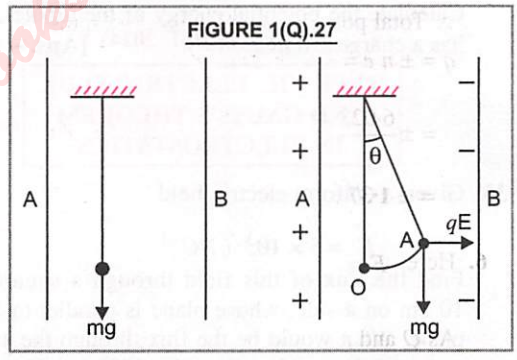
(*i*) Find the electric field at a point $x = 0$ due to this set up of charges.

(*ii*) What will be the electric field if in the above set up, the consecutive charges have opposite signs.

$$\left[\text{Ans. } \frac{q}{3\pi\epsilon_0}, \frac{q}{5\pi\epsilon_0}\right]$$

56. A metal ball suspended from a long thread is held between the plates of a capacitor ; as shown in Fig. 1(Q).27. How will the oscillations of this pendulum change, if the ball and the plates of two capacitor are charged ?

FIGURE 1(Q).27



[Ans. Time period of ball will decrease]

HINTS/EXPLANATIONS For Difficult Questions

1. Here, $n = 10^{22}$, $e = 1{\cdot}6 \times 10^{-19}$ C

$\therefore \quad q = ne = 10^{22} \times 1{\cdot}6 \times 10^{-19} = 1{\cdot}6 \times 10^3$ C

Due to excess of electrons, charge on the comb

$= \mathbf{-1{\cdot}6 \times 10^3}$ **C**

2. Here, no. of molecules in 36 g of water

$$= \frac{6{\cdot}023 \times 10^{23}}{18} \times 36 = 12{\cdot}046 \times 10^{23}$$

A water (H_2O) molecule is made up of two hydrogen atoms and one oxygen atom. Each hydrogen atom contains one electron and each oxygen atom contains 8 electrons. So, each molecule of water has 10 electrons.

No. of free electrons in 36 g of water

$= 12{\cdot}046 \times 10^{23} \times 10$

$= \mathbf{12{\cdot}046 \times 10^{24}}$

Negative charge, $q = ne$

$= 12{\cdot}046 \times 10^{24} \times 1{\cdot}6 \times 10^{-19}$ C

$= \mathbf{1{\cdot}92736 \times 10^6}$ **C**

3. Number of electrons $(n) = \dfrac{\text{total mass}}{\text{mass of each electron}}$

$$= \frac{75{\cdot}0}{9\times10^{-31}} = \frac{25\times10^{31}}{3}$$

$$q = n\,e = \frac{25}{3}\times10^{31}\times(-1{\cdot}6\times10^{-19})$$

$$= -\mathbf{1{\cdot}33\times10^{13}\ C}$$

4. Number of hydrogen molecules in 1·00 mole $= 6{\cdot}023\times10^{23}$.

Each molecule has two electrons/two protons

$\therefore\ n = 2\times6{\cdot}023\times10^{23}$

$q = ne = 2\times6{\cdot}023\times10^{23}\times1{\cdot}6\times10^{-19}$ C

$= 19{\cdot}27\times10^{4}\ C = \mathbf{0{\cdot}1927\ MC}$

5. Number of atoms in penny

$$n' = \frac{6{\cdot}023\times10^{23}}{63{\cdot}5}\times3{\cdot}11$$

As each atom contains 29 protons and 29 electrons

$\therefore$ Total positive/negative charge on the penny

$q = \pm\, n\,e = \pm\, n'\times 29\ e$

$$= \pm\frac{6{\cdot}023\times10^{23}\times3{\cdot}11}{63{\cdot}5}\times29\times1{\cdot}6\times10^{-19}\ C$$

$$= \mathbf{\pm\,1{\cdot}37\times10^{5}\ C}$$

6. Here, $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(Q-q)}{r^2}$

As Q and r are fixed, So F is a function of q

Force F will be maximum, only if $\dfrac{dF}{dq} = 0$

or $\dfrac{1}{4\pi\epsilon_0}\dfrac{d}{dq}[q(Q-q)] = 0$

or $\dfrac{d}{dq}[q(Q-q)] = 0$

or $q(-1) + (Q-q) = 0$

or $Q - 2q = 0$ or $Q = 2q$

$$\frac{Q}{q} = 2$$

7. Here $q_1 = q_2 = q$, $F = 103$ N

$K = 7$, $r = 10$ cm $= 0{\cdot}10$ m

$$F = \frac{1}{4\pi\epsilon_0 K}\frac{q^2}{r^2}$$

$$103 = \frac{9\times10^{9}\,q^2}{7\,(0{\cdot}10)^2} \quad \text{or} \quad q^2 = \frac{103\times7\times(0{\cdot}10)^2}{9\times10^{9}}$$

or $q = \mathbf{28{\cdot}3\times10^{-6}\ C}$

8. Charge on each α particle $= +2\,e$.

9. Use $\dfrac{Gm_1m_2}{r^2} = k\dfrac{qq}{r^2}$

10. Here, $q_1 = q_2 = q = ?$

$F = 3{\cdot}7\times10^{-9}$ N, $r = 5$ Å $= 5\times10^{-10}$ m

$n = ?$

As $F = \dfrac{k\,q_1\,q_2}{r^2} = \dfrac{k\,q^2}{r^2}$

$\therefore\ q = \sqrt{r^2F/k}$

$$= \sqrt{\frac{1}{9\times10^{9}}(5\times10^{-10})^2\times3{\cdot}7\times10^{-9}}$$

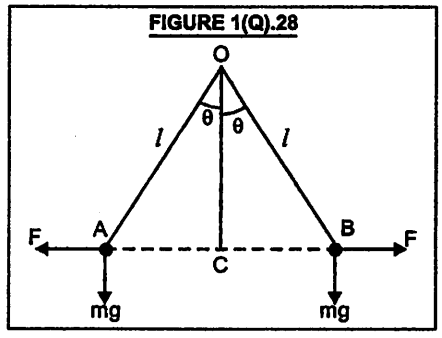
$$= \sqrt{\frac{25\times3{\cdot}7}{9}}\times10^{-19}\,C = 3{\cdot}2\times10^{-19}\ C$$

From $q = ne$

$$n = \frac{q}{e} = \frac{3{\cdot}2\times10^{-19}}{1{\cdot}6\times10^{-19}} = 2$$

11. As is clear from Fig. 1(Q).28,

FIGURE 1(Q).28



$$\frac{F}{AC} = \frac{mg}{OC}$$

$$F = mg\cdot\frac{AC}{OC} \qquad \text{...(i)}$$

$$\frac{q(q)}{4\pi\epsilon_0\,(AB)^2} = mg.\tan\theta$$

Now, $AB = 2\,AC = 2\,l\sin\theta$

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$$\therefore \quad \frac{q^2}{4\pi\epsilon_0 (2l\sin\theta)^2} = mg\tan\theta$$

or $q^2 = 4\,mg\,l^2\sin^2\theta\tan\theta\,.\,4\pi\epsilon_0$

which was to be proved.

12. Here, $q_1 = q_2 = 10^{-7}$ C, $r = 10$ cm $= 10^{-1}$ m

$m = 5\ g = 5\times10^{-3}$ kg, $\mu = ?$

For limiting equilibrium,

$$F = \mu R = \mu\, mg$$

$$\frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \mu\, mg$$

$$\mu = \frac{q_1 q_2}{4\pi\epsilon_0 r^2 mg} = \frac{10^{-7}\times10^{-7}\times9\times10^9}{(10^{-1})^2\times5\times10^{-3}\times9{\cdot}8}$$

$$= \mathbf{0{\cdot}18}$$

13. Here, $q_2 = 3\times10^{-6}$ C, $q_1 = 5\times10^{-6}$ C

$\vec{r_2} = (3\hat{i}+5\hat{j}+\hat{k})$, and $\vec{r_1} = (\hat{i}+3\hat{j}+2\hat{k})$

$$\therefore \quad \vec{r_{12}} = \vec{r_2} - \vec{r_1} = (3\hat{i}+5\hat{j}+\hat{k}) - (\hat{i}+3\hat{j}+2\hat{k})$$

$$= (2\hat{i}+2\hat{j}-\hat{k})$$

$$|\vec{r_{12}}| = \sqrt{2^2+2^2+(-1)^2} = 3\,\text{m}$$

$$\hat{r}_{12} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} = \frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$$

$$\therefore \quad \vec{F_{21}} = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$= \frac{(9\times10^9)\,(5\times10^{-6})\,(3\times10^{-6})}{3^2}\,\frac{(2\hat{i}+2\hat{j}-\hat{k})}{3}$$

$$\vec{F_{21}} = 5\times10^{-3}\,(2\hat{i}+2\hat{j}-\hat{k})\ \text{N}.$$

$$|\vec{F_{21}}| = \sqrt{[2^2+2^2+(-1)^2]}\ 5\times10^{-3}$$

$$= \mathbf{1{\cdot}5\times10^{-2}\ N}$$

$$\vec{F_{12}} = -\vec{F_{21}} = -(\mathbf{5\times10^{-3}})\,(2\hat{i}+2\hat{j}-\hat{k})\,\text{N}$$

14. After transfer of charge dq,

charge on one sphere $= q_1 - dq$

Charge on second sphere $= q_2 + dq$

Coulomb's force between the two spheres is

$$F = \frac{k(q_1-dq)(q_2+dq)}{r^2}$$

$$F = \frac{k}{r^2}(q_1 q_2 + q_1\,dq - q_2\,dq - dq^2)$$

This force will be maximum, when

$$\frac{\partial F}{\partial(dq)} = 0 \quad \therefore \quad \frac{k}{r^2}(q_1 - q_2 - 2\,dq) = 0$$

or $q_1 - q_2 - 2\,dq = 0$ or $dq = \frac{1}{2}(q_1 - q_2)$

$\therefore$ Charge on first sphere $= q_1 - dq$

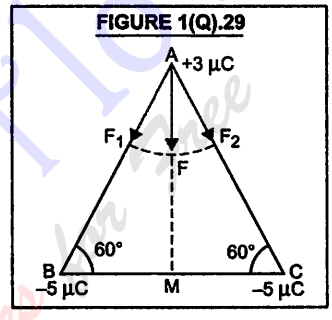
$$= q_1 - \frac{1}{2}(q_1 - q_2) = \frac{1}{2}(q_1 + q_2)$$

and charge on second sphere $= q_2 + dq$

$$= q_2 + \frac{1}{2}(q_1 - q_2) = \frac{1}{2}(q_1 + q_2)$$

15.

FIGURE 1(Q).29



As shown in Fig. 1(Q).29, force exerted on charge + 3 μC by charge at B

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9\times3\times10^{-6}\times5\times10^{-6}}{(0{\cdot}2)^2}$$

$$= 3{\cdot}375\ \text{N along } AB$$

Force exerted on + 3 μC charge by charge at C

$$F_2 = \frac{9\times10^9\times3\times10^{-6}\times5\times10^{-6}}{(0{\cdot}2)^2}$$

$$= 3{\cdot}375\ \text{N along } AC$$

Resultant force of F_1 and F_2

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos60°}$$

$$= 5{\cdot}845\ \text{N}$$

For charge A to be in equilibrium **positive charge** must be placed at M, to exert force along MA

$$AM = \sqrt{(0{\cdot}2)^2 - (0{\cdot}1)^2}$$

$$= 0{\cdot}1\times\sqrt{3}\ \text{m}$$

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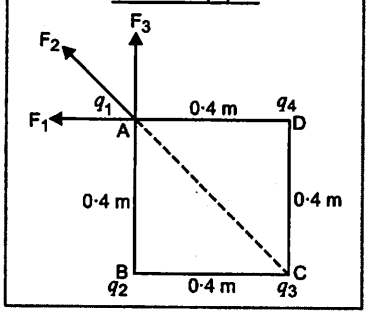
Force on charge + 3 μC will be zero if

$$\frac{9\times10^9\times q\times3\times10^{-6}}{(0{\cdot}1\times\sqrt{3})^2}=5{\cdot}845$$

$$q = 6{\cdot}49\ \mu C$$

16.

FIGURE 1(Q).30



Refer to Fig. 1(Q).30,

$AB = BC = CD = AD = 0{\cdot}4$ m.

Force exerted by q_4 on q_1 = force exerted by q_2 on q_1

$$F_1 = F_3 = \frac{9\times10^9\times16\times10^{-6}\times16\times10^{-6}}{(0{\cdot}4)^2}$$

$$= 14{\cdot}4\ \text{N}$$

Force exerted by q_3 on q_1

$$F_2 = \frac{9\times10^9\times16\times10^{-6}\times16\times10^{-6}}{(0{\cdot}4)^2+(0{\cdot}4)^2} = 7{\cdot}2\ \text{N}$$

F_1 and F_3 are perpendicular to each other, resultant force

$$F = \sqrt{F_1^2+F_3^2} = \sqrt{(14{\cdot}4)^2+(14{\cdot}4)^2} = 20{\cdot}36\ \text{N}$$

Total force on $q_1 = F + F_2 = 20{\cdot}36 + 7{\cdot}2$

$$= \mathbf{27{\cdot}56\ N}$$

17. Let each side of square be x, Fig. 1(Q).31

$\therefore$ Diagonal, $BD = \sqrt{x^2+x^2} = x\sqrt{2}$

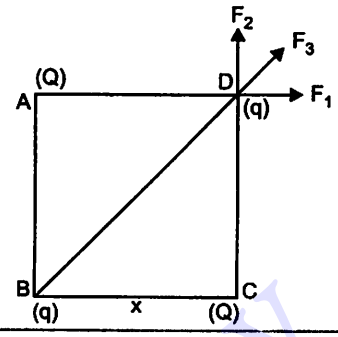
$$F_1 = F_2 = \frac{Qq}{4\pi\epsilon_0 x^2}$$

$$F_3 = \frac{qq}{4\pi\epsilon_0 (x\sqrt{2})^2} = \frac{q^2}{2\times4\pi\epsilon_0 x^2}$$

As F_1 and F_2 are ⊥ to each other, their resultant force

$$= \sqrt{F_1^2+F_2^2} = \sqrt{F_1^2+F_1^2} = F_1\sqrt{2}$$

FIGURE 1(Q).31



As net force on q is zero, therefore,

$$F_1\sqrt{2} = -F_3$$

$$\frac{Qq\sqrt{2}}{4\pi\epsilon_0 x^2} = \frac{-q^2}{2\times4\pi\epsilon_0 x^2}$$

$$\therefore \quad q = -2\sqrt{2}Q$$

18. Force on charge at $x = 2$ cm due to charge at $x = 0$ cm and $x = 4$ cm are equal and opposite. They cancel.

Net force on charge at $x = 2$ cm is resultant of repulsive forces due to two charges at $x = 8$ cm and $x = 16$ cm.

$$\therefore F = \frac{q\times q}{4\pi\epsilon_0}\times\left[\frac{1}{(0{\cdot}08-0{\cdot}02)^2}+\frac{1}{(0{\cdot}16-0{\cdot}02)^2}\right]$$

$$F = 9\times10^9\,(20\times10^{-6})^2\left[\frac{1}{(0{\cdot}06)^2}+\frac{1}{(0{\cdot}14)^2}\right]$$

$$F = \mathbf{1{\cdot}2\times10^3\ N}$$

19. Refer to Fig. 1(Q).16.

$$F_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r_1^2}$$

$$= \frac{1{\cdot}5\times10^{-3}\times0{\cdot}2\times10^{-3}\times9\times10^9}{(1{\cdot}2)^2}$$

$= 1{\cdot}875\times10^3\ N$ .....along AB produced

$$F_2 = \frac{q_2 q_3}{4\pi\epsilon_0 r_2^2}$$

$$= \frac{0{\cdot}2\times10^{-3}\times0{\cdot}5\times10^{-3}\times9\times10^9}{(0{\cdot}6)^2}$$

$= 2{\cdot}5\times10^3\ N$ along BC ⊥ AB.

Resultant force on $q_2 = \sqrt{F_1^2+F_2^2}$

$$= \mathbf{3{\cdot}1\times10^3\ N}$$

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20. $q_1 = 5\ \mu C = 5 \times 10^{-6}\ C$, $q_2 = ?$
$r = 2\ cm = 2 \times 10^{-2}\ m$
$m = 9 \times 10^{-5}\ kg.$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = mg$$

$$\frac{9\times10^9 \times 5\times10^{-6}\, q_2}{(2\times10^{-2})^2} = 9\times10^{-5}\times 9{\cdot}8$$

$$q_2 = 7{\cdot}84 \times 10^{-12}\ C$$

21. Here, $r = 1$ micron $= 10^{-6}$ m, $E = 5{\cdot}1 \times 10^4$ N/C, $n = ?$
From $qE = mg$

$$(ne)\,E = \frac{4}{3}\pi r^3 \rho g,\quad n = \frac{4}{3}\frac{\pi r^3 \rho g}{eE}$$

$$n = \frac{4}{3}\times\frac{22}{7}\frac{(10^{-6})^3\times10^3\times9{\cdot}8}{1{\cdot}6\times10^{-19}\times5{\cdot}1\times10^4} = 5$$

22. From work energy theorem,

$$F \times s = \text{Change in K.E.} = \frac{1}{2}mu^2 - 0$$

$$qE \times s = \frac{1}{2}mu^2$$

$$s = \frac{mu^2}{2qE}$$

23. As $F = qE$

$$a = \frac{F}{m} = \frac{qE}{m}$$

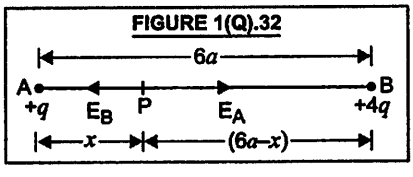
As $v^2 - u^2 = 2as$, and $u = 0$ $\therefore$ $v^2 = 2as$

$$\Rightarrow\ v^2 = 2\left(\frac{qE}{m}\right)x = \frac{2qEx}{m}$$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2qEx}{m}\right) = qEx$$

24. Let electric field be zero at P, where $AP = x$, Fig. 1(Q).32.

FIGURE 1(Q).32



As $E_A = E_B$ $\therefore$ $\dfrac{Kq}{x^2} = \dfrac{K(4q)}{(6a-x)^2}$

$$\frac{(6a-x)^2}{x^2} = 4$$

$$x = 2a \text{ from } +q$$

25. (*a*) $F = qE = 1{\cdot}6 \times 10^{-19} \times 40 = 6{\cdot}4 \times 10^{-18}$ N
(*b*) As separation of electric field lines at B is twice the separation at A, Fig. 1(Q).19,

$$\therefore\quad E_B = \frac{1}{2}\times 40\ N/C = 20\ N/C$$

26. Length of each side $a = 0{\cdot}1$ m $= 10$ cm
$\therefore$ distance of the centre of the cube from each corner,

$$r = \frac{a\sqrt{3}}{2} = \frac{10\sqrt{3}}{2} = 5\sqrt{3}\ cm$$

$$= 5\sqrt{3}\times10^{-2}\ m.$$

When identical point charges $+q$ each are at the eight corners of the cube, net field intensity at the centre is **zero** (cancelling out in four pairs). When one of the corner charges is removed, three pairs of E values cancel. Due to charge on seventh corner,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q\times1}{r^2} = 9\times10^9\,\frac{q}{(5\sqrt{3}\times10^{-2})^2}$$

$$= (1{\cdot}2 \times 10^{12}\, q)\ NC^{-1}$$

It is **towards the corner without charge directed radially away from + q.**

27. $q = 4 \times 10^{-9}$ C, $a = 0{\cdot}3$ m, $r = 0{\cdot}4$ m, $E = ?$

$$E = \frac{qr}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}} = \frac{9\times10^9\times4\times10^{-9}\times0{\cdot}4}{(0{\cdot}4^2 + 0{\cdot}3^2)^{3/2}}$$

$$E = \frac{14{\cdot}4}{(0{\cdot}5)^3} = 115{\cdot}2\ N/C$$

At the centre of ring, $r = 0$, $\therefore$ $E = 0$

28. $F = qE = \dfrac{q(2p)}{4\pi\epsilon_0 r^3}$

$$= \frac{9\times10^9\times(1{\cdot}6\times10^{-19})\times2\times3{\cdot}6\times10^{-29}}{(25\times10^{-9})^3}\ N$$

$$= 6{\cdot}6 \times 10^{-15}\ N$$

30. As is clear from Fig. 1(Q).33, dipole moment $\vec{p}$ is along $\overrightarrow{BA}$. Test charge is to be moved from P to Q along negative X-axis. As electric intensity is perpendicular to displacement, $\vec{ds}$,
$\therefore$ $W = qE\,ds\cos 90° =$ **Zero**

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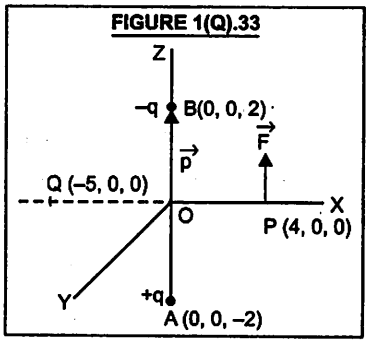
FIGURE 1(Q).33



31. Here, $q = 5\ \mu C = 5 \times 10^{-6}$ C

$2a = 4$ cm $= 4 \times 10^{-2}$ m, $E = ?$

$r = 4$ cm $= 4 \times 10^{-2}$ m

$$E_{axial} = \frac{2pr}{4\pi\epsilon_0 (r^2 - a^2)^2} = \frac{2(q)(2a)r}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

$$= \frac{9\times10^9 \times 2(5\times10^{-6})(4\times10^{-2})\times 4\times10^{-2}}{[0{\cdot}04^2 - 0{\cdot}02^2]^2}$$

$E_{axial} = \mathbf{10^8\ NC^{-1}}$

32. Here, $\tau = pE \sin\theta = 4 \times 10^{-5} \times 10^{-3} \times \sin 30°$

$= \mathbf{2 \times 10^{-8}\ Nm}$

33. Here, $2a = 10$ cm $= 0{\cdot}1$ cm, $q = \pm 6 \times 10^{-3}$ C

$\theta = 30°$, $\tau = 6\sqrt{3}$ N-m, $E = ?$ P.E. $= ?$

As $\tau = pE \sin\theta = q(2a) E \sin\theta$

$$\therefore\ E = \frac{\tau}{q(2a)\sin\theta} = \frac{6\sqrt{3}}{6\times10^{-3}\times0{\cdot}1\sin 30°}$$

$E = \mathbf{2\sqrt{3}\times10^4\ NC^{-1}}$

$U = -pE\cos\theta = -q(2a)E\cos\theta$

$= -6 \times 10^{-3} \times 0{\cdot}1 \times 2\sqrt{3} \times 10^4 \cos 30°$

$U = \mathbf{-18\ J}$

34. Here $2a = 4$ cm $= 4 \times 10^{-2}$ m, $\theta = 60°$,

$\tau = 4\sqrt{3}$ Nm, $q = \pm 8\ nC = 8 \times 10^{-9}$ C

Now, $p = q \times 2a = (8 \times 10^{-9}) \times (4 \times 10^{-2})$ Cm

$\tau = pE \sin\theta$ or $E = \tau/p \sin\theta$

$$PE = -pE\cos\theta = -p \times \frac{\tau}{p\sin\theta} \times \cos\theta$$

$= -\tau\cot\theta = -4\sqrt{3}\cot 60°$

$= -4\sqrt{3} \times 1/\sqrt{3} = \mathbf{-4\ J}$

35. Here, $\vec{E} = 5\times10^3\,\hat{i}\ NC^{-1}$

$A = (10\text{ cm})^2 = (10^{-1}\text{ m})^2 = 10^{-2}\text{ m}^2$

When plane is ‖ to Y-Z plane, $\theta = 0°$

$\therefore\ \phi = EA\cos\theta = (5 \times 10^3) \times 10^{-2} \cos 0°$

$= \mathbf{50\ Nm^2\ C^{-1}}$

When plane makes an angle of 30° with X-axis,

$\theta = 60°$

$\therefore\ \phi' = EA\cos\theta = 5 \times 10^3 \times 10^{-2} \cos 60°$

$= \mathbf{25\ Nm^2\ C^{-1}}$

36. Net outward flux,

$$\phi_E = E_x \pi \left(\frac{r}{100}\right)^2 + E_x . \pi \left(\frac{r}{100}\right)^2 + 0$$

$\phi_E = \mathbf{2\pi r^2 E_x\ (10^{-4})\ Nm^2\ C^{-1}}$

$q = \epsilon_0 \phi_E = \mathbf{2\pi r^2 \epsilon_0 E_x\ (10^{-4})\ C}$

37. $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}\ NC^{-1}$, $\vec{S} = 100\hat{k}\ m^2$

$\phi_E = \vec{E}\cdot\vec{S} = (8\hat{i} + 4\hat{j} + 3\hat{k})\cdot(100\hat{k})$

$= \mathbf{300\ Nm^2\ C^{-1}}$

38. (*i*) $\phi_E = \dfrac{q}{\epsilon_0} = \dfrac{8{\cdot}85\times10^{-8}}{8{\cdot}85\times10^{-12}} = \mathbf{10^4\ Nm^2\ C^{-1}}$

(*ii*) flux will remain the same as charge enclosed by the surface is same as in case (*i*).

39. Here, $E = 25\ Vm^{-1}$,

$s = 10 \times 15\ cm^2 = 150 \times 10^{-4}\ m^2$

$\theta = 60°$, $\phi = ?$

$\phi = Es\cos\theta = 25 \times 150 \times 10^{-4} \cos 60°$

$\phi = \mathbf{1{\cdot}8 \times 10^{-1}\ Nm^2\ C^{-1}}$

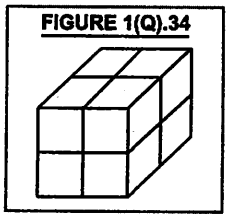
40. $d\phi = \vec{E}.\vec{ds} = (6\hat{i} + 3\hat{j} + 4\hat{k}).\,20\hat{i}$

$= \mathbf{120\ units}$

Note direction of area element is along normal to *YZ* plane, *i.e.*, along *x*-axis.

42. As is clear from Fig. 1(Q).34, eight cubes will be required to form a Gaussian surface-such that the given charge of 2×10^{-9} C appears at the centre of the bigger cube.

FIGURE 1(Q).34



Thus electric flux passing through given small cube

$$\phi = \frac{1}{8}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{8} \times \frac{2\times10^{-9}}{8{\cdot}85\times10^{-12}}$$

$= \mathbf{28{\cdot}25\ NC^{-1}\ m^2}$

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43. Here, number of lines of force emerging from the volume = $\phi = 3000 - 5000 = -2000$

From $\phi = \frac{q}{\epsilon_0}$

$q = \epsilon_0 \phi = 8{\cdot}85 \times 10^{-12} (-2000)$

$= -1{\cdot}77 \times 10^{-8}$ C

44. Here, $q = 17{\cdot}7\ \mu C = 17{\cdot}7 \times 10^{-6}$ C, $r = 0{\cdot}5$ m

Total flux, $\phi = \frac{q}{\epsilon_0} = \frac{17{\cdot}7 \times 10^{-6}}{8{\cdot}85 \times 10^{-12}}$

$= 2 \times 10^6\ Nm^2\ C^{-1}$

Flux density $= \frac{\text{total flux}}{\text{area}} = \frac{\phi}{4\pi r^2}$

$= \frac{2 \times 10^6}{4 \times 3{\cdot}14 (0{\cdot}5)^2} = 6{\cdot}4 \times 10^5\ NC^{-1}$

45. Here, $E = 9 \times 10^4\ NC^{-1}$, $r = 4$ cm $= 4 \times 10^{-2}$ m

$\lambda = ?$

As $E = \frac{\lambda}{2\pi\epsilon_0 r}$

$\therefore \quad \lambda = 2\pi\epsilon_0 r E = 4\pi\epsilon_0 \frac{r}{2} E$

$= \frac{1}{9 \times 10^9} \times \frac{4 \times 10^{-2}}{2} \times 9 \times 10^4$

$E = 2 \times 10^{-7}\ Cm^{-1}$

46. Due to a line charge,

$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda}{4\pi\epsilon_0 r}$

Force exerted by this field on electron

$F = qE = \frac{e \cdot 2\lambda}{4\pi\epsilon_0 r}$

Force exerted by Electric field = Centripetal force

$F = \frac{mv^2}{r}$ or $\frac{e \cdot 2\lambda}{4\pi\epsilon_0 r} = \frac{mv^2}{r}$

So $v^2 = \frac{e \cdot 2\lambda}{4\pi\epsilon_0 m}$

K.E. $= \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{e \cdot 2\lambda}{4\pi\epsilon_0 m} \right) = \frac{e\lambda}{4\pi\epsilon_0}$

$= 9 \times 10^9 \times 1{\cdot}6 \times 10^{-9} \times 2 \times 10^{-8}$

$= 2{\cdot}88 \times 10^{-17}$ J

47. Here, $\sigma = 5 \times 10^{-6}\ C\ m^{-2}$, $r = 0{\cdot}1$ m

$\theta = 60°$, $\phi = ?$

$\phi = Es \cos\theta = \left(\frac{\sigma}{2\epsilon_0} \right) (\pi r^2) \cos\theta$

$= \frac{5 \times 10^{-6}}{2 \times 8{\cdot}85 \times 10^{-12}} \times \frac{22}{7} \times (0{\cdot}1)^2 \cos 60°$

$\phi = 4{\cdot}44 \times 10^3\ Nm^2\ C^{-1}$

48. Here, $m = 9 \times 10^{-5}$ g $= 9 \times 10^{-8}$ kg

$\sigma = 5 \times 10^{-5}\ C\ m^{-2}$, $q = ?$

In equilibrium, $F = qE = mg$

$q \left(\frac{\sigma}{2\epsilon_0} \right) = mg$

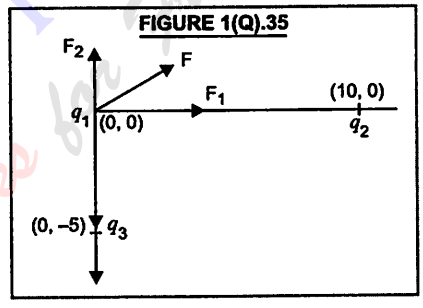
$\therefore \quad q = \frac{2\epsilon_0 mg}{\sigma}$

$= \frac{2 \times 8{\cdot}85 \times 10^{-12} \times 9 \times 10^{-8} \times 9{\cdot}8}{5 \times 10^{-5}}$

$q = 3{\cdot}12 \times 10^{-13}$ C

49.

FIGURE 1(Q).35



Refer to Fig. 1(Q).35.

F_1 is force on q_1 due to q_2

$F_1 = \frac{9 \times 10^9 \times 2 \times 10^{-4} \times 10^{-3}}{(10)^2} = 18$ N

F_2 is force on q_1 due to q_3

$F_2 = \frac{9 \times 10^9 \times 2 \times 10^{-4} \times 1 \times 10^{-4}}{(5)^2} = 7{\cdot}2$ N

As F_1 and F_2 are perpendicular to each other,

resultant force, $F = \sqrt{F_1^2 + F_2^2}$

$= \sqrt{(18)^2 + (7{\cdot}2)^2} = 19{\cdot}4$ N

50. In Fig. 1(Q).36, A and B are two balloons attached to weight W by strings $OA = OB = 1$ m.

$AB = 0{\cdot}6$ m, C is centre of AB.

$\therefore \quad AC = BC = 0{\cdot}3$ m

$OC = \sqrt{OA^2 - AC^2}$

$= \sqrt{1^2 - 0{\cdot}3^2} = 0{\cdot}954$

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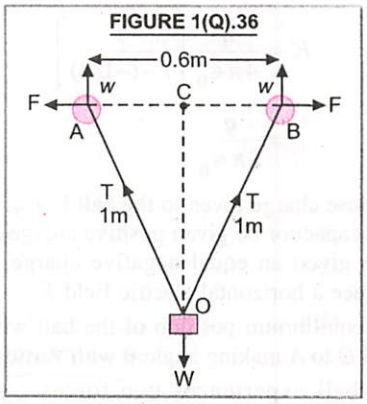
Force of repulsion between the balloons,

$$F = \frac{q \times q}{4\pi\epsilon_0 (AB)^2}$$

FIGURE 1(Q).36



As the balloons float in equilibrium,

∴ Using triangle law of vectors, we get

$$\frac{F}{AC} = \frac{w}{OC},$$

where w = upthrust on each balloon.

As $w + w = W$,

∴ $w = W/2$

$$\therefore \quad F = \frac{AC}{OC} \times \frac{W}{2}$$

$$\frac{q \times q}{4\pi\epsilon_0 (AB)^2} = \frac{AC}{OC} \times \frac{mg}{2}$$

$$q^2 = 4\pi\epsilon_0 (AB)^2 \times \frac{AC}{OC} \times \frac{mg}{2}$$

$$= \frac{(0{\cdot}6)^2 \times 0{\cdot}3 \times (5 \times 10^{-3} \times 9{\cdot}8)}{9 \times 10^9 \times 0{\cdot}954 \times 2}$$

$$q^2 = \frac{0{\cdot}36 \times 0{\cdot}3 \times 49}{9 \times 0{\cdot}954 \times 2} \times 10^{-12}$$

∴ $q = \pm 0{\cdot}56 \times 10^{-6}\ C = \mathbf{\pm 5{\cdot}6 \times 10^{-7}\ C}$

51. Using the law of conservation of mass number and charge number,

(*a*) Let ${}_ZX^A$

As $1 + 9 = A + 1$ ∴ $A = 9$

As $1 + 4 = Z + 0$ ∴ $Z = 5$

Hence, ${}_ZX^A$ is ${}_5X^9$ is ${}_5\boldsymbol{B}^9$

(*b*) Let ${}_ZX^A$

As $12 + 1 = A$ ∴ $A = 13$

As $6 + 1 = Z$ ∴ $Z = 7$

Hence, ${}_ZX^A$ is ${}_7\boldsymbol{N}^{13}$

(*c*) Let ${}_ZX^A$

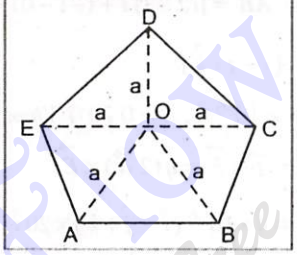
As $15 + 1 = 4 + A$ ∴ $A = 12$

As $7 + 1 = 2 + Z$ ∴ $Z = 6$

Hence, ${}_ZX^A$ is ${}_6\boldsymbol{C}^{12}$

52. In Fig. 1(Q).37, suppose four particles each of charge q are placed at the corners A, B, C and D of regular pentagon $ABCDE$.

FIGURE 1(Q).37



If we were to put the same charge q at E also, then by symmetry, net field at O would be zero. Hence combined field at O due to charges at A, B, C, D is equal and opposite to the field, at O due to charge at E. (which is $q/4\pi\epsilon_0 a^2$, along EO).

Hence the combined field at O due to four charges at A, B, C, D = $\boldsymbol{q/4\pi\epsilon_0 a^2}$ **along** $\boldsymbol{OE}$.

53. Here, density of copper ball, $\rho_1 = 8{\cdot}6$ g/cc $= 8600$ kg/m^3

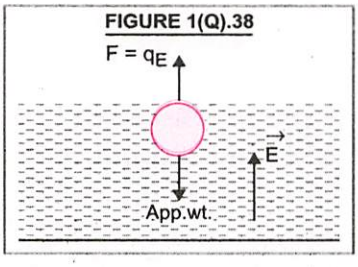
radius of ball, $r = \frac{1}{2}$ cm $= 0{\cdot}5 \times 10^{-2}$ m

density of oil, $\rho_2 = 0{\cdot}8$ g/cc $= 800$ kg/m^3

Charge, q = ?, $E = 3600$ V/m.

The ball will remain first suspended in oil, when force due to electric field (F) is equal to apparent weight of the ball in the oil, [Fig. 1(Q).38]

FIGURE 1(Q).38



i.e., $F = qE$ = apparent weight

= actual wt. – upthrust (wt. of oil displaced)

$$= \frac{4}{3}\pi r^3 \rho_1 g - \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi r^3 g(\rho_1 - \rho_2)$$

$$q = \frac{\frac{4}{3}\pi r^3 g(\rho_1 - \rho_2)}{E}$$

$$= \frac{\frac{4}{3}\times\frac{22}{7}(0{\cdot}5\times10^{-2})^3\times9{\cdot}8(8600-800)}{3600}$$

$q = \mathbf{1{\cdot}1 \times 10^{-5}\ C}$

54. As $A\ (1, 0, 4)$, $B\ (2, -1, 5)$

$$\therefore\ 2\vec{a} = \overrightarrow{AB} = [(2-1)\hat{i} + (-1-0)\hat{j} + (5-4)\hat{k}]$$

$$= [\hat{i} - \hat{j} + \hat{k}]$$

$q = \pm 4\times10^{-6}$ C, $E = 0{\cdot}20\ \hat{i}\ V/cm = 20\ \hat{i}\ V/m$

As $\vec{\tau} = \vec{p}\times\vec{E} = q(2\vec{a})\times\vec{E}$

$$\therefore\ \vec{\tau} = 4\times10^{-6}(\hat{i} - \hat{j} + \hat{k})\times20\hat{i}$$

$$\vec{\tau} = 8\times10^{-5}(\hat{k} + \hat{j})$$

Magnitude of torque,

$$\tau = 8\times10^{-5}\sqrt{1^2+1^2} = \mathbf{1{\cdot}13 \times 10^{-4}\ N\text{-}m}$$

55. (*i*) Electric field at $x = 0$

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \ldots.\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \ldots.\right]^*$$

$$E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1-1/4}\right] = \frac{q}{3\pi\epsilon_0}$$

(*ii*) If consecutive charges have opposite signs, then at $x = 0$,

Electric field, $E' = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1^2} - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots..\right]^*$

$$E' = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1-(-1/4)}\right]$$

$$= \frac{q}{5\pi\epsilon_0}$$

56. Suppose charge given to the ball is $+q$. Let plate A of capacitor be given positive charge and plate B be given an equal negative charge, so as to produce a horizontal electric field E.

The equilibrium position of the ball will change from O to A making angle θ with vertical.

The ball experiences two forces : qE along horizontal and mg along vertical such that

$$\tan\theta = \frac{qE}{mg}$$

The resultant force on the ball

$$F' = mg' = \sqrt{(mg)^2 + (qE)^2}$$

$$g' = \sqrt{g^2 + (qE/m)^2}$$

$\therefore$ Time period of vibration of ball

$$t' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\left[\frac{l}{\sqrt{g^2 + (qE/m)^2}}\right]^{1/2}$$

As $g' > g$, therefore $t' < t$

***i.e.*, time period of vibration of the ball will decrease.**

*Note that it is a geometrical progression, whose sum $= \frac{a}{1-r}$

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NCERT EXERCISES WITH SOLUTIONS

Q. 1. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air ?

Sol. Here, $q_1 = 2 \times 10^{-7}$ C, $q_2 = 3 \times 10^{-7}$ C, $r = 30$ cm $= 30 \times 10^{-2}$ m, $F = ?$

As $$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad \therefore \quad F = \frac{9\times10^9 (2\times10^{-7})(3\times10^{-7})}{(30\times10^{-2})^2} = \mathbf{6 \times 10^{-3}\ N}$$

Q. 2. The electrostatic force on a small sphere of charge 0·4 μ C due to another small sphere of charge – 0·8 μ C in air is 0·2 N. (a) What is the distance between the two spheres ? (b) What is the force on the second sphere due to the first ?

Sol. Here, $q_1 = 0{\cdot}4\ \mu\text{C} = 0{\cdot}4 \times 10^{-6}$ C, $q_2 = -0{\cdot}8\ \mu\text{C} = -0{\cdot}8 \times 10^{-6}$ C, $F = 0{\cdot}2$ N ; $r = ?$

As $$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad \therefore \quad r^2 = \frac{q_1 q_2}{4\pi\epsilon_0 F} \quad \text{or} \quad r^2 = \frac{(0{\cdot}4\times10^{-6})(0{\cdot}8\times10^{-6})\times9\times10^9}{0{\cdot}2}$$

$r^2 = 16 \times 9 \times 10^{-4}$ or $r = 4 \times 3 \times 10^{-2}$ m $=$ **0·12 m**

Force on second sphere due to the first is the same, *i.e.*, **0·2 N**

Q. 3. Check that the ratio $ke^2/Gm_e m_p$ is dimensionless. Look up a table of Physical Constants and determine the value of this ratio. What does the ratio signify ?

Sol. $$\frac{ke^2}{G m_e m_p} = \frac{[N m^2 C^{-2}][C^2]}{[N m^2 kg^{-2}][kg][kg]} = 1 = (M^\circ L^\circ T^\circ)$$ $\therefore$ The given ratio is **dimensionless.**

using $k = 9 \times 10^9$ N m^2 C^{-2}, $e = 1{\cdot}6 \times 10^{-19}$ C, $G = 6{\cdot}67 \times 10^{-11}$ N m^2 kg^{-2},

$m_e = 9{\cdot}1 \times 10^{-31}$ kg, and $m_p = 1{\cdot}66 \times 10^{-27}$ kg, we get, $$\frac{ke^2}{G m_e m_p} = 2{\cdot}29\times10^{39}$$

This is the **ratio of electrostatic force to gravitational force between an electron and a proton.**

Q. 4. (a) Explain the meaning of the statement 'electric charge of a body is quantised.'

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic, i.e., large scale charges ?

Sol. (*a*) Refer to Art. 1.8.

(*b*) As electric charge is quantised, $q = \pm ne$ where $e = 1{\cdot}6 \times 10^{-19}$ C

At macroscopic level, one deals with charges that are enormous compared to the magnitude of charge (e). For example, when $q = 1\ \mu\text{C} = 10^{-6}$ C

$$n = \frac{q}{e} = \frac{10^{-6}}{1{\cdot}6\times10^{-19}} \approx 6{\cdot}2\times10^{12}$$

i.e., a charge of 1 μ C contains $6{\cdot}2 \times 10^{12}$ electrons. The fact that charge can increase or decrease only in units of 'e' when 10^{12} electronic charges are involved has almost no significance. Thus at the macroscopic level, graininess of electric charge or quantisation of electric charge is of no practical consequence. It can be ignored and electric charge can be considered as continuous.

Q. 5. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Sol. When a glass rod is rubbed with a silk cloth, charges appear on both. These charges are equal in magnitude and opposite in sign, so that algebraic sum of the charges produced on both is zero. The net charge on the two bodies was zero even before rubbing them. Thus we find that charges can be created only in equal and unlike pairs. This is consistent with the law of conservation of charge.

Q. 6. Four point charges $q_A = 2\ \mu C$; $q_B = -5\ \mu C$, $q_C = 2\ \mu C$ and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μ C placed at the centre of the square ?

Sol. In Fig. 1(N).1, length of diagonal of the square

$$= AC = BD = \sqrt{10^2 + 10^2} = 10\sqrt{2}\ \text{cm}.$$

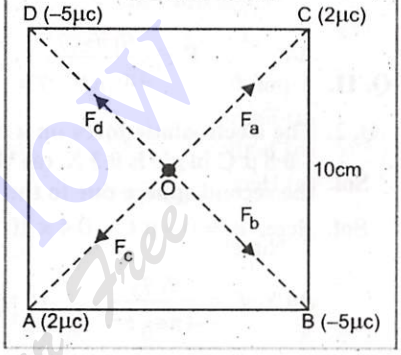
$$OA = OB = OC = OD = \frac{10\sqrt{2}}{2} = \frac{10}{\sqrt{2}}\ \text{cm}.$$

Forces of repulsion on 1 μ C charge at O due to 2 μ C charge, at A and C are equal and opposite. Therefore, they cancel.

Similarly, forces of attraction on 1 μ C charge at O, due to $-5\ \mu$ C charges at B and D are also equal and opposite. Therefore, these also cancel.

Hence the net force on the charge of 1 μ C at O is **zero.**

FIGURE 1(N).1



Q. 7. (a) An electrostatic field line is a continuous curve. That is a field line cannot have sudden breaks. Why not ?

(b) Explain why two field lines never cross each other at any point.

Sol. (*a*) An electrostatic field line is a continuous curve, because it represents the actual path of a unit positive charge, which experiences a continuous force. It cannot have sudden breaks because the moving test charge never jumps from one position to the other.

(*b*) Refer to Art 1.27.

Q. 8. Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum (a) What is the electric field at the mid point O of the line AB joining the two charges ? (b) If a negative test charge of magnitude $1{\cdot}5 \times 10^{-9}$ C is placed at the point, what is the force experienced by the test charge ?

Sol. Here, $q_A = 3\ \mu C = 3 \times 10^{-6}$ C, $q_B = -3\ \mu C = -3 \times 10^{-6}$ C, $AB = 20$ cm, Fig. 1(N).2.

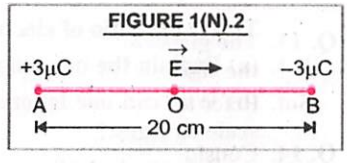
$r = OA = OB = 10\ \text{cm} = 10^{-1}$ m, $E = ?$

Electric field is the force on unit positive charge (+ 1C) at O.

$$E = \frac{2 \times q_A \times 1}{4\pi\epsilon_0 r^2}$$

$$E = \frac{2 \times 3 \times 10^{-6} \times 9 \times 10^9}{(10^{-1})^2} = \mathbf{5{\cdot}4 \times 10^6\ N/C,\ along\ OB}$$

FIGURE 1(N).2



Force on negative charge $q = -1{\cdot}5 \times 10^{-9}$ C at O will be

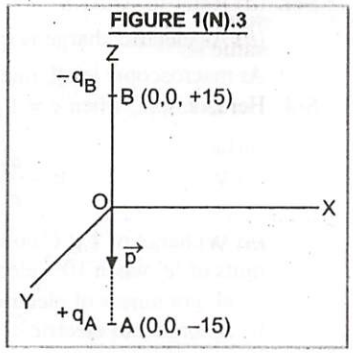
$$F = qE = -1{\cdot}5 \times 10^{-9}\ (5{\cdot}4 \times 10^6)\ \text{N}$$

$$F = \mathbf{-8{\cdot}1 \times 10^{-3}\ N\ ;\ along\ BA}$$

Q. 9. A system has two charges $q_A = 2{\cdot}5 \times 10^{-7}$ C and $q_B = -2{\cdot}5 \times 10^{-7}$ C located at points A (0, 0, – 15 cm) and B (0, 0, +15 cm) respectively. What are the total charge and electric dipole moment of the system ?

Sol. Two charges q_A and q_B are located at points A (0, 0, – 15) and B (0, 0, 15) on Z-axis, as shown in Fig. 1(N).3. They form an electric dipole.

FIGURE 1(N).3



Total charge, $q = q_A + q_B = 2{\cdot}5 \times 10^{-7} - 2{\cdot}5 \times 10^{-7} = 0$

Also, $AB = 15 + 15 = 30$ cm $= 30 \times 10^{-2}$ m

Electric dipole moment,

p = either charge $\times AB = 2{\cdot}5 \times 10^{-7} \times (30 \times 10^{-2}) = 7{\cdot}5 \times 10^{-8}$ C-m

The direction of $\vec{p}$ is along BA, *i.e.*, **along negative Z-axis.**

Q. 10. An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.

Sol. Here, $p = 4 \times 10^{-9}$ C m, $\theta = 30°$, $E = 5 \times 10^4$ NC^{-1}, $\tau = ?$

$$\tau = pE \sin\theta = (4 \times 10^{-9}) \times (5 \times 10^4) \sin 30°$$

$$\tau = 4 \times 5 \times 10^{-5} \times \frac{1}{2} = \mathbf{10^{-4}\ N\text{-}m}$$

Q. 11. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which ?)

(b) Is there a transfer of mass from wool to polythene ?

Sol. (*a*) Here, $q = -3 \times 10^{-7}$ C, Charge on one electron, $e = -1{\cdot}6 \times 10^{-19}$ C

$\therefore$ Number of electrons transferred to polythene piece from wool, $n = \dfrac{q}{e} = \dfrac{-3 \times 10^{-7}}{-1{\cdot}6 \times 10^{-19}} = \mathbf{1{\cdot}875 \times 10^{12}}$

(*b*) Yes, there is a transfer of mass from wool to polythene piece.

As mass of each electron $= 9 \times 10^{-31}$ kg

$\therefore$ Mass transferred to polythene $= 1{\cdot}875 \times 10^{12} \times 9 \times 10^{-31}$ kg $= \mathbf{1{\cdot}687 \times 10^{-18}}$ kg

Q. 12. (a) Two insulated charged copper spheres *A* and *B* have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is $6{\cdot}5 \times 10^{-7}$ C ? The radii of *A* and *B* are negligible compared to the distance of separation.

(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved ?

Sol. (*a*) Here, $q_1 = q_2 = 6{\cdot}5 \times 10^{-7}$ C, $r = 0{\cdot}5$ m

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times (6{\cdot}5 \times 10^{-7})^2}{(0{\cdot}5)^2} = \mathbf{1{\cdot}521 \times 10^{-2}\ N}$$

(*b*) Now q_1, q_2 both are doubled and r is halved,

As $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$ $\therefore$ F becomes 16 times, *i.e.*, $F' = 16F = 16 \times 1{\cdot}521 \times 10^{-2}$ N $= \mathbf{0{\cdot}24\ N}$

Q. 13. The given Fig. 1(Q).15 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio ?

Sol. Refer to Conceptual Problem 17, Page 1/72.

Q. 14. Consider a uniform electric field $E = 3 \times 10^3\,\hat{i}$ N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane ? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis ?

Sol. Here, $\vec{E} = 3 \times 10^3\,\hat{i}$ *N/C*, *i.e.*, field is along + direction of X-axis

surface area, $s = (10\text{ cm})^2 = 10^2\text{ cm}^2 = 10^2 \times 10^{-4}\text{ m}^2 = 10^{-2}\text{ m}^2$.

(*a*) When plane is parallel to yz plane, $\theta = 0°$

$$\phi_E = Es\cos\theta = 3 \times 10^3 \times 10^{-2} \cos 0° = \mathbf{30\ NC^{-1}\,m^2}$$

(*b*) When normal to the plane makes an angle of 60° with X-axis, then $\theta = 60°$

$$\phi_E = Es\cos\theta = 3 \times 10^3 \times 10^{-2} \cos 60° = 30 \times \frac{1}{2} = \mathbf{15\ NC^{-1}\,m^2}$$

Q. 15. What is the net flux of the uniform electric field of Q. 14 above through a cube of side 20 cm oriented so that its faces are parallel to the co-ordinate planes ?

Sol. Net flux over the cube is zero, because the number of lines entering the cube is the same as the number of lines leaving the cube.

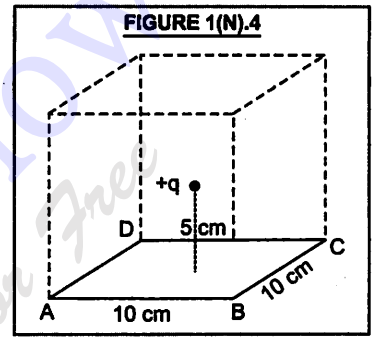
Q. 16. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8{\cdot}0 \times 10^3$ Nm^2/C (a) What is the net charge inside the box ? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box ? Why or why not ?

Sol. (*a*) $\phi_E = 8{\cdot}0 \times 10^3$ NC^{-1} m^2, q = ?

As $\phi_E = \dfrac{q}{\epsilon_0}$ $\quad\therefore\quad q = \epsilon_0\,\phi_E = (8{\cdot}85 \times 10^{-12})\,(8{\cdot}0 \times 10^3) = 0{\cdot}07 \times 10^{-6}$ C $= 0{\cdot}07\ \mu$ C

(*b*) When $\phi_E = 0$, $q = 0$. Infact, $\Sigma q = 0$, *i.e.*, algebraic sum of the charges inside the box must be zero or there may be no charge inside the box.

Q. 17. A point charge +10 μ C is at a distance of 5 cm directly above the centre of a square of side 10 cm as shown in Fig. 1(N).4. What is the magnitude of the electric flux through the square ? (Hint. Think of the square as one face of a cube with edge 10 cm)



Sol. In Fig. 1(N).4, ABCD is a square of side 10 cm. The point charge + 10 μ C is at a distance of 5 cm. directly above the centre of ABCD. We can imagine the square ABCD as one of the six faces of a cube of side 10 cm. The charge + q is at the centre of this cube.

According to Gauss's theorem, total electric flux through all the six faces of the cube = q/ϵ_0

$\therefore$ Electric flux through the square $\phi_E = \dfrac{1}{6}\dfrac{q}{\epsilon_0} = \dfrac{1}{6} \times \dfrac{10 \times 10^{-6}}{8{\cdot}85 \times 10^{-12}} = \mathbf{1{\cdot}88 \times 10^5\ N\ m^2\ C^{-1}}$

Q. 18. A point charge of 2·0 μ C is at the centre of a cubic Gaussian surface 9·0 cm on edge. What is the net electric flux through the surface ?

Sol. Here, $q = 2{\cdot}0\ \mu$ C $= 2 \times 10^{-6}$ C, $l = 9{\cdot}0$ cm.

$$\phi_E = \frac{q}{\epsilon_0} = \frac{2 \times 10^{-6}}{8{\cdot}85 \times 10^{-12}} = \mathbf{2{\cdot}26 \times 10^5\ Nm^2\ C^{-1}}$$

Q. 19. A point charge causes an electric flux of – $1{\cdot}0 \times 10^3$ N m^2/C to pass through a spherical Gaussian surface of 10·0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface ? (b) What is the value of the point charge ?

Sol. Here, $\phi_E = -1{\cdot}0 \times 10^3$ N m^2/C, $r = 10{\cdot}0$ cm.

(*a*) If the radius of Gaussian surface were doubled, flux passing through the surface would remain the same. This is because flux depends only on charge q present inside the surface.

(*b*) As $\phi_E = \dfrac{q}{\epsilon_0}$ $\therefore$ $q = \epsilon_0\,\phi_E = (8{\cdot}85 \times 10^{-12})\,(-1{\cdot}0 \times 10^3) = \mathbf{-8{\cdot}85 \times 10^{-9}\ C}$

Q. 20. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is $1{\cdot}5 \times 10^3$ N/C and points radially inwards, what is the net charge on the sphere ?

Sol. Here, radius of sphere = 10 cm.

Distance of point from the centre of the sphere, $r = 20$ cm $= 0{\cdot}2$ m

Electric field, $E = -1{\cdot}5 \times 10^3$ N/C (Neg. sign for inward field)

Charge, q = ?

As $E = \dfrac{q}{4\pi\epsilon_0\, r^2}$ $\therefore$ $q = (4\pi\epsilon_0\, r^2)\,E = \dfrac{1}{9 \times 10^9}(0{\cdot}2)^2\,(-1{\cdot}5 \times 10^3) = -\dfrac{2}{3} \times 10^{-8}$ C $= \mathbf{-6{\cdot}67 \times 10^{-9}\ C}$

Q. 21. A uniformly charged conducting sphere of 2·4 m diameter has a surface charge density of 80·0 μ C/m^2. (a) Find the charge on the sphere (b) What is the total electric flux leaving the surface of the sphere ?

Sol. Here, D = 2r = 2·4 m, r = 1·2 m, σ = 80·0 μ C/m^2 = 80·0 × 10^{-6} C/m^2.

Charge on the sphere, $Q = \sigma \times 4\pi r^2 = 80\times10^{-6}\times4\times\frac{22}{7}\times(1{\cdot}2)^2$ = **1·45 × 10^{-3} C**

Electric flux, $\phi_E = \frac{Q}{\epsilon_0} = \frac{1{\cdot}45\times10^{-3}}{8{\cdot}85\times10^{-12}}$ = **1·6 × 10^8 NC^{-1} m^2**

Q. 22. An infinite line charge produces a field of 9 × 10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

Sol. Here, E = 9 × 10^4 N/C, r = 2 cm = 2 × 10^{-2} m, λ = ?

As $E = \frac{\lambda}{2\pi\epsilon_0 r}$ ∴ $\lambda = 2\pi\epsilon_0 rE = \frac{1}{2\times9\times10^9}\times2\times10^{-2}\times9\times10^4$ = **10^{-7} C m^{-1}**

Q. 23. Two large thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and magnitude 17·0 × 10^{-22} C/m^2. What is $\vec{E}$: in the outer region of the first plate. (b) in the outer region of the second plate, and (c) between the plates ? See Fig. 1(N).5.

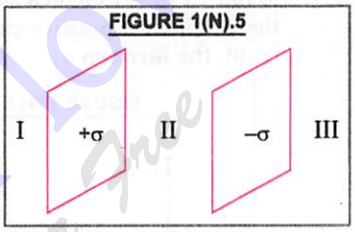


FIGURE 1(N).5

Sol. Here, σ = 17·0 × 10^{-22} C m^{-2}.

As discussed in theory, in region I, to the left of the plates, E = **0.** And in region III to the right of the plates, again, E = **0.** However, in region II, inbetween the plates, $E = \frac{\sigma}{\epsilon_0} = \frac{17{\cdot}0\times10^{-22}}{8{\cdot}85\times10^{-12}}$ = **1·92 × 10^{-10} NC^{-1}**

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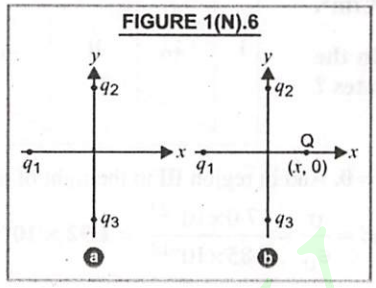
WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. In Fig. 1(N).6, two positive charges q_2 and q_3 fixed along the y-axis, exert a net electric force in the $+x$ direction on a charge q_1 fixed along the x-axis. If a positive charge Q is added at $(x, 0)$, the force on q_1

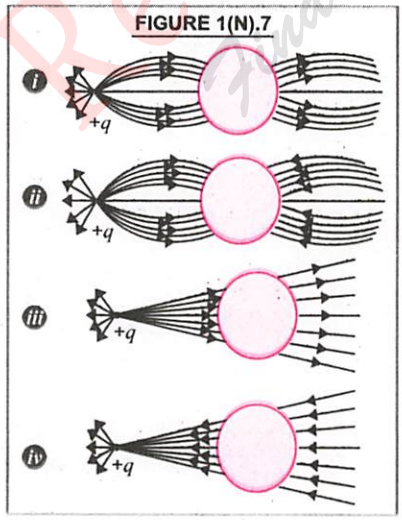
FIGURE 1(N).6



(*a*) shall increase along the positive x-axis
(*b*) shall decrease along the positive x-axis
(*c*) shall point along the negative x-axis
(*d*) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

2. A point positive charge is brought near an isolated conducting sphere, Fig. 1(N).7. The electric field is best given by

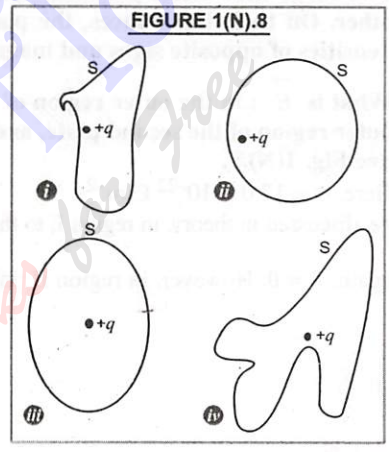
FIGURE 1(N).7



(*a*) Fig. (*i*) (*b*) Fig. (*ii*)
(*c*) Fig. (*iii*) (*d*) Fig. (*iv*)

3. The electric flux through the surface

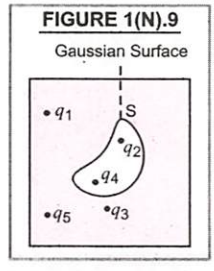
FIGURE 1(N).8



(*a*) in Fig. 1(N).8(*iv*) is the largest
(*b*) in Fig. 1(N).8(*iii*) is the least
(*c*) in Fig. 1(N).8(*ii*) is same as in Fig. 1(N).8(*iii*) but is smaller than that in Fig. 1(N).8(*iv*)
(*d*) is the same for all the figures

4. Five charges q_1, q_2, q_3, q_4 and q_5 are fixed at their positions as shown in Fig. 1(N).9. S is a Gaussian surface. The Gauss's law is given by $\oint_S E.ds = \frac{q}{\varepsilon_0}$

FIGURE 1(N).9



Which of the following statements is correct ?

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(*a*) E on the LHS of the above equation will have a contribution from q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only

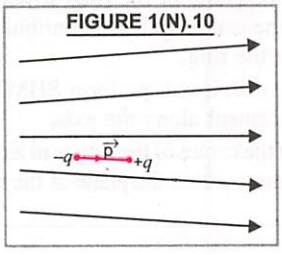
(*b*) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only

(*c*) E on the LHS of the above equation will have a contribution from all charges while q on RHS will have a contribution from q_1, q_3 and q_5 only

(*d*) Both, E on the LHS and q on the RHS will have contributions from q_2 and q_4 only

5. Figure 1(N).10 shows electric field lines in which an electric dipole p is placed as shown. Which of the following statements is correct ?

FIGURE 1(N).10



(*a*) The dipole will not experience any force

(*b*) The dipole will experience a force towards right

(*c*) The dipole will experience a force towards left

(*d*) The dipole will experience a force upwards

6. A point charge + q, is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is

(*a*) directed perpendicular to the plane and away from the plane

(*b*) directed perpendicular to the plane but towards the plane

(*c*) directed radially away from the point charge

(*d*) directed radially towards the point charge

7. A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed

(*a*) perpendicular to the diameter

(*b*) parallel to the diameter

(*c*) at an angle titled towards the diameter

(*d*) at an angle tilted away from the diameter

MULTIPLE CHOICE QUESTIONS-II

8. If $\oint_S E.ds = 0$ over a surface, then

(*a*) the electric field inside the surface and on it is zero

(*b*) the electric field inside the surface is necessarily uniform

(*c*) the number of flux lines entering the surface must be equal to the number of flux lines leaving it

(*d*) all charges must necessarily be outside the surface

9. The Electric field at a point is

(*a*) always continuous

(*b*) continuous if there is no charge at that point

(*c*) discontinuous only if there is a negative charge at that point

(*d*) discontinuous if there is a charge at that point

10. If there were only one type of charge in the universe, then

(*a*) $\oint_S \vec{E}.\vec{ds} \neq 0$ on any surface

(*b*) $\oint_S \vec{E}.\vec{ds} = 0$ if the charge is outside the surface

(*c*) $\oint_S \vec{E}.\vec{ds}$ could not be defined

(*d*) $\oint_S \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface

11. Consider a region inside which there are various types of charges but the total charge is zero. At points outside the region

(*a*) the electric field is necessarily zero

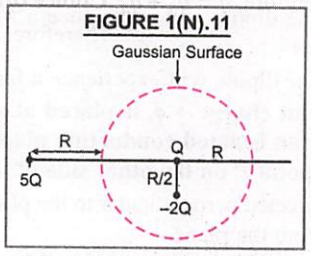
(*b*) the electric field is due to the dipole moment of the charge distribution only

(*c*) the dominant electric field is $\propto \frac{1}{r^3}$, for large r, where r is the distance from a origin in this region

(*d*) the work done to move a charged particle along a closed path, away from the region, will be zero.

12. Refer to the arrangement of charges in Fig. 1(N).11 and a Gaussian surface of radius R with Q at the centre. Then

FIGURE 1(N).11



(*a*) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$

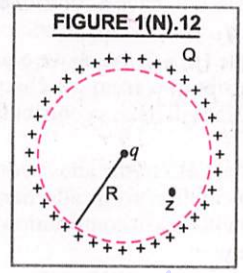
(*b*) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$

(*c*) flux through the surface of sphere due to 5 Q is zero

(*d*) field on the surface of sphere due to – 2 Q is same everywhere

13. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring, Fig. 1(N).12. Then

FIGURE 1(N).12



(*a*) If $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre,

(*b*) If $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring,

(*c*) If $q < 0$, it will perform SHM for small displacement along the axis,

(*d*) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$.

ANSWERS

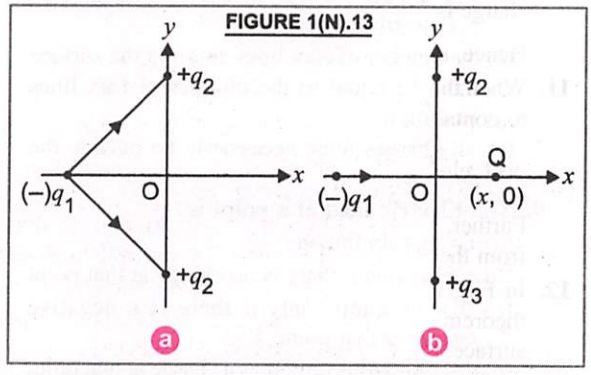
1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*b*) **5.** (*c*) **6.** (*a*) **7.** (*a*) **8.** (*c, d*) **9.** (*b, d*)
10. (*b, d*) **11.** (*c, d*) **12.** (*a, c*) **13.** (*a, b, c*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. As q_2, q_3 are positive charges and net force on q_1 is along + x direction, therefore q_1 must be negative as shown in Fig. 1(N).13(*a*).
When a positive charge Q is added at (x, 0), it will attract ($-q_1$) along + x direc- tion, Fig. 1(N).13(*b*). Therefore, **force on q_1 will increase along the positive x-axis.**

FIGURE 1(N).13

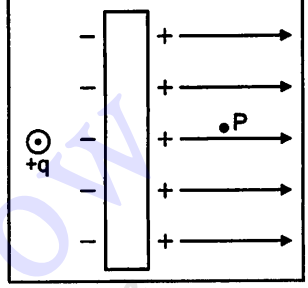


2. When a point positive charge is brought near an isolated conducting sphere, there developes some negative charge on left side of the sphere and an equal positive charge on the right side of the sphere. Electric lines of force emanating from the point positive charge end normally on the left side of the sphere. And due to positive charge on the right side of the sphere, the electric lines of force emanate normally from the right side. The electric field is best given by **Fig. 1(N).7(*i*).**

3. The electric flux through a surface depends only on amount of charge enclosed by the surface. It does not depend on size and shape of the surface, as per Gauss's theorem in electrostatics. Therefore, electric flux through the surface is the same for all the figures, 1(N).8. **Choice (*d*) is correct.**

4. According to Gauss's theorem in electrostatics, $\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$. Here, $\vec{E}$ is due to all the charges q_1, q_2, q_3, q_4 and q_5. As q is charge enclosed by the Gaussian surface, therefore, $q = q_2 + q_4$. **Choice (*b*) is correct.**
5. In Fig. 1(N).10, spacing between electric lines of force increases from left to right. Therefore, E on left is greater that E on right. Force on $+q$ charge of dipole is smaller and to the right. Force on $-q$ charge of dipole is bigger and to the left. Hence, the dipole will experience a **net force towards the left.**
6. When a point charge $+q$ is placed at a distance (d) from an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side of the plane. Hence, the field at a point P on the other side of the plane is directed perpendicular to the plane and away from the plane as shown in Fig. 1(N).14. **Choice (*a*) is correct.**

FIGURE 1(N).14



7. When the point is situated at a point on diameter away from the centre of hemisphere charged uniformly positively, the electric field is **perpendicular to the diameter.** The components of electric intensity parallel to the diameter cancel out.

Multiple Choice Questions – II

8. $\oint \vec{E}.\vec{ds}$ represents electric flux over the closed surface. When $\oint \vec{E}.\vec{ds} = 0$, it means the number of flux lines entering the surface must be equal to number of flux lines leaving it. Further, as $\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$ where q is charge enclosed by the surface. When $\oint \vec{E}.\vec{ds} = 0$, ; $q = 0$, *i.e.*, net charge enclosed by the surface must be zero. Therefore, all other charges must necessarily be outside the surface. This is because charges outside the surface do not contribute to the electric flux. **Choices (*c*) and (*d*) are correct.**
9. From the general knowledge of theory, electric field at a point is continuous if there is no charge at that point. And the field is discontinuous if there is charge at that point. **Choices (*b*) and (*d*) are correct.**
10. As per Gauss's theorem in electrostatics, $\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$, where q is charge enclosed by the surface. If the charge is outside the surface, $q_{inside} = 0$. Therefore, $\oint \vec{E}.\vec{ds} = 0$

    Hence, **choices (*b*) and (*d*) are correct.**
11. When there are various types of charges in a region, but the total charge is zero, the region can be supposed to contain a number of electric dipoles. Therefore, at points outside the region (which may be anywhere w.r.t. electric dipoles), the dominant electric field $\propto \frac{1}{r^3}$ for large r. **Choice (*c*) is correct.**

    Further, as electric field is conservative, work done to move a charged particle along a closed path, away from the region will be zero. **Choice (*d*) is also correct.**
12. In Fig. 1(N).11, charge enclosed by the Gaussian surface = $Q - 2Q = -Q$. Therefore, as per Gauss's theorem, total flux through the surface of the sphere = $-Q/\epsilon_0$. Further, as charge $5Q$ lies outside the surface, it makes no contribution to electric flux through the given surface. **Choices (*a*) and (*c*) are correct.**
13. As is known from theory, at the centre of the ring, $E = 0$ when a positive charge ($q > 0$) is displaced away from the centre in the plane of the ring, say to the right, force of repulsion on q, due to charge on right half increases and due to charge on left half decreases. Therefore, charge q is pushed back towards the centre. **Choice (*a*) is correct.**

    When charge q is negative ($q < 0$), force is of attraction. Therefore, charge q displaced to the right continues moving to the right till it hits the ring. **Choice (*b*) is correct.**

Along the axis of the ring, at a distance r from the centre, $E = \dfrac{Qr}{4\pi\epsilon_0 (r^2 + a^2)^{3/2}}$

If charge q is negative ($q < 0$), it will perform SHM for small displacement along the axis. **Choice (*c*) is correct.**

VERY SHORT ANSWER QUESTIONS

14. An arbitrary surface encloses a dipole. What is the electric flux through this surface ?

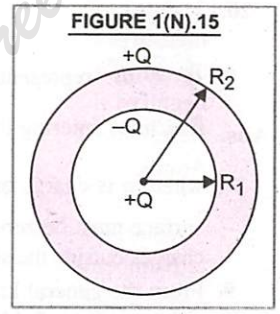
Ans. As net charge on a dipole is $(-q + q) = 0$, therefore, when an arbitrary surface encloses a dipole, as per Gauss's theorem, electric flux through the surface

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0} = \textbf{Zero}$$

15. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (*i*) the inner surface, and (*ii*) the outer surface ?

Ans. When a charge $+Q$ is placed at the centre of spherical cavity, shown in Fig. 1(N).15, charge induced on the inner surface of shell $= -Q$ and charge induced on the outer surface of the shell $= +Q$.

FIGURE 1(N).15



$\therefore$ Surface charge density on the inner surface $= \dfrac{-Q}{4\pi R_1^2}$, and

Surface charge density on the outer surface $= \dfrac{+Q}{4\pi R_2^2}$

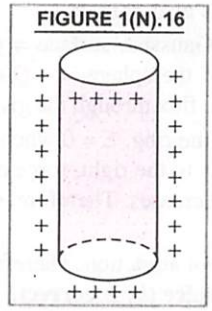
16. The dimensions of an atom are of the order of an Angstrom. Thus there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero ?

Ans. As is known, electrostatic field is caused by excess charges. On the inside of an isolated conductor, there is no excess charge. Therefore, electrostatic field inside a conductor is zero.

17. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero ? Conversely, if the electric field everywhere on a surface is zero, does it imply that net charge inside is zero.

Ans. When the total charge enclosed by a surface is zero, it does not imply that electric field everywhere on the surface is zero. As $\oint \vec{E}.\vec{ds} = \dfrac{q}{\epsilon_0} = 0$, therefore, **the field may be normal to the surface.** The converse is, however true, *i.e.*, when $E = 0$ everywhere on a surface, net charge inside is zero.

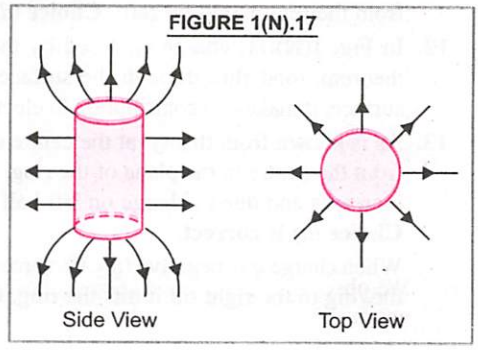
FIGURE 1(N).16



18. Sketch the electric field lines for a uniformly charged hollow cylinder shown in Fig. 1(N).16.

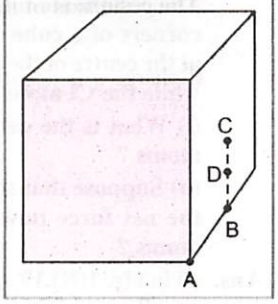
Ans. For a uniformly charged hollow cylinder, the electric field lines are as shown in Fig. 1(N).17.

FIGURE 1(N).17



19. What will be the total flux through the faces of the cube, Fig. 1(N).18 with side of length a if a charge q is placed at (a) A : a corner of the cube (b) B : mid-point of an edge of the cube (c) C : centre of a face of the cube (d) D : mid-point of B and C.

FIGURE 1(N).18



Ans. (a) In Fig. 1(N).18, when a charge q is placed at corner A of the cube, it is being shared equally by 8 cubes. Therefore, total flux through the faces of the given cube = $q/8 \in_0$.

(b) When the charge q is placed at B, middle point of an edge of the cube, it is being shared equally by 4 cubes. Therefore, total flux through the faces of the given cube = $q/4 \in_0$.

(c) When the charge q is placed at C, the centre of a face of the cube, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube = $q/2 \in_0$.

(d) Similarly, when charge q is placed at D, the mid point of B and C, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube = $q/2 \in_0$.

SHORT ANSWER QUESTIONS

20. A paisa coin is made up of Al-Mg alloy and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges. Treating the paisa coin made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude ?

Ans. Here, mass of a paisa coin = 0·75 g

Atomic mass of aluminium = 26·9815 gram

Avogadro's number = $6{\cdot}023 \times 10^{23}$

$\therefore$ Number of aluminium atoms in one paisa coin $N = \dfrac{6{\cdot}023\times10^{23}}{26{\cdot}9815} \times 0{\cdot}75 = 1{\cdot}6742 \times 10^{22}$

As charge number of Al is 13, each atom of Al contains 13 protons and 13 electrons.

$\therefore$ Magnitude of positive and negative charges in one paisa coin = $N Z e$

$= 1{\cdot}6742 \times 10^{22} \times 13 \times 1{\cdot}60 \times 10^{-19}$ C $= 3{\cdot}48 \times 10^4$ C $= 34.8$ KC

Thus, we find that ordinary neutral matter contains enormous amount of ± charges.

21. Consider a coin of Example 20 above. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two point charges separated by (i) 1 cm (~ ½ × diagonal of the one paisa coin), (ii) 100 m (~ length of a long building), and (iii) 10^6 m (radius of the earth). Find the force on each such point charge in each of the three cases. What do you conclude from these results ?

Ans. Here, $q = \pm 34{\cdot}8$ KC $= \pm 3{\cdot}48 \times 10^4$ C, $r_1 = 1$ cm $= 10^{-2}$ m ; $r_2 = 100$ m ; $r_3 = 10^6$ m, $F_1 = ?$, $F_2 = ?$; $F_3 = ?$

According to Coulomb's law,

$$F_1 = \frac{|q|^2}{4\pi\epsilon_0 r_1^2} = \frac{9\times10^9\,(3{\cdot}48\times10^4)^2}{(10^{-2})^2} = 1{\cdot}09 \times 10^{23}\ \text{N},$$

$$F_2 = \frac{|q|^2}{4\pi\epsilon_0 r_2^2} = \frac{9\times10^9\,(3{\cdot}48\times10^4)^2}{(100)^2} = 1{\cdot}09 \times 10^{15}\ \text{N},$$

$$F_3 = \frac{|q|^2}{4\pi\epsilon_0 r_3^2} = \frac{9\times10^9\,(3{\cdot}48\times10^4)^2}{(10^6)^2} = 1{\cdot}09 \times 10^{7}\ \text{N}$$

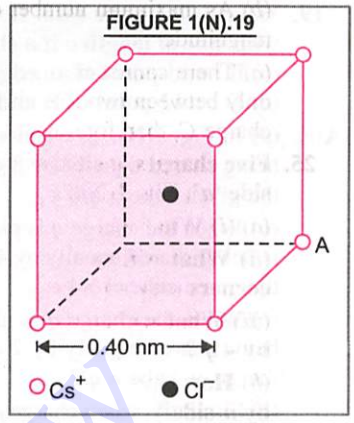
We observe that when ± charges in ordinary neutral matter are separated as point charges, they exert an enormous force. Hence, it is not easy to disturb electrical neutrality of matter.

22. Fig. 1(N).19 represents a crystal unit of cesium chloride, CsCl. The cesium atoms, represented by open circles are situated at the corners of a cube of side 0.40 nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.

(*i*) What is the net electric field on the Cl atom due to eight Cs atoms ?

(*ii*) Suppose that the Cs atom at the corner A is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms ?

FIGURE 1(N).19



Ans. (*i*) In Fig. 1(N).19, Chlorine atom at the centre of the cube is attracted equally by eight cesium atoms at the eight corners of the cube. Symmetry shows that these forces would cancel out in pairs. Therefore, net electric field on Cl atm due to eight Cs atoms is zero.

(*ii*) Removing a Cs atom at the corner A is equivalent to adding a singly charged negative Cs ion at A. Net force on the Cl atom at A would be $F = \dfrac{e^2}{4\pi\epsilon_0 r^2}$, where r = distance between Cl ion and Cs ion, *i.e.*,

$$r = \sqrt{(0{\cdot}20)^2 + (0{\cdot}20)^2 + (0{\cdot}20)^2} \times 10^{-9}\ \text{m} = 0{\cdot}346 \times 10^{-9}\ \text{m}$$

$$\therefore \quad F = \frac{e^2}{4\pi\epsilon_0 r^2} = \frac{9\times10^9\,(1{\cdot}6\times10^{-19})^2}{(0{\cdot}346\times10^{-9})^2} = \mathbf{1{\cdot}92 \times 10^{-9}\ N}$$

The direction of this force is from corner A to Cl^- ion at the centre.

23. Two charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge $2q$ be placed such that it will not experience any force ?

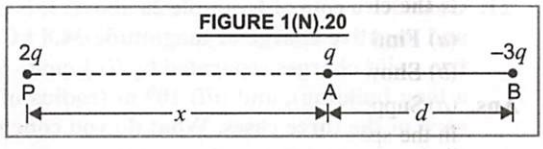
Ans. Let the charge $2q$ be placed at P, at a distance x from A where charge q is placed, Fig. 1(N).20. The charge $2q$ will not experience any force, when force of repulsion on it due to q is balanced by force of attraction on it due to $-3q$, at B, where $AB = d$

i.e., $\dfrac{(2q)q}{4\pi\epsilon_0 x^2} = \dfrac{2q(3q)}{4\pi\epsilon_0 (x+d)^2}$ or $(x+d)^2 = 3x^2$ or $2x^2 - 2xd - d^2 = 0$.

$$x = \frac{2d \pm \sqrt{4d^2 + 8d^2}}{4} = \frac{d}{2} \pm \frac{\sqrt{3}\,d}{2}$$

$$x = \frac{d + \sqrt{3}\,d}{2} \quad \text{(ignoring } - \text{ sign)}$$

FIGURE 1(N).20



The position P of the charge is on the left side of charge q at A.

24. Fig. 1(N).21 shows the electric field lines around three point charges A, B and C.

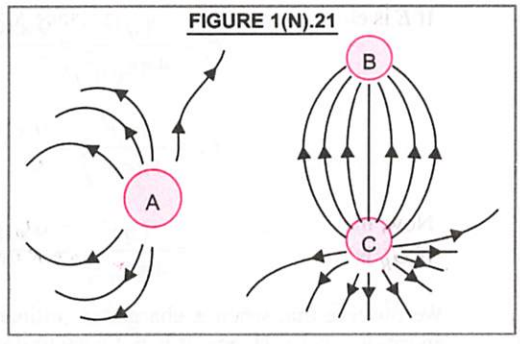
(*a*) Which charges are positive ?

(*b*) Which charge has the largest magnitude ? Why ?

(*c*) In which region or regions of the picture could the electric field be zero ? Justify your answer.

(*i*) near A, (*ii*) near B, (*iii*) near C, (*iv*) nowhere.

FIGURE 1(N).21



Ans. (*a*) In Fig. 1(N).21, the electric lines of force emanate from A and C. Therefore, charges A and C must be positive.

(*b*) As maximum number of electric lines of force are associated with charge *C*, it must have the largest magnitude.

(*c*) There cannot be a neutral point between a positive and a negative charge. Neutral point can be there only between two like charges, *i.e.*, A and C. As magnitude of charge *A* is smaller than the magnitude of charge *C*, therefore, neutral point would lie closer to charge *A*.

25. Five charges, *q* each are placed at the corners of a regular pentagon of side '*a*', Fig. 1(N).22.

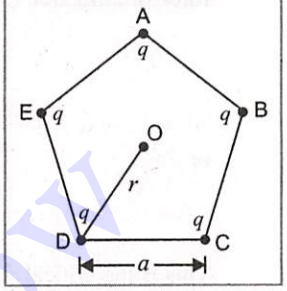
(*a*) (*i*) What will be the electric field at O, the centre of the pentagon ?

(*ii*) What will be the electric field at O if the charge from one of the corners (say A) is removed ?

(*iii*) What will be the electric field at O if the charge *q* at A is replaced by – *q* ?

(*b*) How would your answer to (*a*) be affected if pentagon is replaced by *n*-sided regular polygon with charge *q* at each of its corners ?

FIGURE 1(N).22



Ans. (*a*) (*i*) In Fig. 1(N).22, the electric field at *O*, the centre of pentagon would be zero.

(*ii*) When charge *q* from corner *A* is removed, electric field at *O* would be

$$E_1 = \frac{q \times 1}{4\pi\epsilon_0 r^2} \text{ along } OA.$$

(*iii*) If charge *q* at *A* is replaced by – *q*, it is equivalent to adding charge – 2 *q* at *A*. Therefore, electric field at *O* would be

$$E_2 = \frac{2q}{4\pi\epsilon_0 r^2} \text{ along } OA.$$

(*b*) When pentagon is replaced by *n* sided regular polygon with charge *q* at each of its corners, the electric field at *O* would continue to be zero.

LONG ANSWER QUESTIONS

26. In 1959, Lyttleton and Bondi suggested that the expansion of the Universe could be explained if matter carried a net charge. Suppose that the Universe is made up of hydrogen atoms with a number density *N*, which is maintained a constant. Let the charge on the proton be : $e_p = -(1 + y)\, e$ where *e* is the electronic charge.

(*a*) Find the critical value of *y* such that expansion may start.

(*b*) Show that the velocity of expansion is proportional to the distance from the centre.

Ans. (*a*) Suppose the universe is a sphere of radius *R* and its constituent hydrogen atoms are distributed uniformly in the sphere.

As hydrogen atom contains one proton and one electron, therefore, charge on each hydrogen atom,

$$e_H = e_p + e = -(1 + y)\, e + e = -ye = (ye)$$

If *E* is electric field intensity at distance *R*, on the surface of the sphere, then according to Gauss's theorem,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}, \text{ i.e., } E(4\pi R^2) = \frac{4}{3}\frac{\pi R^3 N |ye|}{\epsilon_0}$$

$$E = \frac{1}{3}\frac{N|ye|R}{\epsilon_0} \qquad ...(i)$$

Now, mass of each hydrogen atom $\simeq m_p$ = mass of a proton.

If G_R is gravitational field at distance *R* on the surface of the sphere, then

$$-4\pi R^2 G_R = 4\pi G . m_p . \left(\frac{4}{3}\pi R^3 N\right) \quad \text{or} \quad G_R = -\frac{4}{3}\pi G m_p N R \qquad ...(ii)$$

$\therefore$ Gravitational force on this atom is $F_1 = m_p \times G_R = -\frac{4\pi}{3} G m_p^2 N R$ ...(iii)

and coulomb force on hydrogen atom at R is $F_2 = (ye)\, E = \frac{1}{3}\frac{N y^2 e^2 R}{\epsilon_0}$ using (i)

The expansion would start when coulomb repulsion (F_2) on hydrogen atom is larger than the gravitational force of attraction (F_1) on the hydrogen atom. The critical value of y to start expansion would be when

$$F_2 = F_1, \text{ i.e., } \frac{1}{3}\frac{N y^2 e^2 R}{\epsilon_0} = \frac{4\pi}{3} G m_p^2 NR$$

or $$y^2 = (4\pi\epsilon_0)\, G \left(\frac{m_p}{e}\right)^2 = \frac{1}{9\times10^9} \times (6{\cdot}67\times10^{-11}) \frac{(1{\cdot}66\times10^{-27})^2}{(1{\cdot}6\times10^{-19})^2} = 79{\cdot}8 \times 10^{-38}$$

$$y = \sqrt{79{\cdot}8\times10^{-38}} = 8{\cdot}9 \times 10^{-19} \simeq 10^{-18}$$

This is the critical value of y corresponding to which expansion of universe would start.

(*b*) Net force experienced by the hydrogen atom, $F = F_2 - F_1 = \frac{1}{3}\frac{N y^2 e^2 R}{\epsilon_0} - \frac{4\pi}{3} G m_p^2 N R$

If acceleration of hydrogen atom is represented by d^2R/dt^2, then

$$m_p \frac{d^2R}{dt^2} = F = \frac{1}{3}\frac{N y^2 e^2}{\epsilon_0} R - \frac{4\pi}{3} G m_p^2 N R = \left(\frac{1}{3}\frac{N y^2 e^2}{\epsilon_0} - \frac{4\pi}{3} G m_p^2 N\right) R$$

$\therefore$ $$\frac{d^2R}{dt^2} = \frac{1}{m_p}\left[\frac{1}{3}\frac{N y^2 e^2}{\epsilon_0} - \frac{4\pi}{3} G m_p^2 N\right] R = \alpha^2 R$$ ...(iv)

where $$\alpha^2 = \frac{1}{m_p}\left[\frac{1}{3}\frac{N y^2 e^2}{\epsilon_0} - \frac{4\pi}{3} G m_p^2 N\right]$$

The general solution of eqn. (*iv*) is $R = A e^{\alpha t} + B e^{-\alpha t}$

As we are looking for expansion, $B = 0$. $\therefore$ $R = A e^{\alpha t}$

velocity of expansion, $v = \frac{dR}{dt} = A e^{\alpha t}(\alpha) = \alpha A e^{\alpha t} = \alpha R$

Hence, $v \propto R$, **which was to be proved.**

27. Consider a sphere of radius R with charge density distributed as $\rho(r) = kr$ for $r \le R$
$= 0$ for $r > R$.

(*a*) Find the electric field at all points r.

(*b*) Suppose the total charge on the sphere is $2e$, where e is the electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

Ans. The given charge density distribution of the sphere of radius R is

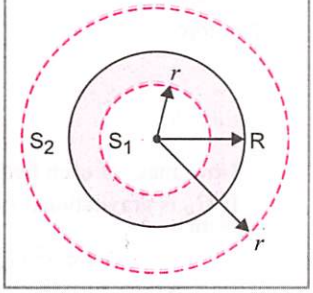
$\rho(r) = k r$ for $r \le R$
$= 0$ for $r > R$

The electric field is obviously radial, as shown in Fig. 1(N).23.

For points $r < R$

let us consider a spherical Gaussian surface S_1 of radius r. Then on the surface,

FIGURE 1(N).23



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$$\oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0}\int \rho\, dV$$

As $V = \frac{4}{3}\pi r^3,\ dV = \frac{4}{3}\pi 3r^2\, dr = 4\pi r^2\, dr$ and $\rho(r) = kr$

$$\therefore \oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} 4\pi k \int_0^r r\, r^2\, dr$$

$$(E)\, 4\pi r^2 = \frac{4\pi k}{\epsilon_0}\frac{r^4}{4} \quad \text{or} \quad E = \frac{1}{4\epsilon_0} k r^2 \qquad ...(i)$$

The direction of $\vec{E}$ is radially outwards (for positive charge density)

For points $r > R$, let us consider a spherical Gaussian surface S_2 of radius r. Then on the surface, when $r = R$

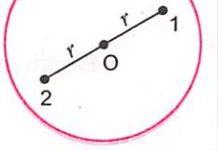
$$\oint \vec{E}.\vec{ds} = \frac{1}{\epsilon_0}\int \rho.dV$$

$$E(4\pi r^2) = \frac{4\pi k}{\epsilon_0}\int_0^R r^3\, dr = \frac{4\pi k}{\epsilon_0}\frac{R^4}{4} \quad \text{or} \quad E = \frac{k}{4\epsilon_0}\frac{R^4}{r^2} \qquad ...(ii)$$

The direction of $\vec{E}$ is radially outwards (for positive charge density).

(*b*) From symmetry, we find that the two protons must be on the opposite sides of the centre, along a diameter of the sphere as shown in Fig. 1(N).24.

FIGURE 1(N).24



Proceeding as above, charge on the sphere,

$$q = \int_0^R \rho\, dV = \int_0^R (kr)\, 4\pi r^2\, dr = 4\pi k\frac{R^4}{4} = 2e$$

$$\therefore \quad k = \frac{2e}{\pi R^4} \qquad ...(iii)$$

If protons 1 and 2 are embedded at distance r from the centre of the sphere as shown, then attractive force on proton 1 due to charge distribution is

$$F_1 = -eE = -e\frac{kr^2}{4\epsilon_0} \qquad ...\text{using } (i)$$

Repulsive force on proton 1 due to proton 2 is $F_2 = \frac{e^2}{4\pi\epsilon_0(2r)^2}$

Net force on proton 1 $F = F_1 + F_2 = -e\frac{kr^2}{4\epsilon_0} + \frac{e^2}{16\pi\epsilon_0 r^2}$

Using (*iii*), $F = \left[-\frac{er^2}{4\epsilon_0}\frac{2e}{\pi R^4} + \frac{e^2}{16\pi\epsilon_0 r^4}\right]$

This net force on proton 1 will be zero, when $\frac{er^2.2e}{4\epsilon_0\pi R^4} = \frac{e^2}{16\pi\epsilon_0 r^2}$ or $r^4 = \frac{R^4}{8}$ or $r = \frac{R}{(8)^{1/4}}$

This is the distance of each of the two protons from the centre of the sphere.

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28. Two fixed, identical conducting plates (α and β), each of surface area S are charged to $-Q$ and q, respectively, where $Q > q > 0$. A third identical plate (γ), free to move is located on the other side of the plate with charge q at a distance d, Fig. 1(N).25. The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.

(*a*) Find the electric field acting on the plate γ before collision.

(*b*) Find the charges on β and γ after the collision.

(*c*) Find the velocity of the plate γ after the collision and at a distance d from the plate β.

Ans. The given set up is shown in Fig. 1(N).26

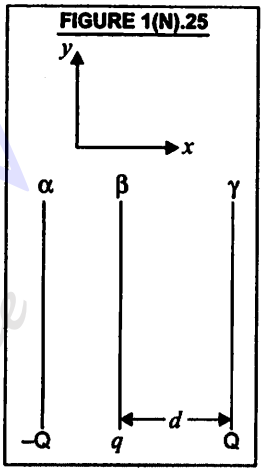

FIGURE 1(N).25

(*a*) The electric field at plate γ due to plate α is $E_1 = -\dfrac{Q}{S(2\epsilon_0)}$; to the left

The electric field at plate γ due to plate β is $E_2 = \dfrac{q}{S(2\epsilon_0)}$; to the right.

Hence, the net electric field at plate γ before collision is

$$E = E_1 + E_2 = \frac{q-Q}{S(2\epsilon_0)},$$ to the left if $Q > q$

(*b*) During collision, plates β and γ are together. Their potentials become same. Suppose charge on plate β is q_1 and charge on plate γ is q_2. At any point O, inbetween the two plates, Fig. 1(N).26, the electric field must be zero.

Electric field at O due to plate $\alpha = \dfrac{(-)Q}{S(2\epsilon_0)}$; to the left

Electric field at O due to plate $\beta = \dfrac{q_1}{S(2\epsilon_0)}$; to the right

Electric field at O due to plate $\gamma = \dfrac{q_2}{S(2\epsilon_0)}$; to the left

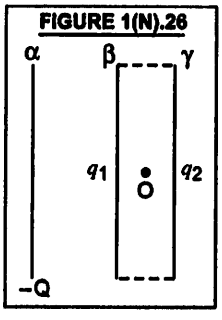

FIGURE 1(N).26

As the electric field at O is zero, therefore, $\dfrac{Q+q_2}{S(2\epsilon_0)} = \dfrac{q_1}{S(2\epsilon_0)}$

$\therefore \quad Q + q_2 = q_1 \quad$ or $\quad Q = q_1 - q_2 \quad$ ...(*i*)

As there is no loss of charge on collision, $Q + q = q_1 + q_2$ ...(*ii*)

On solving (*i*) and (*ii*), we get $q_1 = (Q + q/2)$ = charge on plate β

$q_2 = (q/2)$ = charge on plate γ.

(*c*) After collision, at a distance d from plate β, let the velocity of plate γ be v.

After the collision, electric field at plate γ is $E_2 = \dfrac{-Q}{2\epsilon_0 S} + \dfrac{(Q+q/2)}{2\epsilon_0 S} = \dfrac{q/2}{2\epsilon_0 S}$ to the right.

Just before collision, electric field at plate γ is $E_1 = \dfrac{Q-q}{2\epsilon_0 S}$

If F_1 is force on plate γ before collision, then $F_1 = E_1 Q = \dfrac{(Q-q)Q}{2\epsilon_0 S}$

Similarly, force F_2 on plate γ after collision, $F_2 = E_2 \dfrac{q}{2} = \dfrac{(q/2)^2}{2\epsilon_0 S}$

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Total work done by the electric field in round trip movement of plate γ

$$W = (F_1 + F_2)\, d = \frac{[(Q-q)Q + (q/2)^2]\, d}{2\epsilon_0 S} = \frac{(Q - q/2)^2\, d}{2\epsilon_0 S}$$

If m is mass of plate γ, the KE gained by plate $\gamma = \frac{1}{2} mv^2$

According to work energy principle, $\frac{1}{2} mv^2 = W = \frac{(Q - q/2)^2\, d}{2\epsilon_0 S}$

$$v = (Q - q/2)\left(\frac{d}{m\epsilon_0 S}\right)^{1/2}$$

29. There is another useful system of units, besides the SI/mksA system, called the cgs (centimeter-gram-second) system. In this system, Coloumb's law is given by $F = \frac{Qq}{r^2}\hat{r}$ where the distance r is measured in cm (= 10^{-2} m), F in dyne (= 10^{-5} N) and the charges in electrostatic units (es units), where 1 es unit of charge $= \frac{1}{[3]} \times 10^{-9}$ C

The number [3] actually arises from the speed of light in vacuum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c then is $c = [3] \times 10^8$ m/s.

(*i*) Show that the coulomb law in cgs units yields 1 esu of charge = 1 (dyne)$^{1/2}$ cm.

Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(*ii*) Write 1 esu of charge = xC, where x is a dimenionless number. Show that this gives

$$\frac{1}{4\pi\epsilon_0} = \frac{10^{-9}}{x^2}\frac{\text{N.m}^2}{\text{C}^2} \quad \text{With} \quad x = \frac{1}{|3|} \times 10^{-9}, \text{ we have}$$

$$\frac{1}{4\pi\epsilon_0} = |3|^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \quad \text{or} \quad \frac{1}{4\pi\epsilon_0} = (2.99792458)^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ (exactly).}$$

Ans. (*i*) From $F = \frac{Qq}{r^2}$

$$1 \text{ dyne} = \frac{(1 \text{ esu of charge})^2}{(1 \text{ cm})^2}$$

$\therefore$ 1 esu of charge = (1 dyne)$^{1/2}$ × 1 cm. = $F^{1/2} \cdot L = (MLT^{-2})^{1/2} L$ or 1 esu of charge = $M^{1/2} L^{3/2} T^{-1}$

Thus *esu* of charge is represented in terms of fractional powers : $\frac{1}{2}$ of M and $\frac{3}{2}$ of L.

(*ii*) Let 1 *esu* of charge = x C, where x is a dimensionless number.
Coulomb force on two charges, each of magnitude 1 esu separated by 1 cm is 1 dyne = 10^{-5} N.
This situation is equivalent to two charges of magnitude x C separated by 10^{-2} m.

$\therefore$ $F = \frac{1}{4\pi\epsilon_0}\frac{x^2}{(10^{-2})^2} = 1 \text{ dyne} = 10^{-5} \text{ N}$ $\therefore$ $\frac{1}{4\pi\epsilon_0} = \frac{10^{-9}}{x^2}\frac{\text{N m}^2}{C^2}$

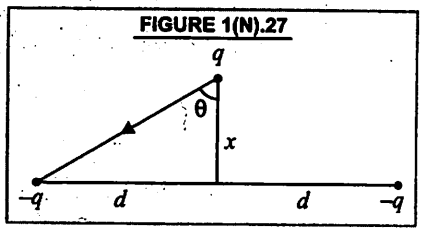
Taking $x = \frac{1}{|3| \times 10^9}$, we get, $\frac{1}{4\pi\epsilon_0} = 10^{-9} \times |3|^2 \times 10^{18} \frac{\text{Nm}^2}{C^2} = 9 \times 10^9 \frac{\text{Nm}^2}{C^2}$

If $|3| \to 2{\cdot}99792458$, we get $\frac{1}{4\pi\epsilon_0} = 8{\cdot}98755 \times 10^9 \text{ Nm}^2 C^{-2}$

30. Two charges – q each are fixed separated by distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by x ($x << d$) perpendicular to the line joining the two fixed charges as shown in Fig. 1(N).27. Show that q will perform simple harmonic oscillation of time period.

$$T = \left[\frac{8\pi^3 \epsilon_0 m d^3}{q^2}\right]^{1/2}$$

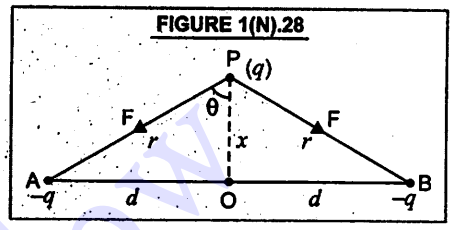
FIGURE 1(N).27



Ans. In Fig. 1(N).28, two charges – q each are held at A and B where AB = 2 d. A third charge + q of mass m is placed at O, middle point of AB. Let it be displaced perpendicular to AB, through a small distance OP = x. Forces of attraction at P towards A and B are each

$$F = \frac{q\,(q)}{4\pi\epsilon_0 r^2}, \text{ where } AP = BP = r$$

FIGURE 1(N).28



Horizontal components of those forces cancel out. Vertical components along PO add.
If $\angle APO = \theta$, the net force on q along PO is $F' = 2F\cos\theta$

$$F' = \frac{2q^2}{4\pi\epsilon_0 r^2}\left(\frac{x}{r}\right) = \frac{2q^2 x}{4\pi\epsilon_0 (d^2 + x^2)^{3/2}}$$

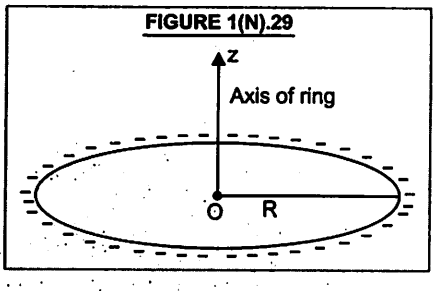
When $x << d$, $\quad F' = \dfrac{2q^2 x}{4\pi\epsilon_0 d^3} = Kx$, where $K = \dfrac{2q^2}{4\pi\epsilon_0 d^3}$

i.e., force on charge q is proportional to its displacement from the centre O and is directed towards O. Hence, motion of charge q would be simple harmonic, where

$$\omega = \sqrt{\frac{K}{m}} \quad \text{and} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m.4\pi\epsilon_0 d^3}{2q^2}} = \left[\frac{8\pi^3\epsilon_0 m d^3}{q^2}\right]^{1/2}$$

31. Total charge – Q is uniformly spread along length of a ring of radius R. A small test charge + q of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.
(a) Show that the particle executes a simple harmonic oscillation.
(b) Obtain its time period.

FIGURE 1(N).29

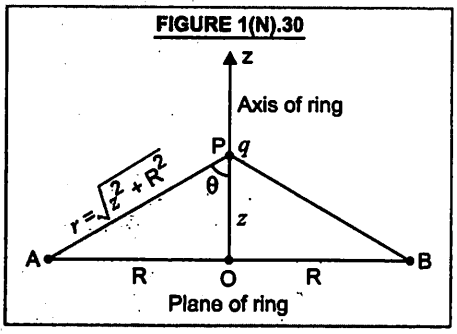


Ans. In Fig. 1(N).29, we have shown – Q charge distributed uniformly over a ring of radius R with centre O. A and B are two points on the ring at the ends of a diameter.
If we take line elements of charge at A and B, having unit length, then charge on each element $= -\dfrac{Q}{2\pi R}$

A small test charge + q of mass m kept at the centre O of the ring is given a gentle push to P where $OP = z$
If $AP = BP = r = (z^2 + R^2)^{1/2}$, and $\angle APO = \theta$, Fig. 1(N).30, then force on q due to the two line elements

$$dF = 2\left(-\frac{Q}{2\pi R}\right) q \times \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\cos\theta$$

FIGURE 1(N).30



Total force on the charge q, due to entire ring

$$F = -\frac{Qq}{\pi R}(\pi R) \cdot \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\cdot\frac{z}{r} = -\frac{Qqz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

When $z << R$, $F = -\frac{Qqz}{4\pi\epsilon_0 R^3} = -Kz$ where $K = \frac{Qq}{4\pi\epsilon_0 R^3}$ = constant

Clearly, force on q is proportional to negative of its displacement. Therefore, motion of q is simple harmonic.

$$\omega = \sqrt{\frac{K}{m}} \quad \text{and} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m4\pi\epsilon_0 R^3}{Qq}}$$

$$= 2\pi\sqrt{\frac{4\pi\epsilon_0 m R^3}{Qq}}$$

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I. Multiple Choice Questions (with One Correct Answer)

I. Electrostatic Charges

1. Four charges, each equal to $-Q$, are placed at four corners of a square and a charge $+q$ is at its centre. If the system is in equilibrium, the value of q is

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

(AIEEE 2004)

2. Three identical spheres, each having a charge q and radius R are kept in such a way that each touches the other two. The magnitude of electric force on any sphere due to other two is given as :

(a) $\frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}\right)^2$ (b) $\frac{1}{4\pi\epsilon_0}\frac{3}{4}\left(\frac{q}{R}\right)^2$

(c) $\frac{1}{4\pi\epsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$ (d) $\frac{1}{4\pi\epsilon_0}\frac{3}{2}\left(\frac{q}{R}\right)^2$

3. An infinite number of charges each of charge 1 μC, are placed on the X-axis with co-ordinates $x = 1, 2, 4, 8$ ..... ∞. If a charge of 1 C is kept at the origin then what is the net force acting on 1 C charge ?

(a) 9000 N (b) 12000 N

(c) 24000 N (d) 36000 N

(DCE 2004)

4. Two point charges A and B, having charges $+Q$ and $-Q$ respectively, are placed at certain distance apart and force acting between them is F. If 25% charge of A is transferred to B, then force between the charges becomes

(a) F (b) $\frac{9F}{16}$

(c) $\frac{16F}{9}$ (d) $\frac{4F}{3}$. **(NEET 2019)**

5. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5\ \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2\times10^{-3}\ \text{ms}^{-1}$. Given, $g = 9.8\ \text{ms}^{-2}$, viscosity of the air $=1.8\times10^{-5}$ Ns m^{-2} and the density of oil = 9000 kg m^{-3}, the magni- tude of q is

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C

(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

(IIT 2010)

6. Two identical charged spheres suspended from a common point by two massless strings of length l, are initially at a distance x ($x \ll l$) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then v varies as a function of distance x between the spheres as

ANSWERS

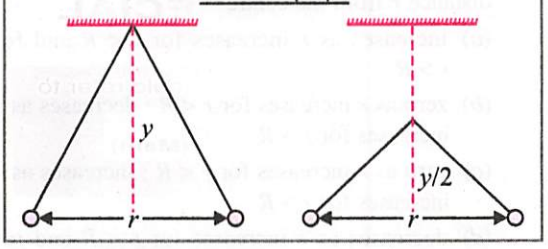
1. (b) 2. (c) 3. (b) 4. (b) 5. (d)

(a) $v \propto x$ (b) $v \propto x^{-1/2}$
(c) $v \propto x^{-1}$ (d) $v \propto x^{1/2}$

(NEET 2016)

7. Two pith balls carrying equal charges are suspended from a common point by strings of equal length, the equilibrium separation between them is r. Now the strings are rigidly clamped at half the height. The equilibrium separation between the balls, now becomes :

FIGURE 1(CF).1



(a) $\left(\frac{2r}{3}\right)$ (b) $\left(\frac{1}{\sqrt{2}}\right)^2$
(c) $\left(\frac{r}{\sqrt[3]{2}}\right)$ (d) $\left(\frac{2r}{\sqrt{3}}\right)$

(AIPMT 2013)

8. Two charges, each equal to q are kept at $x = -a$ and $x = a$ on the X-axis. A particle of mass m and and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to :

(a) y (b) $-y$
(b) $\frac{1}{y}$ (d) $-\frac{1}{y}$

(JEE Main 2013)

9. The capacitance of a parallel plate capacitor with air as medium is 6 μF. With the introduction of a dielectric medium, the capacitance becomes 30 μF. The permittivity of the medium is :

($\epsilon_0 = 8{\cdot}85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$)

(a) $1{\cdot}77 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$
(b) $0{\cdot}44 \times 10^{-10}\ C^2\ N^{-1}\ m^{-2}$
(c) $5{\cdot}00\ C^2\ N^{-1}\ m^{-2}$
(d) $0{\cdot}44 \times 10^{-13}\ C^2\ N^{-1}\ m^{-2}$ **(NEET 2020)**

10. Polar molecules are the molecules

(a) Having a permanent electric dipole moment
(b) Having zero dipole moment
(c) Acquire a dipole moment only in the presence of electric field due to displacement of charges
(d) Acquire a dipole moment only when magnetic field is absent **(NEET 2021)**

11. Two point charges $-q$ and $+q$ are placed at a distance L. The magnitude of electric field intensity at a distance R ($R >> L$) varies as

(a) $\frac{1}{R^2}$ (b) $\frac{1}{R^3}$
(c) $\frac{1}{R^4}$ (d) $\frac{1}{R^6}$ **(NEET 2022)**

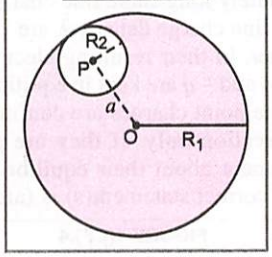
II. Electric Field

12. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (Fig. 1(CF).2) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement is

(a) $\vec{E}$ is uniform, its magnitude is independent of R_2, but its direction depends on $\vec{r}$

(b) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

FIGURE 1(CF).2



(c) $\vec{E}$ is uniform, its magnitude is independent of a, but direction depends on a

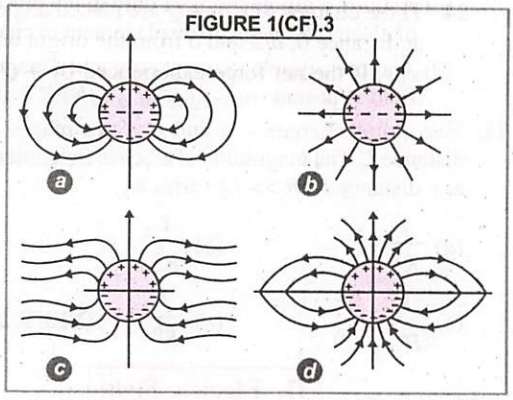
(d) $\vec{E}$ is uniform and both, its magnitude and direction depend on $\vec{a}$ **(JEE Advanced 2015)**

13. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scate).

ANSWERS

6. (b) 7. (c) 8. (a) 9. (b) 10. (a) 11. (b) 12. (d)

FIGURE 1(CF).3



(JEE Main 2015)

14. Two point charges + 8 q and – 2 q are located at $x = 0$ and $x = L$ respectively. The location of a point on X-axis at which the net electric field due to these two point charges is zero is :

(*a*) 2 L (*b*) $L/4$

(*c*) 8 L (*d*) 4 L **(AIEEE 2005)**

15. If potential (in volts) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, the electric field (in N/C) at point (1, 1, 0) is :

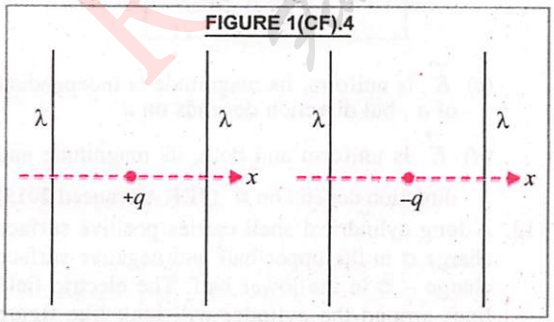
(*a*) $-(6\hat{i} + 9\hat{j} + \hat{k})$ (*b*) $-(3\hat{i} + 5\hat{j} + 3\hat{k})$

(*c*) $-(6\hat{i} + 5\hat{j} + 2\hat{k})$ (*d*) $-(2\hat{i} + 3\hat{j} + \hat{k})$

(AIPMT 2015)

16. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is (are)

FIGURE 1(CF).4



(*a*) both charges execute simple harmonic motion.

(*b*) both charges will continue moving in the direction of their displacement.

(*c*) charge + q executes simple harmonic motion while charge – q continues moving in the direction of its displacement.

(*d*) charge – q executes simple harmonic motion while charge + q continues moving in the direction of its displacement.

(JEE Advanced 2015)

17. A hollow metal sphere of radius R is uniformly charged. The electric field due to the sphere at a distance r from the centre

(*a*) increases as r increases for $r < R$ and for $r > R$

(*b*) zero as r increases for $r < R$; decreases as r increases for $r > R$

(*c*) zero as r increases for $r < R$; increases as r increases for $r > R$

(*d*) decreases as r increases for $r < R$ and for $r < R$. **(NEET 2019)**

18. Two parallel infinite line charges with linear charge densities + λ c/m and – λ c/m are placed at a distance of 2 R in free space. What is the electric field midway between the two line charges ?

(*a*) Zero (*b*) $\frac{2\lambda}{\pi \epsilon_0 R}$ N/C

(*c*) $\frac{\lambda}{\pi \epsilon_0 R}$ N/C (*d*) $\frac{\lambda}{2\pi \epsilon_0 R}$ N/C.

(NEET 2019)

19. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(x) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r (r < R)$ from the origin is given by

(*a*) $\frac{\rho_0 r}{4\epsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (*b*) $\frac{4\rho_0 r}{3\epsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

(*c*) $\frac{\rho_0 r}{3\epsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (*d*) $\frac{4\pi\rho_0 r}{3\epsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

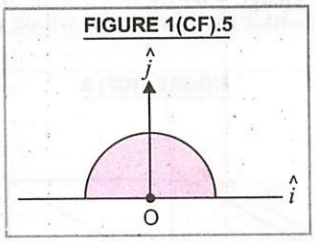
(AIEEE 2010)

20. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it, Fig. 1(CF).5. The net field $\vec{E}$ at the centre O is

ANSWERS

13. (*a*) 14. (*a*) 15. (*c*) 16. (*c*) 17. (*b*) 18. (*c*) 19. (*a*)

FIGURE 1(CF).5



(a) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(AIEEE 2010)

21. Let $\rho(r)=\frac{Qr}{\pi R^4}$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at a distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) $\frac{Q}{4\pi\epsilon_0 r_1^2}$ (b) $\frac{Qr_1^2}{4\pi\epsilon_0 R^4}$

(c) $\frac{Qr_1^2}{3\pi\epsilon_0 R^4}$ (d) zero **(AIEEE 2009)**

22. Two positive charges of magnitude 'q' are placed at the ends of side 1 of a square of side '$2a$'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is

(a) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$ (b) zero

(c) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(d) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$ **(AIEEE 2011)**

23. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is

(a) $R/\sqrt{5}$ (b) $R/\sqrt{2}$

(c) R (d) $R\sqrt{2}$

(JEE Main 2019)

24. Three charges $+Q$, q, $+Q$ are placed respectively at distance 0, $d/2$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x = 0$ is zero, the value of q is

(a) $-Q/4$ (b) $+Q/2$

(c) $+Q/4$ (d) $-Q/2$

(JEE Main 2019)

III. Electric Flux

25. The electrical field in a certain region is acting radially outward and is given by $E = Ar$. A charge contained in sphere of radius 'a' centred at origin of the field will be given by :

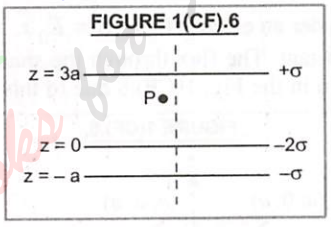
(a) $A\epsilon_0 a^2$ (b) $4\pi\epsilon_0 A a^3$

(c) $\epsilon_0 A a^3$ (d) $4\pi\epsilon_0 A a^2$

(AIPMT 2015)

26. Three infinitely charged sheets are kept parallel to xy plane having charge densities as shown in Fig. 1(CF).6. Then the value of electric field at P is

FIGURE 1(CF).6



(a) $\frac{-4\sigma}{\epsilon_0}\hat{k}$ (b) $\frac{4\sigma}{\epsilon_0}\hat{k}$

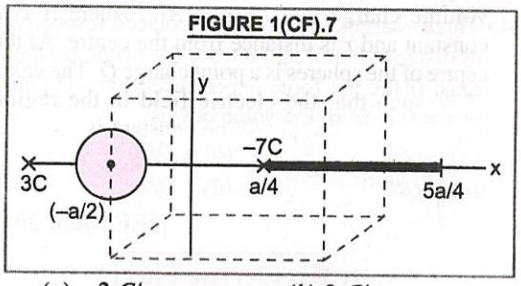
(c) $\frac{-2\sigma}{\epsilon_0}\hat{k}$ (d) $\frac{2\sigma}{\epsilon_0}\hat{k}$ **(IIT 2005)**

27. A disc of radius $a/4$ having a uniformly distributed charge 6 C is placed in X-Y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8 C is placed on the X-axis from $x = a/4$ to $x = 5a/4$. Two point charges -7 C and 3 C are placed at $\left(\frac{a}{4}, -\frac{a}{4}, 0\right)$ and $\left(-\frac{3a}{4}, \frac{3a}{4}, 0\right)$ respectively. Consider a cubical surface formed by six surfaces $x=\pm\frac{a}{2}$, $y=\pm\frac{a}{2}$, $z=\pm\frac{a}{2}$. The electric flux through the cubical surface is

ANSWERS

20. (c) **21.** (c) **22.** (a) **23.** (b) **24.** (a) **25.** (b) **26.** (c)

FIGURE 1(CF).7

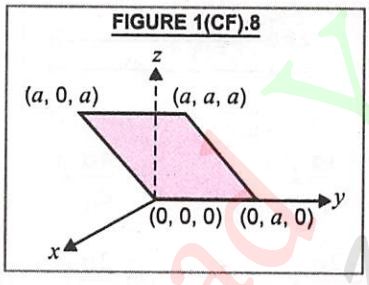


(a) $-2\ C/\epsilon_0$ (b) $2\ C/\epsilon_0$
(c) $10\ C/\epsilon_0$ (d) $12\ C/\epsilon_0$ **(IIT 2009)**

28. Suppose the charge of a proton and an electron differ slightly. One of them is $-e$, the other is $(e + \Delta e)$. If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then Δe is of the order of [Given mass of hydrogen $m_h = 1{\cdot}67 \times 10^{-27}$ kg]
(a) 10^{-20} C (b) 10^{-23} C
(c) 10^{-37} C (d) 10^{-47} C
(NEET 2017)

29. Consider an electric field $\vec{E} = E_0 \hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the Fig. 1(CF).8 due to this field is

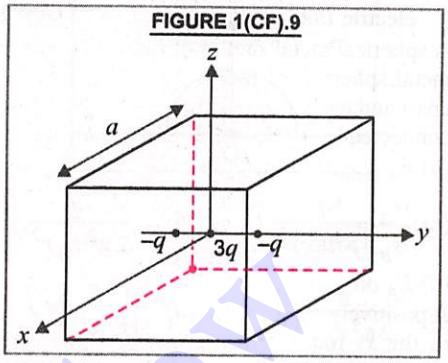
FIGURE 1(CF).8



(a) $2\ E_0\ a^2$ (b) $\sqrt{2}\ E_0\ a^2$
(c) $E_0\ a^2$ (d) $\frac{E_0 a^2}{\sqrt{2}}$ **(IIT 2011)**

30. A cubical region of side a has its centre at the origin. It encloses three fixed point charges $-q$ at $(0, -a/4, 0)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $(0, +a/4, 0)$, Fig. 1(CF).9. Choose the wrong option(s).

FIGURE 1(CF).9



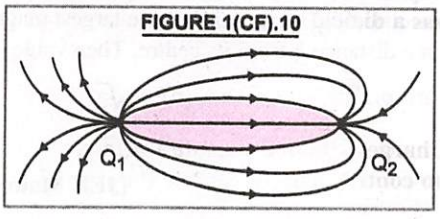
(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$
(c) The net electric flux crossing the entire region is $\frac{q}{\epsilon_0}$
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$ **(IIT 2012)**

31. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{k}$. When subjected to another electric field $\vec{E_2} = \sqrt{3}E_1\hat{j}$, it experiences torque $\vec{T_2} = -\vec{T_1}$. The angle θ is
(a) 60° (b) 90°
(c) 30° (d) 45°
(JEE Main 2017)

II. Multiple Choice Questions (with One or More than One Correct Answers)

32. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on x-axis are shown in the [Fig. 1(CF).10]. These lines suggest that
(a) $|Q_1| > |Q_2|$
(b) $|Q_1| < |Q_2|$

FIGURE 1(CF).10



ANSWERS
27. (a) 28. (c) 29. (c) 30. (b) 31. (a)

(*c*) at finite distance to the left of Q_1, the electric field is zero

(*d*) at a finite distance to the right of Q_2, the electric field is zero **(IIT 2010)**

33. A spherical metal shell *A* of radius R_A and a solid metal sphere *B* of radius $R_B < (R_A)$ are kept far apart and each is given charge + *Q*. Now they are connected by a thin metal wire. Then :

(*a*) $E_{A \text{ inside}} = 0$ (*b*) $Q_A > Q_B$

(*c*) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(*d*) E_A on surface < E_B on surface **(IIT 2011)**

34. A positively charged metal ring of radius *R* is fixed in the *xy*-plane with its centre at origin *O*. A negatively charged particle *P* is released from rest at the point (0, 0, Z_0) where $Z_0 > 0$. Then, the motion of *P* is :

(*a*) Periodic for all the values of Z_0, satisfying $0 < Z_0 < \infty$

(*b*) Simple harmonic for all the values of Z_0 satisfying $0 < Z_0 \le R$

(*c*) approximately simple harmonic provided $Z_0 << R$

(*d*) such that *P* crosses O and continues to move along the negative *Z*-axis towards $Z = -\infty$

35. The region between two concentric spheres of radii *a* and *b* respectively. Fig. 1(CF).11 has volume charge density $\rho = A/r$, where *A* is a constant and *r* is distance from the centre. At the centre of the spheres is a point charge *Q*. The value of *A*, such that the electric field in the region between the spheres will be constant is

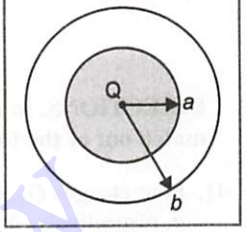
(*a*) $\frac{Q}{2\pi a^2}$

(*b*) $\frac{Q}{2\pi(b^2 - a^2)}$

(*c*) $\frac{2Q}{\pi(a^2 - b^2)}$

(*d*) $\frac{2Q}{\pi a^2}$

FIGURE 1(CF).11



(JEE Main 2016)

36. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance *r* from a point charge *Q*, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(*a*) $Q = 4\sigma\pi r_0^2$

(*b*) $r_0 = \lambda/2\pi\sigma$

(*c*) $E_1(r_0/2) = 2E_2(r_0/2)$

(*d*) $E_2(r_0/2) = 4E_0(r_0/2)$ **(JEE Advanced 2014)**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension **According to Gauss's theorem in electrostatics, total electric flux over a closed surface *S* in vacuum is $1/\epsilon_0$ times the total charge (*Q*) contained inside *S***

i.e., $\phi_E = \oint_{ss} \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}$

The charges enclosed may be distributed any way. If the medium surrounding the charge has a dielectric constant *K*. then

$$\phi_E = \frac{Q}{\epsilon} = \frac{Q}{K\epsilon_0}$$

Charges situated outside the surface make no contribution to electric flux.

37. Charges + 6 *q*, − 2 *q* and +3 *q* are enclosed by a surface in vacuum. The total electric flux over the surface is

(*a*) $6q/\epsilon_0$ (*b*) $-2q/\epsilon_0$

(*c*) $\frac{3q}{\epsilon_0}$ (*d*) $7q/\epsilon_0$

38. A charge − 7 *q* is situated outside the surface in the above question. What will be the total electric flux in that case ?

(*a*) Zero (*b*) $-7q/\epsilon_0$

(*c*) $7q/\epsilon_0$ (*d*) $14q/\epsilon_0$

39. Charges + 2 *q*, − 5 *q* and 8 *q* are enclosed by a surface in a medium of dielectric constant 6. The total electric flux over the surface will be

(*a*) $\frac{5q}{6\epsilon_0}$ (*b*) $\frac{6q}{5\epsilon_0}$

ANSWERS

32. (*a,d*) 33. (*a,b,c,d*) 34. (*a,c*) 35. (*a*) 36. (*c*) 37. (*d*) 38. (*c*)

(c) Zero (d) $\frac{15q}{6\epsilon_0}$

40. A cube of side 1 metre encloses a charge of 1 coulomb in vacuum. What is the electric flux from any one surface of the cube ?

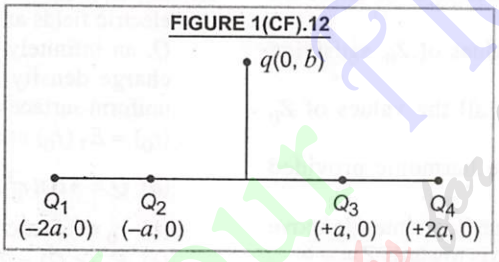
(a) $\frac{1}{\epsilon_0}$ (b) $\frac{1}{6\epsilon_0}$ (c) $\frac{6}{\epsilon_0}$ (d) $\frac{\epsilon_0}{6}$

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II, and select the correct match out of the four given choices.

41. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists.

FIGURE 1(CF).12



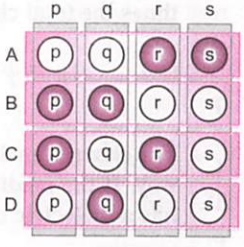
List-I		List-II	
(A)	Q_1, Q_2, Q_3, Q_4 all positive	(p)	$+x$
(B)	Q_1, Q_2 positive; Q_3, Q_4 negative	(q)	$-x$
(C)	Q_1, Q_4 positive; Q_2, Q_3 negative	(r)	$+y$
(D)	Q_1, Q_3 positive ; Q_2, Q_4 negative	(s)	$-y$

(JEE Advanced 2014)

(a) A-r, B-p, C-s, D-q (b) A-s, B-q, C-r, D-p (c) A-r, B-p, C-q, D-s (d) A-s, B-q, C-p, D-r

V. Matrix-Match Type Questions

DIRECTIONS. This question contains statements given in two columns, which have to be matched. The answers to these questions have to be properly bubbled. If the correct matches are A-r, A-s ; B-p, B-q ; C-p, C-r and D-q, then the correctly bubbled matrix will look like the one shown here :



42. Match Column I and Column II :

Column I		Column II	
(a)	unit of electric charge	(p)	$N\,m^2\,C^{-1}$
(b)	unit of electric intensity	(q)	coulomb
(c)	unit of electric flux	(r)	stat coulomb
(d)	charges outside a closed surface	(s)	NC^{-1}
		(t)	do not affect electric flux

ANSWERS

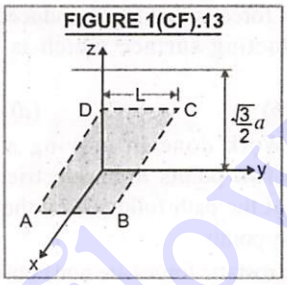
39. (a) 40. (b) 41. (a) 42. (a)–q,r ; (b)–s ; (c)–p ; (d)–t

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

43. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to y-axis in the Y-Z plane at $Z = \frac{\sqrt{3}}{2}a$, Fig. 1(CF).13. If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\epsilon_0}$ (ϵ_0 permitivitly of free space), then the value of n is

FIGURE 1(CF).13



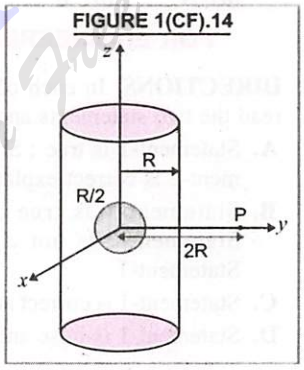
(JEE Advanced 2015)

44. Two charges of values 2 μ C and – 50 μ C are placed at a distance of 6 cm from each other. The distance of the point (in cm) from the bigger charge where the electric intensity will be zero is :

45. A point charge $q = 1$ C and mass 1 kg is projected with speed 10 m/s in the perpendicular direction of uniform electric field $E = 100$ V/m. The value of latus rectum of the path followed by charged particle (in metre) is :

46. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in Fig. 1(CF).14. The magnitude of the electrical field at the point P, which is at a distance 2 R from the axis of this cylinder is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is **(IIT 2012)**

FIGURE 1(CF).14



VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

A. both, Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. both, Assertion and Reason are true, but Reason is not the correct explanation of the Assertion.

C. Assertion is true, but the Reason is false.

D. both, Assertion and Reason are false.

47. Assertion. Dielectric polarisation means formation of positive and negative charges inside the dielectric.
Reason. Free electron are formed in this process.
(*a*) A (*b*) B (*c*) C (*d*) D
(AIIMS 2014)

48. Assertion. When charges are shared between any two bodies, no charge is really lost but some loss of energy does occur.
Reason. Some energy disappears in the form of heat, sparking etc.
(*a*) A (*b*) B (*c*) C (*d*) D
(AIIMS 2014)

49. Assertion. A charge q is placed on a height $h/4$ above the centre of a square of side b. The flux associated with the square is independent of side length.
Reason. Gauss's law is independent of size of the Gaussian surface.
(*a*) A (*b*) B (*c*) C (*d*) D
(AIIMS 2015)

ANSWERS

43. 6 44. 5 45. 2 46. 6 47. (*c*) 48. (*a*) 49. (*a*)

50. Assertion. Insulators do not allow flow of current through themselves.

Reason. They have no free charge carriers.

(*a*) A (*b*) B (*c*) C (*d*) D

51. Assertion. If a point charge q is placed in front of an infinite grounded conducting plane surface, the point charge will experience a force.

Reason. This force is due to the induced charge on the conducting surface which is at zero potential

(*a*) A (*b*) B (*c*) C (*d*) D

52. Assertion. Work done in moving a charge between any two points in an electric field is independent of the path followed by the charge, between these points.

Reason. Electrostatic forces are non conservative

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

A. Statement-1 is true ; Statement-2 is true ; Statement-2 is correct explanation of Statement-1.

B. Statement-1 is true ; Statement-2 is true ; Statement-2 is not a correct explanation of Statement-1.

C. Statement-1 is correct and Statement-2 is false.

D. Statement-1 is false and Statement-2 is true.

53. Statement-1. The number of electrons in one coulomb is $6{\cdot}25 \times 10^{18}$.

Statement-2. $q = ne$, where symbols have their usual meaning.

(*a*) A (*b*) B (*c*) C (*d*) D

54. Statement-1. Units of electric dipole moment are *C-m* and units of torque are *N-m*

Statement-2. $p = q\,(2a)$ and τ = force × distance

(*a*) A (*b*) B (*c*) C (*d*) D

55. Statement-1. When charges are shared between any two bodies, no charge is really lost, but some loss of energy does occur.

Statement-2. Some energy disappears in the form of heat, sparking etc.

(*a*) A (*b*) B (*c*) C (*d*) D

56. Statement-1. The number of electric lines of force emanating from 1 μ C charge in vacuum is $1{\cdot}13 \times 10^6$.

Statement-2. This follows from Gauss's theorem in electrostatics.

(*a*) A (*b*) B (*c*) C (*d*) D

57. Statement-1. For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2. The net work done by a conservative force on an object moving along a closed loop is zero.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIEEE 2009)

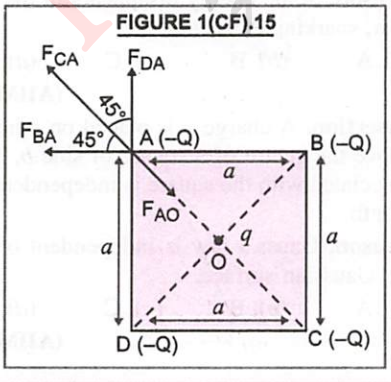
HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Let us consider charge at A to be in equilibrium

FIGURE 1(CF).15



Now $F_{BA} = F_{DA} = \dfrac{KQ^2}{a^2}$

and $F_{CA} = \dfrac{KQ^2}{(\sqrt{2}\,a)^2} = \dfrac{KQ^2}{2a^2}$

Now $F_{AO} = \dfrac{KQq}{(\sqrt{2}a/2)^2} = \dfrac{K\,2Qq}{a^2}$

for equilibrium, all forces along AO should be equal to forces along CA,

$F_{AO} = F_{CA} + F_{BA} \cos 45° + F_{DA} \cos 45°$

$F_{AO} = F_{CA} + 2\,F_{BA} \cos 45°$

ANSWERS

50. (*a*) 51. (*a*) 52. (*c*) 53. (*a*) 54. (*a*) 55. (*a*) 56. (*a*) 57. (*b*)

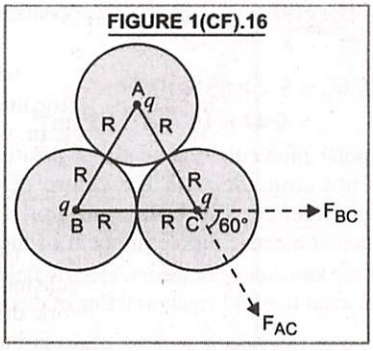
HINTS/EXPLANATIONS
For Difficult Questions

$$\frac{K\,2Q\,q}{a^2}=\frac{KQ^2}{2a^2}+\frac{2KQ^2}{a^2}\times\frac{1}{\sqrt{2}}$$

$$\therefore \qquad q=\frac{Q}{4}(1+2\sqrt{2})$$

2. Force on sphere C due to A, F_{AC} = Force on sphere C due to B. $F_{BC}=\dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{(2R)^2}$, Fig. 1(CF).16.

FIGURE 1(CF).16



Now F_{AC} and F_{BC} are inclined at 60° to each other, therefore resultant force on C is

$$F_R=\sqrt{F_{AC}^2+F_{BC}^2+2F_{AC}F_{BC}\cos 60^\circ}$$

$$=\sqrt{F^2+F^2+2F^2\cos 60^\circ}$$

$$=\sqrt{3}F \qquad [\text{Taking } F_{AC}=F_{BC}=F]$$

$$=\frac{1}{4\pi\epsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$$

3. Net force on charge 1 C at the origin,

$$F=K\left[\frac{1C\times1\mu C}{1^2}+\frac{1C\times1\mu C}{2^2}+\frac{1C\times1\mu C}{4^2}+.....\infty\right]\text{N}$$

$$=9\times10^9\times10^{-6}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.....\infty\right]\text{N}$$

In brackett, we have geometrical progression whose sum $=\dfrac{a}{1-r}$

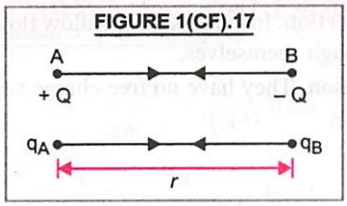
$$F=9\times10^3\left[\frac{1}{1-1/4}\right]=9000\times\frac{4}{3}=\mathbf{12000\ N}$$

4. According to coulomb's Law

$$F=-k\frac{Q^2}{r^2}$$

when 25% of charge of A is transferred to B, then

FIGURE 1(CF).17



$$q_A=Q-\frac{Q}{4}=\frac{3Q}{4} \quad \text{and} \quad q_B=-Q+\frac{Q}{4}=\frac{-3Q}{4}$$

$$\therefore \quad F'=\frac{k\,q_A\,q_B}{r^2}=\frac{k\left(\frac{3Q}{4}\right)\left(-\frac{3Q}{4}\right)}{r^2}$$

$$F'=-k\frac{9}{16}\frac{Q^2}{r^2}=\frac{9}{16}\text{F}$$

5. Here, $E=\dfrac{81\pi}{7}\times10^5\ \text{Vm}^{-1}$, $v=2\times10^{-3}\ \text{ms}^{-1}$, $\eta=1{\cdot}8\times10^{-5}\ \text{N sm}^{-2}$; $\rho=900\ \text{kg m}^{-3}$.

When drop is balanced in still air under the effect of electric field, then

$$qE=\frac{4}{3}\pi r^3\rho g \quad \text{or} \quad q=\frac{4}{3E}\pi r^3\rho g \qquad ...(i)$$

When the electric field is switched off, let the drop fall with terminal velocity v, then

$$v=\frac{2r^2(\rho-\sigma)g}{9\eta} \quad \text{or} \quad r=\left[\frac{9\eta v}{2(\rho-\sigma)g}\right]^{\frac{1}{2}}$$

$$\therefore \qquad q=\frac{1}{E}\times\frac{4}{3}\pi\rho g\times\left[\frac{9\eta v}{2(\rho-\sigma)g}\right]^{\frac{3}{2}}$$

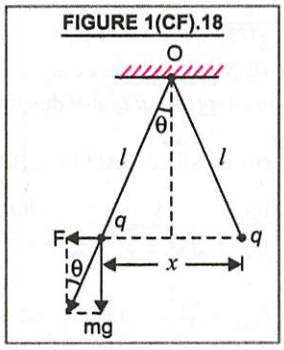
$$=\frac{7}{81\pi\times10^5}\times\frac{4}{3}\times\pi\times900\times9.8$$

$$\times\left[\frac{9\times1.8\times10^{-5}\times2\times10^{-3}}{2\times900\times9.8}\right]^{\frac{3}{2}}$$

On solving, $q=\mathbf{8{\cdot}0\times10^{-19}\ C}$

6. As is clear from Fig. 1(CF).18,

FIGURE 1(CF).18



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$$\tan\theta = \frac{F}{mg} = \frac{kq^2}{x^2\, mg} \quad ...(i)$$

Also, $$\tan\theta = \frac{x/2}{\sqrt{l^2 - x^2/4}} \simeq \frac{x}{2l} \quad ...(ii)$$

From (i) and (ii), $$\frac{kq^2}{x^2\, mg} = \frac{x}{2l}$$

$$q^2 = \frac{mg\, x^3}{2kl}$$

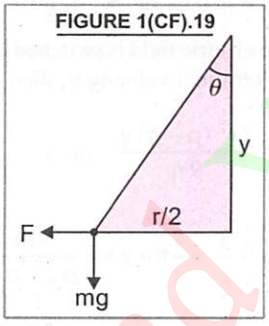
$$\therefore \quad q \propto x^{3/2}$$

$$\frac{dq}{dt} \propto \frac{d}{dx}(x^{3/2})\frac{dx}{dt} \propto \frac{3}{2}x^{1/2}.v$$

As $\frac{dq}{dt}$ is constant, therefore, $v \propto x^{-1/2}$

7. If θ is angle which each pith ball in equilibrium, makes with the vertical, then as is clear from Fig. 1(CF).19,

FIGURE 1(CF).19



$$\tan\theta = \frac{r/2}{y} = \frac{F}{mg} = \frac{kq^2/r^2}{mg} \quad \text{or} \quad y = \frac{mg\, r^3}{2kq^2}$$

or $$y \propto r^3$$

$$\therefore \quad \frac{r'}{r} = \left(\frac{y'}{y}\right)^{1/3} = \left(\frac{y/2}{y}\right)^{1/3} = \left(\frac{1}{2}\right)^{1/3}$$

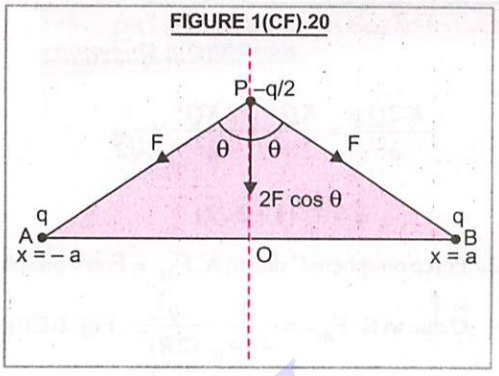
$$r' = \frac{r}{2^{1/3}}$$

8. In Fig. 1(CF).20, let OP = $y << a$.
Net force on charge $(-q/2)$ at P due to the two given charges is
$F_{net} = 2F\cos\theta$, where ∠APO = ∠BPO = θ

$$= \frac{2kq\,(q/2)}{(a^2+y^2)}\frac{y}{\sqrt{a^2+y^2}} = \frac{kq^2}{(a^2+y^2)^{3/2}}$$

$$F_{net} = \frac{kq^2 y}{a^3}, \text{ where } y << a \quad \therefore F_{net} \propto y$$

FIGURE 1(CF).20



9. $$K = \frac{C}{C_0} = \frac{30}{6} = 5$$

$$\epsilon = K\epsilon_0 = 5 \times 8{\cdot}85 \times 10^{-12}$$
$$= 0{\cdot}44 \times 10^{-10}\ C^2\ N^{-1}\ m^{-2}$$

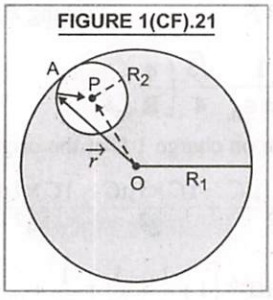
10. In a polar molecule, the centre of positive charge does not coincide with the centre of negative charge. Hence, the polar molecules have a permanent electric dipole moment of their own.

11. From the knowledge of theory, electric field intensity (E) on axial line and equatorial line of electric dipole varies with distance R as $E \propto \frac{1}{R^3}$.

12. Here, total electric field, $\vec{E} = \vec{E_1} + \vec{E_2}$

$$= \frac{\rho}{3\epsilon_0}\overrightarrow{OA} + \frac{\rho}{3\epsilon_0}\overrightarrow{AP} = \frac{\rho}{3\epsilon_0}(\overrightarrow{OA} + \overrightarrow{AP})$$

FIGURE 1(CF).21



$$\vec{E} = \frac{\rho}{3\epsilon_0}(\overrightarrow{OP}) = \frac{\rho}{3\epsilon_0}\vec{a}$$

Hence $\vec{E}$ **is uniform and, both its magnitude and direction depend on** $\vec{a}$.

13. As lower and upper half carry equal and opposite charges, hence electric field around the cylinder will be identical to that of an electric dipole.
Choice (*a*) is correct.

14. Let the net electric field due to two given charges be zero at P, where $OP = x$.

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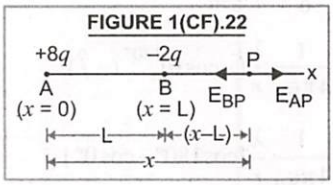
FIGURE 1(CF).22



For net electric field to be zero

$$\therefore \quad E_{AP} = B_{BP} = \frac{K\,8q}{x^2} = \frac{K(2q)}{(x-L)^2}$$

$$\frac{4}{x^2} = \frac{1}{(x-L)^2} \quad \text{or} \quad \frac{2}{x} = \frac{1}{x-L}$$

or $\quad 2x - 2L = x \quad$ or $\quad \boldsymbol{x = 2L}$

15. Given, $V = 6xy - y + 2yz$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\vec{E} = -\left\{\frac{\partial}{\partial x}[6xy - y + 2yz]\,\hat{i} + \frac{\partial}{\partial y}[6xy - y + 2yz]\,\hat{j} + \frac{\partial}{\partial z}[6xy - y + 2yz]\,\hat{k}\right\}$$

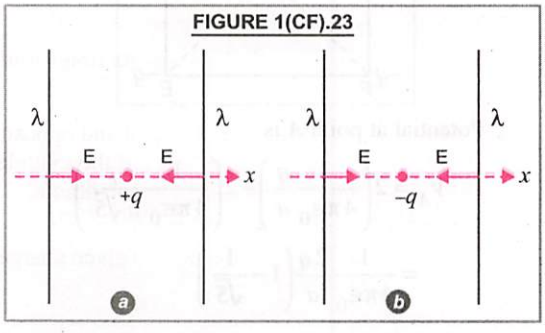
$$= -\{(6y)\,\hat{i} + (6x - 1 + 2z)\,\hat{j} + (2y)\,\hat{k}\}$$

$$\vec{E}_{1,1,0} = -[(6\times1)\,\hat{i} + (6\times1 - 1 + 2\times0)\,\hat{j} + (2\times1)\,\hat{k}]$$

$$= -(\boldsymbol{6\hat{i} + 5\hat{j} + 2\hat{k}})$$

16. Refer to Figs. 1(CF).23 (a) and (b), when point charges $+q$ and $-q$ are displaced a little towards positive x-direction, then $+q$ charge will experience resisting force which will tend to bring it back to its original position. As a result of it the charge $+q$ will execute SHM. The charge $-q$ will experience a force in the direction of displacement, hence charge $-q$ will move in that direction. Therefore, **option (*c*) is true.**

FIGURE 1(CF).23

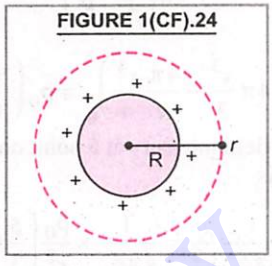


17. Here, charge Q is distributed uniformly over the surface of hollow metal sphere

(*i*) **For $r < R$** (inside the sphere)

$q = 0$. Therefore, E_{in} = Zero.

FIGURE 1(CF).24



(*ii*) **For $r > R$** (outside the sphere)

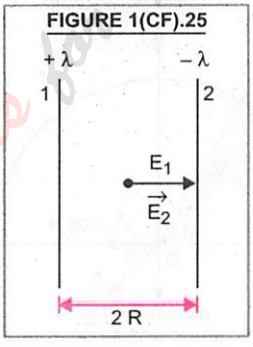
The charge behaves as if concentrated at the centre of the sphere.

$$\therefore \quad E_{out} = \frac{q}{4\pi\epsilon_0 r^2} \quad \text{or} \quad E_{out} \propto \frac{1}{r^2}$$

Choice (*b*) is correct

18. In Fig. 1(CF).25, at the centre of plates,

FIGURE 1(CF).25



Electric field due to line charge (1)

$$\vec{E}_1 = \frac{\lambda}{2\pi\epsilon_0 R}\,\hat{i}\ (N/C)$$

Electric field due to line charge (2) is

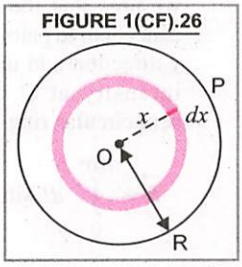
$$\vec{E}_2 = \frac{\lambda}{2\pi\epsilon_0 R}\,\hat{i}\ (N/C)$$

$$\therefore \quad \vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \frac{\lambda}{2\pi\epsilon_0 R}\hat{i} + \frac{\lambda}{2\pi\epsilon_0 R}\hat{i}$$

$$= \frac{\lambda}{\pi\epsilon_0 R}\,\hat{i}\ N/C$$

19. Due to spherically symmetric charge distribution, the charge on the spherical surface of radius r in Fig. 1(CF).26 (when $r < R$) is

FIGURE 1(CF).26



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$$q = \int \rho\, dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{x}{R}\right) 4\pi x^2\, dx$$

$$= \rho_0\left(\frac{5}{4}4\pi\frac{x^3}{3} - \frac{4\pi}{R}\frac{x^4}{4}\right)_0^r = \rho_0\left(\frac{5}{3}\pi r^3 - \frac{\pi r^4}{R}\right)$$

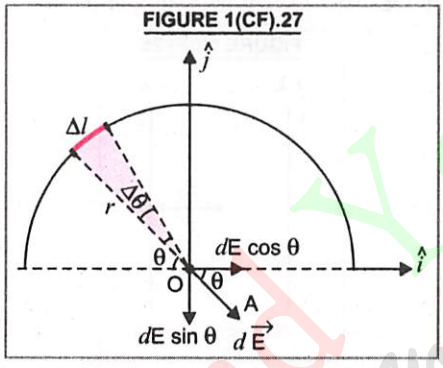
Electric field intensity at a point on this spherical surface is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\times\frac{\rho_0}{r^2}\left(\frac{5}{3}\pi r^3 - \frac{\pi r^4}{R}\right)$$

$$= \frac{\rho_0 r}{4\epsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

20. Let λ be the linear charge density on the semicircular ring. Then $\lambda = q/\pi r$. Take an element of the ring of length Δl as shown in Fig. 1(CF).27.

FIGURE 1(CF).27



Charge on element, $dq = \lambda\,\Delta l = \lambda\,(r\,\Delta\theta)$.
Electric field intensity at O due to charge on element is $dE = \dfrac{1}{4\pi\epsilon_0}\dfrac{\lambda r\Delta\theta}{r^2}$ along OA

Two rectangular components of $d\vec{E}$ are $dE\cos\theta$ along x-direction and $dE\sin\theta$ along negative y-direction. If we find the electric field at O due to charge on all other elements of the ring and resolve them into two rectangular components, we note that the components along X-direction cancel out in pairs and components along negative y-direction add up. Therefore, total electric field intensity at O due to charge on the entire semicircular ring is

$$\vec{E} = \int_0^{180^\circ} dE\sin\theta(-\hat{j})$$

$$= \int_0^{180^\circ} \frac{1}{4\pi\epsilon_0}\frac{\lambda r\Delta\theta\sin\theta}{r^2}(-\hat{j})$$

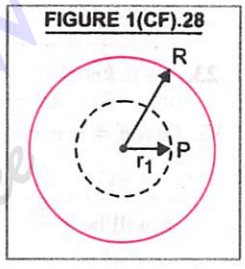
$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}[-\cos\theta]_0^{180^\circ}(-\hat{j})$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}[\cos 180^\circ - \cos 0^\circ]\,\hat{j}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{(q/\pi r)}{r}[-1-1]\,\hat{j} = \frac{q}{2\pi^2\epsilon_0 r^2}\hat{j}$$

21. In Fig. 1(CF).28, dotted sphere of radius r_1 is the Gaussian surface. According to Gauss's theorem,

FIGURE 1(CF).28



$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$E\,ds\cos 0^\circ = \frac{1}{\epsilon_0}\rho(r)\times dV$$

$$E(4\pi r_1^2)\times 1 = \frac{Q r_1}{\epsilon_0\pi R^4}\left(\frac{4}{3}\pi r_1^3\right)$$

$$E = \frac{Q.4\pi r_1^4}{3\epsilon_0\pi R^4(4\pi r_1^2)} = \frac{Q r_1^2}{3\pi\epsilon_0 R^4}$$

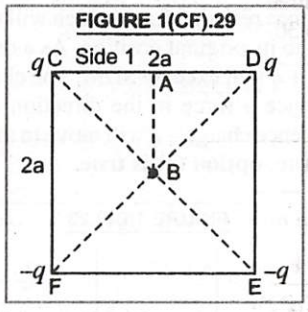
22. Refer to Fig. 1(CF).29 below.

$AC = AD = a$

$AE = AF = \sqrt{a^2 + (2a)^2} = \sqrt{5}\,a$

$BC = BD = BE = BF = \sqrt{2}\,a$

FIGURE 1(CF).29



Potential at point A is

$$V_A = 2\left(\frac{1}{4\pi\epsilon_0}\frac{q}{a}\right) - 2\left(\frac{1}{4\pi\epsilon_0}\frac{q}{a\sqrt{5}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2q}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

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Potential at point B is

$$V_B = 2\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{2}\,a}\right) - 2\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{2}\,a}\right) = 0$$

$$\therefore\ V_A - V_B = \frac{1}{4\pi\epsilon_0}\frac{2q}{a}\left(1-\frac{1}{\sqrt{5}}\right)$$

K.E. at the centre of square of charge Q
= Work done on charge Q from A to B is

$$W = Q\,(V_A - V_B) = \frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$$

23. As is known from theory

$$E = \frac{q\,h}{4\pi\,\epsilon_0\,(h^2+R^2)^{3/2}}$$

E will be maximum, when $\frac{dE}{dh} = 0$.

or $$\frac{d}{dh}\left[\frac{q\,h}{4\pi\,\epsilon_0\,(h^2+R^2)^{3/2}}\right] = 0$$

or $$\frac{q}{4\pi\,\epsilon_0}\left[\frac{-3h^2}{(h^2+R^2)^{5/2}} + (h^2+R^2)^{-3/2}\right] = 0$$

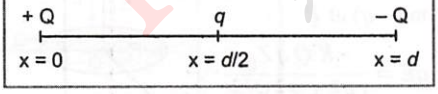
or $$\frac{3h^2}{(h^2+R^2)^{5/2}} = (h^2+R^2)^{-3/2}$$

or $3h^2 = h^2 + R^2$ or $h^2 = \frac{R^2}{2}$ or $h = +\frac{R}{\sqrt{2}}$

Hence, **choice (*b*) is correct.**

24. The three charges are placed as shown in Fig. 1(CF).30.

FIGURE 1(CF).30



As net force on + Q charge at $x = 0$ is zero, therefore,

or $$\frac{kQq}{(d/2)^2} + \frac{KQQ}{d^2} = 0 \quad \text{or} \quad \frac{q}{d^2/4} = -\frac{Q}{d^2}$$

or $q = -Q/4$

25. Flux linked with sphere $= \vec{E}\cdot\vec{ds}$

Since electric field is radial. It is always perpendicular to the surface of the sphere.

So $\phi = EA\cos 0° = EA = Ar\,(4\,\pi\,r^2)$

$\phi = A\,4\,\pi\,a^3$ (as $r = a$)

According to Gauss's law,

$$\phi = \frac{q}{\epsilon_0} \quad \text{or} \quad q = \epsilon_0\,\phi$$

$q = \epsilon_0\,A\,(4\,\pi\,a^3) = 4\,\pi\,\epsilon_0\,A\,a^3$

26. For infinitely charged plane sheet, electric field is given by $E = \frac{\sigma}{2\epsilon_0}$

Due to charge + σ, $E_1 = \frac{\sigma}{2\epsilon_0}$, downwards

Due to charge – 2 σ, $E_2 = \frac{2\sigma}{2\epsilon_0}$, downwards

Due to charge, – σ, $E_3 = \frac{\sigma}{2\epsilon_0}$, downwards

$\therefore$ Total electric field at P,

$$P = \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0},\ \text{downwards}$$

$$\therefore\ \vec{E} = -\frac{2\sigma}{\epsilon_0}\hat{k}$$

27. Now charge on disc contributing to electric flux through cubical surface $= \frac{6C}{2} = 3C$

Charge on rod contributing to electric flux through cubical surface $= \frac{8C}{4} = 2C$

– 7 C charge also contributes to electric flux but 3 C charge is outside the cubical surface, so it does not contribute to electric flux.

Therefore, $q_{\text{enclosed}} = 3\ C + 2\ C - 7\ C = -2\ C$

Hence electric flux, $\phi_C = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{-2C}{\epsilon_0}$

28. Here, electrostatic force = gravitational force

$$\frac{k\,(e+\Delta e)^2}{d^2} = \frac{G\,m_h\,m_h}{d^2}$$

$$(e+\Delta e)^2 = \frac{G\,m_h\,m_h}{k} = \frac{6{\cdot}7\times10^{-11}\,(1{\cdot}67\times10^{-27})^2}{9\times10^9}$$

$$= \frac{6{\cdot}7\times1{\cdot}67\times1{\cdot}67}{9}\times10^{-74}$$

$e + \Delta e = (2{\cdot}076\times10^{-74})^{1/2} = 1{\cdot}44\times10^{-37}$

Hence Δe is of the order of $\mathbf{10^{-37}}$ **C**

29. Here, $\vec{E} = E_0\,\hat{x}$

Let $\hat{x}$, $\hat{y}$ and $\hat{z}$ be the unit vectors along x, y and z axis respectively.

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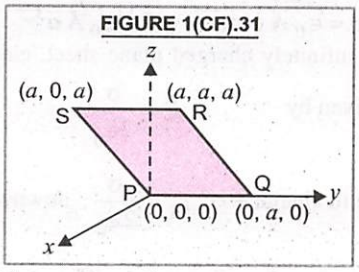
Refer to Fig. 1(CF).31, the shaded area,

FIGURE 1(CF).31



$\vec{A} = \overrightarrow{PQ} \times \overrightarrow{PS}$

$\therefore \vec{A} = (0\hat{x} + a\hat{y} + 0\hat{z}) \times (a\hat{x} + 0\hat{y} + a\hat{z})$

$= (a^2\hat{x} - a^2\hat{z})$

Electric flux through the shaded area,

$\phi = \vec{E}.\vec{A} = (E_0\hat{x}) \cdot (a^2\hat{x} - a^2\hat{z}) = E_0 a^2$

30. In the Fig. 1(CF).10,

we have shown a cubical region of side a.

It encloses three fixed point charges :

$-q$ at $A\ (0, -a/4, 0)$, $+3q$ at $O\ (0, 0, 0)$ and $-q$ at $B\ (0, a/4, 0)$

As $+3q$ is at the origin O, and charges $-q, -q$ are placed symmetrically about planes $x = +a/2$ and $x = -a/2$, therefore, electric flux through them is same. **Choice (*a*) is correct.**

The net electric flux crossing the given region, according to Gauss's theorem is

$$\phi = \frac{\Sigma q}{\epsilon_0} = \frac{-q + 3q - q}{\epsilon_0} = \frac{q}{\epsilon_0}$$

Choice (*c*) is correct.

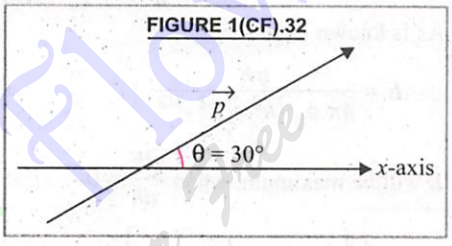
Again, as $+3q$ is at the origin O, and charges $-q, -q$ are placed symmetrically about the planes $z = +a/2$ and $x = -a/2$ so net electric flux crossing these planes is equal. **Choice (*d*) is correct.**

The only wrong option is (*b*).

31. Here, $\vec{\tau}_2 = -\vec{\tau}_1$

From $\vec{\tau} = \vec{p} \times \vec{E}$

FIGURE 1(CF).32



$\vec{\tau}_k - \vec{\tau}_k = (p_x\hat{i} + p_y\hat{j}) \times (E\hat{i} + \sqrt{3}E\hat{j})$

$= p_x \times \sqrt{3}E\hat{k} + p_y E(-\hat{k})$

$0 = E\hat{k}(\sqrt{3}p_x - p_y)$

or $\frac{p_y}{p_x} = \sqrt{3}$ *i.e.*, $\tan\theta = \sqrt{3}$ $\therefore$ $\theta = 60^\circ$

II. Multiple Choice Questions (with One or More than One Correct Answers)

32. Number of field lines from Q_1 is greater than that from Q_2. Therefore $|Q_1| > |Q_2|$. Also, as electric field is less for Q_2, at finite distance to the right of Q_2, electric field would be zero.

33. In a conductor, electric field is zero. Therefore, E_A inside = Zero

After connection, $V_A = V_B \Rightarrow \frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$

$\frac{Q_A}{Q_B} = \frac{R_A}{R_B}$ As $R_A > R_B$ $\therefore$ $Q_A > Q_B$

$\sigma_A = \frac{Q_A}{4\pi\epsilon_0 R_A 2}$, $\sigma_B = \frac{Q_B}{4\pi\epsilon_0 R_B 2} \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

$\frac{E_A \text{ on surface}}{E_B \text{ on surface}} = \frac{\sigma_A/\epsilon_0}{\sigma_B/\epsilon_0} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

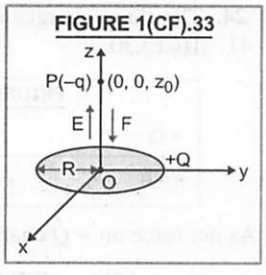
As $R_B < R_A$

$\therefore$ E_A on surface $< E_B$ on surface

34. Electric field at point $(0, 0, Z_0)$

i.e., $E = \frac{K \cdot QZ_0}{(R^2 + Z_0^2)^{3/2}}$

FIGURE 1(CF).33



$\vec{E}$ acts away from O

Force on $(-q)$ at P

$F = -qE = \frac{-KQqZ_0}{(R^2 + Z_0^2)^{3/2}}$

$\vec{F}$ acts towards O.

So, motion of particle P is periodic (for $0 < Z_0 < \infty$) for $Z_0 << R$, $F \propto Z_0$ and such motion is approximately simple harmonic.

35. Let us consider a sphere of radius r and thickness dr lying in the region between two concentric spheres.

According to Gauss's theorem, charge inside alone contributes to electric field/flux.

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$$\therefore \quad \frac{kQ}{a^2} = \frac{k\left[Q + \int_a^b 4\pi r^2\, dr \frac{A}{r}\right]}{b^2}$$

or $$Q\frac{b^2}{a^2} = Q + 4\pi A\left[\frac{r^2}{2}\right]_a^b = Q + 4\pi A\frac{(b^2 - a^2)}{2}$$

$$Q\left(\frac{b^2}{a^2} - 1\right) = 2\pi A(b^2 - a^2)$$

$$\frac{Q(b^2 - a^2)}{a^2} = 2\pi A(b^2 - a^2) \quad \therefore \quad A = \frac{Q}{2\pi a^2}$$

36. Here, $E_1(r_0) = \frac{Q}{4\pi\epsilon_0 r_0^2}$, $E_1(r_0/2) = \frac{4Q}{4\pi\epsilon_0 r_0^2}$

$$E_2(r_0) = \frac{\lambda}{2\pi\epsilon_0 r_0} \; ; \; E_2(r_0/2) = \frac{\lambda}{\pi\epsilon_0 r_0}$$

$$E_3(r_0) = \frac{\sigma}{2\epsilon_0} \; ; \; E_3(r_0/2) = \frac{\sigma}{2\epsilon_0}$$

Given, $E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\therefore \quad \frac{Q}{4\pi\epsilon_0 r_0^2} = \frac{\lambda}{2\pi\epsilon_0 r_0} = \frac{\sigma}{2\epsilon_0}$$

Hence, $Q = 2\pi r_0^2 \sigma$ and $\frac{\lambda}{\pi r_0} = \sigma$ or $r_0 = \frac{\lambda}{\pi\sigma}$

$$E_1\left(\frac{r_0}{2}\right) = 4E_1(r_0) = 2E_2(r_0/8)$$

Also, $E_2\left(\frac{r_0}{2}\right) = 2E_0(r_0) = 2E_3(r_0/2)$

Thus, **option (c) is correct.**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

37. $\phi_E = \frac{\Sigma q}{\epsilon_0} = \frac{+6q - 2q + 3q}{\epsilon_0} = \frac{7q}{\epsilon_0}$

38. $\phi_E = \frac{7q}{\epsilon_0}$. This is because charges present outside the closed surface do not affect the electric flux.

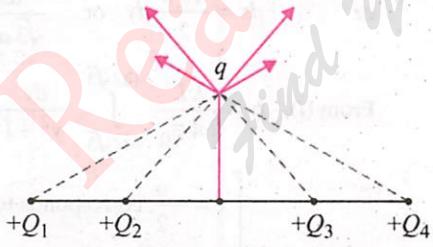
39. $\phi_E = \frac{\Sigma q}{K\epsilon_0} = \frac{2q - 5q + 8q}{6\epsilon_0} = \frac{5q}{6\epsilon_0}$

40. Total electric flux from all the surfaces of the cube $= \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0}$

As the cube has six faces, therefore, electric flux from any one surface of the cube $= \frac{1}{6\epsilon_0}$

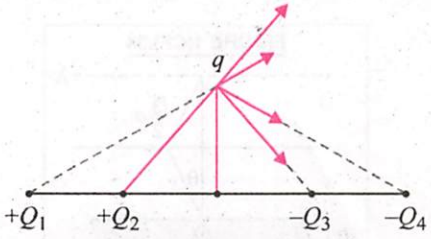
IV. Matching Type Questions

41. (A)



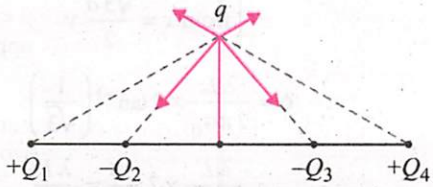
Resultant force along + y direction

(B)



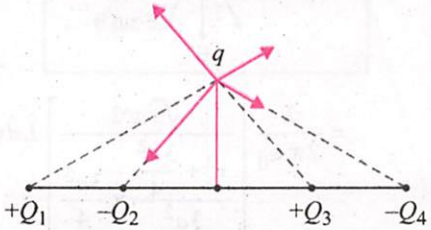
Resultant force along + x direction

(C)



Resultant force along − y direction

(D)



Resultant force along − x direction

Thus **correct option is (a).**

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V. Matrix-Match Type Questions

42. SI unit of electric charge is coulomb and electrostatic unit of charge is stat coulomb. NC^{-1} is the unit of electric intensity. Unit of electric flux $\phi_E = \frac{q}{\epsilon}$ is $Nm^2\ C^{-1}$, charges present outside a closed surface do not affect the electric flux over the surface.

VI. Integer Type Questions

43. Here, $E = \dfrac{\lambda}{2\pi\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0\sqrt{x^2+\frac{3a^2}{4}}}$

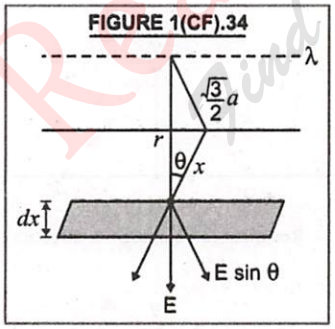
$d\phi = E\sin\theta \cdot A\cos 0 = E\sin\theta \cdot A$

$$= \frac{\lambda}{2\pi\epsilon_0\sqrt{x^2+\frac{3a^2}{4}}} \times \frac{\sqrt{3}\,a/2}{\sqrt{x^2+\frac{3a^2}{4}}} \times L\,dx$$

$$= \frac{(\lambda\sqrt{3}\,a/2)\,L\,dx}{2\pi\epsilon_0\left(x^2+\frac{3a^2}{4}\right)}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\frac{\frac{\sqrt{3}\,a}{2}}{x^2+\frac{3a^2}{4}}\right]L dx$$

FIGURE 1(CF).34



$$= \frac{\lambda}{2\pi\epsilon_0}\left[\frac{\sqrt{3}\,a/2}{\frac{x^2+\frac{3a^2}{4}}{\frac{3a^2}{4}}\times\frac{3a^2}{4}}\right]L dx$$

or $$d\phi = \frac{\lambda}{2\pi\epsilon_0}\left[\frac{\sqrt{3}\,a/2}{\left[\frac{x^2}{3a^2/4}+1\right]\frac{3a^2}{4}}\right]L dx$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\frac{\sqrt{3}\,a/2}{\left[\frac{x^2}{3a^2/4}+1\right]\left[\frac{\sqrt{3}\,a}{2}\right]^2}\right]L dx$$

$$\phi = \int d\phi = \int_{-a/2}^{a/2} \frac{\lambda}{2\pi\epsilon_0}\frac{1}{\left[\frac{x^2}{3a^2/4}+1\right]\left[\frac{\sqrt{3}\,a}{2}\right]}L dx \quad ...(i)$$

Let $\dfrac{x}{\sqrt{3}\,a/2} = y$ or $x = \dfrac{\sqrt{3}\,a}{2}y$

or $dx = \dfrac{\sqrt{3}\,a}{2}dy$ or $\dfrac{dx}{\sqrt{3}\,a/2} = dy$

From (i), $\phi = \dfrac{\lambda L}{2\pi\epsilon_0}\displaystyle\int_{-1/\sqrt{3}}^{+1/\sqrt{3}} \frac{dy}{y^2+1}$

$$\left[\begin{array}{l} x = -\frac{a}{2} \text{ corresponds to } y = -\frac{1}{\sqrt{3}} \\ x = +\frac{a}{2} \text{ corresponds to } y = \frac{1}{\sqrt{3}} \\ \text{from } x = \frac{\sqrt{3}\,a}{2}y \end{array}\right]$$

$$\therefore \quad \phi = \frac{\lambda L}{2\pi\epsilon_0}\times 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\lambda L}{2\pi\epsilon_0}\times 2\times\frac{\pi}{6} = \frac{\lambda L}{6\epsilon_0}$$

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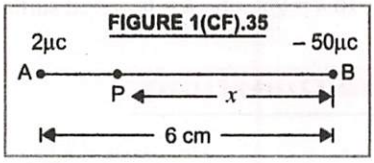
Comparing with $\phi = \dfrac{\lambda L}{n\epsilon_0}$, we get $n = 6$

44. In Fig. 1(CF).35, let the electric intensity be zero at P, where $BP = x$ cm and $AP = (6 - x)$ cm.

FIGURE 1(CF).35



$$\therefore \quad \frac{kq_1}{AP^2} = \frac{kq_2}{BP^2}$$

$$\frac{2}{(6-x)^2} = \frac{50}{x^2}$$

$$\frac{1}{6-x} = \frac{5}{x}$$

$$x = 30 - 5x$$

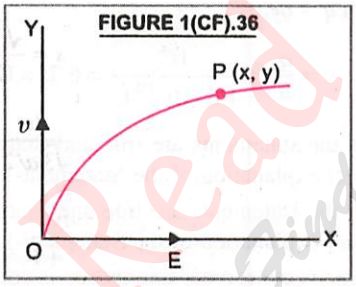
$$6x = 30, \; x = \mathbf{5\ cm}$$

45. Here, $q = 1$ C, $m = 1$ kg

$v = 10$ m/s, $E = 100$ V/m.

In Fig. 1(CF).36, E is along OX and velocity v is along OY.

FIGURE 1(CF).36



Let $P(x, y)$ be the position of point charge at any time t.

$\therefore \quad y = v\,t$

or $\quad t = \dfrac{y}{v} = \dfrac{y}{10}$

Acceleration along X-axis

$$a_x = \frac{qE}{m} = \frac{1 \times 100}{1} = 100\ \text{m/s}^2$$

As $x = \dfrac{1}{2}a_x t^2 \;\therefore\; x = \dfrac{1}{2} \times 100 t^2 = 50\left(\dfrac{y}{10}\right)^2 = \dfrac{y^2}{2}$

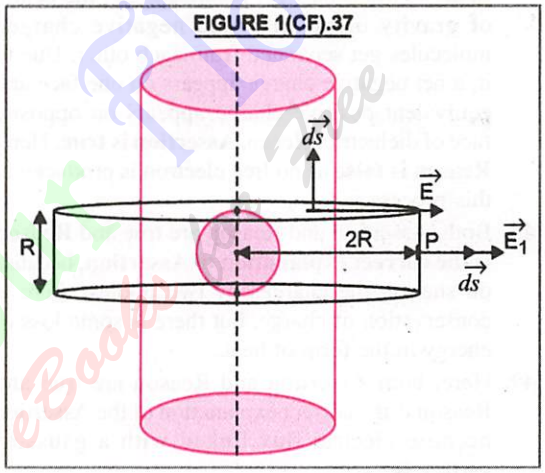
$\therefore \quad y^2 = 2x$

which is the equation of a parabola. Compare with the standard equation of parabola : $y^2 = 4ax$, we find $4a = \mathbf{2\ m}$, which is the length of latus rectum of parabola.

46. Due to symmetry of charge distribution, the electric field at a point outside the cylinder is $\perp r$ to the axis of cylinder, outwards. Let $\vec{E}_1$ be the electric field intensity at P due to charge on solid cylinder.

Draw a cylindrical gaussian surface of radius $2R$ and height R as shown in Fig. 1(CF).37, so that point P lies on the curved surface of gaussian cylinder.

FIGURE 1(CF).37



Electric flux over the gaussian cylindrical surface

$$= \oint \vec{E}_1 \cdot \vec{ds} = \frac{1}{\epsilon_0} \times \text{charge enclosed by the surface}$$

$$\therefore \quad E_1 (2\pi \times 2R) R = \frac{1}{\epsilon_0} (\pi R^2) R \rho$$

$$= \frac{1}{\epsilon_0} \pi R^3 \rho$$

or $$E_1 = \frac{1}{\epsilon_0} \frac{\pi R^3 \rho}{4\pi R^2} = \frac{R\rho}{4\epsilon_0} \qquad \ldots(i)$$

Charge on cavity of radius $R/2$,

$$q = \frac{4}{3}\pi (R/2)^3 \rho = \pi R^3 \rho / 6$$

Electric field intensity at P due to charge on spherical cavity is

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$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{(2R)^2} = \frac{1}{4\pi\epsilon_0}\times\frac{\pi R^3\rho/6}{4R^2}$$

$$= \frac{R\rho}{96\epsilon_0} \quad ...(ii)$$

Hence net electric field, $E = E_1 - E_2$

$$= \frac{R\rho}{4\epsilon_0} - \frac{R\rho}{96\epsilon_0} = \frac{23\rho R}{96\epsilon_0}$$

As per question, $= \frac{23\rho R}{96\epsilon_0} = \frac{23\rho R}{16k\epsilon_0}$

$\therefore \quad 16k = 96 \quad \text{or} \quad k = 6$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

47. When the dielectric is subjected to an electric field, its molecules get polarised, *i.e.*, the centre of gravity of positive and negative charged molecules get separated from each other. Due to it, a net negative charge appears on one face and equivalent positive charge appears on opposite face of dielectric. Hence, **Assertion is true.** Here, **Reason is false** as no free electron is produced in this process.

48. Both, Assertion and Reason are true and **Reason is the correct explanation of Assertion,** because on sharing the charges by two bodies, there is conservation of charge, but there is some loss of energy in the form of heat.

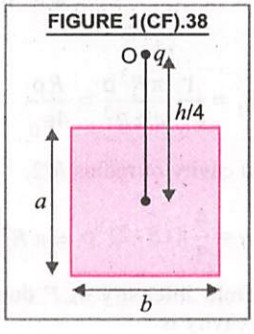
49. Here, both Assertion and Reason are true and Reason is the correct explanation of the Assertion because electric flux linked with a gaussian surface is

$$\phi_E = \oint_S \vec{E}.d\vec{s} = \frac{1}{\epsilon_0}$$

× charge enclosed by the surface

If a charge q is placed at the centre O of the cube then electric flux through all the six faces of the cube is $= \frac{q}{\epsilon_0}$.

FIGURE 1(CF).38



Electric flux through one face $= \frac{1}{6}\frac{q}{\epsilon_0}$

which is independent of side length b.

50. Insulators do not have free charge carriers. That is why flow of current through them is not possible.

51. The conducting surface being connected to earth is at zero potential. The point charge in front induces an equal and opposite charge on the conducting surface. This gives rise to force of attraction.

52. Assertion is true, but reason is false. Electrostatic forces are rather conservative forces.

FOR ENGINEERING STUDENTS

53. From $q = ne$,

$$n = \frac{q}{e} = \frac{1\text{C}}{1{\cdot}6\times10^{-19}\,\text{C}} = 6{\cdot}25\times10^{18}$$

Both the statements are true and Statement-2 is correct explanation of the Statement-1.

54. Both the statements are true and Statement-2 is correct explanation of the Statement-1.

55. Charge is conserved. Energy too is conserved, if we were to take into account the loss of energy in the form of heat etc.

56. From Gauss's theorem in electrostatics,

$$\phi = \frac{q}{\epsilon_0} = \frac{1\times10^{-6}}{8{\cdot}85\times10^{-12}} = 1{\cdot}13\times10^{5}$$

Both the statements are true and statement 2 is correct explanation of statement 1.

57. Statement-1 and 2 both are true, but Statement-2 is not a correct explanation of statement-1.

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2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

2.1. INTRODUCTION

In the previous chapter, we have learnt the concept of electric field due to a point charge and due to a distribution of charges. It is represented in terms of a vector quantity $\vec{E}$, called electric field intensity. In this chapter, we shall learn that electric field can also be represented in terms of a scalar quantity V, called *electrostatic potential*. This concept is of great practical value.

Basically, electrostatic potential of a charged body represents the degree of electrification of the body. It determines the direction of flow of charge between two charged bodies placed in contact with eachother. The charge always flows from a body at higher potential to another body at lower potential. The flow of charge stops as soon as the potentials of the two bodies become equal.

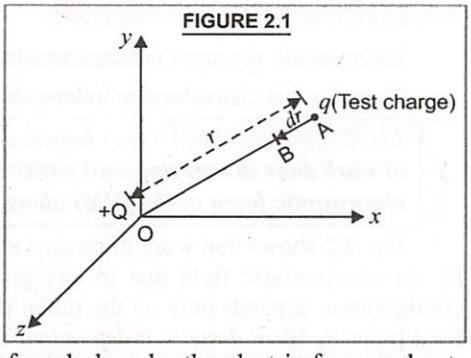
Further, we studied Coulomb's law for calculating electric field intensity. In this chapter, an alternative law called *Gauss's Law* is described for calculating electric field intensity of highly symmetric charge distributions. This law enables us to understand more complicated problems in electrostatics.

2.2. ELECTROSTATIC POTENTIAL ENERGY

Like the potential energy of a mass in a gravitational field, we can define electrostatic potential energy of a charge in an electrostatic field.

For the sake of simplicity, let us assume that electrostatic field $\vec{E}$ is due to charge + Q placed at the origin O. Let a small test charge + q be brought from a point A to a point B, against the repulsive force on it due to charge + Q, Fig. 2.1. We shall assume that the test charge + q is so small that it does not disturb the configuration of charge + Q at the origin. Further, we assume that an external force $\vec{F}_{ext}$ applied is just sufficient to counter the repulsive electric force $\vec{F}_E$ on the test charge q so that net force on test charge q is zero and it moves from A to B without any acceleration.

FIGURE 2.1



In this situation, work done by external force is negative of work done by the electric force and gets fully stored in the charge q in the form of its potential energy.

On reaching B, if the external force applied on q were removed, the electric force will take the test charge + q away from source charge + Q. The stored potential energy at B is used to provide kinetic energy to the charge q in such a way that the sum of kinetic and potential energies at every point is conserved.

Work done by external force in moving the test charge + q from A to B through a distance $AB = dr$ is

$$W_{AB} = \int_A^B \vec{F}_{ext} \cdot d\vec{r} = -\int_A^B \vec{F}_E \cdot d\vec{r} \quad ...(1)$$

This work done against electrostatic force gets stored as potential energy.

Note that charge q possesses a certain amount of electrostatic potential energy at every point in the electric field. Therefore, work done in moving the charge q from A to B increases its potential energy by an amount equal to electrostatic *potential energy difference* between points B and A

i.e., $$\Delta U = U_B - U_A = W_{AB} \quad ...(2)$$

Hence, we define,

Electrostatic potential energy difference ***between two points B and A as the minimum work required to be done by an external force in moving without acceleration a test charge q from A to B.***

This is true for electrostatic field of any charge configuration.

Note that work done by an electrostatic field in moving a given charge from one point to another depends only on the positions of initial and final points. It does not depend on the path chosen in going from one point to the other.

Eqn. (2) defines electrostatic potential energy difference in terms of physically meaningful quantity 'work'. Therefore, it is only the difference of potential energy which is significant. The actual value of potential energy at a point is not relevant. Thus, *there is a freedom in choosing the point where potential energy is zero.* For a charge distribution of finite extent, we choose *zero electrostatic potential energy at infinity.* Therefore, when point A is at infinity, we get from (2)

$$W_{\infty B} = U_B - U_\infty = U_B - 0 = U_B \quad ...(3)$$

Hence we may define

Electrostatic Potential energy of a charge ***q at a point (B) in an electrostatic field due to any charge configuration as the work done by the external force (equal and opposite to electric force) in bringing the charge q from infinity to that point without any acceleration.***

2.3. ELECTROSTATIC POTENTIAL

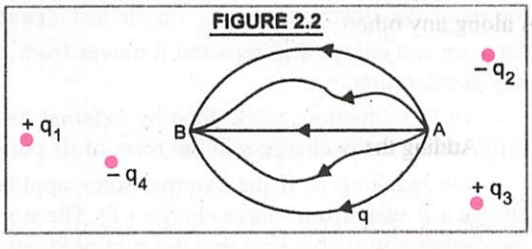
Electrostatic potential is characteristic of electric field associated with a given charge configuration.

To make this characteristic independent of the magnitude of test charge, we define

Electric potential difference ***between two points B and A in an electrostatic field as the amount of work done in carrying unit positive test charge without acceleration from A to B (against the electrostatic force of the field) along any path between the two points.***

Fig. 2.2 shows that work done on a test charge q by the electrostatic field due to any given charge configuration depends only on the initial point A and final point B. ***Work done is independent of the path chosen in going from A to B.***

FIGURE 2.2



If V_A and V_B are the electrostatic potentials at A and B respectively, then from eqn. (2),

$$\frac{U_B - U_A}{q} = \frac{W_{AB}}{q} = V_B - V_A = \Delta V \quad ...(4)$$

The dimensional formula for electrostatic potential difference is

$$\Delta V = \frac{W_{AB}}{q} = \frac{[ML^2T^{-2}]}{[AT]} = [M^1 L^2 T^{-3} A^{-1}]$$

SI unit of electrostatic potential difference is **volt.** That is why potential difference is sometimes referred to as voltage :

$$1\,V = \frac{1\,J}{1\,C} = 1\,JC^{-1} = 1\,Nm\,C^{-1}$$

Thus, ***Electrostatic Potential difference between any two points in an electrostatic field is said to be one volt, when one joule of work is done in moving a positive charge of one coulomb from one point to the other against the electrostatic force of the field without any acceleration.***

As explained already in case of electrostatic potential energy, it is potential difference between any two points, which is physically significant, ... and not the actual value of potential at a point.

We can, however, define electric potential at a point by choosing the potential to be zero at infinity. If we take point A at infinity, $V_A = 0$. From (4),

$$V_B = \frac{W_{\infty B}}{q} \qquad ...(5)$$

Hence, ***Electrostatic potential at any point in a region of electrostatic field is the minimum work done in carrying a unit positive charge (without acceleration) from infinity to that point.***

Obviously, electrostatic potential at a point is a **scalar quantity.** Its SI unit is **volt.**

Electrostatic potential at a point is said to be one volt, when one joule of work is done in moving one coulomb of positive charge from infinity to that point against the electrostatic force of the field without acceleration.

Sample Problem **The potential difference between two points is 10 volt. How much work will be done in carrying a charge of 200 μ C from one point to the other ?**

Sol. As work done = charge × potential

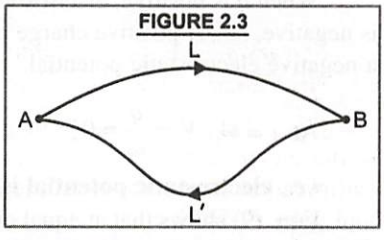
$\therefore \qquad W = (200 \times 10^{-6}) \times 10 = 2 \times 10^{-3}$ J

2.4. ELECTROSTATIC FORCES ARE CONSERVATIVE

To prove that electrostatic forces are conservative in nature, we show that work done in moving a unit positive test charge over a closed path in an electric field is zero. We know that along any path L, Fig. 2.3, work done in carrying unit positive charge from A to B.

$$\frac{W_{AB}}{q_0} = V_B - V_A$$

FIGURE 2.3



Similarly, work done in carrying unit positive charge from B to A along any other path L′

$$\frac{W_{BA}}{q_0} = V_A - V_B$$

Adding the two, we get total work done in carrying unit positive charge over a closed path $A \rightarrow B \rightarrow A$ is

$$\frac{W_{AB}}{q_0} + \frac{W_{BA}}{q_0} = \frac{W_{ABA}}{q_0} = (V_B - V_A) + (V_A - V_B) = \text{Zero}$$

Hence, no work is done in moving a unit positive test charge over a closed path in an electric field. Hence, *electrostatic field is a conservative field* and electrostatic forces are conservative forces in nature.

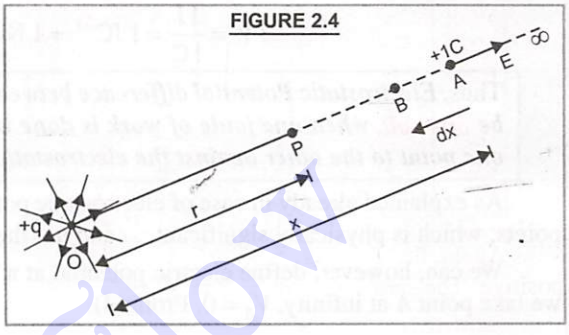
Mathematically, we can write this result as $\oint \vec{E}.\vec{dl} = 0$ ...(6)

i.e. line integral of electric field over a closed path in the electric field is always zero.

2.5. ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

Suppose we have to calculate electrostatic potential at any point P due to a single point charge $+q$ at O ; where $OP = r$, Fig. 2.4.

FIGURE 2.4



By definition, electrostatic potential at P is the amount of work done in carrying a unit positive charge from ∞ to P.

As work done is independent of the path, we choose a convenient path along the radial direction from infinity to the point P without acceleration.

At some intermediate point A on this path, where $OA = x$, the electrostatic force on unit positive charge is

$$E = \frac{1}{4\pi\epsilon_0 x^2}, \text{ along } OA \text{ produced} \quad ...(7)$$

Small amount of work done in moving a unit positive charge from A to B where

$\vec{AB} = \vec{dx}$ is

$$dW = \vec{E}.\vec{dx} = E\,dx \cos 180^\circ = -E\,dx \quad ...(8)$$

$\therefore$ Total work done in moving unit + charge from ∞ to the point P is

$$W = \int_{\infty}^{r} -E\,dx = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\,dx = -\frac{q}{4\pi\epsilon_0}\int_{\infty}^{r} x^{-2}\,dx = -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$W = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{q}{4\pi\epsilon_0 r}$$

By definition, this is the electrostatic potential at P due to charge q

$$V = W = \frac{q}{4\pi\epsilon_0 r} \quad ...(9)$$

When q is positive, electrostatic potential V is positive and when q is negative, electrostatic potential V is negative, *i.e.*, A positive charge produces a positive electrostatic potential and a negative charge produces a negative electrostatic potential.

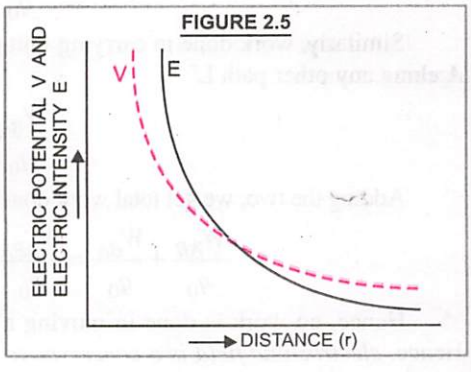
At $r = \infty$, $V = \frac{q}{\infty} = 0$,

i.e., **electrostatic potential is zero at infinity.**

Eqn. (9) shows that at equal distances from a point charge q, value of V is same. ***Hence, electrostatic potential due to a single charge is spherically symmetric.***

Fig. 2.5 shows the variation of electrostatic potential V with distance, *i.e.*, $V \propto \frac{1}{r}$; and also the variation of electric intensity E with distance, *i.e.*, $E \propto \frac{1}{r^2}$.

FIGURE 2.5



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If the charge is negative, the variation of E and V with r is shown in Fig. 2.6.

FIGURE 2.6

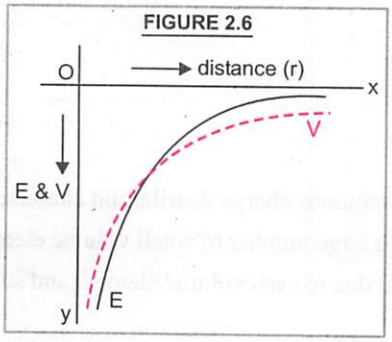
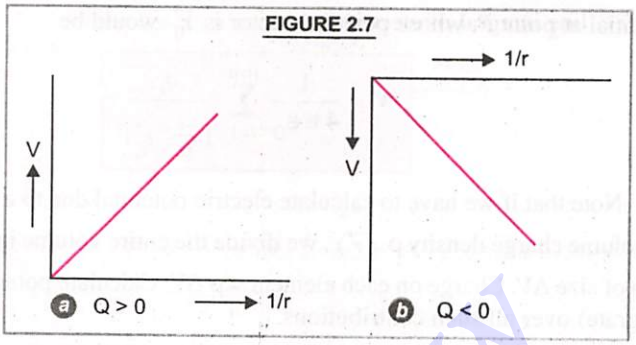


FIGURE 2.7



Again, the variation of electric potential V with $(1/r)$ is shown in Fig.2.7(a). When $Q > 0$, *i.e.*, charge is positive. The same variation is shown in Fig.2.7(b), when $Q < 0$, *i.e.*, charge is negative.

Sample Problem **What is the electrostatic potential at the surface of a gold nucleus of diameter 13·2 fermi. Z for gold is 79 ?**

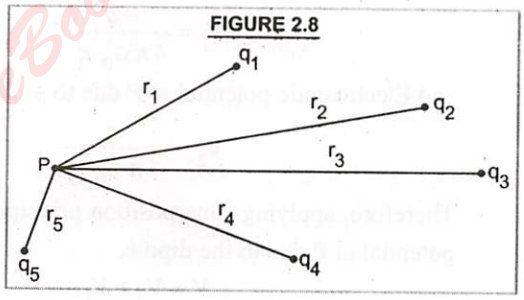
Sol. Here, $V = ?$, $r = \dfrac{13{\cdot}2}{2} = 6{\cdot}6$ fermi $= 6{\cdot}6 \times 10^{-15}$ m, $q = Ze = 79 \times 1{\cdot}6 \times 10^{-19}$ C

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{9\times10^9 \times 79 \times 1{\cdot}6 \times 10^{-19}}{6{\cdot}6 \times 10^{-15}} = \mathbf{1{\cdot}7 \times 10^7 \text{ volt.}}$$

2.6. POTENTIAL AT A POINT DUE TO GROUP OF ELECTRIC CHARGES

Suppose there are a number of point charges q_1, q_2, q_3, ...q_n at distances r_1, r_2, r_3, ...r_n respectively from the point P, where electric potential is to be calculated, Fig. 2.8.

FIGURE 2.8



Now, potential at P due to charge q_1,

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$$

Similarly, values of potential at P due to other charges are

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2},\ V_3 = \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_3}, \text{ and so on } V_n = \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_n} \quad \text{...(10)}$$

Using superposition principle, we obtain resultant potential at P due to total charge configuration as the algebraic sum of the potentials due to individual charges.

$$\therefore \quad V = V_1 + V_2 + V_3 + ... V_n$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_3} + ...\frac{1}{4\pi\epsilon_0}\frac{q_n}{r_n} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ...\frac{q_n}{r_n}\right)$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{q_i}{r_i}} \quad \text{...(11)}$$

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If $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots . \vec{r}_n$ are position vectors of the charges $q_1, q_2, q_3, \ldots . q_n$ respectively, then electrostatic potential at point P, whose position vector is $\vec{r}_0$ would be

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{i=n} \frac{q_i}{|\vec{r}_0 - \vec{r}_i|} \qquad \ldots(12)$$

Note that if we have to calculate electric potential due to a continuous charge distribution characterised by volume charge density $\rho\ (\vec{r})$, we divide the entire volume into a large number of small volume elements, each of size ΔV. Charge on each element $= \rho\, \Delta V$. Calculate potential due to each volume element and sum (or integrate) over all such contributions.

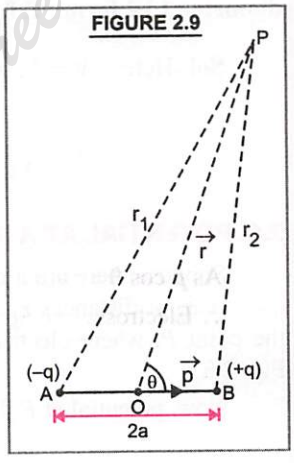
2.7. ELECTROSTATIC POTENTIAL AT A POINT DUE TO AN ELECTRIC DIPOLE

Let an electric dipole consist of two equal and unlike point charges $-q$ at A and $+q$ at B, separated by a small distance $AB = 2a$, with centre at O. The dipole moment

$$|\vec{p}| = q \times 2a \qquad \ldots(13)$$

Let us take the *origin at the centre of the dipole.* We have to calculate electric potential at any point P where $\vec{OP} = \vec{r}$ and $\angle BOP = \theta$, Fig. 2.9.

FIGURE 2.9



Let the distance of P from charge $-q$ at A be r_1, *i.e.*, $AP = r_1$ and distance of P from charge $+q$ at B be r_2, *i.e.*, $BP = r_2$.

Electrostatic Potential at P due to $-q$ charge at A

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1}$$

and Electrostatic potential at P due to $+q$ charge at B

$$V_2 = \frac{q}{4\pi\epsilon_0 r_2}$$

Therefore, applying superposition principle potential at P due to the dipole,

$$V = V_2 + V_1$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \qquad \ldots(16)$$

Now, by geometry, $r_1^2 = r^2 + a^2 + 2ar\cos\theta$, and

$$r_2^2 = r^2 + a^2 + 2ar\cos(180° - \theta) = r^2 + a^2 - 2ar\cos\theta$$

We may rewrite $$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$$

If $a << r$, $\frac{a}{r}$ is small, $\frac{a^2}{r^2}$ can be neglected.

$\therefore$ $$r_1^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$$

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$$r_1 = r\left(1+\frac{2a}{r}\cos\theta\right)^{1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1+\frac{2a}{r}\cos\theta\right)^{-1/2}$$

Similarly, $$\frac{1}{r_2} = \frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-1/2}$$

Putting these values in (16), we get $V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-1/2} - \frac{1}{r}\left(1+\frac{2a}{r}\cos\theta\right)^{-1/2}\right]$

Using Binomial theorem and retaining terms upto the first order in a/r, we get

$$V = \frac{q}{4\pi\epsilon_0 r}\left[1+\frac{a}{r}\cos\theta - \left(1-\frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q}{4\pi\epsilon_0 r}\left[1+\frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$V = \frac{q\times 2a\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}} \quad ...(15)$$

As $p\cos\theta = \vec{p}.\hat{r}$, where $\hat{r}$ is unit vector along the position vector $\vec{OP} = \vec{r}$,

$\therefore$ Electrostatic potential at P due to a short dipole ($a << r$) is

$$\boxed{V = \frac{\vec{p}.\hat{r}}{4\pi\epsilon_0 r^2}} \quad ...(16)$$

On the dipole axis, $\theta = 0^\circ$ or π $\therefore$ $V = \pm\frac{p}{4\pi\epsilon_0 r^2}$

Positive sign for $\theta = 0^\circ$ and negative sign for $\theta = \pi$.

At any point on the **equatorial plane,** $\theta = \frac{\pi}{2}$; $\cos\theta = \cos\frac{\pi}{2} = 0$. $\therefore$ $V = 0$

i.e., electrostatic potential at any point on the equatorial plane of dipole is zero.

Retain in Memory

When we compare electrostatic potential due to a point charge with electrostatic potential due to a dipole, we find that

(*i*) whereas potential due to point charge varies inversely as the distance from the charge, *i.e.*, $V \propto \frac{1}{r}$, potential due to dipole falls off more rapidly, *i.e.*, $V \propto \frac{1}{r^2}$, *i.e.*, electrostatic potential varies inversely as the square of distance from the centre of the dipole.

(*ii*) The potential due to a dipole depends not only on r, but also on the angle between the position vector $\vec{r}$ and dipole moment vector $\vec{p}$.

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2.8. EQUIPOTENTIAL SURFACES

An equipotential surface ***is that surface at every point of which electric potential is the same.***

By definition, potential difference between two points B and A = work done in carrying unit positive test charge from A to B, *i.e.*, $V_B - V_A = W_{AB}$

If points A and B lie on an equipotential surface, then $V_B = V_A \quad \therefore \quad W_{AB} = V_B - V_A = 0$

Hence, ***no work is done in moving the test charge from one point of equipotential surface to the other.***

If dl is the small distance over the equipotential surface through which unit positive charge is carried, then $dW = \vec{E}.\vec{dl} = E\,dl\cos\theta = 0 \quad \therefore \cos\theta = 0$ or $\theta = 90°$, *i.e.*, $\vec{E} \perp \vec{dl}$

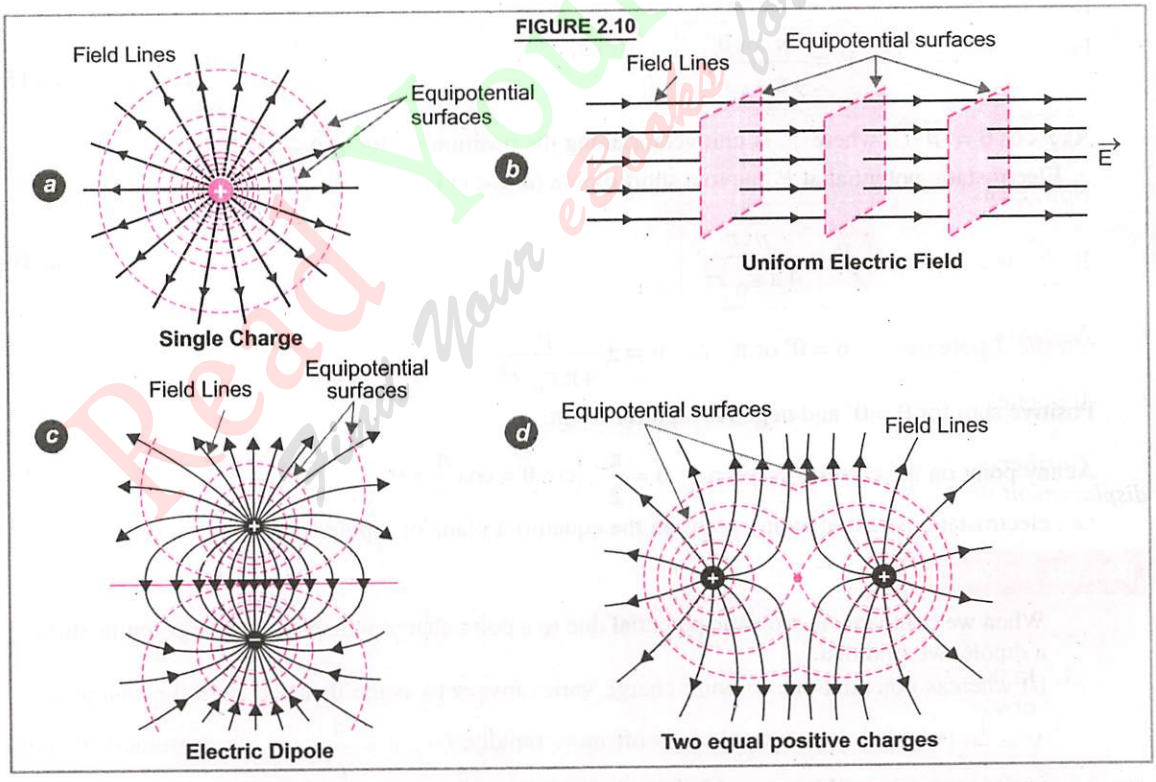
$\therefore$ Electric field intensity $\vec{E}$ is always normal to the equipotential surface, *i.e.*, ***for any charge configuration, equipotential surface through a point is normal to the electric field at that point.***

For a single charge q, the potential is given by $V = \dfrac{q}{4\pi\epsilon_0 r}$

This shows that V is constant if r is constant. Hence, ***equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge,*** as shown in Fig. 2.10(a) by solid curves.

For a uniform electric field, say, along the X-axis, the equipotential surfaces are planes normal to the X-axis, *i.e.*, planes parallel to the Y-Z plane as shown in Fig. 2.10(b).

FIGURE 2.10



The equipotential surfaces for an electric dipole and for two identical positive charges are shown in Fig. 2.10(c) and Fig. 2.10(d) respectively.

The drawings of equipotential surfaces give us an alternative visual picture of electric field around a charge configuration.

Some of the **important properties of equipotential surfaces** are :

(*i*) No work is done in moving a test charge from one point to another point on an equipotential surface.

(*ii*) Electric field is always perpendicular to the equipotential surface.

(*iii*) No two equipotential surfaces can intersect eachother.

(*iv*) Equipotential surfaces are closer in regions of strong electric field and they are farther apart in the regions of weak electric fields.

2.9. RELATION BETWEEN ELECTRIC INTENSITY AND ELECTRIC POTENTIAL

Fig. 2.11 shows cross sections of a set of equipotential surfaces. Let the potential difference between each pair of adjacent surfaces be dV. Let a small test charge $(+q)$ at P move through a distance $\vec{dr}$ from one equipotential surface to the other. The electric field $\vec{E}$ at P is perpendicular to the equipotential surface through point P.

Work done by electric field on the test charge

$$= -q\,dV \qquad ...(i)$$

(Neg. sign for decrease in potential in the direction of $\vec{E}$)

Also, this work done $= \vec{F}.\vec{dr} = q\vec{E}.\vec{dr}$

$$= q\,E\,dr\cos\theta \qquad ...(ii)$$

Here, θ is angle between $\vec{E}$ and $\vec{dr}$.

From (*i*) and (*ii*),

$$-q\,dV = q\,E\,dr\cos\theta \quad \text{or} \quad E\cos\theta = -\frac{dV}{dr}$$

Now, $E\cos\theta$ is component of $\vec{E}$ in the direction of $\vec{dr}$.

If $\vec{dr}$ is along s-axis, we may write $\boxed{E_s = -\frac{\partial V}{\partial r}}$ ...(17)

Negative sign shows that the direction of electric field $\vec{E}$ is the direction of decreasing potential.

It implies that $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$ and $E_z = -\frac{\partial V}{\partial z}$

Further, the magnitude of electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point. This is called potential gradient, i.e.,

$$|\vec{E}| = -\frac{|dV|}{dr} = -\text{(potential gradient)}$$

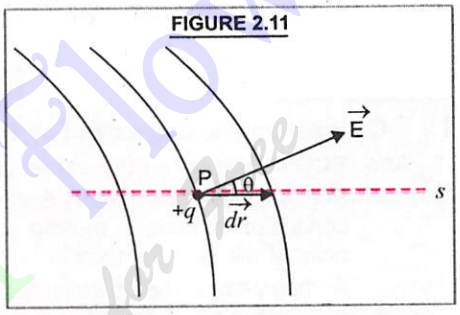
Retain in Memory

1. In the region of strong electric fields, equipotential surfaces are close together and in the region of weak electric fields, equipotential surfaces are far apart. This follows from

$$E = -\frac{dV}{dr},\ i.e.,\ dr = -\frac{dV}{E} \quad \text{or} \quad dr \propto \frac{1}{E}$$

2. Note that electric potential is a scalar quantity, but electric potential gradient is a vector, as it is numerically equal to electric field intensity.

FIGURE 2.11



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Sample Problem 1 **A uniform electric field of 10 N/C exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.**

Sol. Here, $E = -10$ N/C (neg. sign for downward direction)

$$dV = ?\ dr = 50\text{ cm} = \frac{1}{2}\text{m}.$$

As $E = -\dfrac{dV}{dr}$ $\quad\therefore\quad$ $dV = -E\ dr = -(-10)\times\dfrac{1}{2} = \mathbf{5\ volt}$

Sample Problem 2 **The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$ volt. Calculate electric intensity at the point (1 m, 0, 2 m).**

Sol. Here, $V = 4x^2$, $E = ?$, $x = 1$ m, $y = 0$, $z = 2$ m

$$E = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(4x^2) = -8x = -8\,(1) = \mathbf{-8\ V\ m^{-1}}\ \textbf{along negative direction of } x\textbf{-axis.}$$

Curiosity Question

Q. What do you understand by ECG and EEG ? What is their basis ?

Ans. **ECG** stands for electro-cardio graph and **EEG** stands for electro-encephalo graph.

Both are based on the fact that human body is a good conductor of electricity, but not a perfect conductor. Therefore, muscle activity leads to small differences in electric potential from one point of the body to another.

As *human heart* beats, potential differences in the range of 1 milli volt develop. These potential differences are detected and displayed with an instrument known as ECG.

Further, *nerve impulses in the brain* lead to potential differences in the range of 0·1 millivolt. They are detected and displayed with an instrument known as EEG.

2.10. EXPRESSION FOR ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

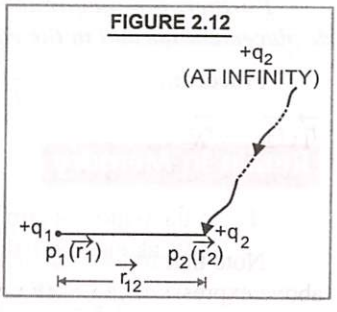
The electrostatic potential energy is the energy possessed by a system of point charges by virtue of their positions. When two charges are infinite distance apart, their fields do not extend upto each other. Their electrostatic potential energy is zero, because no work is done in moving a charge at infinite distance where electric field is zero. Hence we may define

***Electrostatic potential energy** of a system of point charges as the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.*

(a) Electrostatic potential energy of a system of two point charges

Suppose a point charge q_1 is held at a point P_1 with position vector $\vec{r}_1$ in space. Another point charge q_2 is at infinite distance from q_1. This is to be brought to the position P_2 $(\vec{r}_2)$, where $P_1P_2 = \vec{r}_{12}$, Fig. 2.12.

FIGURE 2.12



Now, electrostatic potential at P_2 due to charge q_1 at P_1 is

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{12}}$$

By definition, work done in carrying charge q_2 from ∞ to P_2

W = potential × charge

$$W = \frac{q_1}{4\pi\epsilon_0\ r_{12}} \times q_2$$

This is stored in the system of two point charges q_1 and q_2 in the form of electrostatic potential energy U. Thus

$$U = W = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad \text{...(18)}$$

(b) Electrostatic potential energy of a system of N point charges

It is equal to the total amount of work done in assembling all the charges at the given positions from infinity.

In bringing the first charge q_1 to position P_1 $(\vec{r_1})$, no work is done, because all other charges are still at infinity, and there is no field, *i.e.*, $W_1 = 0$

When we bring charge q_2 from infinity to P_2 $(\vec{r_2})$ at a distance r_{12} from q_1, work done is

$$W_2 = [\text{potential due to } q_1] \times q_2 = \frac{q_1}{4\pi\epsilon_0 r_{12}} (q_2)$$

In bringing q_3 from ∞ to P_3 $(\vec{r_3})$, work has to be done against electrostatic forces of both q_1 and q_2

$$\therefore \quad W_3 = [\text{potential due to } q_1 \text{ and } q_2] \times \text{charge } q_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) \times q_3$$

$$W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Similarly, in bringing q_4 from ∞ to P_4 $(\vec{r_4})$, work has to be done against electrostatic forces of q_1, q_2 and q_3.

$$\therefore \quad W_4 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$$

$\therefore$ Electrostatic Potential energy of a system of four charges,

$$U = W = W_1 + W_2 + W_3 + W_4 = 0 + \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right] + \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] + \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right] \quad \text{...(19)}$$

Proceeding in this way, we can write electrostatic potential energy of a system of N point charges at $\vec{r_1}, \vec{r_2}, \ldots\ldots \vec{r_N}$ as

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\text{all pairs}} \frac{q_j q_k}{r_{jk}} \quad \text{...(20)}$$

Note that in this summation, we should include only one term for each pair of charges. Therefore, the above expression can be rewritten as

$$U = \frac{1}{2} \sum_{j=1}^{j=N} \sum_{\substack{k=1 \\ j \neq k}}^{k=N} \frac{1}{4\pi\epsilon_0} \frac{q_j q_k}{r_{jk}} \quad \text{...(21)}$$

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The factor 1/2 is needed because in this summation set up, we count each pair of charges twice. For example, when $j = 1, k = 2$ and $j = 2, k = 1$, we get contribution from the same pair of charges, 1, 2. As we have to include only one term for each pair, the factor 1/2 is a must.

Rewriting (21) as $$U = \frac{1}{2} \sum_{j=1}^{j=N} q_j \left(\sum_{\substack{k=1 \\ j \neq k}}^{k=N} \frac{1}{4\pi\epsilon_0} \frac{q_k}{r_{jk}} \right)$$

$$\boxed{U = \frac{1}{2} \sum_{j=1}^{j=N} q_j V_j} \quad \text{...(22)}$$

where $V_j = \sum_{\substack{k=1 \\ k \neq j}}^{k=N} \frac{1}{4\pi\epsilon_0} \frac{q_k}{r_{jk}}$ = Potential at $\vec{r}_j$ due to all other charges.

Equation (22) represents the **relation between electrostatic potential energy and electrostatic potential :**

Units of electrostatic potential energy

The SI unit of potential energy is **joule.** One joule is the energy stored in moving a charge of one coulomb through a potential difference of one volt.

Another convenient unit of energy is **electron volt (eV)**, which is the kinetic energy gained or lost by an electron in moving through a potential difference of one volt.

1 electron volt = 1 $eV = 1{\cdot}6 \times 10^{-19}$ coulomb $\times$ 1 $V = 1{\cdot}6 \times 10^{-19}$ joule

Some other related units of energy are

1 milli electron volt = 1 $meV = 10^{-3}\ eV = 1{\cdot}6 \times 10^{-22}$ J

1 kilo electron volt = 1 $keV = 10^{3}\ eV = 1{\cdot}6 \times 10^{-16}$ J

1 Mega or Million electron volt = 1 $MeV = 10^{6}\ eV = 1{\cdot}6 \times 10^{-13}$ J.

1 Giga electron volt = 1 $GeV = 10^{9}\ eV = 1{\cdot}6 \times 10^{-10}$ J

1 Trillion electron volt = 1 TeV = $10^{12}\ eV = 1{\cdot}6 \times 10^{-7}$ J.

Sample Problem **Three charges + q, 2 q and –4 q are placed on the three vertices of an equilateral triangle of each side 0·1 m. Calculate electrostatic potential energy of the system, take $q = 10^{-7}$ C.** (Uttarakhand Board 2013)

Sol. Potential Energy of system of charges $= \frac{1}{4\pi\epsilon_0 r} [q\,(2\,q) + 2\,q\,(-4\,q) + (-4\,q)\,q]$

$$= -\frac{10q^2}{4\pi\epsilon_0 r} = -\frac{9\times10^9 \times 10\,(10^{-7})^2}{0{\cdot}1} = -9 \times 10^{-3}\ \text{J}$$

2.11. POTENTIAL ENERGY OF CHARGES IN AN EXTERNAL ELECTRIC FIELD

(a) Potential energy of a single charge in an external field

We have already obtained an expression for potential energy of a system of charges. In this case, the source of the electric field, *i.e.*, charges and their locations were specified.

We have now to determine potential energy of a charge (or charges) *in an external field* $\vec{E}$ *which is not produced by the given charges whose potential energy we have to calculate.* The sources which produce external field $\vec{E}$ are often unknown and they are of no interest to us.

The external electric field $\vec{E}$ and the corresponding external potential V may change from point to point. If $V(\vec{r})$ is external potential at any point P of position vector $\vec{r}$, then by definition, work done in bringing a unit positive charge from infinity to the point P is equal to V.

Therefore, work done in bringing a charge q from infinity to the point P in the external field $= q \, . \, V(\vec{r})$

This work is stored in the charged particle in the form of its potential energy.

$\therefore$ Potential energy of a single charge q at $\vec{r}$ in an external field $= q \, . \, V(\vec{r})$

(b) Potential energy of a system of two charges in an external field

Suppose q_1, q_2 are two point charges at position vectors $\vec{r_1}$ and $\vec{r_2}$ respectively, in a uniform external electric field of intensity $\vec{E}$. To calculate the potential energy of the system of these two charges in the external field, we find that

Work done in bringing charge q_1 from infinity to position $\vec{r_1}$ is $W_1 = q_1 \, . \, V(\vec{r_1})$ where $V(\vec{r_1})$ is potential at $\vec{r_1}$ due to external field.

Again, work done in bringing charge q_2 from infinity to position $\vec{r_2}$ against the external field is $W_2 = q_2 \, . \, V(\vec{r_2})$, where $V(\vec{r_2})$ is potential at $\vec{r_2}$ due to external field.

While bringing q_2 from infinity to position $\vec{r_2}$, work has also to be done against the field due to q_1. This is $W_3 = \dfrac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$ where r_{12} is the distance between q_1 and q_2.

By the superposition principle, we add up the work done and find P.E. of the system = total work done in assembling the charge configuration

$$U = W_1 + W_2 + W_3$$

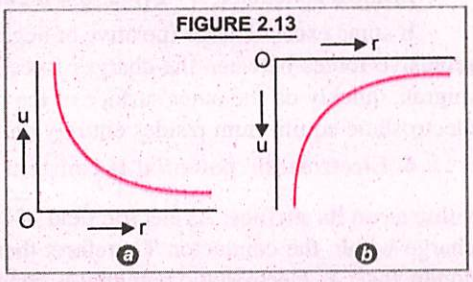
$$\boxed{U = q_1 \, . V(\vec{r_1}) + q_2 \, . V(\vec{r_2}) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}}$$

Retain in Memory

1. Potential energy of any system of charges is defined relative to a point, where its value is zero. Normally, this point would be at infinite distance from the charges.
2. When two charges q_1 and q_2 are of same sign, they repel eachother. Work done to bring them to their respective positions is positive. Therefore, potential energy of a system of two charges of same sign is positive as shown in Fig. 2.13(*a*).

   When the two charges are of opposite sign, they attract eachother. Work done to bring them to their respective positions is negative. The variation of P.E (U) with distance is shown in Fig. 2.13(*b*).

FIGURE 2.13



2.12. ELECTROSTATICS OF CONDUCTORS

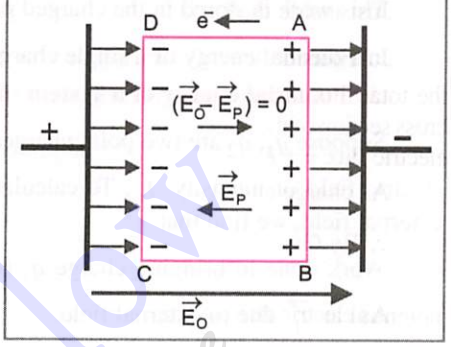
In Art. 1.4, we have learnt that a conductor is a substance that can be used to carry or conduct electric charges from one place to the other. All metals, for example, are good conductors of electricity. This is because conductors contain a large number of free charge carriers (*i.e.*, free electrons).

Some of the important results regarding electrostatics of conductors are discussed below :

1. In electrostatic equilibrium, electric field is zero everywhere inside the conductor. Suppose a conductor *ABCD* is held in an external electric field of intensity $\vec{E}_0$, Fig. 2.14. Free electrons in the conductor move from *AB* to *CD*.

As a result, some net negative charge appears on *CD* and an equal positive charge appears on *AB*. These are called ***induced charges.*** They produce an induced electric field of intensity $\vec{E}_p$ which opposes the flow of free electrons from *AB* to *CD*. The flow, therefore, stops as soon as $\vec{E}_p$ becomes equal to $\vec{E}_0$. As the applied field and induced electric field inside the conductor are equal and opposite, therefore, net electric field in the interior of the conductor is zero. Obviously, there will be no electric field lines inside the conductor held in an external electric field.

FIGURE 2.14



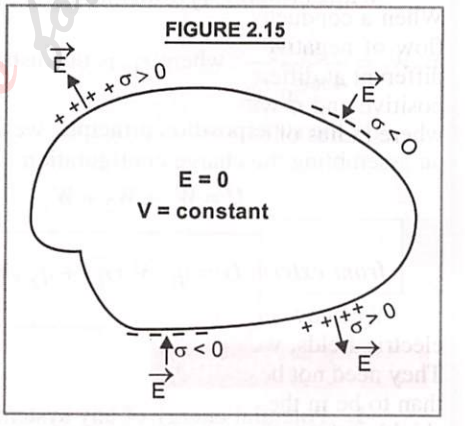
2. Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point. Under electrostatic conditions, once the charges on a conductor are rearranged, the flow of charges stops. Therefore, component of electric field along the tangent to the surface of the conductor must be zero.

i.e. $E \cos\theta = 0$, where θ is the angle which electric field intensity makes with the tangent to the surface.

As $E \neq 0$, $\therefore \cos\theta = 0$ or $\theta = 90°$

Hence the field is perpendicular to the surface of the conductor at every point as shown in Fig. 2.15. If charge on conductor is positive ($\sigma > 0$), $\vec{E}$ is along the outdrawn normal. If charge on conductor is negative ($\sigma < 0$), $\vec{E}$ is along normal drawn inwards.

FIGURE 2.15



3. Net charge in the interior of a conductor is zero.

According to Gauss's theorem ; $\oint \vec{E}.d\vec{s} = \dfrac{q}{\epsilon_0}$

Inside a conductor, $\vec{E} = 0$ $\therefore q = 0$ *i.e.* net charge inside a conductor is zero.

If some excess charge (positive or negative) were placed inside a conductor by any means, the coulomb repulsive forces between the charges force them as far apart as possible. These forces will cause them to migrate quickly on the outer surface of the conductor. Hence any excess charge on an isolated conductor in electrostatic equilibrium resides entirely on the outer surface of the conductor.

4. Electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface. As electric field $\vec{E} = 0$, inside the conductor, no work is done in moving a small test charge within the conductor. Therefore, there is no potential difference between any two points inside the conductor, *i.e.*, electrostatic potential is constant throughout the volume of the conductor.

Further, $\vec{E}$ is perpendicular to the surface of the conductor. Therefore, $\vec{E}$ has no tangential component on its surface, no work is done in moving a test charge on the surface of the conductor. Hence, there is no potential difference between any two points on the surface of the conductor. Hence, the electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface.

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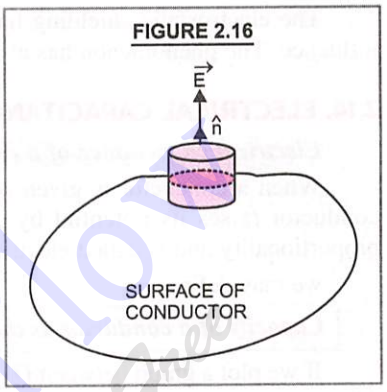
5. Electric field at the surface of a charged conductor is $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$, where σ is surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

Let us consider a short cylinder of small area of cross section *ds* and negligible height partly inside and partly outside the surface of a conductor of surface charge density σ, as shown in Fig. 2.16.

Just inside the surface, $E = 0$.

Just outside, the field $\vec{E}$ is normal to the surface. The contribution to the total flux through the cylinder comes only from the outside circular cross section of the cylinder. Over the small area *ds*, taking *E* to be constant, electric flux $= \pm E\,(ds)$, positive for $\sigma > 0$ and negative for $\sigma < 0$.

FIGURE 2.16



As charge enclosed by the element $= \sigma\, ds$

$\therefore$ By Gauss's theorem, $E\,(ds) = \frac{\sigma\, ds}{\epsilon_0}$ or $E = \frac{\sigma}{\epsilon_0}$

As electric field is normal to the surface, we can write $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$,

which is true for both signs of σ. For $\sigma > 0$, electric field is normal to the surface and outwards as shown in Fig. 2.16. The reverse is also true.

6. Surface density of charge is different at different points of an irregularly shaped conductor. When a conductor is placed in an external electric field, charges on the conductor rearrange on account of flow of negative charges (*i.e.*, electrons). Therefore, the local charge density σ (*i.e.* charge/area) will be different at different points on the surface of the conductor. σ will be positive at places where the charge is positive and σ will be negative at places where the charge on the conductor is negative. Further, at points where radius of curvature of surface is small, area is smaller, therefore, σ will be higher and vice-versa.

2.13. ELECTROSTATIC SHIELDING

***Electrostatic shielding/screening** is the phenomenon of protecting a certain region of space from external electric field.*

We know *electric field inside a conductor is zero.* Therefore, to protect delicate instruments from external electric fields, we enclose them in hollow conductors. Such hollow conductors are called *Faraday Cages.* They need not be earthed. In a thunderstorm accompanied by lightening, it is safer to be inside a car or a bus than to be in the open ground or under a tree etc. The metallic body of the car or bus provides electrostatic shielding from the lightening. The fundamental fact is that electric field inside a cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed. That is, whatever be the charge and field configuration outside, any cavity in a conductor remains protected from outside electrical influence.

Further, an earthed conductor *AB* can also act as a screen against the electric field. For example, in Fig. 2.17, when *AB* is not earthed, field of the charged body *C* due to electrostatic induction continues beyond *AB*. However, when *AB* is earthed, the induced positive charge flows to earth and the field in the region beyond *AB* disappears, Fig. 2.18. A high voltage generator is usually enclosed in such a cage which is *earthed*. This would prevent the electrostatic field of the generator from spreading out of the cage.

FIGURE 2.17

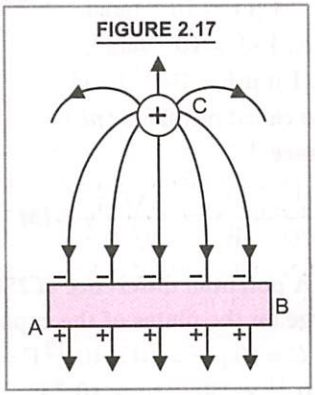
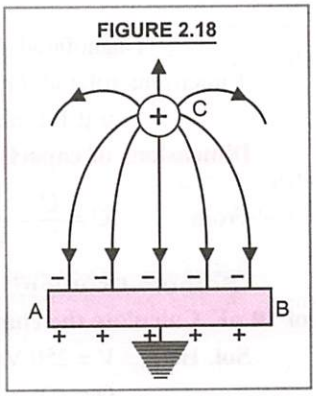


FIGURE 2.18



DO YOU KNOW ?

1. Human body is a good conductor of electricity, *i.e.*, charge can flow easily through the human body.
2. **Grounding or Earthing.** It is the method by which charges are shared between a charged body and earth. Metallic bodies of all electrical appliances are earthed from the safety point of view of the appliance as well as the user.

The electrostatic shielding has been used in protecting sensitive instruments from outside electrical influence. The phenomenon has also been used in the design of T.V. Cables.

2.14. ELECTRICAL CAPACITANCE

Electrical capacitance of a conductor is related to its ability to store the electric charge or energy.

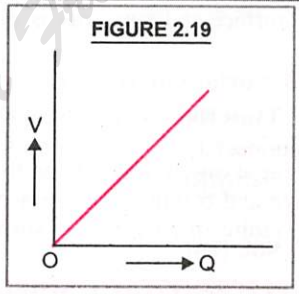
When a conductor is given some charge, its electric potential increases. If a charge Q given to the conductor raises its potential by V, then it is found that $Q \propto V$ or $Q = CV$, where C is a constant of proportionality and is called electrical capacitance of the conductor. From $C = Q/V$, ...(23)

we may define

Capacity of a conductor as the ratio of charge given to the conductor to the rise in its potential.

If we plot a graph between Q (charge given) and V (the potential to which capacitor is raised), we get a straight line as shown in Fig.2.19. It implies that changing Q by a certain factor changes V by the same factor. Therefore, *the capacity of a conductor depends neither on Q nor on V. It depends on the shape and size of the conductor. Further, the capacity of the conductor does not depend upon the nature of material of the conductor. However, the capacity depends on the nature of medium (e.g. its relative permittivity, $\in_r$) in which the conductor is located.*

FIGURE 2.19



If $V = 1$, $C = Q$ ... from eqn. (23). Therefore,

Electrical capacity of a conductor is numerically equal to the charge required to raise its potential by unity. Capacitance is a positive quantity only.

In SI, the unit of capacitance is ***farad.***

$$1 \text{ farad } (F) = \frac{1 \text{ coulomb (C)}}{1 \text{ volt (V)}}, \text{ i.e.,}$$

a conductor is said to have a capacity of one farad, when a charge of one coulomb raises its potential by one volt.

Smaller units of capacitance are 1 micro farad, *i.e.*,

$1\ \mu F = 10^{-6}$ farad.

1 nanofarad, *i.e.*, $1\ nF = 10^{-9}$ farad,

1 micro microfarad, *i.e.*, $1\ \mu\mu F = 10^{-12}$ farad.

$\mu\mu F$ is also called picofarad (pF).

Dimensions of capacitance

From $$C = \frac{Q}{V} = \frac{Q}{W/Q} = \frac{Q^2}{W} = \frac{(AT)^2}{ML^2T^{-2}} = [M^{-1}L^{-2}T^4A^2]$$

Sample Problem **A potential difference of 250 volt is applied across the plates of a capacitor of 10 pF. Calculate the charge on the plates of the capacitor.**

Sol. Here, $V = 250$ V, $C = 10$ p $F = 10 \times 10^{-12}$ F $= 10^{-11}$ F

$\therefore$ $Q = CV = 10^{-11} \times 250 = \mathbf{2{\cdot}5 \times 10^{-9}}$ **C**

2.15. CAPACITY OF AN ISOLATED SPHERICAL CONDUCTOR

Suppose O is the centre and r is radius of an isolated spherical conductor, Fig. 2.20. Let a charge $+q$ be given to the sphere. *This charge spreads uniformly over the outer surface of the sphere whether the sphere is hollow or solid.* Therefore, potential at every point on the surface of the sphere is same. As the sphere behaves as if the entire charge were concentrated at the centre of the sphere, therefore, potential at any point on the surface of the sphere in free space,

$$V = \frac{q}{4\pi\epsilon_0 r} \qquad ...(24)$$

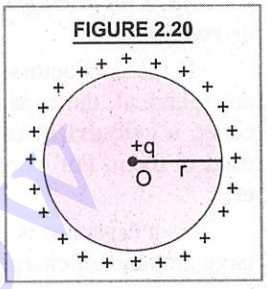


FIGURE 2.20

As $C = \frac{q}{V}$ $\therefore$ $C = \frac{q\,4\pi\epsilon_0 r}{q}$ $\boxed{C = 4\pi\epsilon_0 r}$ ...(25)

When r is in metre, C is in farad.

Clearly, $C \propto r$, *i.e.*, ***larger the radius of the sphere, greater is its capacitance.***

Note that a capacitor is normally made of two conductors. We have calculated here the capacity of a single spherical conductor, assuming the other conductor is at *infinity*.

Capacity of Earth

We can calculate electrical capacity of earth, taking it to be a conducting sphere of radius,

$r = 6400 \text{ km} = 6{\cdot}4 \times 10^6 \text{ m}$

As $C = 4\pi\epsilon_0 r$ $\therefore$ $C = \frac{1}{9\times10^9}(6{\cdot}4\times10^6) = 0{\cdot}711 \times 10^{-3}$ farad $= 0{\cdot}711 \times 10^{-3} \times 10^6\ \mu\text{F} = \mathbf{711\ \mu F}$

Thus the capacity of earth, which is the biggest sphere we come across is only 711 μ F. Therefore, in routine, capacitance will be in nanofarad (nF) and picofarad (pF). ***Farad is too big a unit of capacitance.***

Sample Problem **Diameter of a spherical conductor is 1 metre. What is its capacity ?**

(Uttarakhand Board 2012)

Sol. Here, $r = 1/2\ m = 0{\cdot}5\ m$, C = ?

$$C = 4\pi\epsilon_0 r = \frac{0{\cdot}5}{9\times10^9} \text{ farad} = 55{\cdot}5 \times 10^{-12} \text{ farad} = \mathbf{55{\cdot}5\ pF}$$

2.16. CAPACITOR AND ITS PRINCIPLE

***A capacitor** is an arrangement for storing large amounts of electric charge and hence electric energy in a small space.*

Usually, a capacitor consists of a system of two conductors separated by an insulating medium. Often the two conductors are charged by connecting them to the two terminals of a battery.

To understand the principle of a capacitor, let us consider an insulated metal plate A. Let some positive charge be given to this plate, till its potential becomes *maximum*. No further charge can be given to this plate as it would leak out.

Now, consider another insulated metal plate B held near the plate A. By induction, a negative charge is produced on the nearer face of B and an equal positive charge develops on the farther face of B, Fig. 2.21(a). The induced negative charge tends to decrease the potential of A and the induced positive charge tends to increase the potential of A. As the induced negative charge is closer to A, it is more effective. The overall potential of A reduces and hence some more charge can be given to A to raise its potential to the same maximum value. Thus, capacity of the conductor A has increased, by bringing another uncharged conductor B in its vicinity.

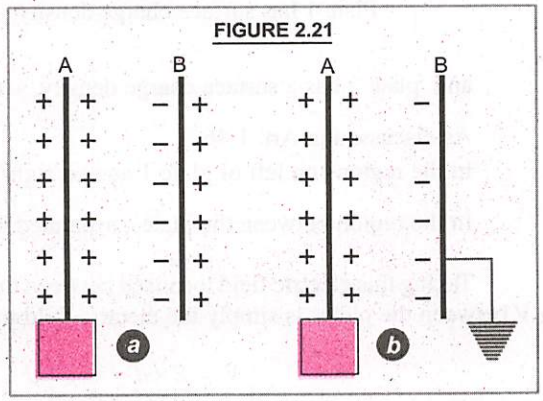


FIGURE 2.21

Now, connect the plate B to earth. The induced positive charge on B being free, flows to earth, Fig. 2.21(b). The induced negative charge on B, however, stays on as it is bound to positive charge on A. Due to the induced negative charge on B, potential of A is greatly reduced. Thus, a large amount of charge can be given to A to raise it to the maximum potential.

Hence, we conclude that the ***capacitance of an insulated conductor is increased considerably by bringing near it an uncharged conductor connected to earth.*** This is the principle of the capacitor.

Such an arrangement of two conductors separated by a dielectric medium is said to form a capacitor or condenser.

If the conductors are plane, the capacitor is called a **parallel plate capacitor.** When the conductors are spherical, the capacitor is called a **spherical capacitor** and when the conductors are cylindrical, it is called **a cylindrical capacitor.** Sometimes, capacitors are also named on the basis of dielectric medium used in them. For example, electrolytic capacitors, paper condenser, mica condenser, ceremic condensers etc.

As a capacitor is used to store large amounts of charge and hence electric energy, it can safely absorb large amounts of charge. Similarly, large amounts of charge can be released by the capacitors.

A capacitor with fixed capacitance is symbolically shown as (⊣⊢), while a capacitor with variable capacitance is represented as (⫤). These symbols are used for capacitors of all geometries.

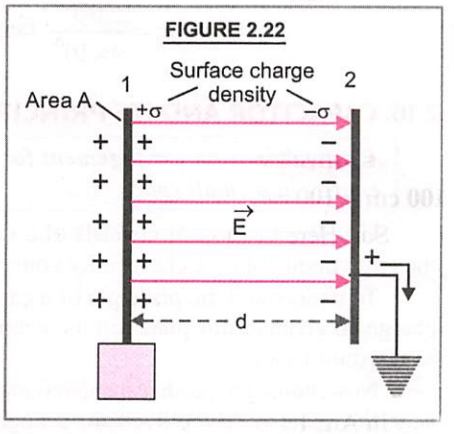
Retain in Memory

Capacitors or condensers are used in many electrical appliances, like electric fans, electric motors, in oscillator circuits, in radio tuning, in ignition system of engines in vehicles, in filter circuits etc. In the microelectronic age we live, microscopic capacitors form the memory banks of computers.

2.17. PARALLEL PLATE CAPACITOR

This is the capacitor which is used most commonly. It consists of two thin conducting plates 1 and 2, each of area A held parallel to each other, suitable distance d apart. The plates are separated by an insulating medium like air, paper, mica, glass etc. One of the plates 1 is insulated and the other plate 2 is earth connected, Fig. 2.22.

FIGURE 2.22



When a charge $+Q$ is given to the insulated plate 1, then a charge $-Q$ is induced on the nearer face of plate 2 and charge $+Q$ is induced on the farther face of plate 2. As plate 2 is earthed, the charge $+Q$ being free, flows to earth.

Plate 1 has surface charge density, $\sigma = \dfrac{Q}{A}$

and plate 2 has a surface charge density, $-\sigma = \dfrac{-Q}{A}$

As discussed in Art. 1.46,

In the regions on left of plate 1 and on right of plate 2, the electric field = 0

In the region between the plates, separated by air/vacuum, electric intensity is, $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{1}{\epsilon_0}\dfrac{Q}{A}$

Taking this electric field localised between the two plates as uniform throughout, the potential difference V between the plates is simply the electric field times the distance between the plates, *i.e.*,

$$V = E \times d = \frac{1}{\epsilon_0}\frac{Q}{A}d$$

The capacity C of parallel plate capacitor is given by

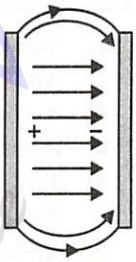
$$C = \frac{Q}{V} = \frac{Q}{\frac{Q\,d}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}, \quad i.e., \quad \boxed{C = \frac{\epsilon_0 A}{d}} \qquad ...(26)$$

Retain in Memory

1. From eqn. (4), we find that capacitance of a capacitor is determined geometrically from the dimensions of the capacitor.
2. The capacitance does not depend upon the charge on the capacitor or potential difference between the plates of the capacitor.
3. Infact, net charge on the capacitor plates is $q + (-q)$ = Zero. However, electric field between the plates of capacitor is not zero. The electric field between the plates of capacitor is as shown in Fig.2.23.

   We observe that $\vec{E}$ is uniform in the central portion of the capacitor and it is non uniform at the edges. This is called ***edge effect*** **or** ***fringing.*** This non uniformity of electric field at the boundaries of the plates becomes negligible, when distance between the plates is very small, compared to the length of the plates.

FIGURE 2.23



4. When plates of capacitor are separated by a dielectric medium of relative permittivity $\epsilon_r = K$, its capacity

$$C_m = \frac{\epsilon A}{d} = \frac{\epsilon_r \epsilon_0 A}{d} = \epsilon_r C_0 = K C_0, \quad i.e., \quad C_m = K C_0 \qquad ...(27)$$

   i.e., capacity becomes K times the capacity with air/vacuum as dielectric.
5. Charged capacitors have electrostatic forces between the walls. Force of attraction between the plates of capacitor

$$F = EQ = \left(\frac{\sigma}{\epsilon_0}\right) Q = \frac{(Q/A)\,Q}{\epsilon_0} = \frac{Q^2}{\epsilon_0 A}$$

Sample Problem **Calculate the capacitance of a parallel plate condenser of two plates 100 cm × 100 cm each separated by 2 mm thick glass sheet of $K = 4$.**

Sol. Here, $A = 100\text{ cm} \times 100\text{ cm} = 10^4\text{ cm}^2 = 1\text{ m}^2$, $d = 2\text{ mm} = 2 \times 10^{-3}\text{ m}$, $K = 4$, $C = ?$

$$C = \frac{K\epsilon_0 A}{d} = \frac{4 \times 8{\cdot}85 \times 10^{-12} \times 1}{2 \times 10^{-3}} = \mathbf{1{\cdot}77 \times 10^{-8}\text{ farad}}$$

2.18. RELATIVE PERMITTIVITY OR DIELECTRIC CONSTANT

In Art. 1.15, we have defined relative permittively or dielectric constant of a medium :

Dielectric constant ***of a medium is the ratio of the electrostatic force of interaction between two given point charges held certain distance apart in vacuum/air to the force of interaction between the same two charges held the same distance apart in the material medium.***

Faraday observed that when an insulating material is **fully occupying the space between the charged plates of a capacitor,** its capacitance increases. The factor by which capacitance is multiplied depends on the nature of the dielectric introduced, and is called relative electrical permittivity (ϵ_r) or dielectric constant (K) of the material. We may write

$$\epsilon_r \text{ or } K = \frac{\text{capacitance of a capacitor with dielectric inbetween the plates}}{\text{capacitance of the same capacitor with vacuum/air inbetween the plates}}$$

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$$\epsilon_r \text{ or } K = \frac{C_m}{C_0} \qquad \text{...(28)}$$

Thus, $C_m = K C_0 = \epsilon_r \times \frac{\epsilon_0 A}{d} = \frac{\epsilon A}{d}$

The values of ϵ_r or K, for common dielectric materials are given in Table 2.1.

TABLE 2.1. Dielectric constant for some common dielectrics

MATERIAL	ϵ_r or K	MATERIAL	ϵ_r or K
1. Vacuum	1·0000	**9.** Paraffined paper	2·1-2·3
2. Air (1 atm)	1·0006	**10.** Porcelain	6-7
3. Air (100 atm)	1·0548	**11.** Germanium	16
4. Hydrogen	1·00026	**12.** Glass	3-4
5. Turpentine	2·23	**13.** Alcohol	25
6. Teflon	2·1	**14.** Transformer oil	2·24
7. Amber	2·8	**15.** Water	81
8. Mica	6-7	**16.** All metals	∞

Sample Problem **The capacity of a capacitor becomes 10 μ *F* when air between the plates is replaced by a dielectric slab of *K* = 2. What is the capacity of the condenser with air inbetween the plates ?**

Sol. Here , $C_m = 10\ \mu\text{F}$, $C_0 = ?$, $K = 2$

As $K = \frac{C_m}{C_0}$ $\quad\therefore\quad C_0 = \frac{C_m}{K} = \frac{10}{2} = 5\,\mu\text{F}$

2.19. SPHERICAL CAPACITOR

Note. A cylindrical capacitor consists of two co-axial cylinders of same length L and radii r_a and r_b. The outer cylinder of radius r_b is earthed and inner cylinder of radius r_a is given some charge.

The capacity of cylindrical air capacitor is given by

$$C = \frac{2\pi\epsilon_0 L}{\log_e (r_b/r_a)} \qquad \text{...(29)}$$

*

2.20. GROUPING OF CAPACITORS/CONDENSERS

In many electrical circuits, capacitors are to be grouped suitably to obtain the desired capacitance. Two most common modes of grouping of condensers are : *Series* grouping and *Parallel* grouping

(*a*) Capacitors in Series

The capacitors are said to be connected in series between two points, when we can proceed from one point to the other only through one path.

In Fig. 2.24, three condensers of **capacities** C_1, C_2, C_3 are connected in series. V is the potential difference applied across the combination.

*Obviously, flow of negative charges (electrons) is in the opposite direction *i.e.* from negative to positive electrode.

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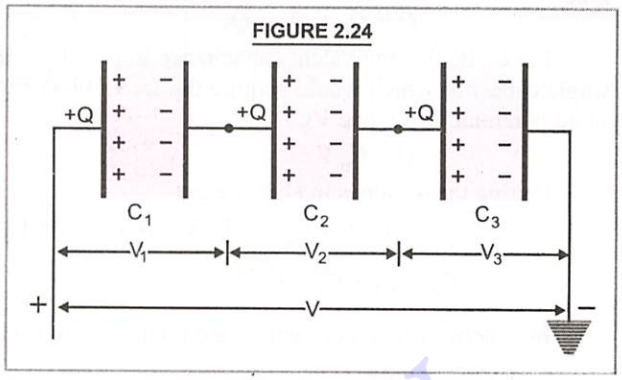
Let + Q units of charge be given to the left plate of C_1. The elctric field of this positive charge (+ Q) pulls negative charge upto the right plates of C_1. Therefore, right plate of C_1 acquires – Q charge. As this – Q charge comes from the left plate of C_2 through the wire connecting them, therefore, left plate of C_2 acquires + Q charge. The right plate of C_2 acquires – Q charge as explained for capacitor C_1. Similarly, capacitor C_3 is charged with + Q charge on left plate and – Q charge on the right plate. Thus, each capacitor receives the same charge of magnitude Q. As their capacities are different, therefore, potential differences across the three capacitors are different :

FIGURE 2.24



$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3} \qquad \text{...(30)}$$

If C_s is the total capacitance of the combination, (*i.e.*, capacitance of a single capacitor that would acquire charge Q when potential difference across its terminals is V), then

$$V = \frac{Q}{C_s} \qquad \text{...(31)}$$

As total potential drop V across the combination is the sum of potential drops V_1, V_2, V_3 across C_1, C_2, C_3 respectively, therefore $V = V_1 + V_2 + V_3$

$$\therefore \quad \frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right), \text{ or } \boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \qquad \text{...(32)}$$

For n capacitors connected in series, total capacitance would be $\boxed{\frac{1}{C_s} = \sum_{i=1}^{i=n} \frac{1}{C_i}}$

i.e., the reciprocal of equivalent capacitance of any number of capacitors joined in series is equal to sum of the reciprocals of individual capacitances.

Obviously, C_s is less than the least of $C_1, C_2, C_3 \ldots\ldots\ldots C_n$.

(*b*) Capacitors in parallel

The capacitors are said to be connected in parallel between any two points, if we can proceed from one point to the other along different paths.

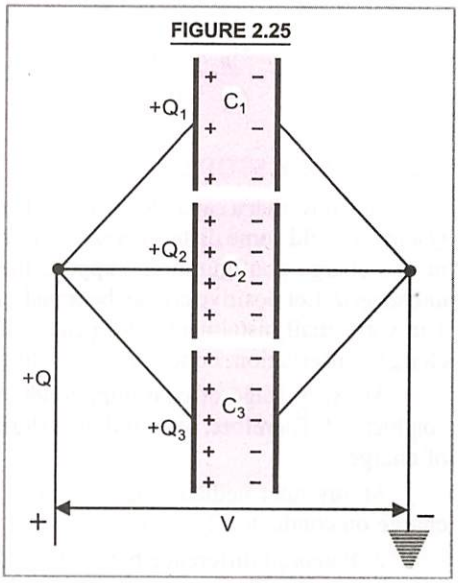
In Fig. 2.25, three capacitors of capacitances C_1, C_2, C_3 are connected in parallel.

Let V be the potential difference applied across the combination.

As potential difference across the three condensers is the same (*i.e.* V), therefore, each capacitor will store charge according to its capacity, say Q_1, Q_2 and Q_3, such that $Q_1 = C_1 V$, $Q_2 = C_2 V$, $Q_3 = C_3 V$

If Q is the total charge on the parallel net work, then as total charge is the sum of charges on all capacitors,

FIGURE 2.25



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$\therefore \quad Q = Q_1 + Q_2 + Q_3$ ...(33)

Let C_p be the equivalent capacitance in parallel (*i.e.*, capacity of a single capacitor which would acquire the same total charge Q under the same potential difference V).

$\therefore \quad Q = C_p V$

Putting these values in (11), we get

$$C_p V = C_1 V + C_2 V + C_3 V = (C_1 + C_2 + C_3) V$$

$\therefore \quad \boxed{C_p = C_1 + C_2 + C_3}$ ...(34)

In general, when n capacitors are connected in parallel,

then $\boxed{C_p = \sum_{i=1}^{i=n} C_i}$ ...(35)

i.e., equivalent capacitance of any number of capacitors joined in parallel is equal to sum of the individual capacitances. Obviously, capacitance in parallel combination increases. Here, C_p > the largest of $C_1, C_2, C_3, \ldots\ldots, C_n$.

Sample Problem **Three capacitors of 1 μF, 2 μF and 3 μF are joined in series. How many times will the capacity become when they are joined in parallel ?**

Sol. Here, $C_1 = 1$ μF, $C_2 = 2$ μF, $C_3 = 3$ μF

The combined capacity (C_s) in series is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}; \quad C_s = \frac{6}{11}\,\mu\text{F}$$

Total capacity in parallel combination,

$$C_p = C_1 + C_2 + C_3 = 1 + 2 + 3 = 6\ \mu\text{F}$$

$\therefore \quad \frac{C_p}{C_s} = \frac{6}{6/11} = 11$ Hence, $C_p = 11\ C_s$

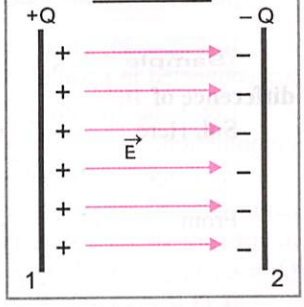
DO YOU KNOW ?

- In a series combination of capacitors, charge can move along only one route. That is why charge on each capacitor is same.
- In a parallel combination, two plates of every capacitor are connected to the same battery. Therefore, potential difference across each capacitor is same. Charge on each would depend upon capacity of the condenser.
- Further, note that battery produces charges on only the two plates to which it is connected. Charges that are produced on the other plates are due merely to the shifting of charges already there.

2.21. ENERGY STORED IN A CAPACITOR

We know that a capacitor is a system of two conductors carrying charges Q and $-Q$ held some distance apart, Fig. 2.26. To calculate the energy stored in this charge configuration, suppose the conductors 1 and 2 are *initially uncharged.* Let positive charge be transferred from conductor 2 to conductor 1 in very small instalments of dq each till conductor 1 gets charge $+Q$. By charge conservation, conductor 2 would acquire charge $-Q$.

FIGURE 2.26



At every stage of charging, conductor 1 is at a higher potential than conductor 2. Therefore, work is done externally in transferring each instalment of charge.

At any intermediate stage, suppose charge on conductor 1 is $+q$ and charge on conductor 2 is $-q$.

$\therefore$ Potential difference between conductors 1 and 2 is q/C, where C is the capacity of the capacitor.

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Small amount of work done in giving an additional charge dq to the conductor 1 is $dW = \frac{q}{C} \times dq$

Total work done in giving a charge Q to conductor 1, $W = \int_{q=0}^{q=Q} \frac{q}{C} dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_{q=0}^{q=Q} = \frac{1}{C}\frac{Q^2}{2}$

As electrostatic force is conservative, this work is stored in the form of potential energy (U) of the condenser.

$$U = W = \frac{1}{2}\frac{Q^2}{C}$$

Put $Q = CV \quad \therefore \quad U = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$ ...(36)

Put $CV = Q \quad \therefore \quad U = \frac{1}{2}QV$

Hence, $$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$ ...(37)

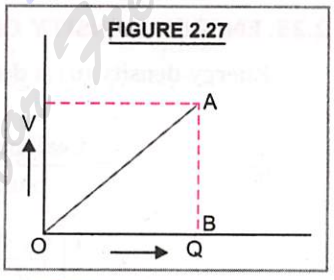
When Q is in coulomb, V is in volt, C is in farad, Energy U is in joule.

If we plot a graph between charge given and potential of the capacitor, we get a straight line. OA as shown in Fig.2.27.

Energy stored in the capacitor = area under $V - Q$ graph

$$U = \frac{1}{2} \times \text{base}\,(OB) \times \text{height}\,(AB) = \frac{1}{2} \times Q \times V = \frac{1}{2} Q \times \frac{Q}{C} = \frac{Q^2}{2C}.$$

FIGURE 2.27



Retain in Memory

1. The potential energy stored in the capacitor is independent of the manner in which charge configuration of the capacitor is built up.
2. The potential energy of charged capacitor can be viewed as stored in the electric field between the plates.
3. The potential energy of capacitor lies in the dielectric medium between the plates.
4. The potential energy of capacitor is obtained at the cost of chemical energy stored in the battery — used for charging the capacitor.
5. Energy spent by the battery in charging the capacitor = QV, whereas energy stored in the capacitor $= \frac{1}{2}QV$

   The rest of the energy is dissipated/lost in charging the capacitor.

Sample Problem **What capacitance is required to store an energy of 100 kWh at a potential difference of 10^4 V ?**

Sol. Here, $C = ?$, $U = 100$ kWh $= 100 \times 10^3 \times 3600$ J $= 3{\cdot}6 \times 10^8$ J

$V = 10^4$ volt

From $U = \frac{1}{2}CV^2$

$$C = \frac{2U}{V^2} = \frac{2 \times 3{\cdot}6 \times 10^8}{(10^4)^2} = \mathbf{7{\cdot}2\ F}$$

2.22. TOTAL ENERGY STORED IN A COMBINATION OF CAPACITORS

In series combination of capacitors, Q is constant., *i.e.*, every capacitor carries the same charge Q.

$$\therefore \text{ Total energy, } U = \frac{1}{2}\frac{Q^2}{C_s} = \frac{Q^2}{2}\cdot\frac{1}{C_s} = \frac{Q^2}{2}\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots\right]$$

$$U = \frac{Q^2}{2C_1}+\frac{Q^2}{2C_2}+\frac{Q^2}{2C_3}+\ldots\ldots \qquad \ldots(38)$$

$$\boxed{U = U_1 + U_2 + U_3 + \ldots\ldots\ldots}$$

In parallel combination of capacitors, V is constant, i.e., potential difference across each capacitor is same.

$$\therefore \text{ Total energy, } U = \frac{1}{2}C_p V^2 = \frac{1}{2}(C_1+C_2+C_3+\ldots\ldots)V^2 = \frac{1}{2}C_1V^2+\frac{1}{2}C_2V^2+\frac{1}{3}C_3V^2+\ldots\ldots$$

$$\boxed{U = U_1 + U_2 + U_3 + \ldots\ldots\ldots} \qquad \ldots(39)$$

i.e., total energy stored in series combination or parallel combination of capacitors is equal to sum of the energies stored in individual capacitors.

2.23. ENERGY DENSITY OF A PARALLEL PLATE CAPACITOR

Energy density (u) is defined as the total energy stored per unit volume of the condenser.

i.e., $$u = \frac{\text{total energy }(U)}{\text{volume }(v)} = \frac{\frac{1}{2}CV^2}{Ad} \quad \text{using} \quad C = \frac{\epsilon_0 A}{d} \text{ and } V = Ed, \text{ we get}$$

$$u = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)\left(\frac{E^2d^2}{Ad}\right) = \frac{1}{2}\epsilon_0 E^2, \quad i.e., \quad \boxed{u = \frac{1}{2}\epsilon_0 E^2} \qquad \ldots(40)$$

Here, E is the strength of electric field in the space between the plates of the capacitor.

SI unit of energy density is J m^{-3}.

Dimensional formula of energy density is $u = \frac{\text{energy}}{\text{volume}} = \frac{ML^2T^{-2}}{L^3} = [ML^{-1}T^{-2}]$

Curiosity Question

Q. What is a defibrillator ? Explain briefly.

Ans. A defibrillator is a device used to save the life of a person suffering a heart stroke.

Due to rapid, unco-ordinated twitching of the heart muscles, the heart undergoes ventricular fibrillation. To save the person's life, regular beating of the heart is to be restored by delivering a jolt to the heart using a defibrillator.

Suppose, a 70 μ F capacitor of a defibrillator is charged to a potential of 5000 V. The energy stored $= \frac{1}{2}CV^2 = \frac{1}{2}(70\times10^{-6})(5000)^2 \text{ J} = 875 \text{ J}$. Typically, about 200 J of energy is allowed to pass through person's body in a pulse lasting 2 milli second. Therefore, power delivered to the body $= \frac{200}{2\times10^{-3}} = 10^5 \text{ W} = 100 \text{ kW}$, which is much larger than the power delivered by the battery.

Thus energy stored in the capacitor discharged at a much higher power provides the much needed jolt to save the person's life.

2.24. COMMON POTENTIAL

When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher potential to the other at lower potential. This flow of charges continues till their potentials become equal. The equal potential of the two capacitors is called common potential. Obviously, no charge is lost in the process of sharing of charges by the two capacitors.

Suppose C_1, C_2 are capacities of two condensers charged to potentials V_1 and V_2 respectively. Total charge before sharing $= C_1 V_1 + C_2 V_2$

If V is the common potential on sharing charges, then total charge after sharing

$$= C_1 V + C_2 V = (C_1 + C_2) V$$

As *no charge is lost* in the process of sharing, therefore $(C_1 + C_2) V = C_1 V_1 + C_2 V_2$

or $$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \qquad \text{...(41)}$$

i.e., common potential $= \dfrac{\text{total charge}}{\text{total capacity}}$

From (19), $C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$ or $C_1 V_1 - C_1 V = C_2 V - C_2 V_2$

i.e., ***charge lost by one capacitor = charge gained by the other capacitor***

This is NOT true for potentials, i.e., potential lost by one is not equal to potential gained by the other, as their capacities are different.

IMPORTANT NOTE

Note that in the derivation of this relation (41), we have assumed that the two capacitors are connected in parallel with their positive plates at one point and negative plates at another point. However, when the two capacitors are connected in series, *i.e.*, with positive plate of one capacitor to the negative plate of other capacitor, eqn. (41) will be modified to $V = \dfrac{C_1 V_1 - C_2 V_2}{C_1 + C_2}$

Thus total charge in this case, would be algebraic sum of the charges on the two capacitors.

2.25. LOSS OF ENERGY ON SHARING CHARGES

When charges are shared between two bodies, no charge is lost. However, some energy is dissipated in the form of heat etc., *i.e.*, some energy is lost on sharing charges. To calculate the loss of energy, suppose two capacitors of capacitances C_1, C_2 charged respectively to potentials V_1, V_2 are connected together and their charges are shared. The charge flows from one capacitor at higher potential to the other at lower potential till their potentials become equal. The common potential reached is given by

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \qquad \text{...(42)}$$

Total energy before sharing charges, $U_1 = \dfrac{1}{2} C_1 V_1^2 + \dfrac{1}{2} C_2 V_2^2$

Total energy after sharing charges, $U_2 = \dfrac{1}{2}(C_1 + C_2) V^2$

Using (42), $$U_2 = \frac{1}{2}(C_1 + C_2) \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)^2} = \frac{(C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

Let us calculate : $U_1 - U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$

$$= \frac{C_1V_1^2(C_1 + C_2) + C_2V_2^2(C_1 + C_2) - (C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{2(C_1 + C_2)}$$

$$U_1 - U_2 = \frac{C_1C_2(V_1^2 + V_2^2 - 2V_1V_2)}{2(C_1 + C_2)}$$

$$\boxed{U_1 - U_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}}, \text{ which is positive} \quad \text{...(43)}$$

i.e., $U_1 - U_2 > 0$ or $U_1 > U_2$ or $U_2 < U_1$

This proves that total energy after contact is somewhat less than total energy before contact. Hence there occurs a loss of energy = $(U_1 - U_2)$ on sharing charges. This loss of energy appears in the form of *sparking and heat produced* in the connecting wire.

Retain in Memory

When charges are shared between any two bodies, their potentials become equal. The charges acquired are in the ratio of their capacities. No charge is really lost, but some loss of energy does occur.

Sample Problem 1 **A 600 *pF* capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 *pF* capacitor. How much electrostatic energy is lost in the process ?**

Sol. Here, $C_1 = C_2 = 600\ pF = 600 \times 10^{-12}\ F = 6 \times 10^{-10}\ F$, $V_1 = 200\ V$, $V_2 = 0$

$$\text{Energy lost} = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)} = \frac{6\times10^{-10}\times6\times10^{-10}(200 - 0)^2}{2\times12\times10^{-10}} = \mathbf{6 \times 10^{-6}\ J}$$

Sample Problem 2 **A 400 pF capacitor, charged by a 100 volt d.c supply is disconnected from the supply and connected to another uncharged 400 pF capacitor. Calculate the loss of energy.**

Sol. Here, $C_1 = 400\ \text{pF} = 400 \times 10^{-12}\ \text{F} = 4 \times 10^{-10}\ \text{F}$, $V = 100\ \text{volt}$

Energy stored, $U = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{1}{2}\times4\times10^{-10}(100)^2 = 2 \times 10^{-6}\ \text{J}$

When this charged capacitor is connected to another uncharged capacitor of same capacity, half the charge ($q/2$) will flow to the unchanrged capacitor.

$\therefore$ Final energy, $U' = \frac{1}{2}\left(\frac{q}{2}\right)V = \frac{1}{2}\times\frac{1}{2}qV = \frac{1}{2}\times2\times10^{-6}\ \text{J} = 10^{-6}\ \text{J}$

Loss of energy = $U - U' = 2 \times 10^{-6} - 10^{-6} = \mathbf{10^{-6}\ J}$

2.26. NON-POLAR AND POLAR DIELECTRICS

As discussed in Art. 1.4, *dielectrics are insulating materials which transmit electric effects without actually conducting electricity.*

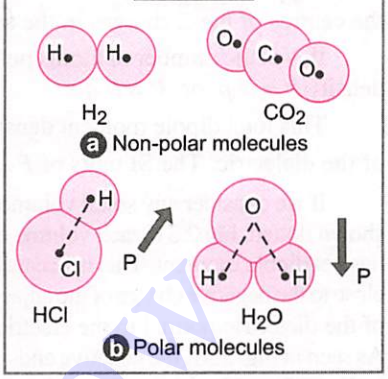
Further, we know that in every atom, there is a positively charged nucleus and a negatively charged electron cloud surrounding it. The two oppositely charged regions have their own *centres of charge*. The centre of positive charge is the centre of mass of positively charged protons in the nucleus. The centre of negative charge is the centre of mass of negatively charged electrons in the atom/molecule.

Dielectrics are of two types : Non-polar dielectrics and Polar Dielectrics.

Non-polar dielectrics like hydrogen, nitrogen, oxygen, CO_2, benzene, methane etc. are made of non-polar molecules. In such molecules, ***the centre of mass of positive charge coincides with the centre of mass of negative charge*** in the molecule. Each molecule has zero dipole moment in its normal state. Such molecules are *symmetric*, in shape as shown in Fig. 2.28(*a*).

Polar dielectrics like water, alcohol, NH_3, HCl etc. are made of polar molecules. In such molecules, (under normal conditions with no external field) ***the centres of mass of positive and negative charges do not coincide because of the asymmetric shape of the molecules*** as shown in Fig. 2.28(*b*). Therefore, each molecule has some spontaneous (or intrinsic) or permanent electric dipole moment.

FIGURE 2.28



2.27. DIELECTRIC POLARIZATION

When a non-polar dielectric is held in an external electric field $\vec{E}_0$, the centre of positive charge (protons) in each molecule is pulled in the direction of $\vec{E}_0$ (towards negative plate) and the centre of negative charge (electrons) is pulled in a direction opposite to $\vec{E}_0$ (towards positive plate). Therefore, the two centres of positive and negative charges in the molecule are separated. The molecule gets distorted, Fig. 2.29, and is said to be polarised as a tiny dipole moment is imparted to each molecule. Infact, the force due to electric field pulling the two centres apart and the force of mutual attraction between the centres of positive and negative charges reach a balance or equilibrium and the molecule gets polarised.

FIGURE 2.29

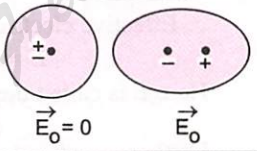
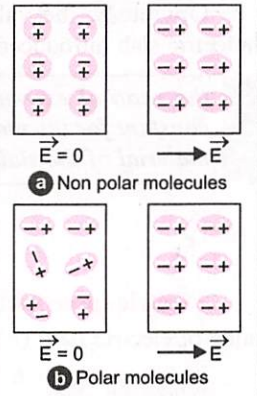


FIGURE 2.30



A dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason. When no external field is applied, the different permanent dipoles of such a dielectric are oriented randomly due to thermal agitation, so the total dipole moment is zero. When an external electric field is applied, the individual dipole moments tend to align with the field. When summed over all the molecules, there appears some net dipole moment in the direction of the external field, *i.e.*, the dielectric is polarised. The extent of polarisation depends on the relative strength of two mutually opposing factors. The applied external electric field tending to align the dipoles with field and thermal energy tending to disrupt the alignment. Fig. 2.30 shows how a dielectric develops a net dipole moment in an external electric field.

Thus each molecule becomes a tiny electric dipole, with a dipole moment parallel to the external field, and proportional to it.

Induced dipole moment p acquired by the molecule may be written as

$$p = \alpha \in_0 E_0 \qquad \text{...(44)}$$

where α is a constant of proportionality and is called ***atomic/molecular polarizability.***

Here, $\in_0$ has been used to give the dimensions of volume to α.

From (44), we can write the dimensions of α :

$$\alpha = \frac{\text{dimensions of } p}{(\text{dimensions of } \in_0)(\text{dimensions of } E_0)} \qquad \alpha = \frac{C\,m}{(C^2\,N^{-1}\,m^{-2})(NC^{-1})} = m^3$$

Thus atomic polarizability has the dimensions of volume. For most of the atoms, α is of the order of 10^{-29} to 10^{-30} m^3 ; which is also the order of atomic volume.

Let us now consider a non-polar dielectric slab *ABCD* placed in an electric field E_0 maintained between the two plates.

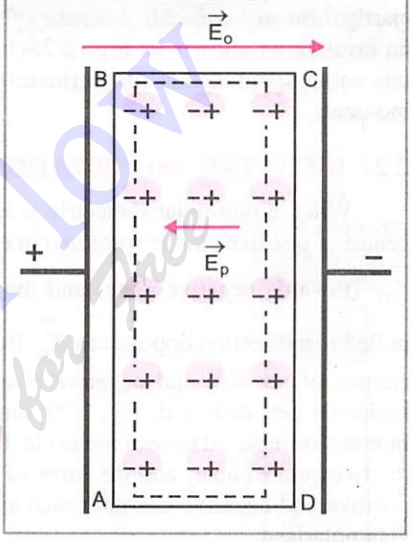
Suppose all its atoms are uniformly polarized in the direction of E_0 and x is the displacement between the centres of the ± charges in the atom. Dipole moment of each atom $p = q\,x$.

If N is the number of atoms per unit volume, then dipole moment per unit volume = total dipole moment density $P = N\,p$ or $P = N\,q\,x$

This total dipole moment density P is called ***electric polarization.*** It measures the extent of polarisation of the dielectric. The SI units of P are $C\text{-}m\ (m^{-3}) = C\text{-}m^{-2}$, which represent σ_i.

If we consider any small volume element *in the interior* of the slab, shown dotted, Fig 2.31, each volume element has no net charge though it has net dipole moment. This is because positive charge of one dipole sits close to the negative charge of the adjacent dipole. However, at the surfaces of the dielectric normal to the electric field, there is net charge density. As seen in Fig. 2.31, the negative ends of the dipoles remain unneutralized at the surface *AB* and positive ends of the dipoles remain unneutralized at the surface *CD*. These are the ***induced charges*** or ***polarization charges*** due to external field. They set up an electric field $\vec{E}_p$ opposite to $\vec{E}_0$.

FIGURE 2.31



∴ Effective electric field in a polarized dielectric is

$$E = E_0 - E_p \qquad \text{...(45)}$$

Here E is called the **reduced value of the electric field.**

Further, $$E_p = \frac{\sigma_i}{\epsilon} = \frac{1}{\epsilon}\left(\frac{Q_i}{A}\right) = \frac{P}{\epsilon} \qquad \text{...(46)}$$

where P is total dipole moment density.

Obviously, the value of E depends upon the nature of the dielectric slab introduced. It is found that

the ratio of external electric field applied (E_0) to the reduced value of electric field (E) is constant for the material. This constant is denoted by K and is called dielectric constant of the material of the slab

i.e., $$\frac{E_0}{E} = K \qquad \text{...(47)}$$

Electric susceptibility. It is found that the electric polarization P is directly proportional to the reduced value of electric field (E),

i.e., $P \propto E$ or $P = \epsilon\,\chi\,E = \chi\,\epsilon\,E$ ...(48)

where χ* is a constant, called ***electric susceptibility*** of the dielectric. Here, ϵ has been used to keep χ dimensionless. The electric susceptibility describes the electrical behaviour of a dielectric. It has different values for different dielectrics. For vacuum, $\chi = 0$

Note that eqn. (48) corresponds to eqn. (44), where ϵ_0 is replaced by ϵ ; E_0 is replaced by E and α is replaced by χ.

From (45), (46) and (48) $E = E_0 - \frac{P}{\epsilon} = E_0 - \frac{\epsilon\,\chi\,E}{\epsilon}$ or $E = E_0 - \chi\,E$

or $E_0 = E + \chi\,E = E\,(1 + \chi)$ or $E_0/E = 1 + \chi$

Using (47), $K = 1 + \chi$ ...(49)

This is the relation between dielectric constant and electric susceptibility of the material.

*χ is pronounced as 'chi'.

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2.28. DIELECTRIC STRENGTH

The maximum electric field that a dielectric medium can withstand without breaking down of its insulating property is called its dielectric strength.

This is of great practical importance.

From $V = \frac{Q}{C}$, we find that a capacitor can hold large amounts of charge Q at a relatively small potential V, only when its capacity is large. High potential difference (V) implies a strong electric field that would ionise the surrounding air, accelerate the charges so produced to oppositely charged plates, thereby neutralizing partly the charge on capacitor plates. It means the charge on capacitor plates would leak away due to reduction in insulating power of the insulating medium.

Hence, for a capacitor to store large amounts of charge, its capacitance should be large enough, so that the potential difference and hence the electric field do not exceed the break down limits.

The dielectric constant and dielectric strength of some common dielectrics are given in Table 2.2.

For reasons of safety, the maximum electric field actually applied is only 10 % of the dielectric strength of the material.

TABLE 2.2. Dielectric constant and Dielectric strength of some common dielectrics

S. No.	Dielectric	Dielectric constant	Dielectric strength (kV/mm)
1.	Natural rubber	2·7-5·0	18-24
2.	Mica	6-7	80
3.	Polystyrene	2·55	20-28
4.	Polythene	2·3	20-160
5.	Teflon	2·1	16-20
6.	PVC	5-6	30
7.	Quartz glass	3·78	—
8.	Pyrex glass	4·6	—
9.	Alumina	8-9·5	—
10.	Zircon	7·1-9·1	—
11.	air	1·0006	0·8
12.	Vacuum	1·00000	∞

2.29. CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A CONDUCTING SLAB

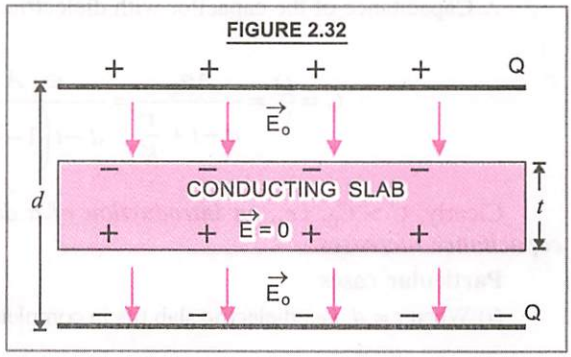
The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum/air inbetween is

$$C_0 = \frac{\epsilon_0 A}{d}$$

Suppose $\pm Q$ are the charges on the capacitor plates, Fig. 2.32.

When a conducting slab of area A and thickness $t < d$ is introduced between the plates, the free charges in the conductor flow. The charges $\pm Q$ appear on the two faces of the slab as shown reducing the electric field in the interior of slab to zero. The original uniform electric field E_0 now exists over a distance $(d - t)$, so that potential difference between the plates

FIGURE 2.32



$$V = E_0 (d - t) = \frac{\sigma}{\epsilon_0}(d - t) = \frac{Q}{A\epsilon_0}(d - t)$$

As $C = \frac{Q}{V}$

$$\therefore \quad C = \frac{QA\epsilon_0}{Q(d-t)} = \frac{\epsilon_0 A}{d\left(1 - \frac{t}{d}\right)} = \frac{\epsilon_0 A/d}{\left(1 - \frac{t}{d}\right)} = \frac{C_0}{\left(1 - \frac{t}{d}\right)}, \quad i.e., \quad \boxed{C = \frac{C_0}{\left(1 - \frac{t}{d}\right)}} \quad ...(50)$$

Clearly, $C > C_0$, *i.e.*, ***Capacitance increases on introducing a conducting slab inbetween the plates.***

When $t = d$, from (50), $C = \infty$, *i.e.*, capacity becomes infinite.

2.30. CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB

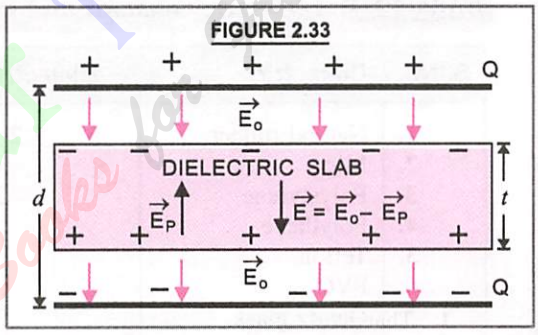
The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum/air inbetween is

$$\boxed{C_0 = \frac{\epsilon_0 A}{d}} \quad ...(51)$$

Suppose $\pm Q$ are the charges on the capacitor plates which produce a uniform electric field E_0 in the space between the plates, Fig. 2.33.

When a dielectric slab of thickness $t < d$ is introduced between the plates, the molecules in the slab get polarized in the direction of E_0. The electric polarization vector P in the direction of E_0 induces an electric field E_p opposite to E_0. Therefore, the effective field inside the dielectric is $E = E_0 - E_p$

FIGURE 2.33



Outside the dielectric, field remains E_0 only.

Therefore, potential difference between the two plates is $V = E_0 (d - t) + Et$

But $\frac{E_0}{E} = \epsilon_r$ or $K \quad \therefore \quad E = \frac{E_0}{K} \quad \therefore V = E_0 (d - t) + \frac{E_0}{K} t$

$$V = E_0 \left[d - t + \frac{t}{K} \right]$$

As $E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \quad \therefore \quad V = \frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K} \right]$

$\therefore$ Capacitance of the capacitor with dielectric inbetween is

$$C = \frac{Q}{V} = \frac{A\epsilon_0}{d - t + \frac{t}{K}} = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}, \quad i.e., \quad \boxed{C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}} \quad ...(52)$$

Clearly, $C > C_0$, *i.e.*, ***on introduction of a dielectric slab inbetween the plates of a capacitor, its capacitance increases.***

Particular cases

(*i*) When $t = d$, *i.e.*, dielectric slab fits in completely between the plates of capacitor, then from eqn. (52)

$$C = \frac{\epsilon_0 A}{d - d\left(1 - \frac{1}{K}\right)} = \frac{K\epsilon_0 A}{d} = KC_0$$

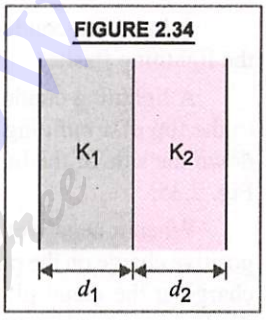
The capacity becomes K times.

(*ii*) *If* $t = d/2$, then from (31), $C = \dfrac{2KC_0}{K+1}$ (*iii*) *If* $t = \dfrac{2d}{3}$, then from (31), $C = \dfrac{3KC_0}{K+2}$

Sample Problem **Find the capacitance of a system of three parallel plates, each of area A metre^2 separated by distances d_1 and d_2 metre respectively. The space between them is filled with dielectrics of relative dielectric constants K_1 and K_2. The dielectric constant of free space is ϵ_0.**

Sol. A system of three parallel plates is forming two capacitors of capacities C_1 and C_2 connected in series as shown in Fig. 2.34.

FIGURE 2.34



$$C_1 = \frac{K_1 \epsilon_0 A}{d_1} \quad \text{and} \quad C_2 = \frac{K_2 \epsilon_0 A}{d_2}$$

The equivalent capacity of the system is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

$$C = \frac{\left(K_1 \frac{\epsilon_0 A}{d_1}\right)\left(K_2 \frac{\epsilon_0 A}{d_2}\right)}{\epsilon_0 A\left(\frac{K_1}{d_1} + \frac{K_2}{d_2}\right)} = \frac{K_1 K_2 A\left(\frac{\epsilon_0}{d_1}\right)\left(\frac{\epsilon_0}{d_2}\right) d_1 d_2}{\epsilon_0 A(K_1 d_2 + K_2 d_1)} = \frac{K_1 K_2 \epsilon_0}{K_1 d_2 + K_2 d_1}$$

Retain in Memory

1. That capacity of condenser increases both, when a conducting slab or an insulating slab is introduced between the plates of the condenser. In the former case, $E = 0$ inside the conductor and in the latter case, $E < E_0$, inside the insulator. Thus, potential difference. $V = E \times d$, decreases in both the cases and hence capacity increases.

   Note that maximum charge induced on the dielectric/conducting slab, $Q_i = -Q\left(1 - \dfrac{1}{K}\right)$.

2. When a dielectric slab is introduced inbetween the plates of a charged capacitor with battery connected across the plates :

   (*i*) Potential V remains constant, $(V = V_0)$ (*ii*) Capacity C increases, $(C = KC_0)$

   (*iii*) Charge Q increases, $(Q = CV)$ (*iv*) Electric field E decreases $(E = E_0/K)$

   (*v*) Energy U increases, $\left(U = \dfrac{1}{2}CV^2 = KU_0\right)$

3. However, when battery across the plates of charged condenser is off and dielectric slab is introduced between its plates :

   (*i*) Charge Q remains constant, $(Q = Q_0)$ (*ii*) Capacity C increases, $(C = KC_0)$

   (*iii*) Potential V decreases, $\left(V = \dfrac{Q}{C} = \dfrac{V_0}{K}\right)$ (*iv*) Electric field E decreases $\left(E = \dfrac{V}{d} = \dfrac{V_0}{Kd}\right)$

   (*v*) Energy U decreases. $\left(U = \dfrac{Q^2}{2C} = \dfrac{U_0}{K}\right)$

2.31. ACTION OF SHARP POINTS

When a spherical conductor of radius r carries a charge q, the surface density of charge is given by

$$\sigma = \frac{\text{charge}}{\text{area}} = \frac{q}{4\pi r^2} \quad ...(53)$$

For a pointed end, r is very very small, therefore σ is very large. Electric field intensity $E = \sigma/\epsilon_0$ becomes large. As a result, air around the sharp end becomes ionized and a **bluish glow,** called **corona** is seen. Though air does not transfer electric charges easily, ionized air is a good conductor and hence the charges leak from the surface into the surrounding air.

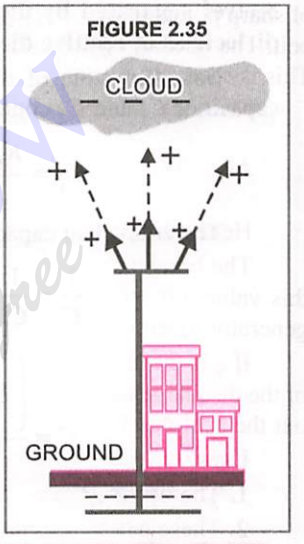
2.32. LIGHTNING CONDUCTOR

A lightning conductor is used to save big buildings from the damage by the lightning flashes.

A lightning conductor consists of a number of pointed conductors fixed to the top of a building and connected to a thick copper wire. This wire runs down the side of the building and ends on a metal plate buried in the ground, Fig. 2.35.

When a negatively charged cloud passes over the building, it induces a positive charge on the pointed ends of lightning conductor and an equal negative charge at the metal plate. Due to discharging action of sharp points (corona discharge), a positively charged electric wind is set up, which neutralises the negative charge on the cloud. The negative charge on the metal plate is immediately neutralized in the earth. If the discharge of the cloud occurs, it passes through the conducting copper strip and the building is saved from the damage.

FIGURE 2.35



2.33. VAN DE GRAAFF ELECTROSTATIC GENERATOR

A van de graaff generator is a device used for building up high potential differences of the order of a few million volts. Such high potential differences are used to accelerate charged particles like electrons, protons, ions etc. needed for various experiments of Nuclear Physics.

It was designed by Van de graaff in the year 1931.

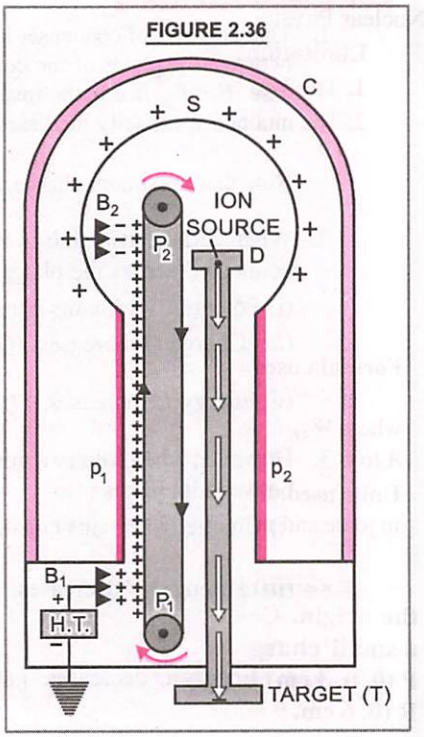
Principle. This generator is based on

(*i*) the action of sharp points , *i.e.*, the phenomenon of **corona discharge.**

(*ii*) the property that charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

Construction. The essential parts of Van de graaff generator are shown in Fig. 2.36. S is a large *spherical conducting shell* of radius equal to a few metres. This is supported at a suitable height (of several metres above the ground) over the *insulating pillars* p_1, p_2. A long narrow belt of insulating material like, silk, rubber or rayon is wound around two pulleys P_1 and P_2. P_1 is at the ground level and P_2 is at the centre of S. The belt is kept moving continuously over the pulleys with the help of a motor (not shown). B_1 and B_2 are two sharply pointed metal combs fixed as shown. B_1 is called the *spray comb* and B_2 is called the *collecting comb*.

FIGURE 2.36



The positive ions to be accelerated are produced in a discharge tube D. The ion source lies at the head of the tube inside the spherical shell. The other end of the tube carrying the target nucleus is earthed.

The generator is enclosed in a steel chamber C filled with nitrogen or methane at high pressure in order to minimise leakage of charge from spherical conductor.

Working. The spray comb B_1 is given a positive potential ($\approx 10^4$ volt) w.r.t. the earth by high tension source H.T. Due to discharging action of sharp points, a positively charged electric wind is set up, which sprays positive charge on the belt (*corona discharge*). As the belt moves, and reaches the comb., a negative charge is induced on the sharp ends of collecting comb B_2 and an equal positive charge is induced on the farther end of B_2. This positive charge shifts immediately to the outer surface of S. Due to discharging action of sharp points of B_2 , a negatively charged electric wind is set up. This neutralises the positive charge on the belt. The uncharged belt returns down, collects the positive charge from B_1, which in turn is collected by B_2. This is repeated. Thus, the postive charge on S goes on accumulating.

Now, the capacity of spherical shell $C = 4\pi\epsilon_0 R$, where R is radius of the shell.

As $V = \dfrac{Q}{C}$ $\quad\therefore\quad V = \dfrac{Q}{4\pi\epsilon_0 R}$

Hence the potential V of the spherical shell goes on increasing with increase in Q.

The breakdown field of air is about 3×10^6 V/m. The moment the potential of spherical shell exceeds this value, air around S is ionised and leakage of charge starts. The leakage is minimised by housing the generator assembly inside a steel chamber filled with nitrogen or methane at high pressures.

If q is the charge on the ion to be accelerated and V is the potential difference developed across the ends of the discharge tube, then energy acquired by the ions = qV. The ions hit the target with this energy and carry out the artificial transmutation, etc.

Uses

1. The beams of accelerated charged particles are often used to trigger nuclear reactions.

2. These beams are also used in the treatment of cancer.

3. The beams of accelerated charged particles are used in variety of experiments in Atomic Physics and Nuclear Physics.

Limitations

1. The Van de graaf generator cannot accelerate uncharged particles like neutrons.

2. The modern day accelerators which produce much higher energies have almost replaced such generators.

SOLVED EXAMPLES

TYPE I. POTENTIAL DIFFERENCE AND WORK DONE

Formula used. $V_B - V_A = \dfrac{W_{AB}}{q_0}$,

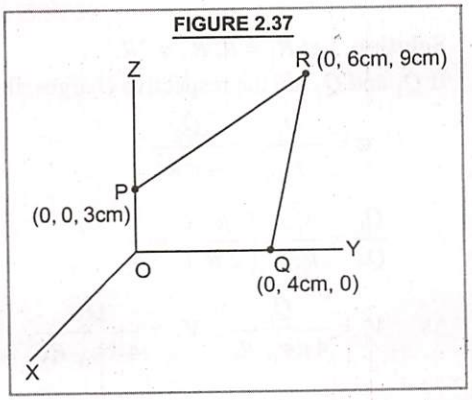
where W_{AB} is work done in carrying q_0 charge from A to B.

Units used. Charge q_0 is in coulomb, work W_{AB} is in joule and potential difference $(V_B - V_A)$ is in volt.

Example 1 **A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0) via a point R (0, 6 cm, 9 cm).**

Solution. In Fig. 2.37, a charge q = 8 mC = 8×10^{-3} C is located at the origin O. Charge to be carried is

FIGURE 2.37



$q_0 = -2 \times 10^{-9}$ C, from P to Q,
where $OP = r_P = 3$ cm $= 3 \times 10^{-2}$ m
and $OQ = r_Q = 4$ cm $= 4 \times 10^{-2}$ m

As electrostatic forces are conservative forces, work done is independent of the path. Therefore, there is no relevance of point R.

$$W_{PQ} = q_0 (V_Q - V_P) = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_Q} - \frac{1}{r_P}\right)$$

$$= 9\times10^9 \times 8\times10^{-3}(-2\times10^{-9})\left[\frac{1}{4\times10^{-2}} - \frac{1}{3\times10^{-2}}\right]$$

$$= -144\times10^{-3}\times10^2\left(-\frac{1}{12}\right) = \mathbf{1{\cdot}2\ joule}$$

TYPE II. ELECTRIC POTENTIAL

Formulae used.

1. Potential at a point due to single charge,

$$V = \frac{q}{4\pi\epsilon_0 r}$$

2. Potential at a point due to group of N charges,

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{i=N}\frac{q_i}{r_i}$$

3. Potential at a point due to electric dipole,

$$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

Units used. charges q, Q are in coulomb, dipole moment, $p = q\ (2\ a)$ in C-m, distance r in metre, potential V in volt.

Example 2 **Two metallic spheres of radii R and $2R$ are charged so that both of these have same surface charge density, σ. If they are connected to eachother with a conducting wire, in which direction will the charge flow and why ?** (CBSE 2016)

Solution. Let $R_1 = R$, $R_2 = 2R$.

If Q_1 and Q_2 are the respective charges, then as

$$\sigma = \frac{Q_1}{4\pi R_1^2} = \frac{Q_2}{4\pi R_2^2}$$

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{R_1^2}{R_2^2} = \left(\frac{R}{2R}\right)^2 = \frac{1}{4}$$

As $V_1 = \frac{Q_1}{4\pi\epsilon_0 R_1}$, $V_2 = \frac{Q_2}{4\pi\epsilon_0 R_2}$,

$$\therefore \quad \frac{V_1}{V_2} = \frac{Q_1}{Q_2}\cdot\frac{R_2}{R_1} = \frac{1}{4}\times\frac{2}{1} = \frac{1}{2}$$

Clearly, $V_2 > V_1$. Hence **charge flows from sphere of radius $2R$ to sphere of radius R.**

Example 3 **Electric field intensity at a point B due to a point charge Q kept at point A is 24 NC^{-1}, and electric potential at B due to the same charge is 12 JC^{-1}. Calculate the distance AB and magnitude of charge.** [CBSE 2003 (C)]

Solution. Here, $E = \frac{Q}{4\pi\epsilon_0 r^2} = 24$ NC^{-1}, and

$$V = \frac{Q}{4\pi\epsilon_0 r} = 12 \text{ JC}^{-1}$$

Dividing, we get $\frac{V}{E} = r = \frac{12}{24} = \mathbf{0{\cdot}5\ m} = AB$

From $V = \frac{Q}{4\pi\epsilon_0 r}$; $Q = 4\pi\epsilon_0 r \times V$

$$Q = \frac{1}{9\times10^9}\times0{\cdot}5\times12 = \mathbf{0{\cdot}667\times10^{-9}\ C}$$

Example 4 **(a) Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away.**

(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought ? (NCERT Solved Example)

Solution. *(a)* Here, $q = 4 \times 10^{-7}$ C,
$r = 9$ cm $= 9 \times 10^{-2}$ m ; $V = ?$

$$V = \frac{q}{4\pi\epsilon_0 r} = 9\times10^9\,\frac{(4\times10^{-7})}{9\times10^{-2}}$$

$$= \mathbf{4\times10^4\ volt}$$

(b) $W = q \times V = (2 \times 10^{-9}) \times 4 \times 10^4$

$$= \mathbf{8\times10^{-5}\ J}$$

The answer does not depend upon the path along which the charge is brought because electrostatic forces are conservative forces.

Example 5 **An infinite number of charges each numerically equal to q and of the same sign are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$ and so on. Find electric potential at $x = 0$.**

Solution. Using superposition principle, we may write electric potential at the origin ($x = 0$) due to various charges as

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+.....\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....\right]$$

As sum of infinite G.P. series, $S = \frac{a}{1-r}$,

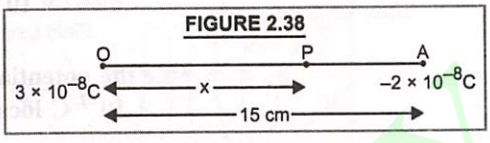
where a is first term and r is common ratio.

$$\therefore \quad V = \frac{q}{4\pi\epsilon_0}\left\{\frac{1}{(1-1/2)}\right\} = \frac{2q}{4\pi\epsilon_0}$$

Example 6 **Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.** (NCERT Solved Example)

Solution. Let the location of positive charge be taken as the origin O, Fig. 2.38 ; and line joining the two charges as x-axis. Let P be required point, where potential is zero. Let OP = x cm.

FIGURE 2.38



Potential at P is

$$V = \frac{3\times10^{-8}}{4\pi\epsilon_0\,(x\times10^{-2})} + \frac{-2\times10^{-8}}{4\pi\epsilon_0\,(15-x)\times10^{-2}} = 0$$

$$\therefore \quad \frac{3}{x} - \frac{2}{15-x} = 0, \text{ which gives } x = 9 \text{ cm.}$$

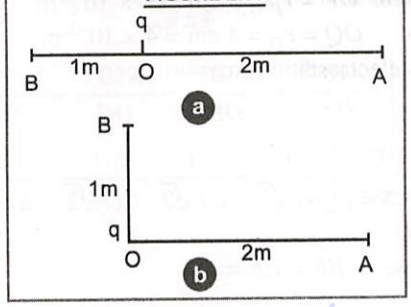
If P lies on extended line OA, the condition for zero electric potential would be $\frac{3}{x} - \frac{2}{x-15} = 0$, which gives $\boldsymbol{x = 45}$ **cm**

Hence electric potential is zero at 9 cm and 45 cm away from positive charge on the side of negative charge.

Potential at infinity is taken zero in the formula used.

Example 7 **A charge $q = +1\ \mu$C is held at O between two points A and B such that AO = 2 m and BO = 1m, Fig. 2.39(a). Calculate the value of potential difference ($V_A - V_B$). What will be the value of potential difference ($V_A - V_B$) if position of B is changed as shown in Fig. 2.39(b) ?**

FIGURE 2.39



Solution. Here, $q = +1\ \mu\text{C} = 10^{-6}$ C,

$AO = 2\ m$, $BO = 1$ m

As shown in Fig. 2.39(a),

$$V_A - V_B = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{AO} - \frac{1}{BO}\right]$$

$$= 9\times10^9\times10^{-6}\left[\frac{1}{2}-1\right] = \mathbf{-4500\ V}$$

If position of B is changed as shown in Fig. 2.39(b), value of p.d. ($V_A - V_B$) ***shall remain the same.*** This is because electric potential is a scalar quantity and its value does not change with change in orientation of the charges.

Example 8 **Four point charges + 1 μC, +1 μC, – 1 μC and – 1 μC are placed at the corners A, B, C and D of a square of each side 0·1 m (i) Calculate electric potential at the centre O of the square (ii) If E is middle point of BC, what is work done in carrying an electron from O to E ?**

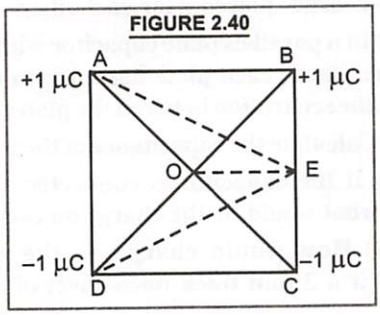
Solution. In Fig. 2.40,

$AB = BC = CD = DA = a = 0{\cdot}1$ m

$$BD = AC = \sqrt{a^2+a^2} = a\sqrt{2}$$

$$OA = OC = OB = OD = \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}}$$

FIGURE 2.40



Potential at O, $V_0 = \frac{1}{4\pi\epsilon_0} \times$

$$\left[\frac{1\times10^{-6}}{OA} + \frac{1\times10^{-6}}{OB} - \frac{1\times10^{-6}}{OC} - \frac{1\times10^{-6}}{OD}\right]$$

$$= \frac{10^{-6}}{4\pi\epsilon_0}\left[\frac{1}{a/\sqrt{2}} + \frac{1}{a/\sqrt{2}} - \frac{1}{a/\sqrt{2}} - \frac{1}{a/\sqrt{2}}\right]$$

$= O$

Now, $BE = CE = a/2$

Again, $AE = DE = \sqrt{DC^2 + CE^2}$

$$= \sqrt{a^2 + (a/2)^2} = \sqrt{\frac{5a^2}{4}} = \frac{a\sqrt{5}}{2}$$

Potential at E, $V_E = \frac{1}{4\pi\epsilon_0} \times$

$$\left[\frac{1\times10^{-6}}{AE} + \frac{1\times10^{-6}}{BE} - \frac{1\times10^{-6}}{CE} - \frac{1\times10^{-6}}{DE}\right]$$

$$= \frac{10^{-6}}{4\pi\epsilon_0}\left[\frac{2}{a\sqrt{5}} + \frac{2}{a} - \frac{2}{a} - \frac{2}{a\sqrt{5}}\right] = 0.$$

Work done in carrying an electron of charge $(-e)$ from O to E

$$W = -e\,[V_E - V_O] = -e\,[0 - 0] = \textbf{Zero.}$$

TYPE III. POTENTIAL GRADIENT AND ELECTRIC FIELD

Formulae used. $E = -\frac{dV}{dr}$. It follows that

$$E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

Units used. E_x, E_y, E_z and E are in N/C or V/m, dV is in volt and dr is in metre.

Example 9 **(a) Draw equipotential surfaces for (i), an electric dipole (ii) two identical positive charges placed near eachother.**

(b) In a parallels plate capacitor with air between the plates, each plate has an area 6×10^{-3} m^2, and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

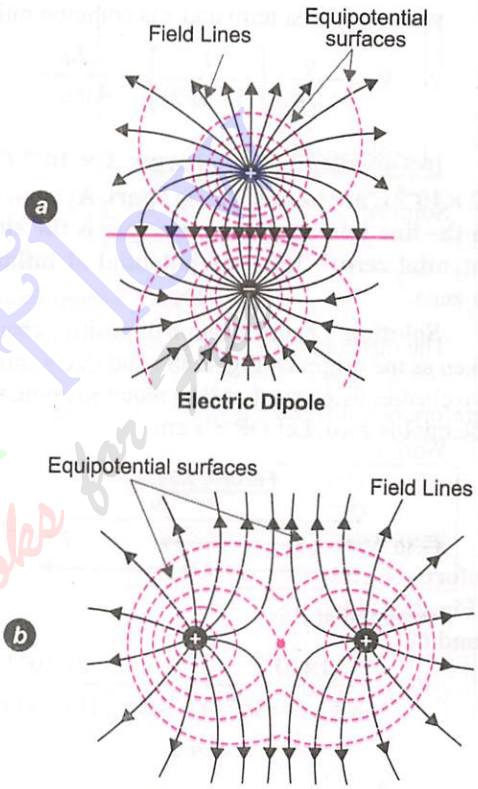
(ii) If the capacitor is connected to 100 V supply, what would be the charge on each plate ?

(iii) How would charge on the plate be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates, while the voltage supply remains connected ?

(CBSE Sample Paper 2022-23)

Solution. (a)

FIGURE 2.41



(b) Here, $A = 6 \times 10^{-3}$ m^2,

$d = 3$ mm $= 3 \times 10^{-3}$ m

(i) $C = \frac{\epsilon_0 A}{d} = \frac{8{\cdot}85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$

$= \mathbf{17{\cdot}7 \times 10^{-12}}$ **farad**

(ii) Charge on each plate

$Q = CV = 17{\cdot}7 \times 10^{-12} \times 100 = \mathbf{17{\cdot}7 \times 10^{-10}}$ **C**

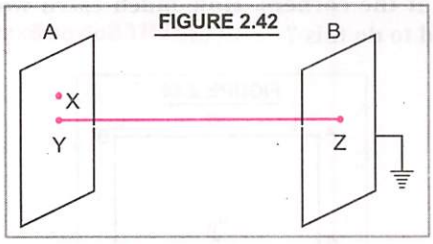
(iii) New charge $Q' = KQ = 6 \times 17{\cdot}7 \times 10^{-10}$ C

$= \mathbf{1{\cdot}062 \times 10^{-8}}$ **C**

Example 10 **Two identical plane metallic surfaces A and B are kept parallel to each other in air separated by a distance of 1·0 cm as shown in Fig. 2.42. Surface A is given a positive potential of**

10 V and the outer surface of B is earthed. (i) What is the magnitude and direction of uniform electric field between points Y and Z ? (ii) What is work done in moving a charge of 20 μC from point X to Y, where X is situated on surface A ?

FIGURE 2.42



Solution. (i) Magnitude of electric field,

$$E = \frac{dV}{dr} = \frac{10\,V}{1\times10^{-2}\,m} = 10^3 \text{ Vm}^{-1}$$

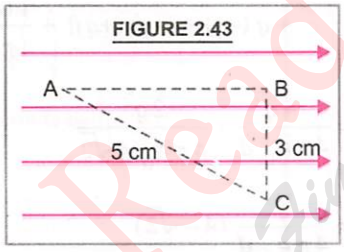
The direction of electric field is from Y to Z.

(ii) As surface A is an equipotential surface, therefore pot. diff. between X and Y is, *i.e.*, $\Delta V = 0$

Work done $= q \,.\, \Delta V = 0$, *i.e.*, **no work is done in moving a charge from X to Y.**

Example 11 Three points A, B, C lie in a uniform electric field E of 5×10^3 N/C, as shown in Fig. 2.43. Find the potential difference between A and C.

FIGURE 2.43



(CBSE (F) 2009)

Solution. In Fig. 2.43, as B and C are in same vertical line, therefore $V_B = V_C$.

Now, potential difference between A and C = potential difference between A and B

$$V_{AC} = V_{AB} = -E\,(\Delta x)$$

Now, $\Delta x = AB = \sqrt{AC^2 - BC^2} = \sqrt{5^2 - 3^2}$

$= 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$

$\therefore \quad V_{AC} = -5 \times 10^3 \times 4 \times 10^{-2}$

$= -200 \text{ V}$

TYPE IV. ELECTROSTATIC POTENTIAL ENERGY

Formulae used. 1. Electrostatic potential energy of a system of two point charges, $U = \dfrac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$

2. Electrostatic potential energy of a system of n point charges, $U = \dfrac{1}{4\pi\epsilon_0} \sum_{\text{all pairs}} \dfrac{q_j q_k}{r_{jk}}$

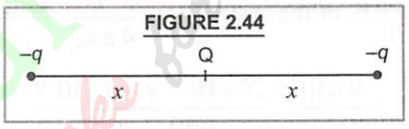
Units used. All charges in coulomb, distance r in metre, energy U in joule.

Example 12 Three charges $-q$, Q, $-q$ are placed at equal distances on a straight line. If the potential energy of the system of these charges is zero, then what is the ratio Q/q ?

(CBSE Sample Paper 2022-23)

Solution. As shown in Fig. 2.44, potential energy of the system of three charges is

FIGURE 2.44



$$\frac{K(-q)\,Q}{x} + \frac{K\,Q(-q)}{x} + \frac{k(-q)(-q)}{(2x)} = 0$$

$$\therefore \quad \frac{K q^2}{2x} = \frac{2K\,q\,Q}{x}$$

$$\therefore \quad q = 4\,Q \quad \text{or} \quad \frac{Q}{q} = \frac{1}{4}$$

Example 13 Two positive point charges of 0·2 μC and 0·01 μC are placed 10 cm apart. Calculate the work done in reducing the distance to 5 cm.

Solution. Here $q_1 = 0{\cdot}2 \times 10^{-6}$ C,

$q_2 = 0{\cdot}01 \times 10^{-6}$ C, $r_2 = 0{\cdot}05$ m, $r_1 = 0{\cdot}10$ m

Work done = Change in P.E. = Final P.E. − Initial P.E.

$$= \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= 9 \times 10^9 \times 0{\cdot}2 \times 10^{-6} \times 0{\cdot}01 \times 10^{-6} \times \left[\frac{1}{0{\cdot}05} - \frac{1}{0{\cdot}10}\right]$$

$$= 1{\cdot}8 \times 10^{-4} \text{ J}$$

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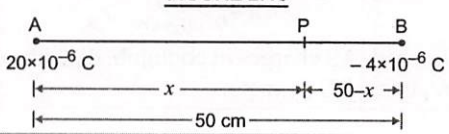
Example 14 **Two point charges 20×10^{-6} C and -4×10^{-6} C are separated by a distance of 50 cm in air. Find (*i*) the point on line joining the charges, where the electrostatic potential is zero (*ii*) Also, calculate the electrostatic potential energy of the system.** (CBSE (OD) 2008)

Solution.

FIGURE 2.45



Suppose the point of zero potential is located at a distance x metre from the charge 20×10^{-6} C, Fig. 2.45.

$$\text{then, } V = \frac{1}{4\pi\epsilon_0}\left[\frac{20\times10^{-6}}{x} - \frac{4\times10^{-6}}{50-x}\right] = 0$$

$x = 0{\cdot}41$ m = 41 cm from 20×10^{-6} C charge

(*ii*) P.E. of the system $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r}$

$$= \frac{9\times10^9 \times 20\times10^{-6} \times (-4\times10^{-6})}{0{\cdot}50}$$

$= -$ **1·44 J**

Example 15 **Two particles have equal masses of 5·0 g each and opposite charges of $+4{\cdot}0 \times 10^{-5}$ C and $-4{\cdot}0 \times 10^{-5}$ C. They are released from rest with a separation of 1·0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.**

Solution. Here, $m_1 = m_2 = 5{\cdot}0\text{g} = 5 \times 10^{-3}$ kg,

$q_1 = 4{\cdot}0 \times 10^{-5}$ C, $q_2 = -4{\cdot}0 \times 10^{-5}$ C

$r_1 = 1{\cdot}0$ m, $r_2 = 50$ cm $= \dfrac{1}{2}$ m

From symmetry, $v_1 = v_2 = v = ?$

As increase in *K.E.* = loss in *P.E.* = final *P.E.* – initial *P.E.*

$$\therefore\ 2\times\frac{1}{2}mv^2 = \frac{|q_1||q_2|}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$5 \times 10^{-3} v^2 = 9 \times 10^9 (4 \times 10^{-5})^2 \left[\frac{1}{1/2} - 1\right]$$

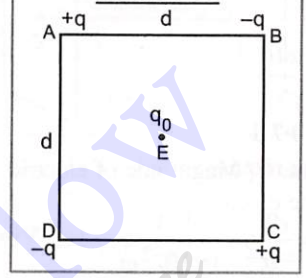
$$= 9 \times 16 \times 10^{-1} (2 - 1)$$

$$v^2 = \frac{14{\cdot}4}{5\times10^{-3}} = 2{\cdot}88\times10^3$$

$v =$ **53·7 m/s**

Example 16 **Four charges are arranged at the corners of a square ABCD of side *d*, as shown in Fig. 2.46. Find the work required to put together this arrangement (*b*) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at the corners. How much extra work is needed to do this ?** (NCERT Solved Example)

FIGURE 2.46



Solution. (*a*) Here, AB = BC = CD = DA = d

$$AC = BD = \sqrt{d^2 + d^2} = d\sqrt{2}$$

Total work done = P.E. of system of 4 charges

$$= \frac{1}{4\pi\epsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

$$= \frac{1}{4\pi\epsilon_0 d}[q(-q) + (-q)(+q) + q(-q) + (-q)(+q)] + \frac{1(q^2+q^2)}{4\pi\epsilon_0 d\sqrt{2}}$$

$$= \frac{-4q^2}{4\pi\epsilon_0 d} + \frac{2q^2}{4\pi\epsilon_0 d\sqrt{2}}$$

$$= \frac{-q^2}{4\pi\epsilon_0 d}(4 - \sqrt{2})$$

(*b*) Extra work needed = charge × potential at E due to charges at four corners

$$W = q_0 \times \frac{(q - q + q - q)}{4\pi\epsilon_0\left(\dfrac{d\sqrt{2}}{2}\right)} = \text{zero}$$

Example 17 **(*a*) Determine the electrostatic potential energy of a system consisting of two charges 7 μ C and – 2 μ C (and with no external field) placed at (– 9 cm, 0, 0) and (9 cm, 0, 0) respectively.**

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(*b*) How much work is required to separate the two charges infinitely away from each other ?

(*c*) Suppose that the same system of charges is now placed in an external electric field $E = A \times 1/r^2$, where $A = 9 \times 10^5$ C m^{-2}.

What would the electrostatic energy of the configuration be ? (NCERT Solved Example)

Solution. (*a*) $U = \dfrac{q_1 q_2}{4\pi\epsilon_0 r}$

$$= \frac{7\times10^{-6}\times(-2\times10^{-6})\times9\times10^9}{(9+9)\times10^{-2}}$$

$$= -0{\cdot}7 \text{ J}$$

(*b*) Work required $= U_2 - U_1 = 0 - U = 0{\cdot}7$ J

(*c*) In the electric field, total P.E.

$$= \frac{q_1 q_2}{4\pi\epsilon_0 r} + q_1 V(\vec{r_1}) + q_2 V(\vec{r_2})$$

From $E = \dfrac{-dV}{dr}$;

$$V = \int -E\,dr = \int -\frac{A}{r^2}dr = \frac{A}{r}$$

$$\therefore \text{ Total P.E.} = \frac{q_1 q_2}{4\pi\epsilon_0 r} + \frac{A q_1}{r_1} + \frac{A q_2}{r_2}$$

$$= (-0{\cdot}7) + \frac{9\times10^5\,(7\times10^{-6})}{0{\cdot}09} + \frac{9\times10^5\,(-2\times10^{-6})}{0{\cdot}09}$$

$$= -0{\cdot}7 + 70 - 20 = 49{\cdot}3 \text{ J}$$

Example 18 **(*a*) Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side l as shown in Fig. 2.47. Obtain the expression for magnitude of the resultant electric force acting on charge q.**

(*b*) Find out the amount of work done to separate the charges at infinite separation.

(CBSE 2018)

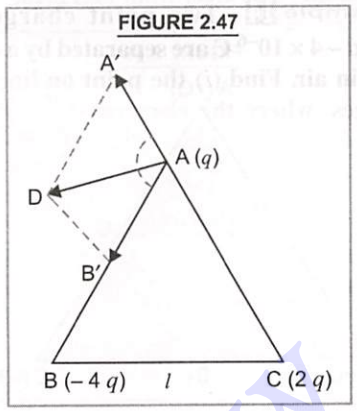
Solution. (*a*) As is clear from Fig. 2.47, force on charge q at A due to charge $2q$ at C is

$$F_1 = \frac{k\,q\,(2q)}{l^2}, \text{ along } AA'$$

Force on charge q at A due to charge $-4q$ at B is

$$F_2 = \frac{k\,q\,(4q)}{l^2}, \text{ along } AB'$$

FIGURE 2.47



Angle between F_1 and $F_2 = \angle A'AB'$

$= 180° - 60°$

$= 120°$

$\therefore$ Resultant force on charge q at A is

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos120°}$$

$$= \sqrt{\left(\frac{2kq^2}{l^2}\right)^2 + \left(\frac{4kq^2}{l^2}\right)^2 + 2\times\frac{2kq^2}{l^2}\times\frac{4kq^2}{l^2}\times\frac{-1}{2}}$$

$$R = \frac{kq^2}{l^2}\sqrt{4+16-8} = \sqrt{12}\,\frac{kq^2}{l^2} \text{ ; along } AD$$

Here, $k = \dfrac{1}{4\pi\epsilon_0}$.

(*b*) Work done to separate the charges

= potential energy of the system

$$= \frac{1}{4\pi\epsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q(2q)}{l} + \frac{2q(-4q)}{l} + \frac{q(-4q)}{l}\right]$$

$$= \frac{-10q^2}{4\pi\epsilon_0 l}$$

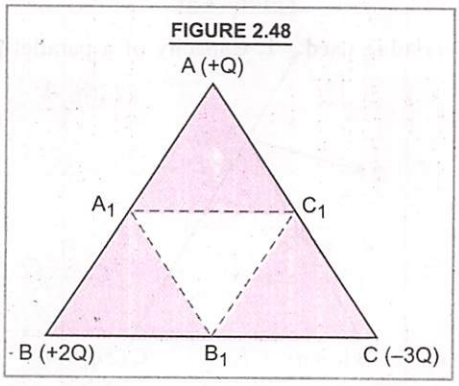
Negative sign for work required to be done.

Example 19 **Three point charges $+Q$, $+2Q$ and $-3Q$ are placed at the vertices of an equilateral triangle ABC of side l. If these charges are displaced to the mid points A_1, B_1 and C_1 respectively, find the amount of work done in shifting the charges to their new positions.**

(CBSE (OD) 2015)

Solution. As is clear from Fig. 2.48,

FIGURE 2.48



$$A_1B_1 = B_1C_1 = A_1C_1$$

$$= \frac{AB}{2} = \frac{l}{2}$$

Initial potential energy of the system,

$$U_i = \frac{1}{4\pi\epsilon_0}\left[\frac{Q(2Q)}{l} + \frac{2Q(-3Q)}{l} + \frac{Q(-3Q)}{l}\right]$$

$$U_i = \frac{-7Q^2}{4\pi\epsilon_0 l}$$

Now $A_1B_1 = B_1C_1 = C_1A_1 = \frac{AB}{2} = \frac{l}{2}$

Therefore, final potential energy of the system,

$$U_f = \frac{1}{4\pi\epsilon_0}\left[\frac{Q(2Q)}{l/2} + \frac{2Q(-3Q)}{l/2} + \frac{Q(-3Q)}{l/2}\right]$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{14Q^2}{l}$$

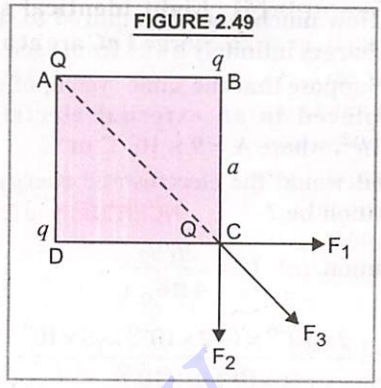
Work done, $W = U_f - U_i$

$$= -\frac{14Q^2}{4\pi\epsilon_0 l} - \left(-\frac{7Q^2}{4\pi\epsilon_0 l}\right) = -\frac{7Q^2}{4\pi\epsilon_0 l}$$

Example 20 **Four point charges Q, q, Q and q are placed at the corners of a square of side a as shown in Fig. 2.49. Find (a) resultant electric force on charge Q, and (b) potential energy of this system.** **(CBSE 2018)**

Solution. (a) As is clear from Fig. 2.49,

FIGURE 2.49



$$F_1 = \frac{kqQ}{a^2}, \text{ along } DC$$

$$F_2 = \frac{kqQ}{a^2}, \text{ along } BC$$

$$F_3 = \frac{kQQ}{(a\sqrt{2})^2}, \text{ along } AC$$

Resultant of F_1 and F_2 is

$$R = \sqrt{F_1^2 + F_2^2} = \frac{kqQ}{a^2}\sqrt{2}.$$

R is along AC produced.

∴ Resultant force on charge Q at C is

$$R' = R + F_3 = \frac{kqQ}{a^2}\sqrt{2} + \frac{kQQ}{(a\sqrt{2})^2}$$

$$= \frac{kQ}{a^2}\left[q\sqrt{2} + \frac{Q}{2}\right]$$

$$R' = \frac{Q}{4\pi\epsilon_0 a^2}\left[q\sqrt{2} + \frac{Q}{2}\right]$$

(b) Potential energy of the system

$$U = (1/4\pi\epsilon_0)\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{Qq}{a} + \frac{qQ}{a} + \frac{Qq}{a} + \frac{Qq}{a} + \frac{QQ}{a\sqrt{2}} + \frac{qq}{a\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{4Qq}{a} + \frac{Q^2+q^2}{a\sqrt{2}}\right]$$

TYPE V. CAPACITANCE

Formulae used.

1. Capacity $C = Q/V$

2. Capacity of a spherical conductor, $C = 4\pi\epsilon_0 r$

Units used. C is in farad ; Q is in coulomb ; V is in volt ; r is in metre.

Standard Value. $\frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{Nm}^2\ \text{C}^{-2}$

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Example 21 **Eight identical spherical drops, each carrying charge 1 *nC* are at a potential of 900 *V* each. All these drops combine together to form a single large drop. Calculate the potential of this large drop. Assume no wastage of any kind. Take capacitance of a sphere of radius *r* as proportional to *r*.** **(CBSE Sample Paper 2015)**

Solution. Here, $C \propto r$ or $C = kr$

Charge on each small drop,

$q = CV = (kr \times 900)$ C

Let R be radius of large drop

As volume of large drop = Volume of 8 small drops,

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3 \quad \text{or} \quad R = 2r.$$

Capacitance of large drop, $C' = kR = 2kr$

Potential of large drop

$$V' = \frac{q'}{C'} = \frac{8q}{C'} = \frac{8 \times kr \times 900}{2kr} = \mathbf{3600\ V}$$

Example 22 **A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC. Calculate (*i*) the potential *V* and the unknown capacitance *C*. (*ii*) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V?** **(CBSE 2013)**

Solution. If capacity is C μF, then as per the statement,

$$C \times V = 360 \qquad ...(i)$$

$$C(V - 120) = 120$$

Dividing, we get

$$\frac{C(V-120)}{C \times V} = \frac{120}{360} = \frac{1}{3}$$

$$3V - 360 = V$$

or $\quad V = \mathbf{180\ volt}$

From (*i*), $\quad C = \dfrac{360}{V} = \dfrac{360}{180} = 2\,\mu F$

New voltage, $V' = V + 120 = 180 + 120$

$= 300$ volt

New charge,

$$Q' = CV' = 2 \times 300 = \mathbf{600\ \mu C}$$

TYPE VI. PARALLEL PLATE CAPACITOR

Formulae used. **1.** Capacity of a parallel plate capacitor with air as dielectric, $C = \dfrac{\epsilon_0 A}{d}$

2. Capacity of a parallel plate capacitor with insulating medium as dielectric, $C = K\dfrac{\epsilon_0 A}{d}$

Units used. C in farads, A in m^2 ; d in metre

Standard Value. $\epsilon_0 = 8{\cdot}85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

Example 23 **Two parallel plate air capacitors have their plate areas 100 and 500 cm^2 respectively. If they have the same charge and potential and the distance between the plates of the first capacitor is 0·5 mm, what is the distance between the plates of second capacitor ?**

Solution. Here, two parallel plate capacitors have same charge q and same potential V, so they have equal capacitances as

$$C = q/V, \quad C_1 = C_2$$

$$\frac{\epsilon_0 A_1}{d_1} = \frac{\epsilon_0 A_2}{d_2} \quad \text{or} \quad d_2 = \frac{A_2}{A_1} d_1$$

Now, $A_1 = 100\ cm^2$, $A_2 = 500\ cm^2$,

$d_1 = 0{\cdot}5$ mm $= 0{\cdot}05$ cm

$$\therefore \quad d_2 = \frac{500 \times 0{\cdot}05}{100} = 0{\cdot}25\ \text{cm} = \mathbf{2{\cdot}5\ mm}$$

Example 24 **A parallel plate capacitor has plate area 25·0 cm^2 and a separation of 2·0 mm between the plates. The capacitor is connected to a battery of 12·0 V (*a*) Find the charge on the capacitor (*b*) The plate separation is decreased to 1·00 mm. Find the extra charge given by the battery to the positive plate.**

Solution. Here, $A = 25{\cdot}0\ cm^2 = 25{\cdot}0 \times 10^{-4}\ m^2$,

$d = 2{\cdot}0$ mm $= 2 \times 10^{-3}$ m, $V = 12$ volt

(*a*) Charge on the capacitor, q = ?

$$q = CV = \frac{\epsilon_0 A}{d} V$$

$$= \frac{8{\cdot}85 \times 10^{-12}(25{\cdot}0 \times 10^{-4}) \times 12}{2 \times 10^{-3}}$$

$$= \mathbf{1{\cdot}35 \times 10^{-10}\ C}$$

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(*b*) When plate separation is decreased to half, capacity becomes twice. The charge ($q' = C'V$), becomes twice. Hence extra charge given by the battery = $q' - q = 2q - q = q = 1{\cdot}35 \times 10^{-10}$ C

Example 25 **A parallel plate capacitor is to be designed with a voltage rating 1 kV using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. [Dielectric strength is the maximum electric field a material can tolerate without break down, i.e., without starting to conduct electricity through partial ionisation. For safety, we should like the field never to exceed say 10% of the dielectric strength]. What minimum area of the plates is required to have a capacitance of 50 pF ?**

Solution. Here, $V = 1$ kV $= 1000$ volt ;

$K = \epsilon_r = 3$; Dielectric strength $= 10^7$ V/m

As electric field at the most should be 10% of dielectric strength, due to reasons of safety,

$\therefore$ $E = 10\%$ of 10^7 V/m $= 10^6$ V/m, $A = ?$,

$C = 50$ pF $= 50 \times 10^{-12}$ F

As, $E = \dfrac{V}{d}$ $\therefore$ $d = \dfrac{V}{E} = \dfrac{10^3}{10^6} = 10^{-3}$ m

Now, $C = \dfrac{\epsilon_0 \epsilon_r A}{d}$

$$A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{50 \times 10^{-12} \times 10^{-3}}{8{\cdot}85 \times 10^{-12} \times 3}$$

$$= 1{\cdot}9 \times 10^{-3} \text{ m}^2$$

TYPE VII. SPHERICAL AND CYLINDRICAL CAPACITORS

Formulae used. 1. Capacity of a spherical air capacitor, $C = 4\pi\epsilon_0 \dfrac{r_a r_b}{(r_b - r_a)}$; $r_b > r_a$

2. Capacity of a cylindrical air capacitor,

$$C = \frac{2\pi\epsilon_0 L}{\log_e \left(\dfrac{r_b}{r_a}\right)} \text{ ; } r_b > r_a$$

Units used. r_a, r_b, L are in metre ;

ϵ_0 is in $C^2 N^{-1} m^{-2}$; C is in farad.

Standard value. $\dfrac{1}{4\pi\epsilon_0} = 9 \times 10^9$ Nm^2 C^{-2}

Example 26 **The thickness of air layer between two coatings of a spherical capacitor is 2 cm. The capacitor has same capacitance as the sphere of 1·2 m diameter. Find the radii of its surfaces.**

Solution. Here, $\dfrac{4\pi\epsilon_0 ab}{b-a} = 4\pi\epsilon_0 R$

$$\frac{ab}{b-a} = R$$

As $b - a = 2$ cm $\quad R = \dfrac{1{\cdot}2}{2}$ m $= 60$ cm

$\therefore$ $\dfrac{ab}{2} = 60$ or $ab = 120$

As $(b + a)^2 = (b - a)^2 + 4ab = (2)^2 + 4 \times 120 = 484$

$b + a = 22$

$2 + a + a = 22$ $\quad [\because b - a = 2$ cm$]$

$\therefore$ $\boldsymbol{a = 10}$ **cm,** $\boldsymbol{b = 12}$ **cm**

Example 27 **Calculate the capacitance of a spherical capacitor consisting of two concentric spheres of radii 0·50 m and 0·60 m. The material filled in the space between the two spheres has a dielectric constant of 6.**

Solution. Here, $C = ?$, $r_a = 0{\cdot}50$ m,

$r_b = 0{\cdot}60$ m, $K = 6$

$$C = KC_0 = K \frac{4\pi\epsilon_0 r_a r_b}{r_b - r_a}$$

$$= \frac{6}{9 \times 10^9} \frac{(0{\cdot}5)(0{\cdot}6)}{(0{\cdot}6 - 0{\cdot}5)} = 2 \times 10^{-9} \text{ F}$$

Example 28 **What is the capacitance of a 1 m long hifi cable where the central conductor is 1 mm in diameter and the shield is 5 mm in diameter ?**

Solution. Here, $C = ?$, $L = 1$ m

$$r_a = \frac{1}{2} \text{mm} = 0{\cdot}5 \text{ mm} = 5 \times 10^{-4} \text{ m},$$

$$r_b = \frac{5}{2} \text{mm} = 2{\cdot}5 \text{ mm} = 2{\cdot}5 \times 10^{-3} \text{ m}$$

For a cylindrical capacitor,

$$C = \frac{2\pi\epsilon_0 L}{\log_e \left(\dfrac{r_b}{r_a}\right)} = \frac{4\pi\epsilon_0 L}{2\log_e (r_b/r_a)}$$

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$$= \frac{1\times1}{9\times10^9\times2\log_e\left(\frac{2{\cdot}5\times10^{-3}}{5\times10^{-4}}\right)}$$

$$C = \frac{1}{18\times10^9\log_e 5} = \frac{10^{-9}}{18\times1{\cdot}609}$$

$$= 3{\cdot}45\times10^{-11}\text{ F} = \mathbf{34{\cdot}5\text{ pF}}$$

Example 29 **The outer cylinder of a cylindrical capacitor of length 0·15 m and radii 1·61 cm and 1·5 cm is earthed while inner cylinder of this capacitor is given a charge of 8 μC. Find the capacitance and potential of inner part of the capacitor.**

Solution. Here, $l = 0{\cdot}15$ m ;

$r_b = 1{\cdot}61\times10^{-2}$ m ; $r_a = 1{\cdot}50\times10^{-2}$ m

$q = 8{\cdot}0\times10^{-6}$ C ; $C = ?$, $V = ?$

For cylindrical capacitor,

$$C = \frac{2\pi\epsilon_0 l}{2{\cdot}303\log_{10}\left(\frac{r_b}{r_a}\right)}$$

$$= \frac{2\pi(8{\cdot}854\times10^{-12})(0{\cdot}15)}{2{\cdot}303\log_{10}\left(\frac{1{\cdot}61\times10^{-2}}{1{\cdot}50\times10^{-2}}\right)}$$

$$C = \mathbf{1{\cdot}18\times10^{-10}\text{ F}}$$

$$V = \frac{q}{C} = \frac{8{\cdot}0\times10^{-6}}{1{\cdot}18\times10^{-10}} = \mathbf{6{\cdot}78\times10^4\text{ V}}$$

TYPE VIII. GROUPING OF CAPACITORS

Formulae used.

1. Series combination :

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

2. Parallel combination :

$$C_p = C_1 + C_2 + C_3 + \ldots$$

3. $Q = CV$

Units used. $C_1, C_2, C_3 \ldots$; C_s, C_p all in farad, V in volt, Q in coulomb.

Example 30 **Seven capacitors, each of capacitance 2 μF are to be connected in a configuration to obtain an effective capacitance of (10/11) μF. Suggest a suitable combination.**

Solution. Suppose a parallel combination of n capacitors is connected in series with a series combination of $(7 - n)$ capacitors.

Capacitance of parallel combination,

$C_1 = 2n$ μF

Capacitance of $(7 - n)$ series combination,

$$C_2 = \frac{2}{7-n}\ \mu\text{F}$$

As these two combinations are in series, and

$$C_s = \frac{10}{11}\ \mu\text{F},\ \text{therefore, from}$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2n} + \frac{7-n}{2} = \frac{11}{10}$$

Multiplying both sides by $10n$, we get

$5 + 5n(7 - n) = 11n$

$5 + 35n - 5n^2 = 11n$

or $5n^2 - 24n - 5 = 0$

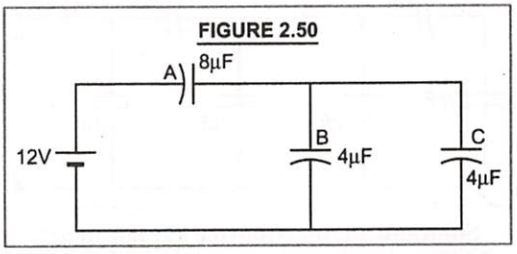
$(n - 5)(5n + 1) = 0$

Rejecting negative value, we get $\boldsymbol{n = 5}$

Hence parallel combination of 5 capacitors must be connected in series with series combination of 2 capacitors.

Example 31 **Find the charge appearing on each of the three capacitors shown in Fig. 2.50.**

FIGURE 2.50



Solution. Capacitors B and C are in parallel. Therefore, their total capacity = 4 + 4 = 8 μ F

The total capacity of 8 μ F and capacitor A which are in series w.r.t. battery is given by

$$\therefore \quad \frac{1}{C_s} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4},\ C_s = 4\ \mu\text{F}$$

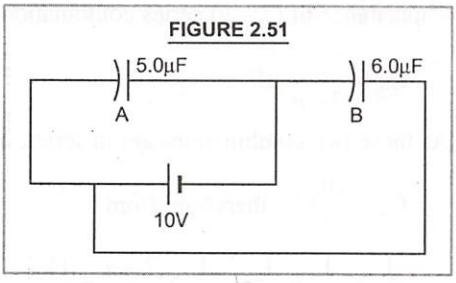
Total charge = $C_s V = 4\times12 = 48$ μ C

This is the charge on A, and also on the combination of B and C. As capacities of B and C are equal, the total charge is shared equally between them.

$\therefore$ Charge on B = charge on $C = \frac{48}{2} = 24\ \mu C$

Example 32 **Find the charge supplied by the battery in the arrangement shown in Fig. 2.51.**

FIGURE 2.51



Solution. As is clear from Fig. 2.51, left plate of capacitor A is + and right plate of capacitor A is negative. Again left plate of B is negative and right plate of B is positive.

The two capacitors are, therefore, in parallel. Their combined capacity is

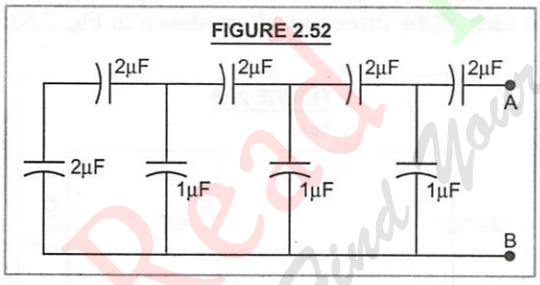
$$C_p = C_1 + C_2 = 5{\cdot}0 + 6{\cdot}0 = 11\ \mu F$$

Charge supplied,

$$q = C_p V = 11 \times 10 = \mathbf{110\ \mu C}$$

Example 33 **Find the capacitance of the combination shown in Fig. 2.52 between A and B.**

FIGURE 2.52



Solution. Starting from the left side of network, two 2 μF capacitors are in series. Therefore,

$$\frac{1}{C_s} = \frac{1}{2} + \frac{1}{2} = 1,\ C_s = 1\ \mu F.$$

This 1 μF and third 1 μF capacitor are in parallel. Therefore, $C_p = 1 + 1 = 2\ \mu F$ and so on.

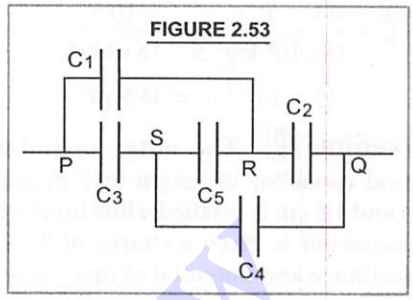
Proceeding in this way, finally two 2 μF capacitors are in series. Therefore, capacitance of the combination between A and B is **1 μ F.**

Example 34 **In Fig. 2.53, the values of capacitances are as follows :**

$$C_1 = C_2 = C_3 = C_4 = 4\ \mu F,\ C_5 = 5\ \mu F$$

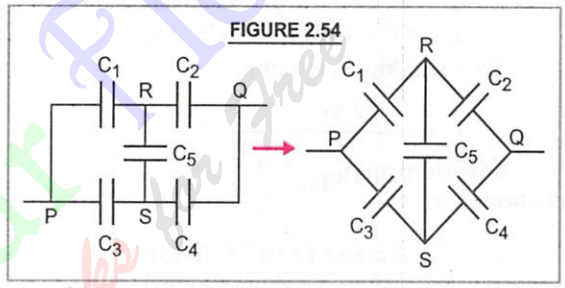
Calculate the equivalent capacitance between the points P and Q. If a battery of 10 volt is connected between these points, what will be the charge on each capacitor ?

FIGURE 2.53



Solution. The given circuit can be redrawn as shown in Fig. 2.54.

FIGURE 2.54



Clearly, the capacitors form wheatstone bridge arrangement.

As $C_1 = C_2 = C_3 = C_4$ $\therefore$ $\frac{C_1}{C_2} = \frac{C_3}{C_4}$

Hence the bridge is balanced. The points R and S are at the same potential. No charge can accumulate on C_5, which thus becomes ineffective.

Between P and Q, two series combinations (C_1, C_2) and (C_3, C_4) are connected in parallel. If C' and C'' are the equivalent capacitances of these combinations, then from

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}\ ;\ C' = 2\ \mu F$$

Similarly, $C'' = 2\ \mu F$

Hence equivalent capacitance between P and Q is $C = C' + C'' = 2 + 2 = \mathbf{4\ \mu F}$

As pot. diff. between P and Q is $V = 10$ volt and equivalent capacitance of C_1 and C_2 in series is $2\ \mu F$.

$\therefore$ Charge on each of capacitors C_1 and C_2 is $= 2 \times 10 = \mathbf{20\ \mu C}$

Similarly, charge on each of capacitors C_3 and C_4 is also **20 μC.** Charge on $C_5 = \mathbf{0}$

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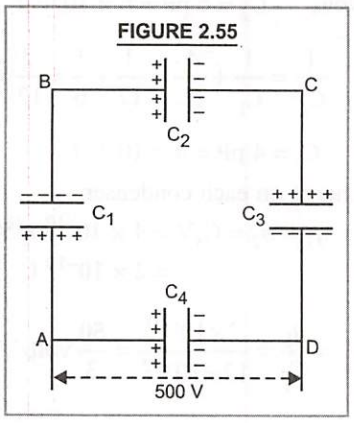
Example 35 **A network of four 10 μF capacitors is connected to a 500 V supply, as shown in Fig. 2.55. Determine the (*a*) equivalent capacitance of the network and (*b*) charge on each capacitor.** **(CBSE 2010)**

(NCERT Solved Example)

FIGURE 2.55



Solution. (*a*) In Fig. 2.55, C_1, C_2, C_3 are connected in series

$$\therefore \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{3}{10}$$

$$C_s = \frac{10}{3} \mu F$$

This combination is in parallel with C_4.

$\therefore$ Equivalent capacitance, $C = C_s + C_4$

$$= \frac{10}{3} + 10 = \mathbf{13{\cdot}3\ \mu F}$$

(*b*) Charge on C_1, C_2, C_3 is same = Q, say.

Let charge on C_4 be Q'.

$$\therefore \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500$$

$$Q\left(\frac{1}{C_s}\right) = 500$$

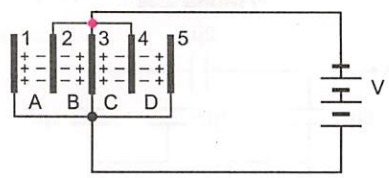
$$\text{or } Q = 500\, C_s = 500 \times \frac{10}{3} \mu C = \mathbf{1{\cdot}7 \times 10^{-3}\ C}$$

$$\text{Also, } \frac{Q'}{C_4} = 500,\ Q' = C_4 \times 500 = 10 \times 500\ \mu C$$

$$= \mathbf{5 \times 10^{-3}\ C}$$

Example 36 **Five identical capacitor plates each of area *A* are arranged such that adjacent plates are at a distance *d* apart. The plates are connected to a source of EMF (*V*) as shown in Fig. 2.56. What is the magnitude and nature of charge on plate 1 and 4 respectively ?**

FIGURE 2.56



Solution. The five plates make up four identical capacitors *A, B, C, D*, each of capacity ($\epsilon_0 A/d$), connected in parallel.

As plate 1 is connected to + of battery and is a part of one capacitor only, therefore, charge on it is

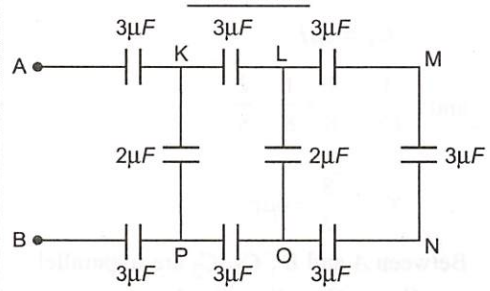
$$q_1 = +\left(\frac{\epsilon_0 A}{d}\right) V$$

However, plate 4 is connected to negative of battery, and is common to two identical capacitors *C* and *D* in parallel, therefore,

$$q_4 = -(q_C + q_D) = -2\, q_1 = \frac{-2\epsilon_0 A}{d} V$$

Example 37 **Find equivalent capacity between *A* & *B*, Fig. 2.57.** **(Bihar Board 2012)**

FIGURE 2.57



Solution. In Fig. 2.57, if C_1 is capacity of LMNO, $\frac{1}{C_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1,\ C_1 = 1\ \mu F$

This is in parallel with $2\mu F$ capacitor in LO.

$\therefore \quad C_2 = 1 + 2 = 3\mu F$

Similarly, capacity across *KP*,

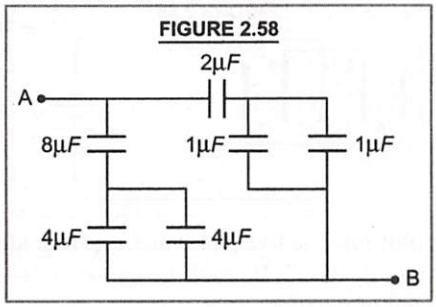
$C_3 = 1 + 2 = 3\mu F$.

$\therefore$ Capacity between *A* and *B* = C_4 is given by

$$\frac{1}{C_4} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1,\ C_4 = \mathbf{1\mu F}$$

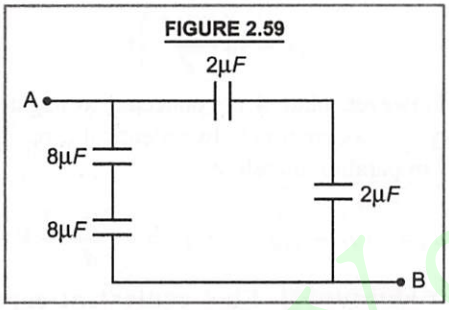
Example 38 **In Fig. 2.58, find equivalent capacity between A and B.** (Uttarakhand Board 2012)

FIGURE 2.58



Solution. The equivalent circuit is shown in Fig. 2.59.

FIGURE 2.59



$$\frac{1}{C_1} = \frac{1}{2} + \frac{1}{2} = 1,$$

$$\therefore \quad C_1 = 1\mu F$$

and $$\frac{1}{C_2} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$$

$$\therefore \quad C_2 = \frac{8}{2} = 4\mu F$$

Between A and B ; C_1, C_2 are in parallel.

$$\therefore \quad C_{AB} = C_1 + C_2 = 1 + 4$$
$$= 5\mu F$$

Example 39 **A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor ? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor.** (CBSE 2017)

Solution. Here, $C_1 = 12$ pF $= 12 \times 10^{-12}$ F

$V = 50$ volt.

Electrostatic energy stored

$$U = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 12 \times 10^{-12} (50)^2$$

$$U = \mathbf{1{\cdot}5 \times 10^{-8}\ J}$$

Now, $C_2 = 6$ pF $= 6 \times 10^{-12}$ F

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

$$C_s = 4 \text{ pF} = 4 \times 10^{-12} \text{ F}$$

Charge on each condenser,

$$q_1 = q_2 = C_s V = 4 \times 10^{-12} \times 50$$
$$= \mathbf{2 \times 10^{-10}\ C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{2 \times 10^{-10}}{12 \times 10^{-12}} = \mathbf{\frac{50}{3}\ volt}$$

$$V_2 = \frac{q_2}{C_2} = \frac{2 \times 10^{-10}}{6 \times 10^{-12}} = \mathbf{\frac{100}{3}\ volt}$$

Example 40 **An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him, each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires minimum number of capacitors.**

Solution. Here, total capacitance, C = 2 μ F,

Potential difference, $V = 1$ kV $= 1000$ volt

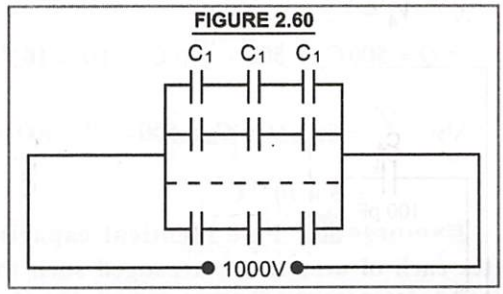
Capacity of each capacitor, $C_1 = 1$ μ F,

Maximum potential difference across each,

$V' = 400$ volt

Let n capacitors of 1 μ F each be connected in series in a row and m such rows be connected in parallel as shown in Fig. 2.60.

FIGURE 2.60



As potential difference across each row = 1000 volt

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$\therefore$ Potential difference across each capacitor

$$= \frac{1000}{n} = V' = 400$$

$$\therefore \quad n = \frac{1000}{400} = 2.5$$

As n has to be a whole number (not less than 2.5), therefore, $\boldsymbol{n = 3}$

Capacitance of each row of 3 condensers of 1 μ F each, in series = 1/3.

Total capacitance of m such rows in parallel $= \frac{m}{3}$ $\quad \therefore \quad \frac{m}{3} = 2(\mu F)$ or $\boldsymbol{m = 6}$.

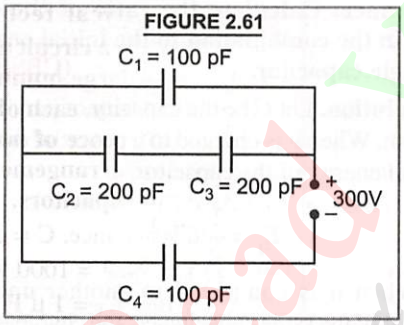
$\therefore$ total number of capacitors $= n \times m = 3 \times 6$

$= 18$

Hence 1 μ F capacitors should be connected in six parallel rows, each row containing three capacitors in series.

Example 41 Obtain equivalent capacitance of the following net work, Fig. 2.61. For a 300 V supply, determine the charge and voltage across each capacitor. (CBSE 2008)

FIGURE 2.61



Solution. The equivalent circuits of network in Fig. 2.71 are shown in Fig. 2.62.

In Fig. 2.62 (c), as 100 pF and 200 pF capacitors are in series, pot. diff. across C_4 is in the ratio 2 : 1.

$$\therefore \quad V_4 = \frac{300 \times 2}{2+1} = \mathbf{200\ V}$$

$q_4 = C_4V_4 = (100 \times 10^{-12})\ 200 = \mathbf{2 \times 10^{-8}\ C}$

Pot. diff. across 200 pF = 300 – 200 = 100 V

This must be the pot. diff. across C_1

i.e. $\quad V_1 = \mathbf{100\ V}$

$\therefore \quad q_1 = C_1V_1 = (100 \times 10^{-12}) \times 100 = \mathbf{10^{-8}\ C}$

Also, pot. diff. across C_2, C_3 in series = 100 V

$$\therefore \quad V_2 = V_3 = \frac{100}{2} = \mathbf{50\ V}$$

$q_2 = C_2V_2 = (200 \times 10^{-12}) \times 50 = \mathbf{10^{-8}\ C}$,

$q_3 = C_3V_3 = (200 \times 10^{-12}) \times 50 = \mathbf{10^{-8}\ C}$

TYPE IX. ENERGY STORED IN A CAPACITOR AND LOSS OF ENERGY ON SHARING CHARGES

Formulae used.

1. Energy, $U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

2. In series as well as parallel combination of capacitors, total energy stored is sum of the energies stored in individual capacitors.

3. Common Potential,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

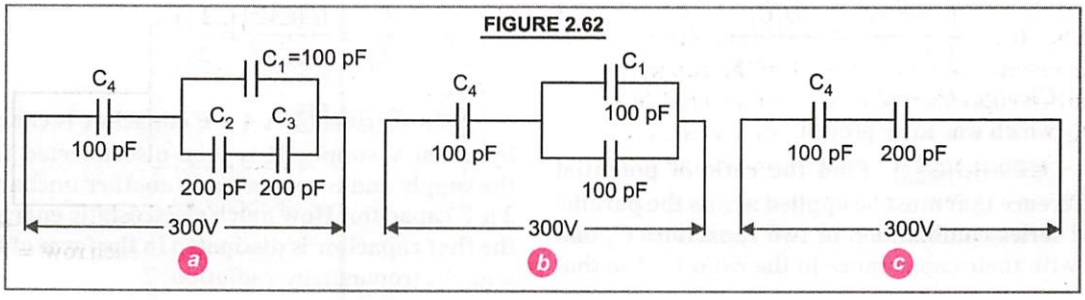
4. Loss of energy on sharing charges,

$$U_1 - U_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

Units used. Charge Q in coulomb, potential V in volt, capacity C in farad and energy U in joule.

Example 42 Find the ratio of potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that energy stored in the two cases becomes the same. (CBSE 2016)

FIGURE 2.62



Solution. Here $\frac{C_1}{C_2} = \frac{1}{2}$ or $C_2 = 2\,C_1$

$$C_p = C_1 + C_2 = C_1 + 2\,C_1 = 3\,C_1$$

$$C_s = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1 \times 2C_1}{C_1 + 2C_1} = \frac{2}{3}\,C_1$$

If V_1 is p.d applied across parallel combination and V_2 is p.d applied across series combination, then as $E_1 = E_2$

$$\frac{1}{2}\,C_p\,.V_1^2 = \frac{1}{2}\,C_s\,V_2^2$$

$$\frac{V_1}{V_2} = \sqrt{\frac{C_s}{C_p}} = \sqrt{\frac{2}{3} \times \frac{C_1}{3C_1}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$$

Example 43 **A capacitor is charged to potential V_1. The power supply is disconnected and capacitor is connected in parallel to another uncharged capacitor. Calculate common potential of the combination of capacitors. Show that total energy of the combination is less than sum of energies stored in them before they are connected.** **(CBSE 2015)**

Solution. Let C_1 and C_2 be the capacitances of the two capacitors and V be their common potential.

$$\text{As } V = \frac{\text{total charge}}{\text{total capacity}} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{C_1V_1}{C_1 + C_2} \quad (\because V_2 = 0)$$

Initial energy stored, $U_1 = \frac{1}{2}\,C_1\,V_1^2$

Final energy after connection,

$$U_2 = \frac{1}{2}\,(C_1 + C_2)\,V^2$$

$$U_2 = \frac{1}{2}\,(C_1 + C_2)\,\frac{(C_1V_1)^2}{(C_1 + C_2)^2}$$

$$= \frac{1}{2}\,\frac{C_1^2\,V_1^2}{C_1 + C_2} = \frac{U_1C_1}{C_1 + C_2}$$

Clearly, $U_2 < U_1$.

which was to be proved.

Example 44 **Find the ratio of potential difference that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitance in the ratio 1 : 3 so that energy stored in the two cases becomes the same.** **(CBSE 2010)**

Solution. Here $\frac{C_1}{C_2} = \frac{1}{3}$

If $C_1 = C,\ C_2 = 3\,C$

$$C_p = C_1 + C_2 = C + 3\,C = 4\,C$$

$$C_s = \frac{C_1\,C_2}{C_1 + C_2} = \frac{C\,(3C)}{C + 3C} = \frac{3C}{4}$$

If V_1 and V_2 are potential differences applied across the parallel and series combination, then as

$$\frac{1}{2}\,C_p\,V_1^2 = \frac{1}{2}\,C_s\,V_2^2$$

$$\frac{V_1}{V_2} = \sqrt{\frac{C_s}{C_p}} = \sqrt{\frac{3C/4}{4C}} = \frac{\sqrt{3}}{4}$$

Example 45 **A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor.** **(CBSE 2019)**

Solution. Let C be the capacity of parallel plate capacitor. When it is charged to a potential difference V, initial energy of the capacitor,

$$E_1 = \frac{1}{2}\,CV^2$$

When it is connected to another uncharged capacitor of the same capacitance, the charge is shared equally between the two. Potential of each capacitor becomes $V/2$.

$\therefore$ Energy stored in the combination,

$$E_2 = \frac{1}{2}\,(C + C)\left(\frac{V}{2}\right)^2 = \frac{1}{4}\,CV^2$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{1/4CV^2}{1/2CV^2} = \frac{1}{2}$$

Example 46 **A 4 μ F capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 2 μ F capacitor. How much electrostatic energy of the first capacitor is dissipated in the form of heat and electromagnetic radiation ?**

Solution. Here, $C_1 = 4\ \mu F = 4 \times 10^{-6}$ F,
$V_1 = 200$ volt

Initial electrostatic energy stored in C_1 is

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 4 \times 10^{-6} \times 200 \times 200$$
$$= \mathbf{8 \times 10^{-2}\ joule}$$

When 4 μ F capacitor is connected to uncharged capacitor of 2 μ F, charge flows and both acquire a common potential

$$V = \frac{\text{total charge}}{\text{total capacity}};$$

$$V = \frac{C_1 V_1}{C_1 + C_2} = \frac{4 \times 10^{-6} \times 200}{(4+2)10^{-6}} = \frac{800}{6} \text{ volt}$$

∴ Final electrostatic energy of both capacitors

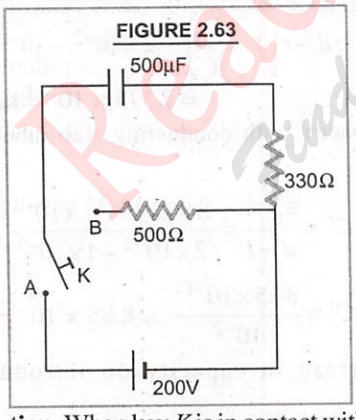
$$U_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2} \times 6 \times 10^{-6} \times \frac{800}{6} \times \frac{800}{6}$$
$$= \mathbf{5.33 \times 10^{-2}\ joule}$$

∴ Energy dissipated in the form of heat and electromagnetic radiation

$$U_1 - U_2 = 8 \times 10^{-2} - 5.33 \times 10^{-2}$$
$$= \mathbf{2.67 \times 10^{-2}\ joule}$$

Example 47 **In the circuit shown in Fig. 2.63, the key *K* was initially in contact with the terminal *A*. What amount of heat will be generated in 500 Ω resistance, when the key *K* is brought in contact with terminal *B* ?**

FIGURE 2.63



Solution. When key *K* is in contact with terminal *A*, the condenser gets charged. Energy stored in condenser

$$U = \frac{1}{2} CV^2 = \frac{1}{2}(500 \times 10^{-6}) \times 200 \times 200$$
$$= 10 \text{ joule.}$$

When key *K* is brought in contact with terminal *B*. The entire energy stored is dissipated as heat energy.

As 500 Ω and 330 Ω resistances are in series, heat produced is in the ratio 500 : 330.

∴ Heat produced in 500 ohm resistance

$$= \frac{500}{500 + 330} \times U = \frac{500 \times 10}{830} = \mathbf{6{\cdot}024\ J}$$

Example 48 **A 900 pF capacitor is charged by 100 V battery.**

How much electrostatic energy is stored by the capacitor ? The capacitor is disconnected from the battery and connected to another 900 pF capacitor. How much is the electrostatic energy stored in the system ? **(NCERT Solved Example)**

Solution. Here, $C = 900$ pF $= 900 \times 10^{-12}$ F
$= 9 \times 10^{-10}$ F, V = 100 V

Energy stored,

$$U_1 = \frac{1}{2} CV^2 = \frac{1}{2} \times 9 \times 10^{-10} (100)^2$$
$$= \mathbf{4{\cdot}5 \times 10^{-6}\ J}$$

On connecting to another 900 pF capacitor, charge is shared equally.

$$\therefore \quad Q' = \frac{1}{2} Q \quad \text{and} \quad V' = \frac{1}{2} V$$

Total energy of the system

$$U_2 = 2 \times \frac{1}{2} Q'V' = \frac{1}{2} Q \times \frac{1}{2} V$$
$$= \frac{1}{2}\left(\frac{1}{2} QV\right) = \frac{1}{2} \times 4{\cdot}5 \times 10^{-6} \text{ J}$$
$$= \mathbf{2{\cdot}25 \times 10^{-6}\ J}$$

Thus half the energy is lost in the form of heat and electromagnetic radiation.

Example 49 **Keeping the voltage of the charging source constant, what would be the percentage change in the energy stored in a parallel plate capacitor, if the separation between its plates were to be decreased by 10% ?**

Solution. Energy stored in a parallel plate capacitor is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{\epsilon_0 A}{d} V^2 \quad ...(i)$$

When separation between the plates is decreased by 10%,

$$d' = d - \frac{10}{100}d = 0{\cdot}9\,d$$

$\therefore$ Change in energy,

$$(U' - U) = \frac{1}{2}\epsilon_0 AV^2\left[\frac{1}{0{\cdot}9} - \frac{1}{1}\right] = \frac{0{\cdot}1}{0{\cdot}9}U = \frac{U}{9}$$

% age change in energy

$$= \frac{(U' - U)}{U} \times 100 = \frac{100}{9} = \mathbf{11{\cdot}1\%}$$

TYPE X. EFFECT OF INTRODUCING A CONDUCTOR/DIELECTRIC SLAB BETWEEN THE PLATES OF A PARALLEL PLATE CAPACITOR

Formulae used. 1. Capacity of a parallel plate capacitor with a conducting slab of thickness t inbetween the plates,

$$C = \frac{\epsilon_0 A}{d - t}$$

2. Capacity of a parallel plate capacitor with dielectric slab of thickness t inbetween the plates,

$$C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

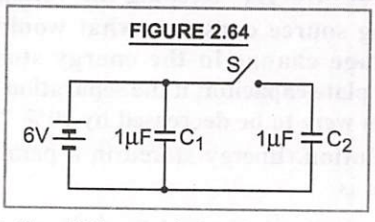
In both cases, $t < d$

Units used. C is in farad ; A in m^2 ; d, t in metre.

Standard Value. $\epsilon_0 = 8{\cdot}85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

Example 50 **Fig. 2.64, shows two identical capacitors C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially, switch S is closed. After some time, the switch S is left open and dielectric slabs of $K = 3$ are inserted to fill completely the space between the plates of two capacitors. How will (*i*) the charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted ?** (CBSE 2011)

FIGURE 2.64



Solution. When switch S is closed,

$V_1 = V_2 = 6\text{V}$

$q_1 = q_2 = CV = 1\ \mu\text{F} \times 6\text{V} = \mathbf{6\ \mu C}$

When dielectric slab of $K = 3$ is inserted,

$$C_2' = C_1' = 1 \times 3\,\mu F = 3\,\mu F$$

p.d across $C_1' = 6\text{ V}$,

$\therefore$ charge on $C_1' = 3\,\mu F \times 6V = \mathbf{18\ \mu C}$

When switch S is left open, charge q_2 on C_2 is still 6 μC, but its potential changes to

$$V_2' = \frac{q_2}{C_2'} = \frac{6\,\mu C}{3\,\mu F} = \mathbf{2\ V}$$

Example 51 **A dielectric slab of thickness 1.0 cm and dielectric constant 5 is placed between the plates of a parallel plate capacitor of plate area 0·01 m^2 and separation 2·0 cm. Calculate the change in capacity on introduction of dielectric. What would be the change, if the dielectric slab were conducting ?**

Solution. Here, $t = 1.0\text{ cm} = 10^{-2}$ m,

$\epsilon_r = K = 5$, $A = 0.01\ m^2 = 10^{-2}\ m^2$,

$d = 2\text{ cm} = 2 \times 10^{-2}$ m

Capacity with air inbetween the plates

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{8{\cdot}85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2}}$$

$$= 4.425 \times 10^{-12}\text{ farad}$$

Capacity with dielectric slab inbetween the plates

$$C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{8{\cdot}85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2} - 10^{-2}\left(1 - \frac{1}{5}\right)}$$

$$= 7.375 \times 10^{-12}\text{ farad}$$

Capacity with conducting slab inbetween the plates

$$C' = \frac{\epsilon_0 A}{d - t} = \frac{8{\cdot}85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2} - 1 \times 10^{-2}}$$

$$C' = \frac{8{\cdot}85 \times 10^{-14}}{10^{-2}} = 8.85 \times 10^{-12}\text{ farad.}$$

Increase in capacity on introduction of dielectric

$$= C - C_0 = 7.375 \times 10^{-12} - 4.425 \times 10^{-12}$$

$$= \mathbf{2.95 \times 10^{-12}\ farad}$$

Increase in capacity on introduction of conducting slab

$$= C' - C_0 = 8.85 \times 10^{-12} - 4.425 \times 10^{-12}$$

$$= \mathbf{4.425 \times 10^{-12}\ farad}$$

Example 52 **A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor, but has a thickness $\left(\frac{3}{4}d\right)$, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates ?** **(NCERT Solved Example)**

Solution. Here, $t = \frac{3}{4}$d

From $$C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{\epsilon_0 A}{d - \frac{3}{4}d\left(1 - \frac{1}{K}\right)}$$

$$= \frac{\epsilon_0 A}{\frac{d}{4} + \frac{3d}{4K}} = \frac{\epsilon_0 A}{\frac{d}{4}\left(1 + \frac{3}{K}\right)}$$

$$C = \frac{\epsilon_0 A}{d}\frac{4K}{K+3} = C_0\left(\frac{4K}{K+3}\right)$$

Example 53 **A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm thick slab is introduced between the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2·4 mm. Find the dielectric constant of the slab.**

Solution. Here, $t = 3$ mm, $x = 2{\cdot}4$ mm, $K = ?$

With air as dielectric, $C_0 = \frac{\epsilon_0 A}{d}$

If d_1 is new distance of separation between the plates when dielectric is introduced, then

$$C_0 = \frac{\epsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)}$$

As $C = C_0$, therefore, $d_1 - t\left(1 - \frac{1}{K}\right) = d$

or $$t\left(1 - \frac{1}{K}\right) = d_1 - d = x$$

As $t = 3$ mm and $x = d_1 - d = 2{\cdot}4$ mm

$$\therefore \quad 3\left(1 - \frac{1}{K}\right) = 2{\cdot}4$$

$$1 - \frac{1}{K} = \frac{2{\cdot}4}{3} = 0{\cdot}8$$

$$\frac{1}{K} = 1 - 0{\cdot}8 = 0{\cdot}2, \quad K = 5$$

TYPE XI. TYPICAL EXAMPLES

Example 54 **An infinitely long positively charged wire has a linear charge density λ Cm^{-1}. An electron is revolving around the wire as its centre with a constant velocity in a circular plane perpendicular to the wire. Deduce the expression for KE of electron. Plot a graph of K.E. as a function of charge density λ.** **(CBSE 2013)**

Solution. The electostatic force exerted by the infinitely long line charge provides the necessary centripetal force to the revolving electron.

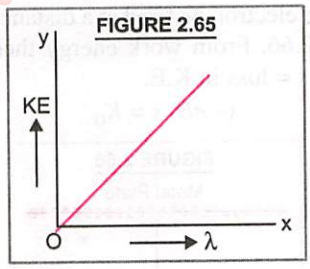
$$\therefore \quad eE = \frac{mv^2}{r}, \text{ but } E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\therefore \quad \frac{e\lambda}{2\pi\epsilon_0 r} = \frac{mv^2}{r} \text{ or } v^2 = \frac{e\lambda}{2\pi\epsilon_0 m}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{e\lambda}{2\pi\epsilon_0 m}\right) = \frac{e\lambda}{4\pi\epsilon_0}$$

As $KE \propto \lambda$, therefore, graph of KE as a function of charge density will be a straight line as shown in Fig. 2.65.

FIGURE 2.65



Example 55 **Two charges 4 μC and – 4μC are placed at (– 3, 0, 0) and (3, 0, 0) cm respectively in an external field given by $E = \frac{9\times10^6}{r^2}$ C m^{-2}. Find the energy of the system in this external field.**

Solution. Here, $q_1 = 4\ \mu C = 4 \times 10^{-6}$ C ;

$r_1 = -3$ cm $= -3 \times 10^{-2}$ m

$q_2 = -4\ \mu C = -4 \times 10^{-6}$ C ;

$r_2 = 3$ cm $= 3 \times 10^{-2}$ m

$r_{12} = 3 - (-3)$ cm $= 6$ cm $= 6 \times 10^{-2}$ m

From $E = \frac{dV}{dr}$, $dV = E\,(dr)$ or $V = E\,(r)$.

$$\text{Energy} = q_1 V_1 + q_2 V_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

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$$= q_1 \times E \times r_1 + q_2 \times E \times r_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

$$\text{Energy} = \frac{4\times10^{-6}\times9\times10^{6}\times r_1}{r_1^2} + \frac{(-4\times10^{-6})\times9\times10^{6}\times r_2}{r_2^2} + \frac{9\times10^{9}\,(4\times10^{-6})\,(-4\times10^{-6})}{6\times10^{-2}}$$

$$\text{Energy} = \frac{36}{3\times10^{-2}} - \frac{36}{3\times10^{-2}} - \frac{16\times9}{60} = -2{\cdot}4 \text{ J}$$

Example 56 **An electron is fired directly towards the centre of a large metal plate that has excess negative charge with surface charge density $= 2{\cdot}0 \times 10^{-6}$ C/m^2. If the initial kinetic energy of electron is 100 eV, and if it is to stop due to repulsion just as it reaches the plate, how far from the plate must it be fired ?**

Solution. Here, $\sigma = -2{\cdot}0 \times 10^{-6}$ C/m^2

Initial K.E. $= K_0 = 100$ eV

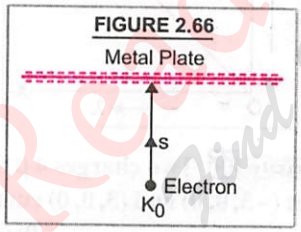
$= 100 \times 1{\cdot}6 \times 10^{-19}$ J

$= 1{\cdot}6 \times 10^{-17}$ J

Let the electron be fired at a distance s from the plate, Fig. 2.66. From work energy theorem, work done $= F \times s =$ loss in K.E.

$$(-eE)\, s = K_0$$

FIGURE 2.66



Now, electric field due to the plate (having some thickness), $E = \sigma/\epsilon_0$

$$\therefore \quad -e\frac{\sigma}{\epsilon_0}\, s = K_0$$

$$s = \frac{\epsilon_0\, K_0}{-e\sigma} = \frac{8{\cdot}85\times10^{-12}\times1{\cdot}6\times10^{-17}}{-1{\cdot}6\times10^{-19}\,(-2\times10^{-6})}$$

$$= 4{\cdot}43 \times 10^{-4} \text{ m}$$

Example 57 **Assuming an expression for the potential of an isolated conductor, show that the capacitance of such a sphere will be increased by a factor n, if it is enclosed within an earthed concentric sphere, the ratio of radii of the spheres being $n/(n-1)$.**

Solution. The capacitance of an isolated conducting sphere of radius a is $C = 4\pi\epsilon_0 a$

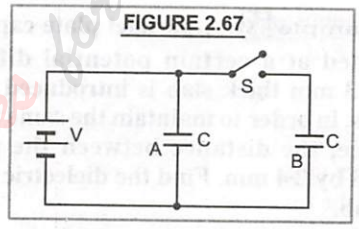
When surrounded by an earthed sphere of radius b, its capacitance becomes

$$C' = 4\pi\epsilon_0 \frac{ab}{a-b}$$

$$\frac{C'}{C} = \frac{ab}{a(b-a)} = \frac{b}{b-a} = \frac{1}{1-\frac{a}{b}} = \frac{1}{1-\frac{n-1}{n}} = n$$

Example 58 **Fig. 2.67 shows two identical parallel plate capacitors connected to a battery with a switch S closed. The switch is now opened and the free space between the plates of the capacitors is now filled with a dielectric of dielectric constant 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of dielectric.**

FIGURE 2.67



Solution. Initially, when the switch S is closed, both the capacitors have same potential difference (V) across them.

$\therefore$ Initial energy stored in both the capacitors is

$$U_i = U_A + U_B = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

When the dielectric ($K = 3$) is introduced, the capacitance of each capacitor becomes 3 C. The p.d. across A is still V as it is still connected across the battery, with switch S open, P.d across B attains new value but charge remains constant

$$q = CV = 3\,C \times V' \quad \text{or} \quad V' = V/3.$$

So final energy $U_f = U_A' + U_B'$

$$= \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{5C}{3}V^2$$

$$\frac{U_i}{U_f} = \frac{CV^2}{\frac{5}{3}CV^2} = 3:5$$

Example 59 **A parallel plate capacitor contains a mica sheet of thickness $d_1 = 10^{-3}$ m and one fibre sheet of thickness $d_2 = 0{\cdot}5\times10^{-3}$m. Values of K for mica and fibre are 8 and 2·5 respectively. Fibre breaks down in electric field of $6{\cdot}4 \times 10^6$ Vm^{-1}. What maximum voltage can be applied to the capacitor ?**

Solution. Let σ be the surface charge density of capacitor plates.

$$\text{for mica } E_1 = \frac{\sigma}{K_1 \epsilon_0}$$

$$\text{and for fibre } E_2 = \frac{\sigma}{K_2 \epsilon_0}$$

$$\text{or} \quad \frac{E_1}{E_2} = \frac{K_2}{K_1}$$

As $E_2 = 6{\cdot}4 \times 10^6$ V/m

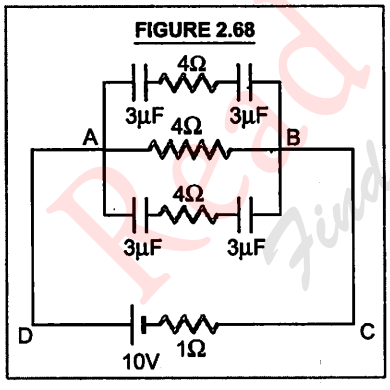
$$E_1 = \frac{K_2}{K_1} \times E_2 = \frac{2{\cdot}5}{8} \times 6{\cdot}4 \times 10^6 = 2 \times 10^6 \text{ V/m}$$

Maximum voltage for capacitor

$$V = E_1 d_1 + E_2 d_2 = 2 \times 10^3 + 3{\cdot}2 \times 10^3$$
$$= 5200 \text{ V}$$

Example 60 **In the circuit shown in Fig. 2.68, each capacitor has a capacity of 3 μF. Calculate the quantity of charge on each capacitor.**

FIGURE 2.68



Solution. Total resistance in the circuit *ABCD*,

$$R = 4 + 1 = 5\ \Omega.$$

$$\therefore \text{ Current } I = \frac{V}{R} = \frac{10}{5} = 2 \text{ A}$$

Potential diff. across *A* and $B = I \times 4 = 2 \times 4 = 8$ V

As two capacitors of 3 μ *F* each are in series,

$$\therefore \text{ pot. diff. across each condenser } = \frac{8}{2} = 4 \text{ V}$$

charge on each condenser, $q = CV = 3 \times 4$
$= 12\ \mu C$

Example 61 **The area of each plate of parallel plate air capacitor is 150 cm^2. The distance between its plates is 0·8 mm. It is charged to a pot. diff. of 1200 volt. What will be its energy ? What will be the energy when it is filled with a medium of $K = 3$ and then charged. If it is charged first as an air capacitor and then filled with this dielectric, what will happen to energy ?**

Solution. Here, $A = 150\ cm^2 = 150 \times 10^{-4}\ m^2$, $d = 0{\cdot}8$ mm $= 8 \times 10^{-4}$ m, $V_0 = 1200$ V, $C_0 = ?$ $E_0 = ?$

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{8{\cdot}85\times10^{-12} \times 150 \times 10^{-4}}{8\times10^{-4}}$$
$$= 1{\cdot}66 \times 10^{-10} \text{ F}$$

$$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \times 1{\cdot}66 \times 10^{-10} \times (1200)^2$$
$$= 1{\cdot}2 \times 10^{-4} \text{ J}$$

With dielectric medium, capacity becomes $C = KC_0 = 3 \times 1{\cdot}66 \times 10^{-10}$ farad

When charged to same potential, $V = 1200$ V,

Energy, $U = \frac{1}{2} CV^2 = \frac{1}{2} (KC_0) V_0^2 = K(E_0)$
$= 3 \times 1{\cdot}2 \times 10^{-4} = 3{\cdot}6 \times 10^{-4}$ J

When capacitor is charged first as air capacitor, then on filling with the dielectric, its potential becomes $V = \frac{V_0}{K} = \frac{1200}{3} = 400$ volt, because capacity becomes 3 times whereas charge remains the same.

$\therefore$ New energy of capacitor

$$= \frac{1}{2} CV^2 = \frac{1}{2} (KC_0) \left(\frac{V_0}{K}\right)^2$$

$$= \frac{1}{2} \frac{C_0 V_0^2}{K} = \frac{1{\cdot}2\times10^{-4}}{3} = 4 \times 10^{-5} \text{ joule}$$

Example 62 **Two identical metal plates are given charges q_1 and q_2 ($< q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance *C*, what will be the potential difference between the plates ?**

Solution. Let *A* be the area of each metal plate. If they are distance *d* apart, then the capacitance of the parallel plate capacitor so formed will be

$$C = \frac{\epsilon_0 A}{d}$$

If E_1, E_2 are electric field intensities due to the two plates (in the space between the plates), then

$$E_1 = \frac{q_1/A}{2\epsilon_0} \quad \text{and} \quad E_2 = \frac{q_2/A}{2\epsilon_0}$$

Net electric field between the two plates

$$E = E_1 - E_2 = \frac{q_1/A}{2\epsilon_0} - \frac{q_2/A}{2\epsilon_0} = \frac{1}{2\epsilon_0 A}(q_1 - q_2)$$

The potential difference between the plates of the capacitor,

$$V = E \times d = \frac{d}{2\epsilon_0 A}(q_1 - q_2)$$

$$V = \frac{q_1 - q_2}{2(\epsilon_0 A/d)} = \frac{q_1 - q_2}{2C}$$

Example 63 **A capacitor of capacitance C_1 = 1·0 microfarad withstands the maximum voltage V_1 = 6·0 kilo volt. While another capacitor of capacitance C_2 = 2·0 microfarad withstands the maximum voltage V_2 = 4·0 kilovolt. What maximum voltage will the system of these two capacitors withstand if they are connected in series ?**

Solution. Here

$C_1 = 1{\cdot}0\ \mu F$, $V_1 = 6{\cdot}0\ kV = 6 \times 10^3\ V$

$\therefore$ Charge on first capacitor

$q_1 = C_1 V_1 = 1{\cdot}0 \times 6 \times 10^3\ \mu C = 6000\ \mu C$

Similarly, charge on 2nd capacitor

$q_2 = C_2 V_2 = 2{\cdot}0 \times 4 \times 10^3\ \mu C = 8000\ \mu C$

In series combination, charge on each capacitor must be the same. As max. charge on C_1 is 6000 μ C ; therefore, max. charge on C_2 must also be 6000 μ C.

Hence maximum voltage for the combination is

$$V' = V_1' + V_2' = \frac{6000}{1{\cdot}0} + \frac{6000}{2{\cdot}0} = 9000\ V = \mathbf{9\ kV}$$

Example 64 **A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C m. A mole of this substance is polarized at low temperature by applying a strong electrostatic field of magnitude 10^6 V m^{-1}. The direction of the field is suddenly changed by an angle of 60°. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation of sample.** (NCERT Solved Example)

Solution. Here, dipole moment of each molecule = 10^{-29} C m

As one mole of substance contains 6×10^{23} molecules, therefore, total dipole moment of all the molecules, $p = 10^{-29} \times 6 \times 10^{23} = 6 \times 10^{-6}$ C m. As polarisation is 100%, therefore, initial potential energy

$U_1 = -pE\cos 0° = -6 \times 10^{-6} \times 10^6 = -6$ J

When θ = 60°, final potential energy

$U_2 = -pE\cos 60°$

$$= -6 \times 10^{-6} \times 10^6 \times \frac{1}{2} = -3\ J$$

Change in *P.E.* $= U_2 - U_1 = -3 - (-6) = \mathbf{3\ J}$

This loss in *P.E.* is the energy released in the form of heat in aligning the dipoles.

Example 65 **A student requires a capacitor of 3 μF in a circuit across a potential of 1000 V. A large number of 2 μF capacitors are available to him, each of which can withstand a potential difference of not more than 300 V. How should the student arrange these capacitors so that he may use minimum number of condensers ?**

Solution. Let the student use minimum number of N capacitors in m rows in parallel and n condensers in series in each row.

$$\therefore \quad N = m \times n \qquad ...(i)$$

As each capacitor in a row can withstand 300 V, therefore, voltage across each row

$$= 300\ n = 1000\ V$$

$$n = \frac{1000}{300} = 3{\cdot}3$$

$\therefore$ There should be **four** capacitors in each row. Capacitance of each row

therefore, $C_s = \frac{C}{4} = \frac{2}{4} = 0{\cdot}5\ \mu F$

As m such rows are in parallel,

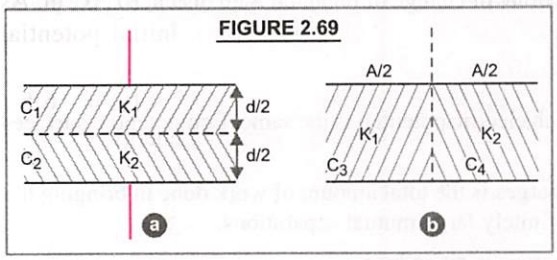
$$m = \frac{C'}{Cs} = \frac{3\mu F}{0{\cdot}5\mu F} = \mathbf{6}$$

From (i), $N = 6 \times 4 = \mathbf{24}$

This is the minimum number of capacitors required by the student.

Example 66 **A capacitor is filled with two dielectrics of same dimensions, but of dielectric constants 2 and 3 respectively. Find the ratio of**

capacitances in the two arrangements shown in Fig. 2.69(a) and (b).



Solution. As is clear from Fig. 2.69(a)

$$C_1 = \frac{\epsilon_0 AK_1}{d/2} \; ; \quad C_2 = \frac{\epsilon_0 AK_2}{d/2}$$

These capacitors are joined in series, therefore, equivalent capacity

$$C = \frac{C_1C_2}{C_1+C_2} = \frac{\left(\frac{2\epsilon_0 K_1A}{d}\right)\left(\frac{2\epsilon_0 K_2A}{d}\right)}{\frac{2\epsilon_0 A}{d}(K_1+K_2)}$$

$$C = \frac{2\epsilon_0 AK_1K_2}{d(K_1+K_2)} \qquad ...(i)$$

In Fig. 2.69(b),

$$C_3 = \frac{\epsilon_0 K_1(A/2)}{d} \; ; \quad C_4 = \frac{\epsilon_0 K_2(A/2)}{d}$$

These capacitors are joined in parallel, therefore equivalent capacity

$$C' = C_3 + C_4 = \frac{\epsilon_0 A}{2d}(K_1+K_2) \qquad ...(ii)$$

Now, $$\frac{C}{C'} = \frac{2\epsilon_0 AK_1K_2}{d(K_1+K_2)} \times \frac{2d}{\epsilon_0 A(K_1+K_2)}$$

$$= \frac{4K_1K_2}{(K_1+K_2)^2} = \frac{4\times2\times3}{(2+3)^2} = \frac{24}{25}$$

Example 67 **A parallel plate capacitor of plate area (1 × 2) m^2 and plate separation 5·0 mm is charged to 10 kV in air. Find charge density and field displacement.**

Solution. Here, $A = 1 \times 2 = 2\ m^2$

$d = 5\ mm = 5 \times 10^{-3}\ m$

$V = 10\ kV = 10^4\ V$, $\rho = ?$

field displacement = ?

$$Q = CV = \frac{\epsilon_0 A}{d}V = \frac{8{\cdot}85\times10^{-12}\times2}{5\times10^{-3}}\times10^4$$

$$Q = 3{\cdot}54 \times 10^{-5}\ C$$

Charge density, $$\rho = \frac{Q}{A} = \frac{3{\cdot}54\times10^{-5}}{2}$$

$$= \mathbf{1{\cdot}77 \times 10^{-5}\ Cm^{-2}}$$

Field displacement = charge density

$$= \mathbf{1{\cdot}77 \times 10^{-5}\ Cm^{-2}}$$

REVISION AT A GLANCE

1. ***Electrostatic potential difference*** between two points B and A in an electrostatic field is the amount of work done in carrying unit positive test charge from A to B (against the electrostatic force of the field) along any path between the two points, *i.e.*, $V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B \vec{E}.\vec{dl}$

If A is at ∞, $V_A = 0$, we may write, $V_B = \frac{W_{\infty B}}{q_0} = -\int_\infty^B \vec{E}.\vec{dl}$

This means electrostatic potential at a point in the electrostatic field is the amount of work done in moving a unit positive test charge from infinity to that point, against the electrostatic forces, along any path.

Electrostatic potential is a ***scalar*** and it is measured in ***volt.***

Electric potential at a point distant r from a point charge q is,

$$V = \frac{q}{4\pi\epsilon_0 r}$$

Electric potential at a point due to a group of charges and due to continuous charge distribution can be calculated similarly using the superposition principle.

Further, electric field intensity at any point is negative rate of change of potential with distance at the point,

i.e., $$|\vec{E}| = -\frac{dV}{dr}$$

2. ***An equipotential surface*** is that at every point of which electric potential is the same. Equipotential surfaces are always perpendicular to the field lines.

3. ***Electrostatic potential energy*** of a system of point charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.

Electric potential energy of a system of two point charges is given by :

$$U = \frac{q_1 q_2}{4\pi \epsilon_0 r_{12}}$$

SI unit of U is joule. Another convenient unit of energy is *electron volt* (eV), where 1 eV = 1.6×10^{-19} joule

4. ***Conductors*** are the materials which possess a large number of free charge carriers whereas *Insulators* or *dielectrics* possess a negligibly small number of free charge carriers.

When a conductor is placed in an external electric field, net electric field in the interior of a conductor is zero, there is no charge in the interior of the conductor, *i.e.*, charge always resides on the outer surface of the conductor. The surface of the conductor is an equipotential surface and electric field over a charged conductor is perpendicular to the surface of the conductor at every point.

5. ***Capacitance*** (C) of a conductor is the ratio of charge (Q) given to conductor and the potential (V) to which it is raised, *i.e.*, $C = Q/V$. The value of C depends on size, shape and relative position of two coatings of the capacitor. It also depends on nature of medium separating the two coatings.

Capacity of an isolated spherical conductor of radius r in vacuum is,

$$C = 4\pi \epsilon_0 r$$

Capacity of a parallel plate capacitor with air as dielectric is

$$C = \frac{\epsilon_0 A}{d}$$

where A is area of each plate and d is distance between them.

When the insulating medium separating the two conductors in a capacitor is other than air, its capacity becomes K times the capacity in air. Here, K is *dielectric constant* or *relative electrical permittivity of the medium. For mica,* K = 3 to 6 ; for paper K = 3.5, for water, K = 81, for ebonite, K = 2.7 and for glass, K varies from 3 to 4.

6. When ***condensers are grouped in series***, total capacity C_s is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

In ***parallel grouping***, total capacity C_p is sum of the individual capacities, *i.e.*, $C_p = C_1 + C_2 + C_3 ....$

7. Work done in charging a capacitor is stored in it in the form of ***electrostatic energy***, given by

$$U = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} \frac{Q^2}{C}$$

when Q is in coulomb, V in volt and C in farad, energy E is in joule.

8. When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher potential to the other at lower potential till their potentials become equal. The equal potential is called ***common potential*** (V), where

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

It should be clearly understood that in ***sharing charges***, there is absolutely no loss of charge. Some energy is, however, lost in the process which is given by :

$$U_1 - U_2 = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}$$

9. ***Dielectrics*** are of two types : *Non polar dielectrics* and *polar dielectrics*. The ***non polar dielectrics*** (like N_2, O_2, benzene, methane) etc. are made up of non polar atoms/molecules, in which the centre of positive charge coincides with the centre of negative charge of the atom/molecule.

The ***polar dielectrics*** (like H_2O, CO_2, NH_3, HCl *etc.*) are made up of polar atoms/molecules, in which the centre of positive charge does not coincide with the centre of negative charge of the atom/molecule.

A non polar dielectric can be polarised by applying an external electric field on the dielectric.

The effective electric field ($\vec{E}$) in a polarised dielectric is given by $\vec{E} = \vec{E}_0 - \vec{E}_p$

where $\vec{E}_0$ is strength of external field applied and $\vec{E}_p$ is intensity of induced electric field set up due to polarisation.

10. ***Capacitance*** of a parallel plate capacitor with a *conducting slab* of thickness t is given by :

$$C' = \frac{\epsilon_0 A}{d - t}$$

Capacitance of a parallel plate capacitor with a *dielectric slab* of thickness t is given by :

$$C'' = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Clearly, capacity increases on introducing a conducting slab as well as dielectric slab between the plates of an air capacitor.

11. ***A Van de graaff generator*** is a powerful machine used for generating high positive potentials $\approx 10^6$ volt. It is based on the action of sharp points and the property that charge always resides on the outer surface of a hollow conductor. Such high positive potentials are needed for accelerating positive ions.

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OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. The electric potential on the axis of an electric dipole at a distance r from its centre is V. Then the potential at a point at the same distance on its equatorial line will be

(a) 2 V (b) $-V$

(c) $V/2$ (d) Zero

(CBSE Sample Paper 2022-23)

2. A car battery is charged by a 12 V supply, and energy stored in it is $7{\cdot}20 \times 10^5$ J. The charge passed through the battery is

(a) $6{\cdot}0 \times 10^4$ C (b) $5{\cdot}8 \times 10^3$ J

(c) $8{\cdot}64 \times 10^6$ J (d) $1{\cdot}6 \times 10^5$ C

(CBSE 2022)

3. Equipotentials at a large distance from a collection of charges, whose total sum is not zero are

(a) spheres (b) planes

(c) ellipsoids (d) parabosides

(CBSE 2022)

4. If net electric flux through a closed surface is zero, we can infer

(a) no net charge is enclosed by the surface

(b) uniform electric field exists within the surface

(c) charge is present inside the surface

(d) electric potential varies from point to point inside the surface. (CBSE 2022)

5. The basic cause of charging is

(a) actual transfer of protons

(b) actual transfer of neutrons

(c) actual transfer of electrons

(d) none of the above

6. A body carries 500 excess electrons. Charge on the body is

(a) -8×10^{-17} C (b) 8×10^{17} C

(c) $1{\cdot}6 \times 10^{-19}$ C (d) $1{\cdot}6 \times 10^{19}$ C

7. The SI unit of electric flux is

(a) NC^{-1} (b) NC^{-2}

(c) $\frac{N}{m^2}C^{-1}$ (d) $\frac{N}{C}m^2$

8. N/C is the SI unit of

(a) electric charge (b) electric potential

(c) electric intensity (d) none of these

9. Due to a single charge, electric field is

(a) spherically symmetric

(b) cylindrically symmetric

(c) asymmetric (d) none of these

10. Three capacitors 2 μF, 3 μF and 6 μF are joined in series with eachother. The equivalent capacitance is

(a) 5 μF (b) 1 μF

(c) 2 μF (d) 1/2 μF

11. The electric potential at a point on the axis of a short electric dipole at a distance x from the mid-point of dipole is proportional to

(a) $\frac{1}{x^4}$ (b) $\frac{1}{x^{3/2}}$

(c) $\frac{1}{x^3}$ (d) $\frac{1}{x^2}$ (CBSE 2022)

12. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from 2 μF to X μF, the decrease in energy of the capacitor is 2×10^{-2} J. The value of X is

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF (CBSE 2022)

13. Let F_1 be the magnitude of force between two small spheres charged to a constant potential in free space and F_2 be the magnitude of force between them in a medium of dielectric constant K. Then F_1/F_2 is

(a) $\frac{1}{K}$ (b) K

(c) K^2 (d) $\frac{1}{K^2}$

14. The capacity of a condenser increases from 6 μF to 30 μF when a medium of dielectric constant K is introduced between the plates. The permittivity of the medium is

(a) $0{\cdot}44 \times 10^{-10}\ C^2\ N^{-1}\ m^{-2}$

(b) $0{\cdot}44 \times 10^{-8}\ C^2\ N^{-1}\ m^{-2}$

(c) $0{\cdot}44 \times 10^{10}\ C^2\ N^{-1}\ m^{-2}$

(d) None of these

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15. The speed acquired by a free electron when accelerated from rest through a potential diff. of 100 V is
(*a*) 6×10^6 m/s (*b*) 3×10^6 m/s
(*c*) 4×10^5 m/s (*d*) 2×10^3 m/s
(CBSE 2022)

16. Electrostatic potential V at a point, distant r from a charge q in air is
(*a*) q/r^2 (*b*) q^2/r
(*c*) q/r (*d*) q^2/r^2

17. Work done in carrying an electron from A to B lying on an equipotential surface of one volt potential is
(*a*) 1 eV (*b*) 10 eV
(*c*) 1 volt (*d*) Zero

18. The correct relation between electric intensity E and electric potential V is
(*a*) $E = -\frac{dV}{dr}$ (*b*) $E = \frac{dV}{dr}$
(*c*) $V = -\frac{dE}{dr}$ (*d*) $V = \frac{dE}{dr}$

19. A charge of 10 μ C lies at the centre of a square. Work done in carrying a charge of 2 μ C from one corner of square to the diagonally opposite corner is
(*a*) 20 J (*b*) 5 J
(*c*) Zero (*d*) 20 μ J

20. A uniform electric field of 100 N/C exists in vertically upward direction. The decrease in electric potential as one goes up through a height of 5 cm is
(*a*) 20 V (*b*) 120 V
(*c*) 5 V (*d*) Zero

21. When a conductor is held in an electric field, the field inside the conductor is always
(*a*) positive (*b*) negative
(*c*) constant (*d*) zero

22. Charge on a capacitor is doubled. Its capacity becomes k times, where
(*a*) $k = 2$ (*b*) $k = 1$
(*c*) $k = \frac{1}{2}$ (*d*) $k = 4$

23. Electrical capacity of earth is .....
(*a*) 1 F (*b*) 1 μ F
(*c*) 711 μ F (*d*) 9×10^9 μ F

24. When air inbetween the plates of a capacitor is replaced by mica of dielectric constant 6, its capacity
(*a*) remains unaffected (*b*) reduced to 1/6th
(*c*) becomes 6 times (*d*) none of the above

25. When a number of capacitors are connected in series between two points, all the capacitors possess same
(*a*) capacity (*b*) potential
(*c*) charge (*d*) none of the above

26. When a number of capacitors are connected in parallel between two points, the equivalent capacitance
(*a*) increases (*b*) decreases
(*c*) remains the same (*d*) none of the above

27. A condenser is charged to double its initial potential. The energy stored in the condenser becomes x times, where x = ...
(*a*) 2 (*b*) 4
(*c*) 1 (*d*) 1/2

28. When two capacitors charged to different potentials are connected by a conducting wire, what is not true ?
(*a*) Charge lost by one is equal to charge gained by the other
(*b*) Potential lost by one is equal to potential gained by the other
(*c*) Some energy is lost
(*d*) Both the capacitors acquire a common potential

29. The relation between dielectric constant K and electric susceptibility χ of a material is
(*a*) $K = \chi$ (*b*) $K = 1 + \chi$
(*c*) $\chi = K + 1$ (*d*) $K^2 = (1 + \chi)(1 - \chi)$

30. When charge is supplied to a conductor, its potential depends upon
(*a*) the amount of charge
(*b*) geometry and size of conductor
(*c*) both (*a*) and (*b*) (*d*) only on (*a*)

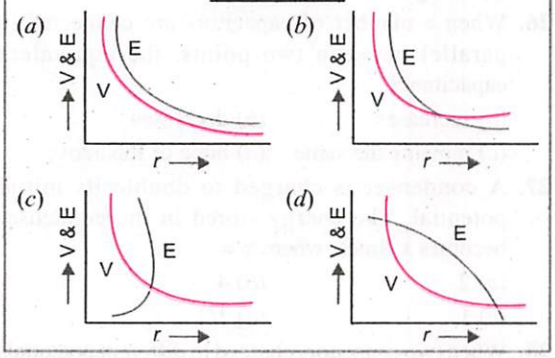
31. A parallel plate capacitor is charged by a battery. Once it is charged, battery is removed. Now a dielectric material is inserted between the plates of the capacitor. Which of the following does not change ?
(*a*) Electric field between the plates
(*b*) Potential difference across the plates
(*c*) Charge on the plates
(*d*) Energy stored in the capacitor.

32. A dipole is placed parallel to electric field. If W is the work done in rotating the dipole from 0° to 60°, then work done in rotating it from 0° to 180° is

(a) 2 W (b) 3 W
(c) 4 W (d) W/2

33. The variation of potential V with r and electric field with r for a point charge is correctly shown in the graphs.

FIGURE 2(Q).1



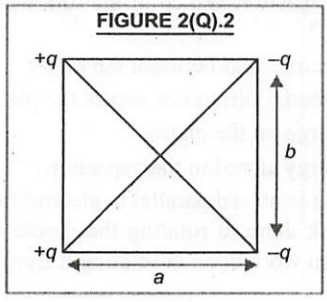
34. A charge Q is supplied to a metallic conductor. Which is true ?
(a) Electric field inside it is same as on the surface
(b) Electric potential inside is zero
(c) Electric potential on the surface is zero
(d) Electric potential inside is constant

35. A parallel plate capacitor C has a charge Q. The actual charges on the plates are
(a) Q, Q (b) $Q/2, Q/2$
(c) $Q, -Q$ (d) $\frac{Q}{2}, \frac{-Q}{2}$

36. Three capacitors of capacitances 1μF, 2μF and 3μF are connected in series and a potential difference of 11 V is applied across the combination, then the potential difference across the plates of 1 μF capacitor is
(a) 2 V (b) 4 V
(c) 1 V (d) 6 V

37. The potential at the centre of the square is–

FIGURE 2(Q).2



(a) zero (b) $\frac{kq}{a\sqrt{2}}$
(c) $\frac{kq}{a^2}$ (d) $\frac{kq}{2a^2}$

38. Two conducting spheres A and B of radii a and b respectively are at the same potential. The ratio of surface charge densities of A and B is
(a) $\frac{b}{a}$ (b) $\frac{a}{b}$
(c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

39. Work done to bring a unit positive charge unaccelerated from infinity to a point inside electric field is called :
(a) Electric field (b) Electric potential
(c) Capacitance (d) Electric flux

40. Electric potential due to a point charge $-q$ at distance x from it is given by :
(a) Kq/x^2 (b) Kq/x
(c) $-Kq/x^2$ (d) $-Kq/x$

41. Electric field is always :
(a) Parallel to equipotential surface
(b) Perpendicular to equipotential surface
(c) It can be perpendicular and parallel as well
(d) It does not depend on distribution of charge

42. Electric field and electric potential inside a charged spherical shell :
(a) $E = 0 ; V = 0$ (b) $E = 0 ; V \neq 0$
(c) $E \neq 0 ; V = 0$ (d) $E \neq 0 ; V \neq 0$

43. Shape of equipotential surface in uniform electric field will be :
(a) Spherical normal to electric field
(b) Random
(c) Circular normal to electric field
(d) Equidistant planes normal to electric field

44. On reducing potential across a capacitor, its capacitance
(a) Decreases
(b) Increases
(c) Remains constant
(d) First increases then decreases

45. Energy stored in a charged capacitor is given by :
(a) $U = CV/2$ (b) $U = CV^2/2$
(c) $2\,CV^2$ (d) $VC^2/2$

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46. If n number of equal capacitors each of capacitance C are connected in series, then equivalent capacitance will be given as :

(*a*) $n \times C$ (*b*) C/n

(*c*) $n + C$ (*d*) n^2C

47. Capacitance of parallel plate capacitor when there is no medium between the plates is C_0. If capacitor is now completely filled with dielectric matter of constant K, the capacitance would be :

(*a*) C_0/K (*b*) KC_0

(*c*) K^2C_0 (*d*) $2\,KC_0$

II. Assertion-Reason Type Questions

Direction. For question numbers 48 to 55, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

48. Assertion. Electrostatic force constant and dielectric constant represent the same thing.

Reason. Electrostatic force constant

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$$= \text{K, dielectric constant}$$

49. Assertion. Electrostatic potential energy of a system of two charges is equal to total amount of workdone in bringing the two charges to their respective positions from infinite separation.

Reason. Potential energy of a system of two charges q_1 and q_2 placed distance r_{12} apart is given by

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}$$

50. Assertion. One electron volt (eV) is the KE gained by an electron when accelerated under a potential difference of 1 volt.

Reason. eV is the SI unit of energy in electrostatics.

51. Assertion. Three charges $+q$, $+2q$ and $-4q$ are placed on the three vertices of an equilateral triangle of each side 0·1 m. If $q = 10^{-7}$ C, then PE of the system of charges is -9×10^{-3} J.

Reason. PE of the system of charges is,

$$U = \frac{1}{4\pi\epsilon_0}\sum_{\substack{\text{all}\\ \text{pairs}}}\frac{q_i q_j}{r_{ij}}.$$

52. Assertion. Electrical capacity of a solid spherical conductor depends upon the radius of conductor and dielectric constant of the material of the conductor.

Reason. The dimensions of the capacitance of a conductor is $[M^{-1} L^{-2} T^4 A^2]$.

53. Assertion. Taking earth to be a sphere of radius 6400 km ; the capacity of earth is 711 μF.

Reason. Capacity of a sphere of radius $R = 4\pi\epsilon_0 R$.

54. Assertion. In a series or parallel combination of capacitors, charge on each capacitor is same.

Reason. The charge is equally shared by capacitors wheather connected in series or parallel.

55. Assertion. There is no loss of energy in the charging process of capacitor.

Reason. Energy spent by the battery in charging a capacitor is QV which is the energy stored in the capacitor.

ANSWERS

I. Multiple Choice Questions

1. (*d*)	**2.** (*a*)	**3.** (*a*)	**4.** (*a*)	**5.** (*c*)	**6.** (*a*)	**7.** (*d*)	**8.** (*c*)	**9.** (*a*)	**10.** (*b*)
11. (*d*)	**12.** (*a*)	**13.** (*b*)	**14.** (*a*)	**15.** (*a*)	**16.** (*c*)	**17.** (*d*)	**18.** (*a*)	**19.** (*c*)	**20.** (*c*)
21. (*d*)	**22.** (*b*)	**23.** (*c*)	**24.** (*c*)	**25.** (*c*)	**26.** (*a*)	**27.** (*b*)	**28.** (*b*)	**29.** (*b*)	**30.** (*c*)
31. (*c*)	**32.** (*c*)	**33.** (*b*)	**34.** (*d*)	**35.** (*c*)	**36.** (*d*)	**37.** (*a*)	**38.** (*a*)	**39.** (*b*)	**40.** (*d*)
41. (*b*)	**42.** (*b*)	**43.** (*d*)	**44.** (*c*)	**45.** (*b*)	**46.** (*b*)	**47.** (*b*)			

II. Assertion-Reason Type Questions

48. (*d*) **49.** (*c*) **50.** (*c*) **51.** (*a*) **52.** (*b*) **53.** (*a*) **54.** (*d*) **55.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. As is known, electric potential on equatorial line of dipole is **zero.**

2. Here, $V = 12$ volt,

energy stored $= W = 7{\cdot}20 \times 10^5$ J

Charge passed, $Q = ?$

$$Q = \frac{W}{V} = \frac{7{\cdot}20 \times 10^5}{12} = 0{\cdot}6 \times 10^5 \text{ C}$$

$$= \mathbf{6 \times 10^4 \text{ C}}$$

3. From a collection of charges, whose total sum is not zero, equipotentials at a large distance as the total charge will act as a point charge, must be **spheres** only.

4. When net electric flux through a closed surface is zero, *i.e.*, $\phi_E = \frac{Q}{\epsilon_0} = 0$, so $Q = 0$. Hence no net charge is enclosed by the surface.

5. The basic cause of charging is actual transfer of electrons.

6. $q = ne = 500\,(-1{\cdot}6 \times 10^{-19})$ C $= \mathbf{-8 \times 10^{-17}}$ **C**

7. $\phi = E.ds = (\text{N/C})\, m^2$

8. N/C is SI unit of electric intensity.

9. Due to a single charge electric field is spherically symmetric.

10. Here, $C_1 = 2\ \mu\text{F}$, $C_2 = 3\ \mu\text{F}$, $C_3 = 6\ \mu\text{F}$, $C_s = ?$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6} = 1$$

$\therefore \quad C_s = \mathbf{1\ \mu F}$

11. On the axis of a short electric dipole,

Electric potential, $V = \pm \frac{p}{4\pi\epsilon_0 x^2} \quad \therefore \quad V \propto \frac{1}{x^2}$

12. Here, $V = 200$ volt, $C_1 = 2\ \mu\text{F} = 2 \times 10^{-6}$ F

$C_2 = \text{X}\ \mu\text{F} = \text{X} \times 10^{-6}$ F, $\Delta U = 2 \times 10^{-2}$ J

Decrease in energy of capacitor

$$\Delta U = \frac{1}{2}C_1V_1^2 - \frac{1}{2}C_2V_2^2 = \frac{1}{2}(C_1 - C_2)V^2$$

$\therefore \quad (2 - x) \times 2 \times 10^4 = 2 \times 10^{-2}$

$2 - x = 10^{-6} = 1\ (\mu\text{F})$

$x = 2 - 1 = \mathbf{1\ \mu F}$

13. If F_1 = force between two small spheres charged to a constant potential in free space

and F_2 = force between the same two spheres in a medium of dielectric constant K, then

$$\frac{F_1}{F_2} = K$$

14. Here, $C_0 = 6\ \mu\text{F}$ and $C_m = 30\ \mu\text{F}$

$\therefore$ Dielectric constant of the medium

$$K = \frac{C_m}{C_0} = \frac{30\ \mu\text{F}}{6\ \mu\text{F}} = 5$$

The permittivity of the medium is

$\epsilon = K\epsilon_0 = 5 \times 8{\cdot}85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$

$\epsilon = \mathbf{0{\cdot}44 \times 10^{-10}\ C^2\ N^{-1}\ m^{-2}}$

15. From, $\frac{1}{2}mv^2 = eV$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1{\cdot}6 \times 10^{-19} \times 100}{9 \times 10^{-31}}}$$

$$= \sqrt{\frac{3{\cdot}2}{9} \times 10^{14}} = \mathbf{6 \times 10^6\ m/s}$$

17. Work done = charge × potential difference

$= (-e) \times 0 =$ **zero.**

19. Diagonally opposite corners of the square are at the same potential. Therefore, potential difference = 0. Hence work done = **Zero.**

20. From $E = \frac{-dV}{dr}$

$$dV = -E\,(dr) = -100 \times \frac{5}{100} = \mathbf{-5\ V}$$

22. Capacity is not affected by the amount of charge.

23. Electrical capacity of earth,

$$C = 4\pi\epsilon_0 R = \frac{1}{9\times10^9}\times(6{\cdot}4\times10^6)\,\text{F}$$
$$= 711\times10^{-6}\,\text{F} = \mathbf{711\ \mu F}$$

24. Capacity of capacitor = KC = **6 C.**

25. In series combination, charge on each capacitor is the same.

27. When V becomes twice, energy stored $= \frac{1}{2}CV^2$, **becomes 4 times.**

31. As battery is removed, **charge on the capacitor does not change.**

32. $W = -pE(\cos 60° - \cos 0°) = -pE\left(\frac{1}{2}-1\right) = \frac{pE}{2}$

$W' = -pE(\cos 180° - \cos 0°) = 2pE = \mathbf{4\ W}$

33. For a point charge, electric potential, $V \propto 1/r$ and Electric intensity, $E \propto 1/r^2$. Thus **option (*b*) is true.**

36. $\frac{1}{C_s} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}$

$$= \frac{6+3+2}{6} = \frac{11}{6} \quad \therefore \quad C_s = \frac{6}{11}$$

$$Q = C_s V = \frac{6}{11}\times 11 = 6 \text{ coulomb}$$

$$V_1 = \frac{Q}{C_1} = \frac{6}{1} = \mathbf{6\ V}$$

38. $\frac{C_1}{C_2} = \frac{a}{b}$

As their potential is same, $\frac{q_1}{q_2} = \frac{C_1 V}{C_2 V} = \frac{a}{b}$.

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1}{4\pi a^2}\times\frac{4\pi b^2}{q_2} = \frac{b^2}{a^2}\left(\frac{a}{b}\right) = \frac{b}{a}$$

39. Knowledge based question.

41. Electric field lines are always perpendicular to equipotential surface.

42. Inside a charged spherical shell, the electric field is zero but electric potential is same as on the surface of spherical shell.

44. The capacitance of capacitor depends upon the dimensions of the plates of capacitor and is independent of change in potential.

45. Knowledge based question.

46. $\frac{1}{C_{eq}} = \frac{1}{C}+\frac{1}{C}$ ..... n terms $= \frac{n}{C}$ or $C_{eq} = \frac{C}{n}$

47. Capacity of parallel plate capacitor with no medium, $C_0 = \frac{\epsilon_0 A}{d}$;

with medium $C_m = \frac{K\epsilon_0 A}{d} = KC_0$

II. Assertion-Reason Type Questions

48. k and K do not represent the same thing. **Thus both Assertion and Reason are false.**

49. Here **Assertion is true but Reason is false** as the P.E. of the system of two charges is given by,

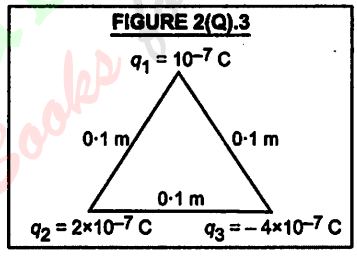
$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}.$$

50. Here, **Assertion is true but Reason is false** as SI unit of energy is joule, where 1 eV = $1{\cdot}6\times10^{-19}$ J.

51. Here **Reason is true.**

Now, PE of the system of three charges shown in Fig. 2(Q).3 is,

FIGURE 2(Q).3



$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}}+\frac{q_1 q_3}{r_{13}}+\frac{q_2 q_3}{r_{23}}\right]$$

$$= 9\times10^9\left[\frac{10^{-7}\times(2\times10^{-7})}{0{\cdot}1}+\frac{10^{-7}\times(-4\times10^{-7})}{0{\cdot}1}+\frac{(2\times10^{-7})\times(-4\times10^{-7})}{0{\cdot}1}\right]$$

$$= 9\times10^9\,[2\times10^{-13} - 4\times10^{-13} - 8\times10^{-13}]$$
$$= 9\times10^9\,[-10\times10^{-13}] = \mathbf{-9\times10^{-3}\ J}$$

Thus Assertion is also true and Reason is the true explanation of Assertion.

52. Here, **both Assertion and Reason are true but Reason is not the correct explanation of Assertion.**

53. Here **Reson is true.** The capacitance of earth

$$C = 4\pi\epsilon_0 R = \frac{1}{(9\times10^9)}\times(6{\cdot}4\times10^6)$$
$$= 711\times10^{-6}\,\text{F} = 711\ \mu\text{F}$$

Thus Assertion is true and Reason is the correct explanation of Assertion.

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54. Both Assertion and Reason are false as in series combination of capacitor, the charge on each capacitor is same and potential across each is different and in a parallel combination of capacitors, the potential across each capacitor is same but charge on each is different.

55. Energy stored in capacitor when charged to potential $V = \frac{1}{2} QV = \frac{1}{2} CV^2$, as some energy is lost in the process of charging.

Thus **both Assertion and Reason are false.**

CONCEPTUAL PROBLEMS

I. Electrostatic Potential, Potential difference related to Electric field

1. Express the unit of electric potential in terms of the base units of S.I. (ICSE 2002)

Sol. As electric potential $= \frac{\text{Work done}}{\text{Charge}}$

$$= \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1} = \text{kg m}^2\,\text{s}^{-3}\,\text{A}^{-1}$$

2. Is the electrostatic potential necessarily zero at a point where the electric field strength is zero ? Give an example to illustrate your answer.

Sol. No. As we know that the electric field is equal to negative of potential gradient :

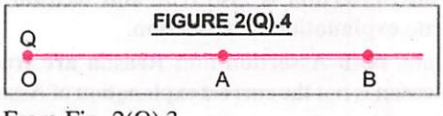
$$E = -\frac{dV}{dr}$$

so, even if electric field at a point is zero, the potential may have some non zero constant value at that point.

Example. Electric field inside a charged conducting sphere is zero but potential at any point inside the sphere is same as that on the surface of sphere.

3. A point charge Q is placed at point O as shown in Fig. 2(Q).4. Is the potential difference $(V_A - V_B)$ positive, negative or zero if Q is (*i*) positive (*ii*) negative ? (CBSE 2011, CBSE (F) 2017)

FIGURE 2(Q).4



Sol. From Fig. 2(Q).3,

$$V_A - V_B = \frac{kQ}{OA} - \frac{kQ}{OB}$$

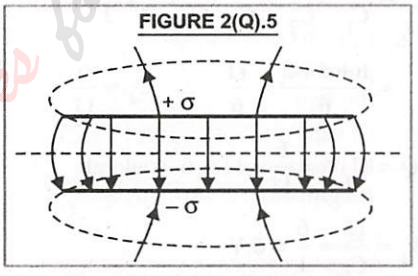
As $OA < OB$,

$\therefore$ When Q is positive, $(V_A - V_B)$ is positive and when Q is negative, $(V_A - V_B)$ is negative.

4. Two large parallel thin plates having uniform charge densities $+\sigma$ and $-\sigma$ are kept in X-Z plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates, what is the magnitude and direction of the field ? (CBSE 2011)

Sol. In Fig. 2(Q).5, we have shown the electric field lines by solid line curves with arrowheads. The corresponding equipotential surfaces are shown by dotted line curves.

FIGURE 2(Q).5



If E is magnitude of the field between the plates, then as particle remains stationary,

upward force due to electric field = weight of particle

$$-qE = mg$$

$$E = -\frac{mg}{q}$$

The direction of field is downwards, *i.e.,* along negative Y-axis.

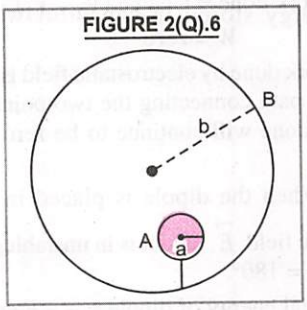
5. A metal sphere A of radius a is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius b and the two are connected by a wire ?

Sol. If q is charge on sphere A, Fig. 2(Q).6, then potential of A is

$$V = \frac{q}{4\pi\epsilon_0 a}$$

$\therefore$ $$q = (4\pi\epsilon_0 a)\, V \quad \ldots(i)$$

FIGURE 2(Q).6



When A is enclosed by a spherical conducting shell B, and the two are connected by a wire, the entire charge q of A shifts to outer surface of B.

$\therefore$ Potential of B will be

$$V_B = \frac{q}{4\pi\epsilon_0 b} = \frac{(4\pi\epsilon_0 a)V}{4\pi\epsilon_0 b}, \quad \text{using } (i)$$

$$V_B = \frac{a}{b}V$$

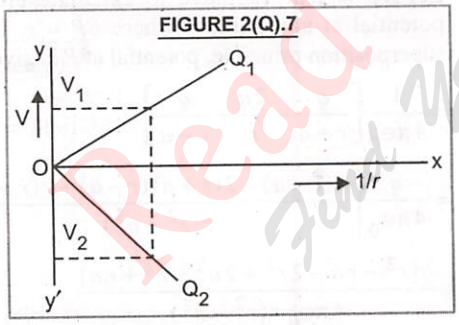
As sphere A is *inside* B, therefore, final potential of A $= V_A = V_B = \frac{a}{b}(V)$, *i.e.*, $V_A < V$.

6. Fig. 2(Q).7 shows the variation of electric potential V with $1/r$, where r is the distance from the two charges Q_1 and Q_2. Determine

(i) signs of two charges Q_1 and Q_2

(ii) which of the two charges has a larger magnitude ? Justify.

FIGURE 2(Q).7



Sol. Electric potential due to a point charge Q at a point distant r from the charge is

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

V is + only when Q is +. Therefore, from Fig. 2(Q).7, Q_1 is + and Q_2 is negative. For a given value of r, we find from Fig. 2(Q).7 that $V_2 > V_1$

$\therefore \qquad Q_2 > Q_1$

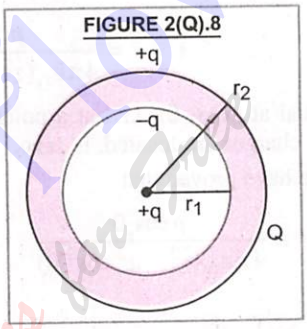
7. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell ?

(b) Is the electric field intensity inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape ? Explain. (CBSE 2010)

Sol. (*a*) The charge + Q resides on the outer surface of the shell. The charge q placed at the centre of the shell induces charge $-q$ on the inner surface and charge $+q$ on the outer surface of the shell, Fig. 2(Q).8. Therefore, total charge on inner surface of the shell is $-q$ and total charge on the outer surface of the shell is $(Q + q)$.

FIGURE 2(Q).8



$$\therefore \quad \sigma_1 = -\frac{q}{4\pi r_1^2} \quad \text{and} \quad \sigma_2 = \frac{Q+q}{4\pi r_2^2}$$

(*b*) Electric field intensity inside a cavity with no charge is zero, even when the shell has any irregular shape. If we were to take a closed loop, part of which is inside the cavity along a field line, and the rest outside it, then net work done by the field in carrying a test charge over the closed loop will not be zero. This is impossible for an electrostatic field. Hence electric field intensity inside a cavity with no charge is always zero.

8. Two charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively.

(i) What is the electrostatic potential at the points $(0, 0, z)$ and $(x, y, 0)$?

(ii) Obtain the dependence of potential on the distance r of the point from the origin, when $\frac{r}{a} >> 1$.

(iii) How much work is done in moving a small test charge from the point (5, 0, 0) to (– 7, 0, 0) along the x-axis. Does the answer change if path of test charge between the same points is not along the x-axis ?

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(iv) If the above point charges are now placed in the same positions in a uniform external electric field $\vec{E}$, what would be the potential energy of the charge system in its orientation of unstable equilibrium ? Justify your answer in each case. [CBSE 2018 (C)]

Sol. Here, $-q$ is at $(0, 0, -a)$ and $+q$ is at $(0, 0, a)$

(*i*) Potential at $(0, 0, z)$ would be

$$V = \frac{1}{4\pi\epsilon_0}\left(-\frac{q}{z+a}\right) + \frac{1}{4\pi\epsilon_0}\frac{q}{(z-a)}$$

$$= \frac{q(z+a-z+a)}{4\pi\epsilon_0(z^2-a^2)} = \frac{q2a}{4\pi\epsilon_0(z^2-a^2)}$$

$$= \frac{p}{4\pi\epsilon_0(z^2-a^2)}$$

Potential at $(x, y, 0)$, *i.e.*, at a point $\perp$ to z-axis where charges are located, is **zero.**

(*ii*) We have proved that

$$V = \frac{p\cos\theta}{4\pi\epsilon_0(r^2 - a^2\cos^2\theta)}$$

If $\frac{r}{a} >> 1$, then $a << r \therefore \quad V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$

$$\therefore \qquad V \propto \frac{1}{r^2}$$

i.e. potential is inversely proportional to square of the distance.

(*iii*) Potential at (5, 0, 0) is

$$V_1 = \frac{-q}{4\pi\epsilon_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}} + \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{(5-0)^2+a^2}}$$

$$= \frac{-q}{4\pi\epsilon_0\sqrt{25+a^2}} + \frac{q}{4\pi\epsilon_0\sqrt{25+a^2}} = \text{zero}$$

Potential at (–7, 0, 0) is

$$V_2 = \frac{-q}{4\pi\epsilon_0}\frac{1}{\sqrt{(-7-0)^2+a^2}} + \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{(-7-0)^2+a^2}}$$

$$= \text{zero}$$

As work done = Charge $(V_2 - V_1)$

$\therefore \qquad W = \text{zero}$

As work done by electrostatic field is independent of the path connecting the two points, therefore, work done will continue to be zero along every path.

(*iv*) When the dipole is placed in an external electric field $\vec{E}$, and it is in unstable equilibrium, then $\theta = 180°$.

Potential energy of dipole $= -pE\cos\theta$

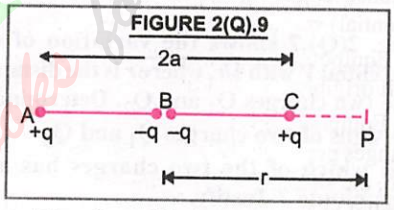
$$= -(q \times 2a)E\cos 180°$$

$$\text{P.E.} = q \times 2aE$$

9. Fig. 2(Q).9 shows a charge array known as an 'electric quadrupole'. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$, and contrast your results with that due to an electric dipole and an electric monopole (*i.e.* a single charge).

Sol. As is clear from Fig. 2(Q).9, an electric quadruple may be regarded as a system of three charges, $+q$, $-2q$ and $+q$ at A, B and C respectively.

FIGURE 2(Q).9



Let $AC = 2a$. We have to calculate electric potential at any point P where $BP = r$. Using superposition principle, potential at P is given by

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r(r-a)-2(r+a)(r-a)+r(r+a)}{r(r^2-a^2)}\right]$$

$$= \frac{q[r^2 - ra - 2r^2 + 2a^2 + r^2 + ra]}{4\pi\epsilon_0 r(r^2-a^2)}$$

$$= \frac{q.2a^2}{4\pi\epsilon_0 r(r^2-a^2)} = \frac{q.2a^2}{4\pi\epsilon_0 r^3(1-a^2/r^2)}$$

when $\frac{r}{a} >> 1$, , $r >> a$ or $a << r$

$\therefore \quad \frac{a^2}{r^2}$ is negligibly small.

$$V = \frac{q.2a^2}{4\pi\epsilon_0 r^3}, \text{ Clearly, } \qquad V \propto \frac{1}{r^3}$$

In case of an electric dipole, $V \propto \frac{1}{r^2}$, and in case of an electric monopole (*i.e.* a single charge),

$$V \propto \frac{1}{r}.$$

10. Describe schematically, the equipotential surfaces corresponding to

(a) a constant electric field in the Z direction

(b) a field that uniformly increases in magnitude but remains in a constant, say z direction

(c) a single positive charge at the origin.

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Sol. By definition, an equipotential surface is that at every point of which potential is the same. In the four cases given above :

(*a*) Equipotential surfaces are planes parallel to $X - Y$ plane. These are equidistant.

(*b*) Equipotential surfaces are planes parallel to $X - Y$ plane. As the field increases uniformly, distance between the planes (differing by fixed potential) decreases.

(*c*) Equipotential surfaces are concentric spheres with origin at the centre.

(*d*) Equipotential surfaces have the shape which changes periodically. At far off distances from the grid, the shape of equipotential surfaces becomes parallel to the grid itself.

11. In a Van de graaf type generator, a spherical metal shell is to be a 15×10^6 volt electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required ? **(CBSE 2008)**

[You will learn from this exercise why one cannot build an electrostatic generator using a very small shell, which requires a small charge to acquire a high potential.]

Sol. Here, $V = 15 \times 10^6$ volt., Dielectric strength $= 5 \times 10^7$ Vm^{-1}, Minimum radius, r = ?

Max. electric field, E = 10% dielectric strength

$$E = \frac{10}{100} \times 5 \times 10^7 = 5 \times 10^6 \text{ Vm}^{-1}.$$

As $$E = \frac{V}{r} \quad \therefore \; r = \frac{V}{E} = \frac{15 \times 10^6}{5 \times 10^6} = 3\text{m}$$

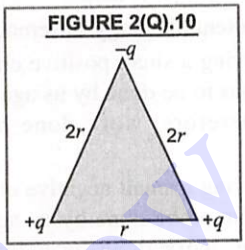
Obviously, we cannot build an electrostatic generator, using a very small spherical shell.

II. Electrostatic Potential Energy

12. Suggest a configuration of three point charges separated by finite distances that has zero electric potential energy.

Sol. The configuration shown in Fig. 2(Q).10 has zero potential energy for three charges.

FIGURE 2(Q).10



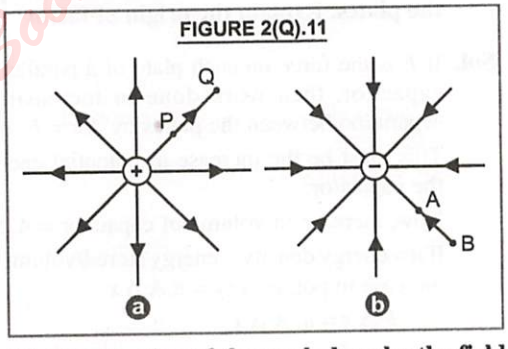
$$U = \frac{Kq \cdot q}{r} + \frac{Kq(-q)}{2r} + \frac{K(-q)q}{2r} = 0$$

13. Figs. 2(Q).11(*a*) and 2(Q).11(*b*) show the field lines of a single positive and negative charge respectively.

(a) Give the signs of potential difference $(V_P - V_Q)$ and $(V_B - V_A)$.

(b) Give the sign of potential energy difference of a small negative charge between the points Q and P ; A and B.

FIGURE 2(Q).11



(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A ? **(NCERT Solved Example)**

Sol. (*a*) As $V \propto \frac{1}{r}$ $\therefore$ $V_P > V_Q$ or $(V_P - V_Q)$ is positive

Also, V_B is less negative than V_A

($\because$ distance of B is more than distance of A)

Thus, $V_B > V_A$ $\therefore$ $(V_B - V_A)$ is positive.

(*b*) A negative charge held at Q would move towards P. As charge moves from higher potential energy to lower potential energy.

$\therefore\ (P.E)_Q > (P.E)_P$

Therefore, sign of potential energy difference of a small negative charge between Q and P is positive.

Similarly, $(P.E)_A > (P.E)_B$

Sign of potential energy difference is positive.

(*c*) In moving a small positive charge from Q to P, work has to be done by us against the electric field. Therefore, work done by the field is negative.

(*d*) In moving a small negative charge from B to A, work has to be done by us against the field. Therefore, work done by external agency is positive.

(*e*) Due to force of repulsion on the negative charge, velocity decreases and hence K.E. decreases in going from B to A.

14. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $\frac{1}{2}$QE, where Q is the charge on the capacitor and E is the magnitude of electric field between the plates. Explain the origin of factor $\frac{1}{2}$.

Sol. If F is the force on each plate of a parallel plate capacitor, then work done in increasing the separation between the plates by $\Delta x = F.\Delta x$.

This must be the increase in potential energy of the capacitor.

Now, increase in volume of capacitor $= A.\Delta x$

If u = energy density = energy stored/volume, then increase in pot. energy $= u.A\,\Delta x$

$\therefore\quad F\,\Delta x = u.A\,\Delta x$

$$F = u.A = \left(\frac{1}{2}\epsilon_0 E^2\right)A = \frac{1}{2}(\epsilon_0 AE)E \quad ;$$

$$F = \frac{1}{2}\left(\epsilon_0 A\frac{V}{d}\right)E \qquad \left(\because C = \frac{\epsilon_0 A}{d}\right)$$

$$F = \frac{1}{2}(CV)E = \frac{1}{2}QE$$

The origin of factor 1/2 in force can be explained by the fact that inside the conductor, field is zero and outside the conductor, the field is E. Therefore, the average value of the field (*i.e.* $E/2$) contributes to the force.

III. Electrical capacitance and capacitors

15. Two copper spheres of same radii, one hollow and other solid are charged to same potential. Which, if any, of the two will have more charge ? **(CBSE 2017)**

Sol. Capacitance of a spherical conductor

$$C = 4\pi\epsilon_0 R,\ i.e.,\ C \propto R$$

As both spheres have equal radii, they have the same capacitance. When charged to same potential, they will have the same charge.

16. Two identical metal plates are given charges q_1 and q_2 respectively. If they are now brought close to form a parallel plate capacitor with capacitance C, what will be the p.d. between the plates ?

Sol. Let A be the area of each plate. When the two plates are placed d distance apart, capacitance of parallel plate capacitor so formed is

$$C = \frac{\epsilon_0 A}{d}$$

If E_1 and E_2 are electric fields due to the two plates, then the net field between the plates

$$E = E_1 - E_2$$

$$\frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2A\epsilon_0}(q_1 - q_2)$$

P.d between the plates

$$V = Ed = \frac{1}{2\epsilon_0 A}(q_1 - q_2)\times d$$

$$V = \frac{1}{\frac{2\epsilon_0 A}{d}}(q_1 - q_2) = \frac{q_1 - q_2}{2C}$$

17. What is the physical significance of capacitance ?

Sol. Capacitance of an arrangement tells us the potential to which the arrangement will be raised when a given charge is placed on it. For a given charge, smaller the potential, larger must be the capacitance and vice-versa.

18. By what factor does the capacity of a metal sphere increase if its volume is tripled ?

Sol. As $\quad V = \frac{4}{3}\pi R^3$

$\therefore\quad V \propto R^3$ or $R \propto V^{1/3}$

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$$\therefore \quad \frac{C_2}{C_1} = \frac{R_2}{R_1} = \left(\frac{V_2}{V_1}\right)^{1/3}$$

$$\therefore \quad C_2 = C_1 \left(\frac{V_2}{V_1}\right)^{1/3} = C_1 (3)^{1/3} = 1{\cdot}44\ C_1$$

19. A parallel plate capacitor of capacitance C is charged to a potential V by a battery. Without disconnecting the battery, distance between the plates of capacitor is tripled and a dielectric medium of $K = 10$ is introduced between the plates of capacitor. Explain giving reasons how will the following be affected ?

(a) Capacitance of capacitor

(b) Charge on capacitor

(c) Energy density of capacitor.

[CBSE 2017 (C)]

Sol. Without the dielectric and with standard notations,

$$C = \frac{\epsilon_0 A}{d}$$

When battery is not disconnected, potential difference

$$V = \text{constant.}$$

$$d' = 3d \text{ and } K = 10$$

$$C' = \frac{K\epsilon_0 A}{d'} = \frac{10\epsilon_0 A}{3d} = \frac{10}{3}\left(\frac{\epsilon_0 A}{d}\right) = \frac{10}{3}\ \text{C}$$

Charge on capacitor, $Q' = C'V = \frac{10}{3}CV = \frac{10}{3}\text{Q}$

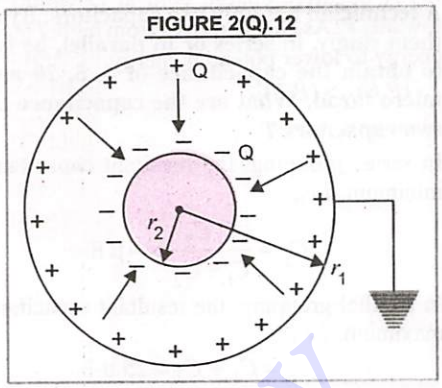
Energy density,

$$\rho' = \frac{1}{2}K\epsilon_0 \frac{V^2}{d'^2} = \frac{K}{3^2}\left(\frac{1}{2}\epsilon_0 \frac{V^2}{d^2}\right) = \frac{10}{9}\rho$$

20. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports. Show that the capacitance of this spherical capacitor is given by $C = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$, where r_1 and r_2 are the radii of outer and inner spheres respectively.

Sol. As is clear from Fig. 2(Q).12, $+Q$ charge spreads uniformly on inner surface of outer sphere of radius r_1. The induced charge $-Q$ spreads uniformly on the outer surface of inner sphere of radius r_2. The outer surface of outer sphere is earthed. Due to electrostatic shielding, $E = 0$ for $r < r_2$ and $E = 0$, for $r > r_1$.

FIGURE 2(Q).12



In the space between the two spheres, electric intensity E exists as shown. Potential difference between the two spheres,

$$V = \frac{Q}{4\pi\epsilon_0 r_2} - \frac{Q}{4\pi\epsilon_0 r_1}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{Q}{4\pi\epsilon_0}\frac{(r_1 - r_2)}{r_1 r_2}$$

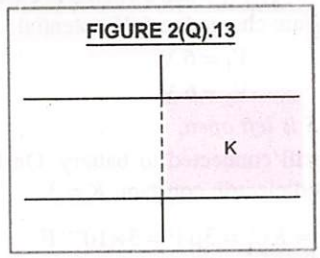
As $C = \frac{Q}{V}$

$$\therefore \quad C = \frac{Q 4\pi\epsilon_0 r_1 r_2}{Q(r_1 - r_2)} = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$$

IV. Grouping of capacitors

21. As shown in Fig. 2(Q).13, a dielectric material of dielectric constant K is inserted in half portion between plates of parallel plate capacitor. If its initial capacitance is C, what is the new capacitance ?

FIGURE 2(Q).13



Sol. The new arrangement is equivalent to two capacitors connected in parallel.

$$C_1 = \frac{\epsilon_0 A/2}{d} \quad ; \quad C_2 = \frac{K\epsilon_0 A/2}{d}$$

$$C_P = C_1 + C_2 = \frac{\epsilon_0 A}{2d} + \frac{K\epsilon_0 A}{2d}$$

$$= \frac{\epsilon_0 A}{2d}(K+1) = \frac{C}{2}(K+1)$$

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22. A technician has only two capacitors. By using them singly, in series or in parallel, he is able to obtain the capacitance of 4, 5, 20 and 25 micro farad. What are the capacitance of the two capacitors ?

Sol. In series grouping, the resultant capacitance is minimum, *i.e.*,

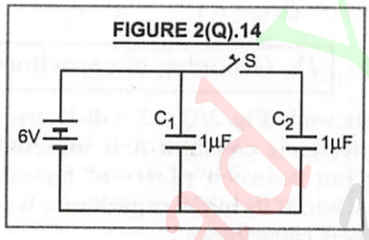
$$C_s = \frac{C_1 C_2}{C_1 + C_2} = 4\,\mu\text{F}$$

In parallel grouping, the resultant capacitance is maximum, *i.e.*,

$$C_p = C_1 + C_2 = 25\ \mu\text{F}$$

It implies that the remaining values **5 μ F** and **20 μ F** represent the individual capacitance values of two capacitors.

23. Fig. 2(Q).14 shows two identical capacitors C_1 and C_2, each of 1 μ F capacitance, connected to a battery of 6 V. Initially, switch S is closed. After sometime, S is left open and dielectric slabs of dielectric constant $K = 3$ are inserted to fill completely the space between the plates of two capacitors. How will the (*i*) charge and (*ii*) potential difference between the plates of the capacitors be affected after the slabs are inserted ? (CBSE 2011)



Sol. In Fig. 2(Q).14, when switch S is closed, both C_1 and C_2 are charged to 6 V potential,

i.e. $V_1 = 6$ V

and $V_2 = 6$ V

When S is left open,

C_1 is still connected to battery. On introducing slab of dielectric constant, $K = 3$,

$$C_1' = KC_1 = 3\mu\text{F} = 3\times10^{-6}\,\text{F}$$

$$V_1' = V_1 = \mathbf{6\,V}$$

$$Q_1' = C_1'V_1' = 3\times10^{-6}\times6 = \mathbf{18\times10^{-6}\,C}$$

However, C_2 is disconnected from battery now, when S is left open. Due to introduction of slab,

$$C_2' = KC_2 = 3\ \mu\text{F} = 3\times10^{-6}\,\text{F}$$

$$Q_2' = Q_2 = C_2V_2 = 1\times10^{-6}\times6 = \mathbf{6\times10^{-6}\,C}$$

$$\therefore\ V_2' = \frac{Q_2'}{C_2'} = \frac{6\times10^{-6}}{3\times10^{-6}} = \mathbf{2\,V}$$

V. Energy stored and loss of energy on sharing charges

24. An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.

Sol. Let capacitance of a capacitor = C

emf of the battery = V

∴ Charge given to the capacitor, $q = CV$

Energy supplied by battery = Work done by battery = qV

$$\text{Energy stored in the capacitor} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

$$\text{Energy lost as heat} = qV - \frac{1}{2}qV = \frac{1}{2}qV$$

So, half energy is lost as heat while charging the capacitor.

25. A spherical shell of radius *b* with charge *Q* is expanded to radius *a*. Find the work done by the electrical forces in the process ?

Sol. Work done by electrical forces in the process = final stored energy – initial stored energy

$$= \frac{Q^2}{2C_2} - \frac{Q^2}{2C_1} = \frac{1Q^2}{2(4\pi\epsilon_0\,a)} - \frac{1}{2}\frac{Q^2}{(4\pi\epsilon_0\,b)}$$

$$= \frac{Q^2}{8\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

26. Two charged spherical conductors of radii R_1 and R_2 when connected by a connecting wire acquire charges q_1 and q_2 respectively. Find the ratio of their charge densities in terms of their radii ? (CBSE 2014)

Sol. Two spherical conductors, when connected by a wire will share charges till, they have common potential. Thus,

$$\frac{q_1}{4\pi\epsilon_0 R_1} = \frac{q_2}{4\pi\epsilon_0 R_2} \quad \text{or} \quad \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

Ratio of surface density of charges is

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \frac{q_1}{q_2}\times\frac{R_2^2}{R_1^2}$$

$$= \frac{R_1}{R_2}\times\frac{R_2^2}{R_1^2} = \frac{R_2}{R_1}$$

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VI. Dielectric Polarization and Van de-graff generator

27. What is the dielectric constant of a metal ?

Sol. The dielectric constant of a metal is infinity. The electric field inside a conductor is zero so the dielectric constant, which is the ratio of applied electric field to the reduced electric field, will be infinite for a metallic conductor.

28. The introduction of dielectric slab between the capacitor plates increases the capacitance. Why ?

Sol. A dielectric slab of dielectric constant K reduces the electric field from E to E/K. This reduces the potential difference from V to V/K. Hence the capacitance increases from C to KC.

29. Where is the knowledge of dielectric strength helpful ?

Sol. Dielectric strength is the maximum strength of electric field that can be tolerated by the dielectric without electric breakdown. Its knowledge helps us in designing a capacitor by determining the maximum potential difference that can be applied across the plates of the capacitor.

30. During lightning, you are safer inside a house than under a tree. Why ?

Sol. When we stand under a tree, we provide an easy path for the lightning to pass through our body. But when we are in the house, discharge due to lightning may be conducted to the ground through iron pipes/wall etc. Therefore, we are safer.

31. The safest way to protect yourself from lightning is to be inside a car. Comment.

Sol. The body of the car is metallic. It provides electrostatic shielding to the person in the car, because electric field inside the car is zero. The discharging due to lightning passes to the ground through the metallic body of the car.

32. A sensitive instrument is to be shifted from the strong electrostatic field in its environment. Suggest a possible way.

Sol. For this, the instrument must be enclosed fully in a metallic cover. This will provide electrostatic shielding to the instrument.

33. Faraday entered a big metallic cage supported on insulating pillars and then charged the cage by a powerful electric machine. He remained quite safe inside the cage. Do you believe on this happening ?

Sol. Yes. As the cage is insulated, the entire charge remained on the outer surface of the cage. The potential at all points inside the cage remained the same. As there was no potential difference between any two points in the cage, Faraday could not get the shock and remained quite safe inside the cage.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

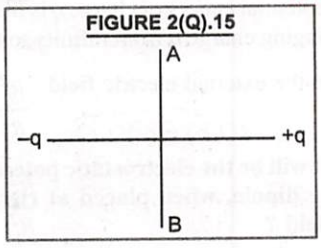
VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Electrostatic Potential, Potential difference related to Electric field

1. A charge q is moved from a point A above a dipole of dipole moment p to a point B below the dipole on equatorial plane without acceleration. Find the work done in the process. (CBSE 2016)

FIGURE 2(Q).15



Ans. At any point on equatorial line of an electric dipole, electric potential is zero. Therefore, work done in moving a charge q from A to B is zero.

2. Define the unit of electric potential.

Ans. The S.I unit of electric potential is volt. The electric potential at a point is said to be 1 volt when 1 J of work is done in moving a positive charge of 1 coulomb from infinity to that point against the electrostatic force.

3. What are the points at which electric potential of a dipole has (*i*) maximum value (*ii*) minimum value ?

Ans. (*i*) At axial points, the electric potential of a dipole has maximum positive or negative value.
(*ii*) At equatorial points, the electric potential of a dipole is zero.

4. Name the physical quantity which has its unit joule coulomb^{-1}. Is it a scalar or vector ?

(Kerala Board 2012)

Ans. joule coulomb^{-1} is the unit of electric potential. It is a scalar quantity.

5. In a certain 0·1 m^3 of space, electric potential is found to be 5 V throughout. What is the electric field in this region ?

Ans. As $E = -\frac{dV}{dr}$,

and V is constant throughout, therefore, $E = 0$.

6. What is the electrostatic potential due to electric dipole at an equatorial point ?

(CBSE 2009)

Ans. Electric potential, $V = 0$.

7. The middle point of a conductor is earthed and its ends are maintained at a potential difference of 220 V. What is the potential at the two ends and at the middle point ?

Ans. Potential at the middle point is zero. The values of potential at the two ends of the conductor must be +110 V and – 110 V, so that pot. diff. across the two ends is 110 – (–110) = 220 V.

8. Do electrons tend to go to regions of high potential or low potential ?

Ans. As electrons are negatively charged, they have tendency to go to regions of high potential.

9. Suppose that the earth has a net charge that is not zero. Is it still possible to adopt the earth as a standard reference point of potential and assign the potential $V = 0$ to it ?

Ans. Yes, earth can still be taken as a standard reference point for potential $V = 0$, because capacity of earth is infinitely large.

10. A hollow metal sphere of radius 5 cm is charged such that potential on its surface is 10 V. What is the potential at the centre of the sphere ? **(CBSE 2011)**

Ans. 10 V, since electric potential is same at all points inside the hollow sphere and at the surface of the sphere.

11. Electric field due to an electric dipole is cylindrically symmetric. Comment.

Ans. It means field due to electric dipole will be same at every point on the surface of a right circular cylinder imagined with the electric dipole as the axis.

12. Write an expression for potential at a point P $(\vec{r})$ due to two point charges q_1 and q_2 at $\vec{r_1}$ and $\vec{r_2}$ respectively.

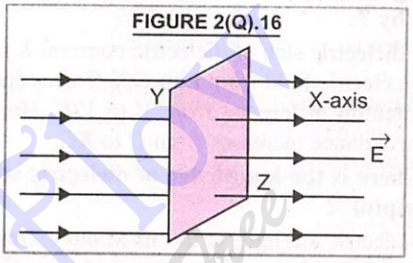
Ans. $$V(\vec{r}) = \frac{q_1}{4\pi\epsilon_0 |\vec{r} - \vec{r_1}|} + \frac{q_2}{4\pi\epsilon_0 |\vec{r} - \vec{r_2}|}$$

13. Draw an equipotential surface for a uniform electric field in X-direction.

(CBSE 2019, 2008)

Ans. For a uniform electric field, say along X-axis, the equipotential surfaces are planes normal to the X-axis, *i.e.*, planes parallel to Y-Z plane as shown in Fig. 2(Q).16.

FIGURE 2(Q).16



14. Can two equipotential surfaces intersect each other ? Give reasons. **(CBSE 2009)**

Ans. No. An equipotential surface is normal to the electric field intensity. If two equipotential surfaces intersect, then at the point of intersection, there will be two directions of electric field intensity, which is not possible.

II. Electrostatic Potential Energy

15. An electron is accelerated through a potential difference V. Write the expression for its final speed if it was initially at rest.

[CBSE 2018 (C)]

Ans. If e is charge on electron, m is mass of electron and v is final speed of electron, then as Kinetic energy acquired = work done on it,

therefore, $\frac{1}{2} m v^2 = e.V$.

$$v = \sqrt{\frac{2eV}{m}}$$

16. Define the term potential energy of charge q at a distance r in an external electric field.

Ans. The potential energy of charge q is the work done in bringing charge q from infinity to the position $\vec{r}$ in the external electric field

$$U(\vec{r}) = q.V(\vec{r})$$

17. What will be the electrostatic potential energy of the dipole, when placed at right angle to the field ?

Ans. Zero. This is because,

$$U = -pE\cos\theta = -pE\cos 90^\circ = \text{Zero.}$$

18. Give expression for 'potential energy' of charge 'q' at a distance 'r' in an external electric field.

Ans. In an external field E, the potential at any point distance r is given by $V = Er$.

$$P.E. = q.V = qEr$$

19. 5 J of work is done in moving a positive charge of 0·5 C between two points. What is the potential difference between these points ?

Ans. Potential difference $= \dfrac{\text{work done}}{\text{charge}} = \dfrac{5}{0{\cdot}5} = 10$ V

20. What is the work done in moving a test charge q through a distance of 1 cm along the equatorial axis of an electric dipole ?

(CBSE 2009)

Ans. On equatorial axis of a dipole, $V = 0$

$\therefore$ $W = q \times V =$ zero.

21. Define the term potential energy of charge q at a distance r in an external electric field.

(CBSE 2009)

Ans. If $V(r)$ is potential at the point due to the external electric field, then $P.E. = W = q.V(r)$.

III. Electrical capacitance and capacitors

22. Write two applications of capacitors in electrical circuits ?

Ans. (*i*) Capacitors are used in radio circuits for tuning purposes.

(*ii*) Capacitors are used in power supplies for smoothening the rectified current.

23. In what form is the energy stored in a charged capacitance ?

Ans. Energy is stored in the form of electrostatic potential energy in the electric field between the plates of capacitor.

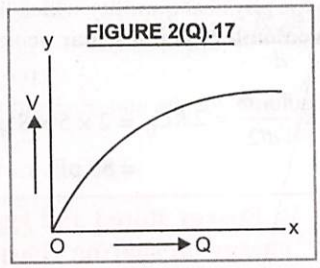
24. Write down the expression for the capacitance of a spherical capacitor.

Ans. $C = 4\pi\epsilon_0 \dfrac{ab}{b-a}$, where a and b are the radii of the inner and outer spheres respectively.

25. Sketch a graph to show how charge Q given to a capacitor of capacity C varies with the potential difference V.

Ans. Initially, the graph between V and Q is a straight line. However, beyond a certain stage, some leakage of charge starts and V does not increase with increasing Q. This variation is shown in Fig. 2(Q).17.

FIGURE 2(Q).17



26. The distance between the plates of a parallel plate capacitor is d. A metal plate of thickness $d/2$ is placed between the plates, what will be the new capacity ?

Ans. As electric field inside the metal plate is zero, d becomes $d/2$. Hence C becomes twice (from $C = \epsilon_0 A/d$).

27. Can there be a potential difference between two conductors of same volume carrying equal positive charges ?

Ans. Yes, because two conductors of same volume may have different shapes and hence different capacitances.

28. What is the net charge on a charged capacitor ? (Bihar Board 2011)

Ans. Zero, because one plate has positive charge and the other carries an equal negative charge.

29. On which factors does the capacitance of a capacitor depend ?

Ans. It depends on geometry of the plates, distance between them and nature of dielectric medium separating the plates.

IV. Grouping of capacitors

30. If two isolated spherical conductors each having a definite capacity are far apart and are connected to eachother by a fine wire, how do you calculate the capacity of the combination ? In joining them with the wire, have you connected them in parallel or in series ?

Ans. On connecting, both the spheres acquire a common potential V. If C is total capacity, then total charge, $q = CV = q_1 + q_2$, as charge is conserved.

$$\therefore \quad C = \frac{q_1}{V} + \frac{q_2}{V} = C_1 + C_2$$

Hence the conductors have been connected in parallel.

31. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the plates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the new value of capacitance.

Ans. $C_0 = \frac{\epsilon_0 A}{d} = 8\,\text{pF}$

$$C = K.\frac{\epsilon_0 A}{d/2} = 2KC_0 = 2 \times 5 \times 8\ \text{pF}$$

$$= 80\ \text{pF}$$

V. Energy stored and loss of energy on sharing charges

32. The electric field inside a parallel plate capacitor is *E*. Find the amount of work done in moving a charge *q* over a closed rectangular loop *abcda*. (CBSE 2014)

Ans. Refer to Fig. 2(Q).18, Work done in moving a charge q on path $abcd$

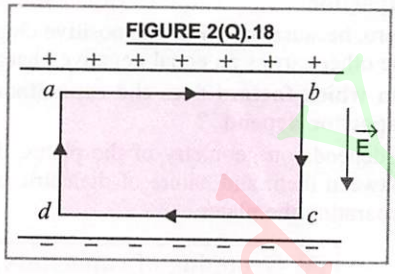
$$= q\vec{E}.(\vec{ab}) + q\vec{E}.(\vec{bc}) + q\vec{E}.(\vec{cd}) + q\vec{E}.(\vec{da})$$

$$= qE\,(ab)\cos 90° + qE\,(bc)\cos 0°$$
$$+ qE\,(cd)\cos 90° + qE\,(da)\cos 180°$$

$$= 0 + qE\,(bc) + 0 - qE\,(da)$$

$$= 0 \qquad (\because bc = da)$$

FIGURE 2(Q).18



33. Where does the energy of a capacitor reside ?

Ans. The energy resides in the dielectric medium separating the two plates.

34. What is the basic difference between a charged capacitor and an electric cell ?

Ans. A capacitor supplies electrical energy stored in it. A cell supplies electrical energy by converting chemical energy into electrical energy at constant potential difference.

35. Why is a space ship entering the ionosphere not sufficiently heated, though temperature at the top of ionosphere is nearly 700 K ?

Ans. This is because density of air in ionosphere is so low that very few molecules only bombard the space ship.

VI. Dielectric Polarization and Van de-graff generator

36. Define polarization density.

Ans. The induced dipole moment developed per unit volme in a dielectric slab on placing it in an external electric field is called polarization density.

37. Define dielectric strength of a medium. What is its values for vacuum?

Ans. The maximum electric field that a dielectric medium can withstand without breaking down of its insulating property is called dielectric strength. Its value for vacuum is infinity.

38. Why does the electric conductivity of earth's atmosphere increase with altitude?

Ans. This is because of ionisation caused by highly energetic cosmic ray particles from cosmos, which are hitting the atmosphere of earth.

39. Why does the electric field inside a dielectric decrease when it is placed in an exte nal electric field?

Ans. It is because the dielectric gets polarised.

SHORT ANSWER QUESTIONS

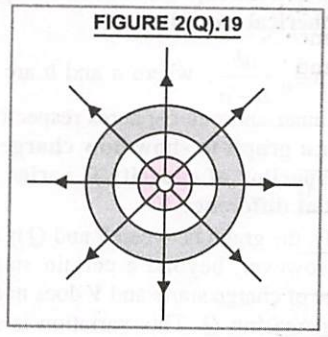
Carrying 2 marks

I. Electrostatic Potential, Potential difference related to Electric field

1. Define an equipotential surface. Draw equipotential surfaces
(*i*) in the case of a single point charge.
(*ii*) in a constant electric field in Z direction.
Why the equipotential surfaces about a single charge are not equidistant ?
(*iii*) Can electric field exist tangential to an equipotential surface ? Give reason.
(CBSE 2016)

Ans.

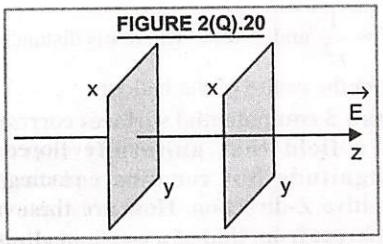
FIGURE 2(Q).19



(*i*) An equipotential surface is that at every point of which electric potential is the same. Due to a single point charge, equipotnetial surfaces are concentric spherical surfaces centred at the charge, Fig. 2(Q).15. As $V \propto \frac{1}{r}$, the equipotential surfaces about a single charge cannot be equidistant.

(*ii*) In a constant electric field in Z-direction, equipotential surfaces lie in XY plane perpendicular to Z-direction as shown in Fig. 2(Q).20.

FIGURE 2(Q).20



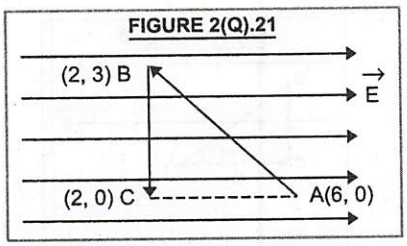
(*iii*) As electric field is perpendicular to the equipotential surface, the field cannot exist tangential to the equipotential surface.

2. Justify that electrostatic potential is constant throughout the volume of charged conductor and has same value on its surface as inside it. (CBSE Sample Paper 2011)

Ans. Since electric field inside the conductor is zero and has no tangential component on its surface, therefore, no work is done in moving a test charge within the conductor or on its surface. It means potential difference between any two points inside or on the surface is zero. Hence, electrostatic potential is constant throughout the volume of charged conductor and has same value on its surface as inside it.

3. A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in Fig. 2(Q).21. (*i*) Calculate the potential difference between A and C (*ii*) At what point [of A and C] is the electric potential more and why ? (CBSE 2012)

FIGURE 2(Q).21



Ans. (*i*) p.d. does not depend upon the path along which the test charge moves.

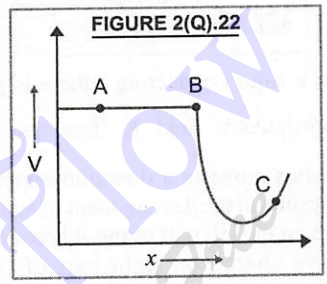
$$\therefore \quad E = -\frac{dV}{dx} = \frac{-(V_C - V_A)}{2-6} = \frac{V_C - V_A}{4}$$

or $V_C - V_A = 4\,E$. Therefore, $V_C > V_A$

(*ii*) Direction of electric field is in the direction of decreasing potential. So, $V_C > V_A$

4. Fig. 2(Q).22 shows the variation of electrostatic potential V with distance x for a given charge distribution.

FIGURE 2(Q).22



From the points marked A, B and C, identify the point at which electric field is (*i*) zero (*ii*) maximum.

Explain your answer in each case.

Ans. As $E = -\frac{dV}{dx}$ = Negative slope of V-x graph,

At A, $\frac{dV}{dx} = 0$; At B, $\frac{dV}{dx} < 0$; At C, $\frac{dV}{dx} > 0$

$\therefore$ (*i*) E is zero at point A.

(*ii*) E is maximum at point B.

5. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ volts. Calculate electric field at the point (1m, 0m, 2m).

Ans. Here $V = 4x^2$

$$E_x = -\frac{dV}{dx} = -8x,$$

$$E_y = -\frac{dV}{dy} = 0,$$

$$E_z = -\frac{dV}{dz} = 0$$

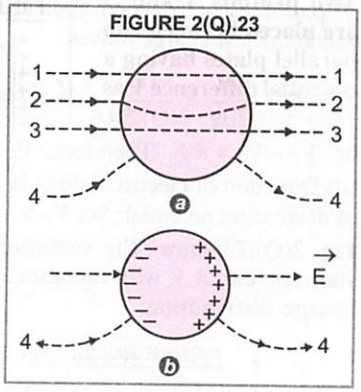
$$\therefore \qquad E = -8x\,\hat{i}$$

At point (1 m, 0, 2 m).

$$\vec{E} = -8 \times 1\,\hat{i} = -8\,\hat{i}\ \text{Vm}^{-1}$$

6. A metallic solid sphere is placed in a uniform electric field, Fig. 2(Q).23(*a*). Which path is followed by the lines of force ?

FIGURE 2(Q).23



Ans. When a solid conducting sphere is placed in a uniform electric field $\vec{E}$, free electrons of the conductor move in a direction opposite to $\vec{E}$. This results in the development of some negative charge on the left part of the sphere and an equal positive charge on right part of the sphere. Electric lines of force end normally on left part, and start normally from the right part. Therefore, path 4 is correct, Fig. 2(Q).23(*b*).

7. Can we create an electric field in which all the lines of force are parallel but their density increases continuously in a direction perpendicular to the lines of force, Fig. 2(Q).24 ?

FIGURE 2(Q).24

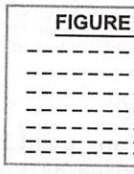
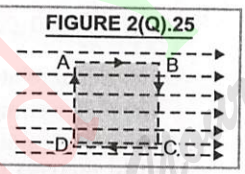


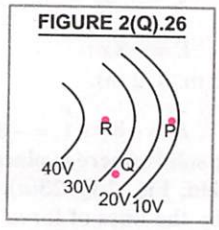
FIGURE 2(Q).25



Ans. No, this is because work done in moving a unit positive charge along the closed path *ABCD* would no longer be zero, being more along *CD*, less along *AB*, zero along *BC* and *DA*, Fig. 2(Q).25. This work done has to be zero, because electrostatic forces are conservative forces.

8. Fig. 2(Q).26 shows lines of constant potential in an electric field. Out of the three given points *P, Q, R* where is electric field intensity maximum and where is it minimum ?

FIGURE 2(Q).26



Ans. Using the relation, $E = -\frac{dV}{dr}$, we find that electric intensity is maximum at *P* and minimum at *R*.

9. Electric charge is distributed uniformly on the surface of a spherical rubber balloon. Show how the value of electric intensity and potential vary (*i*) on the surface (*ii*), inside and (*iii*) outside ?

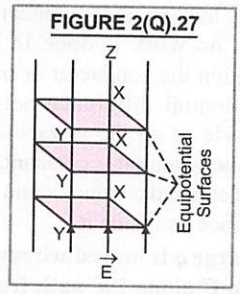
Ans. On the surface, E = constant, V = constant. Inside the surface, $E = 0$, V = constant = potential on the surface. Outside the balloon,

$E \propto \frac{1}{r^2}$ and $V \propto \frac{1}{r}$ where r is distance of point from the centre of the balloon.

10. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along positive Z-direction. How are these surfaces different from that of a constant electric field along Z-direction ? (CBSE 2009)

Ans. The three equipotential surfaces are in XY plane. As magnitude of field increases along positive *Z* direction, distance between successive equipotential surfaces (for given *dV*) decreases as shown in Fig. 2(Q).27. If electric field were constant, distance between successive equipotential surfaces would stay constant.

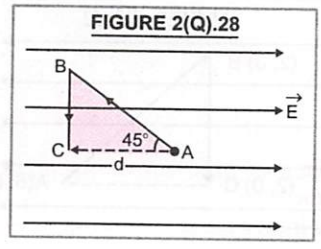
FIGURE 2(Q).27



11. A test charge q_0 is moved without acceleration from *A* to *C* over the path ABC as shown in Fig. 2(Q).28.

Calculate potential difference between *A* and *C*. (CBSE 2012)

FIGURE 2(Q).28



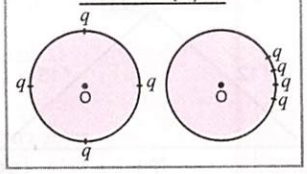
Ans. As is clear from Fig. 2(Q).35, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

As $E = -\frac{dV}{dr} = \frac{-(V_C - V_A)}{d} = \frac{V_A - V_C}{d}$

$\therefore \quad V_A - V_C = E\,d.$

12. Electric potential at a point O due to a number of point charges equidistant from O is V_1 when charges are uniformly distributed and it is V_2 when charges are non uniformly distributed, Fig. 2(Q).29. Is $V_1 = V_2$? Justify.

FIGURE 2(Q).29



Ans. If r is radius of the sphere with centre O, then by definition,

$$V_1 = 4\left(\frac{q}{4\pi\epsilon_0 r}\right), \text{ and } V_2 = 4\left(\frac{q}{4\pi\epsilon_0 r}\right)$$

$\therefore V_1 = V_2$

This is because electric potential is a scalar quantity. It depends only on distance of the point from the charge and not on direction of its location from the charge.

13. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Sol. We know that the length of diagonal of the cube of each side b is $\sqrt{3b^2} = b\sqrt{3}$

$\therefore$ Distance between centre of cube and each vertex, $r = \frac{b\sqrt{3}}{2}$

As $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

and 8 charges each of value q are present at the eight vertices of the cube, therefore,

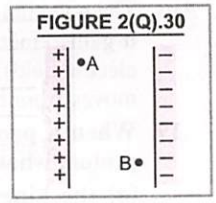
$$V = \frac{1}{4\pi\epsilon_0}\frac{8q}{b\sqrt{3}/2}$$

or $$V = \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$$

Further, electric field intensity at the centre due to all the eight charges is **zero,** because the fields due to individual charges cancel in pairs.

14. Two protons A and B are placed between two parallel plates having a potential difference V as shown in Fig. 2(Q).30. Will these protons experience equal or unequal force ?

FIGURE 2(Q).30



Ans. Electric field E between the two plates is uniform, *i.e.,* E is same in the entire space. As $F = qE$, therefore, two protons A and B will experience equal forces.

15. Define surface density of charge and potential of a charged spherical conductor. Establish a relation between them.

Ans. Surface density of charge is defined as the charge per unit area of a charged conductor. If r is the radius of a spherical conductor having charge q, then its surface density of charge,

$$\sigma = \frac{q}{4\pi r^2} \qquad ...(i)$$

Electric potential of the spherical charged conductor,

$$V = \frac{q}{4\pi\epsilon_0 r} \quad \text{or} \quad q = 4\pi\epsilon_0 r \times V$$

Putting this value in (i), we get

$$\sigma = \frac{4\pi\epsilon_0 rV}{4\pi r^2} = \frac{\epsilon_0 V}{r}$$

16. A charge + Q is lying at the centre of a circle. What is work done in carrying charge q from A to B, where A and B, both lie on the circle.

Ans. The circular path around the charge + Q is an equipotential surface. As A and B lie on the circle *i.e.,* on the equipotential surface, work done = 0.

17. Around a point charge of 1 nC, what is the distance of an equipotential surface of 0·9 V ?

Ans. Here $q = 1\ nC = 10^{-9}$ C, $V = 0{\cdot}9$ V, $r = ?$

As $V = \frac{q}{4\pi\epsilon_0 r}$

$$\therefore \quad r = \frac{q}{4\pi\epsilon_0 r} = \frac{10^{-9}\times 9\times 10^9}{0{\cdot}9} = \mathbf{10\ m}$$

II. Electrostatic Potential Energy

18. A proton is released from rest in a uniform electric field. Does the proton's electric potential energy increase or decrease ? Does the proton move towards a location with a higher or a lower electric potential.

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Ans. The potential energy of the proton decreases (as it gains kinetic energy due to its motion in the electric field). Further the proton (being positive) moves from higher to lower electric potential.

19. When a proton approaches another fixed proton, what happens to :
(*a*) the kinetic energy of the approaching proton
(*b*) the electric potential energy of the system and
(*c*) the total energy of the system ?

Ans. (*a*) The kinetic energy of the approaching proton decreases.
(*b*) The electric potential energy of the system increases because distance between the charges decreases.
(*c*) The total energy of the system remains the same because of the conservation of energy.

20. Write an expression for potential energy of two charges q_1 and q_2 at $\vec{r}_1$ and $\vec{r}_2$ in a uniform electric field $\vec{E}$. (CBSE 2010)

Ans. If $V(\vec{r}_1)$ and $V(\vec{r}_2)$ are electric potentials at points $\vec{r}_1$ and $\vec{r}_2$ where charges q_1 and q_2 are held, then

Potential Energy $= q_1 . V(\vec{r}_1) + q_2 . V(\vec{r}_2) + \dfrac{q_1 q_2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|}$

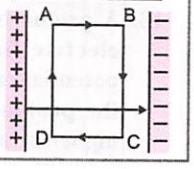
Obviously, the first two terms are the result of interactions of q_1 and q_2 separately with the sources of electric field. The third term is due to mutual interaction between the two charges.

21. Equipotential surfaces are perpendicular to field lines. Why ? (CBSE 2014)

Ans. No work is done in moving a charge from one point on equipotential surface to the other. Therefore, component of electric field intensity along the equipotential surface is zero it means, the electric field intensity is perpendicular to equipotential surface. Hence, the surface is perpendicular to field lines.

22. A uniform electric field *E* exists between two charged plates as shown in Fig. 2(Q).31. What would be work done in moving a charge *q* along the closed rectangular path *ABCDA* ?

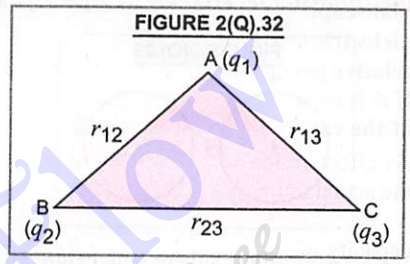
FIGURE 2(Q).31



Ans. Work done in moving a charge along the closed rectangular path would be zero, because field in the entire space is uniform and electrostatic forces are conservative forces.

23. Three charges q_1, q_2 and q_3 are kept respectively at points *A*, *B* and *C* as shown in Fig. 2(Q).32.
Write the expression for electrostatic potential energy of the system.
(CBSE Sample Paper 2019-20)

FIGURE 2(Q).32



Ans. As is known from theory, electrostatic potential energy of the system of three charges can be put as

$$\text{P.E.} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}} + \frac{q_3 q_1}{4\pi\epsilon_0 r_{13}}$$

$$\text{P.E.} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{13}}\right]$$

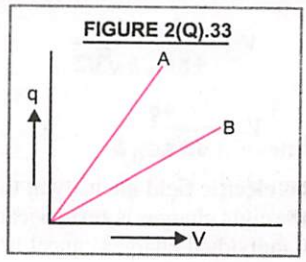
III. Electrical capacitance and capacitors

24. The difference between the radii of the two spheres of a spherical capacitor is increased. Will the capacitance increase or decrease ?

Ans. The capacitance *C* decreases as $C = \dfrac{4\pi\epsilon_0 ab}{b-a}$, where *a*, *b* are radii of two spheres. So if *b*-*a* increases, capacitance *C* will decrease.

25. The given graph in Fig. 2(Q).33 shows the variation of charge q versus potential difference for two capacitors C_1 and C_2. The two capacitors have same plate separation, but the plate area of C_2 is double that of C_1.

FIGURE 2(Q).33



Which of the lines in the graph correspond to C_1 and C_2 and why ?

Ans. As $C = \frac{\epsilon_0 A}{d}$, and plate area of C_2 is double than that of C_1, therefore, $C_2 > C_1$. Now $C = \frac{q}{V}$, which is greater for line A.

$\therefore$ The line A of the graph corresponds to C_2 and line B corresponds to C_1.

26. The space between the plates of a parallel plate capacitor is filled consecutively with two dielectric layers of thickness d_1 and d_2 having relative permittivities ϵ_1 and ϵ_2 respectively. If A is area of each plate, what is the capacity of the capacitor ?

Ans. As effective thickness of dielectric is $1/\epsilon$ times the actual thickness, therefore, we may write the capacity of condenser as $C = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}}$

27. An uncharged insulated conductor A is brought near a charged insulated conductor B. What happens to charge and potential of B ?

Ans. The charge on conductor B remains same, but the potential of B gets lowered because it induces charge of opposite sign on conductor A.

28. n small drops of same size are charged to V volt each. They coalesce to form a bigger drop. Calculate the capacity and potential of the bigger drop.

Ans. Let r be the radius of each small drop and R be the radius of bigger drop.

As volume of bigger drop = volume of n small drops

$$\therefore \quad \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$R = n^{1/3} r$$

As capacity varies directly as the radius,

$\therefore$ Capacity of bigger drop becomes $n^{1/3}$ times the capacity of each small drop.

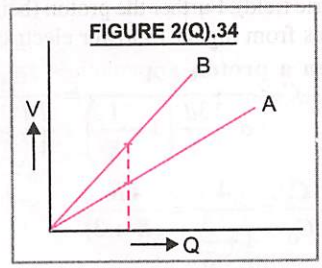
Also, potential of bigger drop $= \frac{\text{total charge}}{\text{capacity}}$

$$\therefore \quad V = \frac{nq}{4\pi\epsilon_0 R} = \frac{nq}{4\pi\epsilon_0 n^{1/3} r} = n^{2/3} \frac{q}{4\pi\epsilon_0 r}$$

$= n^{2/3}$ times the potential of each small drop.

29. The graph in Fig. 2(Q).34 shows variation of voltage V across the plates of two capacitors A and B versus increase of charge Q stored on them. Which of the two has higher capacitance ? Give reason.

FIGURE 2(Q).34



Ans. As $C = Q/V$ and from the figure, for same value of Q ; A has lower value of V and B has higher value of V. Therefore, A has higher capacitance.

30. Two circular metal plates each of radius 10 cm are kept parallel to each other at a distance of 1 mm. What kind of capacitor do they make ? Mention one application of this capacitor. If the radius of each of the plates is increased by a factor of $\sqrt{2}$ and their distance of separation is decreased to half of its initial value, calculate the ratio of capacitance in the two cases. Suggest a possible method by which the capacitance in second case is increased to n times.

Ans. These circular metal plates form a parallel plate capacitor. Such a capacitor is used in filter circuits.

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \pi r^2}{d} \quad ;$$

$$C' = \frac{\epsilon_0 A'}{d'} = \frac{\epsilon_0 \pi (\sqrt{2}\, r)^2}{d/2} = 4\,C$$

$$\therefore \quad \frac{C}{C'} = \frac{1}{4}$$

In the second case, capacitance will become n times by introducing a dielectric of specific inductive capacity $K = n$ inbetween the plates of the capacitor.

31. A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor, but has a thickness $3d/4$. Find the ratio of the capacitance with dielectric inside it to its capacitance without the dielectric.

(CBSE (F) 2017)

Ans. Without the dielectric, capacity of the capacitor

$$= C_0 = \frac{\epsilon_0 A}{d}$$

With dielectric of thickness $t = 3d/4$, the capacity of condenser becomes

$$C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

$$C = \frac{\epsilon_0 A}{d - \frac{3d}{4}\left(1 - \frac{1}{K}\right)} = \frac{\epsilon_0 A}{\frac{d}{4}\left(1 + \frac{3}{K}\right)}$$

$$\therefore \quad \frac{C}{C_0} = \frac{4}{1 + \frac{3}{K}} = \frac{4K}{(K+3)}$$

IV. Grouping of capacitors

32. Given a battery, how would you connect two capacitors, in series or in parallel for them to store the greater (*a*) total charge (*b*) total energy ?

Ans. Total charge $q = CV$ and total energy $U = \frac{1}{2}CV^2$

Now, V is constant and $C_p > C_s$,

therefore, parallel combination is required for storing greater charge and greater energy.

33. A metal foil of negligible thickness is introduced between two plates of a capacitor at the centre. What will be the new capacitance of the capacitor ?

Ans. On introducing a thin metal foil, d is halved. Arrangement is equivalent to two condensers each of capacity $2C$ in series.

$$\therefore \quad \frac{1}{C_s} = \frac{1}{2C} + \frac{1}{2C} = \frac{1}{C}$$

$$\therefore \quad C_s = C$$

Capacity is unchanged.

34. Two capacitors of capacitance 6 μ F and 12 μ F are connected in series with a battery. The voltage across the 6 μ F capacitor is 2 V. Compute the total battery voltage.

Ans. In series, charge on C_1 = charge on C_2

$$6 \times 2 = 12 \times V_2$$

$$V_2 = \frac{12}{12} = 1 \text{ volt}$$

$\therefore$ Battery voltage $V = V_1 + V_2 = 2 + 1 = \mathbf{3\ V}$

V. Energy stored and loss of energy on sharing charges

35. If the potential difference across a capacitor is doubled, what happens to : (*a*) the charge on the capacitor and (*b*) the energy stored in the capacitor

Ans. (*a*) As $Q = CV$, If V doubles, charge also gets doubled.

(*b*) As $U = \frac{1}{2}CV^2$, If V is doubled, U is quadrupled.

36. On charging a parallel-plate capacitor to a potential V, the spacing between the plates is halved and a dielectric medium of $\varepsilon_r = 10$ is introduced between the plates, without disconnecting the dc source. Explain using suitable expression how the (*a*) capacitance (*b*) electric field (*c*) energy density of the capacitor change. (CBSE 2008)

Ans. As the d.c. source remains connected, p.d. (V) between the plates of capacitor remains unchanged even after dielectric is inserted between the plates.

(*a*) Original capacitance $C_0 = \frac{\epsilon_0 A}{d}$

New capacitance $C_0 = \frac{\varepsilon_r \epsilon_0 A}{d/2} = 20 C_0$

(*b*) Changed electric field,

$$E = \frac{V}{d/2} = 2\,(V/d) = 2E_0$$

(*c*) Original energy density, $U_0 = \frac{1}{2}\epsilon_0 E^2$

New energy density, $U = \frac{1}{2}\epsilon E^2$

$$= \frac{1}{2}(\varepsilon_r \varepsilon_0)(2E_0)^2 = 4\varepsilon_r\left(\frac{1}{2} \times \varepsilon_0 E_0^2\right)$$

$$= 4 \times 10\, U_0 = 40\, U_0$$

37. The separation between the plates of a charged capacitor is to be increased. Explain when work done will be more in case battery is removed after charging the capacitor or battery remains connected.

Ans. With increase in separation between the two plates, capacity C decreases.

When battery is removed, charge Q and electric field $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$ would remain constant.

But when battery remains connected, V is constant, Q ($= CV$) decreases and hence E decreases. Clearly, more work is required to be done in the first case.

38. The battery remains connected to a parallel plate capacitor and a dielectric slab is inserted between the plates. What will be the effect on

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its (*i*) capacity (*ii*) charge, (*iii*) potential difference (*iv*) electric field, (*v*) energy stored ? Justify your answer. (CBSE 2009)

Ans. When battery remains connected, pot. diff. V remains constant. Capacity C increases. $Q = CV$ increases. Electric field, $E = \frac{E_0}{K} = \frac{V_0}{K\,d}$ will decrease. Energy stored $= \frac{1}{2}CV^2$ increases.

39. In the above question, if battery is removed after charging the condenser & dielectric slab introduced, how are all the five parameters affected ? (CBSE 2010, 2009)

Ans. When battery is removed, charge Q remains constant, capacity $C = KC_0$ increases. Potential diff. $V = Q/C$ decreases. Electric field $E = \frac{V}{d}$ decreases. Energy stored $= \frac{1}{2}\frac{Q^2}{C}$ decreases.

40. Where does the loss of energy in the above question go ?

Ans. In the process, dielectric slab is polarised. The energy lost is spent in polarising the dielectric.

41. The graph in Fig. 2(Q).35 shows the variation of total energy U stored in the capacitor against the value of the capacitance C itself. Which of the two — the charge on capacitor or potential used to charge is kept constant for this graph ?

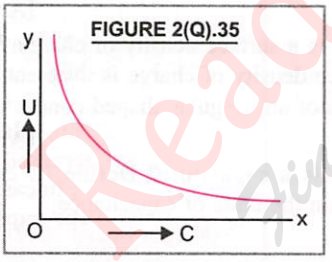

FIGURE 2(Q).35

Ans. It is known that energy stored in capacitor,

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

If V were constant, $U \propto C$. The graph between U and C should be a straight line, which is not.

If Q is constant, $U \propto \frac{1}{C}$,

which is the kind of graph given.

42. Derive an expression for energy stored in a parallel plate capacitor of capacitance C with air as medium between the plates having charges Q and $-Q$. Show that this energy can be expressed in terms of electric field as $\frac{1}{2}\epsilon_0 E^2$ Ad, where A is area of each plate and d is the separation between the plates. How will the energy stored in a fully charged capacitor change when the separation between the plates is doubled and the dielectric medium of constant 4 is introduced between the plates ?

Ans. For energy stored in the capacitor, refer to Art. 2.21.

$$U = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$$

Now, $C = \frac{\epsilon_0 A}{d}$ and $E = \frac{V}{d}$

$$\therefore \quad U = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\epsilon_0 E^2 Ad$$

When separation between the plates is doubled $d' = 2\,d$, and

dielectric medium of constant 4 is introduced,

new capacity, $C' = \frac{4}{2}C = 2C$

As charge Q remains unchanged,

$$\therefore \quad U' = \frac{Q^2}{2C'} = \frac{1}{2}\frac{Q^2}{(2C)} = \frac{1}{2}U$$

VI. Dielectric Polarization and Van de-graff generator

43. An air capacitor is given a charge of 2 μC raising its potential to 200 V. If on inserting a dielectric medium, its potential falls to 50 V, what is the dielectric constant of the medium ?

Ans. As dielectric constant in terms of potential is given by $K = \frac{V_0}{V_m} = \frac{200}{50} = 4.$

44. What is the effect of temperature on dielectric constant ?

Ans. Dielectric constant varies inversely with temperature, *i.e.*, dielectric constant decreases with rise in temperature.

45. What are dielectric substances ? Which of the following is a dielectric : Silicon, mica, carbon ?

Ans. Dielectric substances are basically insulators. However, electric effects can be passed through them without actual conduction. Mica is a dielectric.

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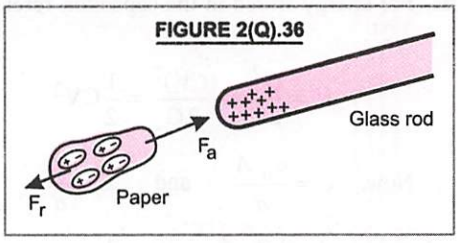
46. Why does a charged glass rod attract a piece of paper ?

Ans. Paper is a dielectric. When a positively charged glass rod is brought near it, atoms of paper get polarised, with centre of negative charge of atoms coming closer to glass rod. Therefore, force of attraction (F_a) between glass rod and piece of paper becomes greater than the force of repulsion (F_r) between the glass rod and the piece of paper, Fig. 2(Q).36. This results in attraction of the piece of paper towards the glass rod.

FIGURE 2(Q).36



47. If dielectric strength of air (minimum field required for ionisation of a medium) is 3 M V/m, can a metal sphere of radius 1 cm hold a charge of 1 coulomb ?

Ans. Here, $q = 1$ C and $r = 1$ cm $= 10^{-2}$ m.

Electric field intensity at the surface of the sphere, $E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{9\times10^9 \times 1}{(10^{-2})^2}$

$= 9 \times 10^{13}$ V/m

As this value of E is much greater than the dielectric strength of air (3 MV/m = 3×10^6 V/m), therefore, air around the sphere will get ionised and the entire charge from the sphere will leak out into air. Hence the sphere of radius 1 cm cannot hold 1 coulomb of charge in air.

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. Explain the physical meaning of potential and potential difference. Find a relation for electrostatic potential at a point due to a point charge. **[Arts. 2.3, 2.5]**

2. What do you understand by potential gradient ? Establish a relation between electric field and potential gradient.

(AP Board 2012) **[Art. 2.9]**

3. Establish that electrostatic forces are conservative. Give two examples of conservative forces. **[Art. 2.4]**

4. What do you understand by electrostatic potential energy ? Find an expression for electrostatic potential energy of a system of two point charges. **[Arts. 2.2 and 2.10]**

5. Depict the equipotential surfaces for

(*i*) a single isolated charge

(CBSE 2012) **[Fig. 2.10(a)]**

(*ii*) a uniform electric field. **[Fig. 2.10(b)]**

(*iii*) an electric dipole.

(CBSE Sample Paper 2020, CBSE 2017)

[Fig. 2.10(c)]

(*iv*) a system of two identical point charges placed distance d apart.

(CBSE Sample Paper 2020, CBSE 2010)

[Fig. 2.10(d)]

6. Explain the term electric field intensity. Establish that electric field inside a charged conductor is zero. **[Art. 2.12]**

7. Explain electrostatic shielding with examples. **[Art. 2.13]**

8. Show that the electric field at the surface of a charged conductor is $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction. **[Art. 2.12 (5)]**

9. What is a surface density of charge ? Show that surface density of charge is different at different points of an irregular shaped conductor. **[Art. 2.12 (6)]**

10. What is a capacitor ? Define its capacitance. Explain the units of capacitance.

(Kerala Board 2012) **[Art. 2.14]**

11. Define capacitance of a conductor. Obtain an expression for capacity of an isolated spherical conductor.

(Chhatisgarh Board 2012)

[Arts. 2.14 & 2.15]

12. Explain the principle of a capacitor ? **[Art. 2.16]**

13. Derive an expression for equivalent capacitance of three capacitors of capacitances C_1, C_2 and C_3 when connected (*i*) in series (*ii*) in parallel.

(Manipur Board 2012, 2011)

[Art. 2.20]

LONG ANSWER QUESTIONS Carrying 5 or more marks

1. Define electrostatic potential at a point. Write its SI. unit. (CBSE Sample Paper 2019-20) [Art. 2.3]

2. Derive an expression for potential at a point

(*i*) due to point charge (Manipur Board 2011) [Art. 2.5]

(*ii*) due to a group of charges [Art. 2.6]

(*iii*) due to an electric dipole.

(CBSE 2019, 2017, Uttarakhand Board 2012, Jharkhand Board 2011, HP Board 2011) [Art. 2.7]

3. Explain the concept of electric potential energy. Derive an expression for potential energy of a system of two point charges. Generalise the expression for N discrete charges. [Arts. 2.2 and 2.10]

4. Derive an expression for the electric potential at any point P at a distance r from the centre of an electric dipole, making angle α with its axis. [CBSE 2017 (C)] [Art. 2.7]

5. Explain the underlying principle of working of a parallel plate capacitor.

If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expressions for

(*i*) the electric field at points between the two plates,

(*ii*) the potential difference between the plates.

(*iii*) the capacitance of the capacitor so formed.

(CBSE 2016, 2014, HP Board 2013, Raj. Board 2011) [Art. 2.17]

6. Derive an expression for the energy stored in a charged parallel plate capacitor. What is the form of this energy and where from it comes ? Find the expression for energy density of a charged capacitor.

(CBSE 2014, 2012, 2008 ; Manipur Board 2012 ; Pb. Board 2011) [Arts. 2.21 and 2.23]

7. A capacitor of capacitance C_1 is charged to a potential V_1, while another capacitor of capacitance C_2 is charged to a potential difference V_2. The capacitors are now disconnected from their respective charging batteries and connected in parallel to eachother.

(*a*) Find the total energy stored in the two capacitors before they are connected.

(*b*) Find the total energy stored in the parallel combination

(*c*) Explain the reason for the difference in energy.

[CBSE 2018 (C)] [Arts 2.21 and 2.25]

8. Explain dielectric polarization, electric susceptibility and dielectric constant. Establish a relation between them. [Art. 2.27]

9. Find the expression for capacitance of a parallel plate capacitor when (*i*) a thin dielectric slab is inserted in between the plates. (*ii*) a thin conducting slab is inserted in between the plates. (*iii*) a dielectric slab fills the entire space between the plates. (CBSE (F) 2017) [Art. 2.30 and 2.29]

10. Describe briefly the process of transferring the charge between the two plates of a parallel plate capacitor when connected to a battery. Derive an expression for the energy stored in a capacitor. (CBSE, 2019) [Art. 2.21]

11. Explain the principle on which Van-de-Graaff generator operates. Draw a labelled schematic sketch and write briefly its working.

(Raj. Board 2012, Bihar Board 2011, Hr. Board 2011, CBSE 2008) [Art. 2.33]

12. What is meant by dielectric constant, and polar and nonpolar dielectrics ? Explain with illustrations. (Kerala Board 2012) [Art. 2.26]

13. What do you understand by energy density of a parallel plate capacitor ? Derive a relation for it. [Art. 2.23]

14. Explain the terms with illustrations

(*i*) Dielectric strength

(*ii*) action of sharp points. [Art. 2.28 and 2.32]

15. What is a lightning conductor ? Explain its working. [Art. 2.32]

16. Three capacitors of capacitances C_1, C_2 and C_3 are connected (*i*) in series, (*ii*) in parallel. Show that the energy stored in the series combination is the same as that in parallel combination. [Art. 2.22]

17. When two charged condensers having different capacities and different potentials are joined together, show that there is always some loss of energy. What is the form of lost energy ? (Uttarakhand Board 2012) [Art. 2.25]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. Electrostatic potential difference between two points B and A in an electrostatic field = amount of work done in carrying unit positive charge from A to B (against the electrostatic force of the field) along any path between the two points, *i.e.*,

$$V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B \vec{E}.\vec{dl}$$

If A is at infinity, then $V_A = 0$

$$\therefore \quad V_B = \frac{W_{\infty B}}{q_0} = -\int_\infty^B \vec{E}.\vec{dl}$$

Electrostatic potential is a **scalar**. It is measured in **volt**.

At any point distant r from a point charge q, electric potential $V = \dfrac{q}{4\pi\epsilon_0 r}$

Electric field intensity at any point is negative rate of change of potential with distance at that point,

i.e., $$|\vec{E}| = \frac{-dV}{dr}$$

Electrostatic potential energy of a system of two point charges q_1 and q_2 distance r apart is,

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

SI unit of U is **joule**. Another unit of energy is **electron volt** (eV), where

$$1 \text{ eV} = 1{\cdot}6 \times 10^{-19} \text{ J}$$

Read the above passage carefully and answer the following very short questions and short questions.

1. At any point distant r from a point charge q, electric potential is given by .....
2. Choose the correct statement out of the following :

   (a) Electric potential is scalar and $V \propto \dfrac{1}{r^2}$

   (b) Electric potential is vector and $V \propto \dfrac{1}{r}$

   (c) Electric potential is vector and $V \propto \dfrac{1}{r^2}$

   (d) Electric potential is scalar and $V \propto \dfrac{1}{r}$
3. What is SI unit of electric potential ?
4. What is electrostatic P.E. of a system of two point charges q_1 and q_2 separated by distance r ? Name any two units of P.E. How are they related ?

CASE 2. The resultant potential at any point due to total charge configuration is

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{i=n} \left(\frac{q_i}{r_i}\right)$$

Electrostatic potential at a point due to short electric dipole of dipole moment p, at a distance r is

$$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

When point is on the dipole axis, $\theta = 0°$ or $180°$

$$\therefore \quad V = \frac{p}{4\pi\epsilon_0 r^2}$$

When point is on equatorial plane, $\theta = \dfrac{\pi}{2}$

$$\cos\theta = \cos\frac{\pi}{2} = 0 \qquad \therefore \ V = 0$$

Note that due to a point charge q,

$$V = \frac{q}{4\pi\epsilon_0 r}$$

$$\therefore \quad V \propto \frac{1}{r},$$

and due to an electric dipole, $V \propto \dfrac{1}{r^2}$

An equipotential surface is that surface, at every point of which electric potential is the same. Therefore no work is done in moving a test charge from one point of equipotential surface to the other. Equipotential surfaces of a single point charge are concentric spherical surfaces centred at the point charge.

For a uniform electric field, say along X-axis, equipotential surfaces are planes normal to X-axis and parallel to YZ plane, as shown in Fig. 2(Q).37(a) and 2(Q).37(b).

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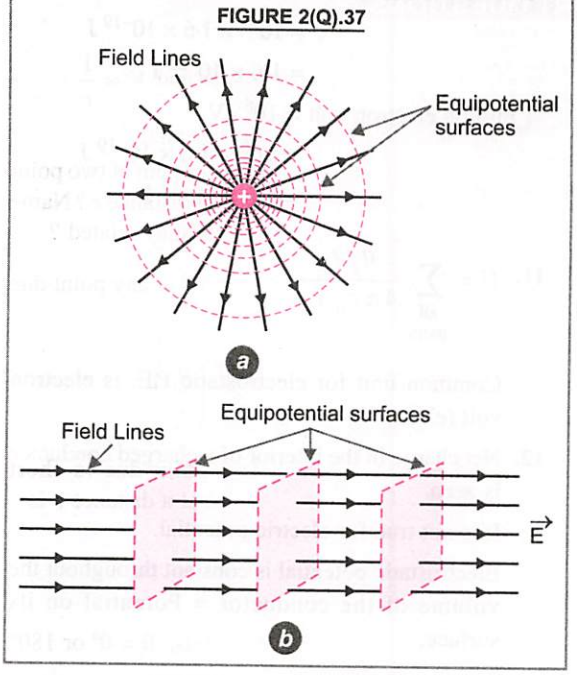
FIGURE 2(Q).37



Read the above passage carefully and answer the following very short questions and short questions.

5. Distance from a point charge is doubled. How is potential due to charge affected ?
6. Distance from the centre of an electric dipole is doubled. How is potential due to the dipole affected ?
7. What is an equipotential surface ? How much work is done in carrying a test charge from one point to the other on such a surface ?
8. Draw equipotential surfaces due to uniform electric field along X-axis.

CASE 3. Electrostatic potential energy of a system of point charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large separations, without acceleration.

Electrostatic P.E. of a system of two point charges q_1 and q_2 separated by a distance r_{12} is

$$U = \frac{q_1}{4\pi\epsilon_0 r_{12}} \times q_2$$

For a system of N point charges at distances, $\vec{r}_1, \vec{r}_2 \ldots \vec{r}_N$ respectively, electrostatic P.E. is given by

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\substack{\text{all} \\ \text{pairs}}} \frac{q_j q_k}{r_{jk}}$$

Common unit of electrostatic potential energy is **electron volt** (eV)

$1\ \text{eV} = 1{\cdot}6 \times 10^{-19}$ joule

$1\ \text{keV} = 10^3\ \text{eV} = 1{\cdot}6 \times 10^{-16}$ J

$1\ \text{MeV} = 10^6\ \text{eV} = 1{\cdot}6 \times 10^{-13}$ J and so on.

Some important results regarding electrostatics of conductors are :

(*i*) In electrostatic equilibrium, electric field $\vec{E} = 0$ everywhere inside the conductor.

(*ii*) Just outside a charged conductor electric field is perpendicular to the surface of the conductor.

(*iii*) Net charge in the interior of a conductor is zero.

(*iv*) Electrostatic potential = constant throughout the volume of the conductor, and has the same value as on its surface.

Read the above passage carefully and answer the following very short questions and short questions.

9. For a system of two point charges q_1, q_2 separated by a distance r_{12}, what is P.E. ?
10. Distinguish between one milli electron volt and one million electron volt.
11. What is the general expression for potential energy of a system of N point charges. What is the common unit used ?
12. What is the net charge in the interior of a charged conductor ? Is the same true for electric potential ?

ANSWERS

1. $V = \dfrac{q}{4\pi\epsilon_0 r}$
2. Electric potential is scalar, and $V \propto \dfrac{1}{r}$.
3. SI unit of electric potential is volt.
4. Electrostatic potential energy in free space is

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

Two units of electrostatic P.E. are electron volt (eV) and joule.

$$1 \text{ eV} = 1{\cdot}6 \times 10^{-19} \text{ J}$$

5. When distance from a point charge is doubled, potential reduces to half, as $V \propto \frac{1}{r}$.

6. Due to an electric dipole, $V \propto \frac{1}{r^2}$

when r is doubled, V becomes one fourth.

7. Equipotential surface is that surface at every point of which electric potential is the same.

W = charge × potential difference

$\therefore$ W = charge × zero = zero.

8. Due to uniform electric field along X-axis, equipotential surfaces are planes normal to X-axis and parallel to YZ plane as shown in Fig. 2(Q).37.

9. P.E. $= \dfrac{q_1 q_2}{4\pi \epsilon_0 r_{12}}$

10. 1 milli electron volt $= 10^{-3}$ eV

$= 10^{-3} \times 1{\cdot}6 \times 10^{-19}$ J

$= 1{\cdot}6 \times 10^{-22}$ J

1 million electron volt $= 10^6$ eV

$= 10^6 \times 1{\cdot}6 \times 10^{-19}$ J

$= 1{\cdot}6 \times 10^{-13}$ J

11. $U = \sum\limits_{\substack{\text{all} \\ \text{pairs}}} \dfrac{q_j q_k}{4\pi \epsilon_0 r_{jk}}$

Common unit for electrostatic P.E. is electron volt (eV).

12. Net charge in the interior of a charged conductor is **zero**.

It is not true for electric potential.

Electrostatic potential is constant throughout the volume of the conductor = Potential on its surface.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. Electrical capacitance of a conductor is related to its ability to store electric charge or electric energy. A charge Q given to a conductor raises its potential by V. It is found that $Q \propto V$ or $Q = CV$, where C is a constant of proportionality and is called **electrical capacitance** of the conductor.

SI unit of capacitance is **farad**.

From $C = \dfrac{Q}{V}$, 1 farad $= \dfrac{1 \text{ coulomb}}{1 \text{ volt}}$

We can show that capacity of an isolated spherical conductor of radius r is

$$C = 4\pi \epsilon_0 r$$

When r is in metre, capacity is in farad. Larger the radius of the sphere, greater is its capacity.

Based on the above paragraph, answer questions no. 1 to 4 :

1. The SI unit of capacitance is

(*a*) coulomb (*b*) volt

(*c*) farad (*d*) none of these

2. Capacity of an isolated spherical conductor of radius R is

(*a*) $4\pi \epsilon_0 R$ (*b*) $1/4\pi \epsilon_0 R$

(*c*) Q/V (*d*) V/Q

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, A and R are true and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

3. Assertion. Capacitance is a positive quantity.

Reason. $C = \dfrac{Q}{V}$,

when Q is positive, V is positive ; and vice-versa.

4. Assertion. Capacity of earth = 711 μF.

Reason. Taking earth as a perfect sphere of radius $R = 6{\cdot}4 \times 10^6$ m.

CASE 2. A capacitor is an arrangement for storing a large amount of electric charge and hence electric energy in a small space. The capacitance of an isolated conductor is increased considerably by bringing near it

an uncharged conductor connected to earth. This is the **principle of capacitor**. Such an arrangement of two conductors separated by a dielectric medium is said to form a **capacitor** or **condenser** Depending on the shape of conductors, we get **parallel plate** capacitor, **spherical** capacitor and **cylindrical** capacitor. Capacity of a parallel plate capacitor is $C = \frac{\epsilon_0 A}{d}$, where A is area of charged plate and d is the distance between the two plates of capacitor.

When plates of a capacitor are separated by a dielectric medium of relative permittivity ϵ_r or K, then

$$C_m = KC_0 \quad \text{or} \quad K = \frac{C_m}{C_0}.$$

Energy stored in a capacitor is

$$U = \frac{1}{2} Q V = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$$

When condensers are joined in parallel

$$C_p = C_1 + C_2 + C_3$$

When condensers are joined in series,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Based on the above paragraph, answer questions no. 5 to 8 :

5. With usual notation, capacity of a parallel plate capacitor is given by

(a) $C = \frac{\epsilon_0 A}{d^2}$ (b) $C = \frac{\epsilon_0}{A}$

(c) $C = \frac{\epsilon_0 A}{d}$ (d) $C = \frac{\epsilon_0 d}{A}$

6. Energy stored in a charged capacitor is

(a) $U = \frac{1}{2} QV^2$ (b) $U = \frac{1}{2} QV$

(c) $U = Q/V$ (d) $U = \frac{QV^2}{C}$

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(a) Both, A and R are true and R is correct explanation of A.

(b) Both, A and R are true, but R is not a correct explanation of A.

(c) A is true, but R is false.

(d) A is false, and R is also false.

7. **Assertion.** Mega Farad is too big a unit of capacity than farad.

Reason. Capacity of condensers increases in series.

8. **Assertion.** For storing larger amount of energy, condensers must be connected in parallel.

Reason. Energy stored $= \frac{1}{2} CV^2 \propto C$, and capacity in parallel increases.

ANSWERS

1. (c) **2.** (a) **3.** (a) **4.** (a) **5.** (c) **6.** (b) **7.** (c) **8.** (a)

HINTS/EXPLANATIONS For Difficult Questions

1. farad is the SI unit of capacitance.
2. $C = 4\pi\epsilon_0 R$
3. **Both, the Assertion and Reason are true, and Reason is correct explanation of Assertion.**
4. **Both, the Assertion and Reason are true and Reason is correct explanation of Assertion.**

Capacity of earth $= 4\pi\epsilon_0 R$

$$= \frac{6{\cdot}4 \times 10^6}{9 \times 10^9}$$

$$= 711 \times 10^{-6}\ \text{F}$$

$$= 711\ \mu\text{F}.$$

5. $C = \epsilon_0 A/d$
6. $U = \frac{1}{2} Q \times V$
7. **Here Assertion is true but Reason is false** as capacity of condensers decreases in series.
8. **Both, Assertion and Reason are true and Reason is the correct explanation of Assertion.**

PROBLEMS FOR PRACTICE

TYPE I. POTENTIAL DIFFERENCE AND WORK DONE

1. Two charges $-q$ and $+q$ are located at points A $(0, 0, -a)$ and B $(0, 0, +a)$ respectively. How much work is done in moving a test charge from point P $(7, 0, 0)$ to Q $(-3, 0, 0)$?

(CBSE 2009) **[Ans. Zero]**

2. If 100 J of work must be done to move electric charge equal to 4 C from a place where potential is –10 V to another place where potential is V volt, find the value of V. **[Ans. 15 V]**

3. If 10 J of work is to be done in moving a charge of –200 C from A to B, which of the two points is at higher potential ? What is the potential difference ?

[Ans. A ; – 0·05 V]

TYPE II. ELECTRIC POTENTIAL

4. The electric field at a point due to a point charge is 20 NC^{-1} and electric potential at that point is 10 JC^{-1}. Calculate the distance of the point from the charge and the magnitude of the charge.

(CBSE 2006)

[Ans. 0·5 m ; $0{\cdot}55 \times 10^{-9}$ C]

5. Two point charges q and $-2q$ are kept 'd' distance apart. Find the location of the point relative to charge q at which potential due to the system of charges is zero. (CBSE 2014)

[Ans. $d/3$ from charge q]

6. A charge of 24 μC is given to a hollow metallic sphere of radius 0·2 m. Find the potential

(*i*) at the surface of sphere

(*ii*) at a distance of 0·1 cm from the centre of sphere. **[Ans. $1{\cdot}08 \times 10^6$ V, $1{\cdot}08 \times 10^6$ V]**

7. Two point charges, one of + 100 μC and another of – 400 μC, are kept 30 cm apart. Find the point of zero potential on the line joining the two charges. **[Ans. 6 cm from 100 μC charge]**

8. Two point charges 4 μ C and – 2 μ C are separated by a distance of 1 m in air. Calculate at what point on the line joining the two charges is the electric potential zero ? (CBSE 2007)

$\left[\frac{2}{3}\text{ m from } 4\mu\text{C}\right]$

9. The electric field at a point due to a point charge is 30 N/C, and the electric potential at that point is 15 J/C. Calculate the distance of the point from the charge and the magnitude of the charge

[Ans. 0·5 m ; $0{\cdot}83 \times 10^{-9}$ C]

10. A point charge of 10^{-8} C is situated at the origin of co-ordinates. Find the potential difference between the points A (4, 4, 2) and B (1, 2, 2).

[Ans. – 15 V]

11. Twenty seven charged water droplets each with a diameter of 2 mm and a charge of 10^{-12} C coalesce to form a single drop. Calculate the potential of the bigger drop. (Pb. Board 2001)

[Ans. 81 V]

12. A charge of 20 μ C produces an electric field. Two points are 10 cm and 5 cm from this charge. Find the values of potentials at these points and calculate work done to take an electron from one point to the other.

[Ans. $1{\cdot}8 \times 10^6$ V, $3{\cdot}6 \times 10^6$ V, $2{\cdot}88 \times 10^{-13}$ J]

13. Calculate the voltage required to balance an oil drop carrying 10 electrons, when located between plates of a capacitor, which are 5 mm apart. Given mass of drop = 3×10^{-16} kg, charge on electron = $1{\cdot}6 \times 10^{-19}$ C and $g = 9{\cdot}8$ m/s^2. **[Ans. 9·2 V]**

TYPE III. POTENTIAL GRADIENT AND ELECTRIC FIELD

14. An infinite plane sheet of charge density 10^{-8} Cm^{-2} is held in air. In this situation how far apart are two equipotential surfaces, whose p.d is 5 V ? **[Ans. 8·85 mm]**

15. An electric field of 20 N/C exists along the X-axis in space. Calculate the potential difference $(V_B - V_A)$ where the co-ordinates of A and B are given by

(*i*) A (0, 0) ; B (4 m, 2 m)

(*ii*) A (4 m, 2 m) ; B (6 m, 5 m)

[Ans. – 80 V ; – 40 V]

16. What is potential gradient at a distance of 10^{-12} m from the centre of the platinum nucleus ? What is the potential gradient at the surface of the nucleus ? Atomic number of platinum is 78 and radius of platinum nucleus is 5×10^{-15} m.

[Ans. $1{\cdot}123 \times 10^{17}$ Vm^{-1} ; $4{\cdot}5 \times 10^{21}$ Vm^{-1}]

17. If the potential in the region of space around the point (– 1m, 2m, 3m) is given by

$V = (10\ x^2 + 5\ y^2 - 3\ z^2)$, calculate the three components of electric field at this point.

[Ans. $E_x = 20$ V m^{-1} ; $E_y = -20$ V m^{-1} ; $E_z = 18$ V m^{-1}]

TYPE IV. ELECTROSTATIC POTENTIAL ENERGY

18. Two charges of magnitude 5 nC and -2 nC are placed at points (2 cm, 0, 0) and (x cm, 0, 0) in a region of space. Where there is no other external field. If the electrostatic potential energy of the system is $-0{\cdot}5$ μJ. What is the value of x ?

(CBSE 2008) [Ans. $x = 4$ cm]

19. Two identical particles, each having a charge of 2×10^{-4} C and mass of 10 g, are kept at a separation of 10 cm and then released. What would be the speeds of particles when the separation becomes large ? **[Ans. 600 ms^{-1}]**

20. Two point charges A and B of value + 15 μ C and + 9 μ C are kept 18 cm apart in air. Calculate the work done when charge B is moved by 3 cm towards A. **[Ans. 1·35 J]**

21. Set up an arrangement of three point charges: $+q$, $+2q$ and xq separated by equal finite distances so that electric potential energy of the system is zero. What is x ?

[Ans. Charges $+q$, $+2q$ and $-\frac{2q}{3}$ at vertices of an equilateral triangle ; $x = -2/3$]

22. Three charges $-q$, Q and $-q$ are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, then what is the ratio of $Q : q$? **[Ans. 1 : 4]**

TYPE V. CAPACITANCE

23. N drops of mercury of equal radii and possessing equal charges combine to form a big drop. Compare the charge, capacitance and potential of bigger drop with the corresponding quantities of individual drops. **(Pb. Board 2001)**

[Ans. N ; $N^{1/3}$; $N^{2/3}$]

24. When electrons equal to Avogadro number are transferred from one conductor to another, a potential difference of 10^6 V appears between them. Calculate the capacity of the system of two conductors. **[Ans. 9·637 × 10^{-2} F]**

25. A charged spherical conductor has a surface density of 0·07 C cm^{-2}. When the charge is increased by 4·4 C, the surface density changes to 0·084 C cm^{-2}. Find the initial charge and capacitance of the spherical conductor.

[Ans. 22 C ; 5·56 × 10^{-12} F]

TYPE VI. PARALLEL PLATE CAPACITOR

26. A parallel plate air capacitor consists of two circular plates of diameter 8 cm. At what distance should the plates be held so as to have the same capacitance as that of a sphere of diameter 20 cm ? **[Ans. 4 mm]**

27. Calculate the area of paper required to construct a parallel plate capacitor of 0·004 μ F, if the dielectric constant of paper be 2·5 and its thickness 0·025 mm. **[Ans. 4·52 × 10^{-3} m^2]**

28. What distance should the two plates each of area $0{\cdot}2 \times 0{\cdot}1$ m^2 of an air capacitor be placed in order to have the same capacitance as a spherical conductor of radius 0·5 m ? **[Ans. 3·18 mm]**

29. The plates of a parallel plate capacitor are 5 mm apart and 2 m^2 in area. The plates are in vacuum. A potential difference of 1000 volt is applied across the capacitor. Calculate (*i*), the capacitance (*ii*) the charge on each plate and (*iii*)electric intensity in the space between the two plates.

[Ans. 3·54 × 10^{-9} F ; 3·54 × 10^{-6} C ; 2 × 10^5 NC^{-1}]

TYPE VII. SPHERICAL AND CYLINDRICAL CAPACITORS

30. A sphere of radius 0·03 m is suspended within a hollow sphere of radius 0·05 m. If the inner sphere is charged to a potential of 1500 volt and outer sphere is earthed. Find the capacitance and the charge of the inner sphere.

[Ans. 8·33 pF, 1·25 × 10^{-8} C]

31. A co-axial cable used in transmission line has inner radius of 0·1 mm and outer radius of 0·6 mm. Calculate capacitance per metre of the cable. **[Ans. 71·5 pF/m]**

32. A spherical capacitor has an outer sphere of radius 0·15 m and the inner sphere of diameter 0·2 m. The outer sphere is earthed and the inner sphere is given a charge of 6 μ C. The space between the concentric spheres is filled with a material of dielectric constant 6. Calculate capacitance and potential of inner sphere.

[Ans. 0·2 × 10^{-9} F ; 18 × 10^4 V]

33. A cable consisting of a wire 3 mm in diameter is insulated with 3 mm thick dielectric of relative permittivity 10. Calculate the capacitance of 1 km length of the cable. **[Ans. 0·506 μF]**

TYPE VIII. GROUPING OF CAPACITORS

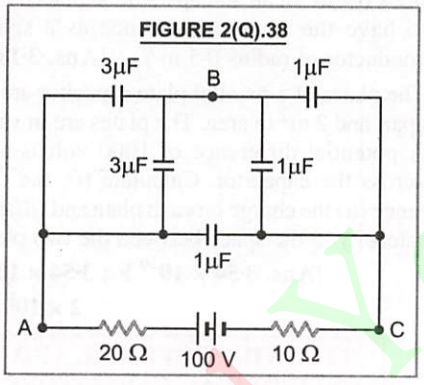
34. Two capacitors of capacitances 3 μF and 6 μF are charged to potentials of 2 V and 5 V respectively. These two charged capacitors are connected in series. Find the potential across each of the capacitors now. (CBSE Sample Paper 2004)

[Ans. 12 V ; 6 V]

35. If $C_1 = 20$ μF, $C_2 = 30$ μF and $C_3 = 15$ μF and the insulated plate of C_1 be at a potential of 90 V, one plate of C_3 being earthed : What is the p.d between the plates of C_2, three capacitors being connected in series. (CBSE 2015) **[Ans. 20 V]**

36. For the network shown in Fig.2(Q).38, find the potential difference between points A and B, and that between B and C in the steady state.

FIGURE 2(Q).38



[Ans. $V_{AB} = 25$ V, $V_{BC} = 75$ V]

37. Two capacitors have a capacitance of 5 μF when connected in parallel and 1·2 μF when connected in series. Calculate their capacitances.

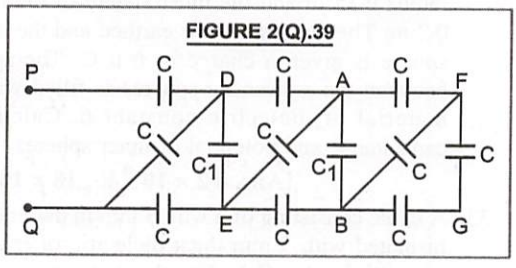
[Ans. 2 μF, 3 μF]

38. Connect three capacitors of 3 μF, 3 μF and 6 μF such that their equivalent capacitance is 5 μF.

[Ans. Series combination of 3 μF and 6 μF in parallel with 3 μF]

39. Find the equivalent capacitance between the points P and Q as shown in Fig. 2(Q).39. Given $C = 18$ μF and $C_1 = 12$ μF.

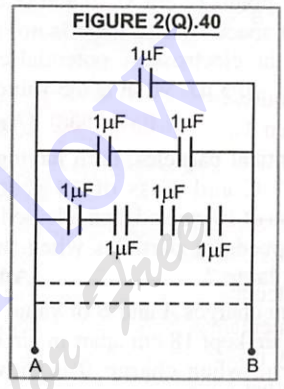
FIGURE 2(Q).39



[Ans. 11 μF]

40. There are infinite number of capacitors each of capacitance 1 μF. They are connected in rows, such that number of capacitors in first, second, third rows are respectively 1, 2, 4, 8 ..... . The rows of these capacitors are connected between A and B, as shown in Fig. 2(Q).40. Determine the equivalent capacitance of the network between points A and B.

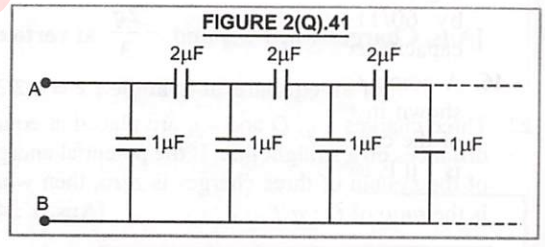
FIGURE 2(Q).40



[Ans. 2 μF]

41. Find the equivalent capacitance of the ladder between points A and B, Fig. 2(Q).41.

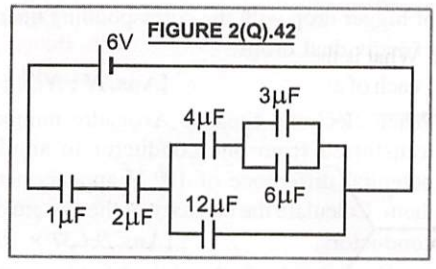
FIGURE 2(Q).41



[Ans. 2 μF]

42. Find out the potential difference across the plates of 1 μ F capacitor in Fig. 2(Q).42.

FIGURE 2(Q).42



[Ans. 3·82 V]

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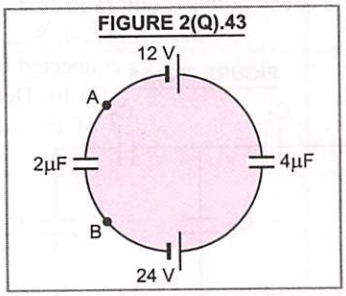
43. Find the p.d. between points A and B of arrangement shown in Fig. 2(Q).43.

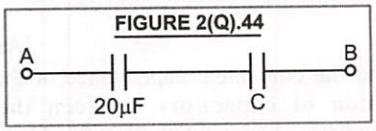
FIGURE 2(Q).43



[Ans. – 8 V]

44. The equivalent capacitance of the combination between A and B in the given Fig. 2(Q).44 is 4 μ F.

FIGURE 2(Q).44



(*i*) Calculate capacitance of capacitor C.

(*ii*) Calculate charge on each capacitor if a 12 V battery is connected across A and B.

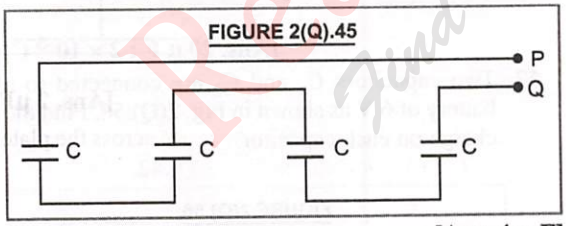
(*iii*) What will be the potential drop across each capacitor ? (CBSE 2009)

[Ans. 5 μ F ; 48 μ C ; 2·4 V, 9·6 V]

45. The capacities of three capacitors are in the ratio of 1 : 2 : 3. Their equivalent capacity in parallel is greater than their equivalent capacity in series by 60/11 pF. Calculate their individual capacitances. **[Ans. 1 pF, 2 pF, 3 pF]**

46. A combination of four identical capacitors is shown in Fig. 2(Q).45. If resultant capacitance of the combination between the points P and Q is 1 μ F, calculate capacitance of each capacitor.

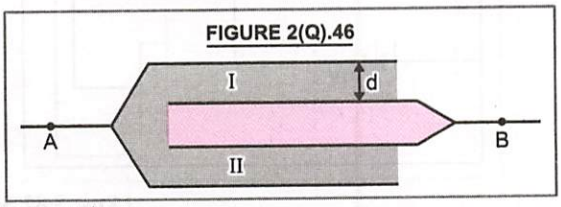
FIGURE 2(Q).45



[Ans. 4 μ F]

47. What is the capacitance of arrangement of 4 plates each of area A at a distance d in air in Fig. 2(Q).46.

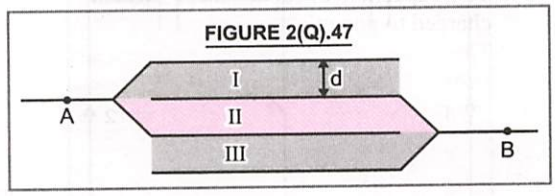
FIGURE 2(Q).46



[Ans. 2 ϵ_0 A/d]

48. What is the capacitance of arrangement of 4 plates of area A at distance d in air in Fig. 2(Q).47.

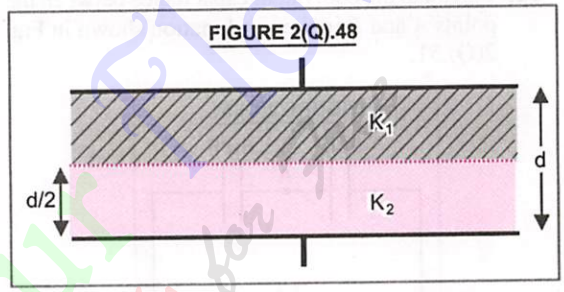
FIGURE 2(Q).47



[Ans. 3 ϵ_0 A/d]

49. A parallel plate capacitor is filled with dielectrics as shown in Fig. 2(Q).48. What is its capacitance ?

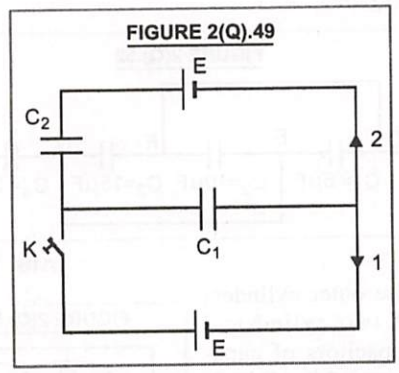
FIGURE 2(Q).48



[Ans. $\frac{2\epsilon_0 A K_1 K_2}{d(K_1 + K_2)}$ **]**

50. Calculate the charges which will flow in sections 1 and 2 in Fig. 2(Q).49, when key K is pressed.

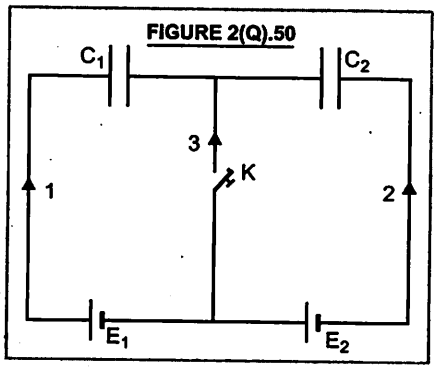
FIGURE 2(Q).49



[Ans. $EC_1 ; \frac{EC_1C_2}{(C_1 + C_2)}$ **]**

51. In the circuit shown in Fig. 2(Q).50, the emf of each battery is E = 12 volt and the capacitances are C_1 = 2·0 μ F and C_2 = 3·0 μ F. Find the charges which flow along the paths 1, 2, 3 when key K is pressed.

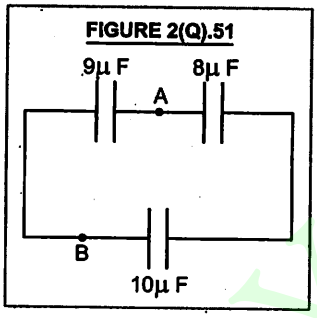
FIGURE 2(Q).50



[Ans. 24 μ C ; – 36 μ C ; 12 μ C]

52. Calculate the equivalent capacitance between the points *A* and *B* in the combination shown in Fig. 2(Q).51.

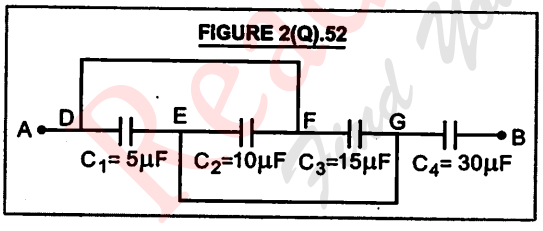
FIGURE 2(Q).51



[Ans. 13·44 μ F]

53. Calculate the equivalent capacitance between the points A and B in the combination shown in Fig. 2(Q).52.

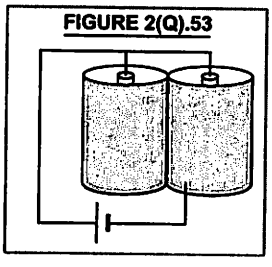
FIGURE 2(Q).52



[Ans. 15 μ F]

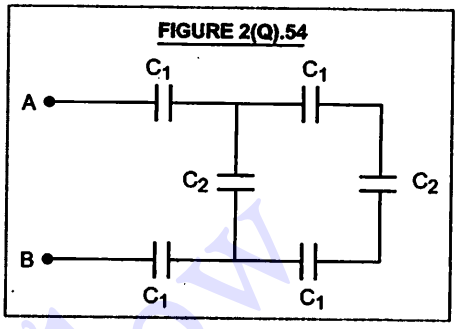
54. The outer cylinders of two cylindrical capacitors of capacitance 2·2 μ F each, are kept in contact and the inner cylinders are connected through a wire. A battery of e.m.f. 10 V is connected as shown in Fig. 2(Q).53. Find the total charge supplied by the battery to the inner cylinders. [Ans. 44 μ C]

FIGURE 2(Q).53



55. If C_1 = 3 pF and C_2 = 2 pF, calculate the equivalent capacitance of the network shown in Fig. 2(Q).54 between points *A* and *B*.

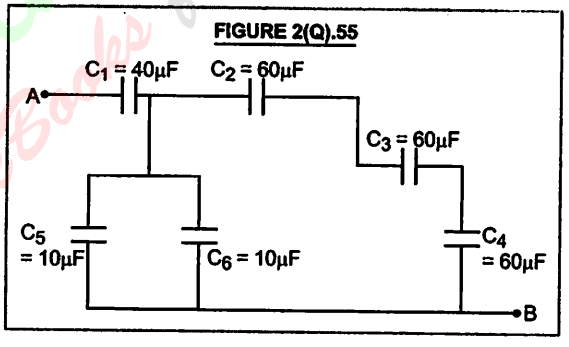
FIGURE 2(Q).54



[Ans. 1 pF]

56. Find the equivalent capacitance of the combination of capacitors between the points *A* and *B* as shown in Fig. 2(Q).55. Also calculate the total charge that flows in the circuit, when a 100 V battery is connected between the points *A* and *B*.

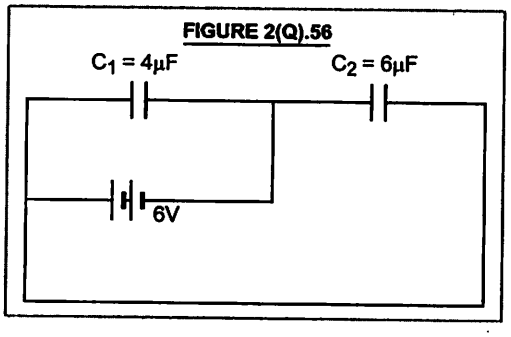
FIGURE 2(Q).55



[Ans. 20 μ F ; 2×10^{-3} C]

57. Two capacitors C_1 and C_2 are connected to a battery of 6 V as shown in Fig. 2(Q).56. Find the charge on each capacitor.

FIGURE 2(Q).56



[Ans. 24 μ C ; 36 μ C]

TYPE IX. ENERGY STORED IN A CAPACITOR AND LOSS OF ENERGY ON SHARING CHARGES

58. A 800 pF capacitor is charged by a 100 V battery. After sometime, the battery is disconnected. The capacitor is then connected to another 800 pF capacitor. What is the electrostatic energy stored ?

(CBSE 2009)

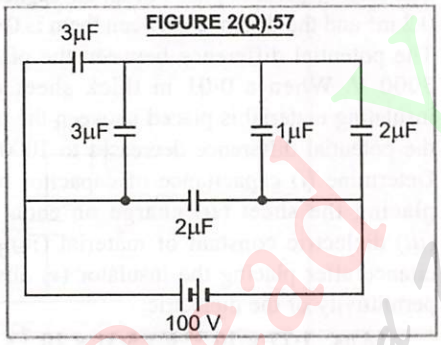
[Ans. 2×10^{-6} J]

59. Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance in parallel ? Find the ratio of energy stored in two configurations if they are connected to the same source. (CBSE 2011)

[Ans. 9 μF : 1 : 9]

60. Fig.2(Q).57, shows a network of five capacitors connected to a 100 V supply. Calculate the total charge and energy stored in the network.

FIGURE 2(Q).57

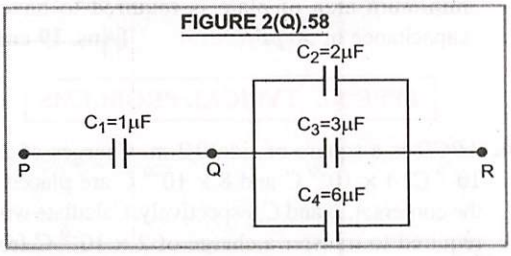


(CBSE Sample Paper 2008)

[Ans. 4×10^{-4} C ; 0·02 J]

61. In Fig. 2(Q).58, the energy stored in C_4 is 27 J. Calculate the total energy stored in the system.

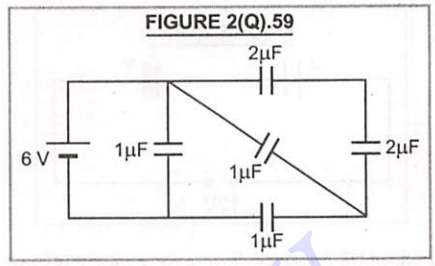
FIGURE 2(Q).58



[Ans. 594·0 J]

62. Find the total energy stored in capacitors in the network shown in Fig. 2(Q).59.

FIGURE 2(Q).59



(CBSE Sample Paper 2019-20)

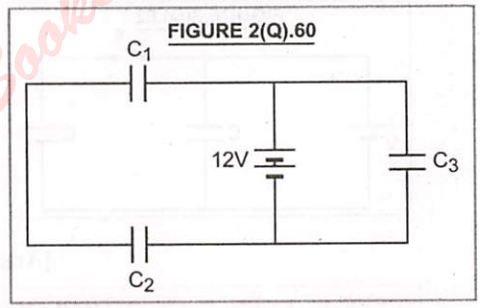
[Ans. $3{\cdot}6 \times 10^{-5}$ J]

63. Net capacitance of three identical capacitors in series is 1 μ F. What will be their net capacitance if connected in parallel ? Find the ratio of energy stored in the two configurations, if they are both connected to the same source. (CBSE 2011)

[Ans. 9 μ F ; 1 : 9]

64. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μ F each are connected to a 12 V battery as shown in Fig. 2(Q).60. Find

FIGURE 2(Q).60



(*i*) charge on each capacitor

(*ii*) equivalent capacitance of the network

(*iii*) energy stored in the network of capacitors.

(CBSE 2009)

[Ans. $q_1 = q_2 = 36$ μ C, $q_3 = 72$ μ C ; 9 μ F ; $E_1 = E_2 = 108$ μ J, $E_3 = 432$ μ J]

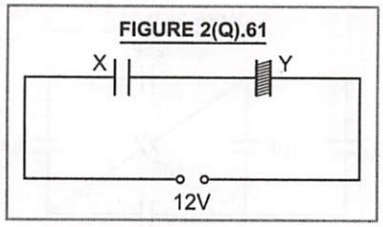
65. A 20 μ F capacitor is charged by a 30 V d.c. supply and then connected across an uncharged 50 μ F capacitor. Calculate (*i*) the final potential diff. across the combination (*ii*) initial and final energies. How will you account for the difference in energies ?

[Ans. 8·57 V ; 9×10^{-3} J, $2{\cdot}57 \times 10^{-3}$ J]

66. Two parallel plate capacitors *X* and *Y* have the same area of plates and same separation between

them. X has air between the plates and Y contains a dielectric medium of $\in_r = 4$, Fig. 2(Q).61.

FIGURE 2(Q).61



Calculate (*i*) capacitance of X and Y if equivalent capacitance of combination is 4 μ F.

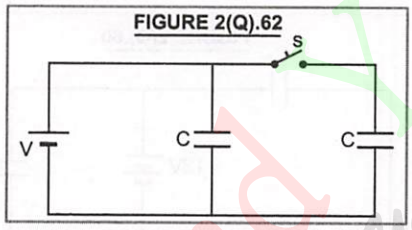
(*ii*) pot. diff. between the plates of X and Y.

(*iii*) What is the ratio of electrostatic energy stored in X and Y ? (CBSE 2009)

[Ans. 5 μ F, 20 μ F, 9·6 V, 2·4 V ; 4 : 1]

67. In the circuit shown in Fig. 2(Q).62, the energy stored in both capacitors is U_1. If switch S is opened and a dielectric slab of constant 5 is put in free spaces of the capacitors, the energy stored is found to be U_2. Calculate U_1/U_2.

FIGURE 2(Q).62



[Ans. 5/13]

TYPE X. EFFECT OF INTRODUCING A CONDUCTOR / DIELECTRIC SLAB BETWEEN THE PLATES OF A PARALLEL PLATE CAPACITOR

68. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with the faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes $\frac{2}{3}$rd of its original value. What is the new distance between the plates ? [CBSE 2008 (C)]

[Ans. 8 mm]

69. An electric field $E_0 = 3 \times 10^4$ Vm^{-1} is established between the plates 0·05 m apart, of a parallel plate capacitor. After removing the charging battery, an uncharged metal plate of thickness $t = 0{\cdot}01$ m is inserted between capacitor plates. Find the p.d across the capacitor. (*i*) before (*ii*) after the introduction of plates (*iii*) what would be the p.d if a dielectric slab ($K = 2$) were introduced in place of metal plate.

[Ans. 1500 V, 1200 V, 1350 V]

70. The two circular plates of a parallel plate capacitor are 8 cm in diameter each and 15 mm apart. An ebonite plate 0·5 *cm* thick is introduced between the plates. Calculate its capacity. If the plate were of copper, what would be the new capacity ? Take $K = 2{\cdot}5$.

[Ans. $3{\cdot}71 \times 10^{-12}$ F ; $4{\cdot}45 \times 10^{-12}$ F]

71. When a slab of insulating material 4 mm thick is introduced between the plates of a parallel plate capacitor, it is found that the distance between the plates has to be increased by 3·2 mm to restore the capacity to its original value. Calculate dielectric constant of the material. **[Ans. 5]**

72. The area of parallel plates of an air capacitor is 0·2 m^2 and the distance between them is 0·01 m. The potential difference between the plates is 3000 V. When a 0·01 m thick sheet of an insulating material is placed between the plates, the potential difference decreases to 1000 volt. Determine (*i*) capacitance of capacitor before placing the sheet (*ii*) charge on each plate (*iii*) dielectric constant of material (*iv*) capacitance after placing the insulator (*v*) absolute permittivity of the dielectric.

[Ans. $1{\cdot}77 \times 10^{-10}$ F ; $5{\cdot}31 \times 10^{-7}$ C ; 3 ; $5{\cdot}31 \times 10^{-10}$ F ; $2{\cdot}66 \times 10^{-11}$ F.m]

73. A parallel plate capacitor is to be designed with voltage rating of 1 kV using a material of dielectric constant 3 and dielectric strength of 10^7 Vm^{-1}. For safety, we like the field never to exceed 10% of dielectric strength. What minimum area of plate is required to have a capacitance of 50 *p*F ? **[Ans. 19 cm^2]**

TYPE XI. TYPICAL PROBLEMS

74. $ABCD$ is a square of side 0·2 m. Charges of 2×10^{-9} C, 4×10^{-9} C and 8×10^{-9} C are placed at the corners A, B and C respectively. Calculate work required to transfer a charge of 2×10^{-9} C from corner D to centre of the square.

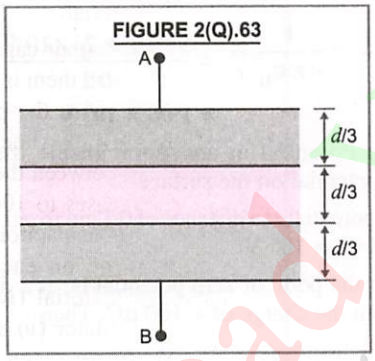
[Ans. $6{\cdot}27 \times 10^{-7}$ J]

75. A point charge q moves from a point P to a point S along the path $PQRS$ in a uniform electric field $\vec{E}$ along positive direction of x-axis. Calculate work done in this process, when co-ordinates of P, Q, R, S are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ respectively. **[Ans. $-qEa$]**

76. The electric field outside a charged long straight wire is given by $E = \frac{1000}{r}\,\text{V m}^{-1}$, and is directed outwards. What is the sign of the charge on the wire ? If two points A and B are situated such that $r_A = 0{\cdot}2$ m and $r_B = 0{\cdot}4$ m, find the value of $(V_B - V_A)$. **[Ans. + charge ; – 693·1 V]**

77. Fig. 2(Q).63 shows a parallel plate capacitor of plate area A and plate separation d. Its entire space is filled with three different dielectric slabs of same thickness. Find the equivalent capacitance of the arrangement.

FIGURE 2(Q).63



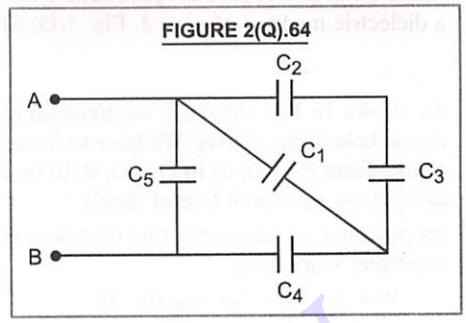
$$\left[\text{Ans. } C = \frac{3\,\varepsilon_0 A K_1 K_2 K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}\right]$$

78. A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has thickness $d/2$, where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates.

(CBSE 2013, 2010) $\left[\text{Ans. } \frac{2K}{K+1} C_0\right]$

79. Five capacitors of capacitances $C_1 = C_5 = 1\,\mu$F ; $C_2 = C_3 = C_4 = 2\,\mu$F are connected as shown in Fig. 2(Q).64. Calculate equivalent capacitance of the system between points A and B.

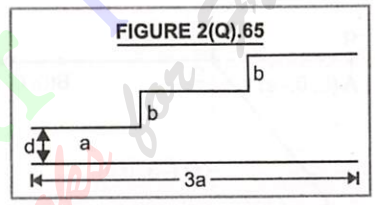
FIGURE 2(Q).64



[Ans. 2 μ F]

80. A capacitor is made of a flat plate of area A and a second plate having a stair like structure as shown in Fig. 2(Q).65. The width of each stair is a and the height is b. Find the capacitance of the assembly.

FIGURE 2(Q).65



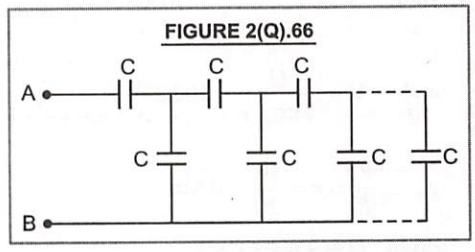
$$\left[\text{Ans. } C = \frac{\epsilon_0 A(3d^2 + 6bd + 2b^2)}{3d(d+b)(d+2b)}\right]$$

81. Find the capacitance of three parallel plates, each of area $A\ m^2$ and separated by d_1 and d_2 metre. The inbetween spaces are filled with dielectrics of relative permittivity ϵ_1 and ϵ_2. The permittivity of free space is ϵ_0.

$$\left[\text{Ans. } C = \frac{\epsilon_1\epsilon_2\epsilon_0 A}{\epsilon_1 d_2 + \epsilon_2 d_1}\right]$$

82. Find the equivalent capacitance between the terminals A and B in the given Fig. 2(Q).66. Take $C = 1\,\mu$F.

FIGURE 2(Q).66



[Ans. 0·62 μF]

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HINTS/EXPLANATIONS

For Difficult Questions

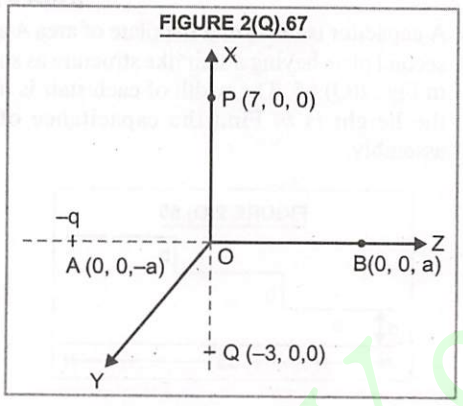
1. As shown in Fig. 2(Q).67, we have an electric dipole held along Z-axis. We have to move a test charge from P (7, 0, 0) to Q (– 3, 0, 0) on the X-axis *i.e.* on equatorial line of dipole.

As potential on equatorial line of dipole is zero, therefore, work done

$$W = q_0 (V_Q - V_P) = q_0 (0 - 0)$$

$$= \textbf{Zero}$$

FIGURE 2(Q).67



2. Here, $W_{AB} = 100$ J, $q = 4$ C,

$V_A = -10$ V, $V_B = V = ?$

As $W_{AB} = q (V_B - V_A)$

$\therefore \quad 100 = 4 (V + 10)$;

$\therefore \quad \textbf{V = 15 volt}$

3. Here, $W_{AB} = 10$ J, $q = -200$ C

$V_B - V_A = ?$

As $W_{AB} = q (V_B - V_A)$

$$\therefore V_B - V_A = \frac{W_{AB}}{q} = \frac{10}{-200} = \textbf{–0·05 V}$$

Clearly, ***A* must be at higher potential.**

4. Here, $E = \dfrac{Q}{4\pi\epsilon_0 r^2} = 20\,\text{NC}^{-1}$

and $V = \dfrac{Q}{4\pi\epsilon_0 r} = 10\,\text{JC}^{-1}$

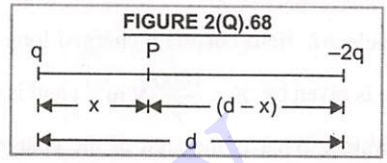
$$\therefore \quad \frac{V}{E} = r = \frac{10}{20} = \textbf{0·5 m}$$

From $Q = 4\pi\epsilon_0 r \times V$

$$= \frac{1}{9\times10^9} \times 0{\cdot}5 \times 10 = \textbf{0·55 × 10}^{\textbf{–9}}\ \textbf{C}$$

5. Let the potential be zero at a point P, distant x from charge q as shown in Fig. 2(Q).68.

FIGURE 2(Q).68



Now, $V = \dfrac{q}{4\pi\epsilon_0 x} + \dfrac{-2q}{4\pi\epsilon_0 (d-x)} = 0$

$$\frac{q}{4\pi\epsilon_0 x} = \frac{2q}{4\pi\epsilon_0 (d-x)}$$

or $\quad 2x = d - x$ or $x = \textbf{\textit{d}/3}$

6. (*i*) $q = 24\ \mu\text{C} = 24 \times 10^{-6}$ C, $r = 0{\cdot}2$ m.

At the surface of the sphere,

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{9\times10^9 \times 24\times10^{-6}}{0{\cdot}2}$$

$$= \textbf{1·08 × 10}^{\textbf{6}}\ \textbf{V}$$

(*ii*) Potential at any point inside the sphere = potential on the surface

$\therefore$ potential at a distance of 0·1 cm from the centre **= 1·08 × 10^6 V**

7. Let the point of zero potential be at a distance x from the charge of + 100 μC. Then

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{100\times10^{-6}}{x} - \frac{400\times10^{-6}}{0{\cdot}30 - x}\right] = 0$$

$x = 0 + 06$ m **= 6 cm from 100 μC charge.**

8. Here, $q_1 = 4\ \mu$ C, $q_2 = -2\ \mu$ C

Let the potential be zero at a point P, at a distance $r_1 = x$ from 4 μ C charge. Therefore, $r_2 = (1 - x)$.

Potential at $P = \dfrac{k q_1}{r_1} + k\dfrac{q_2}{r_2} = 0$

$$\therefore \quad \frac{q_1}{r_1} = \frac{-q_2}{r_2}$$

$$\frac{4}{x} = \frac{-(-2)}{1-x}$$

$$x = 2 - 2x$$

$$3x = 2,\ x = \frac{2}{3}\ \textbf{m from 4 μC charge}$$

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9. Here, $E = \dfrac{q}{4\pi\epsilon_0 r^2} = 30\,\text{NC}^{-1}$ ...(*i*)

$$V = \frac{q}{4\pi\epsilon_0 r} = 15\,\text{JC}^{-1} \quad \text{...(ii)}$$

Divide (*ii*) by (*i*), we get

$$r = \frac{1}{2} = \mathbf{0{\cdot}5\ m}$$

Put in (*ii*), $9\times10^9 \dfrac{q}{1/2} = 15$

$$q = \frac{15}{18}\times10^{-9} = \mathbf{0{\cdot}83 \times 10^{-9}\ C}$$

10. Here, $q = 10^{-8}\,C$, $\vec{r_A} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

so that $r_A = \sqrt{4^2+4^2+2^2} = 6$

$\vec{r_B} = 1\hat{i} + 2\hat{j} + 2\hat{k}$, so that

$r_B = \sqrt{1^2+2^2+2^2} = 3$.

$$V_A - V_B = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$= 9\times10^9\times10^{-8}\left[\frac{1}{6} - \frac{1}{3}\right] = \mathbf{-15\ V}$$

12. Here, $q = 20\ \mu\text{C} = 20 \times 10^{-6}$ C.

If A and B are two points at distances $r_1 = 10$ cm $= 0{\cdot}1$ m and $r_2 = 5$ cm $= 0{\cdot}05$ m

then $V_A = \dfrac{q}{4\pi\epsilon_0 r_1} = \dfrac{20\times10^{-6}\times9\times10^9}{0{\cdot}1}$

$$= \mathbf{1{\cdot}8 \times 10^6\ V}$$

$$V_B = \frac{q}{4\pi\epsilon_0 r_2} = \frac{20\times10^{-6}\times9\times10^9}{0{\cdot}05}$$

$$= \mathbf{3{\cdot}6 \times 10^6\ V}$$

$W_{AB} = (V_B - V_A)\,e$

$= (3{\cdot}6 \times 10^6 - 1{\cdot}8 \times 10^6) \times 1{\cdot}6 \times 10^{-19}$

$= \mathbf{2{\cdot}88 \times 10^{-13}\ J}$

13. Here, $V = ?$, $n = 10$, $d = 5$ mm $= 5 \times 10^{-3}$ m,

$m = 3 \times 10^{-16}$ kg, $e = 1{\cdot}6 \times 10^{-19}$ C,

$g = 9{\cdot}8$ m/s^2

To balance the oil drop

$$F = qE = mg \quad \text{or} \quad q\left(\frac{V}{d}\right) = mg$$

$$\therefore\ V = \frac{mgd}{q} = \frac{mgd}{ne} = \frac{3\times10^{-16}\times9{\cdot}8\times5\times10^{-3}}{10\times1{\cdot}6\times10^{-19}}$$

$$= \mathbf{9{\cdot}2\ volt}$$

14. Electric field intensity of an infinite plane sheet of charge, $E = \dfrac{\sigma}{2\epsilon_0}$

Let Δr be the separation between two equi-potential surfaces having potential difference ΔV,

then $E = \dfrac{\Delta V}{\Delta r}$

$$\frac{\sigma}{2\epsilon_0} = \frac{\Delta V}{\Delta r} \Rightarrow \Delta r = \frac{2\epsilon_0 \Delta V}{\sigma}$$

$$= \frac{2\times8{\cdot}85\times10^{-12}\times5}{10^{-8}} = 8{\cdot}85 \times 10^{-3}\ \text{m}$$

$$= \mathbf{8{\cdot}85\ mm}$$

15. As the electric field is along X-axis, therefore, from $E = -dV/dr$

(*i*) $dV = -E\,(dr) = -E\,(x_2 - x_1)$

$= -20\,(4 - 0) = \mathbf{-80\ V}$

(*ii*) $dV = -20\,(x'_2 - x'_1) = -20\,(6-4) = \mathbf{-40\ V}$

16. Here, $q = Ze = 78 \times 1{\cdot}6 \times 10^{-19}$ C

Potential gradient at a point is numerically equal to electric field at that point, *i.e.*,

$$\frac{dV}{dr} = E = \frac{q}{4\pi\epsilon_0 r^2}$$

At $r = 10^{-12}$ m ;

$$\frac{dV}{dr} = E = \frac{9\times10^9\times78\times1{\cdot}6\times10^{-19}}{(10^{-12})^2}$$

$$= \mathbf{1{\cdot}123 \times 10^{17}\ Vm^{-1}}$$

At $r = 5 \times 10^{-15}$ m ;

$$\frac{dV}{dr} = E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{9\times10^9\times78\times1{\cdot}6\times10^{-19}}{(5\times10^{-15})^2}$$

$$= \mathbf{4{\cdot}5 \times 10^{21}\ Vm^{-1}}$$

17. Here $x = -1$ m, $y = 2$ m, $z = 3$ m

$$E_x = \frac{-\partial V}{\partial x} = \frac{-\partial}{\partial x}\,(10x^2 + 5y^2 - 3z^2)$$

$$= -20x = -20\,(-1) = \mathbf{+20\ Vm^{-1}}$$

$$E_y = \frac{-\partial V}{\partial y} = -\frac{\partial}{\partial y}\,(10x^2 + 5y^2 - 3z^2)$$

$$= -10y = -10\,(2)$$

$$= \mathbf{-20\ V\,m^{-1}}$$

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$$E_z = \frac{-\partial V}{\partial z} = -\frac{\partial}{\partial z}(10x^2 + 5y^2 - 3z^2)$$

$$= 6z = 6(3) = \mathbf{18\ Vm^{-1}}$$

18. $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

$$-5\times10^{-6} = \frac{9\times10^9\times5\times10^{-9}\times(-2)\times10^{-9}}{(x-2)\times10^{-2}}$$

$$x-2 = \frac{9\times10^9\times5\times10^{-9}\times-2\times10^{-9}}{-0{\cdot}5\times10^{-6}\times10^{-2}}$$

On solving, x = **4 cm**

19. Here, $q = 2\times10^{-4}$ C, $m = 10$ g $= 10^{-2}$ kg,

$r = 10$ cm $= 0{\cdot}1$ m.

Let v : speed of each particle at infinite separation

P.E. of two particles at the separation of 10 cm.

= K.E. of two particles at infinite separation

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$v^2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{rm} = \frac{9\times10^9\times2\times10^{-4}\times2\times10^{-4}}{0{\cdot}1\times10^{-2}}$$

$$= 36\times10^4$$

v = **600 ms^{-1}**

20. Here, $q_1 = 15\times10^{-6}$ C, $q_2 = 9\times10^{-6}$ C

$r_1 = 18\times10^{-2}$ m ; $r_2 = (18-3)\ 10^{-2}$ m

work done = final P.E. – initial *P.E.*

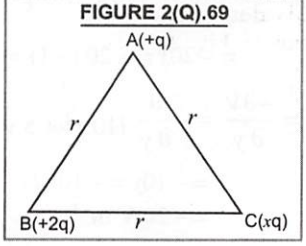
$$W = \frac{q_1 q_2}{4\pi\epsilon_0 r_2} - \frac{q_1 q_2}{4\pi\epsilon_0 r_1} = \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= 9\times10^9\times15\times10^{-6}\times9\times10^{-6}\left[\frac{10^2}{15} - \frac{10^2}{18}\right]$$

W = **1·35 joule**

21. Let the three charges be held at the vertices A, B, C of an equilateral triangle of each side = r, Fig. 2(Q).69.

FIGURE 2(Q).69



Potential energy of the system

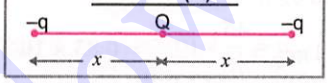
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q(2q)}{r} + \frac{2q(xq)}{r} + \frac{xq(+q)}{r}\right] = 0$$

$$\frac{2q^2}{r} + \frac{3q^2x}{r} = 0$$

$$x = \mathbf{-2/3}$$

22. If x is the distance between the charges as shown in Fig. 2(Q).70, then potential energy of the system of three charges is

FIGURE 2(Q).70



$$\frac{k(-q)Q}{x} + \frac{kQ(-q)}{x} + \frac{k(-q)(-q)}{2x} = 0$$

$$-\frac{2kqQ}{x} + \frac{kq^2}{2x} = 0 \quad \text{or} \quad \frac{kq^2}{2x} = \frac{2kqQ}{x}$$

$$q = 4Q \quad \text{or} \quad \frac{Q}{q} = \mathbf{\frac{1}{4}}$$

23. Let r be the radius of each small drop carrying charge q and potential V. When N such drops combine, there is no loss of charge.

$\therefore$ Charge on bigger drop = $Q = Nq$ or $\frac{Q}{q} = \mathbf{N}$

As volume of big drop = Volume of N small drops,

$$\therefore \quad \frac{4}{3}\pi R^3 = N\times\frac{4}{3}\pi r^3 \quad \text{or} \quad R = N^{1/3}r$$

As capacitance $\propto$ radius, therefore

$$\frac{C'}{C} = \frac{R}{r} = \mathbf{N^{1/3}}$$

$$\text{Potential of big drop} = \frac{Nq}{4\pi\epsilon_0 R} = \frac{Nq}{4\pi\epsilon_0 N^{1/3}r}$$

$$= \mathbf{N^{2/3}} \times \text{pot. of small drop}$$

24. Here, q = Charge on $6{\cdot}023\times10^{23}$ electrons

$= 6{\cdot}023\times10^{23}\times1{\cdot}6\times10^{-19}$ C $= 9{\cdot}637\times10^4$ C

$$C = \frac{q}{V} = \frac{9{\cdot}637\times10^4}{10^6} = \mathbf{9{\cdot}637\times10^{-2}\ F}$$

25. Here, $\sigma_1 = 0{\cdot}07$ C cm^{-2} and $\sigma_2 = 0{\cdot}084$ C cm^{-2}.

Let q be the charge on the spherical conductor of radius r cm.

$$\therefore \quad \sigma_1 = \frac{q}{4\pi r^2} = 0{\cdot}07 \quad ...(i)$$

and $\sigma_2 = \dfrac{q+4{\cdot}4}{4\pi r^2} = 0{\cdot}084$ ...(*ii*)

Dividing (*ii*) by (*i*), we get

$$\frac{q+4{\cdot}4}{q} = \frac{0{\cdot}084}{0{\cdot}07} = 1{\cdot}2$$

or $q = \mathbf{22\ C}$

From (*i*),

$$r = \sqrt{\frac{q}{4\pi\times 0{\cdot}07}} = \sqrt{\frac{22\times 7}{4\times 22\times 0{\cdot}07}} = 5\text{ cm}$$

$r = 0{\cdot}05$ m

$$C = 4\pi\epsilon_0\, r = \frac{0{\cdot}05}{9\times 10^9} = \mathbf{5{\cdot}56\times 10^{-12}\ F}$$

26. $\dfrac{\epsilon_0 A}{d} = 4\pi\epsilon_0 R$

or $\dfrac{\epsilon_0\,\pi\,(D^2)}{d\,4} = 4\pi\epsilon_0 R$

$$d = \frac{D^2}{16R} = \frac{(0{\cdot}08)^2}{16\times 0{\cdot}10} = 4\times 10^{-3}\text{ m} = \mathbf{4\ mm}$$

27. $C = KC_0 = K\dfrac{\epsilon_0 A}{d}$

29. Here, $d = 5\times 10^{-3}$ m, $A = 2$ m^2, $V = 1000$ Volt,

$$C = \frac{\epsilon_0 A}{d} = \frac{8{\cdot}85\times 10^{-12}\times 2}{5\times 10^{-3}} = \mathbf{3{\cdot}54\times 10^{-9}\ F}$$

$$q = CV = 3{\cdot}54\times 10^{-9}\times 1000 = \mathbf{3{\cdot}54\times 10^{-6}\ C}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} = \frac{CV}{A\epsilon_0}$$

$$= \frac{3{\cdot}54\times 10^{-9}\times 1000}{2\times 8{\cdot}85\times 10^{-12}} = \mathbf{2\times 10^5\ NC^{-1}}$$

30. $a = 0{\cdot}03$ m, $b = 0{\cdot}05$ m, $V = 1500$ V

Capacitance of spherical capacitor is

$$C = \frac{4\pi\epsilon_0 ab}{b-a} = \frac{0{\cdot}03\times 0{\cdot}05}{9\times 10^9\,(0{\cdot}05-0{\cdot}03)} = \mathbf{8{\cdot}33\ pF}$$

$$q = CV = 8{\cdot}33\times 10^{-12}\times 1500 = \mathbf{1{\cdot}25\times 10^8\ C}$$

31. $\dfrac{C}{l} = \dfrac{2\pi\epsilon_0}{\log e\left(\dfrac{r_b}{r_a}\right)} = 2\times\dfrac{22}{7}\times\dfrac{8{\cdot}85\times 10^{-12}}{\log_e\left(\dfrac{0{\cdot}6\times 10^{-3}}{0{\cdot}1\times 10^{-3}}\right)}$

$= \mathbf{71{\cdot}5\ pF/m}$

32. Here, $r_a = \dfrac{0{\cdot}2}{2} = 0{\cdot}1$ m, $r_b = 0{\cdot}15$ m

$q = 6\ \mu C = 6\times 10^{-6}$ C, $K = 6$.

$$C = 4\pi\epsilon_0 K\frac{r_a r_b}{r_b - r_a}$$

$$C = \frac{6}{9\times 10^9}\frac{0{\cdot}1\times 0{\cdot}15}{(0{\cdot}15-0{\cdot}1)} = \mathbf{0{\cdot}2\times 10^{-9}\ F}$$

$$V = \frac{q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_b r_a}$$

$$V = 9\times 10^9\times 6\times 10^{-6}\times\frac{0{\cdot}15-0{\cdot}1}{0{\cdot}15\times 0{\cdot}1} = \mathbf{18\times 10^4\ V}$$

33. Here, $r_a = \dfrac{3}{2} = 1{\cdot}50$ mm $= 1{\cdot}5\times 10^{-3}$ m

$r_b = 1{\cdot}5 + 3 = 4{\cdot}5$ mm $= 4{\cdot}5\times 10^{-3}$ m

$L = 1$ km $= 10^3$ m, $K = 10$; $C = ?$

$$C = \frac{2\pi\epsilon_0 LK}{2{\cdot}303\log_{10}\left(\dfrac{r_b}{r_a}\right)}$$

$$= \frac{2\pi\times 8{\cdot}85\times 10^{-12}\times 10^3\times 10}{2{\cdot}303\log\left(\dfrac{4{\cdot}5\times 10^{-3}}{1{\cdot}5\times 10^{-3}}\right)}$$

$= 50{\cdot}6\times 10^{-8}$ F $= \mathbf{0{\cdot}506\ \mu F}$

34. Here, $C_1 = 3\ \mu$F, $C_2 = 6\ \mu$F

$V_1 = 2$ V, $V_2 = 5$ V

Before connecting, $Q_1 = C_1 V_1 = 3\times 2 = 6\ \mu$C

$Q_2 = C_2 V_2 = 6\times 5 = 30\ \mu$C

Total charge, $Q = Q_1 + Q_2 = 6 + 30 = 36\ \mu$C

In series combination, charge is conserved.

$\therefore$ Charge on each capacitor, $Q = 36\ \mu$C

Potential of 3 μF capacitor, $V_1 = \dfrac{Q_1}{C_1} = \dfrac{36}{3} = \mathbf{12\ V}$

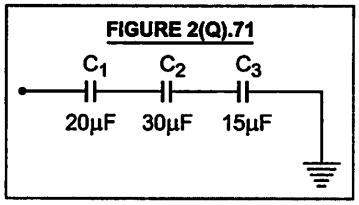
Potential of 6 μF capacitor, $V_2 = \dfrac{Q}{C_2} = \dfrac{36}{3} = \mathbf{6\ V}$

35. The equivalent capacitance of the series combination is, Fig. 2(Q).71

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$

$$= \frac{3}{20},\quad C = \frac{20}{3}\mu F$$

FIGURE 2(Q).71



Total potential diff. $V = 90 - 0 = 90$ V

Total charge, $q = CV = \left(\frac{20}{3}\times10^{-6}\right)\times90$

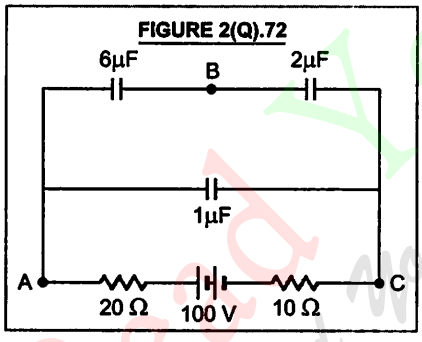
$= 600 \times 10^{-6}$ C

Potential diff. between plates of C_2,

$$V_2 = \frac{q}{C_2} = \frac{600\times10^{-6}}{30\times10^{-6}} = \mathbf{20\ V}$$

36. The two capacitors of 3 μF each are in parallel. Their equivalent capacity = 3 + 3 = 6 μF. Again, two capacitors of 1 μF each are also in parallel. Their equivalent capacity = 1 + 1 = 2 μF

FIGURE 2(Q).72



Therefore, the given netwrok reduces to the equivalent circuit shown in Fig. 2(Q).72. In the steady state, when all the capacitors are charged, there is no current in the circuit. Therefore, potential drop across both the resistances is zero.

∴ Potential diff. across 1 μF capacitor = p.d. between A and $C = 100$ V

As 6 μF and 2 μF capacitors are in series, the p.d of 100 V is divided between them in the inverse ratio of their capacitances, *i.e.*, in the ratio 2 : 6 or 1 : 3.

$$\therefore \quad V_{AB} = \text{p.d. across } 6\ \mu\text{F} = \frac{1}{4}\times100 = \mathbf{25\ V}$$

$$\text{and} \quad V_{BC} = \text{p.d. across } 2\ \mu\text{F} = \frac{3}{4}\times100 = \mathbf{75\ V}$$

37. Let two capacitances be C_1 and C_2

$$C_1 + C_2 = 5\ \mu\text{F} \text{ and } \frac{C_1C_2}{C_1+C_2} = 1{\cdot}2\mu\text{F}$$

On solving, we get

$C_1 = 2\ \mu$F, $C_2 = 3\ \mu$F

38. **We will connect series combination of 3 μF and 6 μF in parallel with the third capacitor of 3 μF**

$$\frac{1}{C_S} = \frac{1}{3} + \frac{1}{6} \text{ or } C_S = 2\ \mu\text{F}$$

$C_P = 2\ \mu\text{F} + 3\ \mu\text{F} = \mathbf{5\ \mu F}$

39. Equivalent capacitance between points F and B is

$$= \frac{18\times18}{18+18} + 18 = 27\ \mu\text{F}$$

Equivalent capacitance between A and B

$$= 12 + \frac{18\times27}{18+27} = 22{\cdot}8 \approx 23\ \mu\text{F}$$

Equivalent capacitance between A and E

$$= \frac{23\times18}{23+18} + 18 = 28\ \mu\text{F}$$

Equivalent capacitance between D and E

$$= \frac{28\times18}{28+18} + 12 = 23\ \mu\text{F}$$

Equivalent capacitance between D and Q

$$= \frac{23\times18}{23+18} + 18 = 28\ \mu\text{F}$$

Equivalent capacitance between P and Q is

$$= \frac{28\times18}{28+18} = \mathbf{11\ \mu F}$$

40. Let C_1, C_2, C_3 ..... be the capacitances of first, second, third row .... respectively. Then

$C_1 = 1\ \mu$F

$$C_2 = \frac{1\times1}{1+1} = \frac{1}{2}\ \mu F$$

$$\frac{1}{C_3} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 4$$

$$C_3 = \frac{1}{4}\ \mu F$$

Similarly $C_4 = \frac{1}{8}\ \mu F$ and so on.

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So equivalent capacitance 'C' as they are connected in parallel is

$$C = C_1 + C_2 + C_3 + ...$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + ...$$

= sum of geometrical progression

$$C = \frac{a}{1-r} = \frac{1}{1-1/2} = \mathbf{2\ \mu F}$$

41. Let C be the equivalent capacitance of the infinite network. It consists of repeating unit of two capacitors of 1 μF and 2 μF. Let equivalent capacitance of that unit be C. The network will appear as shown in Fig. 2(Q).73.

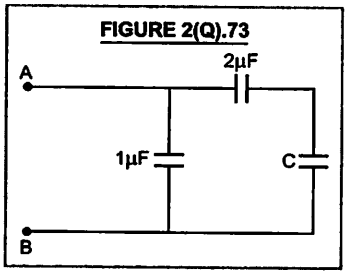


FIGURE 2(Q).73

Equivalent capacitance of new arrangement must be equal to C.

$$C = 1 + \frac{2 \times C}{2 + C}$$

$C^2 - C - 2 = 0 \ \therefore \ C = 2\ \mu F$ or $-1\ \mu F$

As capacitance cannot be negative, so equivalent capacitance is **2 μF.**

42. In Fig. 2(Q).42, $C_{p_1} = 3 + 6 = 9\ \mu F$

$$\frac{1}{C_{s_1}} = \frac{1}{4} + \frac{1}{9} = \frac{13}{36} \qquad C_{s_1} = \frac{36}{13}\mu F$$

$$C_{p_2} = \frac{36}{13} + 12 = \frac{36 + 156}{13} = \frac{192}{13}\mu F$$

$$\frac{1}{C_s} = \frac{1}{1} + \frac{1}{2} + \frac{13}{192} = \frac{192 + 96 + 13}{192} = \frac{301}{192}$$

$$C_s = \frac{192}{301}\mu F \qquad Q = C_s V = \frac{192}{301} \times 6\mu C$$

$$V_1 = \frac{Q}{C_1} = \frac{192 \times 6}{301 \times 1} = \mathbf{3{\cdot}82\ volt}$$

43. Here, $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$, $E_1 = 12\ V$, $E_2 = 24\ V$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}, \quad C_s = \frac{4}{3}\mu F$$

$$q = C_s(E_1 - E_2) = \frac{4}{3}(12 - 24) = -16\ \mu C.$$

P.D. across $2\ \mu F = \frac{q}{C_1} = -\frac{16}{2} = \mathbf{-8\ V}$

44. From $\frac{1}{C_s} = \frac{1}{20} + \frac{1}{C}$,

$$\frac{1}{C} = \frac{1}{C_s} - \frac{1}{20} = \frac{1}{4} - \frac{1}{20} = \frac{4}{20} = \frac{1}{5}, \ C = \mathbf{5\ \mu F}$$

$$q_1 = q_2 = C_s \times V = 4 \times 12 = \mathbf{48\ \mu C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{48}{20} = \mathbf{2{\cdot}4\ volt}$$

$$V_2 = \frac{q_2}{C_2} = \frac{48}{5} = \mathbf{9{\cdot}6\ volt}$$

46. Careful observation shows that in Fig. 2(Q).45, capacitors are connected in series.

47. Suppose the pair of plates A is connected to positive terminal of the battery and the pair of plates B is connected to negative terminal of the battery, Fig. 2(Q).46. As is clear from the figure, we have two capacitors I and II. Positive plate of I is connected to positive plate of II and negative plate to negative therefore, they are in parallel.

As $C_p = C_1 + C_2 \ \therefore \ C_p = 2\,C = \mathbf{2\,\epsilon_0 A/d}$

48. Suppose the pair of plates A is connected to positive terminal of the battery and the pair of plates B is connected to negative terminal of the battery. As is clear from the Fig. 2(Q).47, we have three capacitors, I, II and III. Their positive plates are connected to each other and so are the negative plates. Therefore, these are joined in parallel.

$$C_p = C_1 + C_2 + C_3 = 3\,C = \mathbf{3\,\epsilon_0 A/d}$$

49. The arrangement shown in Fig. 2(Q).48 is equivalent to two condensers of capacities C_1 and C_2, connected in series, where

$$C_1 = \frac{\epsilon_0\ AK_1}{d/2} = \frac{2\epsilon_0\ AK_1}{d}$$

$$C_2 = \frac{\epsilon_0\ AK_2}{d/2} = \frac{2\epsilon_0\ AK_2}{d}$$

Use $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

50. In Fig 2(Q).49, when key K is pressed,

$Q_1 = EC_1$

As C_1 & C_2 are in series

$$\therefore \ \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 C_2}$$

$$C_s = \frac{C_1 C_2}{C_1 + C_2} \quad \therefore \quad Q_2 = EC_s = \frac{EC_1 C_2}{C_1 + C_2}$$

52. In Fig. 2(Q).51, 8 μ F and 10 μ F capacitors are in series and the combination is in parallel with 9 μ F capacitor.

53. In Fig. 2(Q).52, $C_1 = 5\ \mu F$, $C_2 = 10\ \mu F$, and $C_3 = 15\ \mu F$ are in parallel.

$\therefore \quad C_p = C_1 + C_2 + C_3 = 5 + 10 + 15 = 30\ \mu F$

Again, C_p & C_4 are in series. Therefore,

$$\frac{1}{C_s} = \frac{1}{C_p} + \frac{1}{C_4} = \frac{1}{30} + \frac{1}{30} = \frac{1}{15}.$$

$C_s = \mathbf{15\ \mu F}$

54. As is clear from Fig. 2(Q).53, the two cylindrical capacitors are joined in parallel.

$\therefore \quad C_p = C_1 + C_2 = 2{\cdot}2 + 2{\cdot}2 = 4{\cdot}4\ \mu F$

As $q = C_p V \quad \therefore \quad q = 4{\cdot}4 \times 10 = \mathbf{44\ \mu C}$

56. In Fig. 2(Q).55, C_5, C_6 are in parallel.

$\therefore C_{56} = C_5 + C_6 = 10 + 10 = 20\ \mu F.$

Also, C_2, C_3, C_4 are in series

$$\therefore \frac{1}{C_{234}} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{3}{60} = \frac{1}{20}$$

$C_{234} = 20\ \mu F$

Now, C_{234} and C_{56} are in parallel.

$\therefore$ Their combined capacity is

$C' = C_{234} + C_{56} = 20 + 20 = 40\ \mu F.$

Now, C_1 and C' are in series.

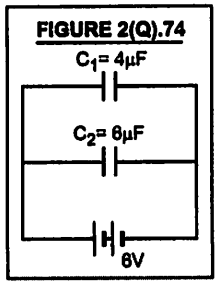
Therefore, capacity of the combination is

$$C = \frac{C_1 C'}{C_1 + C'} = \frac{40 \times 40}{40 + 40} = \mathbf{20\ \mu F}$$

Total charge that flows in the circuit, when a battery of 100 V is connected between A and B is

$q = CV = (20 \times 10^{-6}) \times 100 = \mathbf{2 \times 10^{-3}\ C}$

57. The equivalent circuit shown in Fig. 2(Q).74 reveals that the two capacitors are connected in parallel.



$\therefore$ Charge on C_1 is $q_1 = C_1 V = 4 \times 6 = \mathbf{24\ \mu C}$

Charge on C_2 is $q_2 = C_2 V = 6 \times 6 = \mathbf{36\ \mu C}$

58. Here, $C_1 = C_2 = 800\ pF = 8 \times 10^{-10}\ F$

$V_1 = 100\ V,\ V_2 = 0$

$$\text{Common potential, } V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$= \frac{8 \times 10^{-10} \times 100}{(800 + 800) 10^{-12}} = 50\ V$$

$$\text{Energy stored, } u_f = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2}(800 + 800) 10^{-12} (50)^2$$

$u_f = \mathbf{2 \times 10^{-6}\ J}$

59. Let C be the capacity of each condenser,

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} = 1\mu F \quad \therefore C = 3\ \mu F$$

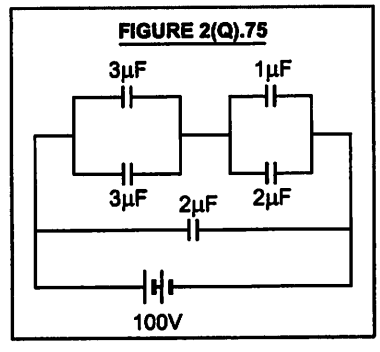
$C_p = C_1 + C_2 + C_3 = 3\ C = 3 \times 3 = \mathbf{9\ \mu F}$

Let V be the p.d of same source,

$$U_s = \frac{1}{2} C_s V^2 \quad \text{and} \quad U_p = \frac{1}{2} C_p V^2$$

$$\therefore \frac{U_s}{U_p} = \frac{C_s}{C_p} = \frac{C/3}{3C} = \frac{1}{9}$$

60. The equivalent circuit diagram of given circuit is shown in Fig. 2(Q).75.



Two capacitors, each of 3 μF are in parallel.

$\therefore \quad C_1 = 3 + 3 = 6\ \mu F$

Two capacitors of 1 μF and 2 μF are in parallel.

$\therefore \quad C_2 = 1 + 2 = 3\ \mu F$

As C_1 and C_2 are in series, therefore

$$C_{12} = \frac{6 \times 3}{6 + 3} = 2\mu F$$

C_{12} is in parallel with 2 μF capacitor.

$\therefore$ Net capacitance, $C = C_{12} + 2 = 2 + 2 = 4\ \mu F.$

Total charge, $q = CV = (4 \times 10^{-6}) \times 100$

$= \mathbf{4 \times 10^{-4}\ C}$

Total energy, $U = \frac{1}{2} CV^2$

$= \frac{1}{2}(4\times10^{-6})(100)^2 = \mathbf{0{\cdot}02\ J}$

61. $U_4 = \frac{1}{2} C_4 V^2 = 27\,J$

$V^2 = \frac{2\times27}{C_4} = \frac{54}{6\times10^{-6}} = 9\times10^6$

Energy stored in C_2,

$U_2 = \frac{1}{2}\times2\times10^{-6}\times9\times10^6 = 9\ J$

Energy sotred in C_3,

$U_3 = \frac{1}{2}\times3\times10^{-6}\times9\times10^6 = 13{\cdot}5\ J$

Total energy stored in C_2, C_3, C_4

$= U_2 + U_3 + U_4 = 9 + 13{\cdot}5 + 27$

$= 49{\cdot}5\ J$

Equivalent capacitance of C_2, C_3, C_4

$= 2 + 3 + 6 = 11\ \mu F$

As $U = \frac{q^2}{2C} = \frac{q^2}{2\times11\times10^{-6}} = 49{\cdot}5$

$\therefore \quad q^2 = 49{\cdot}5\times22\times10^{-6}\ C^2$

$U_1 = \frac{q^2}{2C_1} = \frac{49{\cdot}5\times22\times10^{-6}}{2\times1\times10^{-6}} = 544{\cdot}5\ J$

Total energy stored in the arrangement

$= 544{\cdot}5 + 49{\cdot}5$

$= \mathbf{594{\cdot}0\ J}$

62. In Fig. 2(Q).59, Two 2 μF on right side are in series.

So $C_1 = \frac{2\times2}{2+2} = 1\mu F$

This 1 μF is in parallel with central 1 μF

$C_2 = 1 + 1 = 2\ \mu F$

C_2 is in series with 2 μF at bottom

$C_3 = \frac{2\times2}{2+2} = 1\ \mu F$

C_3 is in parallel with left 1 μF capacitor

$C_{eq} = 1 + 1 = 2\ \mu F$

$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}\times2\times10^{-6}\times(6)^2$

$= \mathbf{3{\cdot}6\times10^{-5}\ J}$

63. Let C be the capacity of each of the three condensers.

$\therefore \quad \frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$

or $\quad C_s = \frac{C}{3} = 1\mu F \ \therefore\ C = 3\ \mu F$

Now, $C_p = C + C + C = 3\ C = 3\times3\ \mu F$

$= \mathbf{9\ \mu F}$

When connected to the same source, V is same.

$E_s = \frac{1}{2} C_s V^2$

and $\quad E_p = \frac{1}{2} C_p V^2$

$\therefore \quad \frac{E_s}{E_p} = \frac{\frac{1}{2} C_s V^2}{\frac{1}{2} C_p V^2} = \frac{C_s}{C_p} = \mathbf{\frac{1}{9}}$

64. As is clear from Fig. 2(Q).60, C_1, C_2 are in series

$\therefore \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3},\ C_s = 3\ \mu F$

$q_1 = q_2 = C_s \times V = 3\times12 = \mathbf{36\ \mu C}$;

$q_3 = C_3\ V = 6\times12 = \mathbf{72\ \mu C}$

(*ii*) C_3 is in parallel with combination of C_1 and C_2.

$\therefore \quad C_p = C_3 + C_s = 3 + 6 = \mathbf{9\ \mu F}$

(*iii*) $U_1 = U_2 = \frac{1}{2} q_1 V = \frac{1}{2} q_2 V$

$= \frac{1}{2}\times36\times6 = \mathbf{108\ \mu J}$;

$U_3 = \frac{1}{2} C_3 V^2 = \frac{1}{2}\times6\times12^2 = \mathbf{432\ \mu J}$

65. Here $C_1 = 20\ \mu F = 20\times10^{-6}\ F$

$V_1 = 30\ V$, $C_2 = 50\ \mu F = 50\times10^{-6}\ F$.

$V_2 = 0$; $V = ?$

$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{20\times10^{-6}\times30}{(20+50)10^{-6}}$

$= \frac{60}{7}\ V = \mathbf{8{\cdot}57\ V}$

Initial energy, $U_1 = \frac{1}{2} C_1 V_1^2$

$= \frac{1}{2}(20\times10^{-6})(30)^2$

$= \mathbf{9\times10^{-3}\ joule}$

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Final energy, $U_2 = \frac{1}{2}(C_1 + C_2)V^2$

$= \frac{1}{2}(20+50)10^{-6}\left(\frac{60}{7}\right)^2$

$= \mathbf{2{\cdot}57 \times 10^{-3}\ joule}$

Loss of energy $= U_1 - U_2$

$= 9 \times 10^{-3} - 2{\cdot}57 \times 10^{-3}$

$= 6{\cdot}43 \times 10^{-3}$ J

This loss of energy appears in the form of heat and e.m. radiations.

66. C_1 is capacity of X; C_2 is capacity of Y, Fig. 2(Q).61

$C_2 = 4\,C_1$.

As $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ $\therefore$ $\frac{1}{4} = \frac{1}{C_1} + \frac{1}{4C_1} = \frac{5}{4C_1}$

$C_1 = \mathbf{5\ \mu F}$; $C_2 = \mathbf{20\ \mu F}$

$q_1 = q_2 = C_s V = 4 \times 12 = 48\ \mu C$

$V_1 = \frac{q_1}{C_1} = \frac{48}{5} = \mathbf{9{\cdot}6\ V}$,

$V_2 = \frac{q_2}{C_2} = \frac{48}{20} = \mathbf{2{\cdot}4\ V}$

$\frac{U_1}{U_2} = \frac{C_2}{C_1} = \mathbf{4}$

67. In Fig. 2(Q).62, *When S is closed,*

$U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$

When S is open and dielectric is introduced.

Capacity of each condenser $= KC = 5$ C. Condenser on the left is connected to battery. Its potential remains V. But condenser on right is not connected to battery. So on introducing dielectric its potential becomes $\frac{V}{5}$

$\therefore$ Total energy,

$U_2 = \frac{1}{2}(5\,C)V^2 + \frac{1}{2}(5C)\left(\frac{V}{5}\right)^2$

$= \frac{1}{2}(5\,C)V^2\left(1 + \frac{1}{25}\right)$

$U_2 = \frac{5CV^2}{2} \times \frac{26}{25} = \frac{13}{5}CV^2$

$\therefore$ $\frac{U_1}{U_2} = \frac{CV^2}{\frac{13}{5}CV^2} = \mathbf{\frac{5}{13}}$

68. Here, $d = 4$ mm, $K = 3$ and $t = 3$ mm

Let d' be the new distance between the plates.

As $\frac{2}{3}$ × Capacitance with air = Capacitance with dielectric

$\therefore$ $\frac{2}{3}\frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{d' - t + \frac{K}{2}}$

or $\frac{2}{3} \times \frac{1}{4} = \frac{1}{d' - 3 + \frac{3}{3}}$

or $6 = d' - 3 + 1$ or $d' = \mathbf{8\ mm}$

69. (*i*) P.d across the capacitor before metal plate was inserted

$V_0 = E_0 d = 3 \times 10^4 \times 0{\cdot}05 = \mathbf{1500\ V}$

(*ii*) After introduction of plate, $V = E_0\,(d - t)$

$= 3 \times 10^4\,(0{\cdot}05 - 0{\cdot}01) = \mathbf{1200\ V}$

(*ii*) When dielectric slab is introduced, $K = 2$

$V = E_0\,(d - t) + \frac{E_0}{K}t$

$= 1200 + \frac{3 \times 10^4 \times 0{\cdot}01}{2} = \mathbf{1350\ V}$

71. Change in distance, $x = t\left(1 - \frac{1}{K}\right)$

72. (*i*) $C = \frac{\epsilon_0 A}{d}$

(*ii*) $Q_0 = CV_0$; $V_0 = 3000$ volt

(*iii*) $K = \frac{E_0}{E} = \frac{V_0}{V} = \frac{3000}{1000} = 3$

(*iv*) $C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$

(*v*) $\epsilon = K\epsilon_0 = 3 \times 8{\cdot}85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$= \mathbf{2{\cdot}66 \times 10^{-11}\ F.m}$

73. Here, $V = 1\ kV = 10^3$ volt, $K = 3$,

Dielectric strength $= 10^7\ Vm^{-1}$

Electric field, $E = 10\% \times 10^7\ V/m = 10^6\ V/m$

$A = ?$, $C = 50\ pF = 50 \times 10^{-12}$ F

As $C = \frac{K\epsilon_0 A}{d}$ and $d = \frac{V}{E}$

$\therefore$ $C = \frac{K\epsilon_0 AE}{V}$

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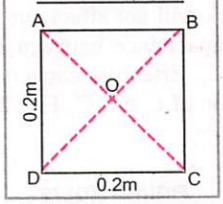
$$A = \frac{CV}{K \in_0 E}$$

$$A = \frac{50 \times 10^{-12} \times 10^3}{3 \times 8{\cdot}85 \times 10^{-12} \times 10^6} = 19 \times 10^{-4} \text{ m}^2$$

$$= \mathbf{19 \text{ cm}^2}$$

74. Here, $q_A = 2 \times 10^{-9}$ C, $q_B = 4 \times 10^{-9}$ C,
$q_C = 8 \times 10^{-9}$ C
$AB = BC = CD = DA = 0{\cdot}2\ m$, Fig. 2(Q).76.

FIGURE 2(Q).76



$$\therefore\ AC = BD = \sqrt{0{\cdot}2^2 + 0{\cdot}2^2} = 0{\cdot}2\sqrt{2} \text{ m}$$

$$AO = BO = CO = DO = \frac{0{\cdot}2\sqrt{2}}{2} = 0{\cdot}1\sqrt{2} \text{ m}$$

Potential at point D is

$$V_D = k\left[\frac{q_A}{AD} + \frac{q_B}{BD} + \frac{q_C}{CD}\right] = 577{\cdot}26 \text{ V}$$

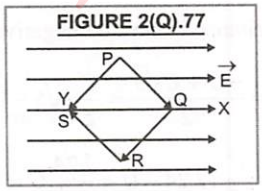
Potential at point O is

$$V_O = k\left[\frac{q_A}{AO} + \frac{q_B}{BO} + \frac{q_C}{CO}\right] = 890{\cdot}82 \text{ V}$$

Work done $= q\,(V_O - V_D)$
$= 2 \times 10^{-9}\,(890{\cdot}82 - 577{\cdot}26)$
$= \mathbf{6{\cdot}27 \times 10^{-7} \text{ joule}}$

75. Here, $P\ (a, b, 0)$, $Q\ (2a, 0, 0)$, $R\ (a, -b, 0)$ and $S\ (0, 0, 0)$, [Fig. 2(Q).77].

FIGURE 2(Q).77



Effective displacement in going from P to S via any path is

$$= \vec{PS} = (0-a)\,\hat{i} + (0-b)\,\hat{j} + (0-0)\,\hat{k}$$

$$= -a\,\hat{i} - b\,\hat{j}$$

force on the charge. $\vec{F} = qE\,\hat{i}$

Work done, $W = \vec{F}.\vec{PS} = qE\,\hat{i}\,.(-a\,\hat{i} - b\,\hat{j})$

$$= \mathbf{-qEa}$$

Negative sign shows that work is done against the electric field by some external agency moving the charge.

76. As the field is directed outwards, charge on the wire must be **positive**

$$V_B - V_A = -\int_A^B \vec{E}.\vec{dl} = -\int_{r=0{\cdot}2m}^{r=0{\cdot}4m} \frac{1000}{r}\,dr$$

$$V_B - V_A = -1000\left[\log_e r\right]_{r=0{\cdot}2m}^{r=0{\cdot}4m}$$

$$= -1000\,[\log_e 0{\cdot}4 - \log_e 0{\cdot}2]$$

$$= -1000\left[\log_e \frac{0{\cdot}4}{0{\cdot}2}\right]$$

$$= -1000 \log_e 2$$

$$V_B - V_A = -1000 \times 0{\cdot}6931$$

$$= \mathbf{-693{\cdot}1 \text{ volt}}$$

77. This arrangement in Fig. 2(Q).63 is equivalent to three capacitors connected in series

$$C_1 = \frac{K_1 \in_0 A}{d/3} = \frac{3K_1 \in_0 A}{d},$$

$$C_2 = \frac{3K_2 \in_0 A}{d},$$

$$C_3 = \frac{3K_3 \in_0 A}{d}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{3 \in_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right)$$

$$C = \frac{\mathbf{3 \in_0 A K_1 K_2 K_3}}{\mathbf{d\,(K_1K_2 + K_2K_3 + K_3K_1)}}$$

78. Without dielectric, $E_0 = \dfrac{V_0}{d}$

With dielectric, $E = \dfrac{E_0}{K}$

$$V = E_0\frac{d}{2} + E.\frac{d}{2} = E_0\frac{d}{2} + \frac{E_0}{K}\frac{d}{2}$$

$$= \frac{E_0 d}{2}\,\frac{(K+1)}{K} = \frac{V_0(K+1)}{2K}$$

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$$C = \frac{q_0}{V_0} = \frac{2Kq_0}{V_0(K+1)} = \frac{2K}{K+1} C_0$$

79. In Fig. 2(Q).64, C_2, C_3 are in series. This combination is in parallel with C_1.

The combined capacity of C_2, C_3, C_1 is in series with C_4. The combination is in parallel with C_5.

80. As total area of flat plate is A and it is divided into three equal parts, forming three capacitors, therefore, area of each capacitor = $A/3$ Starting from left end, Fig. 2(Q).65

$$C_1 = \frac{\epsilon_0 A/3}{d}, \quad C_2 = \frac{\epsilon_0 A/3}{(d+b)},$$

$$C_3 = \frac{\epsilon_0 A/3}{(d+2b)}$$

As the three capacitors are joined in parallel, therefore, total capacity of the assembly

$$C_p = C_1 + C_2 + C_3$$

$$= \frac{\epsilon_0 A/3}{d} + \frac{\epsilon_0 A/3}{d+b} + \frac{\epsilon_0 A/3}{(d+2b)}$$

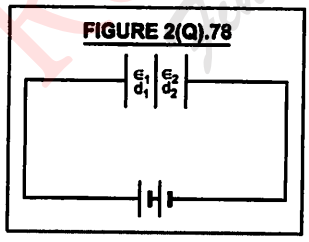
$$C_p = \frac{\epsilon_0 A}{3}$$

$$\times \left[\frac{(d+b)(d+2b) + d(d+2b) + d(d+b)}{d(d+b)(d+2b)}\right]$$

$$= \epsilon_0 A\,(3d^2 + 6bd + 2b^2)/3\,d\,(d+b)\,(d+2b)$$

81. The system is equivalent to two condensers connected in series as shown in Fig. 2(Q).78.

FIGURE 2(Q).78



$$C_1 = \frac{\epsilon_1 \epsilon_0 A}{d_1}; \quad C_2 = \frac{\epsilon_2 \epsilon_0 A}{d_2}$$

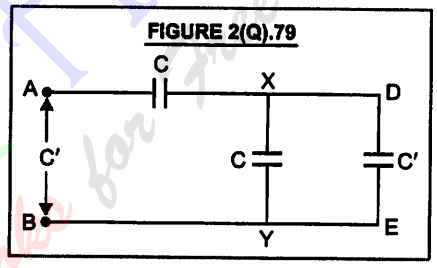
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{\epsilon_1 \epsilon_0 A} + \frac{d_2}{\epsilon_2 \epsilon_0 A}$$

$$= \frac{1}{\epsilon_0 A}\left(\frac{\epsilon_1 d_2 + \epsilon_2 d_1}{\epsilon_1 \epsilon_2}\right)$$

$$= \frac{\epsilon_1 \epsilon_2 \epsilon_0 A}{\epsilon_1 d_2 + \epsilon_2 d_1}$$

82. Suppose the equivalent capacity between A and B is C'. As the network is infinite, therefore addition or removal of one set of capacitors from the network will not affect our result. It implies that the capacitance between D and E can be taken as C'. Effective capacitance of parallel combination of C and C', Fig. 2(Q).79 is $C'' = C + C'$.

FIGURE 2(Q).79



Now C'' and C are in series.

$$\therefore \quad \frac{1}{C'} = \frac{1}{C} + \frac{1}{C''} = \frac{1}{C} + \frac{1}{(C+C')}$$

$$= \frac{C'+2C}{C(C+C')}$$

or $C'^2 + 2CC' = C(C+C') = C^2 + CC'$

or $C'^2 + CC' - C^2 = 0$

$$C' = \frac{-C \pm \sqrt{C^2 + 4C^2}}{2} = \frac{-C \pm \sqrt{5C^2}}{2}$$

As capacitance cannot be negative.

$$\therefore \quad C' = \frac{-C + C\sqrt{5}}{2} = \frac{C}{2}[\sqrt{5} - 1]$$

$$C' = \frac{C}{2}(2{\cdot}24 - 1) = \frac{1{\cdot}24}{2} C = 0{\cdot}62\,C$$

As $C = 1\ \mu F$, therefore, $C' = 0{\cdot}62\ \mu F$

NCERT EXERCISES WITH SOLUTIONS

Q. 1. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what points on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.

Sol. Here $q_1 = 5 \times 10^{-8}$ C, $q_2 = -3 \times 10^{-8}$ C

Let the potential be zero at a distance x cm from the charge $q_1 = 5 \times 10^{-8}$ C

$\therefore \quad r_1 = x \times 10^{-2}$ m ; $r_2 = (16 - x) \times 10^{-2}$ m

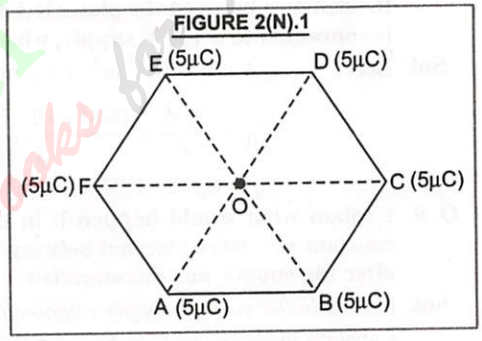
Now, $$V = \frac{q_1}{4\pi\epsilon_0 r_1} + \frac{q_2}{4\pi\epsilon_0 r_2} \quad ; \quad 0 = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$\therefore \quad \frac{q_1}{r_1} = -\frac{q_2}{r_2} = \frac{5\times10^{-8}}{x\times10^{-2}} = \frac{-(-3\times10^{-8})}{(16-x)10^{-2}} \quad \text{or} \quad \frac{5}{x} = \frac{3}{16-x}$$

$3x = 80 - 5x$; $8x = 80$, $x = \mathbf{10\ cm}$

Q. 2. A regular hexagon of side 10 cm has a charge 5 μ C at each of its vertices. Calculate the potential at the centre of the hexagon.

FIGURE 2(N).1



Sol. In Fig. 2(N).1, O is centre of hexagon $ABCDEFA$ of each side 10 cm. As is clear from the figure, OAB, OBC etc. are equilateral triangles. Therefore,

$$OA = OB = OC = OD = OE = OF$$
$$= r = 10 \text{ cm} = 10^{-1} \text{ m.}$$

As potential is scalar, therefore, potential at O is

$$V = 6 \times \frac{q}{4\pi\epsilon_0 r} = \frac{9\times10^9 \times 6 \times 5 \times 10^{-6}}{10^{-1}} = \mathbf{2{\cdot}7 \times 10^6\ V}$$

Q. 3. Two charges + 2 μ C and – 2 μ C are placed at points A and B, 6 cm apart (*a*) Identify an equipotential surface of the system. (*b*) What is the direction of the electric field at every point on this surface ?

Sol. (*a*) The plane normal to AB and passing through its middle point has zero potential everywhere.
(*b*) The direction of electric field at every point on this surface is along normal to the plane in the direction AB.

Q. 4. A spherical conductor of radius 12 cm has a charge of $1{\cdot}6 \times 10^{-7}$ C distributed uniformly on its surface. What is the electric field (a) inside the sphere (b) just outside the sphere (c) at a point 18 cm from the centre of the sphere ?

Sol. Here, $r = 12$ cm $= 12 \times 10^{-2}$ m, $q = 1{\cdot}6 \times 10^{-7}$ C.

(*a*) Inside the sphere, $\mathbf{E = 0}$

(*b*) Just outside the sphere, (say, on the surface of the sphere)

$$E = \frac{q}{4\pi\epsilon_0 r^2} = 9\times10^9 \times \frac{1{\cdot}6\times10^{-7}}{(12\times10^{-2})^2} = \mathbf{10^5\ N/C}$$

(*c*) At $r' = 18$ cm $= 18 \times 10^{-2}$ m, $$E = \frac{q}{4\pi\epsilon_0 r'^2} = \frac{9\times10^9 \times 1{\cdot}6\times10^{-7}}{(18\times10^{-2})^2} = \mathbf{4{\cdot}4 \times 10^4\ N/C}$$

Q. 5. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. (1 pF = 10^{-12} F) What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant 6 ?

Sol. $C_1 = \frac{\epsilon_0 A}{d} = 8\,\text{pF}$ $\qquad C_2 = K\frac{\epsilon_0 A}{d/2} = \frac{6\times 2\epsilon_0 A}{d} = 12\times 8 = 96\,\text{pF}$

Q. 6. Three capacitors each of capacitance 9 pF are connected in series. (*a*) What is the total capacitance of the combination ? (*b*) What is the potential difference across each capacitor if the combination is connected to a 120 V supply.

Sol. $\frac{1}{C_s} = \frac{1}{9}+\frac{1}{9}+\frac{1}{9} = \frac{3}{9} = \frac{1}{3} : C_s = 3\,\text{pF}$

p.d. across each capacitor $= \frac{V}{3} = \frac{120}{3} = 40\,\text{V}$

Q. 7. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel. (*a*) What is the total capacitance of the combination ? (*b*) Determine the charge on each capacitor, if the combination is connected to 100 V supply.

Sol. $C_p = 2 + 3 + 4 = 9$ pF

For each capacitor, V is same = 100 volt

$\therefore \quad q_1 = C_1 V = 2 \times 100 = 200$ pC, $q_2 = C_2 V = 3 \times 100 = 300$ pC, $q_3 = C_3 V = 4 \times 100 = 400$ pC

Q. 8. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor ?

Sol. Here, $A = 6 \times 10^{-3}$ m^2, $d = 3$ mm $= 3 \times 10^{-3}$ m, $C = ?$, $V = 100$ V, $q = ?$

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{(8{\cdot}85\times 10^{-12})(6\times 10^{-3})}{3\times 10^{-3}} = 1{\cdot}77 \times 10^{-11}\ \text{F}$$

$$q = C_0 V = 1{\cdot}77 \times 10^{-11} \times 100 = 1{\cdot}77 \times 10^{-9}\ \text{C}$$

Q. 9. Explain what would happen if in the capacitor in Q. 8, a 3 mm thick mica sheet of (dielectric constant = 6) were inserted between the plates (a) while the voltage supply remained connected (b) after the supply was disconnected.

Sol. (*a*) *While the voltage supply remained connected,* voltage remains constant.

Capacity increases to $C' = KC_0 = 6 \times 1{\cdot}77 \times 10^{-11}$ F

charge increases to $q' = C' V = 6 \times 1{\cdot}77 \times 10^{-11} \times 10^2$ C.

(*b*) *After the supply was disconnected,* charge remains constant. New capacity,

$$C' = KC_0 = 6 \times 1{\cdot}77 \times 10^{-11}\ \text{F}$$

new voltage $V' = \frac{q}{C'} = \frac{1{\cdot}77\times 10^{-9}}{6\times 1{\cdot}77\times 10^{-11}} = 16{\cdot}67\ \text{V}$

Q. 10. A 12 *pF* capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor ?

Sol. Here, $C = 12$ pF $= 12 \times 10^{-12}$ F, $V = 50$ volt, $U = ?$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(12\times 10^{-12})(50)^2 = 1{\cdot}5\times 10^{-8}\ \text{J}$$

Q. 11. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process ?

Sol. Here, $C_1 = C_2 = 600$ pF $= 600 \times 10^{-12}$ F $= 6 \times 10^{-10}$ F, $V_1 = 200$ V, $V_2 = 0$

$$\text{Loss of energy} = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)} = \frac{6\times 10^{-10}\times 6\times 10^{-10}(200-0)^2}{2(6\times 10^{-10} + 6\times 10^{-10})} = \frac{36\times 10^{-20}\times 4\times 10^4}{24\times 10^{-10}} = 6\times 10^{-6}\ \text{J}$$

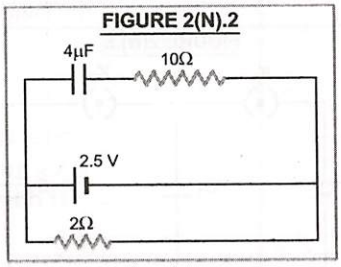
NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. A capacitor of 4 μ F is connected as shown in the circuit, Fig. 2(N).2. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be

FIGURE 2(N).2



(*a*) 0 (*b*) 4 μ C

(*c*) 16 μ C (*d*) 8 μ C

2. A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge

(*a*) remains a constant because the electric field is uniform

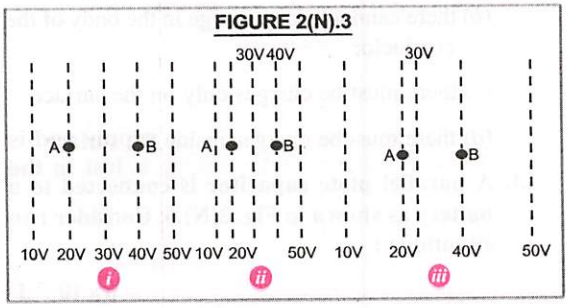
(*b*) increases because the charge moves along the electric field

(*c*) decreases because the charge moves along the electric field

(*d*) decreases because the charge moves opposite to the electric field

3. Figure 2(N).3 shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

FIGURE 2(N).3



(*a*) The work done in Fig. (*i*) is the greatest

(*b*) The work done in Fig. (*ii*) is least

(*c*) The work done is the same in Fig. (*i*), Fig. (*ii*) and Fig. (*iii*)

(*d*) The work done in Fig. (*iii*) is greater than Fig. (*ii*) but equal to that in Fig. (*i*)

4. The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard :

S_1 : At any point inside the sphere, electric intensity is zero.

S_2 : At any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statement ?

(*a*) S_1 is true but S_2 is false

(*b*) Both S_1 and S_2 are false

(*c*) S_1 is true, S_2 is also true and S_1 is the cause of S_2

(*d*) S_1 is true, S_2 is also true but the statements are independent

5. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

(*a*) spheres

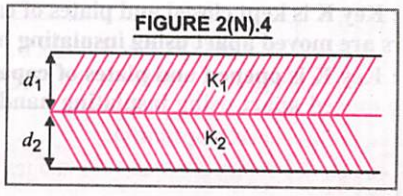
(*b*) planes

(*c*) paraboloids

(*d*) ellipsoids

6. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant k_1 and the other has thickness d_2 and dielectric constant k_2 as shown in Fig. 2(N).4. This arrangement can be thought as a capacitor with dielectric slab of thickness d $(= d_1 + d_2)$ and effective dielectric constant k. The k is

FIGURE 2(N).4



(a) $\frac{k_1d_1 + k_2d_2}{d_1 + d_2}$ (b) $\frac{k_1d_1 + k_2d_2}{k_1 + k_2}$

(c) $\frac{k_1k_2(d_1 + d_2)}{(k_1d_2 + k_2d_1)}$ (d) $\frac{2k_1k_2}{k_1 + k_2}$

MULTIPLE CHOICE QUESTIONS-II

7. Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant

(a) in all space

(b) for any x for a given z

(c) for any y for a given z

(d) on the x-y plane for a given z.

8. Equipotential surfaces

(a) are closer in regions of large electric fields compared to regions of lower electric fields

(b) will be more crowded near sharp edges of a conductor

(c) will be more crowded near regions of large charge densities

(d) will always be equally spaced

9. The work done to move a charge along an equipotential from A to B

(a) cannot be defined as $-\int_A^B E.dl$

(b) must be defined as $-\int_A^B E.dl$

(c) is zero

(d) can have a non-zero value.

10. In a region of constant potential

(a) the electric field is uniform

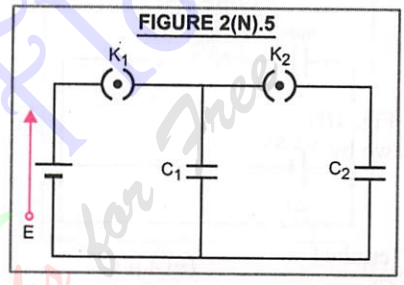
(b) the electric field is zero

(c) there can be no charge inside the region

(d) the electric field shall necessarily change if a charge is placed outside the region

11. In the circuit shown in Fig. 2(N).5, initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively.] Then

FIGURE 2(N).5



(a) charge on C_1 gets redistributed such that $V_1 = V_2$

(b) charge on C_1 gets redistributed such that $Q_1' = Q_2'$

(c) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$

(d) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

12. If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then

(a) there must be charges on the surface or inside itself

(b) there cannot be any charge in the body of the conductor

(c) there must be charges only on the surface

(d) there must be charges inside the surface

13. A parallel plate capacitor is connected to a battery as shown in Fig. 2(N).6. Consider two situations :

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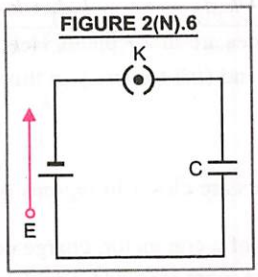
FIGURE 2(N).6



A : Key K is kept closed and plates of capacitors are moved apart using insulating handle
B : Key K is opened and plates of capacitors are moved apart using insulating handle
Choose the correct option(s)
(*a*) In A : Q remains same but C changes
(*b*) In B : V remains same but C changes
(*c*) In A : V remains same and hence Q changes
(*d*) In B : Q remains same and hence V changes.

ANSWERS

1. (*d*) **2.** (*c*) **3.** (*c*) **4.** (*c*) **5.** (*a*) **6.** (*c*) **7.** (*b,c,d*) **8.** (*a,b,c*) **9.** (*b,c*)
10. (*b,c*) **11.** (*a,d*) **12.** (*a,b*) **13.** (*c,d*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. In Fig. 1(N).2, no current is allowed to flow through 10 Ω resistance because of capacitor. The current drawn by 2 Ω res.

$$I = \frac{E}{R+r} = \frac{2{\cdot}5}{2+0{\cdot}5} = 1 \text{ A.}$$

Terminal pot. diff. of battery, $V = E - Ir = 2{\cdot}5 - 1 \times 0{\cdot}5 = 2$ volt

$\therefore$ Charge on capacitor plates, $Q = CV = 4 \times 2 = 8$ μC

Choice (*d*) is correct.

2. In a uniform electric field, when a positively charged particle is released from rest, it moves along the electric field (*i.e.*, from higher potential to lower potential). Therefore, electric potential energy of charge decreases. **Choice (*c*) is correct.**

3. In Fig. 1(N).3, we observe that in all the three parts, $V_A = 20$ V and $V_B = 40$ V. Work done in carrying a charge q from A to B is $W = q\,(V_B - V_A)$ = same in all the three figures. **Choice (*c*) is correct.**

4. As is known from theory, at any point inside a charged conducting sphere, $E = 0$ and $V = \dfrac{kq}{R}$ = potential on the surface = 100 V

Infact, $E = -\dfrac{dV}{dr} = -\dfrac{d}{dr}$ (constant) = Zero.

Choice (*c*) is correct.

5. From a collection of charges, whose total sum is not zero, equipotentials at large distances must be **spheres** only.

Choice (*a*) is correct.

6. The capacities of two individual condensers are $C_1 = \dfrac{k_1 \epsilon_0 A}{d_1}$ and $C_2 = \dfrac{k_2 \epsilon_0 A}{d_2}$

The arrangement is equivalent to two capacitors joined in series. Therefore, the combined capacity (C_s) is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{k_1 \epsilon_0 A} + \frac{d_2}{k_2 \epsilon_0 A} = \frac{1}{\epsilon_0 A}\left[\frac{d_1}{k_1} + \frac{d_2}{k_2}\right] = \frac{d_1 k_2 + d_2 k_1}{\epsilon_0 A k_1 k_2}$$

or $$C_s = \frac{\epsilon_0 A k_1 k_2}{d_1 k_2 + d_2 k_1} = \frac{k \epsilon_0 A}{(d_1 + d_2)} \qquad \therefore \qquad \boldsymbol{k = \frac{k_1 k_2 (d_1 + d_2)}{(d_1 k_2 + d_2 k_1)}}$$

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Multiple Choice Questions – II

7. As electric field in the $\hat{z}$ direction is uniform, equipotential surfaces are in XY plane. Hence the potential for given z_1 is constant (*i*) on x-y plane (*ii*) for any x in this plane and (*iii*) for any y in this plane.
Hence **Choices (*b*), (*c*), (*d*) are correct.**

8. From $E = -\frac{dV}{dr}$, we find that $dr \propto \frac{1}{E}$, *i.e.*, equipotential surfaces are closer in regions of large electric fields compared to regions of lower electric fields. At sharp edges of a conductor, charge density is more. Therefore, electric field is stronger. Hence equipotential surfaces are more crowded. Same argument is true for option (*c*). **The choices (*a*), (*b*), (*c*) are correct.**

9. From the knowledge of theory, work done to move a charge along an equipotential from A to B,

$$W = -\int_A^B \vec{E}.\vec{dl}$$

On equipotential surface, $\vec{E} \perp \vec{dl}$

$$W = -\int_A^B E(dl)\cos 90° = \text{Zero.}$$

Choices (*b*) and (*c*) are correct.

10. In a region of constant potential (V = constant) ; $E = -\frac{dV}{dr} = 0$, *i.e.*, electric field is zero.
As $E = 0$, there can be no charge inside the region.
Choices (*b*) and (*c*) are correct.

11. In Fig. 1(N).5, when K_1 is closed and K_2 is open, C_1 is charged to potential E acquiring a total charge $Q = C_1E$.
When K_1 is opened and K_2 is closed, battery is cut off. C_1 and C_2 are in parallel. The charge on C_1 is shared between C_1 and C_2, such that $V_1 = V_2$.
As there is no loss of charge, $Q_1' + Q_2' = Q$
Choices (*a*) and (*d*) are correct.

12. From the knowledge of theory, if a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then there must be charges on the surface of conductor or inside the conductor. However, there cannot be any charge in the body of the conductor.
Choices (*a*) and (*b*) are correct.

13. In Fig. 1(N).6, when key K is kept closed, condenser C is charged to potential E. When plates of capacitor are moved apart, its capacity $C = \frac{\epsilon_0 A}{d}$ decreases. As potential of condenser remains same, charge $Q = CE$ decreases. **Choice (*c*) is correct.**
Once key K is closed, condenser gets charged : $Q = CE$. Now, if key K is opened, battery is disconnected, no more charging can occur, *i.e.*, Q remains same. As plates of capacitor are moved apart, its capacity $C = \frac{\epsilon_0 A}{d}$ decreases. Therefore, its potential $V = Q/C$ increases. **Choice (*d*) is correct.**

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VERY SHORT ANSWER QUESTIONS

14. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

Ans. Here, $V_1 = V_2$

$$\frac{Q_1}{4\pi\epsilon_0 R_1} = \frac{Q_2}{4\pi\epsilon_0 R_2} \quad \therefore \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2} \qquad ...(i)$$

As $R_1 > R_2$, $\therefore$ $Q_1 > Q_2$

i.e., larger sphere has more charge than the smaller sphere.

Now, $\sigma_1 = \dfrac{Q_1}{4\pi R_1^2}$ and $\sigma_2 = \dfrac{Q_2}{4\pi R_2^2}$ $\therefore$ $\dfrac{\sigma_2}{\sigma_1} = \dfrac{Q_2}{R_2^2} \cdot \dfrac{R_1^2}{Q_1}$

Using (i), $\dfrac{\sigma_2}{\sigma_1} = \dfrac{R_2}{R_1} \cdot \dfrac{R_1^2}{R_2^2} = \dfrac{R_1}{R_2}$

As $R_1 > R_2$, therefore, $\sigma_2 > \sigma_1$

i.e., charge density of smaller sphere is **more** than the charge density of larger sphere.

15. Do free electrons travel to region of higher potential or lower potential ?

Ans. As free electrons are negatively charged, they would move to regions of **higher potential.**

16. Can there be a potential difference between two adjacent conductors carrying the same charge ?

Ans. Yes, there can be potential difference between two adjacent conductors carrying the same charge. This is because the sizes of the conductors might be different, and hence their capacities may be different.

17. Can the potential function have a maximum or minimum in free space ?

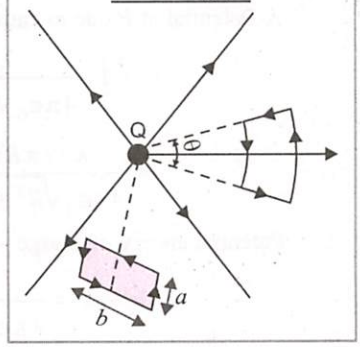
Ans. No. The potential function cannot have a maximum or minimum in free space. This is because in that event,

$$E = -\frac{dV}{dr} = \text{Zero}$$

18. A test charge q is made to move in the electric field of a point charge Q along two different closed paths, Fig. 2(N).7. First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases ?

Ans. From the knowledge of theory, we understand that electric field is conservative. Therefore, work done in moving a test charge over any closed path is zero.

FIGURE 2(N).7



SHORT ANSWER QUESTIONS

19. Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

Ans. Suppose a closed equipotential surface with no charge within itself does not enclose an equipotential volume. Therefore, potential just inside the surface would be different from potential at the surface, resulting in some potential gradient. Therefore, there would be field lines pointing inwards or outwards from the surface. This is possible only if other end of the lines are at charges inside. As there is no charge inside, therefore, the entire volume inside the equipotential surface must be at the same potential.

20. A capacitor has some dielectric between its plates, and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

Ans. When the capacitor is connected to dc source, it gets charged. The battery is now disconnected. So no more charge can flow in, *i.e.*, charged stored will stay constant. On removing dielectric, capacitance decreases.

Energy stored $= \frac{Q^2}{2C}$ will increase. Pot, $V = \frac{Q}{C}$ would increase $E = \frac{V}{d}$ will increase.

21. Prove that, if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

Ans. When an insulated uncharged conductor is placed near a charged conductor, the uncharged conductor acquires some induced charge. Along any path from the charged conductor to uncharged conductor along the electric field, potential will continually decrease. Another path from uncharged conductor to infinity will lower the potential further. Hence the required result.

22. Calculate potential energy of a point charge $-q$ placed along the axis due to a charge $+Q$ uniformly distributed along a ring of radius R. Sketch P.E. as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis) ?

Ans. In Fig. 2(N).8(a), we have shown a charge $+Q$ distributed uniformly along a ring of radius R, with centre at O.

Let λ be the linear charge density of the ring. Therefore,

$$\lambda = Q/2\pi R \quad ...(i)$$

Consider a small element of length dl of the ring with centre at A.

$\therefore$ Charge on the element, $dq = \lambda\, dl$

At any point P on the axis of ring,

where $OP = z$, potential due to the charge element,

$$dV = \frac{dq}{4\pi\epsilon_0 (AP)} = \frac{\lambda\, dl}{4\pi\epsilon_0 \sqrt{R^2 + z^2}}$$

$\therefore$ Potential at P due to entire ring,

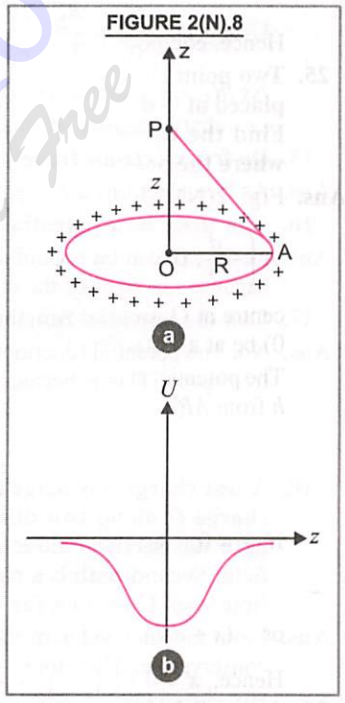
$$V = \oint \frac{\lambda\, dl}{4\pi\epsilon_0 \sqrt{R^2 + z^2}}$$

$$= \frac{\lambda.(2\pi R)}{4\pi\epsilon_0 \sqrt{R^2 + z^2}} = \frac{Q}{4\pi\epsilon_0 \sqrt{R^2 + z^2}}$$

Potential energy of charge $-q$ held at P

$$U = -qV = \frac{-qQ}{4\pi\epsilon_0 \sqrt{R^2 + z^2}}$$

FIGURE 2(N).8



The variation of potential energy with axial distance z from the centre of the ring is shown in Fig. 2(N).8(b).

If charge $-q$ is displaced slightly from the centre of the ring along the axis of the ring, and left, the charge would perform oscillations. However, by just looking at the graph, we cannot come to a firm conclusion about the nature of these oscillations.

23. Calculate potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Ans. Refer to Q. 22 above for the derivation of potential on the axis of the charged ring.

LONG ANSWER QUESTIONS

24. Find the equation of the equipotentials for an infinite cylinder of radius r_0, carrying charge of linear density λ.

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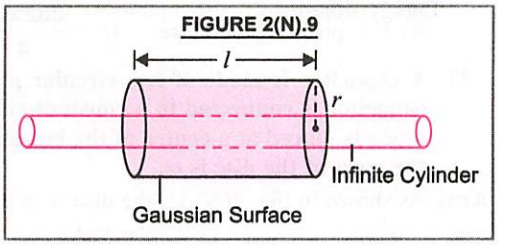
Ans. Fig. 2(N).19 shows an infinite cylinder of radius r_0 carrying charge of linear density λ. From symmetry, we find that the field lines must be radially outwards. Imagine a cylindrical Gaussian surface of radius r and length l. According to Gauss's theorem,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\epsilon_0 r} \qquad ...(i)$$

$$\therefore \; V(r) - V(r_0) = -\int_{r_0}^{r} \vec{E}.\vec{dl} = \frac{\lambda}{2\pi\epsilon_0} \log_e \frac{r_0}{r}$$



For an equipotential surface of given $V(r)$,

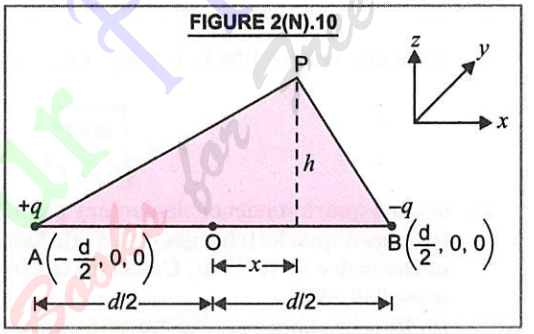
$$\log_e \frac{r}{r_0} = -\frac{2\pi\epsilon_0}{\lambda}[V(r) - V(r_0)] \quad \therefore \quad r = r_0 e^{-2\pi\epsilon_0 [V(r)-V(r_0)]/\lambda} \qquad ...(ii)$$

Hence, equipotential surfaces are cylinders of radius r given by *(ii)*.

25. Two point charges of magnitude + q and – q are placed at (– d/2, 0, 0) and (d/2, 0, 0), respectively. Find the equation of the equipotential surface where the potential is zero.

Ans. Fig. 2(N).10 shows two point charges + q at $A\left(-\frac{d}{2}, 0, 0\right)$ and charge – q at $B\left(\frac{d}{2}, 0, 0\right)$ with centre at O. Let the equipotential surface (with V = 0) be at a distance x from the origin O.

The potential at any point P on the surface at a height h from AB is



$$V = \frac{q}{4\pi\epsilon_0[(x+d/2)^2 + h^2]^{1/2}} - \frac{q}{4\pi\epsilon_0[(x-d/2)^2 + h^2]^{1/2}}$$

If $V = 0$, then $\dfrac{q}{4\pi\epsilon_0[(x+d/2)^2 + h^2]^{1/2}} = \dfrac{q}{4\pi\epsilon_0[(x-d/2)^2 + h^2]^{1/2}}$

or $(x + d/2)^2 + h^2 = (x - d/2)^2 + h^2$ or $x^2 + \dfrac{d^2}{4} + xd = x^2 + \dfrac{d^2}{4} - xd$ or $2xd = 0$ or $x = 0$

Hence, **x = 0 is the equation of the equipotential surface where potential is zero.**

26. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $\alpha = 2\ V^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78$ V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.

Ans. On connecting the two given capacitors, let the final voltage be U.

If capacity of capacitor without the dielectric is C, then the charge on this capacitor is $Q_1 = CU$

The other capacitor with dielectric has capacity ϵC. Therefore, charge on it is $Q_2 = \epsilon CU$.

As $\epsilon = \alpha U$, therefore, $Q_2 = \alpha C U^2$

The initial charge on the capacitor (without dielectric) that was charged is $Q_0 = CU_0$

From the conservation of charge, $Q_0 = Q_1 + Q_2$

$$CU_0 = CU + \alpha CU^2 \quad \text{or} \quad \alpha U^2 + U - U_0 = 0 \quad \therefore \quad U = \frac{-1 \pm \sqrt{1 + 4\alpha U_0}}{2\alpha}$$

using $\alpha = 2\ V^{-1}$ and $U_0 = 78$ volt, we get $U = \dfrac{-1 \pm \sqrt{1 + 4 \times 2 \times 78}}{2 \times 2} = \dfrac{-1 \pm \sqrt{625}}{4}$

As U is positive, therefore, $U = \dfrac{\sqrt{625} - 1}{4} = \dfrac{24}{4} = \mathbf{6\ volt}$

27. A capacitor is made of two circular plates of radius R each, separated by a distance $d << R$. The capacitor is connected to a constant voltage. A thin conducting disc of radius $r << R$ and thickness $t << r$ is placed at a centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.

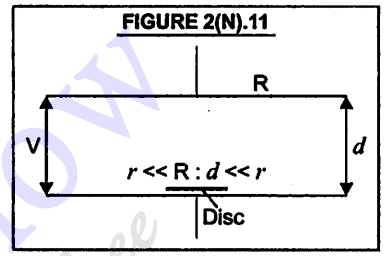
Ans. As shown in Fig. 2(N).11, the disc is in touch with the bottom plate. The electric field on the disc is

$$E = V/d$$

Therefore, charge q' transferred to the disc

$$q' = C.V = \left(-\frac{\epsilon_0 A}{d}\right)V = -\epsilon_0 \frac{V}{d}\pi r^2$$

FIGURE 2(N).11



Force acting on the disc, $F = -q'E = \left(\epsilon_0 \dfrac{V}{d}\pi r^2\right)\dfrac{V}{d} = \epsilon_0 \dfrac{V^2}{d^2}\pi r^2$

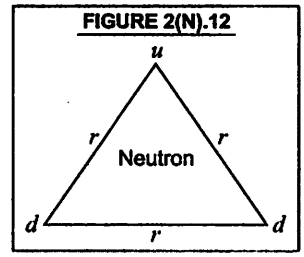
If the disc is to be lifted, $F \geq mg$, *i.e.*, $\epsilon_0 \dfrac{V^2}{d^2}\pi r^2 = mg$ (for minimum V)

$$V = \sqrt{\frac{mgd^2}{\pi \epsilon_0 r^2}}$$

28. (*a*) In a quark model of elementary particles, a neutron is made of one up quarks [charge (2/3) *e*] and two down quarks [charges – (1/3) *e*]. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate electrostatic potential energy of neutron and compare it with its mass 939 MeV.

(*b*) Repeat above exercise for a proton which is made of two up and one down quark.

FIGURE 2(N).12



Ans. Fig. 2(N).12 shows quark model of a neutron, where $r = 10^{-15}$ m. Potential energy of neutron

$$U = \frac{1}{4\pi\epsilon_0 r}[q_d q_d + q_u q_d + q_u q_d]$$

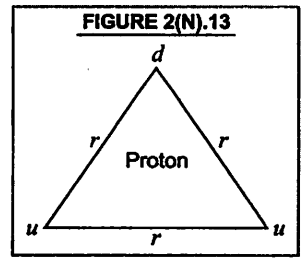
$$= \frac{9 \times 10^9}{10^{-15}}\left[\left(-\frac{e}{3}\right)\left(-\frac{e}{3}\right) + \left(\frac{2e}{3}\right)\left(-\frac{e}{3}\right) + \left(\frac{2e}{3}\right)\left(-\frac{e}{3}\right)\right]$$

$$U = \frac{9 \times 10^9}{10^{-15}}\left(\frac{1}{9} - \frac{4}{9}\right)(1{\cdot}6 \times 10^{-19})^2 = -7{\cdot}68 \times 10^{-14}\ \text{J}$$

$$U = \frac{-7{\cdot}68 \times 10^{-14}}{1{\cdot}6 \times 10^{-19}}\ \text{eV} = -4{\cdot}8 \times 10^5\ \text{eV} = \mathbf{-0{\cdot}48\ MeV}$$

$m_n c^2 = 939$ MeV. $\therefore$ $\dfrac{U}{m_n c^2} = \dfrac{0{\cdot}48}{939} = 5{\cdot}11 \times 10^{-4}$

FIGURE 2(N).13



Proceed similarly for a proton whose quark model is shown in Fig. 2(N).13.

29. Two metal spheres, one of radius R and the other of radius $2R$, both have same surface charge density s. They are brought in contact and separated. What will be new surface charge densities on them ?

Ans. Let σ be the surface charge density of each of two metal spheres of radii R and 2 R.

$\therefore$ Before contact, charges on the two spheres are

$$Q_1 = \sigma \,.\, 4\pi R^2, \quad \text{and} \quad Q_2 = \sigma\, 4\pi (2R)^2 = 4\, Q_1$$

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When the two spheres are brought in contact, their charges are shared till their potentials become equal. If Q_1' and Q_2' are the final charges on the two spheres after contact, then as $V_1 = V_2$

$$\frac{Q_1'}{4\pi\epsilon_0 R} = \frac{Q_2'}{4\pi\epsilon_0 (2R)} \quad \therefore \quad Q_2' = 2\,Q_1' \qquad ...(i)$$

As there is no loss of charge in the process, therefore,

$$Q_1' + Q_2' = Q_1 + Q_2 = Q_1 + 4\,Q_1 = 5\,Q_1 = 5\,(\sigma \cdot 4\,\pi\,R^2) \quad \text{using } (i),\ Q_1' + 2\,Q_1' = 5\,\sigma\,4\,\pi\,R^2$$

$$Q_1' = \frac{5}{3}\,\sigma\,4\pi R^2 \quad \text{and} \quad Q_2' = 2Q_1' = \frac{10}{3}\,\sigma 4\pi R^2$$

Hence, $$\sigma_1 = \frac{Q_1'}{4\pi R^2} = \frac{5}{3}\,\sigma \quad \text{and} \quad \sigma_2 = \frac{Q_2'}{4\pi(2R)^2} = \frac{5}{6}\,\sigma$$

30. In the circuit shown in Fig. 2(N).14, initially K_1 is closed and K_2 is open. What are the charges on each capacitor.

Then K_1 was opened and K_2 was closed (order is important). What will be the charge on each capacitor now ? [$C = 1\ \mu F$]

Ans. In Fig. 2(N).14, when K_1 is closed and K_2 is open, C_3 is out of contact.

$\therefore \quad Q_3 = 0.$

C_1, C_2 are in series with the battery $E = 9$ V,

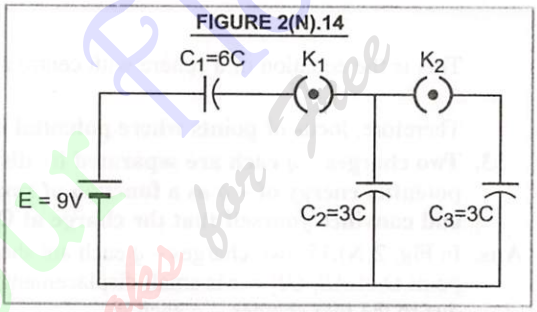
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$C_s = 2\ \mu F$$

$\therefore \quad Q_1 = E\,.\,C_s = 9 \times 2 = \mathbf{18\ \mu C},$

$$Q_2 = E\,.\,C_s = 9 \times 2 = \mathbf{18\ \mu C}$$

FIGURE 2(N).14



Later, K_1 is opened and K_2 is closed, charge on C_1 remains unchanged, *i.e.*, $Q_1' = Q_1 = \mathbf{18\ \mu C}$

Charge on C_2 is now shared between C_2 and C_3. As $C_2 = C_3$, therefore, $Q_2' = Q_3' = \frac{Q_2}{2} = \frac{18}{2} = 9\ \mu C$

31. Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.

Ans. When a charge Q is distributed uniformly on the surface of a circular disc of radius R, surface charge density

$$\sigma = \frac{Q}{\pi R^2} \qquad ...(i)$$

Consider a small element of the circular disc in the form of a circular strip of radius r and thickness/width dr, Fig. 2(N).15,

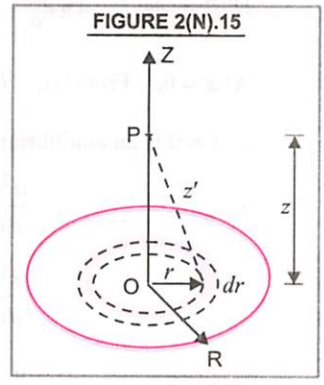
= Area of this element = $2\,\pi\,r\,dr$

Charge on this element, $q = \sigma\,(2\,\pi\,r\,dr)$

Potential due to this element at any point P on the axis of the disc, where $OP = z$ is

$$dV = \frac{q}{4\pi\epsilon_0 z'} = \frac{\sigma 2\pi r\,dr}{4\pi\epsilon_0\sqrt{r^2+z^2}}$$

FIGURE 2(N).15



Potential at P due to charge on the entire circular disc

$$V = \frac{\pi\sigma}{4\pi\epsilon_0}\int_0^R \frac{2r\,dr}{\sqrt{r^2+z^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R = \frac{2\pi\sigma}{4\pi\epsilon_0}\left[\sqrt{R^2+z^2} - z\right]$$

Using (i), $$V = \frac{2Q}{4\pi\epsilon_0 R^2}\left[\sqrt{R^2+z^2} - z\right]$$

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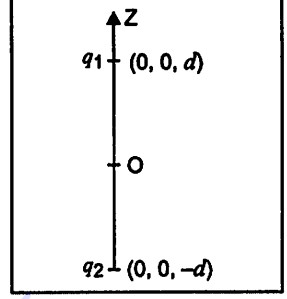
32. Two charges q_1 and q_2 are placed at (0, 0, d) and (0, 0, $-d$) respectively. Find locus of points where the potential is zero.

FIGURE 2(N).16



Ans. In Fig. 2(N).16, we have shown two charges q_1 (0, 0, d) and q_2 (0, 0, $-d$). For potential to be zero at (x, y, z) we should have

$$\frac{q_1}{4\pi\epsilon_0\sqrt{x^2+y^2+(z-d)^2}}+\frac{q_2}{4\pi\epsilon_0\sqrt{x^2+y^2+(z+d)^2}}=0$$

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}}=\frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}} \qquad ...(i)$$

Clearly, total potential can be zero when q_1, q_2 have **opposite signs.**

Squaring both sides of (i), we get $q_1^2\,[x^2+y^2+(z+d)^2]=q_2^2\,[x^2+y^2+(z-d)^2]$

on simplifying, we get $$x^2+y^2+z^2+\left[\frac{(q_1/q_2)^2+1}{(q_1/q_2)^2-1}\right](2zd)+d^2=0.$$

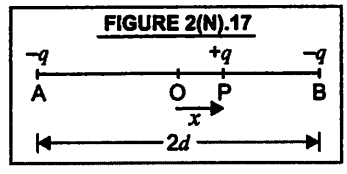
This is the equation of a sphere with centre at $\left[0,\,0,\,-2d\left(\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right)\right]$. If $q_1=-q_2$, then $z=0$

Therefore, **locus of points where potential is zero is the plane through mid point of the two charges.**

33. Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid point O. Find potential energy of $+q$ as a function of small distance x from O due to $-q$ charges. Sketch P.E. v/s x and convince yourself that the charge at O is in an unstable equilibrium.

Ans. In Fig. 2(N).17, two charges $-q$ each are shown at A and B, where AB = 2 d. A charge $+q$ is kept at mid point O of AB. $OP = x$ is small displacement of $+q$ charge. Therefore, potential energy of $+q$ charge at P due to the two charges $-q$ each is

FIGURE 2(N).17

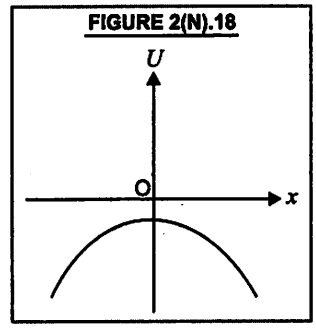


$$U=\frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{d+x}-\frac{q^2}{d-x}\right]=-\frac{q^2}{4\pi\epsilon_0}\,\frac{2d}{(d^2-x^2)} \qquad ...(i)$$

The potential energy (u) versus x graph is as shown in Fig. 2(N).28.

Now $$\frac{dU}{dx}=\frac{-q^2(2d)}{4\pi\epsilon_0}\,\frac{2x}{(d^2-x^2)^2} \qquad ...(ii)$$

FIGURE 2(N).18



At $x = 0$, From (i), $U_0=\frac{-2q^2}{4\pi\epsilon_0 d}$, and from ($ii$), $\frac{dU}{dx}=0$

$\therefore$ $x = 0$ is an equilibrium point

Now $$\frac{d^2U}{dx^2}=\left(\frac{-2dq^2}{4\pi\epsilon_0}\right)\left[\frac{2}{(d^2-x^2)^2}-\frac{8x^2}{(d^2-x^2)^3}\right]$$

$$\frac{d^2U}{dx^2}=\left(\frac{-2dq^2}{4\pi\epsilon_0}\right)\frac{1}{(d^2-x^2)^3}\,[2(d^2-x^2)-8x^2]$$

At $x = 0$, $$\frac{d^2U}{dx^2}=\left(\frac{-2dq^2}{4\pi\epsilon_0}\right)\frac{1}{d^6}\,(2d^2)=-\frac{q^2}{\pi\epsilon_0 d^3}$$ which is less than zero.

Hence, the equilibrium of charge q at O is **unstable.**

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I. Multiple Choice Questions (with One Correct Answer)

I. Electric Flux

1. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point at a distance $R/2$ from the centre of the shell is :

(a) $\frac{2Q}{4\pi\epsilon_0 R}$ (b) $\left(\frac{2Q}{4\pi\epsilon_0 R}\right)-\left(\frac{2q}{4\pi\epsilon_0 R}\right)$

(c) $\frac{2Q}{4\pi\epsilon_0 R}+\frac{q}{4\pi\epsilon_0 R}$

(d) $\frac{(q+Q)^2}{4\pi\epsilon_0 R}$ **(AIEEE 2003)**

2. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_0$ where V_0 is the potential at the origin and V_A, the potential at $x = 2$ m is

(a) – 80 V (b) 80 V
(c) 120 V (d) – 120 V **(JEE Main 2014)**

3. The potential at a point distant x (measured in μm) due to some charges situated on the x-axis is given by $V(x) = \frac{20}{x^2-4}$ Volt.

The electric field at $x = 5$ μ m is given by

(a) $\frac{5}{3}$ volt/μm and in + x direction,

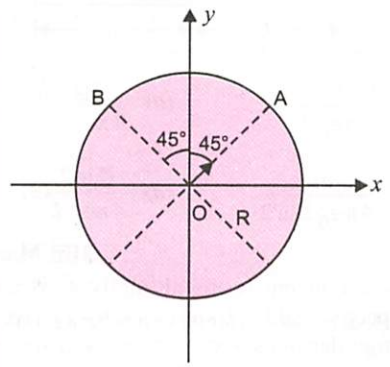
(b) $\frac{10}{9}$ V/μm and in neg. x direction,

(c) $\frac{10}{9}$ V/μm and in + x direction

(d) $\frac{5}{3}$ V/μm and in neg. x direction. **(AIEEE 2007)**

4. An electric dipole with dipole moment $\frac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of a uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centred at the origin as shown in Fig. 2(CF).1 ? The correct statement is/are :

FIGURE 2(CF).1



ANSWERS
1. (c) 2. (a) 3. (c)

ϵ_0 is permittivity of free space and $R >>$ dipole size.

(a) $R = \left[\frac{P_0}{4\pi \epsilon_0 E_0}\right]$

(b) The magnitude of total electric field on any two points of the circle will be same.

(c) Total electric field at point A is

$\vec{E}_A = \sqrt{2}\,\vec{E}_0\,(\hat{i} + \hat{j})$

(d) Total electric field at point B is $\vec{E}_B = 0$.

(JEE (Advanced) 2019)

5. An electric charge 10^{-3} μC is placed at the origin (0, 0) of XY Co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be

(a) 4·5 V (b) 9 V

(c) zero (d) 2 V **(AIEEE 2007)**

6. Three concentric spherical shells have radii a, b and c $(a < b < c)$ and have surface charge densities $\sigma, -\sigma + \sigma$ respectively. If V_A, V_B and V_C denote the potentials of the three shells, then for $c = a + b$, we have

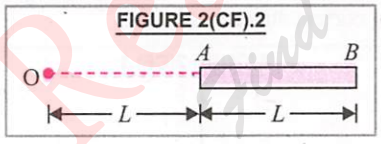
(a) $V_C = V_B = V_A$ (b) $V_A = V_C \neq V_B$

(c) $V_C = V_B \neq V_A$ (d) $V_C = V_B \neq V_A$

(AIPMT 2009)

7. A charge Q is uniformly distributed over a long rod AB of length L as shown in the Fig. 2(CF).2. The electric potential at the point O lying at distance L from the end A is :

FIGURE 2(CF).2



(a) $\frac{Q}{8\pi\epsilon_0 L}$ (b) $\frac{3Q}{4\pi\epsilon_0 L}$

(c) $\frac{3Q}{4\pi\epsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\epsilon_0 L}$

(JEE Main 2013)

8. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(a) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$ (b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$

(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$ (d) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$

(JEE Main 2018)

9. In a certain region of space with volume 0·2 m^3, the electric potential is found to be 5 V throughout. The magnitude of electric field in this region is :

(a) 0·5 N/C (b) 1 N/C

(c) 5 N/C (d) zero **(NEET 2020)**

10. A short electric dipole has a dipole moment of 16 $\times 10^{-9}$ Cm. The electric potential due to the dipole at a point at a distance of 0·6 m from the centre of the dipole, situated on a line making an angle of 60° with the dipole axis is :

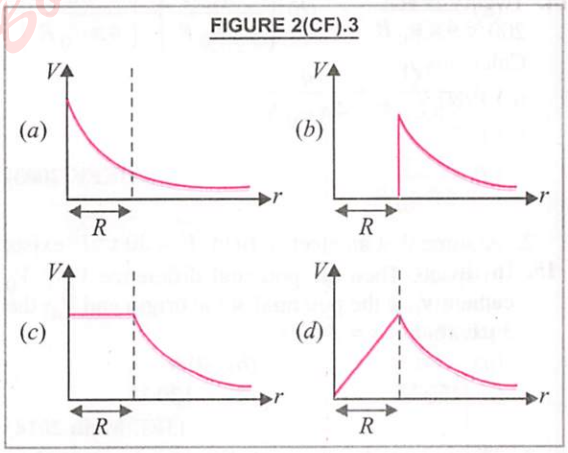
$\left(\frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{Nm}^2/\text{C}^2\right)$

(a) 200 V (b) 400 V

(c) zero (d) 50 V **(NEET 2020)**

11. The variation of electrostatic potential with radial distance r from the centre of a positively charged metallic thin shell of radius R is given by the graph :

FIGURE 2(CF).3



(NEET 2020)

12. Two charged spherical conductors of radius R_1 and R_2 are connected by a wire. Then the ratio of surface charge densities of the spheres $[\sigma_1/\sigma_2]$ is

(a) $\frac{R_1^2}{R_2^2}$ (b) $\frac{R_1}{R_2}$

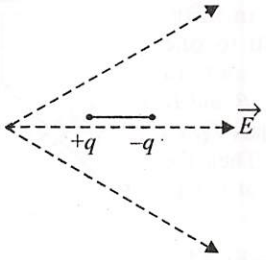
ANSWERS

4. (a,d) 5. (c) 6. (b) 7. (d) 8. (b) 9. (d) 10. (a) 11. (c)

(c) $\frac{R_2}{R_1}$ (d) $\sqrt{\frac{R_1}{R_2}}$ **(NEET 2021)**

13. A dipole is placed in an electric field as shown. In which direction will it move ?

FIGURE 2(CF).4



(a) Towards the right as its potential energy will increase.

(b) Towards the left as its potential energy will increase.

(c) Towards the right as its potential energy will decrease.

(d) Towards the left as its potential energy will decrease. **(NEET 2021)**

14. Twenty seven drops of same size are charged at 200 V each. They combine to form a bigger drop. Calculate the potential of the bigger drop.

(a) 1980 V (b) 660 V

(c) 1320 V (d) 1520 V

(NEET 2021)

II. Capacitance and Capacitors

15. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of C is given properly by :

FIGURE 2(CF).5

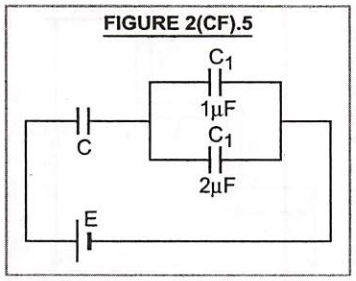
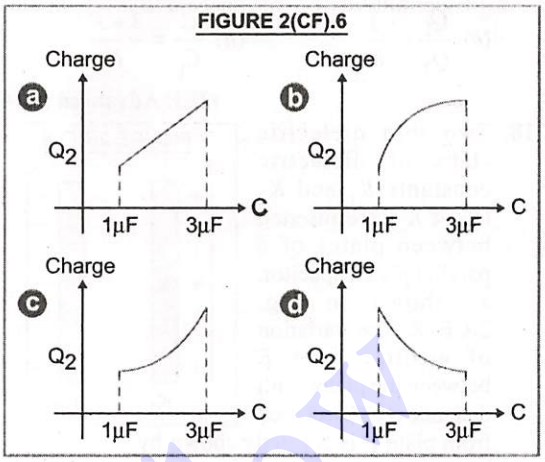


FIGURE 2(CF).6



(JEE Main 2015)

16. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1·0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V.

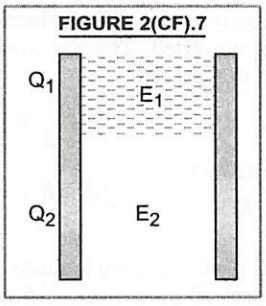
The minimum number of capacitors required to achieve this is

(a) 24 (b) 32 (c) 2 (d) 16

(JEE Main 2017)

17. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in Fig. 2(CF).7. The total capacitance of capacitor is C while that of the portion with dielectric inbetween is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.

FIGURE 2(CF).7



(a) $\frac{E_1}{E_2} = 1$ (b) $\frac{E_1}{E_2} = \frac{1}{K}$

ANSWERS

12. (c) 13. (c) 14. (a) 15. (b) 16. (b)

(c) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (d) $\frac{C}{C_1} = \frac{3+K}{K}$

(JEE Advanced 2014)

18. Two thin dielectric slabs of dielectric constants K_1 and K_2 ($K_1 < K_2$) are inserted between plates of a parallel plate capacitor, as shown in Fig. 2(CF).8. The variation of electric field E between the plates with distance d as measured from plate P is correctly shown by

FIGURE 2(CF).8

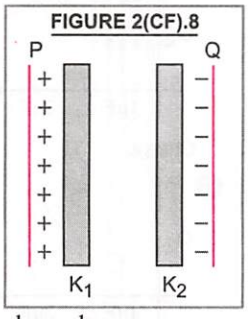
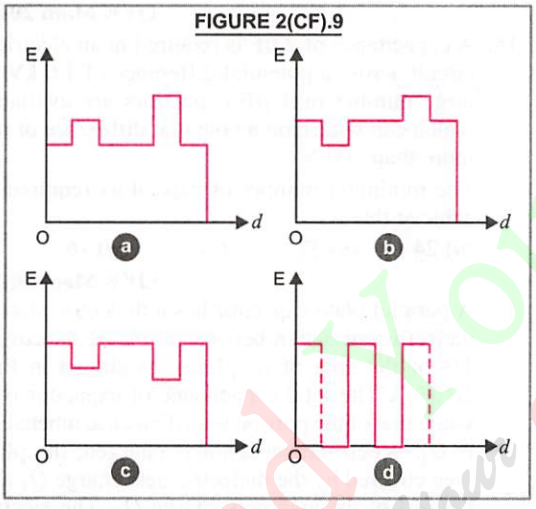


FIGURE 2(CF).9



(AIPMT 2014)

19. A parallel plate condenser with a dielectric of constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is :

(a) zero (b) $\frac{1}{2}(K-1)CV^2$

(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)CV^2$

(AIEEE 2009)

20. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

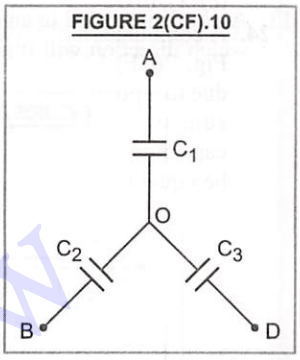
(a) 3×10^4 C/m^2 (b) 6×10^4 C/m^2

(c) 6×10^{-7} C/m^2 (d) 3×10^{-7} C/m^2

(JEE Main 2014)

21. Three uncharged capacitors of capacitance C_1, C_2 and C_3 are connected as shown in Fig. 2(CF).10 to one another and to points A, B and D at potentials V_A, V_B and V_D. Then the potential at O will be :

FIGURE 2(CF).10



(a) $\frac{V_A + V_B + V_D}{C_1 + C_2 + C_3}$

(b) $\frac{V_A C_1 + V_B C_2 + V_D C_3}{C_1 + C_2 + C_3}$

(c) $\frac{V_A V_B + V_B V_D + V_D V_A}{C_1 + C_2 + C_3}$

(d) $\frac{V_A V_B V_D}{C_1 C_2 + C_2 C_3 + C_3 C_1}$

22. A capacitor is charged by a battery. The battery is removed and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system

(a) increases by a factor of 4

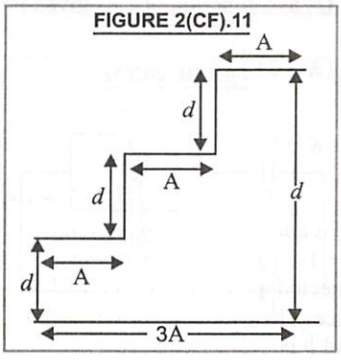
(b) decreases by a factor of 2

(c) remains the same

(d) increases by a factor of 2 **(NEET 2017)**

23. The expression for the equivalent capacitance of the system shown in Fig. 2(CF).11 is (A is the cross-sectional area of one of the plates) :

FIGURE 2(CF).11



ANSWERS

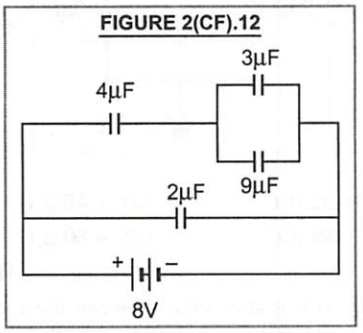
17. (a) **18.** (c) **19.** (a) **20.** (c) **21.** (b) **22.** (b)

(*a*) $\varepsilon_0 A/3d$ (*b*) $\dfrac{3\varepsilon_0 A}{d}$

(*c*) $\varepsilon_0 A/6d$ (*d*) none of these

24. A combination of capacitors is set up as shown in Fig. 2(CF).12. The magnitude of electric field, due to a point charge Q (having a charge equal to sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it would be equal to

FIGURE 2(CF).12



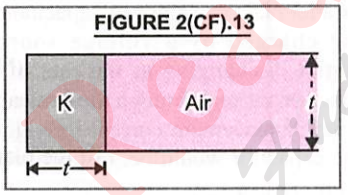
(*a*) 240 N/C (*b*) 360 N/C

(*c*) 420 N/C (*d*) 480 N/C

(JEE Main 2016)

25. A parallel plate capacitor with air as the dielectric has capacitance C. A slab of dielectric constant K and having the same thickness as the separation between the plates is introduced so as to fill one-fourth of the capacitor as shown in Fig. 2(CF).13. The new capacitance will be

FIGURE 2(CF).13



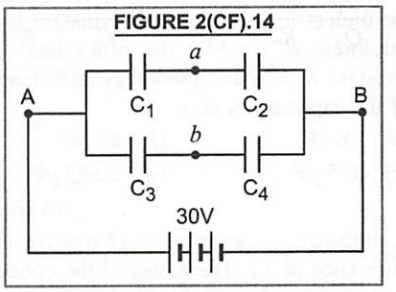
(*a*) $(K+3)\dfrac{C}{4}$ (*b*) $(K+2)\dfrac{C}{4}$

(*c*) $(K+1)\dfrac{C}{4}$ (*d*) $\dfrac{KC}{4}$

(CET Karnataka 2007)

26. Four capacitors with capacitances C_1 = 1 μF, C_2 = 1·5 μF, C_3 = 2·5 μF and C_4 = 0·5 μF are connected as shown in Fig. 2(CF).14 to a 30 volt source. The potential difference between points *a* and *b* is

FIGURE 2(CF).14



(*a*) 5 V (*b*) 9 V

(*c*) 10 V (*d*) – 13 V

27. The plates of a parallel plate capacitor with air as medium are separated by a distance of 8 mm. A medium of dielectric constant 2 and thickness 4 mm having the same area is introduced between the plates. For the capacitance to remain the same, the distance between the plates is

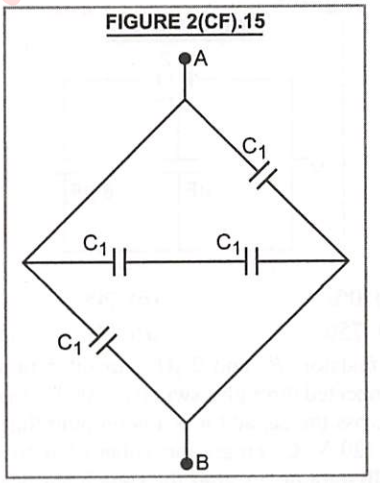
(*a*) 8 mm (*b*) 6 mm

(*c*) 4 mm (*d*) 12 mm

(*e*) 10 mm **(Kerala PET 2007)**

28. Four identical capacitors are connected as shown in Fig. 2(CF).15. When a battery of 6 V is connected between *A* and *B*, the charge stored is found to be 1·5 μ C. The value of C_1 is

FIGURE 2(CF).15



(*a*) 2·5 μ F (*b*) 15 μ F

(*c*) 1·5 μ F (*d*) 0·1 μ F

29. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is *d*. The space between the

ANSWERS

23. (*d*) **24.** (*c*) **25.** (*a*) **26.** (*d*) **27.** (*e*) **28.** (*d*)

plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$, while the other has dielectric constant $K_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now

(a) 1·8 pF (b) 45 pF
(c) 40·5 pF (d) 20·25 pF

(AIEEE 2008)

30. A condenser of capacity C is charged to a potential difference of V_1. The plates of the condenser are then connected to an ideal inductor of inductance L. The current through the inductor when the potential difference across the condenser reduces to V_2 is

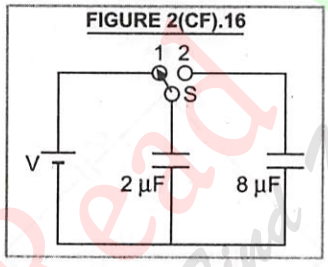
(a) $\left(\frac{C(V_1 - V_2)^2}{L}\right)^{\frac{1}{2}}$ (b) $\frac{C(V_1^2 - V_2^2)^2}{L}$

(c) $\frac{C(V_1^2 + V_2^2)}{L}$ (d) $\left(\frac{C(V_1^2 - V_2^2)}{L}\right)^{\frac{1}{2}}$

(AIPMT (Main) 2010)

31. A 2 μF capacitor is charged as shown in fig. 2(CF).16. The percentage of its stored energy dissi- pated after the switch S is turned to position 2 is

FIGURE 2(CF).16



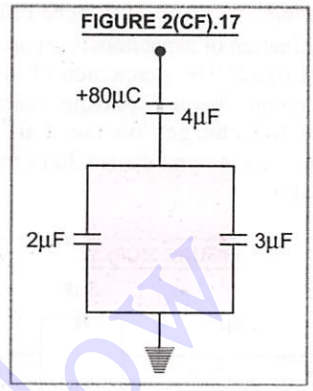
(a) 0% (b) 20%
(c) 75% (d) 80% **(IIT 2011)**

32. A resistor 'R' and 2 μF capacitor in series are connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2{\cdot}5 = 0{\cdot}4$)

(a) $1{\cdot}7 \times 10^5\ \Omega$ (b) $2{\cdot}7 \times 10^6\ \Omega$
(c) $3{\cdot}3 \times 10^7\ \Omega$ (d) $1{\cdot}3 \times 10^4\ \Omega$

33. In given circuit, Fig. 2(CF).17, a charge of + 80 μ C is given to the upper plate of the 4 μ F capacitor. Then in the steady state, the charge on the upper plate of the 3 μ F capacitor is

FIGURE 2(CF).17



(a) + 32 μ C (b) + 40 μ C
(c) + 48 μ C (d) + 80 μ C

(IIT 2012)

34. The electrostatic force between the metal plates of an isolated parallel plate capacitor C having a charge Q and area A, is

(a) Proportional to the square root of the distance between the plates
(b) Linearly proportional to the distance between the plates
(c) Independent of the distance between the plates
(d) Inversely proportional to the distance between the plates **(NEET 2018)**

35. A parallel plate capacitor of capacitance 20 μF is being charged by a voltage source, whose potential is changing at the rate of 3V/s. The conduction current through the connecting wires, and the displacement current through the plates of the capacitor would be, respectively

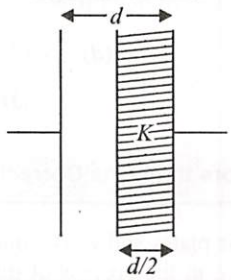
(a) zero, 60 μA
(b) 60 μA, 60 μA
(c) 60 μA, zero
(d) zero, zero **(NEET 2019)**

36. A parallel plate capacitor having cross-sectional area A and separation d has air inbetween the plates. Now an insulating slab of same area but thickness $d/2$ is inserted between the plates as shown in Fig. 2(CF).18 having dielectric constant K (= 4). The ratio of new capacitance to its original capacitance will be :

ANSWERS

29. (c) 30. (d) 31. (d) 32. (b) 33. (c) 34. (c) 35. (b)

FIGURE 2(CF).18



(*a*) 4 : 1 (*b*) 2 : 1
(*c*) 8 : 5 (*d*) 6 : 5 **(NEET 2020)**

37. A parallel plate capacitor has a uniform electric field '*E*' in the space between the plates. If the distance between the plates is '*d*' and the area of each plate is '*A*', the energy stored in the capacitor is (ε_0 = permittivity of free space)

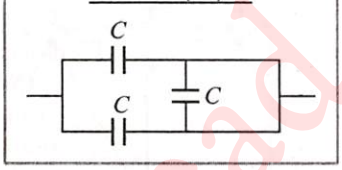
(*a*) $\frac{E^2 A d}{\varepsilon_0}$ (*b*) $\frac{1}{2}\varepsilon_0 E^2$

(*c*) $\varepsilon_0 E A d$ (*d*) $\frac{1}{2}\varepsilon_0 E^2 A d$

(NEET 2021)

38. The equivalent capacitance of the combination shown in Fig. 2(CF).19 is

FIGURE 2(CF).19



(*a*) $\frac{3C}{2}$ (*b*) 3 *C*

(*c*) 2 *C* (*d*) $\frac{C}{2}$ **(NEET 2021)**

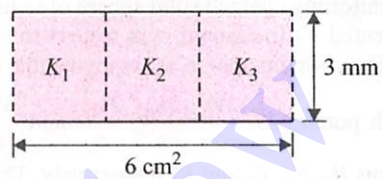
39. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be

(*a*) 3·2 μF (*b*) 8·4 μF
(*c*) 1·6 μF (*d*) 4·2 μF

(JEE Main 2020)

40. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material, which when fully inserted in above capacitor, gives same capacitance would be

FIGURE 2(CF).20



(*a*) 36 (*b*) 14
(*c*) 12 (*d*) 4 **(JEE Main 2020)**

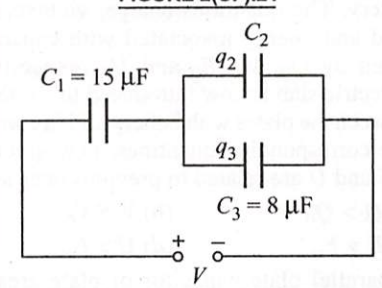
41. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply, and is connected in series to another uncharged 2·5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$, then value of *X* to the nearest integer is

(*a*) 3 (*b*) 5
(*c*) 4 (*d*) 6

(JEE Main 2020)

42. In the circuit shown in Fig. 2(CF).21, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is

FIGURE 2(CF).21



(*a*) 590 μC (*b*) 450 μC
(*c*) 650 μC (*d*) 160 μC

(JEE Main 2020)

ANSWERS

36. (*c*) **37.** (*d*) **38.** (*c*) **39.** (*c*) **40.** (*c*) **41.** (*c*) **42.** (*a*)

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43. Two capacitors of capacitances C and $2\ C$ are charged to potential differences V and $2\ V$ respectively. These are then connected in parallel in such a way that positive terminal of one is connected to negative terminal of the other. The final energy of this configuration is

(a) $\frac{9}{2}CV^2$ (b) $\frac{25}{6}CV^2$

(c) Zero (d) $\frac{3}{2}CV^2$

(JEE Main 2020)

II. Multiple Choice Questions (with One or More than One Correct Answers)

44. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(b) $R_1 \neq 0$ and $R_2 > (R_4 - R_3)$

(c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(d) $2\ R < R_4$ **(JEE Main 2015)**

45. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3\ x_0$, $x = 5\ x_0$ ..... ∞ on the X-axis and a charge $-q$ is fixed at each of the points $x = 2\ x_0$, $x = 4\ x_0$, $x = 6\ x_0$ .... to ∞. Here x_0 is a positive constant. Take the electric potential at a point due to charge Q at a distance r from it to be $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}$.

Then the potential at the origin due to above system of charges is :

(a) 0 (b) ∞

(c) $\frac{q}{4\pi\epsilon_0 x_0 \ln 2}$ (d) $\frac{q \ln 2}{4\pi\epsilon_0 x_0}$

46. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities, now given by Q, V, E and U are related to previous ones as :

(a) $Q > Q_0$ (b) $V > V_0$

(c) $E > E_0$ (d) $U > U_0$

47. A parallel plate capacitor of plate area A and separation d is charged to a potential difference V and then battery is disconnected. A slab of dielectric constant K is then inserted between the plates so as to fill space between the plates. If $Q_1 E_1 W$ denote charge on each plate, electric field between the plates and work done on the system respectively, in the process of inserting the slab then :

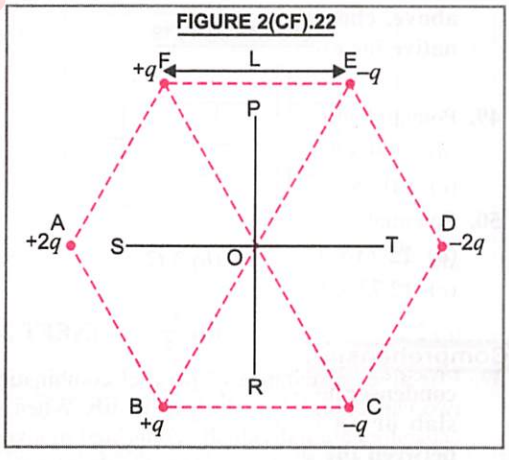
(a) $Q = \frac{\epsilon_0 AV}{d}$ (b) $Q = \frac{\epsilon_0 KAV}{d}$

(c) $E = \frac{V}{Kd}$

(d) $W = \frac{\epsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$

48. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in Fig. 2(CF).22. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct ?

FIGURE 2(CF).22



(a) The electric field at O is 6 K along OD

(b) The potential at O is zero

(c) The potential at all points on the line PR is same

(d) The potential at all points on the line ST is same **(IIT 2012)**

ANSWERS

43. (d) 44. (c,d) 45. (d) 46. (a,d) 47. (a,c,d) 48. (a,b,c)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1

We may define electrostatic potential at a point in an electrostatic field as the amount of work done in moving a unit positive test charge from infinity to that point against the electrostatic forces, along any path. Due to a single charge q, potential at a point distant r from the charge is $V = \frac{q}{4\pi\epsilon_0 r}$. The potential can be positive or negative. However, it is a scalar quantity. The total amount of work done in bringing various charges to their respective positions from infinitely large mutual separations gives us the electric potential energy of the system of charges. Whereas electric potential is measured in volt, electric potential energy is measured in joule. You are given a square of each side 1·0 metre with four charges $+1 \times 10^{-8}$ C, -2×10^{-8} C, $+3 \times 10^{-8}$ C and $+2 \times 10^{-8}$ C placed at the four corners of the square. With the help of the passage given above, choose the most appropriate alternative for each of the following questions :

49. Potential at the centre of the square is

(*a*) $5{\cdot}09 \times 10^2$ V (*b*) $5{\cdot}09 \times 10^3$ V

(*c*) 5·09 V (*d*) $8{\cdot}23 \times 10^2$ V

50. Potential energy of the system of four charges is

(*a*) $12{\cdot}73 \times 10^7$ J (*b*) $-6{\cdot}4 \times 10^{-7}$ J

(*c*) $12{\cdot}73 \times 10^{-9}$ J (*d*) $-12{\cdot}73 \times 10^{-9}$ J

Comprehension 2

The capacity of a condenser increases both, when a conducting slab or an insulating slab is introduced between the plates of the condenser. In the former case, electric field $E = 0$ inside the conductor and in the latter case, $E < E_0$, inside the insulator. Thus, potential difference $V = E \times d$ decreases and hence capacity $C = Q/V$ increases.

It should be clearly understood that when a dielectric slab is introduced inbetween the plates of a charged capacitor with battery connected across the plates,

(*i*) Capacity C increases, (*ii*) Potential V remains constant, (*iii*) Charge $Q = CV$, increases, (*iv*) Electric field E decreases,

(*v*) Energy stored $U = \frac{1}{2}CV^2$ increases.

However, when battery across the plates of charged capacitor is put off and dielectric slab is introduced inbetween the plates of the capacitor, (*i*) Capacity C increases, (*ii*) Charge Q remains constant,

(*iii*) Potential $V = \frac{Q}{C}$ decreases, (*iv*) Electric field, $E = V \times d$ decreases, (*v*) Energy stored $U = \frac{Q^2}{2C}$ decreases.

Consider a parallel plate air capacitor with area of each plate = 150 cm^2 and distance between its plates = 0·8 mm. With the help of the passage given above, choose the most appropriate alternative for each of the following questions :

51. Energy stored in the capacitor, when charged to a potential difference of 1200 V is

(*a*) $1{\cdot}2 \times 10^{-4}$ J (*b*) $1{\cdot}2 \times 10^4$ J

(*c*) $3{\cdot}6 \times 10^{-4}$ J (*d*) $3{\cdot}6 \times 10^4$ J

52. If the air capacitor is filled with a medium of $K = 3$ and then charged to the same potential, the energy stored will be

(*a*) $1{\cdot}2 \times 10^{-4}$ J (*b*) $3{\cdot}6 \times 10^{-4}$ J

(*c*) $3{\cdot}6 \times 10^4$ J (*d*) $1{\cdot}2 \times 10^4$ J

53. If the capacitor is charged first as an air capacitor and then filled with this dielectric, energy stored will be

(*a*) $3{\cdot}6 \times 10^{-4}$ J (*b*) $1{\cdot}2 \times 10^{-4}$

(*c*) 4×10^{-5} J (*d*) 4×10^5 J

ANSWERS

49. (*a*) **50.** (*b*) **51.** (*a*) **52.** (*b*) **53.** (*c*)

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II, and select the correct match out of the four given choices.

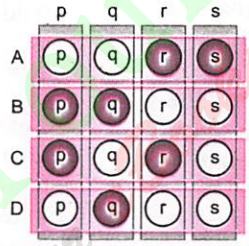
54.

	Column I		Column II
A.	Electric intensity (E) due to a single charge	(p)	$\propto 1/r$
B.	Electric intensity (E) due to an electric dipole	(q)	$\propto 1/r^4$
C.	Electric potential (V) due to a single charge	(r)	$\propto 1/r^3$
D.	Electric intensity (E) due to a quadrupole	(s)	$\propto \frac{1}{r^2}$, where r is the distance

(*a*) A-*s*, B-*r*, C-*q*, D-*p* (*b*) A-*p*, B-*q*, C-*r*, D-*s* (*c*) A-*r*, B-*s*, C-*p*, D-*q* (*d*) A-*s*, B-*r*, C-*p*, D-*q*

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following two questions contains statements given in two columns, which have to be matched. The answers to these questions have to be properly bubbled. If the correct matches are A-*r*, A-*s* ; B-*p*, B-*q* ; C-*p*, C-*r* and D-*q*, then the correctly bubbled matrix will look like the one shown here :



55. Match Column I and Column II :

	Column I		Column II
(A)	In series combination of capacitors	(p)	pot. diff. across each capacitor is same
(B)	In parallel combination of capacitors	(q)	charge on each capacitor is same
(C)	On inserting dielectric slab inbetween plates of a capacitor	(r)	capacity decreases
(D)	On replacing mica by air inbetween plates of a capacitor	(s)	capacity increases

56. Match Column I and Column II :

	Column I		Column II
(A)	same + dimension of mass	(p)	Electric charge
(B)	negative dimension of mass	(q)	Electric potential
(C)	zero dimension of mass	(r)	Electrical capacity
(D)	dimension of length is ± 2	(s)	energy density

ANSWERS

54. (*d*) **55.** A-*q*, A-*r* ; B-*p*, B-*s*, C-*s*, D-*r* **56.** A-*q*, A-*s* ; B-*r*, C-*p*, D-*q*, D-*r*

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

57. Eight drops of water, each having a charge of 3×10^{-9} C having surface potential 2 V coalesce to form a single drop. What is the surface potential (in volt) of new drop ?

58. An electric field is described as $\vec{E} = \hat{i}\,x + \hat{k}\,z$.

The potential difference V_{AB} (in volt) between A (0, 0, 0) and B (2, 2, 0) is :

59. Two balls with charges 5 μ C and 10 m C are at a distance of 90 m from each other. In order to reduce the distance between them to 45 m, the amount of work to be performed in joule is :

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

A. both, Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. both, Assertion and Reason are true, but Reason is not the correct explanation of the Assertion.

C. Assertion is true, but the Reason is false.

D. both, Assertion and Reason are false.

60. Assertion. A spherical equipotential surface is not possible for a point charge.

Reason. A spherical equipotential surface is possible inside a spherical capacitor.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

61. Assertion. Electrons move away from a region of lower potential to a region of higher potential.

Reason. Since an electron has a negative charge

(*a*) A (*b*) B (*c*) C (*d*) D

62. Assertion. Work done in moving any charge through any distance on an equipotential surface is zero.

Reason. An equipotential surface is very smooth.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

A. Statement-1 is true ; Statement-2 is true ; Statement-2 is correct explanation of Statement-1.

B. Statement-1 is true ; Statement-2 is true ; Statement-2 is not a correct explanation of Statement-1.

C. Statement-1 is correct and Statement-2 is false.

D. Statement-1 is false and Statement-2 is true.

63. Statement-1. In a series combination of capacitors, charge on each capacitor is same.

Statement-2. In such a combination, charge cannot move only along one route.

(*a*) A (*b*) B (*c*) C (*d*) D

64. Statement-1. For practical purposes, the Earth is used as a reference at zero potential in electrical circuits.

Statement-2. The electrical potential of a sphere of radius. R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\epsilon_0 R}$

(*a*) A (*b*) B (*c*) C (*d*) D

(IIT 2008)

ANSWERS

57. 8 58. 2 59. 5 60. (*d*) 61. (*a*) 62. (*c*) 63. (*a*) 64. (*a*)

HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Potential due to charge on the shell, $V_1 = \dfrac{Kq}{R}$ and potential due to charge at the centre

$$V_2 = \frac{KQ}{R/2} = \frac{K2Q}{R}$$

Net potential, $V = V_1 + V_2$

$$= \frac{Kq}{R} + \frac{K2Q}{R}$$

$$= \frac{q}{4\pi\epsilon_0 R} + \frac{2Q}{4\pi\epsilon_0 R}$$

2. $dV = -\vec{E}.\vec{dx} = -30x^2\,\hat{i}.dx\,\hat{i} = -30x^2\,dx$

$$\int_{V_0}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_0 = -\left[10x^3\right]_0^2 = \mathbf{-80\ V}$$

3. Here, $V(x) = \dfrac{20}{x^2-4} = 20(x^2-4)^{-1}$.

$$E = -\frac{dV}{dx} = -20(-1)(x^2-4)^{-2}(2x)$$

$$= \frac{40x}{(x^2-4)^2}$$

At $x = 4\ \mu$m, $E = \dfrac{40\times4}{(4^2-4)^2} = \dfrac{160}{144} = \mathbf{\dfrac{10}{9}\ V/\mu m}$

$\vec{E}$ is along the **positive x-direction.**

4. Here, $\vec{P} = \dfrac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$

As the circle is equipotential, electric field at B along tangent should be zero, Fig. 2(CF).23.

i.e., $\vec{E}_B = 0$

and $\vec{E}_0 = \dfrac{K|\vec{P}_0|}{R^3}$

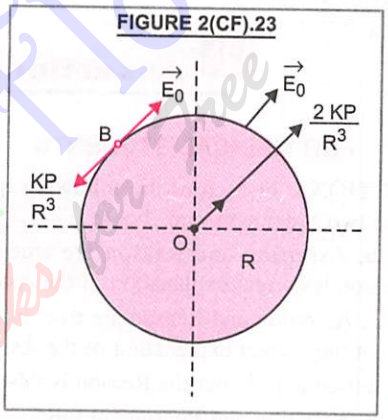
or $R^3 = \dfrac{K|\vec{P}_0|}{E_0} = \dfrac{KP_0}{E_0} = \dfrac{P_0}{4\pi\epsilon_0 E_0}$

$$\therefore \quad R = \left[\frac{P_0}{4\pi\epsilon_0 E_0}\right]^{1/3}$$

Choices (a) and (d) are correct.

As E_0 is uniform and electric field due to dipole is different at different points, so magnitude of total electric field will also be different at different points.

FIGURE 2(CF).23



Therefore, choice (b) is incorrect.

Further, as $E_A = \dfrac{2KP}{R^3} + \dfrac{KP}{R^3} = \dfrac{3KP}{R^3}$

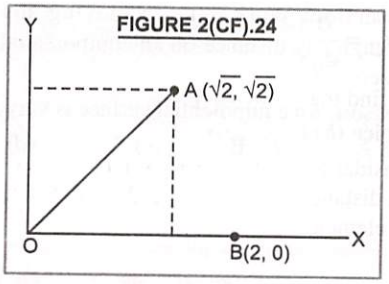
$$= \frac{3KP}{R^3}\frac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$$

$\therefore$ Choice (c) is also wrong.

Hence, the correct choices are (a) and (d).

5. Here, $q = 10^{-3}\ \mu C$ is at O. $A(\sqrt{2},\sqrt{2})$, and B (2, 0) are as shown in Fig. 2(CF).24.

FIGURE 2(CF).24



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$$r_1 = OA = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$$

$$r_2 = OB = 2$$

$$V_A - V_B = \frac{q}{4\pi\epsilon_0 r_1} - \frac{q}{4\pi\epsilon_0 r_2} = \mathbf{0}$$

$(\because\ r_1 = r_2 = 2)$

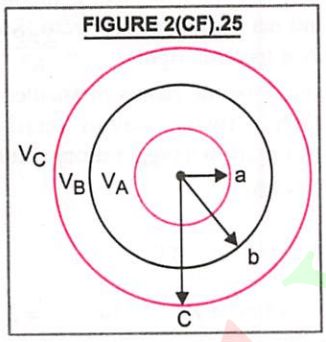
6. As is clear from Fig. 2(CF).25

$$V_A = \frac{\sigma a}{\epsilon_0} - \frac{\sigma b}{\epsilon_0} + \frac{\sigma c}{\epsilon_0} = \frac{\sigma}{\epsilon_0}(a - b + c)$$

As $c = a + b$

$$\therefore\ V_A = \frac{\sigma}{\epsilon_0}(2a)$$

FIGURE 2(CF).25



$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right] = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + a + b\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} + a\right] = \frac{\sigma a}{\epsilon_0}\frac{(a+b)}{b} = \frac{\sigma_0 ac}{b}$$

$$V_C = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{c} - \frac{b^2}{c} + c\right] = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2 + c^2}{c}\right]$$

$$= \frac{\sigma}{\epsilon_0}\left[\frac{(a+b)(a-b) + (a+b)^2}{c}\right]$$

$$V_C = \frac{\sigma}{\epsilon_0}\frac{(a+b)}{c}(a - b + a + b) = \frac{\sigma 2a}{\epsilon_0}$$

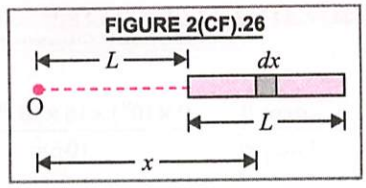
We find that $V_A = V_C \neq V_B$

Choice (*b*) is correct.

7. Consider a small element of length dx of the rod at a distance x from O, Fig. 2(CF).26. Charge on the element

$$dq = \frac{Q}{L}\,dx$$

FIGURE 2(CF).26



Potential at O due to this element $= \dfrac{k\,dq}{x}$

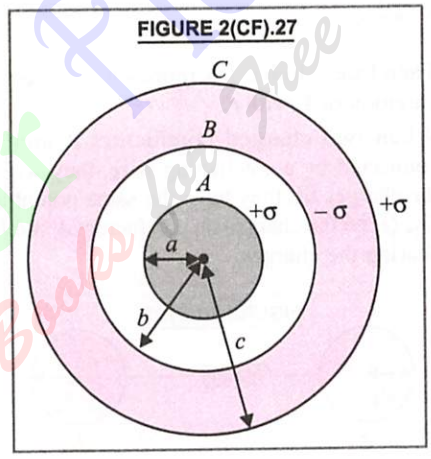
Potential at O due to the rod,

$$V = \int_L^{2L} \frac{k\,dq}{x} = \frac{kQ}{L}\int_L^{2L}\frac{dx}{x} = k\,\frac{Q}{L}\,[\log x]_L^{2L}$$

$$= \frac{\frac{Q}{L}\ln 2}{4\pi\epsilon_0} = \frac{Q \ln 2}{4\pi\epsilon_0 L}$$

8. (*b*) : Refer to Fig. 2(CF).27

FIGURE 2(CF).27



Charge on sphere $A = \sigma\, 4\,\pi\, a^2$

Charge on sphere $B = -\,\sigma\, 4\,\pi\, b^2$

Charge on sphere $C = \sigma\, 4\,\pi\, c^2$

The potential of shell B due to charges on all the shells is

$$V_B = \frac{\sigma 4\pi a^2}{4\pi\epsilon_0 b} + \frac{(-\sigma 4\pi b^2)}{4\pi\epsilon_0 b} + \frac{\sigma 4\pi c^2}{4\pi\epsilon_0 c}\Bigg]$$

$$= \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

9. Here, volume, $v = 0{\cdot}2\ \text{m}^3$

Electric potential, $V = 5$ volt (constant)

$$dV = 0$$

As $E = -\dfrac{dV}{dr}$ $\quad\therefore\quad$ $E =$ **zero**

10. Here, $p = 16 \times 10^{-9}$ Cm, $V = ?$

$r = 0{\cdot}6$ m, $\theta = 60°$

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$$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} = \frac{(9\times10^9)\times16\times10^{-9}\cos 60°}{(0{\cdot}6)^2}$$

$$V = \frac{9\times16\times1/2}{0{\cdot}6\times0{\cdot}6} = \mathbf{200\ volt.}$$

11. In case of a thin shell of radius R, which is positively charged, the charge can be assumed to be at the centre of the shell. Inside the shell, $r \le R$, the potential is constant,

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{q}{4\pi\epsilon_0 R}.$$

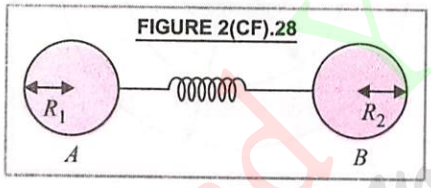
Outside the shell, at any distance r from the centre,

$$V \propto \frac{1}{r}.$$

Therefore, **choice (*c*)** represents correctly the variation of V with r.

12. When two charged conductors A and B are connected by a conducting wire, they will share the charges till they have the same potential. Let Q_1, Q_2 be the charges on conductors A and B after sharing the charges.

FIGURE 2(CF).28



As, $V_A = V_B$ so, $\frac{1}{4\pi\epsilon_0}\frac{Q_1}{R_1} = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{R_2}$

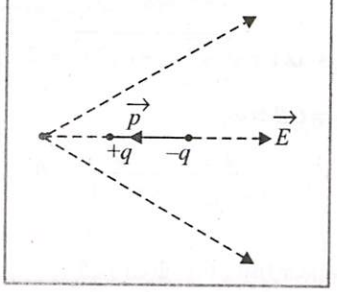
or $\frac{Q_1}{4\pi R_1^2}\frac{R_1}{\epsilon_0} = \frac{Q_2}{4\pi R_2^2}\frac{R_2}{\epsilon_0}$

or $\frac{\sigma_1 R_1}{\epsilon_0} = \frac{\sigma_2 R_2}{\epsilon_0}$

or $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

13. Refer to Fig. 2(CF).29, the direction of electric dipole moment $\vec{p}$ is from –ve charge to +ve charge. Angle between electric field $\vec{E}$ and electric dipole moment $\vec{p}$ is 180°.

FIGURE 2(CF).29



Potential energy of electric dipole in external electric field is

$$U = -\vec{p}.\vec{E} = -pE\cos180° = pE$$

On moving towards right, the strength of electric field decreases, therefore potential energy decreases. Net force on electric dipole is towards right and net torque on it is zero. So the dipole will move towards right.

14. Let r and R be the radius of smaller and bigger drop. When twenty seven small drops are combined to form a bigger drop, volume remains the same, *i.e.*,

$$\left(\frac{4}{3}\pi r^3\right)\times27 = \frac{4}{3}\pi R^3$$

or $(3r)^3 = R^3$ or $R = 3r$

Let q be the charge on each small drop and Q be the charge on bigger drop formed. According to law of conservation of charge,

$$Q = 27q.$$

Electric potential due to small charged drop,

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r} = 220\text{ V (given)}$$

Electric potential due to bigger charged drop is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{1}{4\pi\epsilon_0}\frac{27q}{3r} = 9\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) = 9\times220$$

$$= \mathbf{1980\ V}$$

15. Here $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$, $C_P = 2 + 1 = 3\ \mu F$
This combination is in series with C

$$\therefore\ C_{eq} = \frac{3C}{3+C}$$

Total charge, $q = C_{eq}E = \left(\frac{3C}{3+C}\right)E$

HINTS/EXPLANATIONS
For Difficult Questions

Charge on 2 μF capacitor

$$q_2 = \frac{2}{3} \times \frac{3CE}{3+C} = \frac{2CE}{3+C} = \frac{2E}{1+\frac{3}{C}} = Q, \text{ say}$$

As $\frac{dQ}{dC} > 0 \,.\, \frac{d^2Q}{dC^2} < 0$,

therefore Q_2 versus C graph is shown correctly in **Fig. 2(CF).6(b).**

16. Capacity of each condenser $C = 1$ μF.

Potential difference it can withstand = 300 V

Required capacitance = 2 μF across a potential difference of 1·0 kV *i.e.*, 1000 V

Minimum no. of capacitors in series.

$$x = \frac{1000}{300} = 3{\cdot}3 \implies 4$$

Capacitance of each row, $\frac{1}{C_s} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$

$$C_s = \frac{1}{4}\,\mu\text{F}$$

If m such rows of capacitors are required, then

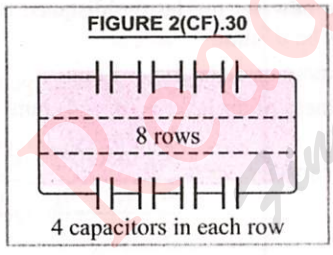
$$C_{eq} = 2 = m\,C_s = m \times \frac{1}{4} \text{ or } m = 8$$

Hence minimum number of capacitors

$$= 8 \times 4 = \mathbf{32}$$

The arrangement is shown in Fig. 2(CF).30 here.

FIGURE 2(CF).30



17. When capacitor is charged, both parts of capacitor have common potential difference V. So

$$E_1 = \frac{V}{d} = E_2 \quad \text{or} \quad \frac{E_1}{E_2} = \mathbf{1}$$

$$C_1 = \frac{K\epsilon_0 A/3}{d} \;;\; C_2 = \frac{\epsilon_0 2A/3}{d}$$

$$C = C_1 + C_2 = \frac{K\epsilon_0 A}{3d} + \frac{2\epsilon_0 A}{3d} = \frac{(K+2)\epsilon_0 A}{3d}$$

$$\frac{C}{C_1} = \frac{K+2}{K}$$

$$Q_1 = C_1 V = \frac{K\epsilon_0 A}{3d} \;;\; Q_2 = C_2 V = \frac{2\epsilon_0 A}{3d} V$$

$$\frac{Q_1}{Q_2} = \frac{K}{2}$$

18. Electric field inside the parallel plate capacitor having charge Q at place in the absence of dielectric between the plates is $E = \frac{Q}{A\epsilon_0}$

When dielectric is filled, then $E' = \frac{Q}{KA\epsilon_0}$

As $K_2 > K_1$, therefore, **option (*c*) is correct.**

19. Initial energy $U_i = \frac{Q^2}{2KC}$

When the slab is removed, $U = \frac{Q^2}{2C}$.

When dielectric slab is introduced, energy becomes $U_f = \frac{Q^2}{2KC} = U_i$.

$\therefore$ Net work done by the system $= U_f - U_i =$ **zero**

20. $E = \frac{\sigma}{K\epsilon_0}$ or $\sigma = K\epsilon_0 E$

$$= 2{\cdot}2 \times (8{\cdot}85 \times 10^{-12}) \times (3 \times 10^4)$$
$$\approx \mathbf{6 \times 10^{-7}\ C/m^2}$$

21. Let q_1, q_2, q_3 be the charges on C_1, C_2, C_3 respectively. From Fig. 2(CF).10,

$$V_A - V_0 = \frac{q_1}{C_1} \quad \text{or} \quad q_1 = (V_A - V_0)\,C_1$$

$$V_B - V_0 = \frac{q_2}{C_2} \quad \text{or} \quad q_2 = (V_B - V_0)\,C_2$$

$$V_D - V_0 = \frac{q_3}{C_3} \quad \text{or} \quad q_3 = (V_D - V_0)\,C_3$$

As $q_1 = q_2 + q_3$

$\therefore\ (V_A - V_0)\,C_1 = (V_B - V_0)\,C_2 + (V_D - V_0)\,C_3$

which gives, $\mathbf{V_0 = \frac{V_A C_1 + V_B C_2 + V_D C_3}{C_1 + C_2 + C_3}}$

22. If C is capacity and V is potential of battery, then initial energy stored

$$E_1 = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

When uncharged capacitor of capacity C is connected in parallel,

$$C_p = C + C = 2\,C$$

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Charge on each capacitor = $Q/2$

Total energy $E_2 = \frac{(Q/2)^2}{2C} + \frac{(Q/2)^2}{2C}$

$$= \frac{2Q^2}{4(2C)} = \frac{1}{2}\left(\frac{Q^2}{2C}\right)$$

$$E_2 = E_1/2$$

i.e., **total electrostatic energy of resulting system decreases by a factor of 2.**

23. $C_1 = \frac{\epsilon_0 A}{d}$, $C_2 = \frac{\epsilon_0 A}{2d}$, $C_3 = \frac{\epsilon_0 A}{3d}$

As the three condensers are in parallel, Fig. 2(CF).11.

$$\therefore \quad C_p = C_1 + C_2 + C_3 = \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{2d} + \frac{\epsilon_0 A}{3d}$$

$$C_p = \frac{\epsilon_0 A}{d}\left[1 + \frac{1}{2} + \frac{1}{3}\right] = \frac{11\epsilon_0 A}{6d}$$

Choice (*d*) is correct.

24. In Fig. 2(CF).12, 3 μF and 9 μF capacitors are in parallel.

$\therefore \quad C_p = 3 + 9 = 12\ \mu F$

This combination is in series with 4 μF capacitor

$$\frac{1}{C_s} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}, \quad C_s = 3\ \mu F$$

Charge on 4 μF and combination 3 μF is

$$Q = CV = 3 \times 8 = 24\ \mu C$$

This charge distributes itself in the direct ratio of capacitors 3 μF and 9 μF.

$$\therefore \quad Q_{9\mu F} = \frac{(24)\times 9}{3+9} = 18\mu C$$

Now $Q_{9\mu F} + Q_{4\mu F} = 18 + 24 = 42\ \mu C$

Electric field intensity due to $Q = 42\ \mu C$ at a distance of 30 m from it

$$E = \frac{kQ}{r^2} = \frac{9\times 10^9 \times 42\times 10^{-6}}{30\times 30} = \mathbf{420\ N/C}$$

25. $C_0 = \frac{\epsilon_0 A}{d}$

When one fourth of capacitor is filled with dielectric of constant K, Fig. 2(CF).13, then

$$C_1 = \frac{K\epsilon_0 A/4}{d} \quad \text{and} \quad C_2 = \frac{\epsilon_0 3A/4}{d},$$

which are in parallel.

$$\therefore \quad C_p = C_1 + C_2 = \frac{K\epsilon_0 A}{4d} + \frac{3\epsilon_0 A}{4d}$$

$$= (K+3)\frac{\epsilon_0 A}{4d} = (K+3)\frac{C}{4}$$

26. In Fig. 2(CF).14, let $q\ \mu C$ be charge on C_1 and C_2.

$$\therefore \quad \frac{q}{C_1} + \frac{q}{C_2} = 30 \quad \text{or} \quad \frac{q}{1} + \frac{q}{1{\cdot}5} = 30$$

$$q = 18\ \mu C$$

$$V_A - V_a = \frac{q}{C_1} = \frac{18}{1} = 18\,V$$

Similarly, $\frac{q'}{C_3} + \frac{q'}{C_4} = 30$ or $\frac{q'}{2{\cdot}5} + \frac{q'}{0{\cdot}5} = 30$

$$q' = \frac{25}{2}\mu C$$

$$V_A - V_b = \frac{q'}{C_3} = \frac{25}{2\times 2{\cdot}5} = 5$$

$$V_a - V_b = (V_A - V_b) - (V_A - V_a) = 5 - 18$$

$$= \mathbf{-13\ volt}$$

27. With air as dielectric,

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{8\times 10^{-3}} = \frac{\epsilon_0 A\times 10^3}{8} \quad ...(i)$$

In the new set up, let the distance between the plates be d' mm.

Thickness of dielectric = 4 mm

Thickness of air layer = $(d' - 4)$ mm

New capacity, $C' = \frac{\epsilon_0 A\times 10^3}{\frac{4}{2} + \frac{(d'-4)}{1}} = \frac{\epsilon_0 A\times 10^3}{(d'-2)}$

As $C' = C \quad \therefore \quad \frac{\epsilon_0 A\times 10^3}{(d'-2)} = \frac{\epsilon_0 A\times 10^3}{8}$

$$d' - 2 = 8$$

$$d' = 8 + 2 = \mathbf{10\ mm}$$

28. As is clear from Fig. 2(CF).15, three rows of capacity C_1; $\frac{C_1}{2}$ and C_1 are connected in parallel between the points A and B.

Total capacity, $C = C_1 + \frac{C_1}{2} + C_1 = \frac{5}{2}C_1$

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As $q = CV \therefore 1{\cdot}5\times10^{-6} = \frac{5}{2}C_1\times6$

$$C_1 = \frac{3}{30}\times10^{-6}\text{ F} = \mathbf{0{\cdot}1\,\mu F}$$

29. Here, $C = \frac{\epsilon_0 A}{d} = 9\text{pF}$

$$C_1 = \frac{\epsilon_0 K_1 A}{d/3} = \frac{3K_1\epsilon_0 A}{d} = 3\times3\times9 = 81\text{ pF}$$

$$C_2 = \frac{\epsilon_0 K_2 A}{2d/3} = \frac{3K_2}{2}\frac{\epsilon_0 A}{d} = \frac{3\times6}{2}\times9 = 81\text{pF}$$

As C_1, C_2 are connected in series,

$$\therefore \frac{1}{C_s} = \frac{1}{C_1}+\frac{1}{C_2} = \frac{1}{81}+\frac{1}{81} = \frac{2}{81}$$

$$C_s = \frac{81}{2} = \mathbf{40{\cdot}5\,pF}$$

30. Here, $q_0 = CV_1$ and $q = CV_2$

When a charged capacitor is connected to ideal inductor, the discharge of capacitor is oscillatory. The charge on capacitor at an instant t is given by, $q = q_0 \sin\omega t$ where, $\omega = \frac{1}{\sqrt{LC}}$.

Therefore, $\sin\omega t = \frac{q}{q_0} = \frac{CV_2}{CV_1} = \frac{V_2}{V_1}$

Current through inductor is

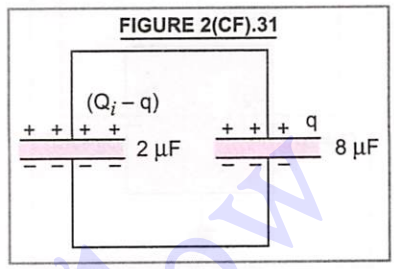
$$I = \frac{dq}{dt} = \frac{d}{dt}(q_0\sin\omega t)$$
$$= q_0\,\omega\cos\omega t$$
$$= q_0\,\omega\,[1-\sin^2\omega t]^{1/2}$$
$$= CV_1\times\frac{1}{\sqrt{LC}}\left[1-\left(\frac{V_2}{V_1}\right)^2\right]^{1/2}$$
$$= \left[\frac{C(V_1^2-V_2^2)}{L}\right]^{1/2}$$

31. Initially, the energy stored in 2 μF capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(2\times10^{-6})V^2 = V^2\times10^{-6}\text{ J}$$

Initially, the charge stored in 2 μF capacitor is $Q_i = CV = (2\times10^{-6})V = 2V\times10^{-6}$ coulomb. When switch S is turned to position 2, the charge flows and both the capacitors share charges till a common potential V_C is reached, Fig. 2(CF).31. Therefore,

FIGURE 2(CF).31



$$V_C = \frac{\text{total charge}}{\text{total capacitance}} = \frac{2V\times10^{-6}}{(2+8)\times10^{-6}} = \frac{V}{5}\text{ volt.}$$

Finally, the energy stored in both the capacitors

$$U_f = \frac{1}{2}[(2+8)\times10^{-6})\left(\frac{V}{5}\right)^2 = \frac{V^2}{5}\times10^{-6}\text{J}$$

$$\%\text{ loss of energy, } \Delta U = \frac{U_i-U_f}{U_i}\times100$$
$$= \frac{(V^2-V^2/5)\times10^{-6}}{V^2\times10^{-6}}\times100 = \mathbf{80\%}$$

32. As, $V = V_0(1-e^{-t/RC}) \therefore 120 = 200(1-e^{-5/RC})$

or $\frac{120}{200} = 1-e^{-5/RC}$

or $e^{-5/RC} = 1-\frac{120}{200} = \frac{2}{5}$ or $e^{5/RC} = \frac{5}{2} = 2{\cdot}5$

Taking log of both the sides, we get

$$\frac{5}{RC}\log_e e = \log_e 2{\cdot}5$$

$$\frac{5}{RC}\times1 = 2{\cdot}3026\log_{10}2{\cdot}5 = 2{\cdot}3026\times0{\cdot}4$$

$$R = \frac{5}{2{\cdot}3026\times0{\cdot}4\times C} = \frac{5}{2{\cdot}3026\times0{\cdot}4\times(2\times10^{-6})}$$
$$= \mathbf{2{\cdot}7\times10^6\,\Omega}$$

33. In Fig. 2(CF).32, if Q_2, Q_3 are the charges on C_2, C_3 respectively, then for the isolated system marked, with dotted boundary.

$$Q_2 + Q_3 - 80 = 0$$

or $Q_2 + Q_3 = 80$ ...(*i*)

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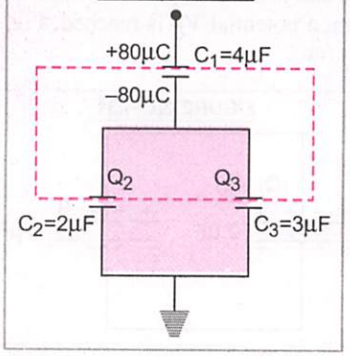
FIGURE 2(CF).32



As C_2 and C_3 are in parallel, therefore, potential difference across C_2 and C_3 is same, *i.e.*,

$$\frac{Q_2}{C_2} = \frac{Q_3}{C_3} \quad \text{or} \quad \frac{Q_2}{Q_3} = \frac{C_2}{C_3} = \frac{2}{3}$$

Put in (*i*), $\frac{2}{3}Q_3 + Q_3 = 80$

$$Q_3 = \frac{80 \times 3}{5} = \mathbf{48\ \mu\ C}$$

34. For an isolated capacitor, charge Q is constant. The electrostatic force between the metal plates of || plate capacitor is

$$F_{\text{plates}} = \frac{Q^2}{2\epsilon_0 A}$$

Clearly, F is **independent of the distance between the plates.**

35. Here, $C = 20\ \mu F = 20 \times 10^{-6}\ F$

Rate of change of potential, $\frac{dV}{dt} = 3$ V/s.

From $q = CV$

$$\frac{dq}{dt} = C \cdot \frac{dV}{dt}$$

$$I_C = 20 \times 10^{-6} \times 3 = 60 \times 10^{-6}\ A = \mathbf{60\ \mu A}$$

As is known, $I_C = I_d = \mathbf{60\ \mu A}$.

36. Original capacity,

$$C = \frac{\epsilon_0 A}{d} \quad ...(1)$$

When insulating slab of same area but thickness $d/2$ is introduced between the plates as shown in Fig. 2(CF).33, the new capacitance would be

$$C' = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

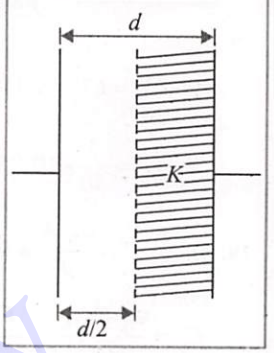
$$C' = \frac{\epsilon_0 A}{d - d/2\left(1 - \frac{1}{4}\right)}$$

$$= \frac{\epsilon_0 A}{d/2\left(2 - 1 + \frac{1}{4}\right)} = \frac{\epsilon_0 A}{\frac{5}{8}d}$$

$$\frac{C'}{C} = \frac{\epsilon_0 A}{\frac{5}{8}d} \times \frac{d}{\epsilon_0 A} = \frac{8}{5}$$

$$\therefore \frac{C'}{C} = \mathbf{8:5}$$

FIGURE 2(CF).33



37. Volume of the space between the two plates of parallel plate capacitor, $V = A \times d$.

Energy density associated with electric field E in between the two plates of capacitor is

$$u = \frac{1}{2}\epsilon_0 E^2$$

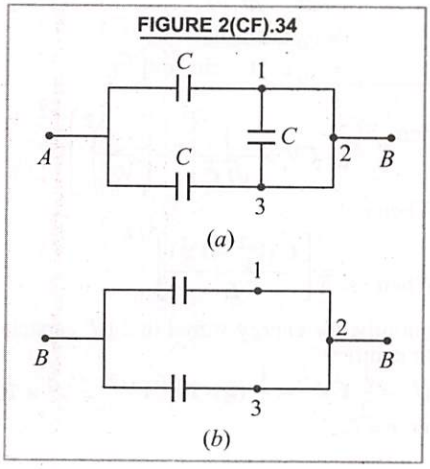
Total energy stored in the space between the plates of capacitor will be

$$U = u \times V = \frac{1}{2}\epsilon_0 E^2 \times (A \times d) = \mathbf{\frac{1}{2}\epsilon_0 E^2 Ad}$$

38. Refer to Fig. 2(CF).34(*a*) the points 1, 2 and 3 are at the same potential (since they are connected by the conducting wire), so the capacitor between points 1 and 3 is short circuited. Hence it will not store any charge, *i.e.*, it becomes ineffective. The effective circuit will be as shown in Fig. 2(CF).34(*b*). The equivalent capacitance of the parallel combination of two capacitors is,

$$C_{AB} = C + C = 2\ \mathbf{C}$$

FIGURE 2(CF).34



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39. Here, $C_1 + C_2 = 10\ \mu F$ ...(*i*)

Again $E_2 = 4\,E_1$

$$\frac{1}{2}C_2V^2 = 4\times\frac{1}{2}C_1V^2$$

$$C_2 = 4\,C_1$$

From (*i*),

$C_1 + 4\,C_1 = 10,\ C_1 = 2\ \mu F + C_2 = 8\ \mu F$

For series combination of these capacitors,

$$C_s = \frac{C_1C_2}{C_1+C_2} = \frac{2\times8}{2+8} = \mathbf{1{\cdot}6\ \mu F}$$

40. The three capacitors so formed are connected in parallel. Therefore, total capacity of the combination,

$$C = C_1 + C_2 + C_3$$

$$\frac{\epsilon_0 KA}{d} = \frac{\epsilon_0 K_1A}{3d} + \frac{\epsilon_0 K_2A}{3d} + \frac{\epsilon_0 K_3A}{3d}$$

$$K = \frac{K_1+K_2+K_3}{3} = \frac{10+12+14}{3} = \mathbf{12\#}$$

41. Here, $C_1 = 5\ \mu F = 5\times10^{-6}$ F, $V_1 = 220$ V

Initial energy $u_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times(5\times10^{-6})(220)^2$

$= \mathbf{0{\cdot}121\ J}$

$C_2 = 2{\cdot}5\ \mu F = 2{\cdot}5\times10^{-6}$ F, $V_2 = 0$

Final common potential

$$V = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$$

$$V = \frac{5\times220 + 2{\cdot}5\times0}{5+2{\cdot}5} = 220\times\frac{2}{3}\text{ volt}$$

Final value of energy,

$$u_2 = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}(5+2{\cdot}5)\times10^{-6}\left(220\times\frac{2}{3}\right)^2$$

$u_2 = 0{\cdot}0806$

$$\Delta u = u_2 - u_1 = 0{\cdot}0806 - 0{\cdot}1210 = -0{\cdot}0404 = \frac{X}{100}$$

$\therefore\ X = 0{\cdot}0404\times100 = 4{\cdot}04$

Hence, to the nearest integer, $X = \mathbf{4}$

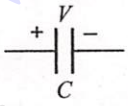
42. Here, total charge $Q = 750\ \mu C$

As voltage across C_3 = voltage across $C_2 = 20$ V

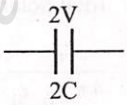
$\therefore$ Charge on $C_3 = q_3 = C_3\times V = 8\times20 = 160\ \mu C$

Hence, charge on $C_2 = (750 - 160)\ \mu C = \mathbf{590\ \mu C}$

43. Here $Q_1 = CV$

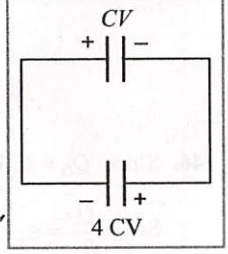


$$Q_2 = 2\,C\times2\,V = 4\ CV$$



The two condensers are connected in parallel as shown in Fig. 2(CF).35.

FIGURE 2(CF).35



If V' is common potential, then by charge conservation, we get

$$q_i = q_f$$

$$Q_2 + Q_1 = q_f$$

$$4\,CV - CV = (C + 2\,C)\,V'$$

$$\therefore\quad V' = V$$

Final energy of this configuration $= \frac{1}{2}(3C)V'^2$

$$= \frac{3}{2}CV^2$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

44. Here, $V_0 = \frac{kQ}{R}$ where $k = \frac{1}{4\pi\epsilon_0}$

When $r > R$, $V_{(r>R)} = \frac{kQ}{r}$

When $r < R$, $V_{(r<R)} = \frac{kQ}{2R^3}(3R^2 - r^2)$

At centre, $r = 0$, $V_{\text{centre}} = \frac{3}{2}\frac{kQ}{r} = \frac{3}{2}V_0$

When $r = R_2$, $V_{R_2} = \frac{5V_0}{4} = \frac{5}{4}\frac{kQ}{R} = \frac{kQ}{2R^3}(3R^2 - R_2)$

On solving, $R_2 = \frac{R}{\sqrt{2}}$

When $r = R_3$, $V_{R_3} = \frac{3V_0}{4} = \frac{3}{4}\frac{kQ}{R} = \frac{kQ}{R_3}$ or $R_3 = \frac{4}{3}R$

When $r = R_4$, $V_{R_4} = \frac{V_0}{4} = \frac{1}{4}\frac{kQ}{R} = \frac{kQ}{R_4}$

or $R_4 = 4\,R$, *i.e.*, $\boldsymbol{R_4 > 2\,R}$

$$R_4 - R_3 = 4R - \frac{4}{3}R = \frac{\mathbf{8R}}{\mathbf{3}} > \mathbf{R_2}$$

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45. Potential due to positive charges

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{x_0} + \frac{1}{4\pi\epsilon_0}\frac{q}{3x_0} + \ldots\ldots$$

$$= \frac{q}{4\pi\epsilon_0 x_0}\left[1 + \frac{1}{3} + \ldots\ldots\right]$$

potential due to negative charge

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{-q}{2x_0} + \frac{1}{4\pi\epsilon_0}\frac{-q}{4x_0} + \ldots\ldots$$

$$= \frac{+q}{4\pi\epsilon_0 x_0}\left[-\frac{1}{2} - \frac{1}{4} - \frac{1}{6} + \ldots\ldots\right]$$

Total potential $V = V_1 + V_2$

$$\frac{q}{4\pi\epsilon_0 x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\right]$$

$$\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots = \ln 2\right]$$

$$V = \frac{q \ln 2}{4\pi\epsilon_0 x_0}$$

46. Since $Q_0 = C_0 V_0$ and $Q = CV_0 = \epsilon_r C_0 V_0$

So $\frac{Q}{Q_0} = \epsilon_r$, *i.e.*, $Q > Q_0$

Now $U_0 = \frac{1}{2} C_0 V_0^2$,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \epsilon_r C_0 V^2$$

$$\frac{U}{U_0} = \epsilon_r, \text{ i.e., } U > U_0$$

V and E remain same.

47. As battery is disconnected, charge remains same

$$Q = \sigma A = \epsilon_0 E_0 A = E_0 \frac{V}{d} A = \frac{\epsilon_0 AV}{d}$$

As $E = \frac{E_0}{K}, \frac{V/d}{K} = \frac{V}{Kd}$

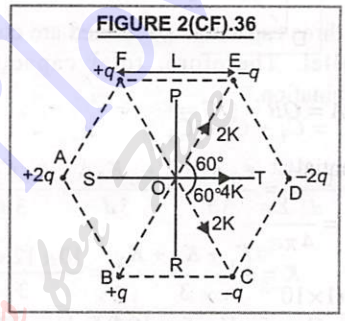
Work done $W = \Delta U = \frac{1}{2} CV^2 - \frac{1}{2} C'V'^2$

$$= \frac{1}{2}\frac{\epsilon_0 AV^2}{d} - \frac{1}{2}\frac{K\epsilon_0 A}{d}\left(\frac{V}{K}\right)^2$$

$$= \frac{\epsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$$

48. In Fig. 2(CF).36, L is the length of each side of a regular hexagon $ABCDEF$, at the vertices of which six point charges are held.

FIGURE 2(CF).36



Imagine a unit positive charge at O.

Electric intensity at O due to + 2 q charge at A

$$= \frac{2q \times 1}{4\pi\epsilon_0 L^2} = 2\,\text{K}, \text{ along } OD$$

Electric intensity at O due to – 2 q charge at D = 2 K, along OD.

Electric intensity at O due to + q charge at B and – q charge at E = 2 K, along OE

Electric intensity at O due to – q charge at C and + q charge at F = 2 K along OC

As $\angle COE = 120°$, therefore, resultant of 2 K along OC and 2 K along OE is 2 K along OD.

$\therefore$ Net electric intensity at O = 4 K + 2 K = **6 K** along OD

Potential at O, $V = -\frac{dE}{dr} = -\frac{d}{dr}(6K)$ = **Zero**

A closer look at figure shows that PR is equatorial line of three electric dipoles (– q, + q) at E, F ; (– 2 q, + 2 q) at D, A and (– q, + q) at C, B.

Hence potential at all points on ***PR* is zero (*i.e.*, same).**

The **choices (*a*), (*b*), (*c*) are correct.**

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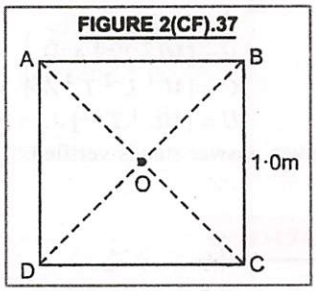
III. Multiple Choice Questions (Based on the given Passage/Comprehension)

49. As is clear from Fig. 2(CF).37

FIGURE 2(CF).37



$$OA = OB = OC = OD = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}\text{m}$$

Potential at O

$$V = \frac{\Sigma q}{4\pi\epsilon_0 r}$$

$$= \frac{9\times10^9\,(1\times10^{-8} - 2\times10^{-8} + 3\times10^{-8} + 2\times10^{-8})}{1/\sqrt{2}}$$

$$= 9\times10^9 \times 4\times10^{-8}\ \sqrt{2} = \mathbf{5\cdot09\times10^2\ V}$$

50. As Potential energy $= \frac{q_1 q_2}{4\pi\epsilon_0 r}$

$\therefore$ Potential energy of the system of charges

$$= 9\times10^9 \times \left[\frac{(1\times10^{-8})(-2\times10^{-8})}{1\cdot0} + \frac{(-2\times10^{-8})(3\times10^{-8})}{1\cdot0} + \frac{(3\times10^{-8})(2\times10^{-8})}{1\cdot0} + \frac{(2\times10^{-8})(1\times10^{-8})}{1\cdot0}\right]$$

$$+ 9\times10^9 \times \left[\frac{(1\times10^{-8})(3\times10^{-8})}{\sqrt{2}} + \frac{(2\times10^{-8})(-2\times10^{-8})}{\sqrt{2}}\right]$$

$$= 0 + \frac{9\times10^9\,(-10^{16})}{\sqrt{2}} = \mathbf{-6\cdot4\times10^{-7}\ J}$$

51. Here, $A = 150\ \text{cm}^2 = 150\times10^{-4}\ \text{m}^2$,

$d = 0\cdot8\ \text{mm} = 8\times10^{-4}\ \text{m}$,

$V_0 = 1200\ \text{V}$

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$= \frac{8\cdot85\times10^{-12}\times150\times10^{-4}}{8\times10^{-4}}$$

$$= 1\cdot66\times10^{-10}\ \text{F}$$

$$E_0 = \frac{1}{2}C_0 V_0^2 = \frac{1}{2}\times1\cdot66\times10^{-10}\,(1200)^2$$

$$= \frac{1}{2}\times1\cdot66\times10^{-10}\,(1200)^2$$

$$= \mathbf{1\cdot2\times10^{-4}\ J}$$

52. With dielectric medium, new capacity becomes

$$C = KC_0 = 3\times1\cdot66\times10^{-10}\ \text{F}$$

Energy, $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)V_0^2$

$$= K\times\frac{1}{2}C_0 V_0^2 = 3\times1\cdot2\times10^{-4}\ \text{J}$$

$$= \mathbf{3\cdot6\times10^{-4}\ J}$$

53. When the capacitor is charged first as an air capacitor, and then filled with dielectric, its potential becomes

$$V = \frac{V_0}{K} = \frac{1200}{3} = \mathbf{400\ volt}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)\left(\frac{V_0}{K}\right)^2$$

$$= \frac{1}{2}\frac{C_0 V_0^2}{K}$$

$$= \frac{1\cdot2\times10^{-4}}{3} = \mathbf{4\times10^{-5}\ J}$$

IV. Matching Type Questions

54. Due to a single charge, $E \propto \frac{1}{r^2}$; for an electric dipole, $E \propto \frac{1}{r^3}$; for a quadrupole, $E \propto \frac{1}{r^4}$. The electric potential (V) for a single charge $\propto \frac{1}{r}$. **Choice (*d*) is correct.**

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V. Matrix-Match Type Questions

55. In series combination of capacitors, charge on each capacitor is same ; and C_s decreases. In parallel combination of capacitors, pot. diff. across each capacitor is same and C_p increases.
On inserting dielectric slab inbetween plates of a capacitor, its capacity increases.
On replacing mica by air inbetween plates of a capacitor, its capacity decreases.

56. Dimensional formulae of various quantities in column II are

$$Q = [AT]\ ;$$
$$V = [ML^2 T^{-3} A^{-1}]$$
$$C = [M^{-1} L^{-2} T^4 A^2]$$
and $$U = [ML^{-1} T^{-2}]$$

The given answer stands verified.

VI. Integer Type Questions

57. Let r be the radius of each small drop and R be the radius of one big drop.

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$$R^3 = (2r)^3 \quad \text{or} \quad R = 2r$$

Now $q = CV = 4\pi\epsilon_0 r V$

$$r = \frac{q}{4\pi\epsilon_0 V} = \frac{9\times10^9 \times 3\times10^{-9}}{2} = \frac{27}{2}\text{ m}$$

$$R = 2r = 2\times\frac{27}{2} = 27\text{ m}.$$

Surface potential of new drop

$$V = \frac{\text{total charge}}{\text{capacity}} = \frac{8q}{4\pi\epsilon_0 R}$$

$$V = \frac{8\times3\times10^{-9}\times9\times10^9}{27} = \mathbf{8\ volt}$$

58. From $E = \dfrac{-dV}{dr}$

$$dV = -\vec{E}.d\vec{r}$$

$$\int_B^A dV = -\int_{(2,2,0)}^{(0,0,0)} (\hat{i}x + \hat{k}z).(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$$

$$V_A - V_B = -\int_{(2,2,0)}^{(0,0,0)} (x\,dx + z\,dz)$$

$$V_{AB} = -\left[\frac{x^2}{2} + \frac{z^2}{2}\right]_{(2,2,0)}^{(0,0,0)}$$

$$= \frac{-1}{2}[(0^2 - 2^2) + (0^2 - 0^2)] = \mathbf{2\ volt}$$

59. Here, $q_1 = 5\ \mu\text{C} = 5\times10^{-6}$ C
$q_2 = 10\text{ m C} = 10\times10^{-3}$ C
$r_1 = 90$ m, $r_2 = 45$ m, $W = ?$

$$W = U_2 - U_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r_2} - \frac{q_1 q_2}{4\pi\epsilon_0 r_1}$$

$$= \frac{q_1 q_2}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= (5\times10^{-6})\times(10\times10^{-3})\times9\times10^9\left[\frac{1}{45} - \frac{1}{90}\right]$$

$$W = 450\times\frac{1}{90} = \mathbf{5\ J}$$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

60. Here Assertion is wrong and Reason is also wrong.

61. As electron has a negative charge. That is why it moves from a region of lower potential to a region of higher potential.

62. Assertion is true but the reason is false.

FOR ENGINEERING STUDENTS

63. Both the statements are true and statement-2 is correct explanation of statement-1.

64. As $V = \dfrac{Q}{4\pi\epsilon_0 R}$

As radius of earth (R) is very large compared to the size of bodies we deal with, therefoe, potential of earth is taken as zero. **Choice (a) is correct.**.

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UNIT-2

CURRENT ELECTRICITY

CHAPTER 3.
CURRENT ELECTRICITY

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3

CURRENT ELECTRICITY

3.1. INTRODUCTION

In the first unit on 'Electrostatistics', we have studied various characteristics of electric charges at rest. However, most of the practical applications of electricity involve charges in motion. For example, an electric bulb glows when charge flows through it, an electric fan rotates when charge is made to pass through its coil. Again, our radio and T.V. sets operate on flowing charges, and so on. *The flowing charges are said to constitute electric current.* We shall study later that electric current is of two types : direct current (d.c.) and 'alternating current (a.c.). Most of the practical devices use alternating current where charges flow back and forth through conductors. However, we start studying 'charges in motion' through conductors in one direction. In this chapter, we shall confine ourselves to the study of direct current circuits.

The branch of Physics, which deals with the study of charges in motion is called **Current Electricity.**

3.2. CURRENT CARRIERS

The charged entities whose flow in a definite direction constitutes the electric current are called current carriers. For example,

(*i*) In solids, which are ***good conductors,*** *e.g.*, metals, the current carriers are the *valence electrons.* These electrons are not firmly attached to individual atoms, and are free to move throughout the volume of the conductor.

In ***semiconductors*** like silicon and germanium, current carriers are negatively charged electrons and positively charged holes.

In ***insulators*** like glass, ebonite etc., there are no free charge/current carriers.

(*ii*) In liquids like NaCl, current carriers are positively charged Na^+ ion (called cation) and negatively charged Cl^- ion (called anion). Similarly, in $CuSO_4$ solution, the current carriers are Cu^{++} ion and SO_4^{--} ions. Such liquids through which electricity can pass are called ***electrolytes.*** Thus, in an electrolyte, electric current is due to simultaneous drifting of cations and anions in opposite directions.

(*iii*) In Gases under normal conditions, there are no charge/current carriers as they are insulators. However, gases can be ionized by applying high potential difference at low pressures or by exposing them to X-rays. In that event, current carriers in gases would be the electrons and positive ions of the gas.

3.3 ELECTROMOTIVE FORCE

Electric current is possible in a closed circuit if there is a source of external force (infact a source of energy) which compels the current carriers (electrons or ions) to move in a definite direction (*i.e.* from lower potential energy to higher potential energy.) This external force which makes the current carriers to move in a definite direction is called **electromotive force (emf).** *The emf is not a force at all. It is a special case of potential difference.* Due to this, it has the nature of workdone per unit charge.

Thus the electromotive force is associated with an arrangement or mechanism which can supply energy or does work to move the electric charge from lower potential energy to higher potential energy. Such an arrangement is called a source of emf, which may be a cell, a battery, a generator or dynamo. It is a seat of power.

> ***E.M.F. of a cell is defined as the maximum potential difference between the two electrodes of the cell when no current is drawn from the cell or cell is in the open circuit.***

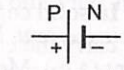
It may also be defined as the energy supplied by the cell to drive a unit positive charge once around the complete circuit and is given by

$$\varepsilon = \oint \vec{E}.d\vec{l}$$

where, $\vec{E}$ is the electric field (*i.e.*, force per unit charge) and $d\vec{l}$ is the small path segment vector.

Symbolically a cell is represented by two unequal parallel lines having a small separation, Fig. 3.1. The taller line represents the positive electrode P and smaller line represents the negative electrode N.

FIGURE 3.1



The SI unit of emf of a cell is volt (V) or joule per coulomb (JC^{-1}). The emf of a cell is said to be one volt, if 1 joule energy is supplied by the cell to drive one coulomb of charge once around the whole circuit (including the cell).

The emf of a cell depends upon the nature of electrodes, nature and the concentration of electrolyte used in the cell and its temperature.

3.4. DIFFERENCE BETWEEN ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

ELECTROMOTIVE FORCE	POTENTIAL DIFFERENCE
1. It is related with the source of current.	1. It is related with any two points of a closed circuit.
2. It is a cause of potential difference between two points of a circuit.	2. It is an effect.
3. It is the amount of workdone in taking a unit charge once round the complete circuit.	3. It is the amount of workdone in taking a unit charge from one point to another.
4. It is also defined as the maximum potential difference between two electrodes of a cell when no current is drawn from the cell or cell is in the open circuit.	4. It will exist between any two points in a circuit, when current is sent by the cell in the circuit.
5. It is equal to sum of potential differences across all the components of a closed circuit, including to that of cell.	5. In a closed circuit, the different components of a circuit will have different potential differences.
6. It is independent of the external resistance and current drawn from the cell.	6. It depends on the resistance between two points of circuit and current flowing through it.

3.5. ELECTRIC CURRENT

As is known, the flowing charges are said to constitute electric current. Quantitatively, *electric current is defined as the rate of flow of electric charge through any cross-section of a conductor, i.e.,*

$$\text{Electric current} = \frac{\text{total charge flown}}{\text{time taken}} = \frac{q}{t}$$

Oftenly, the currents are not usually steady. Therefore, more generally, we say that if ΔQ = net charge flowing across a cross section of a conductor in a particular direction during the time interval Δt [*i.e.,* between times t and $(t + \Delta t)$], then the current

$$I(t) = \underset{\Delta t \to 0}{\text{Limit}} \left(\frac{\Delta Q}{\Delta t} \right) = \frac{dQ}{dt}$$

Here, $\frac{dQ}{dt}$ is first derivative of Q w.r.t. t. Thus, **current through a conductor at anytime is defined as the first derivative of charge with respect to time passing through a cross section of the conductor in a particular direction.**

SI unit of electric current is **Ampere.** It is also called the practical unit of current.

$$1 \text{ ampere (A)} = \frac{1 \text{ coulomb } (C)}{1 \text{ second } (s)} = 1 \text{ Cs}^{-1}.$$

Thus, current through a conductor is said to be 1 ampere, if one coulomb of charge is flowing per second through any cross section of the conductor.

1 coulomb (the standard unit of charge) = charge due to $6{\cdot}25 \times 10^{18}$ electrons.

In solid conductor, there are free electrons called conduction electrons, which are called current carriers. They carry the charge through the circuit because they are free to move throughout the atomic network of the conductor. In solid conductor, the protons are bound inside the atomic nuclei which are more or less locked in fixed positions. Under the effect of electric field applied, the conduction electrons move in a fixed direction and constitutes the current.

The current carrying wire does not have a net electric charge because the number of electrons (negatively charged electrons) in the wire is equal to number of positive protons in the atomic nuclei of the current carrying wire. When electrons flow in a wire, the number of electrons entering per second at one end of wire is equal to the number of electrons leaving per second at the other end. As a result of it, the net charge of the wire is normally zero at every moment.

Total current in a circuit is equal to the sum of the currents due to motion of all charge carriers. In a circuit if n_1 number of electrons move from B to A per second through a conductor and n_2 number of protons (or positively charged particles) move from A to B per second through the same conductor. If e is the charge of electron or proton, then the electric current, $I = n_1 e + n_2 e = (n_1 + n_2)\, e$

In case an electron is revolving in a circle of radius r with speed v, period of revolution of electron is

$$T = \frac{2\pi r}{v}$$

Frequency of revolution, $\nu = \frac{1}{T} = \frac{v}{2\pi r}$

Current at any point of the circular path is ;

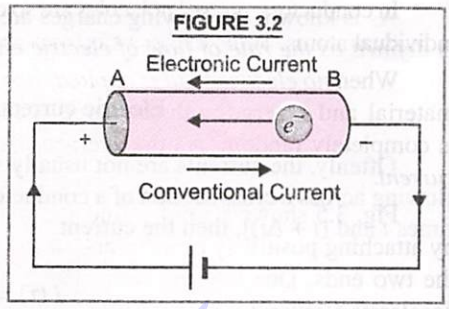
I = charge flowing in one revolution × number of revolutions per second

$$= e\nu = \frac{ev}{2\pi r}$$

Direction of Current.

As is known, current carriers in conductors such as metals are valence electrons, which carry negative charge. When a source of potential difference such as a cell is connected to a conductor *AB*, Fig. 3.2 free electrons flow from negative end *B* to positive end *A*. They are said to form *electronic current* which is along *BA, i.e.,* from lower potential to higher potential. However, normally water flows from higher level to lower level and heat flows from a body at higher temperature to another at lower temperature and so on. To maintain this convention, we define what is called **conventional current** ; which is equal and opposite to the electronic current. It would flow along *AB* from higher potential to lower potential.

FIGURE 3.2



In all our discussions, electric current will mean only the conventional current which flows from positive end (higher potential) to negative end (lower potential) in the closed circuit.

Retain in Memory

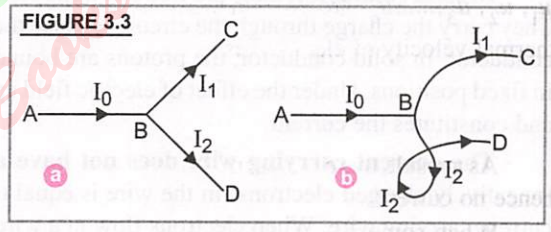
When cross sectional area is different at different points of the conductor, the magnitude of current through the conductor, $I = dQ/dt$ remains the same, at every cross-sections.

Current is a scalar quantity. Though the electric current represents the direction of flow of positive charge, yet it is treated as a scalar quantity because current follows the laws of scalars addition and not the laws of vectors addition. The angle between the wires carrying currents does not affect the total current in the circuit.

Fig. 3.3 shows a conductor *AB* with current I_0. It is splitted at a junction *B* into two branches *BC* and *BD*, carrying currents I_1 and I_2. Since the charge is conserved, the sum of the magnitudes of the currents in the two branches must be equal to the magnitude of the current in the original conductor. So

$$I_0 = I_1 + I_2$$

FIGURE 3.3



The above relation is valid even when there is bending or reorienting the conducting wires in space as shown in Fig. 3.3(*b*).

Sample Problem 1 **What is the current flowing through a conductor if 1 million electrons are crossing in 1 millisecond through a cross section of it from *A* to *B* ? Charge on an electron = $1{\cdot}6 \times 10^{-19}$ C.**

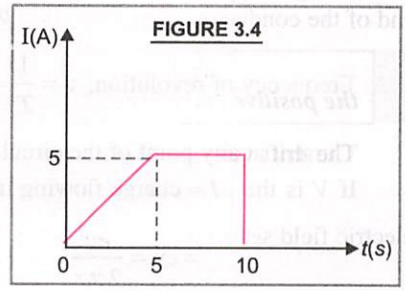
Sol. Here, $q = 10^6 \times 1{\cdot}6 \times 10^{-19}\ \text{C} = 1{\cdot}6 \times 10^{-13}\ \text{C}$; $t = 10^{-3}$ s

Now, current $I = \dfrac{q}{t} = \dfrac{1{\cdot}6 \times 10^{-13}}{10^{-3}} = \mathbf{1{\cdot}6 \times 10^{-10}\ A}$

The direction of current in conductor is from *B* to *A*.

Sample Problem 2 **Fig. 3.4 shows a plot of current *I* through the cross-section of a wire over a time interval of 10 s. Find the amount of charge that flows through the wire during this time period.** [CBSE OD 2015 (C)]

FIGURE 3.4



Sol. Amount of charge that flows in 10 seconds is

= area under the *I*-*t* graph

$$= \frac{1}{2} \times 5 \times 5 + (10 - 5) \times 5 = 12{\cdot}5 + 25 = \mathbf{37{\cdot}5\ C}$$

3.6 ELECTRIC CURRENT IN CONDUCTORS

In conductors, atoms/molecules are so closely packed, that valence electrons are no longer attached firmly to individual atoms. Some of these electrons are free to move within the bulk material, like molecules in a gas.

When *no electric field is applied,* free electrons have thermal motion. They collide with fixed ions in the material and emerge with the same speed. However, the direction of electron velocities after collision is completely random. As the average thermal random velocity of electrons is zero, therefore, there is *no current.*

Fig. 3.5 shows a solid cylinder across which an electric field is applied by attaching positively charged and negatively charged discs of a dielectric at the two ends. Due to this electric field, free electrons in the conductor are accelerated towards the positive disc, and a *transient current* (i.e., conventional current) is set up along *BA*. This current persists till the charges ± *Q* on the discs are neutralized, and there is no current there after.

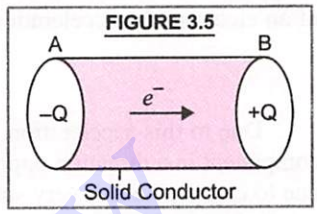


FIGURE 3.5

If we apply a steady electric field in the body of conductor by connecting a cell or a battery across the two ends of the conductor, there will be a steady current in the conductor.

3.7 DRIFT VELOCITY

A metal conductor contains a large number of free electrons or conduction electrons, whose number density (*i.e.*, no. of electrons per unit volume) is about 10^{29} m^{-3}. At room temperature these electrons move at random within the body of the conductor, like the molecules of a gas. The average thermal speed of the free electrons in random motion at room temperature is of the order of 10^5 ms^{-1}. The directions of motion of these free electrons are so randomly distributed that the average thermal velocity of the electrons is zero. If $\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots\ldots \vec{u}_n$ are random thermal velocities of n free electrons in the metal conductor, then the average thermal velocity of electrons is

$$\frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots + \vec{u}_n}{n} = \vec{0} \qquad \ldots(2)$$

As a result, there will be no net flow of electrons or charge in one particular direction in a metal conductor, hence no current.

When some potential difference is applied across the two ends of conductor, say a copper wire ; an electric field is set up inside the conductor. The free electrons in the conductor experience a force in a direction opposite to that of electric field and are accelerated from negative end to positive end of the conductor. On their way, the accelerated free electrons suffer frequent collisions against the copper ions/atoms, lose their gained kinetic energy and again get accelerated due to electric field and lose the gained kinetic energy in the next collision and so on. The process continues till the electrons reach the positive end of the conductor.

Thus under the effect of electric field applied, the free electrons acquire small velocities by virtue of acceleration due to electric field in addition to their random thermal velocities. These velocities are in different directions. Therefore, each free electron describes a curved path between two successive collisions, Fig. 3.6. The average velocity of all the free electrons is no longer zero. It has some little value towards the positive end of the conductor, which is called **drift velocity.** Thus,

> ***Drift velocity** is defined as the average velocity with which the free electrons get drifted towards the positive end of the conductor under the influence of an external electric field applied.*

The drift velocity of electrons is of the order of 10^{-4} ms^{-1}.

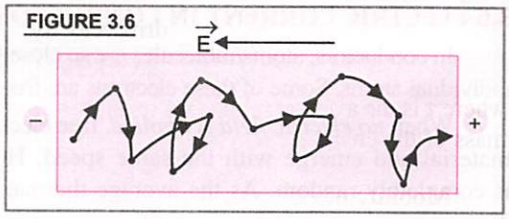
If V is the potential difference applied across the ends of a conductor of length l, the magnitude of electric field set up is $E = \dfrac{\text{potential difference}}{\text{length}} = \dfrac{V}{l}$

The direction of this electric field is from positive end to negative end of conductor. Since the charge on electron is $-e$, each free electron in the conductor experiences a force,

$$\vec{F} = -e\vec{E}$$

Here the negative sign shows that the direction of force is opposite to that of electric field applied. If m is the mass of an electron, the acceleration of each electron is

FIGURE 3.6



$$\vec{a} = -\frac{e\vec{E}}{m} \quad ...(3)$$

Due to this acceleration, the free electron, apart from its thermal velocity, acquires additional velocity component in a direction opposite to the direction of electric field. However, the gain in velocity of electron due to electric field is very small and it is lost in the next collision with the ion/atom of the conductor. As a result, the acceleration of an electron is not proportional to the external electric field applied [with reference to equation (3)]. The positive ions also experience a force due to electric field but they cannot move as they are heavy and tightly bound in the metal.

At any instant of time, the velocity acquired by electron having thermal velocity $\vec{u}_1$ will be

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$$

where τ_1 is the time elapsed since it has suffered its last collision with ion/atom of the conductor. Similarly, the velocities acquired by other electrons in the conductor will be

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \quad \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, ..., \quad \vec{v}_n = \vec{u}_n + \vec{a}\tau_n.$$

The average velocity of all the free electrons in the conductor under the effect of external electric field is the drift velocity $\vec{v}_d$ of the free electrons.

Thus, $$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + ... + \vec{v}_n}{n} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + ...(\vec{u}_n + \vec{a}\tau_n)}{n}$$

$$= \left(\frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_n}{n}\right) + \vec{a}\frac{(\tau_1 + \tau_2 + ... + \tau_n)}{n} = 0 + \vec{a}\tau = \vec{a}\tau \quad \text{[from (2)]}$$

where, $\tau = \frac{\tau_1 + \tau_2 + ... + \tau_n}{n}$ = average time that has elapsed since each electron suffered its last collision with the ion/atom of conductor and is called **average relaxation time.** Its value is of the order of 10^{-14} second.

Putting the value of $\vec{a}$ in the above relation, we have

$$\vec{v}_d = \frac{-e\vec{E}\tau}{m} \quad ...(4)$$

Average drift speed, $$v_d = \frac{eE}{m}\tau$$

In (4), the negative sign shows that $\vec{v}_d$ is opposite to the direction of $\vec{E}$.

Note that, **Average Relaxation time = mean free path of electron/drift speed of electron.**

3.8. MOBILITY

***Mobility** of charge carrier (μ), responsible for current is defined as the magnitude of drift velocity of charge per unit electric field applied, i.e.*

$$\mu = \frac{\text{drift velocity}}{\text{electric field}} = \frac{v_d}{E} = \frac{qE\tau/m}{E} = \frac{q\tau}{m}$$

where τ is the average relaxation time of the charge while drifting towards the opposite electrode, and m is the mass of the charged particle.

Mobility of electron, $\mu_e = \frac{e\tau_e}{m_e}$

Mobility of holes (*i.e.* positive charge carriers present in semiconductors) will be given by, $\mu_h = \frac{e\tau_h}{m_h}$

where, τ_e and τ_h are average relaxation time for electrons and holes respectively. m_e and m_h refer to mass of electron and mass of hole respectively. Charge on either is e.

Mobility is positive for both, positive current carriers and negative current carriers, although their drift velocities are opposite to each other.

The convential current in a conducting material due to motion of negative current carriers will be in the same direction as that due to positive current carriers. Therefore ***the total current in the conducting material is the sum of the currents due to positive current carriers and negative current carriers.***

If there are only electrons in a conducting material as current carrier, then the drift velocity of electron is

$$v_d = \mu_e E$$

SI unit of mobility is $m^2 s^{-1} V^{-1}$ or $m\ s^{-1} N^{-1} C$

Mobilities of electrons and holes in some materials, at room temperature are given in Table 3.1.

TABLE 3.1. Mobility of electrons and holes in some materials

Materials	Electrons mobilities ($cm^2\ V^{-1}\ s^{-1}$)	Holes mobilities ($cm^2\ V^{-1}\ s^{-1}$)
Diamond	1800	1200
Silicon	1350	480
Germanium	3600	1800
InSb	800	450
GaAs	8000	300

3.9. RELATION BETWEEN CURRENT AND DRIFT VELOCITY

Consider a conductor (say a copper wire) of length l and uniform area of cross-section A.

$\therefore$ Volume of the conductor = Al.

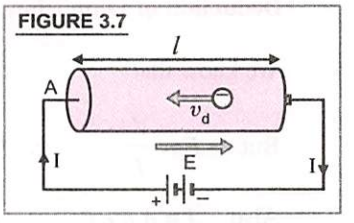
If n is the number density of electrons, *i.e.*, the number of free electrons per unit volume of the conductor, then total number of free electrons in the conductor = Aln. If e is the charge on each electron, then total charge on all the free electrons in the conductor, $\boxed{q = Alne}$

Let a constant potential difference V be applied across the ends of the conductor with the help of a battery.

The electric field set up across the conductor is given by $E = V/l$ (In magnitude)

Due to this field, the free electrons present in the conductor will begin to move with a drift velocity v_d towards the left end A, as shown in Fig. 3.7.

FIGURE 3.7



$\therefore$ Time taken by the free electrons to cross the conductor,

$$t = l/v_d$$

Hence, current, $I = \frac{q}{t} = \frac{Alne}{l/v_d}$ or $\boxed{I = A\,n\,e\,v_d}$ ...(5)

Because, $v_d = \mu_e E$, so $I = Ane\,\mu_e E$

Equation (5), gives the relation between the current flowing through the conductor and drit velocity of the electrons.

Putting the value of $v_d \left(= \frac{eE\tau}{m}\right)$ in (5), we have $I = \frac{Ane^2\tau E}{m}$

Sample Problem **Estimate the average drift velocity of conduction electrons in a copper wire of cross-sectional area $2{\cdot}5 \times 10^{-7}$ m^2, carrying a current of 2·7 A. Assume the density of conduction electrons to be 9×10^{28} m^{-3}.** **(CBSE OD 2014)**

Sol. Here, $A = 2{\cdot}5 \times 10^{-7}$ m^2 ; $I = 2{\cdot}7$ A ; $n = 9 \times 10^{28}$ m^{-3} ; $I = n A e v_d$ or $v_d = \dfrac{I}{n A e}$

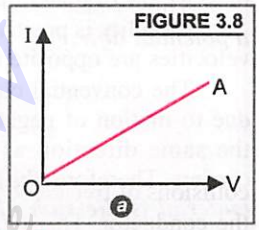
$$\therefore \quad v_d = \frac{2{\cdot}7}{(9\times10^{28})\times(2{\cdot}5\times10^{-7})\times(1{\cdot}6\times10^{-19})} = 0{\cdot}75 \times 10^{-3} \text{ ms}^{-1} = \mathbf{0{\cdot}75 \text{ mms}^{-1}}$$

3.10. OHM'S LAW

It is the most fundamental law of electricity.

***Ohm's law** states that the current (I) flowing through a conductor is directly proportional to the potential difference (V) applied across the ends of the conductor, provided physical conditions of the conductor such as temperature, mechanical strain etc. are kept constant,*

FIGURE 3.8



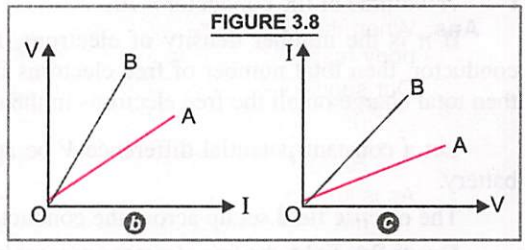
$$i.e., \quad I \propto V \quad \text{or} \quad V \propto I \quad \text{or} \quad V = RI \quad \text{or} \quad \frac{V}{I} = R = \text{a constant} \qquad \text{...(6)}$$

where R is known as resistance of the conductor. It depends upon the length, shape and the nature of material of the conductor. R also depends on temp. of the conductor. Thus, from Ohm's law, the variation between potential difference (V) and current (I) through a conductor is represented by a straight line OA having a constant slope as shown in Fig. 3.8(a).

If the V–I graph for the two conductors A and B is of the type as shown in Fig.3.8(b), then the slope of graph $= \dfrac{V}{I} = R$ (resistance). It shows that the higher is the slope, the higher is the resistance. So the resistance of conductor B is higher than that of conductor A.

If the I-V graph for two conductors A and B is of the type as shown in Fig.3.8(c), then the slope of gaph $= \dfrac{I}{V} = \dfrac{1}{R}$. It shows that the smaller is the slope, the higher is the resistance. So the resistance of conductor A is higher than that of conductor B.

FIGURE 3.8



Deduction of Ohm's law.

We know that $v_d = \dfrac{eE}{m}\tau$

But $E = \dfrac{V}{l}$ $\quad \therefore \quad v_d = \dfrac{eV}{ml}\tau$

Also, $I = A n e v_d$ $\quad \therefore \quad I = Ane\left(\dfrac{eV}{ml}\tau\right) = \left(\dfrac{Ane^2\tau}{ml}\right)V$ ...(7)

or $\dfrac{V}{I} = \dfrac{ml}{Ane^2\tau} = R =$ a constant for a conductor for a given value of n, l, A and at a given temperature.

It is known as the electrical resistance of the conductor.

Thus, $V = RI$

This is Ohm's law.

3.11. ELECTRICAL RESISTANCE

The electrical resistance of a conductor is the obstruction posed by the conductor to the flow of electric current through it.

Electrical resistance (R) of a conductor is defined as the ratio of the potential difference (V) applied across the ends of the conductor to the current (I) flowing through it,

i.e., $R = V/I$

The SI unit of electrical resistance is ohm (Ω), where $1\,\text{ohm} = \dfrac{1\,\text{volt}}{1\,\text{ampere}} = \dfrac{1V}{1A}$

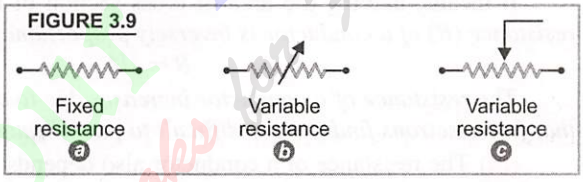
Thus, *1 ohm is the electrical resistance of a conductor through which a current of 1 ampere flows when a potential difference of 1 volt is applied across the ends of the conductor.*

Dimensions of electric resistance $= \dfrac{\text{pot. diff.}}{\text{current}} = \dfrac{\text{work done/charge}}{\text{current}} = \dfrac{ML^2T^{-2}/AT}{A} = [M^1L^2T^{-3}A^{-2}]$

Cause of resistance of a conductor. Resistance of a given conducting wire arises on account of frequent collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor. This would depend upon the arrangement of atoms in the conducting material, *i.e.*, on the nature of material (silver, copper etc.) as well as on the length and area of cross section of the conducting wire.

Any material which offers the obstruction to the flow of current through it is called a **resistor.** Some of the symbols of the various resistors are shown in Fig.3.9(*a*), (*b*) and (*c*).

FIGURE 3.9



Curiosity Question

Q. What is the biological connection of resistance and current to the human body ?

Ans. When the skin is dry, resistance of human body is ≈ 10^5 ohm ; and when the skin is wet, the body resistance is ≈ 1500 ohm.

Our supply voltage is 220 V. Therefore, when the skin is dry, current through the body,

$I = \dfrac{V}{R} = \dfrac{220}{10^5}\,\text{A} = 2{\cdot}2\,\text{mA}$ and current through the wet body, $I = \dfrac{V}{R} = \dfrac{220}{1500}\,\text{A} = 147{\cdot}0\,\text{mA}.$

As is known, a current of 100 mA to 3 A through the human body is fatal ; current of 20 mA to 100 mA causes paralysis ; current of 10 mA to 20 mA causes muscular disorder and a current of 2 mA through the body causes tingling. Hence touching a 220 V live wire, especially when the body is wet, may have serious consequences.

3.12. ELECTRICAL RESISTIVITY OR SPECIFIC RESISTANCE

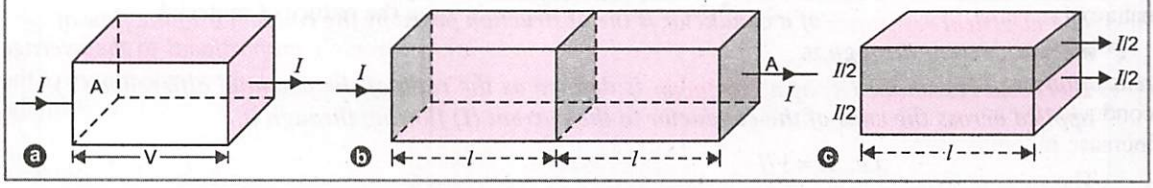
The resistance of a conductor depends upon the following factors :

(i) Length (*l*). Consider two identical conductors in the form of slabs, each of length l and area of cross-section A. Let V be the potential difference applied across either slab and I be the current through it, Fig. 3.10(*a*). The resistance of each slab is $R = V/I$

Let the two slabs be placed side by side so that the total length of the slabs becomes $2l$ as shown in Fig. 3.10(*b*). If same potential difference is applied across the ends of two slabs, then the current through them becomes $I/2$. Therefore, the resistance of the combination is

$$R_c = \frac{V}{I/2} = 2\frac{V}{I} = 2R$$

FIGURE 3.10



Thus, doubling the length of a conductor, doubles the resistance. Therefore, *the resistance (R) of a conductor is directly proportional to its length (l), i.e.,* $R \propto l$

Note that resistance of a conductor increases due to increase in its length on account of the fact that electrons have to travel a longer distance through the conductor suffering large number of collisions.

(ii) **Area of cross-section (A).** Divide a conductor slab into two equal parts cutting it length wise, so that the slab may be considered as a combination of two identical slabs each of length l, having a cross-sectional area $A/2$, Fig. 3.10(c). When the same potential difference V is applied across each of the two half slabs, the current flowing through each is $I/2$, which is half of the current through the full slab. The resistance of each of the half slab is

$$R' = \frac{V}{I/2} = 2\frac{V}{I} = 2R$$

It means, halving the area of cross-section of the conductor, doubles the resistance. Therefore, *the resistance (R) of a conductor is inversely proportional to the area of cross-section (A) of the conductor, i.e.,*

$$R \propto 1/A$$

The resistance of a conductor increases due to decrease in cross sectional area on account of the fact that free electrons find it more difficult to pass through smaller cross sectional area.

(iii) The resistance of a conductor also depends upon the **nature of material** and **temperature of the conductor.**

From above ; $R \propto \frac{l}{A}$ or $R = \frac{\rho l}{A}$ ...(8)

where ρ is constant of proportionality and is known as **specific resistance** or **electrical resistivity** of the material of the conductor

If $l = 1$, $A = 1$, then from (8), $R = \rho$

Thus, ***specific resistance (or electrical resistivity) of the material of a conductor is defined as the resistance of a unit length with unit area of cross section of the material of the conductor, i.e., it is also defined as the resistance of unit cube of a material of the given conductor.***

Unit of resistivity ; From (8), $\rho = RA/l$

In SI, unit of resistivity $= \frac{\text{ohm} \times m^2}{m} =$ ohm-m or Ω m.

Dimensions of resistivity $= \frac{RA}{l} = \frac{ML^2T^{-3}A^{-2} \times L^2}{L} = [ML^3T^{-3}A^{-2}]$

Factors affecting electrical resistivity

From (7), we have, $R = \frac{V}{I} = \frac{ml}{Ane^2\tau} = \frac{m}{ne^2\tau} \times \frac{l}{A}$

Comparing the above relation with the relation, $R = \rho\frac{l}{A}$, we find $\rho = \frac{m}{ne^2\tau}$ ...(9)

Thus the resistivity of the material of conductor depends upon the following factors.

(*i*) $\rho \propto 1/n$, *i.e., the resistivity of the material of conductor is inversely proportional to the number density of electrons* (n), *i.e., number of free electrons per unit volume of the conductor.* Since the value of n depends upon the nature of material, hence the **resistivity of the conductor depends upon the nature of material.**

(*ii*) $\rho \propto 1/\tau$, *i.e., the resistivity of the material of conductor is inversely proportional to the average relaxation time* (τ) *of the free electrons in the conductor.* Since the value of τ depends upon temperature of the conductor (which is discussed in article 3.16), so resistivity of conductor changes with temperature. With the increase in temperature of metal conductor, τ decreases, hence its electrical resistivity increases.

The value of electrical resistivity of different materials varies over a very wide range as shown in Table 3.2 :

TABLE 3.2. Resistivity of different materials

NAME OF THE MATERIAL	RESISTIVITY AT 0°C (Ωm)	NAME OF THE MATERIAL	RESISTIVITY AT 0°C (Ωm)
A. Conductors		**B. Semiconductors**	
(*a*) **Metals**			
Silver	$1{\cdot}6 \times 10^{-8}$	Carbon	$3{\cdot}5 \times 10^{-5}$
Copper	$1{\cdot}7 \times 10^{-8}$	Germanium	0·46
Aluminium	$2{\cdot}7 \times 10^{-8}$	Silicon	2300
Tungsten	$5{\cdot}6 \times 10^{-8}$		
Iron	10×10^{-8}		
Platinum	11×10^{-8}		
Mercury	98×10^{-8}	**C. Insulators**	
		Glass	$10^{10} - 10^{14}$
(*b*) **Alloys**		Hard rubber	$10^{13} - 10^{16}$
Nichrome (alloy of Ni, Fe, Cr)	100×10^{-8}	Mica	$10^{11} - 10^{15}$
Manganin	44×10^{-8}	Wood	$10^{8} - 10^{11}$
Constantan	49×10^{-8}	Amber	5×10^{14}

From the given table, we note that the value of electrical resistivity is very *low for* **conductors,** *very high for* **insulators** and in between those of conductors and insulators for **semi-conductors.**

Note. 1. Since electrical resistivity of copper is low, its electrical conductivity is high, therefore **copper wire is used as connecting wire.**

2. Since electrical resistivity of alloys like manganin (Cu – 84% + Mn – 12% + Ni – 4%) and constantan (Cu – 60% + Ni – 40%) is high, the lengths required for the wires of given diameter in making the standard resistances of given values are smaller. It follows from $R = \rho\, l/A$. For a given value of R and for a given value of A, as ρ is high, l is low.

DO YOU KNOW ?

1. Resistance of a conductor depends upon the dimensions (*i.e.* length and area of cross-section) nature of the material of conductor and temperature of the conductor but resistivity depends on nature of the material and temperature but is independent of the dimensions of the conductor.
2. Resistance is a property of object. Resistivity is a property of material.
3. ohm-metre is the unit of resistivity, where as Ohm-meter is an instrument to measure the resistance.

Sample Problem **What is the length of a nichrome wire of radius 0·32 mm, resistance 9·3 Ω and resistivity 15×10^{-6} Ω m ? If a potential difference of 10 V is applied across the wire, what will be the current in the wire ?**

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Sol. Here, $l = ?$; $r = 0{\cdot}32 \times 10^{-3}$ m ; $R = 9{\cdot}3\ \Omega$; $\rho = 15 \times 10^{-6}\ \Omega$m ; $V = 10$V

$$R = \frac{\rho l}{\pi r^2} \quad \text{or} \quad l = \frac{R\pi r^2}{\rho} = \frac{(9{\cdot}3)\times 3{\cdot}14\times(0{\cdot}32\times10^{-3})^2}{(15\times10^{-6})} = 0{\cdot}199 \text{ m} \simeq \mathbf{0{\cdot}20\ m}$$

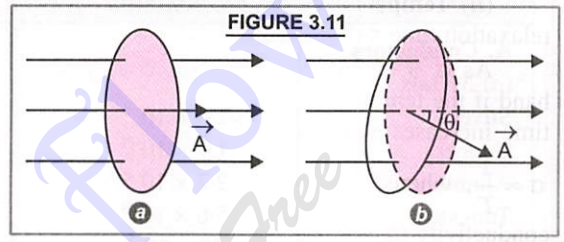
Current $$I = \frac{V}{R} = \frac{10}{9{\cdot}3} = 1{\cdot}075 = \mathbf{1{\cdot}08\ A}$$

3.13. CURRENT DENSITY, CONDUCTANCE AND ELECTRICAL CONDUCTIVITY

Current density (J) at a point in a conductor is defined as the amount of current flowing per unit area of the conductor around that point provided the area is held in a direction normal to the current.

Let I be the current flowing uniformly and normally through a cross-sectional area A of the conductor Fig. 3.11(a), then magnitude of the current density at any point on the cross-section of the conductor is

FIGURE 3.11



$$J = \frac{I}{A} = \frac{q/t}{A}$$

If the current I is not flowing normally through a cross-sectional area A as shown in Fig.3.11(b) then component of normal area to the direction of current is, $A_n = A\cos\theta$

Current density, $$J = \frac{I}{A_n} = \frac{I}{A\cos\theta} \quad \text{or} \quad I = JA\cos\theta = \vec{J}\cdot\vec{A}$$

This relation shows that the **current being the dot product of two vectors is a scalar quantity.**

Current density is a vector quantity. Its direction is the direction of motion of positive charge.

Relation between current density and Drift velocity

We know that $I = nAev_d$

$\therefore$ Current density, $$J = \frac{I}{A} = nev_d \qquad \text{...(10)}$$

The *SI* unit of current density is *ampere (metre)*$^{-2}$ or $\mathbf{Am^{-2}}$

Note. For a particular surface of a conductor, the current is the flux of current density $\vec{J}$ over that surface S and is given by $$I = \int_S \vec{J}.d\vec{S}$$

where $d\vec{S}$ is elementary surface area vector of an element taken over the particular surface S and integral is taken over the surface in question.

Conductance (G) of a resistor *is the measure of ease with which the charges flow through the conductor.* **It is equal to the inverse of its resistance (R) and is denoted by G.** Thus

Conductance, $$G = \frac{1}{R}$$

The *SI* unit of conductance is $\mathbf{ohm^{-1}}$ or **mho** or **siemen (symbol S).**

Electrical conductivity **(σ)** ***of a conductor is defined as the reciprocal of its resistivity. It is represented by*** **σ,** *i.e.*,

$$\sigma = 1/\rho \qquad \text{...(11)}$$

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The *SI* unit of electrical conductivity is **ohm^{-1} m^{-1}** or **mho m^{-1}** or **S m^{-1}.**

The dimensions of electrical conductivity are $\sigma = \frac{1}{\rho} = \frac{n e^2 \tau}{m} = \frac{L^{-3} \times (AT)^2 \times T}{M} = [M^{-1} L^{-3} T^3 A^2]$

Factors affecting electrical conductivity (σ) of a conductor

As $\rho = \frac{m}{ne^2\tau}$ and $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

where the different terms have their usual meanings. Thus electrical conductivity depends on the following factors :

(*i*) **Nature of material of conductor.** As number density of electrons in different materials is different and $\sigma \propto n$ (number density of electrons) in a conductor, so conductivity of different material is different.

(*ii*) **Temperature of the conductor.** When temperature of the conductor increases then the value of relaxation time τ decreases.

As $\sigma \propto \tau$, so, conductivity of a conductor decreases. On the other hand if the temperature of a conductor decreases, then the value of relaxation time increases and hence the conductivity of the conductor increases. Thus $\sigma \propto \frac{1}{T}$, where T is the temperature of the conductor. The variation of electrical conductivity (σ) of a conductor with temperature (T) is shown in Fig. 3.12.

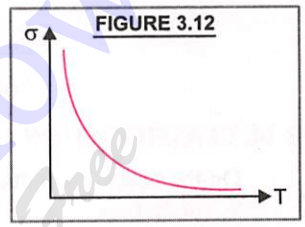

FIGURE 3.12

Relation between J, σ and E [Microscopic form of Ohm's law]

We know, $I = n A e v_d = n Ae\left(\frac{eE}{m}\tau\right) = \frac{n A e^2 \tau E}{m}$ or $\frac{I}{A} = \frac{ne^2\tau E}{m} = \frac{E}{m/ne^2\tau}$

or $J = \frac{1}{\rho}E$ $\left[\because J = \frac{I}{A} \text{ and } \rho = \frac{m}{ne^2\tau}\right]$

$\therefore$ $\boxed{J = \sigma E}$ $\left(\because \sigma = \frac{1}{\rho}\right)$

It is a **microscopic form of Ohm's law.**

As the direction of current density $\vec{J}$ is the same as that of electric field $\vec{E}$, hence

$$\boxed{\vec{J} = \sigma \vec{E}}$$

This relation is the **vector form of Ohm's law.**

Relation between electrical resistivity and electron mobility.

We know that $I = n A e v_d$; $v_d = \mu E$ and $J = \frac{I}{A} = \sigma E = \frac{E}{\rho}$

$\therefore$ $\frac{E}{\rho} = \frac{I}{A} = ne v_d = ne\mu E$ or $\boxed{\rho = \frac{1}{ne\mu}}$ ...(12)

Sample Problem 1 **Find the conductance of a conductor of resistance 2 milli ohm.**

Sol. Here, $R = 2 \times 10^{-3}\ \Omega$; conductance, $G = \frac{1}{R} = \frac{1}{2\times10^{-3}} = 500$ **S**

Sample Problem 2 **Potential difference of 100 V is applied to the ends of a copper wire one metre long. Calculate the average drift velocity of the electrons ? Compare it with thermal velocity at 27°C. Consider there is one conduction electron per atom. The density of copper is $9{\cdot}0 \times 10^3$ kg/m^3 ; Atomic mass of copper is 63·5 g. Avogadro's number = $6{\cdot}0 \times 10^{23}$ per gram–mole. Conductivity of copper is $5{\cdot}81 \times 10^7\ \Omega^{-1}$ m^{-1}. Boltzmann constant = $1{\cdot}38 \times 10^{-23}$ JK^{-1}.**

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Sol. Here ; $V = 100$ V, $l = 1$ m, $M = 63{\cdot}5$ g ; $d = 9{\cdot}0 \times 10^3$ kg/m^3 $= 9{\cdot}0$ g/m^3
$N = 6{\cdot}0 \times 10^{23}$ per gram-mole ; $\sigma = 5{\cdot}81 \times 10^7\ \Omega^{-1}\ m^{-1}$.

Since $6{\cdot}0 \times 10^{23}$ copper atoms have a mass of 63·5 g, and there is one conduction electron per atom, so number density of electrons , $n = \dfrac{6{\cdot}0 \times 10^{23}}{63{\cdot}5} \times 9{\cdot}0 = 8{\cdot}5 \times 10^{22}\ cm^{-3} = 8{\cdot}5 \times 10^{28}\ m^{-3}$

Electric field, $E = \dfrac{V}{l} = \dfrac{100}{1} = 100\ Vm^{-1}$

As $J = \sigma E = ne\, v_d \quad \therefore \quad v_d = \dfrac{\sigma E}{ne} = \dfrac{(5{\cdot}81 \times 10^7) \times (100)}{(8{\cdot}5 \times 10^{28}) \times (1{\cdot}61 \times 10^{-19})} = \mathbf{0{\cdot}43\ ms^{-1}}$

Thermal velocity, $v_{rms} = \sqrt{\dfrac{3k_B T}{m_e}} = \sqrt{\dfrac{3 \times 1{\cdot}38 \times 10^{-23} \times 300}{9{\cdot}1 \times 10^{-31}}} = \mathbf{1{\cdot}17 \times 10^5\ m/s}$

$$\frac{v_d}{v_{rms}} = \frac{0{\cdot}43}{1{\cdot}17 \times 10^5} = \mathbf{3{\cdot}67 \times 10^{-6}}$$

3.14. CLASSIFICATION OF MATERIALS IN TERMS OF CONDUCTIVITY

On the basis of conductivity, the different materials have been classified into following categories.

1. Insulators.

These are the materials whose electrical conductivity is either very very small or nil.

Insulators do not allow the flow of charges. When a small potential difference is applied across the two ends of an insulator, the current through the insulator is zero.

Examples of insulators are glass, rubber, wood etc.

2. Conductors.

These are the materials whose electrical conductivity is very high.

Conductors conduct charges very easily. When a small potential difference is applied across the two ends of conductor, a strong current flows through the conductor. For super-conductor, the value of electrical conductivity is infinite and electrical resistivity is zero.

Examples of conductors are all metals like copper, silver, aluminium, tungsten etc.

3. Semiconductors.

These are the materials whose electrical conductivity lies inbetween that of insulators and conductors.

Semiconductors can conduct charges but not so easily as is in case of conductors. When a small potential difference is applied across the ends of a semiconductor, a weak current flows through semiconductor due to motion of electrons and holes.

Examples of semiconductors are germanium, silicon etc.

3.15. EFFECT OF TEMPERATURE ON RESISTANCE

We know that resistance of a **metallic conductor** is given by

$$R = \frac{m}{ne^2\tau} \cdot \frac{l}{A}$$

For a given conductor, $R \propto \dfrac{1}{\tau}$

When the temperature of metal conductor is raised, the number density (n) of electrons in metal conductor does not change appreciably. However, the ions/atoms of the metal vibrate with greater amplitudes about their mean positions (*i.e.* lattice sites). Due to increase in thermal energy, the frequency of collision

of free electrons with atoms/ions while drifting towards positive end of the conductor also increases. This reduces the average relaxation time τ. Hence, ***the value of electrical resistance R increases with rise of temperature.***

The resistance R_t of a metal conductor at temperature $t^\circ C$ is given by

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$

where α and β are constants, R_0 is the resistance of conductor at 0°C. The values of α and β vary from metal to metal. If the temperature $t°C$ is not sufficiently large which is so in most of the practical cases, the above relation may be simplified as

$$R_t = R_0 (1 + \alpha t) \quad \text{...(13)}$$

or $$R_t = R_0 + R_0 \alpha t$$ or $$\alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\text{increase in resistance}}{\text{original resistance} \times \text{rise of temp.}}$$

Thus, temperature coefficient of resistance (α) is defined as the increase in resistance per unit original resistance per degree celsius or kelvin rise of temperature.

For metals like silver, copper etc., the value of **α is positive,** therefore, resistance of a metal increases with rise in temperature. The unit of α is K^{-1} *or* $°C^{-1}$.

For insulators and semiconductors, α is negative, therefore, the resistance decreases with rise in temperature.

Note : (*i*) Let α be the temperature coefficient of resistance of a conductor at 0°C. R_1 and R_2 be the resistances of that conductor at t_1°C and t_2°C respectively, then

$$R_1 = R_0 (1 + \alpha t_1) \text{ and } R_2 = R_0 (1 + \alpha t_2)$$

$$\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2} \text{ or } R_1 (1 + \alpha t_2) = R_2 (1 + \alpha t_1) \text{ or } \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

(*ii*) The value of α is different at different temperatures. Temperature coefficient of resistance averaged over the temperature range $t_1\ °C$ to $t_2\ °C$ is given by

$$\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$$

where R_1, R_2 are the resistances at $t_1\ °C$ and $t_2\ °C$ respectively. ...(14)

The **alloys like manganin, eureka** and **constantan are used for making standard resistance coils*** due to following facts :

(*i*) These alloys have high value of resistivity.

(*ii*) They have small values of temperature coefficient of resistance. That is why their resistance does not change appreciably even for several degrees rise in temperature.

(*iii*) Their contact potential with copper is small.

(*iv*) Their resistance is least affected by atmospheric conditions like air, moisture, etc.

Sample Problem **The resistance of a conductor at 30°C is 3·25 Ω and at 100°C is 3·95 Ω. Calculate the temperature coefficient of resistance of the conductor and the resistance of the conductor at 0°C.**

Sol. $R_1 = 3{\cdot}25\ \Omega$; $t_1 = 30°C$; $R_2 = 3{\cdot}95\ \Omega$; $t_2 = 100°C$

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} = \frac{3{\cdot}95 - 3{\cdot}25}{(3{\cdot}25 \times 100) - (3{\cdot}95 \times 30)} = \frac{0{\cdot}70}{325 - 118{\cdot}5} = \frac{0{\cdot}70}{206{\cdot}5} = \mathbf{0{\cdot}0034\ °C^{-1}}$$

$$R_0 = \frac{R_1}{1 + \alpha t_1} = \frac{3{\cdot}25}{1 + (0{\cdot}0034) \times 30} = \frac{3{\cdot}25}{1{\cdot}102} = \mathbf{2{\cdot}95\ \Omega}$$

*$\alpha_{Cu} \approx 10^{-3}\ °C^{-1}$ and *$\alpha_{manganin} \approx 10^{-5}\ °C^{-1}$

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3.16. VARIATION OF ELECTRICAL RESISTIVITY WITH TEMPERATURE

Electrical Resistivity of a material, $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$ [Eqn. (9)] or $\rho \propto \frac{1}{n\tau}$

This shows that the electrical resistivity is related to two parameters of the material, namely the number density n of free electrons in the material and relaxation time, τ. The variation of electrical resistivity of material with temperature is different for different materials.

(*a*) In most metals, number density n of free electrons does not change with temperature but an increase in temperature increases the amplitude of vibrations of lattice ions of the metal and thermal speed of the electrons which act as carriers of current. Due to it, the collisions of free electrons with ions/atoms while drifting towards the positive end of the conductor become more frequent, resulting in a decrease in relaxation time τ. Hence, **electrical resistivity of conductor increases with increase in temperature.** At low temperature, resistivity increases with a higher power of T.

The temperature dependence of electrical resistivity of a metal can be obtained from (13), as $\rho \propto R$. It is given by the relation $\rho = \rho_0 [1 + \alpha (T - T_0)]$.

where ρ and ρ_0 are the values of resistivity at temperatures T and T_0 respectively and α is called **temperature coefficient of resistivity.**

Clearly, $$\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)} = \frac{d\rho}{\rho_0} \cdot \frac{1}{dT} \quad ...(15)$$

Thus temperature coefficient of electrical resistivity is also defined as the fractional change in electrical resistivity ($d\rho / \rho_0$) per unit change in temperature (dT). The unit of α is $^oC^{-1}$ or K^{-1}.

For metals. The value of α is positive, showing that their electrical resistivity increases with increase in temperature.

For many metallic conductors, the value of α is nearly 4×10^{-3} $^oC^{-1}$. For such metallic conductors, the temperature dependence of ρ at low temperature is non-linear. At low temperatures, the resistivity (ρ) of a pure metal increases as a higher power of temperature. The variation of electrical resistivity ρ of copper with temperature T is shown in Fig. 3.13(*a*).

For alloys, the resistivity is high. The resistivity of nichrome (which is alloys of nickel, iron and chromium) has weak temperature dependence as shown in Fig. 3.13(*b*). The resistivity of manganin is almost independent of temperature, Fig. 3.13(*c*). That is why wire bound standard resistors are made of manganin. At temperature 0K, a pure metal has negligible small resistivity while an alloy (like nichrome) has some residual resistivity. This concept has been utilized to distinguish a pure metal from an alloy.

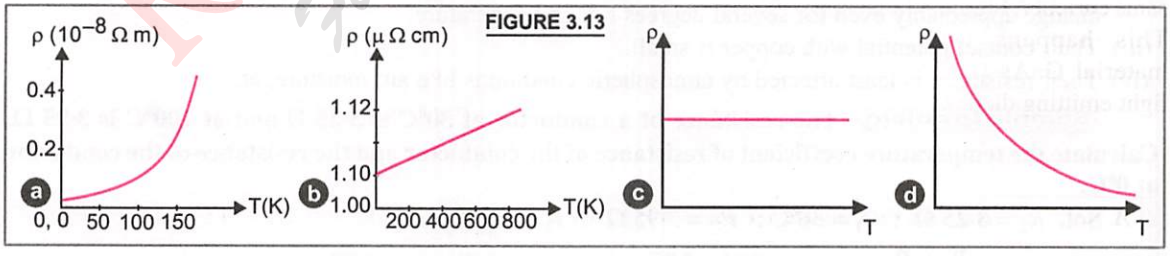


FIGURE 3.13

(*b*) For semiconductors like carbon, germanium, silicon etc., with increase in temperature, the number density n increases exponentially and time of relaxation τ decreases. But the increase in n compensates more than decrease in τ. That is why the resistivity of semiconductor decreases exponentially with increase in temperature.

The number density of electrons at temperature T is given by

$$n(T) = n_0 e^{-E_g / k_B T}$$

where k_B is Boltzmann constant and E_g is the energy gap between conduction band and valence band of the resistor. As, $\rho \propto 1/n$, so $\frac{1}{\rho(T)} = \frac{1}{\rho_0} e^{-E_g/kT}$ or $\rho(T) = \rho_0 e^{E_g/kT}$

This relation shows that resistivity of semiconductor decreases rapidlly with increase in temperature.

For semiconductors, the coefficient of resistivity (α) is negative. Therefore, their **resistivity decreases with increase in temperature.** For semiconductor, the variation of ρ with temperature T is shown in Fig. 3.13(d).

(*c*) **For insulators.** The coefficient of resistivity (α) is negative. The resistivity ρ increases exponentially with decrease in temperature. It becomes infinitely large at temperature near absolute zero, and the **electrical conductivity of insulator is almost zero at 0K.**

(*d*) **For electrolytes,** the coefficient of resistivity (α) is negative. With the rise in temperature, the resistivity ρ of electrolytes decreases because with the rise in temperature, interionic attractions decrease and also viscous forces decrease in electrolytes. Due to it, the ions move more freely, *i.e.*, drift velocity of ions increases and the time of relaxation (τ) increases. As $\rho \propto \frac{1}{\tau}$, so **$\rho$ decreases with increase in temperature.**

3.17. NON-OHMIC DEVICES OR LIMITATIONS OF OHM'S LAW

Those devices which do not obey Ohm's law are called **non-ohmic devices.** For example, vacuum tubes, semiconductor diode, liquid electrolyte, transistor etc. are all non-ohmic devices.

The relation; $V/I = R$, is valid for both ohmic and non-ohmic devices. The value of R is constant for ohmic conductors but R is not constant for non-ohmic devices, *i.e.*, Ohm's law fails when V/I is not constant.

For all non-ohmic devices (where there will be failure of Ohm's law), *V-I* graph has anyone of the following characteristics : *V-I* graph is not a straight line. That is why non-ohmic *devices are called non-linear* devices.

(*a*) *The relation between V and I is non-linear,* Fig. 3.14(*a*).

(*b*) *The relation between V and I depends on the sign of V.* It means, if I is the current for a certain value of V, then reversing the direction of V, keeping its magnitude fixed, does not produce a current of same magnitude I, in the opposite direction, Fig. 3.14(*b*). This happens in case of junction diode which we shall study in chapter IX (*b*).

(*c*) *The relation between V and I is not unique*, *i.e.*, there is more than one value of V for the same current I, Fig. 3.14(*c*). This happens for the material GaAs (*i.e.*, in a light emitting diode).

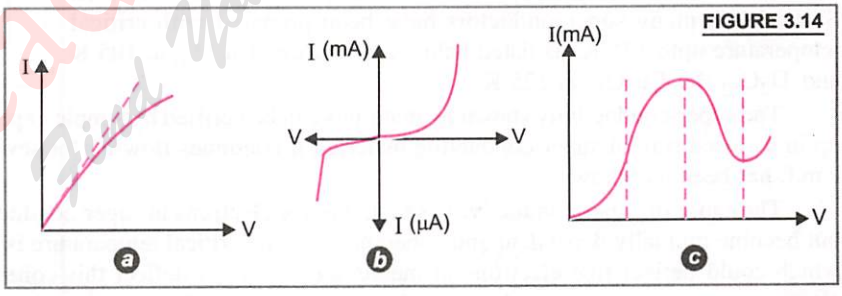


FIGURE 3.14

3.18. THERMISTORS

A thermistor is a heat sensitive device whose resistivity changes very rapidly with change of temperature.

The temperature coefficient of resistivity of a thermistor is very high, which may be positive or negative.

Thermistors are usually prepared from oxides of various metals such as nickel, iron, copper, cobalt etc. These compounds are also semi-conductors. The size of thermistor is very small. The thermistors are generally in the form of beads, discs or rods. A pair of platinum leads are attached at the two ends of thermistor for electric connections. This arrangement is sealed in a small glass bulb.

A thermistor can have a resistance in the range of 0·1 Ω to 10^7 Ω, depending upon its composition. A thermistor can be used over a wide range of temperatures.

A thermistor differs from an ordinary resistance in the following ways :

1. The resistance of thermistor changes very rapidly with change in temperature, therefore, the temperature coefficient of resistivity of a thermistor is very high.

2. The temperature coefficient of resistivity of a thermistor can be both positive and negative.

Important applications of thermistors

1. Thermistors are used to detect small temperature changes and to measure very low temperature ($\approx$ 10 K). A typical thermistor can easily measure a change in temperature of 10^{-3} °C.

2. Thermistors are used to safe-guard the filament of picture tube of a television set against the variation of current.

3. Thermistors are used in temperature control units of industry.

4. Thermistors are used in the protection of windings of generators, transformers and motors.

5. Thermistors are used for voltage stablisation and remote sensing.

3.19. SUPER-CONDUCTIVITY

Prof. K. Onnes, in 1911, discovered that certain metals and alloys at very low temperature lose their resistance considerably and almost becomes zero. This phenomenon is known as **super-conductivity.** As the temperature decreases, the resistance of the material also decreases, but when the temperature reaches a certain critical value (called **critical temperature or transition temperature**), the resistance of the material completely disappears *i.e.,* it becomes zero. Then the material behaves as if it is a **super-conductor** and on applying small-electric field, there will be flow of electrons without any resistance whatsoever. **Thus superconductivity is the property by virtue of which a metal alloy or oxide etc. shows zero resistance at a very low temperature.**

The variation of resistance (R) and temperature (T) for mercury is shown in Fig. 3.15. This graph shows that the resistance of mercury drops abruptly to zero below 4·2 K. The critical temperature is different for different materials. It has been found that mercury at critical temperature 4·2 K, lead at 7·25 K and niobium at critical temperature 9·2 K become super-conductors.

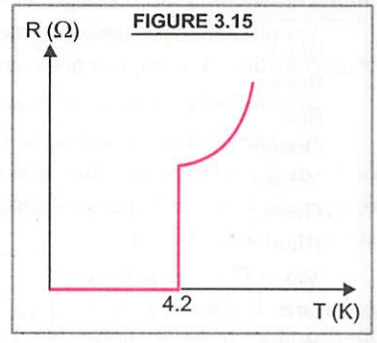


FIGURE 3.15

A team of scientists discovered that an alloy of plutonium, cobalt and gallium exhibits super conductivity at temperatures below 18·5 K. Since 1987, many superconductors have been prepared with critical temperature upto 125 K, as listed below : $Bi_2\ Ca_2Sr_2\ Cu_3O_{10}$ at 105 K and $Tl_2Ca_2\ Ba_2\ Cu_3O_{10}$ at 125 K.

The super-conductivity shown by materials can be verified by simple experiment. If a current is once set up in a closed ring of super-conducting material, it continues flowing for several weeks after the source of e.m.f. has been withdrawn.

The cause of super-conductivity is that, the free electrons in super-conductor are no longer independent but become mutually dependent and coherent when the critical temperature is reached. The ionic vibrations which could deflect free electrons in metals are unable to deflect this coherent or co-operative cloud of electrons in super-conductors. It means the coherent cloud of electrons makes no collisions with ions of the super-conductor and, as such, there is no resistance offered by the super-conductor to the flow of electrons.

Super-conductivity is a very interesting field of research all over the world these days. The scientists have been working actively to prepare super-conductors at room temperature and they have met with partial success only.

Applications of super conductors

1. Super conductors are used for making very strong electromagnets.

2. Super conductivity is playing an important role in material science research and high energy particle physics.

3. Super conductivity is used to produce very high speed computers.

4. Super conductors are used for the transmission of electric power.

3.20. MEISSNER EFFECT

In 1933, Meissner and Orchsenfold found that if a conductor is cooled in a magnetic field to a temperature equal to or lower than the transition temperature T_c, then at the transition temperature, the magnetic lines of induction are pushed out as has been shown in Fig. 3.16.

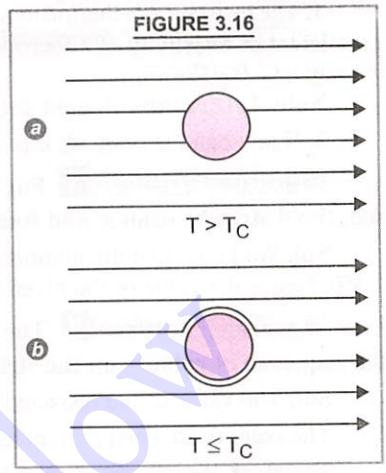


FIGURE 3.16

Thus the Meissner effect states that *at or below the transition temperature, there is no magnetic field or magnetic flux density inside the super-conductor when the specimen is placed inside a uniform magnetic field.*

3.21. COLOUR CODE FOR CARBON RESISTORS

In the market, the commercial resistors are carbon resistors. These are made by moulding carbon into a cylinder with wire leads at the two ends. Every such carbon cylinder is encased in a ceramic or plastic jacket. Carbon resistors are thus compact and cost less. The value of the resistance of these resistors and their percentage accuracy are indicated by a colour code printed on them. The colour code for carbon resistances is given in Table 3.3 :

TABLE 3.3. Colour code of carbon resistors.

COLOUR	LETTER AS AN AID TO MEMORY	NUMBER	MULTIPLIER	COLOUR	TOLERANCE
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No colour	20%
Orange	O	3	10^3		
Yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
Violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		
Gold			10^{-1}		
Silver			10^{-2}		

To remember the value of colour coding used for carbon resistor, the following sentences are found to be of great help (where bold letters stand for colours)

B B ROY Great **B**ritain **V**ery **G**ood **W**ife wearing **Gold Silver** necklace.

Black **B**rown **R**ods **O**f **Y**our **G**ate **B**ecame **V**ery **G**ood **W**hen **G**iven **S**ilver colour.

Way of finding the resistance of carbon resistor from its colour coding.

In the system of colour coding, strips of different colours are given on the body of the resistor, Fig. 3.17. The colours on strips are noted from left to right.

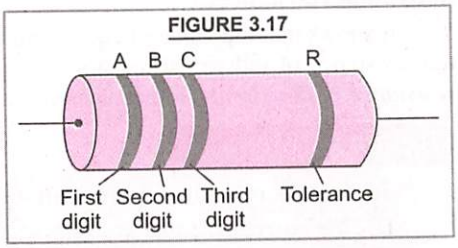


FIGURE 3.17

(*i*) ***Colour of the first strip A*** from the end indicates the *first significant figure* of resistance in ohm.

(*ii*) ***Colour of the second strip B*** indicates the *second significant figure* of resistance in ohm.

(*iii*) ***The colour of the third strip C*** indicates the multiplier, *i.e., the number of zeros that will follow after the two significant figures.*

(*iv*) ***The colour of fourth strip R*** indicates the tolerance limit of the resistance value or *percentage accuracy of resistance.*

Note. 1. If fourth coloured strip does not exist then tolerance is to be taken as ± 20%.

2. If any band is omitted, it is of the same colour as the body of resistor.

Sample Problem 1 **For the given carbon resistor, let the first strip be yellow, second strip be red, third strip be orange and fourth be gold. What is its resistance ?**

Sol. We know that the numbers for yellow, red and orange are 4, 2 and 3. Gold represents tolerance of ± 5%. Hence the value of the given resistance = $\mathbf{42 \times 10^3\ \Omega \pm 5\% = 4{\cdot}2 \times 10^4\ \Omega \pm 5\%}$.

Sample Problem 2 **The resistance of the given carbon resistor is $2{\cdot}4 \times 10^6\ \Omega \pm 5\%$. What is the sequence of colours on the strips provided on resistor ?**

Sol. The value of the resistance of given carbon resistor, $R = 2{\cdot}4 \times 10^6\ \Omega \pm 5\% = 24 \times 10^5\ \Omega \pm 5\%$.

The colours attached to numbers 2, 4 and 5 are red, yellow and green. The colour for 5% is gold.

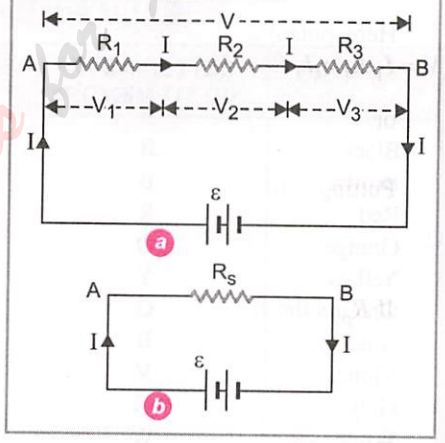
Therefore, the colours of strips in sequence for the given carbon resistor are **red, yellow, green** and **gold.**

3.22. RESISTANCES IN SERIES

Resistors are said to be connected in series, if the same current is flowing through each resistor when some potential difference is applied across the combination. Here the resistors are connected end to end with one another.

If the three resistances R_1, R_2 and R_3 are connected end to end as shown in Fig 3.18(*a*), they are said to be connected in series. Let V be the potential difference applied across A and B using the battery ε. In series combination, the same current (say I) will be passing through each resistance. Let V_1, V_2, V_3 be the potential differences across R_1, R_2 and R_3 respectively. According to Ohm'law

FIGURE 3.18



$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$$

Here, $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$

$$= I(R_1 + R_2 + R_3) \qquad \text{...(17)}$$

If R_s is the equivalent resistance of the given series combination of resistances, Fig. 3.18(*b*), then the potential difference across A and B is,

$$V = IR_s. \qquad \text{...(18)}$$

From (17) and (18), we have $IR_s = I(R_1 + R_2 + R_3)$ or $\boxed{R_s = R_1 + R_2 + R_3}$

Thus, **the equivalent resistance of a number of resistors connected in series is equal to the sum of individual resistances.**

It shows that the value of equivalent resistance of series combination of resistors is always greater than the resistance of individual resistors. Therefore, the *resistors are to be connected in series, if the effective resistance in the circuit is to be increased.*

Retain in Memory

In a series resistor circuit, it should be noted that :

(*i*) *the current is same in every resistor, because there is single path along which current can flow.*

(*ii*) *the current in the circuit is independent of the relative positions of the various resistors in the series.*

(*iii*) *the voltage across any resistor is directly proportional to the resistance of that resistor.*

(*iv*) *the total resistance of the circuit is equal to the sum of the individual resistances, plus the internal resistance of a cell if any.*

(*v*) *The total resistance in the series circuit is obviously more than the greatest resistance in the circuit.*

(*vi*) *Connecting resistors in series is equivalent to increasing the length of the conductor. That is why resistance of the series combination increases.*

3.23. RESISTANCES IN PARALLEL

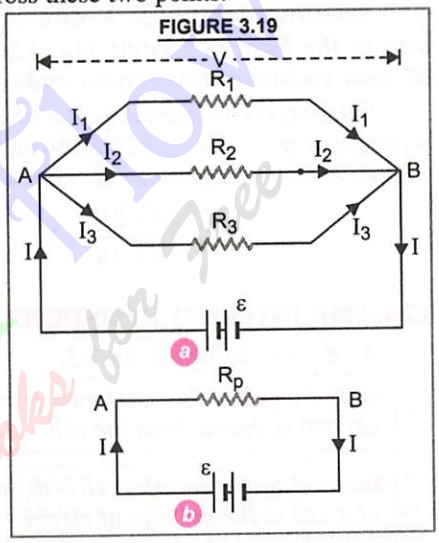
Any number of resistors are said to be connected in parallel if potential difference across each of them is the same and is equal to the applied potential difference.

Here, one end of all the resistors is connected to one point and other end of all the resistors is connected to another point. The source of potential difference is connected across these two points.

If three resistances R_1, R_2 and R_3 are connected as shown in Fig. 3.19(*a*), they are said to be connected in parallel. Let V be the potential difference applied across A and B with the help of a battery ε. Let I be the main current in the circuit from battery. I divides itself into three unequal parts because the resistances of these branches are different. If I_1, I_2, I_3 be the currents through the resistances R_1, R_2 and R_3 respectively. Then,

$$I = I_1 + I_2 + I_3 \qquad \text{...(19)}$$

FIGURE 3.19



Here, potential difference across each resistor is V, therefore

$$V = I_1 R_1 = I_2 R_2 = I_3 R_3$$

or $$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

Putting values in (19), we get

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad \text{...(20)}$$

If R_p is the equivalent resistance of the given parallel combination of resistances, Fig. 3.19(*b*), then

$$V = IR_p \quad \text{or} \quad I = \frac{V}{R_p} \qquad \text{...(21)}$$

From (20) and (21), we have $$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad \text{...(22)}$$

Thus, **the reciprocal of equivalent resistance of a number of resistors connected in parallel is equal to the sum of the reciprocals of the individual resistances.**

It shows that the value of equivalent resistance of parallel combination of resistors is always less than the resistance of individual resistors. Therefore, *the resistors are connected in parallel, if the effective resistance in the circuit is to be decreased.*

Note. If two resistors of resistances R_1 and R_2 are connected in parallel, then their effective resistance, R_p is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \quad \text{or} \quad R_p = \frac{R_1 R_2}{R_1 + R_2}$$

Retain in Memory

1. In a parallel resistor circuit, it should be noted that :

(*i*) *the potential difference across each resistor is the same and is equal to the applied potential difference.*

(*ii*) *the current through each resistor is inversely proportional to the resistance of that resistor.*

(iii) *total current through the parallel combination is the sum of the individual currents through the various resistors.*

(iv) *The reciprocal of the total resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.*

(v) *The total resistance in parallel combination is less than the least resistance used in circuit.*

(vi) *Connecting a number of resistors in parallel is equivalent to increasing the cross sectional area of the conductor. As* $R \propto \frac{1}{A}$, *therefore, total resistance of parallel combination decreases.*

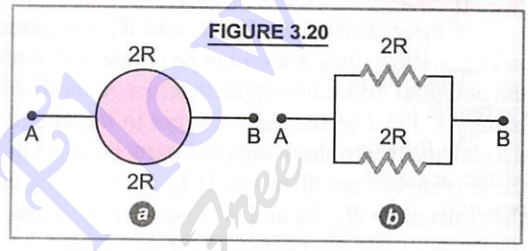
2. When resistances are connected in series, the current through each resistance is same. When the resistances are in parallel, the pot-diff. across each resistance is the same and not the current.

Sample Problem **A wire of resistance $4R$ is bent in the form of a circle Fig. 3.20(a). What is the effective resistance between the ends of the diameter ?**

Sol. Here two resistances each of value $2R$ are connected in parallel as shown in Fig. 3.20(b). So the resistance between the ends A and B of a diameter of circle is

$$R' = \frac{2R \times 2R}{2R + 2R} = R$$

FIGURE 3.20



3.24. EMF, INTERNAL RESISTANCE AND TERMINAL POTENTIAL DIFFERENCE OF A CELL

As already defined in Art 3.3,

***emf of a cell** is the maximum potential difference between two electrodes of the cell when no current is drawn from the cell.*

***Internal resistance** of a cell is defined as the resistance offered by the electrolyte and electrodes of a cell to the passage of electric current through the cell.*

The value of internal resistance of a freshly prepared cell is generally low and its value goes on increasing as the cell is put to more and more use.

Internal resistance of a cell depends upon the following factors : (i) Distance between the electrodes, (ii) The nature, concentration and temperature of the electrolyte, (iii) The nature of electrodes, (iv) Area of the electrodes, immersed in the electrolyte. If area increases, internal resistance decreases.

***Terminal Potential difference** of a cell is defined as the potential difference between the two electrodes of a cell in a closed circuit* (*i.e.*, when some current is drawn from the cell). It is generally denoted by V and is measured in volt.

When the electric cell (*source of emf*) is in a closed circuit, the current flows through the circuit. There is a fall of potential across the internal resistance of the cell. The terminal potential difference between the two electrodes of the cell becomes less than the emf of the cell by an amount equal to potential drop across the internal resistance of the cell. Thus, ***in a closed circuit the terminal potential difference of a cell is always less than the emf of the cell.***

Consider a cell of emf ε, and internal resistance r connected to an external resistor of resistance R and a one way key K as shown in Fig. 3.21(a). A high resistance voltmeter V is connected across the two electrodes (terminals) of the cell.

If the key K is not closed, the circuit remains open. No current flows through the circuit. The reading of the voltmeter is equal to emf ε of the cell, as per definition of the emf of the cell.

When the key K is closed, the current flows through the circuit as shown in Fig. 3.21(b). Now, total resistance of the circuit = $R + r$.

∴ Magnitude of current,

$$I = \frac{\varepsilon}{R+r} \qquad ...(23)$$

Potential difference across internal resistance r

$= I\,r$

It is found that the reading of voltmeter (V) now is less than ε by Ir. So

$$V = \varepsilon - Ir \qquad ...(24)$$

If $I = 0$, then

$V = \varepsilon - 0\,(r) = \varepsilon$

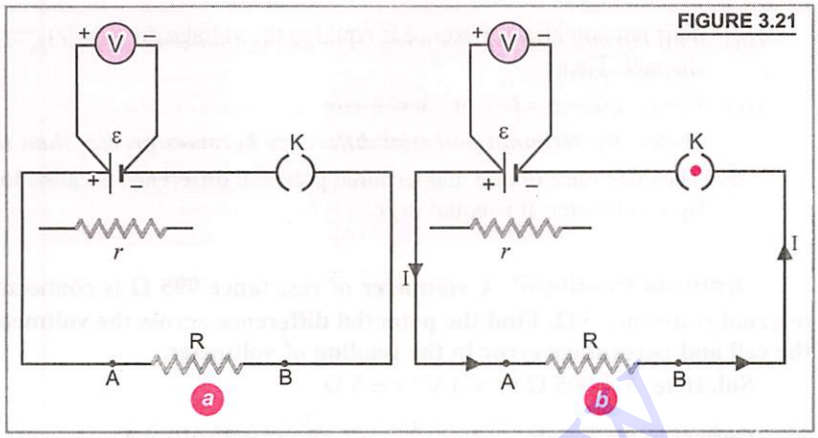


FIGURE 3.21

Thus, *the terminal potential difference of two electrodes of a cell is equal to emf of the cell when no current is drawn from the cell.*

As the point A is at the same potential as the positive electrode and the point B is at the same potential as the negative electrode of the cell, therefore, *the terminal potential difference of a cell is equal to the potential difference across the external resistance R of the circuit i.e. between points A and B,* so

$$V = IR \qquad \text{or} \qquad I = V/R$$

From (24),

$$r = \frac{\varepsilon - V}{I} = \left(\frac{\varepsilon - V}{V}\right) \times R \qquad ...(25)$$

Graph between (*a*) ε and *R* (*b*) *V* and *R* and (*c*) *V* and *I*.

(*a*) EMF of a cell is independent of external resistance R connected across the cell. So the graph between ε and R is a st. line as shown in Fig. 3.22(*a*).

(*b*) As $V = \varepsilon - Ir = \varepsilon - \frac{V}{R}r \qquad (\because I = V/R)$

or $V\left(1+\frac{r}{R}\right) = \varepsilon$ or $V = \frac{\varepsilon}{(1 + r/R)}$

Above relation shows that as R increases V also increases. A graph between V and R is a curve of the type as shown in Fig. 3.22(*b*). When $R = \infty$, then $V = \varepsilon$. In this situation the curve becomes parallel to resistance axis.

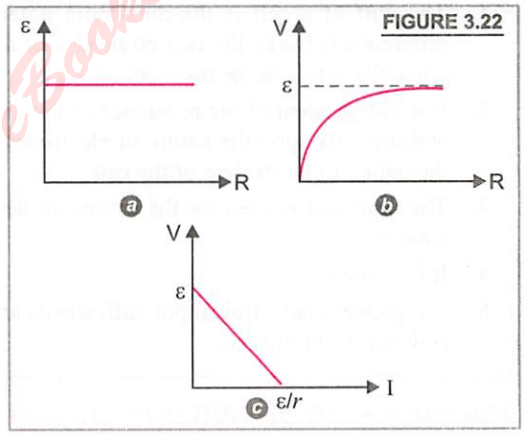


FIGURE 3.22

(*c*) As $V = \varepsilon - Ir = -Ir + \varepsilon$

This relation shows that as I increases, V decreases. It is similar to the equation of straight line, $y = mx + C$. Therefore, the graph between V and I is a st. line as shown in Fig. 3.22(*c*), whose slope ($= -r$) tells the internal resistance of the cell and intercept is ε (*i.e.*, emf of the cell)

Retain in Memory

1. In an open circuit, terminal voltage across a secondary cell is equal to its emf.
2. It is important to note that **during charging of a cell**, the positive electrode of the cell is connected to positive terminal of battery charger and negative electrode of the cell is connected to negative terminal of battery charger, Fig. 3.23. In this case, net potential difference of circuit = $V - \varepsilon$

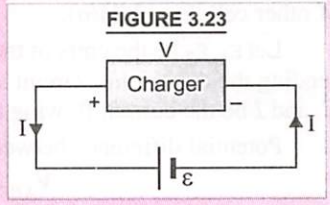


FIGURE 3.23

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This net potential difference is equal to the voltage drop (= Ir) across the internal resistance (r) of the cell. Then

$$V - \varepsilon = Ir \quad \text{or} \quad V = \varepsilon + Ir$$

Hence, the ***terminal potential difference becomes greater than the emf of the cell.***

3. The difference of emf and terminal potential difference is called **lost voltage** as it is not indicated by a voltmeter. It is equal to Ir.

Sample Problem **A voltmeter of resistance 995 Ω is connected across a cell of *emf* 3V and internal resistance 5 Ω. Find the potential difference across the voltmeter, that across the terminals of the cell and percentage error in the reading of voltmeter.**

Sol. Here, $R = 995\ \Omega$; $\varepsilon = 3$ V ; $r = 5\ \Omega$

Current in the circuit, $I = \dfrac{\varepsilon}{R+r} = \dfrac{3}{995+5} = 3 \times 10^{-3}$ A

Potential difference across the voltmeter $V = IR = (3 \times 10^{-3}) \times 995 =$ **2·985 V**

The potential difference across the terminals of the cell, $V' = \varepsilon - Ir = 3 - (3 \times 10^{-3}) \times 5 =$ **2·985 V**

As the voltmeter used is to measure the *emf* of the cell but will read 2·985 V. Hence percentage error is

$$= \frac{\varepsilon - V}{\varepsilon} \times 100 = \frac{3 - 2{\cdot}985}{3} \times 100 = \mathbf{0{\cdot}5\%}$$

3.25. DISTINCTION BETWEEN E.M.F. AND TERMINAL POTENTIAL DIFFERENCE OF A CELL

E.M.F. OF A CELL	TERMINAL POTENTIAL DIFFERENCE
1. The emf of a cell is the maximum potential difference between the two electrodes of a cell when the cell is in the open circuit.	1. The terminal potential difference of a cell is the difference of potentials between the two terminals of the cell in a closed circuit.
2. It is independent of the resistance of the circuit and depends upon the nature of electrodes and the nature of electrolyte of the cell.	2. It depends upon the resistance between the two terminals of the cell and current flowing through the cell.
3. The term emf is used for the source of electric current.	3. The terminal potential difference is measured between two terminals of the cell.
4. It is a cause.	4. It is an effect.
5. It is greater than terminal pot. diff. when current is drawn from the cell.	5. It is greater than *emf* of a cell, when the cell is being charged.

3.26. GROUPING OF TWO UNIDENTICAL CELLS IN SERIES AND PARALLEL

(*a*) Two cells in series

The two cells are said to be connected in series between two points A and C, when negative terminal of one cell is connected to positive terminal of other cell, Fig. 3.24(*a*).

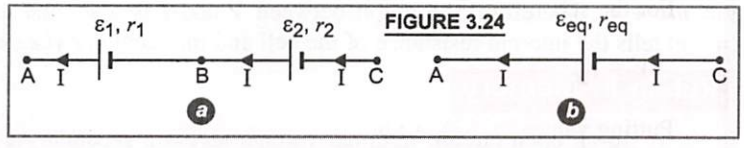


FIGURE 3.24

Let ε_1, ε_2 be the emfs of the two cells and r_1, r_2 be their internal resistances respectively. Let the cells be sending the current in a circuit shown in Fig. 3.24(*a*). Let V_A, V_B and V_C be the potentials at points A, B and C, and I be the current flowing through them.

Potential difference between positive and negative terminals of the first cell is

$$V_{AB} = V_A - V_B = \varepsilon_1 - I r_1 \qquad \text{...(26)}$$

Potential difference between positive and negative terminals of second cell is

$$V_{BC} = V_B - V_C = \varepsilon_2 - I r_2 \quad ...(27)$$

Potential difference between A and C of the series combination of the two cells is

$$V_{AC} = V_A - V_C = (V_A - V_B) + (V_B - V_C) = (\varepsilon_1 - I r_1) + (\varepsilon_2 - I r_2) = (\varepsilon_1 + \varepsilon_2) - I (r_1 + r_2) \quad ...(28)$$

If the series combination of two cells is replaced by a single cell between A and C of emf ε_{eq} and internal resistance r_{eq} as shown in Fig. 3.24(b) ; then

$$V_{AC} = \varepsilon_{eq} - I r_{eq} \quad ...(29)$$

Comparing (28) and (29), we get $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ ...(30)

and $r_{eq} = r_1 + r_2$ ...(31)

If n cells of emfs $\varepsilon_1, \varepsilon_2$ ..... ε_n and of internal resistance $r_1, r_2, ...r_n$ respectively, are connected in series between A and C, then equivalent emf, $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_n$

equivalent internal resistance of the cells is $r_{eq} = r_1 + r_2 + ... + r_n$.

Rules for the series combination of cells are as follows :

(i) The equivalent emf of a series combination of cells is equal to the sum of their individual emfs.

(ii) The equivalent internal resistance of a series combination of cells is equal to sum of their individual internal resistances.

Note that in the series combination of two cells, if negative terminal of first cell is connected to the negative terminal of the second cell between points *A* and *C*, as shown in Fig. 3.25, then

$$V_{AB} = V_A - V_B = \varepsilon_1 - I r_1 ; \quad V_{BC} = V_B - V_C = -\varepsilon_2 - I r_2$$

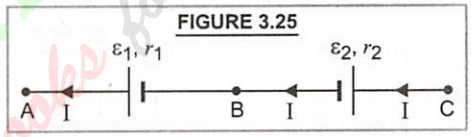
Now, $V_{AC} = V_{AB} + V_{BC} = (\varepsilon_1 - Ir_1) + (-\varepsilon_2 - Ir_2)$

$= (\varepsilon_1 - \varepsilon_2) - I (r_1 + r_2)$

Then equivalent emf of the two cells is, $\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$

But equivalent internal resistance is, $r_{eq} = r_1 + r_2$.

FIGURE 3.25



(*b*) Two cells in parallel

The two cells are said to be connected in parallel between two points A and C, if positive terminal of each cell is connected to one point and negative terminal of each cell is connected to the other point, as shown in Fig. 3.26(a). Let the cells be sending the current in a circuit shown in Fig. 3.26(a). Let $\varepsilon_1, \varepsilon_2$ be the emfs of the two cells and r_1, r_2 be their internal resistances respectively. Let I_1, I_2 be the currents from the two cells flowing towards point B_1 and I be the current flowing out of B_1. Then

$$I_1 + I_2 = I \quad ...(32)$$

Let V_{B_1}, V_{B_2} be the potentials at B_1 and B_2 respectively and V be the potential difference between B_1 and B_2. Here, the potential difference across the terminals of first cell is equal to the potential difference across the terminals of the second cell. So, for the first cell,

$$V = V_{B_1} - V_{B_2} = \varepsilon_1 - I_1 r_1 \quad \text{or} \quad I_1 = \frac{\varepsilon_1 - V}{r_1}$$

For the second cell,

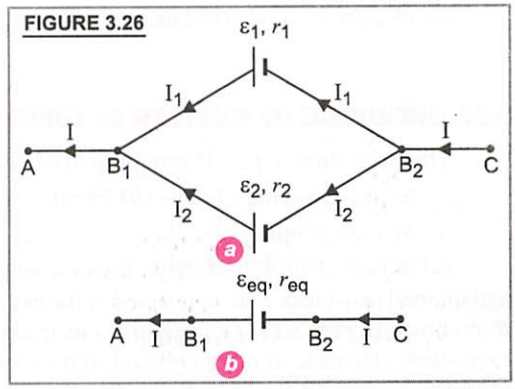
$$V = V_{B_1} - V_{B_2} = \varepsilon_2 - I_2 r_2 \quad \text{or} \quad I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Putting values in (32), we have

$$I = \left(\frac{\varepsilon_1 - V}{r_1}\right) + \left(\frac{\varepsilon_2 - V}{r_2}\right) = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

FIGURE 3.26



or $$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - \frac{I r_1 r_2}{r_1 + r_2} \quad ...(33)$$

If the parallel combination of cells is replaced by a single cell between B_1 and B_2 of emf ε_{eq} and internal resistance r_{eq} [(Fig. 3.26(*b*)], then

$$V = \varepsilon_{eq} - I r_{eq} \quad ...(34)$$

Comparing (33) and (34), we have $$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} = \frac{(\varepsilon_1/r_1) + (\varepsilon_2/r_2)}{(1/r_1) + (1/r_2)} \quad ...(35)$$

and $$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad ...(36)$$

or $$\frac{1}{r_{eq}} = \frac{r_1 + r_2}{r_1 r_2} = \frac{1}{r_1} + \frac{1}{r_2} \quad ...(37)$$

Dividing (35) by (36), we have $$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

If n cells of emfs $\varepsilon_1, \varepsilon_2, ...\varepsilon_n$ and internal resistances $r_1, r_2, ...r_n$ are connected in parallel, whose equivalent emf is ε_{eq} and equivalent internal resistance is r_{eq}, then

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n} \quad \text{and} \quad \frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + ... + \frac{\varepsilon_n}{r_n}.$$

Note that (*i*) If the two cells connected in parallel are of the same emf ε and same internal resistance r, then

from (35), $$\varepsilon_{eq} = \frac{\varepsilon r + \varepsilon r}{r + r} = \varepsilon$$

From (37), $$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r} \quad \text{or} \quad r_{eq} = \frac{r}{2}$$

Rules for parallel combination of cells are as follows :

(i) The equivalent emf of parallel combination of cells of same emf is equal to emf of one cell as this combination only increases the sizes of the electrodes and not the effective emf of all the cells.

(ii) The reciprocal of equivalent internal resistance of parallel combination of cells is equal to sum of the reciprocals of the internal resistance of each cell.

If **three cells of different emfs and internal resistances are connected in parallel,** then equivalent emf of cells $$\varepsilon_{eq} = \frac{(\varepsilon_1/r_1) + (\varepsilon_2/r_2) + (\varepsilon_3/r_3)}{(1/r_1) + (1/r_2) + (1/r_3)}$$

Equivalent internal resistance of cells, $$r_{eq} = \left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right]^{-1}$$

3.27. GROUPING OF NUMBER OF IDENTICAL CELLS

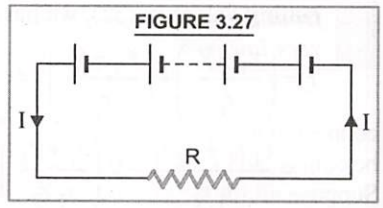
There are three types of grouping of identical cells.

(*a*) Series grouping of cells (*b*) Parallel grouping of cells

(*c*) Mixed grouping of cells.

(a) Series grouping of cells. When n identical cells each of emf ε and internal resistance r are connected to the external resistor of resistance R as shown in Fig. 3.27, *i.e.*, negative electrode of one cell is connected to positive electrode of other cell and so on, then the cells are connected in series.

FIGURE 3.27



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Equivalent emf of n cells in series, $\varepsilon_{eq} = \varepsilon + \varepsilon + ...$upto n terms $= n\varepsilon$.

Equivalent internal resistance of n cells in series,

$$r_{eq} = r + r + ...\text{upto } n \text{ terms} = nr$$

Total resistance of the circuit $= nr + R$

Current in the resistance R is given by, $I = \dfrac{n\varepsilon}{nr + R}$ ...(38)

Special cases :

Case (*i*). If $R << nr$, then R can be neglected in comparison to nr. Then from (38), $I = \dfrac{n\varepsilon}{nr} = \dfrac{\varepsilon}{r}$

Thus, the current in the external resistor is the same as current due to single cell.

Case (*ii*). If $R >> nr$, then nr can be neglected as compared to R. Then from (38), $I = \dfrac{n\varepsilon}{R}$

Thus, the current in the external resistor is n times the current due to a single cell.

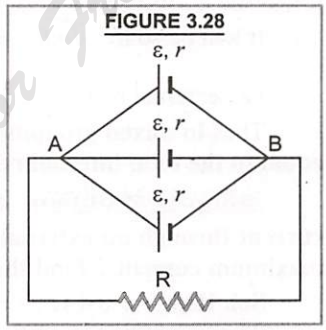
From above, we conclude that

from the series combination of cells current drawn is maximum if the value of external resistance is very high as compared to the total internal resistance of the cells.

(b) Parallel grouping of cells. When m identical cells each of emf ε and internal resistance r are connected to the external resistor of resistance R as shown in Fig. 3.28, *i.e.*, positive terminals of all the cells are connected at one point and their negative terminals to other point, then the cells are connected in parallel. As the cells are connected in parallel, their equivalent internal resistance r_p is given by

$$\frac{1}{r_p} = \frac{1}{r} + \frac{1}{r} + ...\text{upto } m \text{ terms} = \frac{m}{r} \quad \text{or} \quad r_p = \frac{r}{m}$$

FIGURE 3.28



As R and r_p are in series, so total resistance in the circuit $= R + r/m$

In **parallel combination of identical cells, the effective emf in the circuit is equal to the emf due to a single cell,** because in this combination, only the size of the electrodes increases, but not emf.

$\therefore$ Current in the resistance R is given by, $I = \dfrac{\varepsilon}{R + r/m} = \dfrac{m\varepsilon}{mR + r}$ ...(39)

Special Cases :

Case (*i*) If $R << r/m$, then R can be neglected as compared to r/m. $\therefore$ from (39), $I = \dfrac{m\varepsilon}{r}$

Thus the current in the external resistance is m times the current due to a single cell.

Case (*ii*). If $r << R$, then r can be neglected as compared to mR $\therefore$ $I = \dfrac{m\varepsilon}{mR} = \dfrac{\varepsilon}{R}$

Thus the current in the external resistance is same as current due to a single cell.

From above, we conclude that

from the parallel combination of cells current drawn is maximum if the external resistance is very low as compared to the total internal resistance of the cells.

3. Mixed Grouping of cells. If the cells are connected as shown in Fig. 3.29, *i.e.*, some cells are connected in series to form a row and number of such rows of cells are connected in parallel, they are said to be connected in mixed grouping. Let there be n cells in series in one row and m such rows of cells in parallel. Suppose all the cells are identical. Let each cell be of *emf* ε and internal resistance r.

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In each row, there are n cells in series, therefore their total internal resistance = $n\,r$

Their total *emf* = $n\varepsilon$

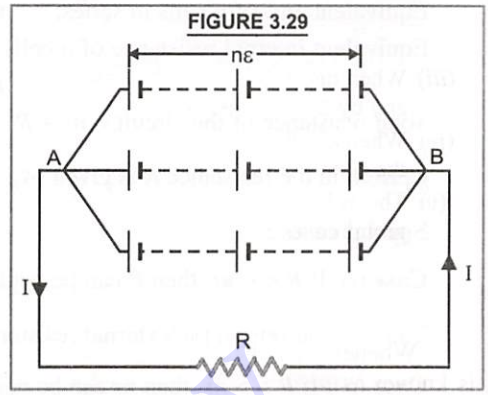
Since there are m rows of cells in parallel, therefore, total internal resistance (r_p) of all the cells is given by

$$\frac{1}{r_p} = \frac{1}{nr} + \frac{1}{nr} + \text{.....upto } m \text{ terms} = \frac{m}{nr} \quad \text{or} \quad r_p = \frac{nr}{m}$$

Total resistance in the circuit = $R + nr/m$

Since the parallel combination of rows of cells does not affect the *emf* of each row of the cells but simply increases the sizes of the electrodes, therefore, effective *emf* of all the cells = $n\varepsilon$

FIGURE 3.29



The current in the external resistance R is given by, $I = \dfrac{\text{effective } e.m.f.}{\text{total resistance}} = \dfrac{n\varepsilon}{R + nr/m} = \dfrac{mn\varepsilon}{mR + nr}$

The current I will be maximum if $(mR + nr)$ is minimum, *i.e.*

$$[(\sqrt{mR})^2 + (\sqrt{nr})^2 - 2\sqrt{mnRr}] + 2\sqrt{mnRr} = \text{minimum or } (\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnRr} = \text{minimum}$$

It will be so if, $\sqrt{mR} - \sqrt{nr} = 0$ or $\sqrt{mR} = \sqrt{nr}$ or $mR = nr$ or $\boxed{R = \dfrac{nr}{m}}$.

i.e., external resistance = total internal resistance of all the cells.

Thus **in mixed grouping of cells, current drawn is maximum if the value of external resistance is equal to the total internal resistance of all the cells.**

Sample Problem **45 cells each of internal resistance 0·4 Ω and emf 2·0 V are used to send current through an external circuit of resistance 2·0 Ω. What is the best mode of grouping them for maximum current ? Find the current in the external circuit.**

Sol. Here, $r = 0{\cdot}4\ \Omega$; $\varepsilon = 2{\cdot}0$ V, $R = 2{\cdot}0\ \Omega$

As internal resistance r (= 0·4 Ω) of a cell is comparable of external resistance R (= 2·0 Ω), so for maximum current, the cells are to be connected in mixed grouping.

Let there be n cells in a row and m rows of cells. Then

$$mn = 45 \qquad \ldots(i)$$

For maximum current in the mixed grouping

$$\frac{nr}{m} = R \quad \text{or} \quad \frac{n \times 0{\cdot}4}{m} = 2 \quad \text{or} \quad \frac{n}{m} = 5 \quad \text{or} \quad n = 5m \qquad \ldots(ii)$$

From (i) and (ii),

$$m \times 5m = 45 \quad \text{or} \quad m^2 = 9 \quad \text{or} \quad m = 3$$

$$\therefore \quad n = 5 \times 3 = 15$$

Therefore for maximum current there should be **3 rows of cells and each row should have 15 cells.**

Maximum current, $I_{max} = \dfrac{mn\varepsilon}{mR + nr} = \dfrac{45 \times 2{\cdot}0}{3 \times 2 + 15 \times 0{\cdot}4} = \mathbf{7{\cdot}5\ A}$

DO YOU KNOW ?

(*i*) When n identical cells each of emf ε and internal resistance r are connected in parallel, the total *emf* of the cells is ε, as the parallel combination of cells increases only the size of the electrodes and does not change the *emf* of the cell.

(*ii*) When n identical cells each of *emf* ε are connected in series, the total *emf* = $n\varepsilon$.

DO YOU KNOW ?

(*iii*) When one cell is wrongly connected in series of n identical cells, each of *emf* ε, it will reduce the total *emf* by 2 ε *i.e.*, the effective $emf = n\varepsilon - 2\varepsilon$.

(*iv*) When n cells each of internal resistance r are wrongly connected in series, the total internal resistance of the cells = nr, *i.e.*, there is no effect on the total internal resistance of the cells.

(*v*) The cells are connected in series to get more voltage, in parallel to get more current and in mixed grouping to get more power.

3.28. HEATING EFFECT OF CURRENT ; JOULE'S LAW

Whenever an electric current is passed through a conductor, it becomes hot after some time. This effect is known as ***heating effect of current or Joule heating effect.*** It forms the basis of working of various electrical appliances such as electric bulb, electric furnace, electric press, geyser, heat convector, immersion rod etc.

Joule in 1941, studied experimentally the heating effect of current and came to the conclusion that the amount of heat produced (H) when a current I flows through a conductor of resistance R for a time t is given by

$$H \propto I^2Rt \qquad \text{or} \qquad H = I^2 Rt \text{ joule} = \frac{I^2 Rt}{4{\cdot}2} \text{ cal.} \qquad \text{...(40)}$$

This equation (40) is known as ***Joule's law of heating.***

3.29. CAUSE OF HEATING EFFECT OF CURRENT

When a potential difference is applied across the ends of a conductor, an electric field is set up inside the conductor. The large number of free electrons present in the conductor get accelerated towards the positive end of the conductor, *i.e.*, in a direction opposite to the electric field applied and acquire kinetic energy. This is in addition to the kinetic energy due to their thermal motion. These accelerated electrons on their way suffer frequent collisions with the ions or atoms of the conductor and transfer their gained kinetic energy to them. As a result, the amplitude of thermal vibrations of the crystal lattice increases and consequently the temperature of the conductor rises. That is why ***the conductor gets heated due to flow of electric current through it.*** Obviously, the electrical energy supplied by the source of emf is converted into heat energy.

3.30. HEAT PRODUCED BY ELECTRIC CURRENT

Consider a conductor AB of resistance R, Fig. 3.30.

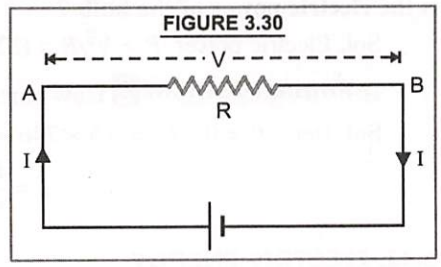


FIGURE 3.30

Let

V = potential difference (in volt) applied across the ends of AB,

I = current (in ampere) flowing through AB,

t = time (in second) for which the current is flowing.

$\therefore$ Total charge flowing from A to B in time t is

$$q = It \qquad \text{...(41)} \quad [\because \text{charge} = \text{current} \times \text{time}]$$

By definition of potential difference, work done in carrying unit charge from A to B = V

Total work done in carrying a charge q from A to B is, $W = V \times q = VIt$ joule $= I^2Rt$ joule ($\because V = IR$)

This work done is called ***electric work done.*** If this electric work done appears as heat, then amount of heat produced (H) is given by

$$H = W = I^2Rt \text{ joule} = \frac{I^2Rt}{4.18} \text{ calorie} \qquad \text{...(42)} \quad [\because 1 \text{ calorie} = 4.18\ J]$$

Equation (42) is a statement of **Joule's law of heating.** Thus, Joule's law of *heating states that the amount of heat produced in a conductor, when current is passed through it is directly proportional to*

(*i*) *square of current flowing through the conductor,* (*ii*) *resistance of the conductor and* (*iii*) *time for which the current is passed.*

3.31. ELECTRIC POWER

The rate at which electric work is done by the source of emf in maintaining the current in electric circuit is called electric power of the circuit.

If a current I ampere flows through a conductor of resistance R ohm for a time t second under a potential difference of V volt, then electric work done to maintain the current is given by $W = VIt$ joule.

$\therefore$ Electric power, $P = \frac{W}{t} = \frac{VIt}{t} = VI$ watt or Js^{-1}

Power (in watt) = V (in volt) $\times$ I (in ampere)

If, $V = 1\ V$ and $I = 1\ A$ then $P = 1$ watt. $\therefore$ 1 watt = 1 $V \times 1\ A$ = 1 ampere-volt

Thus the power of an electric circuit is said to be one watt or 1 ampere – volt if one ampere current flows in it against a potential difference of 1 volt.

Bigger units of power are **kilowatt** (*kW*) and **megawatt** (MW).

where, $1\ kW = 10^3\ W$ and $1\ MW = 10^6\ W$

Commercial unit of power is **horse power** (*hp*), where 1 *hp* = 746 watt.

As, $V = IR$ or $I = V/R$, so $P = VI = I^2R = V^2/R$

The source of this power is an external source to keep a steady current through the conductor. In a simple circuit, where the current is supplied to a conductor by the cells, it is the chemical energy of the cells which supplies this power.

Note : 1. The rate of transfer of electric energy from a source to a circuit, $P = VI$

2. The rate of transfer of electric energy to thermal or heat energy in a resistor of resistance R is

$$P = I^2R = V^2/R$$

Sample Problem 1 A bulb of 484 Ω is producing light when connected to 220 V supply. What is the electric power of the bulb ?

Sol. Electric power, $P = V^2/R = (220)^2/484$ **= 100 watt.**

Sample Problem 2 How much current is drawn by the motor of 0·5 hp from 220 volt supply ?

Sol. Here, $P = 0{\cdot}5\ hp = 0{\cdot}5 \times 746$ watt.

$$I = \frac{P}{V} = \frac{0{\cdot}5 \times 746}{220} = \mathbf{1{\cdot}7\ A}$$

3.32. ELECTRIC ENERGY

The total electric work done or energy supplied by the source of emf in maintaining the current in an electric circuit for a given time is called electric energy consumed in the circuit.

$\therefore$ Electric energy, $W = VIt = P.\ t$ $\therefore$ Electric energy = electric power $\times$ time

SI unit of electric energy is **joule**, where 1 joule = 1 volt $\times$ 1 ampere $\times$ 1 second = 1 watt $\times$ 1 second

The commercial unit of electric energy is called a **kilowatt-hour (kWh)** or **Board of Trade Unit** (***BOT***) or **UNIT of electricity** (in brief), where 1 *kWh* =1 kilo watt $\times$ 1 hour = 1000 watt $\times$ 1 hour

Thus 1 kilo watt hour is the total electric energy consumed when an electrical appliance of power 1 kilo-watt works for one hour.

$$1\ kWh = 1000\ \text{Wh} = (1000\ \text{W}) \times (60 \times 60\ \text{s}) = 3{\cdot}6 \times 10^6\ \text{J}.$$

Note that the number of units of electricity consumed = No. of kWh $= \dfrac{\text{watt} \times \text{hour}}{1000}$

$$\text{Electric energy} = VI\,t = I^2Rt = V^2t/R$$

Cost of electric consumption = No. of units of electricity consumed × cost of energy per unit.

Sample Problem **A bulb of 100 W is operating for 6 hours a day. Find the units of energy consumed in 7 days.**

Sol. Energy consumed = power × time = $100 \times 6 \times 7\ Wh = \dfrac{100 \times 6 \times 7}{1000}\ kWh$ **= 4·2 unit of electricity.**

3.33. SOME ASPECTS OF HEATING EFFECTS OF CURRENT

1. The wires supplying current to an electric lamp are not practically heated while the filament of lamp becomes white hot.

We know that in series connections, the heat produced due to current in a conductor is proportional to its resistance (*i.e.*, $H \propto R$). The filament of the lamp and the supply wires are in series. The resistance of the wires supplying the current to the lamp is very small as compared to that of the filament of the lamp. Therefore, there is more heating effect in the filament of the lamp than that in the supply wires. Due to it, the filament of the lamp becomes white hot whereas the wires remain practically unheated.

2. Electric Iron (i.e. Electric press), electric heater and heating rod are some of the important household electrical appliances, based on the heating effect of electric current. In all such appliances, the heating element used is of nichrome (an alloy of Ni and Cr). The wire of nichrome is used because

(*i*) it has high value of specific resistance. (*ii*) its melting point is high.

(*iii*) it can be easily drawn into wires. (*iv*) it is not oxidised easily when heated in air.

Electric iron, electric heater and heating rod are high power instruments. As electric power, $P = VI$. Therefore, for the given voltage V, $P \propto I$.

Thus higher the power of the electric appliance, the larger is the current drawn by it. Since the heat produced, $H \propto I^2$, hence the heat produced will be large.

It should be noted that electric power, $P = V^2/R$. Therefore, for the given voltage V, $P \propto 1/R$. This shows that the ***resistance of high electric power instrument is smaller than that of low electric power.***

The heater wire must be of high resistivity and of high melting point. Heat produced, $H = \dfrac{V^2t}{R} = \dfrac{V^2}{\rho l}At$. The resistivity is kept high so that the length l, used for the given area of cross section of the wire may be small. Nichrome wire is used in heater due to its high resistivity as compared to platinum, tungsten and copper.

3. Electric Arc. It consists of two carbon rods with a small suitable separation between them. When a high voltage (40 to 60 volt) is applied, a spark jumps across the gap and a very **bright light** is emitted in the gap, Fig. 3.31.

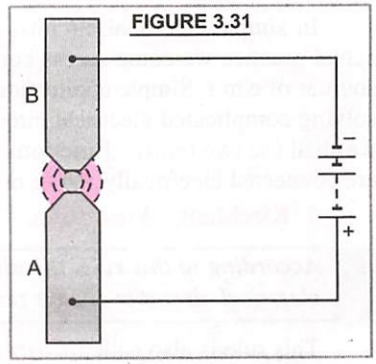
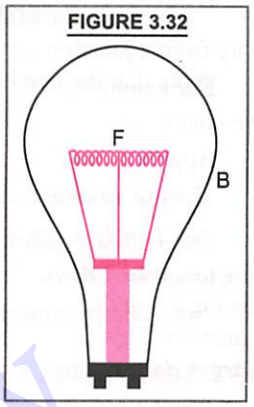


FIGURE 3.31

4. Incandescent electric lamp. It consists of a metal filament F of fine wire of tungsten enclosed in a glass bulb B with some inert gas at suitable pressure, Fig. 3.32. The metal filament used must be of very high melting point. When voltage is applied across the bulb, the current is passed through the filament. The filament gets heated to very high temperature. It then becomes white hot (*i.e.* incandescent state) and then starts emitting white light at once.

5. Fuse wire. A fuse wire is a safety device generally prepared from tin-lead alloy (63% tin + 37% lead). It should have **high resistivity, low melting point** and **suitable current rating** corresponding to the load in the circuit. It means the value of the melting point of the fuse wire should be slightly above the temperature that fuse wire can attain when current corresponding to maximum safe value flows through it.

FIGURE 3.32



The fuse wire is used in series with the electrical installations and protects them from the strong currents. All of a sudden, if strong current flows, the fuse wire melts away, causing the breakage in the circuit, thereby saving the main intallations from being damaged. Thus, very cheap fuse wire is capable of saving very costly appliances.

6. Efficiency of an electric device (η)

Efficiency of an electric device is defined as the ratio of its output power to the input power, i.e.,

$$\eta = \frac{\text{output power}}{\text{input power}}$$

In case of an electric motor, $\text{efficiency} = \frac{\text{output mechanical power}}{\text{input electric power}}$.

Here, Input electric power = output mechanical power + power lost in heat

Sample Problem **An electric motor operates on a 110 V supply and draws a current of 10 A. If the motor yields a mechanical power of 330 watt, what is the efficiency of the motor ?**

Sol. Here, input power = $VI = 110 \times 10 = 1100$ W

Output power = 330 W

$$\text{Efficiency of motor} = \frac{\text{output power}}{\text{input power}} \times 100 = \frac{330}{1100} \times 100 = \mathbf{30\%}$$

3.34. MAXIMUM POWER TRANSFER THEOREM

It states that the power output across a certain load due to a cell or battery is maximum if the load resistance is equal to the effective internal resistance of cell or battery.

It means, when the effective internal resistance of cell or battery is equal to external load resistance in a circuit, the efficiency of a battery or cell is maximum.

3.35. KIRCHHOFF'S RULES

In simple electrical circuits, resistance and potential difference are calculated using Ohm's Law. In actual practice we come across complicated electrical circuits consisting of a large number of resistors and sources of e.m.f. Simple application of Ohm's Law fails to solve these circuits. Kirchhoff gave two rules for solving complicated electrical circuits. These are known as ***Kirchhoff's Rules.*** While studying these rules, we shall use two terms : **Junction** or a **node** is a point in the electrical circuit, where two or more conductors are connected electrically. **Loop** or **Mesh** is any closed path in an electrical circuit.

1. Kirchhoff's First Rule.

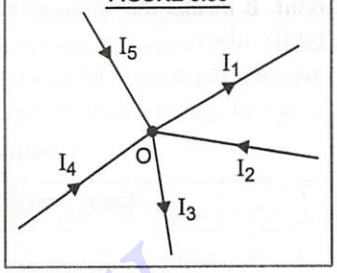
According to this rule, the algebraic sum of various currents meeting at a junction in a closed electrical circuit is always zero, i.e., $\Sigma I = 0$.

This rule is also called *Current Rule* or *Junction Theorem.*

Sign Convention. The currents flowing towards a junction are taken as positive, and the currents flowing away from a junction are taken as negative.

For example, in Fig. 3.33, O is a junction in an electrical circuit, where five conductors carrying currents I_1, I_2, I_3, I_4 and I_5 are meeting.

FIGURE 3.33



Applying Kirchhoff's first rule at junction O, we get

$$-I_1 + I_2 - I_3 + I_4 + I_5 = 0, \quad i.e., \quad \Sigma I = 0.$$

$$\therefore \quad I_2 + I_4 + I_5 = I_1 + I_3$$

i.e., total current flowing towards the junction is equal to total current flowing out of the junction. It implies that electric current cannot be stored at a junction. **This rule supports the *law of conservation of charges, i.e., the charges do not remain accumulated at a junction of a circuit.***

Proof : Multiplying the above equation with small interval of time t, we have

$$I_2 t + I_4 t + I_5 t = I_1 t + I_3 t \quad \text{or} \quad Q_2 + Q_4 + Q_5 = Q_1 + Q_3 \qquad [\because Q = It]$$

$$\text{or} \quad Q_2 + Q_4 + Q_5 - Q_1 - Q_3 = 0 \quad \text{or} \quad \Sigma Q = 0$$

Thus the total incoming charges to a junction are equal to the total outgoing charges from that junction. It shows that a junction of a circuit can not act as a source or sink of charges, *i.e.*, during a given time, the total charge entering a junction is equal to the total charge leaving that junction. This is what stated by law of conservation of charge.

Note : 1. Kirchhoff's first rule applies equally well when we consider any point on a current carrying conductor, then current entering a point is always equal to current leaving that point.

2. When motion of charge takes place across a junction momentum of charge is not conserved. It is due to accumulation of charges on the surface of wires at a junction which produces an electric field causing the variation in the direction of momentum of charge.

2. Kirchhoff's Second Rule.

Around any closed loop of a network, the algebraic sum of changes in potential must be zero, i.e., $\Sigma dV = 0$. Or, in a closed electrical loop or mesh, algebraic sum of e.m.fs is equal to algebraic sum of the products of various resistances and the respective currents flowing through them, i.e., $\Sigma \varepsilon = \Sigma IR$

This rule is called **Voltage Rule** or the **Loop Theorem.**

Sign Conventions

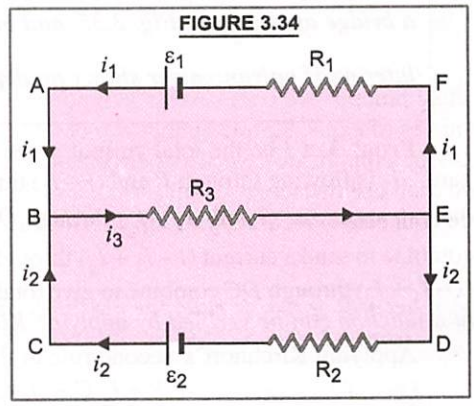
Select any point on a closed electrical loop and traverse the loop once in any direction, clockwise or anticlockwise.

(*i*) If negative terminal of a cell is encountered first, its e.m.f. is taken as positive and vice-versa.

(*ii*) If direction of current through a resistance is the same along which the loop is being traversed, then the product of resistance and current is taken as positive and vice-versa.

For example : Fig. 3.34 shows a section of electrical circuit. The current distribution is shown according to Kirchhoff's first rule. Applying Kirchhoff's second rule to the closed loop ABCDEFA, we get

FIGURE 3.34



$$-\varepsilon_2 + \varepsilon_1 = -i_2 R_2 + i_1 R_1, \quad i.e., \quad \Sigma \varepsilon = \Sigma iR$$

Similarly, for closed path *BEDCB* $\quad \varepsilon_2 = i_3 R_3 + i_2 R_2$

Similarly, for closed path *ABEFA* $\quad \varepsilon_1 = i_3 R_3 + i_1 R_1$

Kirchhoffs second rule supports the law of conservation of energy

Proof : In a closed circuit (or loop) the charges circulate around a closed loop in a particular direction due to electrostatic force which is a conservative in nature. The workdone by the electrostatic force on the circulating charge in a closed loop is zero. Due to it, the net change in the energy of charge, when goes on a closed loop is zero. It means there is no gain in energy by circulating charges around a closed loop in a particular direction. This is what stated by law of conservation of energy. Thus Kirchhoff's loop law supports the law of conservation of energy.

Comparison between Kirchhoff's first and second rules

FIRST RULE	SECOND RULE
1. According to this law $\Sigma I = 0$.	1. According to this law $\Delta V = 0$ or $\Sigma E = \Sigma IR$.
2. It can be applied to open or closed circuit.	2. It can be applied to a closed circuit.
3. It supports the law of conservation of charges.	3. It supports the law of conservation of energy.
4. It is also known as junction law or current law.	4. It is also known as mesh law or voltage law.

3.36. WHEATSTONE BRIDGE PRINCIPLE

***Wheatstone bridge* is an arrangement of four resistances in the form of a bridge which is used for measuring one unknown resistance in terms of other three known resistances.**

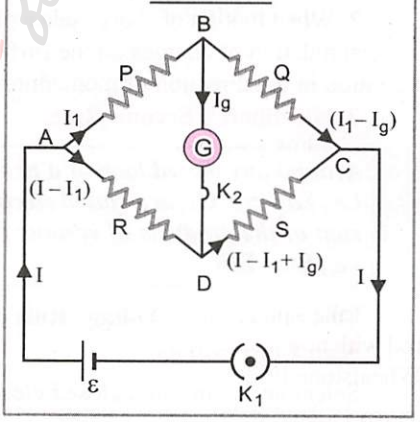
Fig. 3.35 shows a standard Wheatstone bridge. It is used for measuring unknown resistances. Resistance P in arm AB and resistance Q in the arm BC are said to form **Ratio Arms.** Arm AD contains a known resistance R which can be varied. The unknown resistance S is put in arm DC. A cell ε and one way key K_1 is connected between the points A and C. A galvanometer G and a tapping key K_2 is put between the points B and D.

On closing key K_1 first and tapping key K_2 later, if galvanometer shows no deflection, the Wheatstone bridge is said to be balanced. In that case, we shall get

$$\frac{P}{Q} = \frac{R}{S}$$

This is called *Wheatstone Bridge Principle.*

FIGURE 3.35



Wheatstone bridge principle ***states that if four resistances P, Q, R and S are connected to form a bridge as shown in Fig. 3.35, and bridge is balanced (i.e., on closing key K_1 first and key K_2 later on if galvanometer shows no deflection) then*** $\frac{P}{Q} = \frac{R}{S}$

Proof. Let I be the total current given out by the cell. On reaching the point A, it is divided into two parts : I_1 is flowing through P and $(I - I_1)$ through R. At B, the current I_1 is divided into two parts, I_g through the galvanometer G and $(I_1 - I_g)$ through Q. The current I_g through the arm BD and $(I - I_1)$ through AD, combine to send a current $(I - I_1 + I_g)$ through S. On reaching the point C, the current $(I_1 - I_g)$ through BC and $(I - I_1 + I_g)$ through DC combine to give total current I, thus completing the circuit. The *values of the currents at a junction can be verified by applying Kirchhoff's first rule at that junction.*

Applying Kirchhoff's Second rule to the closed circuit $ABDA$, we get

$$I_1 P + I_g G - (I - I_1) R = 0 \qquad \text{...(43)}$$

where G is the resistance of galvanometer. Again applying Kirchhoff's Second rule to the closed circuit $BCDB$, we get $(I_1 - I_g)\,Q - (I - I_1 + I_g)\,S - I_g G = 0$ ...(44)

The value of R is adjusted such that the galvanometer shows no deflection, *i.e.*, $I_g = 0$. The bridge is said to be balanced. Putting $I_g = 0$ in (1) and (2) we have

$$I_1 P - (I - I_1)\,R = 0 \quad \text{or} \quad I_1 P = (I - I_1)\,R \qquad ...(45)$$

and $$I_1 Q - (I - I_1)\,S = 0 \quad \text{or} \quad I_1 Q = (I - I_1)\,S \qquad ...(46)$$

Dividing (45) by (46), we get $\dfrac{P}{Q} = \dfrac{R}{S}$

This proves Wheatstone Bridge principle.

It is important to note that

(*i*) Wheatstone bridge method is a null method, hence the measurement of unknown resistance is very accurate.

(*ii*) Wheatstone bridge method remains unaffected by the internal resistance of the source and resistance of the galvanometer.

(*iii*) Wheatstone bridge becomes very sensitive when resistances in all the four arms of bridge are almost same.

(*iv*) Wheatstone bridge method can not be used to measure very high or very low resistances.

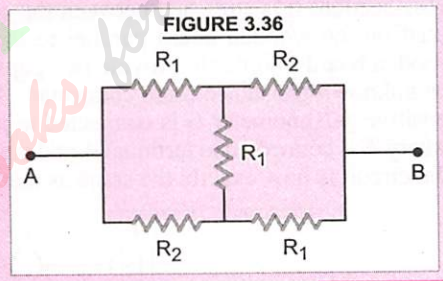
Retain in Memory

The equivalent resistance between points A and B for the circuit shown in Fig. 3.36 is

$$R_{AB} = \frac{R_1\,(R_1 + 3R_2)}{(R_2 + 3R_1)}$$

For solution. Refer to Solved Example 86, page 3/64.

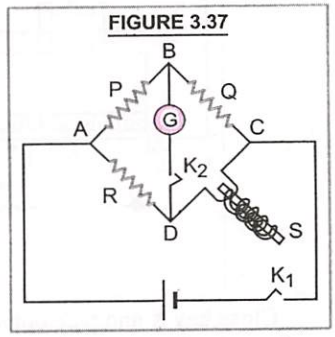
FIGURE 3.36



3.37. TEMPERATURE MEASUREMENT USING WHEATSTONE BRIDGE

Take a platinum wire about 50 cm in length. Wind it on non-conducting rod with non inductive winding. Connect this platinum wire in the arm CD of Wheatstone bridge circuit. Make other connections as shown in Fig. 3.37.

FIGURE 3.37



Working and Theory

1. Keep the platinum wire S in ice at $0°C$. Make $P = Q$, adjust the value of R so that on closing key K_1 first and key K_2 later on, galvanometer shows no deflection, *i.e.*, the bridge is balanced. If R_0 is the resistance of platinum wire at $0°C$, then

$$\frac{P}{Q} = \frac{R}{R_0} \quad \text{or} \quad R_0 = \frac{RQ}{P} = R \qquad ...(47)$$

2. Keep the platinum wire into steam at $100°C$. Repeat the experiment. Let R' be the resistance of arm AD for which the bridge is balanced. If R_{100} is the resistance of platinum wire, at 100°C, then

$$\frac{P}{Q} = \frac{R'}{R_{100}} \quad \text{or} \quad R_{100} = \frac{R'Q}{P} = R' \qquad ...(48)$$

3. Keep the platinum wire into the bath whose temperature $t°C$ is to be determined. Again repeat the experiment. Let R'' be the resistance of arm AD for which the bridge is balanced. If R_t is the resistance of platinum wire at t°C, then

$$\frac{P}{Q} = \frac{R''}{R_t} \quad \text{or} \quad R_t = \frac{R''Q}{P} = R'' \qquad ...(49)$$

If α is the temperature coefficient of resistance of platinum wire, then

$$R_{100} = R_0 (1 + \alpha \times 100) \quad \text{or} \quad \alpha = \frac{R_{100} - R_0}{R_0 \times 100} \qquad ...(50)$$

Also $$R_t = R_0 (1 + \alpha \times t) \quad \text{or} \quad \alpha = \frac{R_t - R_0}{R_0 \times t} \qquad ...(51)$$

From (50) and (51), we have

$$\frac{R_t - R_0}{R_0 \times t} = \frac{R_{100} - R_0}{R_0 \times 100}$$

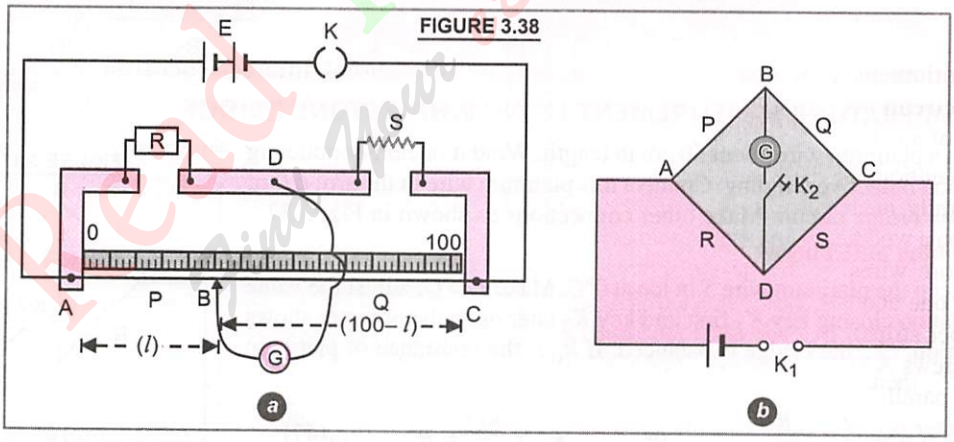
or $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{R'' - R}{R' - R} \times 100 \quad \text{[from (47), (48) and (49)]}$$

Thus, unknown temperature can be calculated.

3.38. SLIDE WIRE BRIDGE OR METER BRIDGE

A meter bridge is a practical form of Wheatstone bridge which is used for measuring unknown resistances. It consists of a wire AC of constantan or manganin of 1 metre length and of uniform area of cross-section. It is stretched between the copper strips on a horizontal wooden board. A metre scale is also fitted on the wooden board parallel to the length of the wire. There is another copper strip fitted on the wooden board in order to provide two gaps in strips. Across one gap, a resistance box R and in another gap, the unknown resistance S are connected. A jockey B can be slided on the bridge wire. One terminal of the sensitive galvanometer G is connected to jockey B and the other to the terminal D. The positive pole of the battery E is connected to terminal A and the negative pole of the battery to terminal C through one way key K. The circuit is now exactly the same as that of the Wheatstone bridge, Fig. 3.38.

FIGURE 3.38



Close key K and take out suitable resistance R from resistance box. Adjust the position of jockey on the wire (say at B) where on pressing the jockey, galvanometer shows no deflection. Note the length AB ($= l$ cm) of the wire. Find the length BC [$= (100 - l)$ cm] of the wire. Now as the bridge is balanced, therefore, according to Wheatstone bridge principle.

$$\frac{P}{Q} = \frac{R}{S}$$

If r is the resistance per cm length of wire, then P = resistance of the length l cm of the wire $AB = l\,r$

Q = resistance of the length $(100 - l)$ cm of the wire $BC = (100 - l)\,r$.

$$\therefore \quad \frac{l\,r}{(100-l)\,r} = \frac{R}{S} \quad \text{or} \quad S = \left(\frac{100-l}{l}\right) \times R$$

Knowing l and R, we can calculate S.

End error in meter bridge. It arises due to the following reasons :

(*i*) The zero mark of the scale provided along the bridge may not start from the position where the bridge wire leaves the copper strip and 100 cm mark of the scale may not be at position, where the bridge wire just touches the other copper strip.

(*ii*) The resistances of the connecting wires and copper strips of meter bridge have not been taken into account.

Removal of End error. The end error in meter bridge can be eliminated by changing R with S and S with R and repeating the experiment. The average value of S determined earlier and now is free from end error of meter bridge.

Sample Problem **In a Wheatstone bridge circuit, $P = 5\ \Omega$, $Q = 6\ \Omega$, $R = 10\ \Omega$ and $S = 5\ \Omega$. Find the additional resistance to be used in series with S, so that the bridge is balanced.**

Sol. Let the bridge be balanced when additional resistance x is put in series with S. Then,

$$(S+x) = \frac{Q}{P}R \quad \text{or} \quad x = \frac{Q}{P}R - S = \frac{6}{5} \times 10 - 5 = 7\ \Omega$$

DO YOU KNOW ?

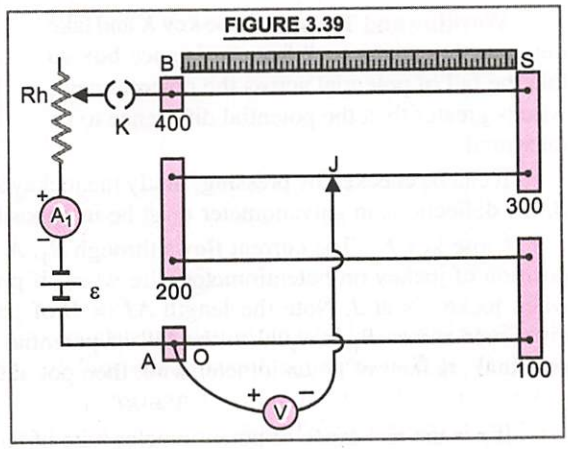
1. Meter bridge or slide wire bridge is the practical form of Wheat stone bridge.
2. The Wheat stone bridge is most sensitive when the resistance of all the four arms of the bridge is the same, *i.e.*, null point is obtained at the middle of bridge wire.
3. The Meter bridge can not be used to measure very high or very low resistances.
4. The balanced position of Meter bridge is not affected on interchanging the positions of battery and galvanometer.

3.39. POTENTIOMETER AND ITS PRINCIPLE OF WORKING

Potentiometer is an apparatus used for measuring potential difference between two points in an electrical circuit accurately. It is also used for comparing the *e.m.fs* of two cells and for measuring internal resistance of a cell.

A potentiometer consists of a long uniform wire generally made of manganin or constantan, stretched on a wooden board. (Sometimes there are four or more wires, each one metre long, fixed parallel to one another and connected in series by thick copper strips). Its ends are connected to the binding screws A and B. A metre scale is fixed on the board parallel to the length of the wire. The potentiometer is provided with a jockey J with the help of which, the contact can be made at any point on the wire, Fig. 3.39. A battery ε, an ammter A_1, a key K and a rheostat R_h are connected in series with potentiometer wire.

FIGURE 3.39



Principle. ***The working of a potentiometer is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross-section and a constant current is flowing through it.***

Suppose A is the area of cross-section and ρ is specific resistance of the material of the wire. Let V be the potential difference across the portion of the wire of length l whose resistance is R. If I is the current flowing through the wire, then from Ohm's law; $V = IR$; As, $R = \rho l/A$

$$\therefore \quad V = I\rho\frac{l}{A} = Kl, \qquad \left(\text{where } K = \frac{I\rho}{A}\right)$$

or $V \propto l$ (if I and A are constant)

i.e., potential difference across any portion of potentiometer wire is directly proportional to length of the wire of that portion.

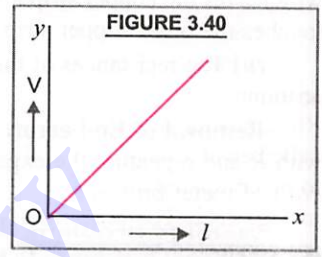


FIGURE 3.40

Here, $V/l = K =$ ***is called potential gradient, i.e., the fall of potential per unit length of wire.***

Obviously, the graph between length (l) and potential difference (V) across it will be a straight line, passing through the origin, as shown in Fig. 3.40.

Sample Problem **A current of 1·0 mA is flowing through a potentiometer wire of length 4 m and of resistance 4 Ω, find the potential gradient of potentiometer wire.**

Sol. Here ; $I = 1{\cdot}0\ mA = 10^{-3}\ A$; $L = 4\ m$; $R = 4\ \Omega$

Potential drop across potentiometer wire, $V = IR = 10^{-3} \times 4\ V$

$$\text{Potential gradient, } K = \frac{V}{L} = \frac{4\times10^{-3}}{4} = 10^{-3}\ \text{V/m}$$

3.40. DETERMINATION OF POTENTIAL DIFFERENCE USING POTENTIOMETER

A battery of *emf* ε is connected between the end terminals A and B of potentiometer wire with ammeter A_1, resistance box R and key K in series. This circuit is called an auxiliary circuit. The ends of resistance R_1 are connected to terminals A and jockey J through galvanometer G. A cell ε_1 and key K_1 are connected across R_1 as shown in Fig. 3.41.

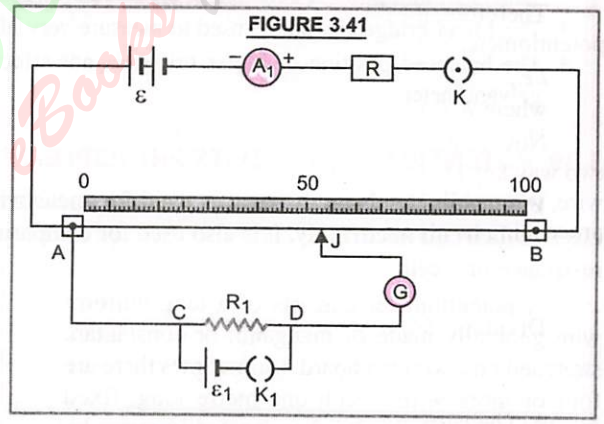


FIGURE 3.41

Working and Theory. Close key K and take out suitable resistance R from resistance box so that the fall of potential across the potentiometer wire is greater than the potential difference to be measured.

It can be checked by pressing, firstly the jockey J on potentiometer wire near end A and later on near end B, the deflections in galvanometer must be in opposite directions.

Close key K_1. The current flows through R_1. A potential difference is developed across R_1. Adjust the position of jockey on potentiometer wire where if pressed, the galvanometer shows no deflection. Let it be when jockey is at J. Note the length AJ $(= l)$ of potentiometer wire. This would happen when potential difference across R_1 is equal to the fall of potential across the potentiometer wire of length l. If K is the potential gradient of potentiometer wire, then pot. diff. across R_1, *i.e.*,

$$V = Kl \qquad \text{...(52)}$$

If r is the resistance of potentiometer wire of length L, then current through potentiometer wire is

$$I = \frac{\varepsilon}{R+r}$$

$$\text{Potential drop across potentiometer wire } = Ir = \left(\frac{\varepsilon}{R+r}\right) r$$

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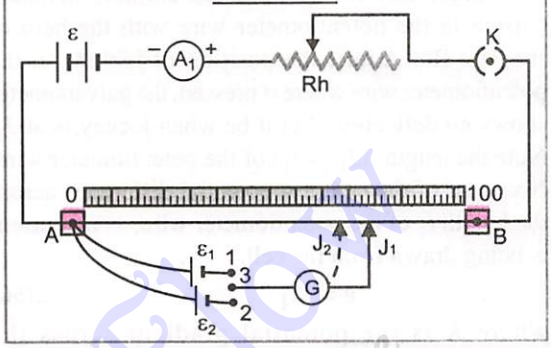
Potential gradient of potentiometer wire, *i.e.*, fall of potential per unit length is

$$K = \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L} \qquad \text{From (10),} \qquad \boxed{V = \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L}l} \qquad ...(53)$$

Hence, V can be calculated.

3.41. COMPARISON OF EMFs OF TWO CELLS USING POTENTIOMETER

FIGURE 3.42



A battery of emf ε is connected between the end terminals A and B of potentiometer wire with rheostat Rh, ammeter A_1 and key K in series. This circuit is called an auxiliary circuit. Two cells of emfs ε_1 and ε_2 to be compared are connected to the circuit as shown in Fig. 3.42. The positive terminals of both the cells are connected to point A of the potentiometer. Their negative terminals are connected to two terminals 1 and 2 of two way key, while its common terminal 3 is connected to jockey J through a galvanometer G.

Close key K and adjust a suitable constant current in the potentiometer wire with the help of rheostat. Insert the plug in the gap between the terminals 1 and 3 of two way key so that the cell of emf ε_1 is in the circuit. Adjust the position of jockey on potentiometer wire, where if pressed, the galvanometer shows no deflection. Let it be when jockey is at J_1. Note the length AJ_1 ($= l_1$ say) of the wire. As the galvanometer is showing no deflection, therefore, there is no current in the arm $A\varepsilon_1 J_1$. It means the potential of positive terminal of cell = potential of the point A, and the potential of negative terminal of cell = potential of the point J_1.

Therefore, the e.m.f. of the cell ($= \varepsilon_1$) is equal to potential difference between the points A and J_1 of the potentiometer wire.

i.e.,
$$\varepsilon_1 = Kl_1 \qquad ...(54)$$

where K is the potential gradient across the potentiometer wire.

Now remove the plug from the gap between 1 and 3 and insert the same in the gap between 2 and 3 of two way key so that cell of emf ε_2 comes into the circuit. Again, find the position of jockey on potentiometer wire, where galvanometer shows no deflection. Let it be at J_2. Note the length of the wire AJ_2 ($= l_2$ say). Then proceeding as above,

$$\varepsilon_2 = Kl_2 \qquad ...(55)$$

Dividing (54) by (55), we get $\boxed{\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{l_1}{l_2}}$

Precautions of experiment

1. The current in the potentiometer wire from driving cell ε must be kept constant during experiment.

2. While adjusting the position of jockey on potentiometer wire, the edge of jockey should not be rubbed on the wire, otherwise area of cross-section of wire may not remain uniform and constant.

3. The current in the potentiometer wire from driving cell should not be passed continuously for long time as this would cause heating effect, resulting in the change in resistance of wire.

4. The *emf* of the driving cell must be greater than the *emf* of the cells whose values are to be compared, otherwise null point will not be obtained on potentiometer wire.

5. The positive terminals of all the cells (ε, ε_1 and ε_2) are to be connected at zero end A of potentiometer wire, otherwise null point will not be obtained on potentiometer wire. All lengths are measured from this end.

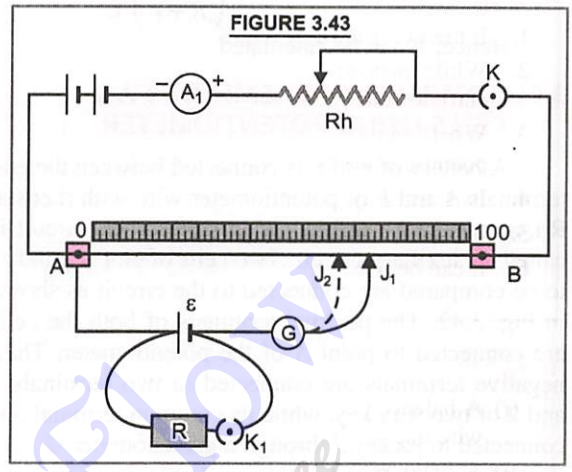
DO YOU KNOW ?

The potentiometer method is a null method. It is a very accurate method. Its working does not depend on the accuracy of galvanometer reading. Potentiometer acts like an ideal voltmeter of infinite resistance, because when galvanometer shows no deflection, no current is drawn from the cell.

3.42. DETERMINATION OF INTERNAL RESISTANCE OF A CELL BY POTENTIOMETER METHOD

To find the internal resistance r of a cell of emf ε using potentiometer, set up the circuit as shown in Fig. 3.43.

FIGURE 3.43



Close key K and maintain suitable constant current in the potentiometer wire with the help of rheostat Rh. Adjust the position of jockey on the potentiometer wire where if pressed, the galvanometer shows no deflection. Let it be when jockey is at J_1. Note the length AJ_1 ($= l_1$) of the potentiometer wire. Now emf of the cell, ε = potential difference across the length l_1 of the potentiometer wire, as no current is being drawn from the cell.

$$\therefore \qquad \varepsilon = Kl_1 \qquad \text{...(56)}$$

where K is the potential gradient across the potentiometer wire.

Take out suitable resistance R from the resistance box in the cell circuit and close key K_1. Again find the position of the jockey on the potentiometer wire where galvanometer shows no deflection. Let it be at J_2. Note the length of the wire AJ_2 ($= l_2$ say). As current is being drawn from the cell, its terminal potential difference V and not emf ε is balanced across AJ_2. Therefore, potential difference between two terminals of the cell, V = potential difference across the length l_2 of the potentiometer wire

i.e.,
$$V = Kl_2 \qquad \text{...(57)}$$

Dividing (56) by (57), we have
$$\frac{\varepsilon}{V} = \frac{l_1}{l_2} \qquad \text{...(58)}$$

We know that the internal resistance r of a cell of emf ε, when a resistance R is connected in its circuit is given by

$$r = \frac{\varepsilon - V}{V} \times R = \left(\frac{\varepsilon}{V} - 1\right) R \qquad \text{...(59)}$$

Using (58), we get
$$r = \left(\frac{l_1}{l_2} - 1\right) R = \frac{l_1 - l_2}{l_2} \times R$$

Thus, knowing the values of l_1, l_2 and R, the internal resistance r of the cell can be determined.

3.43. SENSITIVENESS OF POTENTIOMETER

The sensitiveness of potentiometer means the smallest potential difference that can be measured with its help.

The sensitiveness of a potentiometer can be increased by decreasing its potential gradient (V/l). The same can be achieved

(*i*) by increasing the length of potentiometer wire.

(*ii*) If the potentiometer wire is of fixed length, the potential gradient can be decreased by reducing the current in the potentiometer wire circuit by increasing the resistance, with the help of rheostat.

3.44. DIFFERENCE BETWEEN POTENTIOMETER AND VOLTMETER

POTENTIOMETER	VOLTMETER
1. It measures the emf of a cell very accurately.	1. It measures the emf of a cell approximately.
2. While measuring emf, it does not draw any current from the source of known emf.	2. While measuring emf, it draws some current from the source of emf.
3. While measuring emf, the resistance of potentiometer becomes infinite.	3. While measuring emf the resistance of voltmeter is high but finite.
4. Its sensitivity is high.	4. Its sensitivity is low.
5. It is based on null deflection method.	5. It is based on deflection method.
6. It can be used for various purposes.	6. It can be used only to measure emf or potential difference.

DO YOU KNOW ?

(*i*) A balance point is obtained on the potentiometer wire if the fall of potential along the potentiometer wire due to driving cell is greater than the *emf* of the cell to be balanced.

(*ii*) A balance point can not be obtained on the potentiometer wire if the fall of potential along the potentiometer wire due to driving cell is less than the *emf* of the cell to be balanced.

(*iii*) The potentiometer is an ideal voltmeter as it measures the *emf* of a cell very accurately whereas voltmeter measures the *emf* of cell approximately.

(*iv*) The positive terminals of driving cell and the cell whose *emf* is to be determined must be connected to the zero end of the potentiometer wire, otherwise null point is never obtained. Zero end is the end from where balancing length of potentiometer wire is to be measured.

SOLVED EXAMPLES

TYPE I. ELECTRIC CURRENT

Formulae used. (*i*) $I = \frac{Q}{t} = \frac{ne}{t} = ev = \frac{ev}{2\pi r}$

(*ii*) $I = Q/t = (n_e + n_p)\, e/t$.

where Q is the charge flowing, t is the time. n_e is the number of electrons and n_p is the number of positive ions or holes crossing through a cross section of conductor and e is the charge of electron or proton or hole = $1{\cdot}6 \times 10^{-19}$ C.

v is the frequency of revolution of electron in an orbit of nucleus of radius r with speed v.

Units used. Q is in coulomb, t in seconds, I in ampere. v in s^{-1} or Hz and v in ms^{-1}.

Example 1 **How many electrons pass through a lamp in one minute, if the current is 300 mA ?** **(Pb. Board 2002)**

Solution. Here, $I = 300$ mA $= 300 \times 10^{-3}$ A ;

$t = 60$ s, $n = ?$

$$I = \frac{ne}{t} \quad \text{or} \quad n = \frac{It}{e} = \frac{(300\times10^{-3})\times60}{1{\cdot}6\times10^{-19}}$$

$$= \mathbf{1{\cdot}125 \times 10^{20}}$$

Example 2 **In an atom an electron revolves around the nucleus in a circular orbit at the rate of 6×10^{15} revolutions per second. Calculate the equivalent current in milliampere. Take value of electronic charge = $1{\cdot}6 \times 10^{-19}$ C.**

Solution. Here, $v = 6 \times 10^{15}$ s^{-1} ;

$e = 1{\cdot}6 \times 10^{-19}$ C

Equivalent current, $I = ev$

$= (1{\cdot}6 \times 10^{-19}) \times 6 \times 10^{15} = \mathbf{9{\cdot}6 \times 10^{-4}}$ **A**

$= 9{\cdot}6 \times 10^{-4} \times 10^{3}$ mA

$= \mathbf{0{\cdot}96}$ **mA**

Example 3 **In Bohr model of hydrogen atom, the electron revolves around the nucleus in a circular orbit of radius $5{\cdot}0 \times 10^{-11}$ m with a speed $2{\cdot}2 \times 10^6$ ms^{-1}. Find the equivalent current.**

(Electronic charge = $1{\cdot}6 \times 10^{-19}$ coulomb)

Solution. Here, $r = 5{\cdot}0 \times 10^{-11}$ m ;

$v = 2{\cdot}2 \times 10^6$ ms^{-1}, $e = 1{\cdot}6 \times 10^{-19}$ C.

Let ν be the frequency of revolution of electron in hydrogen, then

$$\nu = \frac{1}{T} = \frac{1}{(2\pi r/v)} = \frac{v}{2\pi r}$$

Current,

$$I = e\nu = \frac{ev}{2\pi r} = \frac{(1{\cdot}6\times10^{-19})\times(2{\cdot}2\times10^6)}{2\times\left(\frac{22}{7}\right)\times5{\cdot}0\times10^{-11}}$$

$$= \mathbf{1{\cdot}12 \times 10^{-3}\ A}$$

Example 4 **If 0·8 mole of electrons flow through a wire in 55 minutes. What is (*a*) the total charge in kilocoulomb that passes through the wire and (*b*) the magnitude of the current ? Avogadro's number = 6×10^{23} per mole.**

Solution. No. of electrons in 0·8 mole,

$$N = 0{\cdot}8 \times 6 \times 10^{23}$$

(*a*) Charge flowing,

$q = Ne = (0{\cdot}8 \times 6 \times 10^{23}) \times (1{\cdot}6 \times 10^{-19})$ C

$= 7{\cdot}68 \times 10^4$ C $= 76{\cdot}8 \times 10^3$ C

= 76·8 kilo coulomb

(*b*) Current $= \frac{q}{t} = \frac{76{\cdot}8\times10^3\ \text{C}}{(55\times60)\ \text{s}} =$ **23·3 A**

Example 5 **In a conductor, 10^{16} electrons move from a point A towards point B in 1 milli second. 10^{14} positive ions move from a point B towards point A in 1 milli second. What is the current in ampere and its direction ? Charge on electron = charge on positive ion = $1{\cdot}6 \times 10^{-19}$ C.**

Solution. Here, $n_e = 10^{16}$; $n_{\text{ion}} = 10^{14}$, $t = 10^{-3}$ s, charge on electron or ion, $q = 1{\cdot}6 \times 10^{-19}$ C.

Current, $I = \frac{n_e \times q + n_{\text{ion}} \times q}{t}$

$$= \frac{10^{16}\times1{\cdot}6\times10^{-19} + 10^{14}\times1{\cdot}6\times10^{-19}}{10^{-3}}$$

$$= \frac{10^{16}\times1{\cdot}6\times10^{-19}}{10^{-3}} = \mathbf{1{\cdot}6\ A}$$

The direction of current is from **B to A.**

Example 6 **The charge flowing in a conductor varies with time as,**

$$q = at - \frac{1}{2}bt^2 + \frac{1}{6}ct^3$$

where *a*, *b*, *c* are positive constants. Then, find (i) the initial current (ii) the time after which the value of current reaches a maximum value (iii) the maximum or minimum value of current.

Solution. (*i*) Current, $i = \frac{dq}{dt}$

$$= \frac{d}{dt}\left(at - \frac{1}{2}bt^2 + \frac{1}{6}ct^3\right)$$

$$= a - bt + \frac{1}{2}ct^2 \qquad \text{...(i)}$$

When $t = 0$, initial current, $i = a$

(*ii*) $\frac{di}{dt} = -b + ct$

For i to be maximum or minimum,

$$\frac{di}{dt} = 0 = -b + ct \quad \text{or} \quad t = \frac{b}{c}$$

(*iii*) Putting this value of t in (*i*), we have

$$i = a - b\times\frac{b}{c} + \frac{1}{2}c\times\frac{b^2}{c^2} = a - \frac{b^2}{c} + \frac{b^2}{2c} = a - \frac{b^2}{2c}$$

As this value of i is less than that at $t = 0$, it must be minimum. So

$$\text{minimum value of current} = a - \frac{b^2}{2c}$$

Example 7 **A steady beam of α-particles travelling with kinetic energy $E = 83{\cdot}5$ keV carries a current of $I = 0{\cdot}2$ μA. Mass of α-particle = $6{\cdot}68 \times 10^{-27}$ kg.**

(i) If this beam strikes a plane surface at an angle $\theta = 60°$ with normal to the surface, how many α-particles strike the surface in t = 4 second ?

(ii) How many α-particles are there in length $l = 20$ cm of the beam ?

Solution. Here, $I = 0{\cdot}2\ \mu\text{A} = 0{\cdot}2 \times 10^{-6}$ A,

$E = 83{\cdot}5$ keV $= 83{\cdot}5 \times 10^3$ eV

$= 83{\cdot}5 \times 10^3 \times 1{\cdot}6 \times 10^{-19}$ J

$= 83{\cdot}5 \times 1{\cdot}6 \times 10^{-16}$ J

(*i*) Charge on α-particle,

$q = 2e = 2 \times 1{\cdot}6 \times 10^{-19}$ C

No. of α-particles crossing per second a cross-section of beam,

Read Your Flow Back to Free

$$n = \frac{I}{q} = \frac{0{\cdot}2\times10^{-6}}{2\times1{\cdot}6\times10^{-19}} = 6{\cdot}25\times10^{11}\ s^{-1}$$

No. of α-particles striking the surface in 4 second = $nt = 6{\cdot}25 \times 10^{11} \times 4 = \mathbf{2{\cdot}5 \times 10^{12}}$

[**Note.** There is no significance of angle θ (= 60°) for finding the number of α-particles striking the surface.]

(ii) If v is the velocity of α-particle while travelling towards a surface, then $E = \frac{1}{2}mv^2$

$$\text{or}\quad v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times83{\cdot}5\times1{\cdot}6\times10^{-16}}{6{\cdot}68\times10^{-27}}}$$

$$= 2 \times 10^6\ ms^{-1}$$

It means a beam of length $v = 2 \times 10^6$ m crosses a section in one second. But number of α-particle passing through a section in one second is,

$$n = 6{\cdot}25 \times 10^{11}$$

$\therefore$ No. of α-particles in unit length of the beam $= \frac{n}{v}$

No. of α-particles in length l of the beam

$$= \frac{nl}{v} = \frac{6{\cdot}25\times10^{11}}{2\times10^6}\times0{\cdot}20 = \mathbf{6{\cdot}25 \times 10^4}$$

TYPE II. DRIFT VELOCITY AND ELECTRON MOBILITY

Formulae used

(i) $I = ne\,A\,v_d$ *(ii)* $\mu_e = v_d/E = q\tau/m$

where, n = number density of electrons *i.e.* number of conduction electrons per unit volume of conductor, A = area of cross section of the conductor, v_d = drift velocity of electron,

E = strength of electric field applied = V/l,

V = potential difference across the conductor of length l.

Units used. n in m^{-3}, A in m^2, v_d in ms^{-1}, E in N/C or V/m and l in m.

Example 8 **A copper wire of diameter 1·0 mm carries a current of 0·2 A. Copper has $8{\cdot}4 \times 10^{28}$ atoms per cubic metre. Find the drift velocity of electrons, assuming that one charge carrier of $1{\cdot}6 \times 10^{-19}$ C is associated with each atom of the copper.**

Solution. Here, $D = 1{\cdot}0$ mm $= 1 \times 10^{-3}$ m,

$I = 0{\cdot}2$ A, $n = 8{\cdot}4 \times 10^{28}\ m^{-3}$; $e = 1{\cdot}6 \times 10^{-19}$ C.

$$v_d = \frac{I}{nAe} = \frac{I}{n(\pi D^2/4)e}$$

$$= \frac{0{\cdot}2}{(8{\cdot}4\times10^{28})\times\frac{22}{7}\times\frac{(10^{-3})^2}{4}\times1{\cdot}6\times10^{-19}}$$

$$= \mathbf{1{\cdot}895 \times 10^{-5}\ ms^{-1}}$$

Example 9 **A 60 coulomb of charge flows through a wire in half minute. The radius of the wire is 1 mm. The wire contains 5×10^{22} electrons per cubic centimetre. Calculate the current and drift velocity.**

Solution. Here, $q = 60$ C, $t = 30$ seconds ;

$r = 10^{-3}$ m ; $n = 5 \times 10^{22}$/cc $= 5 \times 10^{28}\ m^{-3}$

Current, $I = \frac{q}{t} = \frac{60}{30} = 2$ A

Drift velocity, $v_d = \frac{I}{neA} = \frac{I}{ne\pi r^2}$

$$= \frac{2}{(5\times10^{28})\times(1{\cdot}6\times10^{-19})\times3{\cdot}142\times(10^{-3})^2}$$

$$= \mathbf{7{\cdot}95 \times 10^{-5}\ ms^{-1}}$$

Example 10 **A current of 3 A is flowing through a wire of length 2 m and cross-sectional area 1 mm^2. If wire contains 10^{29} electrons/m^3, calculate the average time taken by an electron to cross the length of the wire.**

Solution. Here, $I = 3$ A, $l = 2$ m,

$A = 1\ mm^2 = 10^{-6}\ m^2$, $n = 10^{29}\ m^{-3}$

Drift velocity, $v_d = \frac{I}{neA}$

$$= \frac{3}{10^{29}\times(1{\cdot}6\times10^{-19})\times10^{-6}}$$

$$= 1{\cdot}875 \times 10^{-4}\ ms^{-1}$$

Time taken, $t = \frac{l}{v_d} = \frac{2}{1{\cdot}875\times10^{-4}} = \mathbf{1{\cdot}07 \times 10^4\ s}$

Example 11 **If the free electron density of copper is $8{\cdot}6 \times 10^{28}\ m^{-3}$ and resistivity of copper at room temperature is $1{\cdot}7 \times 10^{-6}\ \Omega$ cm, find the relaxation time for the free electrons of copper. Given, mass of electron = $9{\cdot}1 \times 10^{-31}$ kg ; and charge of electron = $1{\cdot}6 \times 10^{-19}$ C.**

Solution. Here, $m = 8{\cdot}6 \times 10^{28}\ m^{-3}$;
$\rho = 1{\cdot}7 \times 10^{-6}\ \Omega$ cm $= 1{\cdot}7 \times 10^{-8}\ \Omega$m ;
$m = 9{\cdot}1 \times 10^{-31}$ kg, $e = 1{\cdot}6 \times 10^{-19}$

As, $\rho = \dfrac{m}{ne^2\tau}$; so, $\tau = \dfrac{m}{ne^2\rho}$

$$\therefore\ \tau = \frac{9{\cdot}1\times10^{-31}}{(8{\cdot}6\times10^{28})\times(1{\cdot}6\times10^{-19})^2\times(1{\cdot}7\times10^{-8})}$$

$$= 2{\cdot}43 \times 10^{-14}\ s$$

Example 12 **What is the drift velocity for the electrons in a conductor when an electric field of strength 200 V/m is applied on it and mobility of electrons is $4{\cdot}5 \times 10^{-6}\ m^2\ V^{-1}\ s^{-1}$?**

Solution. Here, $E = 200$ V/m ;
$\mu = 4{\cdot}5 \times 10^{-6}\ m^2\ V^{-1}\ s^{-1}$

Drift velocity, $v_d = \mu E = (4{\cdot}5 \times 10^{-6}) \times 200$

$$= 9{\cdot}0 \times 10^{-4}\ ms^{-1}$$

Example 13 **If the current flowing through a copper wire of 1 mm diameter is 1·1 A. The density of copper is 9 g cm^{-3} and atomic mass of copper is 63 u. One free electron is contributed by each atom of copper. Find the drift velocity of electron. Avogadro's number = $6{\cdot}0 \times 10^{26}$ per kg-atom.**

Solution. Here, $2r = 1$ mm $= 10^{-3}$ m

or $r = \dfrac{1}{2}\times10^{-3}$ m ; $I = 1{\cdot}1$ A

$d = 9$ g $cm^{-3} = 9 \times 10^3$ kg/m^3.

Therefore, mass of 1 m^3 of copper $= 9 \times 10^3$ kg

Atomic mass, $M = 63$ u $= 63$ kg.

Avogadro's number,

$N = 6{\cdot}0 \times 10^{26}$ per kg atom.

So $6{\cdot}0 \times 10^{26}$ atoms of copper has a mass = 63 kg

No. of atoms per m^3 of copper $= \dfrac{N}{M/d} = \dfrac{Nd}{M}$

As 1 atom contributes one conduction electron, therefore, number of conduction electrons per m^3 of copper is

$$n = \frac{Nd}{M} = \frac{(6{\cdot}0\times10^{26})\times(9\times10^3)}{63}$$

$$= 8{\cdot}57 \times 10^{28}\ m^{-3}$$

$$v_d = \frac{I}{neA}$$

$$= \frac{1{\cdot}1}{(8{\cdot}57\times10^{28})\times(1{\cdot}6\times10^{-19})\times\left[\dfrac{22}{7}\times\left(\dfrac{1}{2}\times10^{-3}\right)^2\right]}$$

$$\simeq 10^{-4}\ m/s$$

Example 14 **Find the current flowing through a copper wire of length 0·2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that electron mobility = $4{\cdot}5 \times 10^{-6}\ m^2\ V^{-1}\ s^{-1}$ and charge on electron = $1{\cdot}6 \times 10^{-19}$ C. The number density of electron in copper is $8{\cdot}5 \times 10^{28}\ m^{-3}$.**

Solution. Here, $l = 0{\cdot}2$ m ;
$A = 1\ mm^2 = 10^{-6}\ m^2$; $V = 4$ volt ;
$\mu = 4{\cdot}5 \times 10^{-6}\ m^2\ V^{-1}\ s^{-1}$;
$e = 1{\cdot}6 \times 10^{-19}$ C and $n = 8{\cdot}5 \times 10^{28}\ m^{-3}$.

We know that electric field set up across the conductor, $E = \dfrac{V}{l} = \dfrac{4}{0{\cdot}2} = 20\ Vm^{-1}$

Now, current through wire is

$$I = nAev_d = nAe\mu E = (8{\cdot}5 \times 10^{28}) \times (10^{-6}) \times (1{\cdot}6 \times 10^{-19}) \times (4{\cdot}5 \times 10^{-6}) \times 20$$

$$= 1{\cdot}22\ A$$

Example 15 **(*a*) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1{\cdot}0 \times 10^{-7}\ m^2$ carrying a current of 1·5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is $9{\cdot}0 \times 10^3$ kg/m^3, and its atomic mass is 63·5 u.**

(*b*) Compare the drift speed obtained above with (*i*) thermal speed of electrons carrying the current at 27°C and (*ii*) speed of propagation of electric field along the conductor which causes the drift motion.

Avogadro's number = $6{\cdot}0 \times 10^{26}$ per kg-atom. Boltzmann constant $k = 1{\cdot}38 \times 10^{-23}$ J/K ; mass of electron = $9{\cdot}1 \times 10^{-31}$ kg.

(NCERT Solved Example)

Solution. *(a)* Here, $A = 1{\cdot}0 \times 10^{-7}\ m^2$;
$I = 1{\cdot}5\ A$; $d = 9{\cdot}0 \times 10^3$ kg/m^3
Atomic mass, M = 63·5 u = 63·5 kg
Avogadro's number, $N = 6{\cdot}0 \times 10^{26}$ per kg atom
Number of atoms per unit volume in the given copper wire $= \dfrac{N}{M/d} = \dfrac{Nd}{M}$

As one atom contributes one conduction electron, therefore, number of conduction electrons per unit volume or number density of conduction electrons in the given copper wire is

$$n = \frac{N}{M}d = \frac{6{\cdot}0\times10^{26}\times9{\cdot}0\times10^3}{63{\cdot}5} = 8{\cdot}5 \times 10^{28}\ m^{-3}$$

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$$v_d = \frac{I}{neA}$$

$$= \frac{1{\cdot}5}{(8{\cdot}5\times10^{28})\times(1{\cdot}6\times10^{-19})\times(1{\cdot}0\times10^{-7})}$$

$$= \mathbf{1{\cdot}1 \times 10^{-3}\ ms^{-1}}$$

(*b*) (*i*) The thermal speed of electron of mass m at temperature T is,

$$v = \sqrt{\frac{3kT}{m}}\ ;$$

where k is Boltzmann constant.

Here, $T = 300\ K$;

$k = 1{\cdot}38 \times 10^{-23}$ J mole^{-1} K^{-1} ;

$m = 9{\cdot}1 \times 10^{-31}$ kg

$$\therefore \quad v = \left[\frac{3\times(1{\cdot}38\times10^{-23})\times300}{9{\cdot}1\times10^{-31}}\right]^{1/2}$$

$$= 1{\cdot}17 \times 10^5\ ms^{-1}$$

$$\therefore \quad \frac{v_d}{v} = \frac{1{\cdot}1\times10^{-3}}{1{\cdot}17\times10^5} \approx \mathbf{10^{-8}}$$

(*ii*) The speed of propagation of electric field is equal to the speed of an electromagnetic wave,

$c = 3 \times 10^8$ m/s.

$$\therefore \quad \frac{v_d}{c} = \frac{1{\cdot}1\times10^{-3}}{3\times10^8} = 0{\cdot}36\times10^{-11} = \mathbf{3{\cdot}6 \times 10^{-12}}$$

TYPE III. OHM'S LAW, RESISTANCE, RESISTIVITY

Formulae used. (*i*) $V = IR$

(*ii*) $R = \rho l/A$ or $\rho = RA/l$ (*iii*) $\rho = \dfrac{m}{e^2 n\tau}$

where ρ is the resistivity of the material of the wire conductor, l is the length and A is the area of cross-section of the wire, e is the charge of electron ($= 1{\cdot}6 \times 10^{-19}$ C), n is the number density of electron in the conductor and τ is the average time of relaxation.

Units used. V in volt, I in amp, R in ohm, l in metre, A in m^2, ρ in Ω m and n in m^{-3}

Example 16 **A wire of resistance 5·0 Ω is used to wind a coil of radius 5 cm. The wire has a diameter 2·0 mm and the specific resistance of its material is $2{\cdot}0 \times 10^{-7}$ Ωm. Find the number of turns in the coil.**

Solution. Here, $R = 5{\cdot}0\ \Omega$; $r_1 = 5 \times 10^{-2}$ m ; $D = 2{\cdot}0 \times 10^{-3}$ m ; $\rho = 2{\cdot}0 \times 10^{-7}$ Ωm.

$$R = \frac{\rho l}{\pi D^2/4} \quad \text{or} \quad l = \frac{R\pi D^2}{4\rho}$$

Let n be the number of turns in the coil. Then total length of the wire used, $l = 2\pi r_1 n$

$$\text{or} \quad n = \frac{l}{2\pi r_1} = \frac{R\pi D^2}{4\rho\times2\pi r_1} = \frac{RD^2}{8\rho r_1}$$

$$= \frac{5{\cdot}0\times(2{\cdot}0\times10^{-3})^2}{8\times(2{\cdot}0\times10^{-7})\times(5\times10^{-2})} = \mathbf{250}$$

Example 17 **A wire of 10 Ω resistance is stretched to thrice its original length. What will be its (*i*) new resistivity and (*ii*) new resistance ?**

Solution. (*i*) Since resistivity depends on the nature of material of wire and is independent of dimensions of wire, so the **resistivity remains unchanged.**

(*ii*) In both the cases, the volume of the wire is same.

$$\text{So} \quad V = Al = A'l' \quad \text{or} \quad A' = \frac{lA}{l'} = \frac{lA}{3l} = \frac{A}{3}$$

$$\text{Now} \quad R = \frac{\rho l}{A}$$

$$\text{and} \quad R' = \frac{\rho l'}{A'} = \frac{\rho 3l}{A/3} = \frac{9\rho l}{A} = 9R$$

$$\therefore \quad R' = 9 \times 10 = \mathbf{90\ \Omega}$$

Example 18 **A wire has a resistance of 16 Ω. It is melted and drawn into a wire of half its length. Calculate the resistance of new wire. What is the percentage change in its resistance ?**

Solution. In both the cases, the volume of the wire is same. So

$$V = Al = A'l' \quad \text{or} \quad A' = \frac{Al}{l'} = \frac{Al}{l/2} = 2A$$

$$\text{Now} \quad R = \frac{\rho l}{A}$$

$$\text{and} \quad R' = \frac{\rho l'}{A'} = \frac{\rho\times(l/2)}{2A} = \frac{1}{4}\frac{\rho l}{A} = \frac{R}{4}$$

$$\therefore \quad R' = \frac{16}{4} = 4\ \Omega$$

% decrease in resistance

$$= \frac{R - R'}{R}\times100 = \frac{16-4}{16}\times100 = \mathbf{75\%}$$

Example 19 **There are two wires A and B of same mass and of the same material. The diameter of wire A is one-third the diameter of wire B. If the resistance of wire A is 30 Ω, find the resistance of wire B.**

Solution. Here, $R_A = 30\ \Omega$. Let l_A, D_A be the length and diameter of wire A and l_B, D_B be the length and diameter of wire B. Let d be the density of the material of wires A and B.

Mass of the wire = volume × density

= area of cross-section × length × density

As mass of two wires is same so

$$m = \frac{\pi D_A^2}{4} \times l_A \times d = \frac{\pi D_B^2}{4} \times l_B \times d$$

or $D_A^2 l_A = D_B^2 l_B$

or $\frac{l_B}{l_A} = \frac{D_A^2}{D_B^2} = \frac{(D_{B/3})^2}{D_B^2} = \frac{1}{9}$

$$\frac{R_B}{R_A} = \frac{4\rho l_B/(\pi D_B^2)}{4\rho l_A/(\pi D_A^2)} = \frac{l_B}{l_A} \times \frac{D_A^2}{D_B^2} = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$$

$$R_B = \frac{R_A}{81} = \frac{30}{81} = 0{\cdot}37\ \Omega$$

Example 20 **In a discharge tube the number of hydrogen ions (*i.e.*, protons) drifting across a cross-section per second is $1{\cdot}0 \times 10^{18}$, while number of electrons drifting in opposite direction across another cross-section is $2{\cdot}5 \times 10^{18}$ per second. If the supply voltage is 220 V, what is the effective resistance of the tube ?**

Solution. Here, $n_p = 1{\cdot}0 \times 10^{18}\ s^{-1}$,

$n_e = 2{\cdot}5 \times 10^{18}\ s^{-1}$

The current due to motion of negative charged electron is equivalent to the current due to motion of proton in the opposite direction. Therefore, total current in the direction of motion of proton is

I = total charge flowing per second

$= (n_p + n_e) \times e$

$= (1{\cdot}0 \times 10^{18} + 2{\cdot}5 \times 10^{18}) \times 1{\cdot}6 \times 10^{-19}$

$= 0{\cdot}56$ A

Effective resistance,

$$R = \frac{V}{I} = \frac{220}{0{\cdot}56} = 3{\cdot}93 \times 10^2\ \Omega$$

Example 21 **A potential difference of 3 V is applied across a conductor of resistance 1·5 Ω. Calculate the number of electrons flowing through it in one second. Given charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C.**

Solution. Here, $V = 3$ volt ; $R = 1{\cdot}5\ \Omega$;

$e = 1{\cdot}6 \times 10^{-19}$ C, $t = 1$ s.

Now, $I = \frac{V}{R} = \frac{3}{1{\cdot}5} = 2\ A$ and $I = \frac{Q}{t} = \frac{ne}{t}$

or $n = \frac{It}{e} = \frac{2 \times 1}{1{\cdot}6 \times 10^{-19}} = 1{\cdot}25 \times 10^{19}$

Example 22 **Find the relaxation time for free electrons in copper, if the density of mobile electrons is $8{\cdot}4 \times 10^{28}\ m^{-3}$. The resistivity of copper at room temperature is $1{\cdot}7 \times 10^{-8}\ \Omega m$. Given : mass of electron = $9{\cdot}11 \times 10^{-31}$ kg and charge on electron = $1{\cdot}6 \times 10^{-19}$ C.**

Solution. Here, $n = 8{\cdot}4 \times 10^{28}\ m^{-3}$;

$\rho = 1{\cdot}7 \times 10^{-8}\ \Omega$ m ; $m = 9{\cdot}11 \times 10^{-31}$ kg ;

$e = 1{\cdot}6 \times 10^{-19}$ C ; $\tau = ?$

We know that, $\rho = \frac{m}{e^2 n \tau}$

or $\tau = \frac{m}{e^2 n \rho}$

$$= \frac{9{\cdot}11 \times 10^{-31}}{(1{\cdot}6 \times 10^{-19})^2 \times 8{\cdot}4 \times 10^{28} \times 1{\cdot}7 \times 10^{-8}}$$

$= 2{\cdot}49 \times 10^{-14}$ s

Example 23 **A copper wire is stretched to make it 0·2% longer. What is the percentage change in its resistance ?**

Solution. The mass m of the wire of length l, area of cross section A and density d is given by

$$m = Ald \qquad \text{or} \qquad A = \frac{m}{ld}$$

The resistance R of the wire of resistivity ρ is given by

$$R = \frac{\rho l}{A} = \frac{\rho l^2 d}{m} = k l^2$$

where $k = \rho\, d/m$ is a constant of the wire.

$\therefore \quad \frac{dR}{R} = \frac{2\,dl}{l}$;

% increase in resistance

$$= \frac{dR}{R} \times 100 = \frac{2 \times 0{\cdot}2}{100} \times 100 = 0{\cdot}4\%$$

Example 24 **When 5 V potential difference is applied across a wire of length 0·1 m. The drift speed of electrons is $2{\cdot}5 \times 10^{-4}\ ms^{-1}$. If the electron**

density in the wire is 8×10^{28} m^{-3}, calculate the resistivity of the material of wire. (CBSE OD 2016)

Solution. Here, $V = 5$ V ; $l = 0{\cdot}1$ m ;

$v_d = 2{\cdot}5 \times 10^{-4}$ ms^{-1}, $n = 8 \times 10^{28}$ ms^{-3}

As, $V = RI = \dfrac{\rho l}{A} \times (n A e v_d) = \rho\, l\, n\, e\, v_d$

or $\rho = \dfrac{V}{l n e v_d}$

$$= \frac{5}{0{\cdot}1 \times (8 \times 10^{28}) \times (1{\cdot}6 \times 10^{-19})(2{\cdot}5 \times 10^{-4})}$$

$= \mathbf{1{\cdot}56 \times 10^{-8}\ \Omega m}$

Example 25 **Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter ? Hence explain why aluminium wires are preferred for overhead power cables. Given, For Al, $\rho_1 = 2{\cdot}63 \times 10^{-8}\ \Omega$ m, For *Cu*, $\rho_2 = 1{\cdot}72 \times 10^{-8}\ \Omega$ m. Relative density of Al = 2·7, of Cu = 8·9.**

Solution. Given, ***for aluminium wire*** ;

$R_1 = R$; $l_1 = l$; Relative density $d_1 = 2{\cdot}7$

For copper wire,

$R_2 = R$; $l_2 = l$; $d_2 = 8{\cdot}9$

Let A_1, A_2 be the area of cross section for aluminium wire and copper wire.

We know, $R_1 = \rho_1 \dfrac{l_1}{A_1} = \dfrac{2{\cdot}63 \times 10^{-8} \times l}{A_1}$ ...(*i*)

and mass of the aluminium wire,

$m_1 = A_1 l_1 \times d_1 = A_1 l \times 2{\cdot}7$ ...(*ii*)

Also, $R_2 = \rho_2 \dfrac{l_2}{A_2} = \dfrac{1{\cdot}72 \times 10^{-8} \times l}{A_2}$ ...(*iii*)

Mass of copper wire,

$m_2 = A_2 l_2 \times d_2 = A_2 l \times 8{\cdot}9$ ...(*iv*)

Since two wires are of equal resistance

$R_1 = R_2$

$$\frac{2{\cdot}63 \times 10^{-8} \times l}{A_1} = \frac{1{\cdot}72 \times 10^{-8} \times l}{A_2}$$

or $\dfrac{A_2}{A_1} = \dfrac{1{\cdot}72}{2{\cdot}63}$ ...(*v*)

From (*ii*) and (*iv*), we have

$$\frac{m_2}{m_1} = \frac{A_2 l \times 8{\cdot}9}{A_1 l \times 2{\cdot}7} = \frac{8{\cdot}9}{2{\cdot}7} \times \frac{A_2}{A_1} = \frac{8{\cdot}9}{2{\cdot}7} \times \frac{1{\cdot}72}{2{\cdot}63} = \mathbf{2{\cdot}16}$$

[From (*v*)]

It shows that copper wire is 2·16 times heavier than aluminium wire. Since for the same value of length and resistance, aluminium wire has lesser mass than copper wire, therefore aluminium wire is preferred for overhead power cables. A heavy cable may sag down owing to its own weight.

TYPE IV. CURRENT DENSITY, CONDUCTANCE AND CONDUCTIVITY

Formulae used. (*i*) $J = I/A$;

(*ii*) $G = \dfrac{1}{R}$ and (*iii*) $\sigma = \dfrac{1}{\rho} = n e \mu_e$

where J is the current density, G is the conductance, σ is the electrical conductivity and μ_e is the electron mobility.

Units used. J in Am^{-2} ; G in Ω^{-1} or S and σ in $\Omega^{-1}\ m^{-1}$ or Sm^{-1}.

Example 26 **A wire carries a current of 0·5 A, when a potential difference of 1·5 V is applied across it. What is its conductance ? If the wire is of length 3 m and area of cross-section 5·4 mm^2, calculate its conductivity.**

Solution. Here, $I = 0{\cdot}5$ A ; $V = 1{\cdot}5$ V, $l = 3$ m ;

$A = 5{\cdot}4\ mm^2 = 5{\cdot}4 \times 10^{-6}\ m^2$

Now, resistance, $R = \dfrac{V}{I} = \dfrac{1{\cdot}5}{0{\cdot}5} = 3\,\Omega$

Conductance, $G = \dfrac{1}{R} = \dfrac{1}{3} = \mathbf{0{\cdot}33\ S}$

Electrical conductivity, $\sigma = \dfrac{1}{\rho} = \dfrac{l}{RA}$

$$= \frac{3}{3 \times 5{\cdot}4 \times 10^{-6}} = \mathbf{1{\cdot}85 \times 10^{5}\ Sm^{-1}}$$

Example 27 **Calculate the electric field in a copper wire of cross-sectional area 2·0 mm square carrying a current of 2 A. The resistivity of copper is $1{\cdot}7 \times 10^{-8}\ \Omega$m.**

Solution. Here, $A = 2{\cdot}0$ mm sq $= 2 \times 2\ mm^2$

$= 4 \times 10^{-6}\ m^2$,

$I = 2$A ; $\rho = 1{\cdot}7 \times 10^{-8}\ \Omega$m.

Electric field $E = \dfrac{V}{l} = \dfrac{IR}{l} = \dfrac{I \rho\, l/A}{l} = \dfrac{I\rho}{A}$

$$= \frac{2 \times (1{\cdot}7 \times 10^{-8})}{4 \times 10^{-6}} = \mathbf{8{\cdot}5 \times 10^{-3}\ V/m}$$

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Example 28 **An aluminium wire of diameter 2·5 mm is connected in series with a copper wire of diameter 1·6 mm. A current of 2·0 A is passed through them. Find (a) current density in aluminium wire, (b) drift velocity of electrons in copper wire. Given the number density of conduction electrons in copper is 10^{29} m^{-3}.**

Solution. For Al, $r_1 = \frac{2.5}{2}$ mm $= 1.25$ mm

$= 1.25 \times 10^{-3}$ m ; $I = 2.0$ A

Area of cross-section, $A_1 = \pi r_1^2$

$= 3.14\ (1.25 \times 10^{-3})^2 = 4.9 \times 10^{-6}$ m^2

For Cu, $r_2 = \frac{1.6}{2} = 0.8$ mm $= 0.8 \times 10^{-3}$ m ;

$n = 10^{29}$ m^{-3}

Area of cross-section, $A_2 = \pi r_2^2$

$= 3.14\ (0.8 \times 10^{-3})^2 = 2.01 \times 10^{-6}$ m^2

(*a*) Current density in aluminium wire,

$$J = \frac{I}{A_1} = \frac{2}{4.9 \times 10^{-6}} = \mathbf{4.08 \times 10^5\ A\ m^{-2}}$$

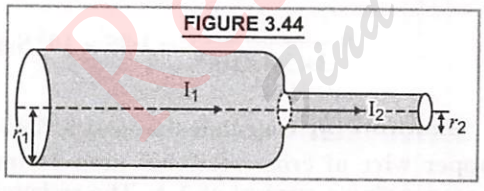
(*b*) Drift velocity in copper wire,

$$v_d = \frac{I}{ne A_2} = \frac{2.0}{10^{29} \times 1.6 \times 10^{-19} \times 2.01 \times 10^{-6}}$$

$$= \mathbf{6.2 \times 10^{-5}\ ms^{-1}}$$

Example 29 **Current flows through a constricted conductor as shown in Fig. 3.44. The radius and the current density to the left of constriction are 2 mm and 4.2×10^5 Am^{-2}.**

FIGURE 3.44



(*a*) How much current flows through the constriction ? (*b*) If the current density is doubled as it emerges from the right side of the constriction, what is the radius r_2 ?

Solution. Here, $r_1 = 2$ mm $= 2 \times 10^{-3}$ m ;

$J_1 = 4.2 \times 10^5$ Am^{-2}

(*a*) Current flowing through the constriction

$I_1 = J_1 A_1 = J_1 \times \pi r_1^2$

$= (4.2 \times 10^5) \times (22/7)\ (2 \times 10^{-3})^2 = \mathbf{5.28\ A}$

(*b*) For a steady flow of current, $I_1 = I_2$

or $J_1 A_1 = J_2 A_2 = 2 J_1 A_2$

or $A_2 = \frac{A_1}{2}$ or $\pi r_2^2 = \frac{1}{2} \pi r_1^2$ or $r_2 = \frac{r_1}{\sqrt{2}}$

or $r_2 = \frac{2 \times 10^{-3}}{\sqrt{2}} = \sqrt{2} \times 10^{-3}$ m $= \mathbf{1.414\ mm}$

TYPE V. VARIATION OF RESISTANCE WITH TEMPERATURE

Formulae used

(*i*) $R_t = R_o (1 + \alpha t)$

(*ii*) $\alpha = (R_2 - R_1)/R_1 (t_2 - t_1)$

where R_o, R_t = resistance of conductor at temperature 0°C and *t*°C respectively.

α = temperature coeff. of resistance

In relation (*ii*), α is temperature coeff. of resistance of the conductor averaged over the temperature range t_1°C to t_2°C.

(*iii*) $\alpha = (R_2 - R_1)/(R_1 t_2 - R_2 t_1)$ where α is the temperature coefficient of resistance at 0°C.

Units used. R_0, R_t, R_1 and R_2 is ohm, α in °C^{-1} or K^{-1}.

Example 30 **A tungsten coil has a resistance 16 Ω at 20°C. If the temperature coefficient of resistance of tungsten is 0·004 °C^{-1}, calculate the resistance of the coil at 80°C.**

Solution. Here, $R_{20} = 16\ \Omega$; $t_1 = 20$°C,

$t_2 = 80$°C, $\alpha = 0.004$ °C^{-1} $= 4 \times 10^{-3}$ °C^{-1},

$R_{80} = ?$

$$R_{20} = R_0 (1 + \alpha \times 20)$$

and $$R_{80} = R_0 (1 + \alpha \times 80)$$

$$\therefore \quad \frac{R_{80}}{R_{20}} = \frac{1 + \alpha \times 80}{1 + \alpha \times 20} = \frac{1 + (4 \times 10^{-3}) \times 80}{1 + (4 \times 10^{-3}) \times 20}$$

$$= \frac{1.32}{1.08}$$

$$R_{80} = \frac{1.32}{1.08} \times R_{20} = \frac{1.32}{1.08} \times 16 = \mathbf{19.6\ \Omega}$$

Example 31 **A conductor has a cross-section of 15 mm^2 and resistivity of 7.6×10^{-8} Ωm at 0°C. If the temperature coefficient of resistance of the material of the conductor is 5×10^{-3} °C^{-1}, calculate its resistance for 2 km length of conductor when its temperature is 60°C.**

Solution. Here, $A = 15\ mm^2 = 15 \times 10^{-6}\ m^2$;
$\rho = 7{\cdot}6 \times 10^{-8}\ \Omega\ m$, $\alpha = 5 \times 10^{-3}\ °C^{-1}$,
$l_0 = 2\ km = 2000\ m$; $t_1 = 0°C$, $t_2 = 60°C$,

$$R_0 = \frac{\rho l}{A} = \frac{(7{\cdot}6\times10^{-8})\times2000}{15\times10^{-6}} = 10{\cdot}12\ \Omega$$

$$R_{60} = R_0\,[1 + \alpha\,(t_2 - t_1)]$$
$$= 10{\cdot}12\,[1 + 5 \times 10^{-3} \times 60]$$
$$= 10{\cdot}12 \times 1{\cdot}3 = \mathbf{13{\cdot}16\ \Omega}$$

Example 32 **The heating element of an electric toaster is of nichrome. When a very small current passes through it, at room temperature 27°C, its resistance is 75·3 Ω. When toaster is connected to a 230 V supply, the current settles after a few seconds to a steady value of 2·68 A. What is the steady temperature of nichrome element ? The temperature coefficient of resistance of nichrome over averaged temperature range is $1{\cdot}7 \times 10^{-4}\ °C^{-1}$.** **(NCERT Solved Example)**

Solution. Here, $R_{27} = 75{\cdot}3\ \Omega$,
$\alpha = 1{\cdot}7 \times 10^{-4}\ °C^{-1}$, $V = 230\ V$, $I = 2{\cdot}68\ A$

$$R_t = \frac{230}{2{\cdot}68} = 85{\cdot}82\,\Omega\ ;$$

As, $$t_2 - t_1 = \frac{R_2 - R_1}{R_1 \times \alpha}$$

$$\therefore\ t - 27 = \frac{R_t - R_{27}}{R_{27} \times \alpha} = \frac{85{\cdot}82 - 75{\cdot}3}{75{\cdot}3\times1{\cdot}7\times10^{-4}} = 822°C$$

or $t = 822 + 27 = \mathbf{849°C}$

Example 33 **The resistance of a platinum wire of platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5·23 Ω. When thermometer is inserted in a hot bath, the resistance of the platinum wire is 5·795 Ω. Calculate the temperature of the bath ?**

(NCERT Solved Example)

Solution. Here, $R_0 = 5\ \Omega$, $R_{100} = 5{\cdot}23\ \Omega$,
$R_t = 5{\cdot}795\ \Omega$, $t = ?$

As, $$\alpha = \frac{R_{100} - R_0}{R_0 \times 100} = \frac{R_t - R_0}{R_0 \times t}$$

$$\therefore\ t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{5{\cdot}795 - 5}{5{\cdot}23 - 5} \times 100$$

$$= \frac{0{\cdot}795}{0{\cdot}23} \times 100 = \mathbf{345{\cdot}65°C}$$

Example 34 **A heating element using nichrome connected to a 115 V supply draws a current of 1·6 A which settles after a few seconds to a steady value of 1·4 A. What is the steady temperature of the heating element if the room temperature is 30°C ? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1{\cdot}7 \times 10^{-4}\ °C^{-1}$.**

Solution. Here, initial room temperature,
$t_1 = 30°C$; $I_1 = 1{\cdot}6\ A$, $V = 115\ V$, $\alpha = 1{\cdot}7 \times 10^{-4}\ °C^{-1}$

Let $t_2°C$ be the final steady temperature when current settles down to, $I_2 = 1{\cdot}4\ A$.

Initial resistance, $R_{t_1} = \frac{V}{I_1} = \frac{115}{1{\cdot}6} = 71{\cdot}875\ \Omega$;

Final resistance, $R_{t_2} = \frac{V}{I_2} = \frac{115}{1{\cdot}4} = 82{\cdot}143\ \Omega$;

$\alpha = 1{\cdot}7 \times 10^{-4}\ °C^{-1}$

As, $$t_2 - t_1 = \frac{R_{t_2} - R_{t_1}}{R_{t_1} \times \alpha}$$

$$\therefore\ t - 30 = \frac{R_{t_2} - R_{t_1}}{R_{t_1}\,\alpha} = \frac{82{\cdot}143 - 71{\cdot}875}{71{\cdot}875\times1{\cdot}7\times10^{-4}}$$
$$= 840{\cdot}3°C$$

Hence, $t = 840{\cdot}3 + 30 = \mathbf{870{\cdot}3°C}$

Example 35 **A standard coil marked 5 Ω is found to have a resistance of 5·128 Ω at 30°C. Calculate the temperature at which the marking is correct. The temperature coefficient of resistance of the material of the coil is ·0042 $°C^{-1}$.**

Solution. $R_{t_1} = 5\,\Omega$, $R_{t_2} = 5{\cdot}128\,\Omega$,
$t_1 = ?$, $t_2 = 30°C$.

$$\frac{R_{t_1}}{R_{t_2}} = \frac{R_0\,(1+\alpha t_1)}{R_0\,(1+\alpha t_2)} = \frac{1+\alpha t_1}{1+\alpha t_2}$$

or $$\frac{5}{5{\cdot}128} = \frac{1+0{\cdot}0042\times t_1}{1+0{\cdot}0042\times30}$$

On solving, $t_1 = \mathbf{23{\cdot}3°C}$

Example 36 **The resistances of iron and copper wires at 20°C are 4·1 Ω and 4·3 Ω respectively. At what temperature will the resistance be equal ? Temperature coefficient of resistance for iron is $5{\cdot}0 \times 10^{-3}\ K^{-1}$ and for copper is $4{\cdot}0 \times 10^{-3}\ K^{-1}$. Neglect any thermal expansion.**

Solution. For iron, $R_{20} = 4{\cdot}1\ \Omega$, $\alpha = 5{\cdot}0 \times 10^{-3}\ K^{-1}$

From copper, $R'_{20} = 4{\cdot}3\,\Omega$, $\alpha' = 4{\cdot}0 \times 10^{-3}\ K^{-1}$

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Let at temperature t°C, the resistance of iron wire be equal to the resistance of copper wire. So

$$R_{20}[1+\alpha(t-20)] = R'_{20}(1+\alpha'(t-20)]$$

or $4{\cdot}1[1+5{\cdot}0\times10^{-3}(t-20)] = 4{\cdot}3[1+4{\cdot}0\times10^{-3}\times(t-20)]$

$$(t-20)\times10^{-3}[4{\cdot}1\times5{\cdot}0-4{\cdot}3\times4{\cdot}0] = 4{\cdot}3-4{\cdot}1$$

or $$t-20 = \frac{0{\cdot}2}{3{\cdot}3\times10^{-3}} = 60{\cdot}6$$

$$t = 20+60{\cdot}6 = \mathbf{80{\cdot}6°C}$$

Example 37 **Resistivity of the material of a conductor of uniform cross-section varies along its length as $\rho = \rho_0(1+\alpha x)$. Find its resistance if its length is L and area of cross-section is A.**

Solution. Resistance of the material of a conductor of length dx at distance x from one end is

$$dR = \rho\frac{dx}{A} = \rho_0(1+\alpha x)\frac{dx}{A}$$

[Given, $\rho = \rho_0(1+\alpha x)$]

The resistance of the material of conductor of length L is

$$R = \int_0^L \rho_0(1+\alpha x)\frac{dx}{A} = \frac{\rho_0}{A}\left[L+\frac{1}{2}\alpha L^2\right]$$

TYPE VI. COLOUR CODES OF CARBON RESISTORS

Formulae used. The number assigned to different colours from 0 to 9 can be recollected by the sentence **B.B.ROY Great Britain Very Good Wife Wearing Gold Silver necklace.**

Black	Brown	Red	Orange	Yellow
0	1	2	3	4
Green	**Blue**	**Violet**	**Grey**	**White**
5	6	7	8	9

Multiplier for gold is 10^{-1} and for silver is 10^{-2}. For tolerance or % accuracy, the colour used are : Gold for ± 5%, Silver for ± 10% and No colour for ± 20%.

Example 38 **What is the colour of the third band of a coded resistor of resistance $2{\cdot}3\times10^2\ \Omega$?** (Karnataka Board 2012)

Solution. Resistance of resistor = $2{\cdot}3\times10^2\ \Omega = 23\times10^1\ \Omega$

Therefore, the colour of third band of a colour coded resistor will be related to number 1, *i.e.*, **brown.**

Example 39 **A current of 2mA is passed through a colour coded carbon resistor with first, second and third rings of yellow, green and orange colours. What is the voltage drop across the resistor ?**

Solution. For colour coded carbon resistor, the numbers attached for yellow, green and orange are 4, 5 and 3 respectively. Therefore, the resistance of the given carbon resistor, $R = 45\times10^3\ \Omega$.

Voltage $V = RI = (45\times10^3)\times(2\times10^{-3}) = \mathbf{90\ V}$

Example 40 **A voltage of 5 V is applied across a colour coded carbon resistor. A current of 5 mA flows through it. What are the colours of second and third band of coded colour resistor ?**

Solution. Here, $V = 5$V, $I = 5$mA $= 5\times10^{-3}$ A

Resistance of carbon resistor,

$$R = \frac{V}{I} = \frac{5}{5\times10^{-3}} = 10^3\ \Omega = 10\times10^2\ \Omega$$

Therefore, the colours of second and third bands will be related to numbers 0 and 2, *i.e.*, **black** and **red.**

TYPE VII. COMBINATION OF RESISTANCES IN SERIES AND PARALLEL

Formulae used.

(*i*) In series, $R_S = R_1 + R_2 + R_3 + \ldots$

(*ii*) In parallel, $\frac{1}{R_p} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$.

where R_s is the total resistance of the various resistances $R_1, R_2, R_3, \ldots$ connected in series and R_p is the total resistance in parallel combination.

Units used. $R_1, R_2, \ldots, R_s, R_p$ all in ohm.

Example 41 **Two resistances are in the ratio 1 : 4. If these are connected in parallel, their total resistance becomes 20 ohm. Find the value of each resistance.** (Pb. Board 2000)

Solution. Let R_1, R_2 be the two given resistance.

Then $$\frac{R_1}{R_2} = \frac{1}{4} \quad \text{or} \quad R_1 = \frac{R_2}{4} \qquad \ldots(i)$$

When these resistances are connected in parallel then

$$R_p = \frac{R_1R_2}{R_1+R_2} \quad \text{or} \quad 20 = \frac{(R_2/4)R_2}{(R_2/4)+R_2} = \frac{R_2}{5}$$

or $R_2 = \mathbf{100\ \Omega}$; From (*i*), $R_1 = \frac{100}{4} = \mathbf{25\ \Omega}$

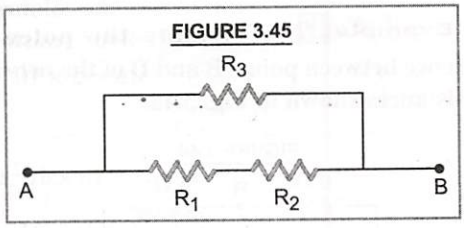
Example 42 **Two conductors of conductances G_1 and G_2 are connected in series. They are connected in parallel to another conductor of conductance G_3. Determine their equivalent conductance.**

Solution. Let R_1, R_2 and R_3 be the resistances of conductors of conductances G_1, G_2 and G_3 respectively.

Here $R_1 = \frac{1}{G_1}$, $R_2 = \frac{1}{G_2}$, $R_3 = \frac{1}{G_3}$

Refer to Fig. 3.45, the effective resistance between A and B is

FIGURE 3.45



$$R = \frac{R_3 \times (R_1 + R_2)}{R_3 + (R_1 + R_2)}$$

$\therefore$ Effective conductance,

$$G = \frac{1}{R} = \frac{R_3 + R_1 + R_2}{R_3 \times (R_1 + R_2)}$$

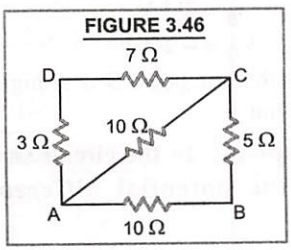
$$\therefore \quad G = \frac{\frac{1}{G_3} + \frac{1}{G_1} + \frac{1}{G_2}}{\frac{1}{G_3} \times \left(\frac{1}{G_1} + \frac{1}{G_2}\right)}$$

$$= \frac{G_1G_2 + G_2G_3 + G_1G_3}{(G_2 + G_1)}$$

Example 43 **In the given network of resistors, Fig. 3.46, find the equivalent resistance (*i*) between the points A and B (*ii*) between the points A and D and (*iii*) between the points A and C.**

Solution. (*i*) The resistors 3 Ω and 7 Ω are in series. Their total resistance = 3 + 7 = 10 Ω.

FIGURE 3.46



This resistance is in parallel with 10 Ω. The effective resistance between A and $C = \frac{10 \times 10}{10 + 10} = 5\ \Omega$

This resistance is in series with resistance 5 Ω, the effective resistance = 5 + 5 = 10 Ω

This resistance is in parallel with resistor 10 Ω in arm AB. Thus equivalent resistance between A and $B = \frac{10 \times 10}{10 + 10} = 5\ \Omega$

(*ii*) The resistance of arm ABC (= 5 + 10 = 15 Ω) is in parallel with resistance 10 Ω of arm AC.

Their effective resistance $= \frac{15 \times 10}{15 + 10} = 6\ \Omega$

This resistance is in series with resistance 7 Ω of arm CD. Their effective resistance = 6 + 7 = 13 Ω. This resistance is in parallel with resistance 3 Ω of arm AD.

The equivalent resistance between A and D

$$= \frac{13 \times 3}{13 + 3} = \frac{39}{16}\ \Omega$$

(*iii*) The resistance of arm ADC = 3 + 7 = 10 Ω

Resistance of arm ABC = 10 + 5 = 15 Ω

These resistance are in parallel.

They are also in parallel with resistance 10 Ω of arm AC.

Therefore, equivalent resistance between A and C is

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} + \frac{1}{15} = \frac{8}{30} \quad \text{or} \quad R_{eq} = \frac{30}{8} = \frac{15}{4}\ \Omega$$

Example 44 **A resistor of 5 Ω is connected in series with a parallel combination of a number of resistors each of 5 Ω. If the total resistance of the combination is 6 Ω, how many resistors are in parallel ?**

Solution. Let n resistors each of 5 Ω be connected in parallel. Their effective resistance is

$$\frac{1}{R_p} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \ldots n \text{ times} = \frac{n}{5} \quad \text{or} \quad R_p = \frac{5}{n}$$

As this parallel combination of resistances is connected in series with 5 Ω resistance, the total resistance of the combination is

$$R = R_p + 5 = \frac{5}{n} + 5 = 6$$

$$\text{or} \quad \frac{5}{n} = 6 - 5 = 1 \quad \text{or} \quad n = 5$$

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Example 45 **A uniform wire is cut into three parts with their lengths in the ratio 2 : 3 : 6. The ends of each of these three parts are connected across an ideal battery of 10 V. If a current of 5 A is drawn from the battery, find the initial resistance of the wire.** **(CBSE 2020)**

Solution. Let $11l$ be the initial length of the wire of area of cross-section A and resistance R_i. Then

$$R_i = \frac{\rho(11l)}{A}$$

When length of the wire is cut into three parts of lengths $2\,l$, $3\,l$ and $6\,l$. Let R_1, R_2 and R_3 be their resistances. Then

$$R_1 = \frac{\rho(2l)}{A}\ ;\ R_2 = \frac{\rho(3l)}{A} \text{ and } R_3 = \frac{\rho(6l)}{A}$$

These three resistances are connected in parallel, then equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{A}{\rho(2l)} + \frac{A}{\rho(3l)} + \frac{A}{\rho(6l)}$$

$$= \frac{A}{\rho l}\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\right] = \frac{A}{\rho l}\left[\frac{3+2+1}{6}\right] = \frac{A}{\rho l}$$

$$\therefore \quad R_{eq} = \frac{\rho l}{A}$$

But $R_{eq} = \frac{V}{I} = \frac{10}{5} = 2\,\Omega \quad \therefore \quad \frac{\rho l}{A} = 2\,\Omega$

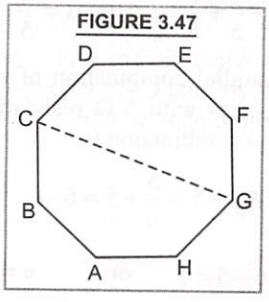
Now, initial resistance, $R_i = \frac{\rho(11l)}{A} = 11\frac{\rho l}{A}$

$$= 11 \times 2 = 22\ \Omega$$

Example 46 **A uniform wire of resistance R is shaped into a regular n sided polygon where n is even. Find the equivalent resistance between (*i*) opposite corners of polygon (*ii*) adjacent corners of polygon.**

Solution. Let the polygon be as shown in Fig. 3.47. The resistance of each side of polygon = R/n.

FIGURE 3.47



(*i*) For resistance between opposite corners C and G, we have two resistances in parallel each of value $R/2$.

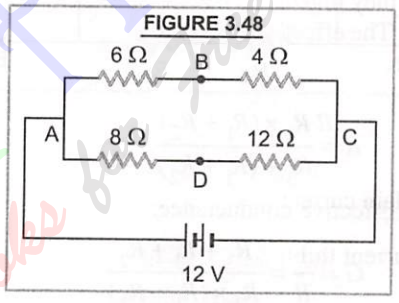
Therefore, the equivalent resistance between opposite corners is $= \frac{(R/2)(R/2)}{(R/2)+R/2} = \frac{R}{4}$

(*ii*) For resistance between adjacent corners A and H, we have two resistances of R/n and $(n-1)\,R/n$ in parallel. The equivalent resistance is

$$= \frac{(R/n)(n-1)\,R/n}{(R/n)+(n-1)\,R/n} = R\frac{(n-1)}{n^2}$$

Example 47 **Calculate the potential difference between points B and D of the network of resistances shown in Fig. 3.48.**

FIGURE 3.48



Solution. Here, potential difference across ABC = potential difference across ADC = 12V

Current in arm ABC,

$$I_1 = \frac{12}{6+4} = 1{\cdot}2\ \text{A}$$

Pot. diff. between A and B,

$$V_A - V_B = I_1 \times 6 = 1{\cdot}2 \times 6 = 7{\cdot}2\ \text{V}$$

Current in arm ADC,

$$I_2 = \frac{12}{8+12} = 0{\cdot}6\ \text{A}$$

Pot. diff. across A and D,

$$V_A - V_D = I_2 \times 8 = 0{\cdot}6 \times 8 = 4{\cdot}8\ \text{V}$$

$$\therefore\ (V_A - V_B) - (V_A - V_D) = 7{\cdot}2 - 4{\cdot}8 = 2{\cdot}4\ \text{V}$$

or $\quad V_D - V_B = \mathbf{2{\cdot}4\ V}$

$\therefore \quad V_B - V_D = \mathbf{-2{\cdot}4\ V}$

It shows that the point D is at higher potential than that of point B.

Example 48 **In the circuit shown in Fig. 3.49, find the potential difference across capacitor.**

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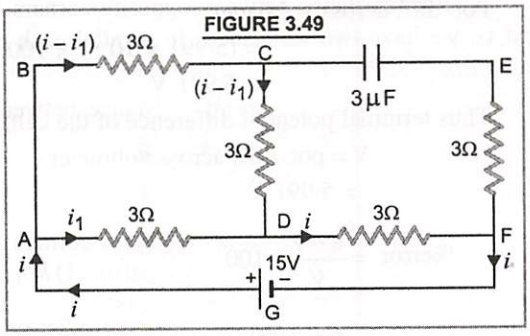
FIGURE 3.49



Solution. When capacitor is fully charged, it draws no current, and hence provides infinite resistance, then no current flows in arm *EF*. The potential difference across the capacitor is equal to the potential difference across *C* and *F*. The resistance of arm *ABCD* is in parallel with the resistance of arm *AD* and they together are in series with resistance of arm *DF*. The effective resistance between *A* and *F* of circuit is,

$$R = \frac{3\times 6}{3+6} + 3 = 2 + 3 = 5\ \Omega$$

Main current in the circuit, $i = \frac{15}{5} = 3\text{ A}$

Current through arm *BCD*

$$= \frac{i\times 3}{6+3} = \frac{3\times 3}{9} = 1\text{ A}$$

Potential difference across *C* and $D = 1 \times 3 = 3$ V

Current through arm *DF*, $i = 3$ A

Potential difference across *D* and $F = 3 \times 3 = 9$ V

So, potential difference across *C* and $F = 3 + 9 = 12$ V

Thus, potential difference across the capacitor is **12 V**

TYPE VIII. EMF, TERMINAL P.D. AND INTERNAL RESISTANCE OF A CELL

Formulae used. (*i*) $\varepsilon = V + Ir$, where $V = IR$.

Here, ε is the emf of a cell ; *V* is the terminal pot. diff., *r* is the internal resistance of the cell, *R* is the external resistance in circuit, *I* is the current.

Units used. ε in volt, *V* in volt, *I* in ampere and *r* or *R* in ohm.

Example 49 **A resistor is connected to a battery of *emf* 10V and internal resistance 0·3 Ω. What is the resistance of the resistor to be inserted in the circuit for the circuit current 1·2 A ?**

Solution. Here, $\varepsilon = 10$ V, $r = 0{\cdot}3\ \Omega$,

$R = ?$, $I = 1{\cdot}2$ A

Current, $I = \frac{\varepsilon}{R+r}$

or $R = \frac{\varepsilon}{I} - r = \frac{10}{1{\cdot}2} - 0{\cdot}3$

$= 8{\cdot}3 - 0{\cdot}3 = \mathbf{8{\cdot}0\ \Omega}$

Example 50 **A cell of emf ε and internal resistance r gives a current of 0·5 A with an external resistance of 12 Ω and a current of 0·25 A with an external resistance of 25 Ω. Calculate (a) internal resistance of the cell and (b) emf of the cell.** [CBSE 2013 (C)]

Solution. Let *R* be external resistance in series with the cell of emf ε and internal resistance *r*. The current in circuit is

$$I = \frac{\varepsilon}{R+r}$$

In first case, $I = 0{\cdot}5\ A$, $R = 12\ \Omega$

$$0{\cdot}5 = \frac{\varepsilon}{12+r}$$

or $\varepsilon = (12 + r) \times 0{\cdot}5 = 6{\cdot}0 + 0{\cdot}5\ r$ ...(*i*)

In second case, $I = 0{\cdot}25\ A$, $R = 25\ \Omega$

$\therefore$ $0{\cdot}25 = \frac{\varepsilon}{25+r}$

or $\varepsilon = (25 + r) \times 0{\cdot}25 = 6{\cdot}25 + 0{\cdot}25\ r$ ...(*ii*)

From (*i*) and (*ii*),

$6{\cdot}0 + 0{\cdot}5\ r = 6{\cdot}25 + 0{\cdot}25\ r$

or $r = \mathbf{1\ \Omega}$

From (*i*), $\varepsilon = 6{\cdot}0 + 0{\cdot}5 \times 1 = \mathbf{6{\cdot}5\ V}$

Example 51 **A uniform wire of resistance 12 Ω is cut into three pieces in the ratio 1 : 2 : 3 and the three pieces are connected to form a triangle. A cell of emf 8 V and internal resistance 1 Ω is connected across the highest of the three resistors. Calculate the current through each part of the circuit.**

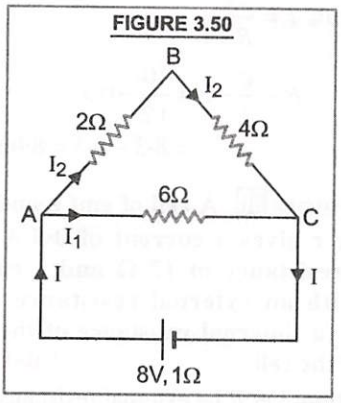
Solution. When 12 Ω resistance wire is cut into three pieces *AB*, *BC* and *CA* in the ratio 1 : 2 : 3, then resistances of these parts will be,

$$\frac{1}{6}\times 12 = 2\ \Omega\ ;\ \frac{2}{6}\times 12 = 4\ \Omega$$

and $\frac{3}{6}\times 12 = 6\ \Omega$

respectively. As per question, the cell of emf 8 V and internal resistance 1 Ω is connected across *CA* (the highest resistor) as shown in Fig. 3.50.

FIGURE 3.50



Here 2 Ω and 4 Ω are in series. Their effective resistance = 2 + 4 = 6 Ω. This is in parallel with 6 Ω resistance. The effective resistance between *A* and *C*,

$$R = \frac{6\times 6}{6+6} = 3\,\Omega$$

Total current from the cell,

$$I = \frac{\varepsilon}{R+r} = \frac{8}{3+1} = 2\,\text{A}$$

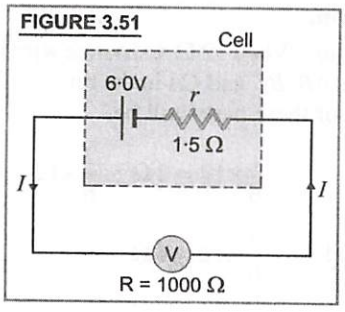
Let I_1, I_2 = currents through arms *AC* and *ABC* respectively. Since the resistance of arm *AC* = resistance of arm *ABC*, so $I_1 = I_2 = \frac{I}{2} = \frac{2}{2} = 1\,\text{A}$

Example 52 **The *emf* of a battery is 6·0 V and its internal resistance is 1·5 Ω. Its potential difference is measured by a voltmeter of resistance 1000 Ω. Calculate the percentage error in the reading of *emf* shown by voltmeter.**

Solution. Here, ε = 6·0 V ; *r* = 1·5 Ω ;
Resistance of voltmeter, *R* = 1000 Ω
Refer to Fig. 3.51, Current in the circuit is

$$I = \frac{\varepsilon}{R+r} = \frac{6\ V}{1000+1{\cdot}5} = 5{\cdot}991\times 10^{-3}\ \text{A}$$

FIGURE 3.51



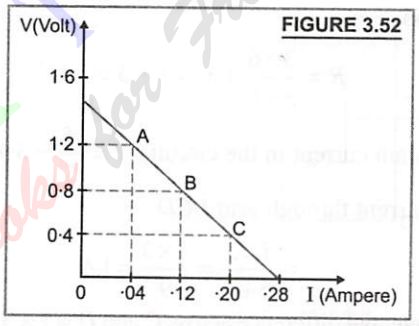
Pot. diff. across voltmeter
$= (5{\cdot}991\times 10^{-3})\times 1000$
$= 5{\cdot}991$ V
Thus terminal potential difference of the cell,
V = pot. diff. across voltmeter
= 5·991 V

$$\%\text{error} = \frac{\varepsilon - V}{V}\times 100$$

$$= \frac{6-5{\cdot}991}{6}\times 100 = \mathbf{0{\cdot}15\%}$$

Example 53 **Potential differences across the terminals of a cell were measured (in volt) against different currents (in ampere) flowing through the cell. A graph was drawn which was a straight line *ABC* as shown in Fig. 3.52.**

FIGURE 3.52



Determine from the graph (*i*) emf of the cell (*ii*) maximum current obtained from the cell and (*iii*) internal resistance of the cell.

[CBSE 2011 (C)]

Solution. (*i*) *EMF* of the cell is equal to maximum potential difference across the two electrodes of cell corresponding to zero current. Thus *emf* of the cell, **ε = 1·4 V.**

(*ii*) Max. current is drawn from the cell when the terminal pot. diff. is zero. Therefore

I_{max} **= 0·28 A**

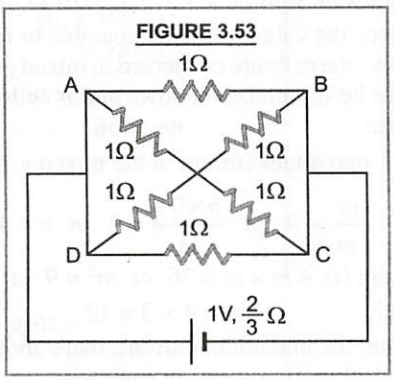
(*iii*) Internal resistance,

$$r = \frac{\varepsilon}{I_{max}} = \frac{1{\cdot}4\,V}{0{\cdot}28\,A} = 5\,\Omega$$

Example 54 **Find the current drawn from a cell of emf 1 V and internal resistance (2/3) Ω connected to the network shown in Fig. 3.53.**

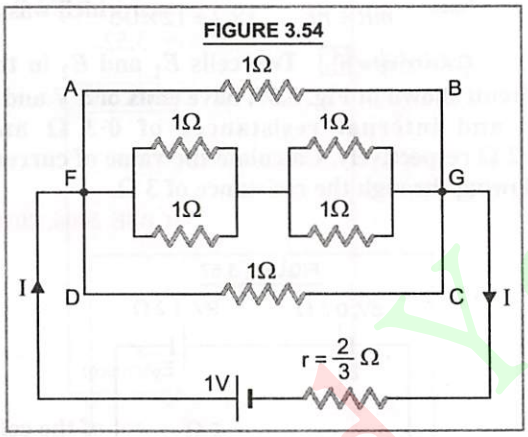
[CBSE 2001 (C)]

FIGURE 3.53



Solution. The equivalent circuit is shown in Fig. 3.54.

FIGURE 3.54



Resistance in arm $AB = 1\ \Omega$

Resistance in arm FG

$$= \frac{1\times1}{1+1} + \frac{1\times1}{1+1} = \frac{1}{2} + \frac{1}{2} = 1\Omega$$

Resistance in arm $DC = 1\ \Omega$

As resistance of these arms are in parallel, their equivalent resistance is

$$\frac{1}{R_e} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{3}{1}$$

or $$R_e = \frac{1}{3}$$

Total resistance of circuit,

$$R = R_e + r = \frac{1}{3} + \frac{2}{3} = 1\Omega$$

$$\therefore \text{ Current, } I = \frac{\in}{R} = \frac{1}{1} = \mathbf{1\,A}$$

TYPE IX. GROUPING OF CELLS

Formulae used.

(*i*) Cells in series, $I = \dfrac{n\varepsilon}{R + nr}$

(*ii*) Cells in parallel, $I = \dfrac{m\varepsilon}{mR + r}$

(*iii*) Cells in mixed grouping, $I = \dfrac{mn\varepsilon}{mR + nr}$

where n = no. of cells in series in each row, m = no. of rows of cells in parallel.

Units used. I in ampere, ε in volt, R or r in ohm.

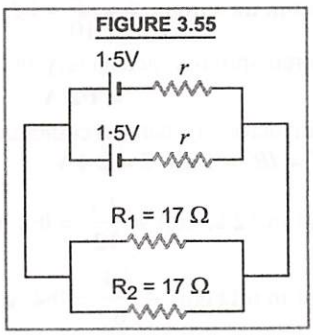
Example 55 **Two identical cells of *emf* 1·5 V each joined in parallel provide supply to an external circuit consisting of two resistances of 17 Ω each joined in parallel. A very high resistance voltmeter reads. The terminal voltage of cells to be 1·4 V. Calculate the internal resistance of each cell.**

Solution. Here, $\varepsilon = 1{\cdot}5$ V, $R_1 = R_2 = 17\ \Omega$, $V = 1{\cdot}4$ V

Refer to Fig. 3.55, resistance of external circuit,

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{17\times17}{17+17} = 8{\cdot}5\ \Omega$$

FIGURE 3.55



As the two cells are connected in parallel, their effective *emf*, $\varepsilon = 1{\cdot}5$ V,

their effective internal resistance,

$$r' = \frac{r\times r}{r+r} = \frac{r}{2}.$$

Then, $$r' = \left(\frac{\varepsilon - V}{V}\right) R$$

So $$\frac{r}{2} = \left(\frac{1{\cdot}5 - 1{\cdot}4}{1{\cdot}4}\right) 8{\cdot}5$$

or $$r = \frac{0{\cdot}1\times8{\cdot}5}{2}\times2 = \mathbf{1{\cdot}2\ \Omega}$$

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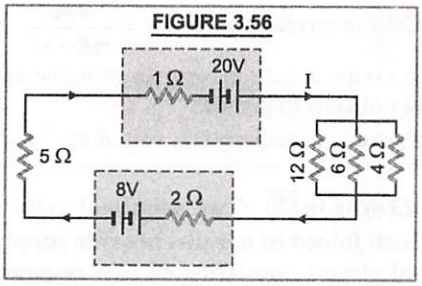
Example 56 **A 20 V battery of internal resistance 1 Ω is connected to three coils of 12 Ω, 6 Ω and 4 Ω in parallel, a resistor of 5 Ω and a reversed battery (*emf* = 8 V and internal resistance = 2 Ω) as shown in Fig. 3.56. Calculate the current in each resistor and the terminal potential difference across each battery.**

[CBSE 2001 (C)]

FIGURE 3.56



Solution. Equivalent resistance R' of 12 Ω, 6 Ω and 4 Ω connected in parallel is

$$\frac{1}{R'}=\frac{1}{12}+\frac{1}{6}+\frac{1}{4}=\frac{6}{12}=\frac{1}{2} \quad \text{or} \quad R'=2\ \Omega$$

Total resistance of circuit, $R = 1 + 5 + 2 + 2$

$= 10\ \Omega$

Effective *emf* of circuit = 20 – 8 = 12 V

Current in the circuit $I=\frac{12}{10}=1{\cdot}2\ \text{A}$

So, current through each battery and 5 Ω resistor

$= \mathbf{1{\cdot}2\ A}$

Pot. diff. across the parallel combination of three resistors, $V' = IR' = 1{\cdot}2 \times 2 = 2{\cdot}4$ V

Current in 12 Ω coil $=\frac{2{\cdot}4}{12}=\mathbf{0{\cdot}2\ A}$

Current in 6 Ω coil $=\frac{2{\cdot}4}{6}=\mathbf{0{\cdot}4\ A}$

Current in 4 Ω coil $=\frac{2{\cdot}4}{4}=\mathbf{0{\cdot}6\ A}$

Terminal pot. diff. across 20 V battery

$V = \varepsilon - Ir = 20 - 1{\cdot}2 \times 1 = \mathbf{18{\cdot}8\ V}$

Terminal pot. diff. across 8 V battery

$V' = \varepsilon' + Ir' = 8 + 1{\cdot}2 \times 2 = \mathbf{10{\cdot}4\ V}$

Example 57 **Thirty six cells each of *emf* 1·5 V and internal resistance 0·5 Ω are used to send current through an external resistor of resistance 2 Ω. What is the best mode of grouping them and the current through the external resistor.**

Solution. Here, $\varepsilon = 1{\cdot}5$ V, $r = 0{\cdot}5\ \Omega$, $R = 2\ \Omega$.

Since the value of r is comparable to R, for the best mode, the cells are connected in mixed grouping. Let there be m number of rows and n cells in each row. Then $nm = 36$ ...(*i*)

For maximum current in the mixed grouping

$$\frac{nr}{m}=R \quad \text{or} \quad \frac{n\times 0{\cdot}5}{m}=2 \quad \text{or} \quad n=4m$$

From (*i*), $4m \times m = 36$ or $m^2 = 9$ or $m = 3$

and $n = 4 \times 3 = 12$

Thus for maximum current, there should be 3 rows and 12 cells in series in each row.

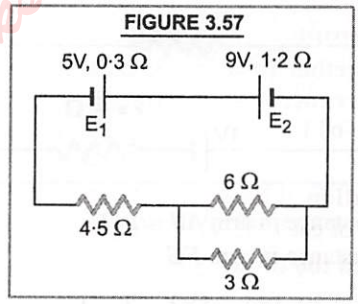
Maximum current,

$$I_{\max}=\frac{mn\varepsilon}{mR+nr}=\frac{36\times 1{\cdot}5}{3\times 2+12\times 0{\cdot}5}=\mathbf{4{\cdot}5\ A}$$

Example 58 **Two cells E_1 and E_2 in the circuit shown in Fig. 3.57, have emfs of 5 V and 9 V and internal resistances of 0·3 Ω and 1·2 Ω respectively. Calculate the value of current flowing through the resistance of 3 Ω.**

(CBSE 2006, 2010)

FIGURE 3.57



Solution. Net *emf* $= E_2 - E_1 = 9 - 5 = 4$ V.

Total resistance $= 0{\cdot}3 + 1{\cdot}2 + 4{\cdot}5 + \frac{6\times 3}{6+3} = 8\ \Omega$

Current through the circuit, $I=\frac{4V}{8\ \Omega}=0{\cdot}5\ \text{A}$

Current through 3 Ω resistance $=\frac{6\times 0{\cdot}5}{6+3}=\frac{1}{3}\ \text{A}$

Example 59 **(a) Three cells of emfs, 1·5 V, 2·0 V and 2·5 V are connected in series. Their internal resistances are 0·20 Ω, 0·15 Ω and 0·15 Ω respectively. The battery is connected to an external resistor of 5·5 Ω via a very low resistance ammeter, what would be the reading of ammeter ?**

(b) If the three cells above were joined in parallel, would they be characterised by a definite

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emf and internal resistance (independent of their individual internal resistance) ? If not, how will you obtain currents in different branches of the circuit ?

Solution. *(a)* Here, $\varepsilon_1 = 1{\cdot}5$ V ; $\varepsilon_2 = 2{\cdot}0$ V ; $\varepsilon_3 = 2{\cdot}5$ V ; $r_1 = 0{\cdot}2\ \Omega$; $r_2 = 0{\cdot}15\ \Omega$; $r_3 = 0{\cdot}15\ \Omega$; $R = 5{\cdot}5\ \Omega$

Since the cells are joined in series, the effective emf of all the cells is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1{\cdot}5 + 2{\cdot}0 + 2{\cdot}5 = 6{\cdot}0 \text{ V}$$

Total internal resistance of all the cells is

$$r = r_1 + r_2 + r_3 = 0{\cdot}20 + 0{\cdot}15 + 0{\cdot}15 = 0{\cdot}50\ \Omega$$

Total resistance of the circuit $= R + r$

$$= 5{\cdot}5 + 0{\cdot}5 = 6{\cdot}0\ \Omega$$

$$\text{Current, } I = \frac{\varepsilon}{R+r} = \frac{6}{6{\cdot}0} = \mathbf{1\ A}$$

(b) No, they will not be characterised by a definite emf and internal resistance, since there is no simple formula for total emf and total internal resistance of cells joined in parallel. However, current in different branches of the circuit can be obtained using Kirchhoff's laws.

Example 60 **Two identical cells, whether joined together in series or in parallel give the same current, when connected to external resistance of 1 Ω. Find the internal resistance of each cell.**

Solution. Let ε, r be the emf and internal resistance of each cell. External resistance $R = 1\ \Omega$.

When the two cells are connected in series

Total emf of cells $= \varepsilon + \varepsilon = 2\varepsilon$

Total resistance of circuit $= R + r + r = 1 + 2r$

$$\text{Current in the circuit, } I_1 = \frac{2\varepsilon}{1+2r}$$

When the two cells are connected in parallel

Effective emf of two cells = emf of single cell $= \varepsilon$

Total internal resistance of two cells

$$= \frac{r \times r}{r+r} = \frac{r}{2}$$

Total resistance of the circuit $= R + r/2 = 1 + r/2$.

$$\text{Current in the circuit, } I_2 = \frac{\varepsilon}{1+r/2}$$

As per question, $I_1 = I_2$

$$\text{so } \frac{2\varepsilon}{1+2r} = \frac{\varepsilon}{1+r/2} = \frac{2\varepsilon}{2+r}$$

$$\text{or } 1+2r = 2+r \quad \text{or} \quad r = \mathbf{1\ \Omega}$$

Example 61 **8 cells, each of internal resistance 0·5 Ω and emf 1·5 V are used to send a current through an external resistance of (a) 200 Ω (b) 0·002 Ω (c) 1·0 Ω. How would you arrange them to get the maximum current in each case ? Find the value of current in each case.**

Solution. Here, total number of cells = 8 ;

$r = 0{\cdot}5\ \Omega$; $\varepsilon = 1{\cdot}5$ V.

(a) When $R = 200\ \Omega$; then $R >> r$, so for maximum current, the cells are to be connected in series in circuit.

Total internal resistance of 8 cells $= 8r$

Current in circuit,

$$I = \frac{8\varepsilon}{R+8r} = \frac{8 \times 1{\cdot}5}{200 + 8 \times 0{\cdot}5} = \frac{12{\cdot}0}{204} = \mathbf{0{\cdot}059\ A}$$

(b) When $R = 0{\cdot}002\ \Omega$, then $R << r$; so for maximum current, the cells are to be connected in parallel in circuit.

Total internal resistance of 8 cells $= r/8$

Total resistance of circuit $= R + r/8$

$$= 0{\cdot}002 + 0{\cdot}5/8 = 0{\cdot}0645\ \Omega$$

Effective emf of all cells = emf of each cell

$$= 1{\cdot}5 \text{ V}$$

$$\text{Current in circuit, } I = \frac{1{\cdot}5}{0{\cdot}0645} = \mathbf{23{\cdot}26\ A}$$

(c) When $R = 1{\cdot}0\ \Omega$, then R is comparable to r. For maximum current, the cells are to be connected in mixed grouping. Let there be m rows of cells in parallel with n cells in series, in each row. Then,

$$m\,n = 8 \qquad \ldots(i)$$

$$\text{For maximum current, } R = \frac{n\,r}{m}$$

$$\text{or } \quad 1{\cdot}0 = \frac{n \times 0{\cdot}5}{m} \quad \text{or} \quad n = 2\,\text{m}$$

From (i), $m \times 2\,\text{m} = 8$ or $m^2 = 4$ or $m = 2$.

So $n = 2 \times 2 = 4$

Thus, **4 cells in series in a row and 2 such rows of cells in parallel.**

$$\text{Max. current, } I = \frac{n\varepsilon}{R + nr/m} = \frac{mn\varepsilon}{mR + nr}$$

$$= \frac{8 \times 1{\cdot}5}{2 \times 1 + 4 \times 0{\cdot}5} = \mathbf{3\ A}$$

Example 62 **In the given circuit as shown in Fig. 3.58, in the steady state, obtain the expressions for (*i*) potential drop (*ii*) the charge and (*iii*) the energy stored in the capacitor *C*.**

(CBSE (F) 2015)

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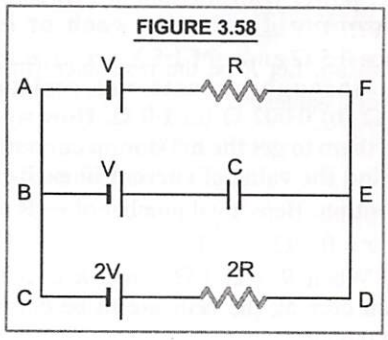
FIGURE 3.58



Solution. In the steady state, the capacitor C is the fully charged. So no current flows through the arm BE of the circuit.

$\therefore$ Effective *emf* of the circuit $= 2V - V = V$

Effective resistance of the circuit $= 2R + R$

$= 3R$

Current in the circuit, $I = \dfrac{V}{3R}$

Potential difference across B and E

$$= 2V - I \times 2R = 2V - \frac{V}{3R} \times 2R = \frac{4}{3}V$$

(*i*) Potential difference across C

$$= \frac{4}{3}V - V = \frac{\mathbf{V}}{\mathbf{3}}$$

(*ii*) Charge on capacitor

$$Q = CV = C \times \frac{V}{3} = \frac{\boldsymbol{CV}}{\mathbf{3}}$$

(*iii*) Energy stored in the capacitor

$$= \frac{1}{2}C\left(\frac{V}{3}\right)^2 = \frac{\boldsymbol{CV^2}}{\mathbf{18}}$$

Example 63 **(a) Six lead-acid type of secondary cells each of emf 2·0 V and internal resistance 0·015 Ω are joined in series to provide a supply to a resistance of 8·5 Ω. What are the current drawn from the supply and its terminal voltage ?**

(b) A secondary cell after long use has an emf of 1·9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell ? Could the cell drive the starting motor of a car ?

Solution. (*a*) Here, $\varepsilon = 2{\cdot}0$ V ; $n = 6$;

$r = 0{\cdot}015\ \Omega$; $R = 8{\cdot}5\ \Omega$

Current, $I = \dfrac{n\varepsilon}{R + nr} = \dfrac{6 \times 2{\cdot}0}{8{\cdot}5 + 6 \times 0{\cdot}015} = \mathbf{1{\cdot}4\ A}$

Terminal voltage, $V = IR = 1{\cdot}4 \times 8{\cdot}5 = \mathbf{11{\cdot}9\ V}$.

(*b*) Here, $\varepsilon = 1{\cdot}9\ V$; $r = 380\ \Omega$.

$$I_{max} = \frac{\varepsilon}{r} = \frac{1{\cdot}9}{380} = \mathbf{0{\cdot}005\ A}$$

This amount of current cannot start a car because to start the motor, the current required is **100 A for few seconds.**

TYPE X. ELECTRIC POWER

Formulae used. Electric power ;

$$P = VI = I^2R = \frac{V^2}{R}$$

where P is lower, V is voltage, I is current and R is resistance.

Units used. P in watt, V in volt, I in A, and R in Ω.

Example 64 **How many electrons flow through the filament of 220 V and 100 W electric lamp per second. Given, electronic charge $= 1{\cdot}6 \times 10^{-19}$ C.**

Solution. Here, $V = 220$ V, $P = 100$ W, $t = 1$ s

$$I = \frac{P}{V} = \frac{100}{220} = \frac{5}{11}\text{A}$$

But $I = \dfrac{q}{t} = \dfrac{ne}{t}$ or $n = \dfrac{It}{e} = \dfrac{(5/11) \times 1}{1{\cdot}6 \times 10^{-19}}$

$= \mathbf{2{\cdot}84 \times 10^{18}}$

Example 65 **A heating element is marked 210 V, 630 W. What is the current drawn by the element when connected to a 210 V dc mains ? What is the resistance of an element ?**

(NCERT Solved Example)

Solution. Here, $V = 210$ V, $P = 630$ W,

current drawn by element, $I = \dfrac{P}{V} = \dfrac{630}{210} = \mathbf{3\ A}$

Resistance of the element, $R = \dfrac{V}{I} = \dfrac{210}{3} = \mathbf{70\ \Omega}$

Example 66 **An electric motor operates on a 50 V supply and draws a current of 12 A. If the motor yields a mechanical power of 150 W, what is the efficiency of the motor ?**

(NCERT Solved Example)

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Solution. Here $V = 50$ V, $I = 12$ A

Input power $= VI = 50 \times 12 = 600$ W

Output power = 150 W

Efficiency of motor,

$$\eta = \frac{\text{output power}}{\text{input power}} \times 100 = \frac{150 \times 100}{600} = 25\%$$

Example 67 **A 1 kW electric heater is designed to operate from 220 V lines. (i) What is its resistance ? (ii) What current does it draw ? (iii) What is the rate of dissipation of energy in calories per second ? (iv) If the line voltage drops to 200 volt, what power does the heater take in watts ? [Assume the resistance is constant. Actually, it will change because of the change in temperature.]**

Solution. Here, power of electric heater,

$P = 1\text{kW} = 1000$ W ; $V = 220$ V

(*i*) Resistance of heater,

$$R = \frac{V^2}{P} = \frac{(220)^2}{1000} = 48{\cdot}4\ \Omega$$

(*ii*) Current drawn by heater,

$$I = \frac{P}{V} = \frac{1000}{220} = 4{\cdot}545\ \text{A}$$

(*iii*) Rate of dissipation of energy,

$$H = \frac{P}{J} = \frac{1000}{4{\cdot}2} = 238{\cdot}1\ \text{cal.s}^{-1}$$

(*iv*) New power of heater $= \frac{V_1^2}{R} = \frac{(200)^2}{48.4}$

$= 826{\cdot}45$ W

Example 68 **Which of the two has greater resistance a 1 kW heater or a 100 W tungsten bulb, both marked for 230 V ?** (NCERT Solved Example)

Solution. Here, $V = 230$ V,

$P_1 = 1$ kW $= 1000$ W ; $P_2 = 100$ W

$$R_1 = \frac{V^2}{P_1} = \frac{(230)^2}{1000} = 52{\cdot}9\ \Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{(230)^2}{100} = 529\ \Omega$$

As $R_2 > R_1$, so **100 W bulb has greater resistance.**

Example 69 **An electric power station (100 MW) transmits power to a distant load through long and thin cables. Which of the two modes of transmission would result in lesser power wastage : power transmission of : (*i*) 20,000 V or (*ii*) 200 V ?** (NCERT Solved Example)

Solution. Let R be the resistance (in ohm) of transmission cables.

Power of electric station,

$P = 100$ MW $= 100 \times 10^6$ W $= 10^8$ W

(*i*) $V_1 = 20{,}000$ V,

Current $I_1 = \frac{P}{V_1} = \frac{10^8}{20{,}000} = 5000$ A

Rate of heat dissipation at 20,000 V is

$$P_1 = I_1^2 R = (5000)^2 R = 25 \times 10^6\ \text{R watt}$$

(*ii*) $V_2 = 200$ V,

Current $I_2 = \frac{P}{V_2} = \frac{10^8}{200} = 5 \times 10^5$ A

Rate of heat dissipation at 200 V is

$$P_2 = I_2^2 R = (5 \times 10^5)^2 \times R$$

$$= 25 \times 10^{10}\ R\ \text{watt}$$

As $P_1 < P_2$, therefore, there will be **lesser power wastage when the power is transmitted at 20,000 V.**

TYPE XI. ELECTRIC ENERGY AND HEATING EFFECT OF CURRENT

Formulae used. (*i*) Electric energy,

$$E = VIt = \frac{V^2}{R}t = I^2Rt$$

(*ii*) Electric energy spent in heating a liquid from θ_1 to θ_2°C is $E = ms\,(\theta_2 - \theta_1)$

(*iii*) Units of electricity consumed

$$= \frac{V_{\text{volt}} \times I_{\text{amp}} \times t_{\text{hour}}}{1000}$$

(*iv*) Cost of electricity consumption = units of electricity consumed × cost per unit of electricity

(*v*) 1 unit of electricity = 1 kWh = $3{\cdot}6 \times 10^6$ J

Units used. *E*lectric energy in joule, m in kg, s in J $\text{kg}^{-1}\ \text{K}^{-1}$, $(\theta_2 - \theta_1)$ in °C or K.

Example 70 **The potential difference applied across a given resistor is altered so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change ?** (CBSE 2017)

Solution. As heat produced, $H = VIt = \frac{V^2}{R}t$

$$\therefore \quad H_1 = \frac{V_1^2 t}{R} \text{ and } H_2 = \frac{V_2^2 t}{R}$$

$$\frac{H_1}{H_2} = \frac{V_1^2}{V_2^2} \text{ or } \frac{V_2}{V_1} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{9H_1}{H_1}} = 3$$

or $V_2 = 3V_1$

Hence potential difference **changes by a factor of 3.**

Example 71 **A 10 V storage battery of negligible internal resistance is connected across a 50 Ω resistor made of alloy manganin. How much heat energy is produced in the resistor in 1 h ? What is the source of this energy ?** **(NCERT Solved Example)**

Solution. Here, $V = 10$ V, $R = 50\ \Omega$,

$t = 1$ h $= 60 \times 60$ s ;

Heat energy produced in 1 h is

$$H = \frac{V^2 t}{R} = \frac{(10)^2 \times (60 \times 60)}{50} = 7200 \text{ J}$$

The source of this energy is the chemical energy stored in the cell.

Example 72 **A heater coil is rated 1000 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.** **(CBSE 2000)**

Solution. Here, $P = 1000$ W, $V = 200$ V

Resistance of heater coil,

$$R = \frac{V^2}{P} = \frac{200 \times 200}{1000} = 40\ \Omega$$

Resistance of either part = 20 Ω

Equivalent resistance when both parts are connected in parallel, $R' = \frac{20 \times 20}{20 + 20} = 10\ \Omega$

Energy liberated per second in the new combination when connected to a source of 200 V

$$= \frac{V^2}{R'} = \frac{200 \times 200}{10} = 4000 \text{ Js}^{-1}$$

Example 73 **Two heaters are marked 200 V, 300 W and 200 V, 600 W. If the heaters are combined in series and the combination connected to a 200 V dc supply, which heater will produce more heat ?** **(NCERT Solved Example)**

Solution. Resistances of the two heaters are

$$R_1 = \frac{V^2}{P_1} = \frac{200 \times 200}{300} = \frac{400}{3}\ \Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{200 \times 200}{600} = \frac{200}{3}\ \Omega$$

For series combination, the effective resistance is

$$R = R_1 + R_2 = \frac{400}{3} + \frac{200}{3} = \frac{600}{3} = 200\ \Omega$$

Current, $I = \frac{V}{R} = \frac{200}{200} = 1$ A

Power dissipated in 1st heater,

$$P_1 = I^2 R_1 = 1^2 \times \frac{400}{3} = \frac{400}{3} \text{ W}$$

Power dissipated in second heater,

$$P_2 = I^2 R_2 = 1^2 \times \frac{200}{3} = \frac{200}{3} \text{ W}$$

$$\therefore \quad P_1 = 2P_2 \text{ or } P_1 > P_2$$

Example 74 **A heating coil of 2000 watt is immersed in an electric kettle. How much time will it take in raising the temperature of 1 litre of water from 4°C to 100°C ? Only 80% part of the thermal energy produced is used in raising the temperature of water.**

Solution. Here, $P = 2000$ W, $t = ?$ (in seconds)

Volume of water = 1 litre = 1000 c.c.

Mass of water, m = volume × density

$= 1000 \times 1 = 1000$ gram

Rise in temperature, $= \theta_2 - \theta_1 = 100 - 4$

$= 96°C$

We know sp. heat of water, $s = 1$ cal g^{-1} °C^{-1}

$\therefore$ Heat taken by water $= ms(\theta_2 - \theta_1)$

$= 1000 \times 1 \times 96 = 96000$ cal.

Energy spent in heating coil $= Pt = 2000 \times t$

Useful thermal energy produced = 80%

$$= 2000 \times t \times \frac{80}{100} = \frac{2000 \times t \times 80}{100 \times 4 \cdot 2} \text{ cal.}$$

As this thermal energy is taken by water, therefore, $\frac{2000 \times t \times 80}{100 \times 4 \cdot 2} = 96000$

or $t = \dfrac{96000 \times 100 \times 4{\cdot}2}{2000 \times 80} = 252$ **seconds.**

Example 75 **The resistance of each of the three wires joined as shown in the Fig. 3.59 is 4 Ω, and each one can have a maximum power of 20 watt (otherwise it will melt). What maximum power will the whole circuit dissipate ?**

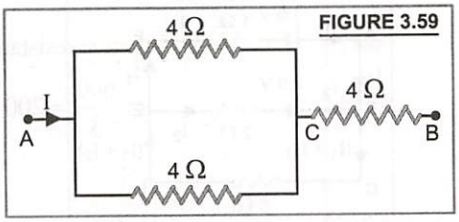


FIGURE 3.59

(CBSE 2003)

Solution. Let I be the current through a resistance wire of maximum power 20 watt.

$\therefore \quad I^2R = 20 \quad$ or $\quad I^2 \times 4 = 20 \quad$ or $\quad I^2 = 5$

Effective resistance R_p between A and C will be

$$R_p = \frac{4 \times 4}{4+4} = 2\,\Omega$$

Electric power dissipated as heat by the resistance between A and $C = I^2 R_p = 5 \times 2$

$= 10$ watt.

Electric power dissipated as heat by the resistance between C and $B = I^2 R = 5 \times 4 = 20$ watt.

$\therefore$ Total electric power dissipated as heat by the whole circuit $= 10 + 20 =$ **30 watt.**

Example 76 **Two uniform wires of same material each weighting 1g but one having double the length of the other, are connected in series, carrying a current of 10 A. The length of longer wire is 20 cm. Calculate the rate of consumption of energy in each of the two wires. Which one wire gets hotter ? Density of the material of wire = 11g/cc, specific resistance of the material = 20×10^{-5} Ω cm.**

Solution. Let a_1 and a_2 be the area of cross-section of shorter and longer wire respectively.

As, mass = volume × density = $a\,l\,\rho$

$\therefore \quad 1 = a_1 \times 10 \times 11 = a_2 \times 20 \times 11$

or $a_1 = \dfrac{1}{10 \times 11}$ cm^2 and $a_2 = \dfrac{1}{20 \times 11}$ cm^2

$$\therefore \quad R_1 = 20 \times 10^{-5} \times \frac{10}{1/(10 \times 11)}$$

$$= 20 \times 10^{-5} \times 10 \times 10 \times 11 = 22 \times 10^{-2}\ \Omega$$

$$\therefore \quad R_2 = 20 \times 10^{-5} \times \frac{20}{1/(20 \times 11)} = 88 \times 10^{-2}\ \Omega$$

And rate of heat produced,

$$H_1 = I^2 R_1 = (10)^2 \times 22 \times 10^{-2} = 22\ \text{W}$$

and $\quad H_2 = I^2 R_2 = (10)^2 \times 88 \times 10^{-2} = 88$ W

Thus the wire **of longer length gets hotter.**

Example 77 **A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected to two resistances of 10 Ω and R Ω, Fig. 3.60. If the heater is now operating with a power of 62·5 W, calculate the value of R.**

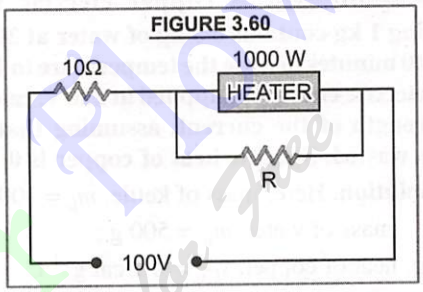


FIGURE 3.60

(CBSE 2003)

Solution. Resistance of heater,

$$R' = \frac{V^2}{P} = \frac{100^2}{1000} = 10\,\Omega$$

When heater is in the circuit, let pot. diff. across heater be V, then $62{\cdot}5 = V^2/10$ or $V^2 = 625$

or $\quad V = 25$ volt

Therefore, current through heater,

$I_2 = V/R' = 25/10 = 2{\cdot}5$ A.

Voltage drop across 10 Ω resistor connected in series with the heater $= 100 - 25 = 75$ volt.

$$\text{Main current} = \frac{75}{10} = 7{\cdot}5\ \text{A}$$

$\therefore$ current through resistance $R = 7{\cdot}5 - 2{\cdot}5 = 5$ A.

$$\therefore \quad \text{Resistance, } R = \frac{25}{5} = 5\,\Omega$$

Example 78 **The resistance of a 220 V and 200 W electric bulb when hot is 12 times the resistance at room temperature (= 30°C). If the working temperature of the filament is 2000°C, find the temperature coefficient of resistance of the filament.**

Solution. For hot bulb at temperature,

$t = 2000$°C,

Resistance, $R_t = \frac{V^2}{P} = \frac{(220)^2}{200} = 242\ \Omega$

Resistance of bulb at room temperature

$$R = \frac{R_t}{12} = \frac{242}{12} \approx 20\ \Omega$$

Since $R_t = R\,[1 + \alpha \times (1000 - 30)]$

$242 = 20\,[1 + \alpha + 1970]$

$14{\cdot}1 = 1 + \alpha \times 1970$

$$\alpha = \frac{13{\cdot}1}{1970} = 6{\cdot}6 \times 10^{-3}\ ^\circ C^{-1}$$

Example 79 **A copper electric kettle weighing 1 kg contains 0·5 kg of water at 20°C. It takes 10 minutes to raise the temperature to 100°C. If the electric energy is supplied at 220 V, calculate the strength of the current, assuming that 20% heat is wasted. Specific heat of copper is 0·1.**

Solution. Here, mass of kettle, $m_k = 1000$ g ;

mass of water, $m_w = 500$ g ;

sp. heat of copper, $s_{Cu} = 0{\cdot}1$ cal $g^{-1}\ C^{-1}$;

sp. heat of water, $s_w = 1$ cal $g^{-1}\ ^\circ C^{-1}$

Rise in temperature $= \theta_2 - \theta_1 = 100 - 20 = 80^\circ C$

Heat required $= (m_k\, s_{cu} + m_w\, s_w)\,(\theta_2 - \theta_1)$

$= (1000 \times 0{\cdot}1 + 500 \times 1) \times 80 = 48000$ cal

Heat produced,

$$H = \frac{VIt}{J} = \frac{220 \times I \times (10 \times 60)}{4{\cdot}2}\ \text{cal.}$$

Useful heat produced $= (100 - 20) = 80\%$

$$= \frac{80}{100} \times \frac{220 \times I \times 10 \times 60}{4{\cdot}2}\ \text{cal.}$$

$$\therefore \quad \frac{80}{100} = \frac{220 \times I \times 10 \times 60}{4{\cdot}2} = 48000$$

On solving, $I = 1{\cdot}9$ A

TYPE XII. KIRCHHOFF'S LAWS

Formulae used. (*i*) $\Sigma i = 0$, and (*ii*) $\Sigma \varepsilon = \Sigma\, IR$.
where i is current, R is resistance and ε is the *emf*.
Units used. i in ampere ; R in ohm, and ε in volt.

Example 80 **Two cells of emfs 1·5 V and 2·0 V internal resistances 1 Ω and 2 Ω are connected in parallel so as to send current in the same direction through an external resistance of 5 Ω.**

(*i*) Draw the circuit diagram.

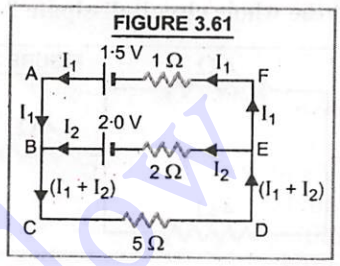
(*ii*) Using Kirchhoff's rules, calculate,

(*a*) current through each branch of the circuit

(*b*) potential difference across the 5 Ω resistance. **(CBSE 2005)**

Solution. (*i*) The circuit diagram is shown in the Fig. 3.61.

FIGURE 3.61



(*ii*) (*a*) Let the currents through the various arms of net work be as shown in figure.

Using Kirchhoff's second rule for the loop *ABCDEFA*, we have

$$5\,(I_1 + I_2) + 1\,I_1 = 1{\cdot}5$$

or $$6\,I_1 + 5\,I_2 = 1{\cdot}5 \qquad ...(i)$$

For the loop *BCDEB*, we have

$$5\,(I_1 + I_2) + 2\,I_2 = 2$$

or $$5\,I_1 + 7\,I_2 = 2 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get ;

$$I_1 = \frac{1}{34}A\,, \quad I_2 = \frac{9}{34}A$$

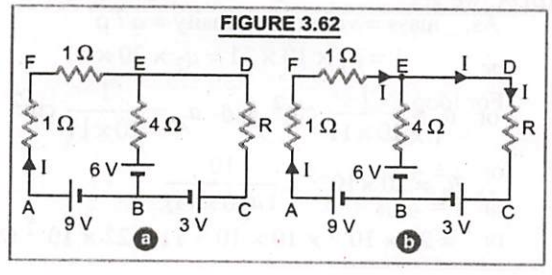
and $$I_1 + I_2 = \frac{1}{34} + \frac{9}{34} = \frac{10}{34}A$$

(*b*) Pot. diff. across 5 Ω resistance

$$= (I_1 + I_2) \times 5 = \frac{10}{34} \times 5 = 1{\cdot}47\ \text{V}$$

Example 81 **Using Kirchhoff's rules deter- mine the value of unknown resistance R in the circuit shown in Fig. 3.62(*a*) so that no current flows through 4 Ω resistance. Also find the potential difference between A and D.** **(CBSE 2012)**

FIGURE 3.62



Solution. The current I flows in the circuit as shown in Fig. 3.66(*b*).

Applying Kirchhoff's second rule to loop *AFEBA*, we have

$$1 \times I + 1 \times I = -6 + 9 = 3 \text{ or } I = 1{\cdot}5 \text{ A}$$

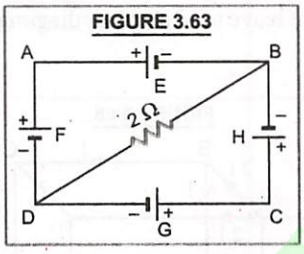
From the loop *BEDCB*, we have

$$IR = 6 - 3 = 3 \text{ or } R = \frac{3}{1{\cdot}5} = \mathbf{2\ \Omega}$$

$$V_{AD} = (1 + 1) \times I = 2 \times 1{\cdot}5 = \mathbf{3\ V}$$

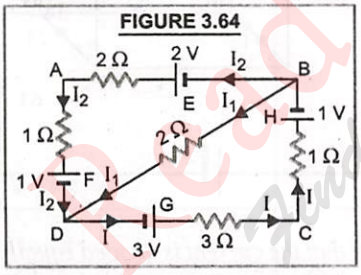
Example 82 **In the circuit shown in Fig. 3.63, *E*, *F*, *G* and *H* are cells of emf 2 V, 1 V, 3 V and 1 V and their internal resistances are 2Ω, 1Ω, 3Ω and 1Ω respectively. Calculate (*i*) the potential difference between *B* and *D* and (*ii*) the potential difference across the terminals of each of the cells *G* and *H*.** **[CBSE 2004 (C)]**

FIGURE 3.63



Solution. (*i*) The complete network has been shown in Fig. 3.64 with currents in the various arms.

FIGURE 3.64



Applying Kirchhoff's first rule at junction *B*,

$$I = I_1 + I_2 \qquad \text{...(i)}$$

Applying Kirchhoff's second rule to loop *ABDA*, we get

$$-2I_2 + 2I_1 - 1I_2 = -2 + 1$$

or $$2I_1 - 3I_2 = -1 \qquad \text{...(ii)}$$

For loop *BCDB*, we get

$$-1 \times I - 3 \times I - 2I_1 = 1 - 3 = -2$$

or $2 = 4I + 2I_1 = 4(I_1 + I_2) + 2I_1$

or $1 = 2(I_1 + I_2) + I_1 = 3I_1 + 2I_2$

or $$3I_1 + 2I_2 = 1 \qquad \text{...(iii)}$$

On solving (*i*), (*ii*) and (*iii*), we get

$$I_1 = \frac{1}{13}A,\ I_2 = \frac{5}{13}A \text{ and } I = \frac{6}{13}A$$

Pot. diff. between points *B* and *D*

$$= I_1 \times 2 = \frac{1}{13} \times 2 = \mathbf{\frac{2}{13}\ V}$$

(*ii*) Pot. diff. across the terminals of G (giving current)

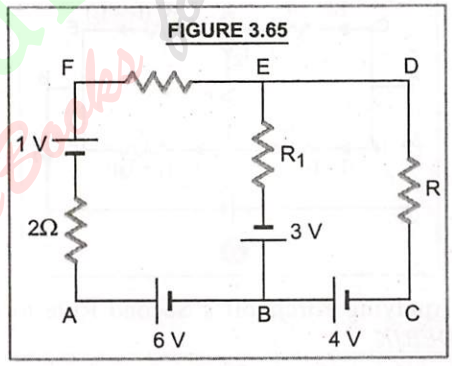
$$V_G = \varepsilon - Ir = 3 - \frac{6}{13} \times 3 = \mathbf{1{\cdot}615\ V}$$

Pot. diff. across the terminals of *H* (receiving current for charging)

$$V_H = \varepsilon' + Ir' = 1 + \frac{6}{13} \times 1 = \mathbf{1{\cdot}46\ V}$$

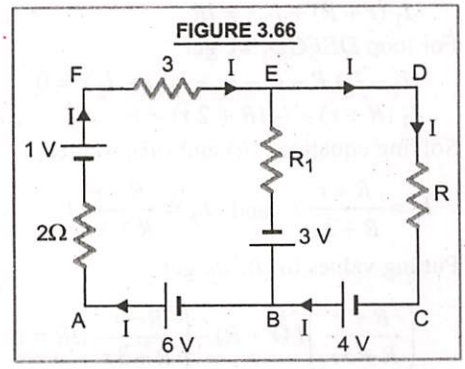
Example 83 **Use Kirchhoff's rules to determine the potential difference between the points *A* and *D* when no current flows in the arm *BE* of the electric network shown in Fig. 3.65.** **(CBSE 2015)**

FIGURE 3.65



Solution. Since no current flows in the arm *BE*, so the current I flows along the outer loop as shown in Fig. 3.66.

FIGURE 3.66



Using Kirchhoff's loop rule to the closed circuit *AFEBA*, we have

$$2I - 1 + 3I + R_1 \times 0 - 3 - 6 = 0$$

or $5I = 10$ or $I = 2\text{A}$

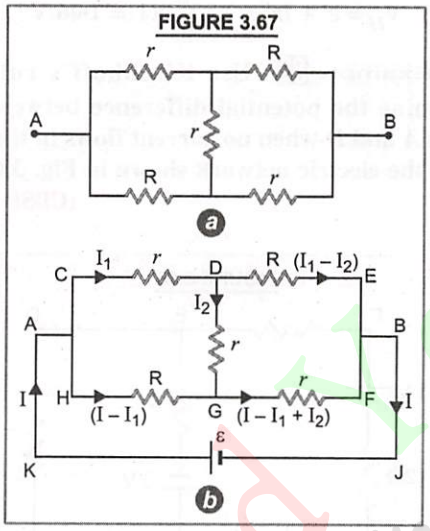
Potential difference across *A* to *D* while following the path *AFD*, we have

$$V_{AD} = 2 \times 2 - 1 + 3 \times 2 = \mathbf{9V}$$

Example 84 Calculate the equivalent resistance between the points *A* and *B* of the network shown in Fig. 3.67(*a*).

Solution. Let a cell of emf ε be connected between points *A* and *B*. Let the currents in the various branches be as shown in Fig. 3.67(*b*).

FIGURE 3.67



Applying Kirchhoff's Second Rule to loop *KACDEBJK*

$$I_1 r + (I_1 - I_2) R = \varepsilon$$

or $I_1 (r + R) - I_2 R = \varepsilon$ ...(*i*)

For loop *CDGHC*, we get

$$I_1 r + I_2 r - (I - I_1) R = 0$$

$$I_1 (r + R) + I_2 r = IR \quad \text{...(ii)}$$

For loop *DEFGD*, we get

$$(I_1 - I_2) R - (I - I_1 + I_2) r - I_2 r = 0$$

$$I_1 (R + r) - I_2 (R + 2r) = Ir \quad \text{...(iii)}$$

Solving equations (*ii*) and (*iii*), we get

$$I_1 = \frac{R + r}{R + 3r} I \quad \text{and} \quad I_2 = \frac{R - r}{R + 3r} I$$

Putting values in (*i*), we get

$$\left(\frac{R + r}{R + 3r}\right) I (r + R) - \left(\frac{R - r}{R + 3r}\right) IR = \varepsilon$$

or $$\left(\frac{3rR + r^2}{R + 3r}\right) I = \varepsilon$$

Equivalent resistance between *A* and *B*

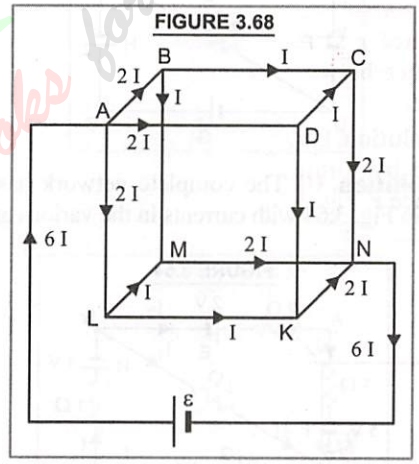
$$R_{eq} = \frac{\varepsilon}{I} = \frac{3rR + r^2}{R + 3r} = \frac{r(3R + r)}{(R + 3r)}$$

Example 85 A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of resistance 1 Ω. Determine equivalent resistance of the network and the current through the battery.

(NCERT Solved Example)

Solution. Let *ABCDKLMN* be the skeleton cube formed by joining twelve equal wires each of resistance *r*, Fig. 3.68. Let the current enter the cube at corner *A* and after passing through all twelve wires, let the current leave at *N*, a corner diagonally opposite to corner *A*.

FIGURE 3.68



Note *that the current is divided equally amongst paths of equal resistances provided their end points are situated symmetrically w.r.t. the exit point. However, along paths of different resistances, current is divided unequally in the inverse ratio of resistances.*

For the sake of convenience, let us suppose that the total current is 6 *I*. At *A*, this current is divided into three equal parts each (2 *I*) along *AB*, *AD* and *AL* as the resistances along these paths are equal. At the points *B*, *D* and *L*, each part is further divided into two equal parts each equal to *I* because resistances along the two paths are equal. Thus, the distribution of current in the various arms of skeleton cube is

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shown according to Kirchhoff's First rule. The current leaving the cube at N is again $6I$. If ε is the emf of the battery used, then applying Kirchhoff's Second rule to the closed circuit $ALKN\varepsilon A$, we get

$$2Ir + Ir + 2Ir = \varepsilon \quad \text{or} \quad 5Ir = \varepsilon \qquad ...(i)$$

where ε is the e.m.f. of the cell of negligible internal resistance.

If R is the resistance of the cube between the diagonally opposite corners A and N, then according to Ohm's Law, we have

$$6I \times R = \varepsilon \qquad ...(ii)$$

From (i) and (ii), $6IR = 5Ir$ or $R = \frac{5}{6}r$

Here, $r = 1\ \Omega$, so, $R = \frac{5}{6} \times 1 = \frac{5}{6}\Omega$

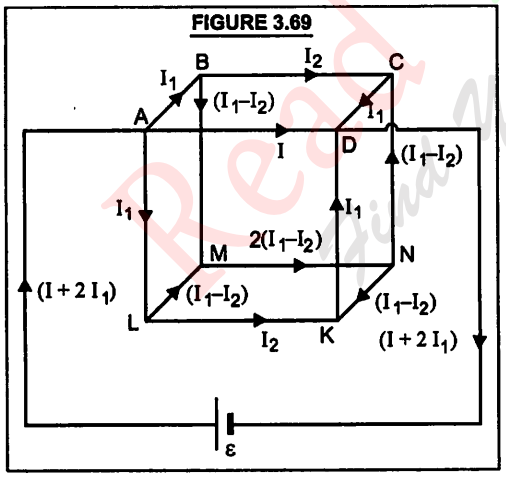
$$\text{Current through battery} = \frac{\text{voltage of battery}}{\text{total resistance}}$$

$$= \frac{10}{5/6} = 12\ \text{A}$$

Example 86 Twelve equal wires each of resistance r Ω form a skeleton cube. Find the resistance between the corners of the same edge of cube.

Solution. Let $ABCDKLMN$ be the skeleton cube formed by joining twelve equal wires each of resistance r, Fig. 3.69.

FIGURE 3.69



Let the current enter the cube at corner A and after passing through all the twelve wires, let the current leave at D, other corner of the same edge of the cube.

For the sake of convenience, let us suppose that the total current is $(I + 2I_1)$. At A, this current is divided into three parts : I along AD and I_1 each along AB and AL as the resistances along AB and AL are the same but along AD, it is different. At the points B and L, current is further divided into two unequal parts. The distribution of current in the various arms of skeleton cube is shown according to Kirchhoff's First rule. The current leaving the cube at D is again $(I + 2I_1)$. Applying Kirchhoff's Second rule to the closed mesh $ABCDA$, we get

$$I_1 r + I_2 r + I_1 r - Ir = 0$$

$$\text{or} \quad 2I_1 + I_2 = I \qquad ...(i)$$

For closed mesh $BCNMB$;

$$I_2 r - (I_1 - I_2) r - 2(I_1 - I_2) r - (I_1 - I_2) r = 0$$

$$\text{or} \quad I_2 r = 4(I_1 - I_2) r$$

$$\text{or} \quad I_2 = 4I_1 - 4I_2 \qquad ...(ii)$$

$$\text{or} \quad 5I_2 = 4I_1 \quad \text{or} \quad I_2 = \frac{4}{5}I_1$$

$$\text{From } (i), \quad I = 2I_1 + \frac{4}{5}I_1 = \frac{14}{5}I_1 \qquad ...(iii)$$

For closed mesh $AD\varepsilon A$, $Ir = \varepsilon$ ...(iv)

where ε is the emf of the cell of negligible internal resistance.

If R is the total resistance between the two corners of the same edge of the cube, then applying Kirchhoff's second rule for the closed mesh $AD\varepsilon A$, we get

$$(I + 2I_1) R = \varepsilon \quad \text{or} \quad \left(\frac{14}{5}I_1 + 2I_1\right) R = \varepsilon$$

$$\text{or} \quad \frac{24}{5} I_1 R = \varepsilon \qquad ...(v)$$

$$\text{From } (iii) \text{ and } (iv), \quad \frac{14}{5} I_1 r = \varepsilon \qquad ...(vi)$$

From (v) and (vi),

$$\frac{24}{5} I_1 R = \frac{14}{5} I_1 r \quad \text{or} \quad R = \frac{14}{24} r = \frac{7}{12} r$$

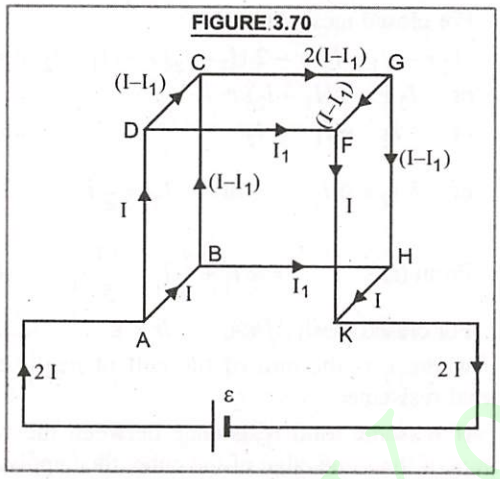
Example 87 Eleven equal wires each of resistance 2 Ω form the edges of an incomplete cube. Find the total resistance from one end of vacant edge to the other end of the same edge.

Solution. Let $ABCDFGHK$ be the open cube formed by joining eleven equal wires each of resistance 2 Ω. Fig. 3.70.

Let the current from the cell of *emf* ε enter the cube at corner A and after passing through all the wires leave at K, the other end of the open edge of the cube.

For the sake of convenience, let us suppose that the total current is 2 *I*. At *A*, this current is divided into two parts : *I* along *AB* and *I* along *AD*. This is because the resistances of the two paths are equal. At the points *B* and *D*, each part of current is further divided into two unequal parts. The distribution of current in the various arms of skeleton cube is shown according to Kirchhoff's first rule. The current leaving the cube at *K* is again 2 *I*. Let *R* be the equivalent resistance of skeleton cube between edges *A* and *K*.

FIGURE 3.70



From Ohm's law, $\varepsilon = 2\,I\,R$ ...(*i*)

Applying Kirchhoff's second rule to mesh $\varepsilon ABHK\varepsilon$, we have

$$\varepsilon = 2 \times I + 2 \times I_1 + 2 \times I$$

or $\varepsilon = 4\,I + 2\,I_1$ ...(*ii*)

Applying Kirchhoff's Second rule to mesh of DFGCD, we have

$$2\,I_1 - 2\,(I - I_1) - 2 \times 2\,(I - I_1) - 2\,(I - I_1) = 0$$

$$2\,I_1 - 2\,I + 2\,I_1 - 4\,I + 4\,I_1 - 2\,I + 2\,I_1 = 0$$

or $10\,I_1 - 8\,I = 0$ or $8\,I = 10\,I_1$

or $$I_1 = \frac{8}{10} I = \frac{4}{5} I$$ ...(*iii*)

From (*ii*) and (*iii*),

$$\varepsilon = 4\,I + 2\left(\frac{4}{5} I\right) = \frac{28\,I}{5}$$ ...(*iv*)

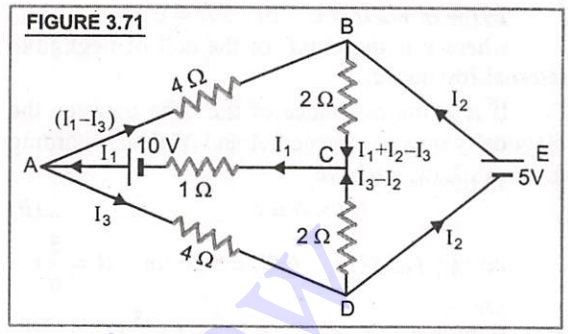
From (*i*) and (*iv*),

$$2\,IR = \frac{28\,I}{5}$$

or $$R = \frac{28}{5 \times 2} = \mathbf{2{\cdot}8\ \Omega}$$

Example 88 **Determine current in each branch of the network shown in the Fig. 3.71.** (NCERT Solved Example)

FIGURE 3.71



Solution. In the closed circuit *ABCA*,

$$(I_1 - I_3)\,4 + (I_1 + I_2 - I_3)\,2 + I_1 \times 1 = 10$$

or $$7\,I_1 + 2\,I_2 - 6\,I_3 = 10$$ ...(*i*)

In the closed circuit *ADCA*,

$$I_3 \times 4 + (I_3 - I_2)\,2 + I_1 \times 1 = 10$$

or $$I_1 - 2\,I_2 + 6\,I_3 = 10$$ ...(*ii*)

In the closed current *ABEDA*

$$(I_1 - I_3)\,4 - I_3 \times 4 = -5$$

or $$4\,I_1 - 8\,I_3 = -5$$ ...(*iii*)

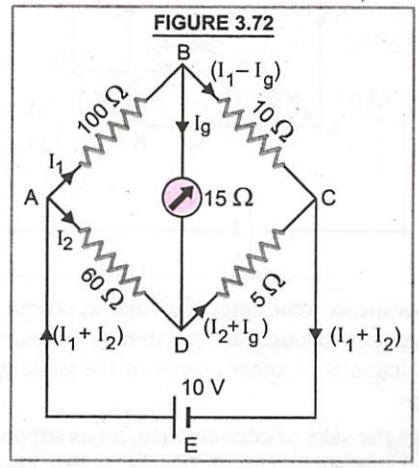
On solving (*i*), (*ii*) and (*iii*), we get

$$I_1 = \mathbf{2{\cdot}5\ A},$$

$$I_2 = \mathbf{1{\cdot}875\ A} = I_3.$$

Example 89 **The Wheat stone bridge circuit have the resistances in various arms as shown in Fig 3.72. Calculate the current through the galvanometer.** (NCERT Solved Example)

FIGURE 3.72



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Solution. In the closed loop $ABDA$;

$$100\, I_1 + 15\, I_g - 60\, I_2 = 0$$

or $$20\, I_1 + 3\, I_g - 12\, I_2 = 0 \quad ...(i)$$

In the closed loop $BCDB$;

$$10\,(I_1 - I_g) - 5\,(I_2 + I_g) - 15\, I_g = 0$$

or $$10\, I_1 - 30\, I_g - 5\, I_2 = 0$$

or $$2\, I_1 - 6\, I_g - I_2 = 0 \quad ...(ii)$$

In the closed loop $ADCEA$;

$$60\, I_2 + 5\,(I_2 + I_g) = 10$$

or $65\, I_2 + 5\, I_g = 10$ or $13\, I_2 + I_g = 2$ ...(*iii*)

On solving (*i*), (*ii*) and (*iii*), we get

$$I_g = \frac{4}{821} A = \mathbf{4{\cdot}87\ mA}$$

TYPE XIII. WHEATSTONE BRIDGE PRINCIPLE AND SLIDE WIRE BRIDGE

Formulae used. (*i*) $\frac{P}{Q} = \frac{R}{S}$ (*ii*) $S = \left(\frac{100 - l}{l}\right) R$

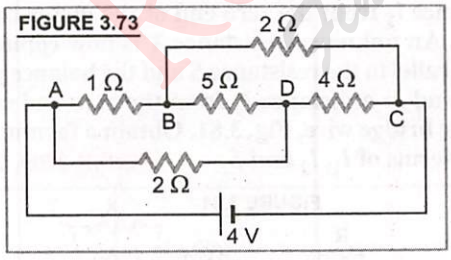
where, P, Q, R, S = resistance in four arms of Wheatstone bridge.

l = length of meter bridge wire where galvanometer shows no deflection.

Units used. P, Q, R and S are in *ohm*, l is in *cm*.

Example 90 **Calculate the current drawn from the battery by the network of resistors shown in Fig. 3.73.** [CBSE 2015 (C), 2009]

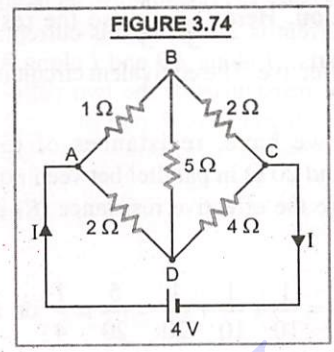
FIGURE 3.73



Solution. The equivalent circuit for the given network is shown in Fig. 3.74, which is a Wheatstone bridge. Here $P = 1\ \Omega$, $Q = 2\ \Omega$, $R = 2\ \Omega$ and $S = 4\ \Omega$

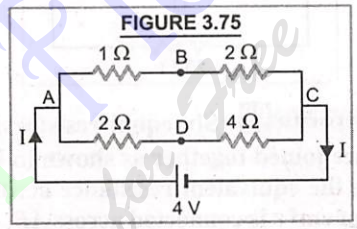
and $\frac{1\Omega}{2\Omega} = \frac{2\Omega}{4\Omega}$ *i.e.*, $\frac{P}{Q} = \frac{R}{S}$

FIGURE 3.74



Thus the bridge is a balanced Wheatstone bridge. The resistance 5 Ω in arm BD is ineffective. Therefore, the equivalent circuit reduces to the circuit as shown in Fig. 3.75.

FIGURE 3.75



The resistance of arm $ABC = 1 + 2 = 3\ \Omega$

Resistance of arm $ADC = 2 + 4 = 6\ \Omega$

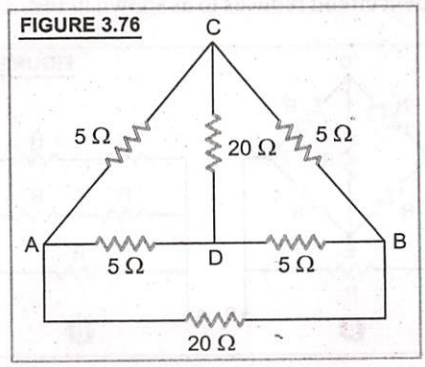
The resistances of these two arms are in parallel. The equivalent resistance R_{eq} between A and C is

$$R_{eq} = \frac{3 \times 6}{3 + 6} = 2\Omega$$

$$\text{Current } I = \frac{V}{R_{eq}} = \frac{4}{2} = \mathbf{2\ A}$$

Example 91 **Calculate the resistance between the points A and B of the network shown in Fig. 3.76.**

FIGURE 3.76

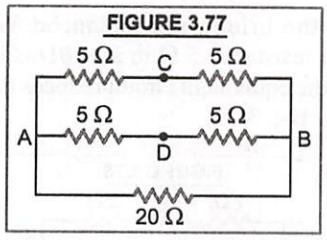


Solution. Here, $\frac{5}{5}=\frac{5}{5}$, so the resistance of 20 Ω is ineffective. The equivalent circuit is as shown in Fig. 3.77.

Now we have, resistances of (5 + 5) Ω, (5 + 5) Ω and 20 Ω in parallel between points *A* and *B*. Therefore the effective resistance (*R*) between *A* and *B* is

$$\frac{1}{R}=\frac{1}{10}+\frac{1}{10}+\frac{1}{20}=\frac{5}{20}=\frac{1}{4} \quad \text{or} \quad R=4\ \Omega$$

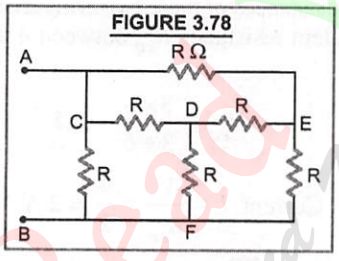
FIGURE 3.77



Example 92 **Six equal resistors, each of value *R* are joined together as shown in Fig. 3.78. Calculate the equivalent resistance across *AB*. If a supply of emf ε is connected across *AB*, compute the current through the arms *DE* and *AB*.**

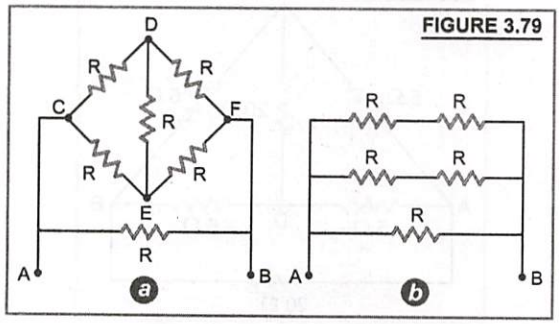
(CBSE Sample Paper 2003)

FIGURE 3.78



Solution. The equivalent circuit is shown in Fig. 3.78(*a*), where resistance *R* of arm *DE* is ineffective because the wheatstone bridge is balanced. Thus the equivalent circuit reduces to as shown in Fig. 3.79(*b*).

FIGURE 3.79



The equivalent resistance R_{eq} across *AB* is

$$\frac{1}{R_{eq}}=\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}=\frac{4}{2R}=\frac{2}{R} \quad \text{or} \quad R_{eq}=\frac{R}{2}$$

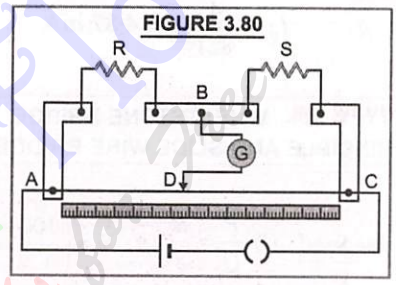
If a battery of emf ε is connected to *AB*, then

$$\text{current through arm } AB=\frac{\varepsilon}{R_{eq}}=\frac{\varepsilon}{R/2}=\frac{2\varepsilon}{R}$$

Current through arm *DE* = **0** (as Wheatstone bridge is in balanced state)

Example 93 **In meter bridge, the null point is found at a distance of 60·0 cm from end *A*. If now a resistance of 5 Ω is connected in series with *S*, the null point occurs at 50 cm. Determine the values of *R* and *S*, Fig. 3.80.** **(CBSE 2010)**

FIGURE 3.80



Solution. In first case, $\frac{R}{S}=\frac{60}{100-60}=\frac{3}{2}$ ...(*i*)

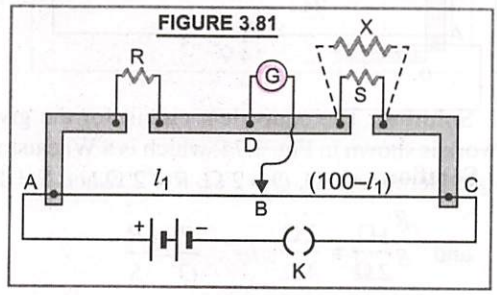
In second case, $\frac{R}{S+5}=\frac{50}{50}=1$ or $R=S+5$

From (*i*), $\frac{S+5}{S}=\frac{3}{2}$ or $2S+10=3S$

or $S=\mathbf{10\ \Omega}$ and $R=10+5=\mathbf{15\ \Omega}$

Example 94 **When two known resistances *R* and *S* are connected in the left and right gaps of a meter bridge, the balance point is found at a distance l_1 from the zero end of the meter bridge wire. An unknown resistance *X* is now connected in parallel to the resistance *S* and the balance point is found at a distance l_2 from the zero end of the meter bridge wire, Fig. 3.81. Obtain a formula for *X* in terms of l_1, l_2 and *S*.** **(CBSE 2009, 2017)**

FIGURE 3.81



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Solution. When resistance R and S are connected to the left and right gaps of meter bridge and bridge is balanced at length l_1 from zero end, then

$$\frac{R}{S} = \frac{l_1}{(100 - l_1)} \quad ...(i)$$

When unknown resistance X is connected in parallel to S, then effective resistance in right gap is

$$S' = \frac{SX}{S + X} \quad ...(ii)$$

Now, balance point is obtained at length l_2,

$$\therefore \quad \frac{R}{S'} = \frac{l_2}{(100 - l_2)}$$

Putting the value of S', we have

$$\frac{R(S + X)}{SX} = \frac{l_2}{(100 - l_2)} \quad ...(iii)$$

Dividing (*iii*) by (*i*), we get

$$\frac{S + X}{X} = \frac{l_2}{(100 - l_2)} \times \frac{(100 - l_1)}{l_1}$$

or $$\frac{S}{X} + 1 = \frac{l_2 (100 - l_1)}{l_1 (100 - l_2)}$$

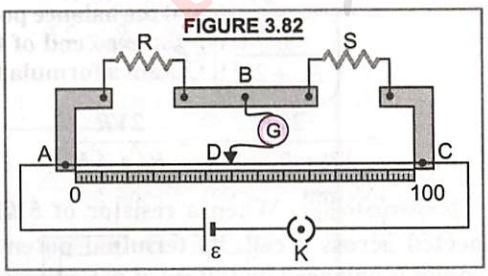
or $$\frac{S}{X} = \frac{100 l_2 - l_1 l_2 - 100 l_1 + l_1 l_2}{l_1 (100 - l_2)} = \frac{100 (l_2 - l_1)}{l_1 (100 - l_2)}$$

or $$X = \frac{l_1 (100 - l_2)}{100 (l_2 - l_1)} S$$

Example 95 **In a meter bridge Fig. 3.82, the null point is found at a distance of 33·7 cm from A. If now a resistance of 12 Ω is connected in parallel with S, the null point occurs at 51·9 cm. Determine the values of R and S.**

(NCERT Solved Example)

FIGURE 3.82



Solution. For the first balanced bridge situation,

$$\frac{R}{S} = \frac{33·7}{100 - 33·7} = \frac{33·7}{66·3} \quad ...(i)$$

When 12 Ω is connected in parallel with S, the equivalent resistance is

$$S_{eq} = \frac{12 S}{12 + S}$$

For the second balanced bridge situation,

$$\frac{R}{S_{eq}} = \frac{51·9}{100 - 51·9} = \frac{51·9}{48·1}$$

or $$\frac{R(12 + S)}{12 S} = \frac{51·9}{48·1} \quad ...(ii)$$

Putting the value of $\frac{R}{S}$ from (*i*) in (*ii*),

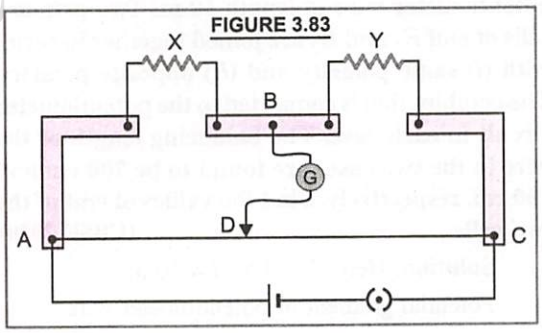
we have $$\frac{(12 + S)}{12} \times \frac{33·7}{66·3} = \frac{51·9}{48·1}$$

On solving we get, $S = \mathbf{13·5\ \Omega}$

From (*i*), $$R = \frac{33·7}{66·3} S = \frac{33·7}{66·3} \times 13·5 = \mathbf{6·86\ \Omega}$$

Example 96 **The Fig. 3.83 shows the experimental set up of a meter bridge. The null point is found to be 60 cm away from the end A with X and Y in positions as shown. When a resistance of 15 Ω is connected in series with Y, the null point is found to shift by 10 cm towards the end A of the wire. Find the position of the null point if a resistance of 30 Ω were connected in parallel with Y.** **(CBSE Sample Paper 2008)**

FIGURE 3.83



Solution. *In first case,* $$\frac{X}{Y} = \frac{60}{40} = \frac{3}{2} \quad ...(i)$$

In second case, $$\frac{X}{Y + 15} = \frac{50}{50} = 1 \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{X}{Y} \times \frac{(Y + 15)}{X} = \frac{3}{2 \times 1} \quad \text{or} \quad 1 + \frac{15}{Y} = \frac{3}{2}$$

or $Y = 30\ \Omega$

From (*i*), $X = \frac{3}{2}Y = \frac{3}{2} \times 30 = 45\,\Omega$

When a resistance of 30 Ω is connected in parallel with Y, the total resistance in the right gap of bridge will be

$$Y' = \frac{30 \times Y}{30 + Y} = \frac{30 \times 30}{30 + 30} = 15\,\Omega$$

If the null point is obtained at length l from end A of the meter bridge, then

$$\frac{X}{15} = \frac{l}{(100 - l)} \quad \text{or} \quad \frac{45}{15} = \frac{l}{(100 - l)}$$

or $300 - 3l = l$ or $4l = 300$ or $l = 75$ cm

TYPE XIV. POTENTIOMETER

Formulae used. (*i*) $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$ (*ii*) $r = \left(\frac{l_1 - l_2}{l_2}\right) R$

where ε_1, ε_2 = emfs of two cells,
l_1, l_2 = lengths of the potentiometer wire against which emfs ε_1 and ε_2 are balanced.
Units used. ε_1, ε_2 in volt, R in ohm, l_1, l_2 in metre.

Example 97 **In a potentiometer, a standard source of emf 5 V and negligible internal resistance maintains a steady current through the potentiometer wire of length 10 m. Two primary cells of emf E_1 and E_2 are joined together in series with (*i*) same polarity and (*ii*) opposite polarity. The combination is connected to the potentiometer circuit in each case. The balancing length of the wire in the two cases are found to be 700 cm and 100 cm, respectively. Find the values of emf of the two cells.** (CBSE 2020)

Solution. Here, $E = 5$ V, $l = 10$ m

Potential gradient of potentiometer wire

$$k = \frac{E}{l} = \frac{5}{10} = 0{\cdot}5\ \text{V/m}$$

Case (i). $E_1 + E_2 = kl_1 = 0{\cdot}5 \times 7{\cdot}00 = 3{\cdot}5$ ...(1)

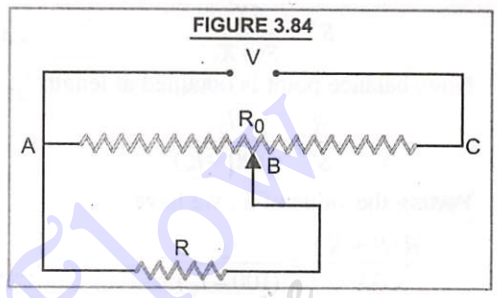
Case (ii). $E_1 - E_2 = kl_2 = 0{\cdot}5 \times 1{\cdot}00 = 0{\cdot}5$ ...(2)

Solving (1) and (2), we get

$$E_1 = 2\ \text{V}, E_2 = 1{\cdot}5\ \text{V}$$

Example 98 **A resistance of R Ω draws current from a potentiometer. The potentiometer has a total resistance R_0 Ω [Fig. 3.84]. A voltage V is supplied to the potentiometer. Derive an expression for the voltage fed into the circuit when the slide contact is in the middle of potentiometer.** (CBSE 2014, 2017) (NCERT Solved Example)

FIGURE 3.84



Solution. When slide is in the middle of the potentiometer wire, only half of the resistance of potentiometer wire ($= R_0/2$) will be between the points A and B. Hence effective resistance (R_1) between A and B is

$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{(R_0/2)} \quad \text{or} \quad R_1 = \frac{R_0 R}{R_0 + 2R}$$

Total resistance between A and $C = R_1 + \frac{R_0}{2}$

Current through the potentiometer wire will be

$$I = \frac{V}{R_1 + R_0/2} = \frac{2V}{2R_1 + R_0}$$

The voltage V_1 taken from the potentiometer will be the product of current I and resistance R_1,

$$i.e.,\ V_1 = IR_1 = \left(\frac{2V}{2R_1 + R_0}\right) \times R_1$$

$$= \frac{2V}{2\left(\frac{R_0 R}{R_0 + 2R}\right) + R_0} \times \frac{R_0 R}{R_0 + 2R}$$

$$= \frac{2VR}{2R + R_0 + 2R} = \frac{2VR}{R_0 + 4R}$$

Example 99 **When a resistor of 5 Ω is connected across a cell, its terminal potential difference is balanced by 140 cm of potentiometer wire and when a resistance of 8 Ω is connected across the cell, the terminal potential difference is balanced by 160 cm of the potentiometer wire. Find the internal resistance of the cell.**

Solution. Here, $R_1 = 5\ \Omega$; $l_1 = 140$ cm ;
$R_2 = 8\ \Omega$; $l_2 = 160$ cm

Let emf of the cell be balanced by length l of the potentiometer wire. Then as per question,

In the first case,

$$r = R_1\left(\frac{l - l_1}{l_1}\right) \quad \text{or} \quad \frac{rl_1}{R_1} = l - l_1 \qquad ...(i)$$

In the second case,

$$r = R_2\left(\frac{l - l_2}{l_2}\right) \quad \text{or} \quad \frac{rl_2}{R_2} = l - l_2 \qquad ...(ii)$$

Subtracting (ii) from (i), we get

$$r\left[\frac{l_1}{R_1} - \frac{l_2}{R_2}\right] = (l - l_1) - (l - l_2) = l_2 - l_1$$

or $$r = \frac{l_2 - l_1}{\frac{l_1}{R_1} - \frac{l_2}{R_2}} = \frac{160 - 140}{\frac{140}{5} - \frac{160}{8}} = 2{\cdot}5\ \Omega$$

Example 100 **A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a battery of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of external resistance ?**

Solution. Resistance of 40 cm length of potentiometer wire $= \frac{10}{100} \times 40 = 4\ \Omega$

Current through potentiometer wire,

$$I = \frac{\text{pot. diff.}}{\text{resistance}} = \frac{10\ \text{mV}}{4\ \Omega} = \frac{10 \times 10^{-3}\ \text{V}}{4\ \Omega}$$
$$= 2{\cdot}5 \times 10^{-3}\ \text{A}$$

The same current flows through the potentiometer wire and the external circuit having resistance R.

Total resistance of potentiometer circuit
$= (R + 10)\ \Omega$

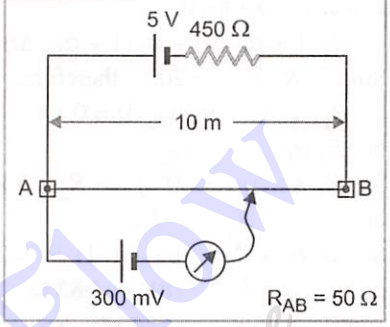
$$\therefore \quad 2{\cdot}5 \times 10^{-3}\ \text{A} = \frac{2\ \text{V}}{(R + 10)\ \Omega}$$

or $$R + 10 = \frac{2}{2{\cdot}5 \times 10^{-3}} = 800$$

or $$R = 800 - 10 = \mathbf{790\ \Omega}$$

Example 101 **In the potentiometer circuit given below, calculate the balancing length l. Give reason whether the circuit will work, if the driver cell of emf 5 V is replaced with a cell of 2 V, keeping all other factors constant.** **(CBSE 2019)**

FIGURE 3.85



Solution. Here, $\varepsilon = 5$ V, $R = 450\ \Omega$, $r = 50\ \Omega$, $L = 10$ m.

Current in the potentiometer circuit,

$$I = \frac{\varepsilon}{R + r} = \frac{5}{450 + 50} = \frac{1}{100}\ \text{A}$$

Fall of potential across wire $AB = Ir$

Potential gradient across wire AB,

$$k = \frac{Ir}{L} = \frac{1}{100} \times \frac{50}{10} = 0{\cdot}05\ \text{V/m}$$

Let the balancing length be l for the cell of emf 300×10^{-3} V. Then

$$300 \times 10^{-3} = k\,l = 0{\cdot}05 \times l$$

or $$l = \frac{300 \times 10^{-3}}{0{\cdot}05} = \mathbf{6\ m}$$

If the driver cell of emf 5 V is replaced with a cell of emf 2 V, then fall of potential across the potentiometer wire is

$$= \frac{\varepsilon'}{(R + r)} \times r = \frac{2}{450 + 50} \times 50 = 200 \times 10^{-3}\ \text{V}$$

As this value is less than 300×10^{-3} V, so there will be no balancing length on potentiometer wire.

TYPE XV. TYPICAL EXAMPLES

Example 102 **It is desired to make a 20·0 Ω coil of wire whose temperature coefficient of resistance is zero. To do this, a carbon resistor of resistance R_1 is placed in series with an iron**

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resistor of resistance R_2. The proportion of iron and carbon are so chosen that $R_1 + R_2 = 20\ \Omega$ for all temperatures near 20°C. Find the values of R_1 and R_2. Temperature coefficient of resistance for carbon, $\alpha_C = -0{\cdot}5 \times 10^{-3}$/°C and that of iron is $\alpha_{Fe} = 5 \times 10^{-3}$/°C.

Solution. Let Δt°C be the rise in temperature. As per question we need,

$$R_1(1 + \alpha_C \Delta t) + R_2(1 + \alpha_{Fe} \Delta t) = 20$$

Since, $R_1 + R_2 = 20$, therefore,

$$R_1 \alpha_C \Delta t + R_2 \alpha_{Fe} \Delta t = 0$$

or $R_1 \alpha_C = -R_2 \alpha_{Fe}$

or $R_1 \times (-0{\cdot}5 \times 10^{-3}) = -R_2 \times 5 \times 10^{-3}$

or $R_1 = 10\ R_2$

So, $10\ R_2 + R_2 = 20$ or $11\ R_2 = 20$

or $R_2 = 20/11 = \mathbf{1{\cdot}82\ \Omega}$

and $R_1 = 10 \times 1{\cdot}82 = \mathbf{18{\cdot}2\ \Omega}$

Example 103 A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1·5 V. When the batteries are connected in series, the galvanometer records a current of 1 A and when the batteries are in parallel, the current is 0·6 A. What is the internal resistance of the battery ?

Solution. Here, emf of each cell, $\varepsilon = 1{\cdot}5$ V

Let r be the internal resistance of each cell and G be the resistance of galvanometer.

The circuits are shown in Figs. 3.86(*a*) and (*b*).

In circuit (a), effective emf = 1·5 + 1·5 = 3·0 V

total resistance = $G + R + r + r = G + R + 2r$

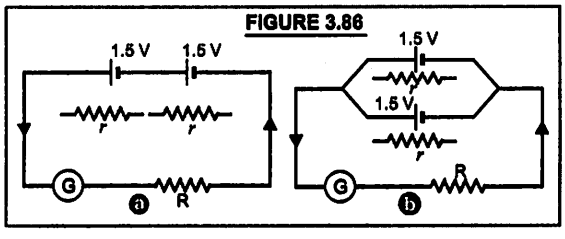
Current, $I = 1$ A

$$\therefore \quad \text{Current, } I = \frac{\text{total emf}}{\text{total resistance}}$$

or $$1 = \frac{3{\cdot}0}{G + R + 2r}$$

or $G + R + 2r = 3$ or $G + R = 3 - 2r$ ...(*i*)

FIGURE 3.86



In circuit (b), effective emf = 1·5 V

∵ When cells are in parallel, only the size of the electrodes increases, the total emf does not change.

Total internal resistance of the two cells will be

$$R_p = \frac{r \times r}{r + r} = \frac{r}{2} = 0{\cdot}5\,r$$

Total resistance of the circuit = $G + R + 0{\cdot}5\ r$

Here, current, $I = 0{\cdot}6$ A

As, $$\text{current} = \frac{\text{effective emf}}{\text{total resistance}}$$

$$\therefore \quad 0{\cdot}6 = \frac{1{\cdot}5}{G + R + 0{\cdot}5\,r}$$

or $0{\cdot}6\ (G + R + 0{\cdot}5\ r) = 1{\cdot}5$

or $G + R + 0{\cdot}5\ r = 1{\cdot}5/0{\cdot}6 = 2{\cdot}5$

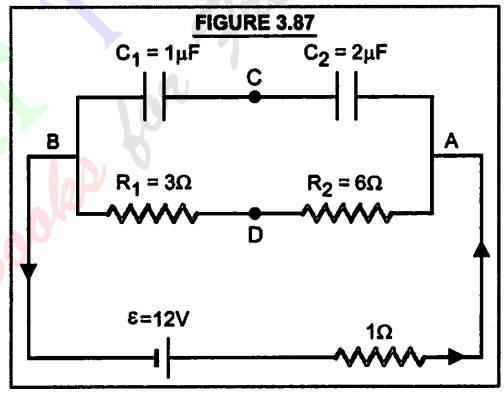
or $G + R = 2{\cdot}5 - 0{\cdot}5\ r$ ...(*ii*)

From (*i*) and (*ii*), $3 - 2\ r = 2{\cdot}5 - 0{\cdot}5\ r$

or $0{\cdot}5 = 1{\cdot}5\ r$ or $r = 0{\cdot}5/1{\cdot}5 = \mathbf{0{\cdot}333\ \Omega}$

Example 104 Determine the potential difference between the points C and D in Fig. 3.87.

FIGURE 3.87



Solution. Since the capacitors are in series, charge on each capacitor is the same. At steady state, the charge on the capacitor is due to terminal potential difference of cell ε.

When steady state is reached, no current flows through the capacitors. The current in the circuit is,

$$I = \frac{12}{1 + 3 + 6} = 1{\cdot}2\ \text{A}$$

Terminal pot. diff. across A and B,

$$V = \varepsilon - I\,r = 12 - 1{\cdot}2 \times 1 = 10{\cdot}8\ \text{V}$$

Capacitors are in series, their effective capacitance $C = \frac{1 \times 2}{1 + 2} = \frac{2}{3}\ \mu\text{F}$

∴ Charge on each capacitor,

$$q = \frac{2}{3} \times 10^{-6} \times 10{\cdot}8 = 7{\cdot}2 \times 10^{-6}\ \text{C}$$

Charge on capacitor C_2, $q = 7{\cdot}2 \times 10^{-6}$ C.

Pot. diff. between A and C

$$= V_A - V_C = \frac{q}{C_2} = \frac{7{\cdot}2\times10^{-6}}{2\times10^{-6}} = 3{\cdot}6\text{ V}$$

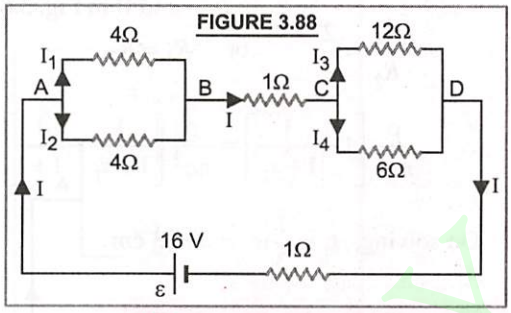
Pot. diff. between A and $D = V_A - V_D = IR_2$
$= 1{\cdot}2 \times 6 = 7{\cdot}2$ V.

Pot. diff. between C and $D = V_C - V_D$
$= (V_A - V_D) - (V_A - V_C) = 7{\cdot}2 - 3{\cdot}6 = \mathbf{3{\cdot}6\text{ V}}$

Example 105 **A network of resistances is connected to a 16 V battery with internal resistance of 1 Ω as shown in Fig. 3.88. (*a*) Compute the equivalent resistance of the network. (*b*) Obtain the current in each resistor and (*c*) Obtain the voltage drop V_{AB}, V_{BC} and V_{CD}.**

(NCERT Solved Example)

FIGURE 3.88



Solution. (*a*) In the circuit shown, 4 Ω and 4 Ω are in parallel ; 12 Ω and 6 Ω are in parallel. These two combination of resistances are in series with 1 Ω resistance in the circuit. Total resistance of circuit between A and D is

$$R = \frac{4\times4}{4+4} + 1 + \frac{12\times6}{12+6} = 2 + 1 + 4 = 7\ \Omega$$

(*b*) Total current, $I = \dfrac{\varepsilon}{(R+r)} = \dfrac{16}{7+1} = \mathbf{2\text{ A}}$

Current at A is divided equally in each of 4 Ω resistances in parallel. So,

$$I_1 = I_2 = \frac{2}{2} = \mathbf{1\text{ A}}$$

Pot. diff. across C and D,

$$V_{CD} = I \times R_{CD} = 2 \times 4 = 8\text{ V}$$

$$I_3 = \frac{V_{CD}}{12} = \frac{8}{12} = \frac{2}{3}\text{A} \ ; \ I_4 = \frac{V_{CD}}{6} = \frac{8}{6} = \frac{4}{3}\text{A}$$

(*c*) $V_{AB} = I \times R_{AB} = 2 \times 2 = \mathbf{4\text{ V}}$;

$V_{BC} = I \times 1 = 2 \times 1 = \mathbf{2\text{ V}}$;

$V_{CD} = I \times R_{CD} = 2 \times 4 = \mathbf{8\text{ V}}$

Example 106 **The resistance of a 200 V and 100 W electric bulb when hot is 10 times the resistance when cold. Find its resistance at room temperature. If the working temperature of the filament is 2000°C, find the temperature coefficient of resistance of filament.**

Solution. Resistance of hot bulb,

$$R_t = \frac{V^2}{P} = \frac{200\times200}{100} = 400\ \Omega$$

Resistance of bulb at room temperature,

$$R_0 = \frac{400}{10} = \mathbf{40\ \Omega}$$

Now, $R_t = R_0(1 + \alpha t)$

$\therefore \quad 400 = 40(1 + \alpha \times 2000)$

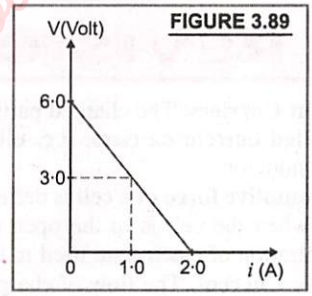
or $\quad 10 = 1 + \alpha \times 2000$

On solving, we get

$$\alpha = \frac{9}{2000} = \mathbf{4{\cdot}5 \times 10^{-3}\ °C^{-1}}$$

Example 107 **The following graph shows the variation of terminal potential difference V, across a combination of three cells in series to a resistor versus the current i :**

FIGURE 3.89



(*i*) Calculate the emf of each cell.

(*ii*) For what current i, will the power dissipation of the circuit be maximum ? **(CBSE 2008)**

Solution. (*i*) Total emf of 3 cells in series = 6·0 V

$$\text{EMF of each cell} = \frac{6{\cdot}0}{3} = \mathbf{2{\cdot}0\text{ V}}$$

(*ii*) When i = 1·0 A, V = 3·0/3 = 1·0 V

Internal resistance of a cell

$$r = \frac{\varepsilon - V}{i} = \frac{2{\cdot}0 - 1{\cdot}0}{1{\cdot}0} = 1{\cdot}0\ \Omega$$

The output power is maximum when external resistance = internal resistance = 3 r

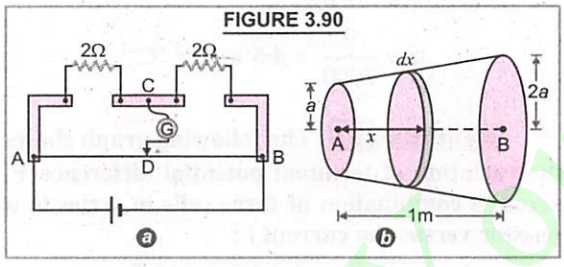
$$I_{max} = \frac{\text{total emf}}{\text{total resistance}} = \frac{3\varepsilon}{3r+3r} = \frac{\varepsilon}{2r}$$

$$= \frac{2{\cdot}0}{2\times 1{\cdot}0} = 1{\cdot}0 \text{ A}$$

Example 108 The wire AB of a meter bridge changes linearly from radius a to 2 a from left end to right end ; Fig. 3.90(a). Where should the free end of the galvanometer be connected on AB so that the deflection in the galvanometer be zero?

Solution. Let the free end of galvanometer be connected on wire AB at point D, then galvanometer shows no deflection. Let length of wire from A to D be x, where $x = 0$ at end A of wire. Let R_1 be the resistance of left part (*i.e.*, AD) of wire of length x_1 and R_2 be the resistance of right part (*i.e.*, DB) of wire of length $(1 - x_1)$.

FIGURE 3.90



Consider an element of wire of thickness dx at a distance x from end A of wire. Its radius be a_x. Then

$$a_x = \left[a + \frac{(2a-a)}{1}x\right] = a(1+x)$$

Resistance of this element is

$$dR = \frac{\rho\, dx}{\pi a_x^2} = \frac{\rho\, dx}{\pi a^2 (1+x)^2}$$

$$\therefore \quad R_1 = \int_0^{x_1} \frac{\rho\, dx}{\pi a^2 (1+x)^2} = \frac{\rho}{\pi a^2}\left[1 - \frac{1}{1+x_1}\right]$$

$$\text{and } R_2 = \int_{x_1}^{1} \frac{\rho\, dx}{\pi a^2 (1+x)^2} = \frac{\rho}{\pi a^2}\left[\frac{1}{1+x_1} - \frac{1}{1+1}\right]$$

For no deflection in galvanometer

$$\frac{R_1}{R_2} = \frac{2}{2} \quad \text{or} \quad R_1 = R_2$$

$$\therefore \quad \frac{\rho}{\pi a^2}\left[1 - \frac{1}{1+x_1}\right] = \frac{\rho}{\pi a^2}\left[\frac{1}{1+x_1} - \frac{1}{1+1}\right]$$

On solving, $x_1 = \frac{1}{3}\text{m} = 33{\cdot}33 \text{ cm}$

REVISION AT A GLANCE

1. **Current Carriers.** The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. *e.g.* electrons in conductors, ions in electrolyte, electrons and holes in semiconductor.
2. **Electromotive force** of a cell is defined as the maximum potential difference between the two electrodes of a cell when the cell is in the open circuit. emf of a cell depends upon nature of electrodes, nature and concentration of electrolyte used in the cell and its temperature.
3. **Electric Current.** The flow of charge in a definite direction constitutes the electric current.

$$\text{Electric current} = \frac{\text{charge flowing}}{\text{time taken}} = \frac{q}{t}$$

SI unit of current is ampere (denoted by A). The arrow head marked in circuit represents the direction of conventional current, *i.e.*, direction of flow of positive charge, whereas the direction of flow of electrons gives the direction of electronic current which is opposite to that of conventional current. ***Current is a scalar quantity.***

4. **Drift Velocity :** It is defined as the average velocity with which free electrons get drifted towards the positive end of the conductor under the influence of an external electric field. Drift velocity of electrons is given by :

$$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

where e is the charge on electron, m is the mass, $\vec{E}$ is the electric field applied and τ is the average time of relaxation, *i.e.*, the average time that has elapsed since each electron suffered its last collision with the ion or atom of the conductor, while drifting towards the positive end of the conductor under the effect of external electric field applied.

The value of drift velocity of electron is about 10^{-4} m/s and value of average relaxation time is about 10^{-14} second.

5. **Mobility.** Mobility of electron (μ_e) is defined as the drift velocity (v_d) per unit electric field applied, *i.e.*,
$$\mu_e = v_d / E$$
6. **Relation between current and drift velocity/mobility.**
$$I = nAe\, v_d = A\, ne\, \mu_e\, E$$
where n is the electron density or no. of electrons per unit volume of the conductor and A is the area of cross-section of the conductor. The small value of the drift velocity (~ $10^{-4}\ ms^{-1}$) produces a large amount of current due to presence of large number of free electrons in a conductor (~ $10^{29}\ m^{-3}$). The propagation of current in a conductor under the effect of electric field applied is at the speed of light.
7. **Ohm's law :** It states that the current (I) flowing through a conductor is directly proportional to the potential difference (V) across the ends of the conductor, provided physical conditions of the conductor such as temperature, mechanical strain etc. are kept constant, *i.e.*, $V \propto I$ or $V = IR$
where R is known as resistance of the conductor, which depends upon the nature and dimensions of the conductor. The SI unit of R is ohm.
8. **Resistance of a conductor.** It is the obstruction posed by the conductor to the flow of current through it. Resistance of a conductor is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor. The resistance of a conductor can be given by the expression :
$$R = \frac{m}{ne^2\tau}\frac{l}{A} = \rho\frac{l}{A}$$
where m is the mass of electron, e is charge of electron, n is the number density of electron, τ is the average relaxation time, l is the length of conductor and A is its area of cross section, ρ is the specific resistance of the conductor.
9. **Specific resistance or electrical resistivity** of the material of a conductor is defined as the resistance of unit length and unit area of cross-section of the conductor. The SI unit of resistivity is Ω m. Resistivity of a conductor depends upon the nature of the conductor but is independent of the length or area of cross section of the conductor. In fact resistivity, $\rho = \frac{m}{ne^2\tau}$.
10. **Current density** (J) at a point is defined as the amount of current flowing per unit area of cross-section of the conductor, provided the area is held in a direction normal to the current.
$$J = \frac{i}{A} = nev_d$$ The SI unit of current density is A m^{-2}
11. **Electrical conductivity** (σ) of a conductor is the inverse of its resistivity (ρ), i.e., $\sigma = 1/\rho$. The SI unit of σ is $\Omega^{-1}\ m^{-1}$ or $S\ m^{-1}$.
12. **Effect of temperature on resistance.** The resistance of a metal conductor at a temperature $t°C$ is given by
$$R_t = R_0\,(1 + \alpha t)$$
where R_0 is the resistance of a conductor at $0°C$ and α is the temperature co-efficient of resistance.
For metals α is positive, *i.e.*, resistance increases with rise in temperature.
For semi conductors and insulators, α is negative, *i.e.*, resistance decreases with rise in temperature.
For alloys like manganin, eureka and constantan, the value of α is very small as compared to that of conductors. That is why these alloys are used in making standard resistances.
If R_{t_1} and R_{t_2} are the resistances of the same conductor at temperature t_1°C and t_2°C, then $R_{t_2} = R_{t_1}\,[1 + \alpha(t_2 - t_1)]$. Here α is the temperature coeff. of resistance averaged over the temperature range t_1°C and t_2°C.
13. **Non-ohmic conductors.** Those conductors which do not obey Ohm's law are called non-ohmic conductors *e.g.* vacuum tube, liquid electrolyte etc.
14. **Super-conductors.** Those materials which offer least resistance to the flow of current through them are called super-conductors. *Examples :* mercury at temp 4.2 K, lead at 7.25 K and niobium at temperature 9.2 K become super-conductors. The conductivity of superconductor is called superconductivity.

15. **Colour code for carbon resistors.** The number attached from 0 to 9, 10^{-1} and 10^{-2} to the various colours can be recollected by the sentence ***B.B. ROY*** Great Britain Very Good Wife wearing Gold, Silver necklace.

Black–0, Brown–1, Red–2, Orange–3, Yellow–4, Green–5, Blue–6, Violet–7, Grey–8, White–9, Gold–10^{-1}, Silver–10^{-2}. The separate strip of gold, silver and no colour shows the accuracy of ± 5%, ± 10% and ± 20% of the given carbon resistor.

16. **Resistance in series.** The total resistance (R_s) is given by $$R_s = R_1 + R_2 + R_3 + ...$$

17. **Resistance in parallel.** The total resistance (R_p) is given by $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

18. **Internal resistance of a cell** is defined as the resistance offered by the electrolyte and electrodes of a cell when electric current flows through it. Internal resistance of a cell depends upon : (*i*) distance between the electrodes, (*ii*) the nature of electrodes (*iii*) nature of electrolyte and (*iv*) area of the electrodes immersed in the electrolyte.

19. **Terminal potential difference of a cell** is defined as the potential difference between the two electrodes of a cell in a closed circuit. Terminal potential difference of a cell decreases if the current drawn from the cell increases. Terminal potential difference of a cell (V) is less than the emf of a cell (ε) by an amount equal to potential drop across the internal resistance of the cell *i.e.*

$$V = \varepsilon - Ir \quad \text{or} \quad r = \frac{\varepsilon - V}{I} = \left(\frac{\varepsilon - V}{V}\right) R \qquad (\because V = IR)$$

where R is the external resistance in the circuit and r is the internal resistance of a cell.

20. **Joule's law of heating.** It states that the amount of heat produced in a conductor is directly proportional to the : (*i*) square of the current flowing through the conductor, (*ii*) resistance of the conductor and (*iii*) time for which the current is passed.

21. **Electric power.** It is defined as the rate at which work is done in maintaining the current in electric circuit.

Electric power, $P = VI = I^2R = V^2/R$ watt or joule/second

22. **Electric energy.** The electric energy consumed in a circuit is defined as the total work done in maintaining the current in an electric circuit for a given time.

Electric energy = $VIt = Pt = I^2Rt = V^2t/R$

SI unit of electric energy is joule (denoted by J)

where 1 joule = 1 watt × 1 second = 1 volt × 1 ampere × 1 second

Commercial unit of electric energy is kilowatt hour (*kWh*)

where $1\ kWh = 1000\ Wh = 3{\cdot}6 \times 10^6$ J = one unit of electricity.

23. **Kirchhoff's rules**

First rule : The algebraic sum of the current meeting at a junction is zero, *i.e.*, $\Sigma i = 0$. The current reaching a function if taken positive then the current leaving the junction is taken negative. This law supports the concept that moving charges are not accumulated at a junction, *i.e.*, the law of conservation of charge.

Second rule : In a closed loop, the algebraic sum of the *emf*s is equal to algebraic sum of the products of current and resistance in the various arms of the loop.

While traversing a closed loop (in clockwise or anti clock wise direction), if negative pole of the cell is encountered first then its emf is +ve, otherwise –ive. The product of resistance and current in an arm of the circuit is taken positive if the direction of current in that arm is in the same sense as one moves in a closed loop and is taken negative if the direction of current in that arm is opposite to the sense as one moves in the closed loop. This rule supports the law of conservation of energy.

24. **Wheatstone bridge Principle** : It states that if four resistances P, Q, R, S are arranged to form a bridge as shown in Fig. 2(b).4, on pressing battery key K_1 first and then galvanometer key K_2, if the galvanometer shows no deflection, then the bridge is balanced. In that case.

$$\frac{P}{Q} = \frac{R}{S}$$

The practical application of Wheatstone bridge principle is in meter bridge or slide wire bridge and post office box which are used to find the unknown resistance or specific resistance of the given metallic wire.

25. Expression for unknown temperature using Wheatstone bridge principle is

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

where R_t, R_{100}, R_0 are the resistance of platinum wire at t°C, 100°C and 0°C respectively.

26. **Principle of potentiometer.** It is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross section and a constant current is flowing through it., *i.e.*, $V \propto l$ (if I and A are constant) or $V = Kl$,
where K is called potential gradient, i.e., fall of potential per unit length of the given wire.

27. **Expression for comparison of emf of two cells by using potentiometer,**

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

where l_1, l_2 are the balancing lengths of potentiometer wire for the emfs ε_1 and ε_2 of two cells.

28. **Expression for the determination of internal resistance of a cell (r) by potentiometer method**

$$r = \left(\frac{l_1 - l_2}{l_2}\right) R$$

where, l_1 = balancing length of potentiometer wire corresponding to emf of the cell.

l_2 = balancing length of potentiometer wire corresponding to terminal potential difference of the cell when a resistance R is connected in series with the cell whose internal resistance is to be determined.

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

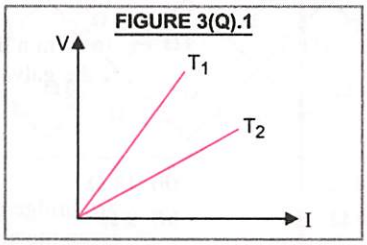
I. Multiple Choice Questions

1. Dimensions of a block are 1 cm × 1 cm × 100 cm. If specific resistance of its material is 3×10^{-7} Ω m, then the resistance between the opposite rectangular faces is
   (*a*) 3×10^{-9} Ω (*b*) 3×10^{-7} Ω
   (*c*) 3×10^{-5} Ω (*d*) 3×10^{-3} Ω

2. The temperature coefficient of resistance for a wire is 0·00125 °C^{-1}. At 27°C its resistance is 1 Ω. The temperature at which the resistance becomes 2 Ω is
   (*a*) 1154 K (*b*) 1100 K
   (*c*) 1400 K (*d*) 1127 K

3. The voltage V and current I graph for a conductor at two different temperatures T_1 and T_2 are shown in the Fig. 3(Q).1. The relation between T_1 and T_2 is

FIGURE 3(Q).1

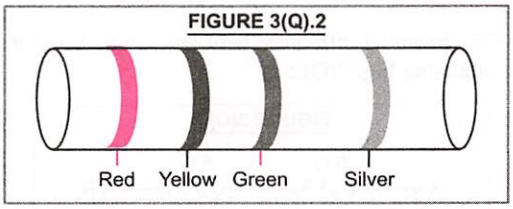


   (*a*) $T_1 > T_2$ (*b*) $T_1 \approx T_2$
   (*c*) $T_1 = T_2$ (*d*) $T_1 < T_2$

4. Masses of three wires of copper are in the ratio 1 : 3 : 5 and their lengths are in the ratio of 5 : 3 : 1. The ratio of their electrical resistances are
   (*a*) 1 : 3 : 5 (*b*) 5 : 3 : 1
   (*c*) 1 : 15 : 125 (*d*) 125 : 15 : 1

5. In the Fig. 3(Q).2, a carbon resistor has bands of different colours on its body as mentioned in the figure. The value of the resistance is

FIGURE 3(Q).2



   (*a*) 24×10^6 Ω ± 5% (*b*) 35×10^6 Ω ± 10%
   (*c*) 5·6 kΩ (*d*) 24×10^6 Ω ± 10%

6. Two wires of same material have length L and 2 L and cross-sectional area 4 A and A respectively. The ratio of their specific resistance would be
   (*a*) 1 : 2 (*b*) 8 : 1
   (*c*) 1 : 8 (*d*) 1 : 1

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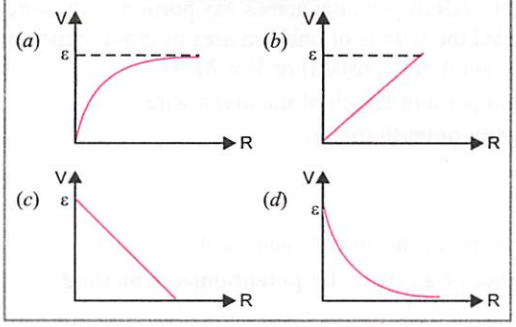
7. A cell of emf ε and internal resistance r is connected across an external resistance R. The graph showing the variation of *P.D.* across R (*i.e.*, V) versus R is

FIGURE 3(Q).3



(CBSE 2020)

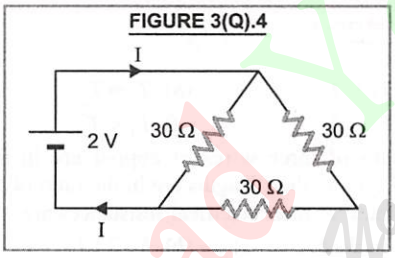
8. A cell supplies a current of 0·9 A through a 2 Ω resistor and a current of 0·3 A through 7 Ω resistor. The internal resistance of the cell is :

(*a*) 2·0 Ω (*b*) 1·5 Ω

(*c*) 1·0 Ω (*d*) 0·5 Ω

9. The current in the adjoing circuit will be :

FIGURE 3(Q).4

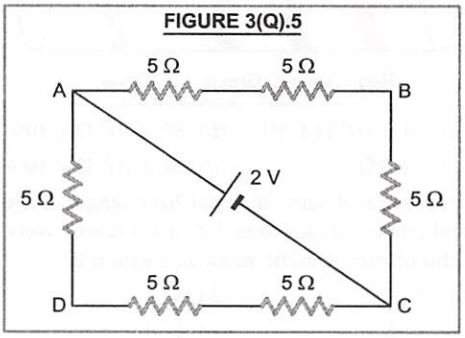


(*a*) $\frac{1}{45}$ ampere (*b*) $\frac{1}{15}$ ampere

(*c*) $\frac{1}{10}$ ampere (*d*) $\frac{1}{5}$ ampere

10. The potential difference between points A and B of adjoining Fig. 3(Q).5 is

FIGURE 3(Q).5

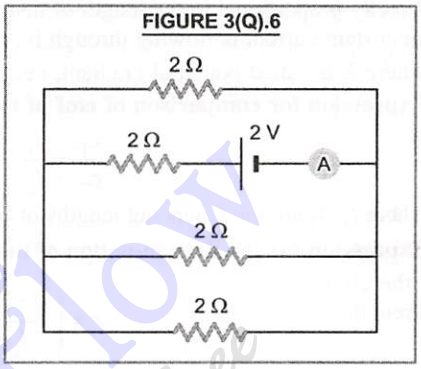


(*a*) $\frac{2}{3}$ V (*b*) $\frac{8}{9}$ V

(*c*) $\frac{4}{3}$ V (*d*) 2 V

11. The reading of the ammeter as per Fig. 3(Q).6 is :

FIGURE 3(Q).6

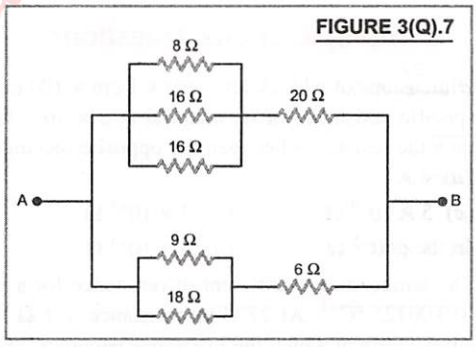


(*a*) $\frac{1}{8}$ A (*b*) $\frac{3}{4}$ A

(*c*) $\frac{1}{2}$ A (*d*) 2 A

12. The equivalent resistance of the arrangement of resistances shown in Fig. 3(Q).7 between the points A and B is :

FIGURE 3(Q).7

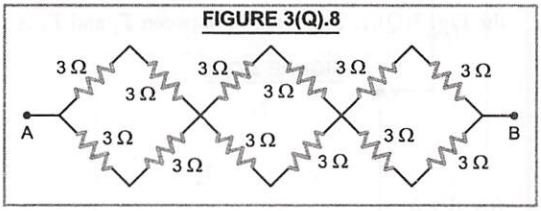


(*a*) 6 Ω (*b*) 8 Ω

(*c*) 16 Ω (*d*) 24 Ω

13. In the network of resistors shown in Fig. 3(Q).8, the equivalent resistance between A and B is :

FIGURE 3(Q).8



(*a*) 54 Ω (*b*) 18 Ω

(*c*) 36 Ω (*d*) 9 Ω

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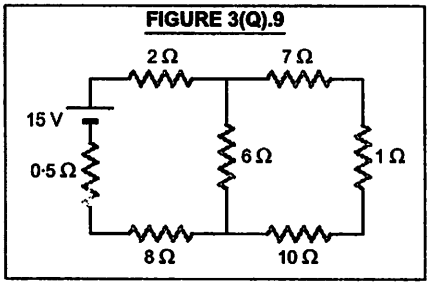
14. The current from the battery in circuit diagram shown in Fig. 3(Q).9 is :

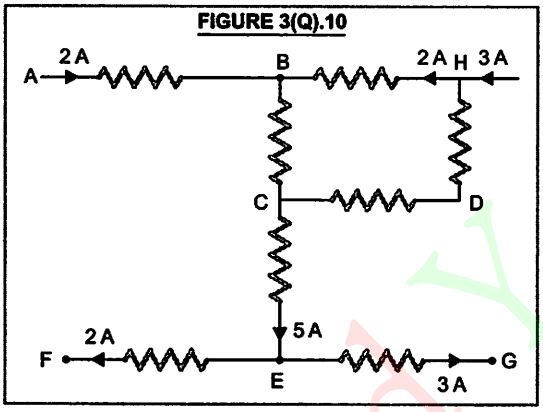
FIGURE 3(Q).9



(*a*) 1 A (*b*) 2 A
(*c*) 1·5 A (*d*) 3 A

15. In the circuit diagram, Fig. 3(Q).10, the electric current through branch *BC* is :

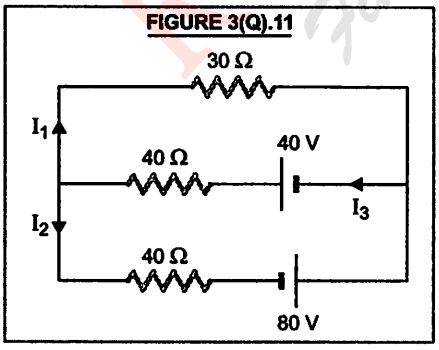
FIGURE 3(Q).10



(*a*) 4 A (*b*) 2 A
(*c*) 5 A (*d*) 10 A

16. In the given circuit Fig. 3(Q).11, the current I_1 is :

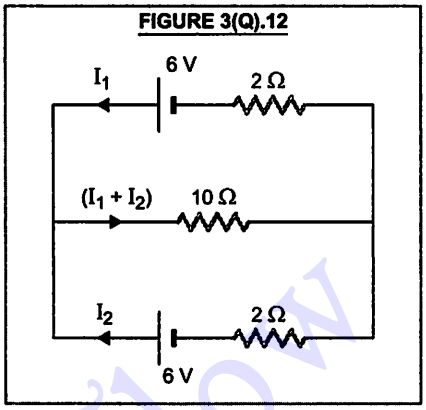
FIGURE 3(Q).11



(*a*) 0·4 A (*b*) – 0·4 A
(*c*) 0·8 A (*d*) – 0·8 A

17. Electric current through resistance 10 Ω in the given circuit is :

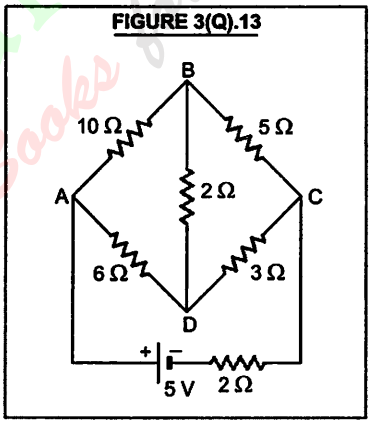
FIGURE 3(Q).12



(*a*) 0 A (*b*) 0·5 A
(*c*) 6/11 A (*d*) 2 A

18. Determine the electric current through branch *BD* of the electric network :

FIGURE 3(Q).13



(*a*) 0·6 A (*b*) 0 A
(*c*) 1 A (*d*) 10 A

19. In Wheatstone's bridge, all the four arms have equal resistance *R*. If resistance of the galvanometer arm is also *R*, then equivalent resistance of the combination is

(*a*) *R* (*b*) 2 R
(*c*) *R*/2 (*d*) *R*/4

20. In a meter bridge experiment, resistance box (with $R = 2\ \Omega$) is connected in the left gap and the unknown resistance *S* in the right gap. If the balancing length be 40 cm, calculate the value of *S*

(*a*) 2 Ω (*b*) 3 Ω
(*c*) 4 Ω (*d*) 2·5 Ω

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21. For a cell of emf 2 V, a balance is obtained for 50 cm of the potentiometer wire. If the cell is shunted by a 2 Ω resistor and the balance is obtained across 40 cm of the wire, then the internal resistance of the cell is :

(*a*) 1 Ω (*b*) 0·5 Ω
(*c*) 1·2 Ω (*d*) 2·5 Ω

22. Two cells when connected in series are balanced on 8 m on a potentiometer. If the cells are connected with polarities of one of the cell is reversed, they balance on 2 *m*. The ratio of the emf's of the two cells is :

(*a*) 3 : 5 (*b*) 5 : 3
(*c*) 3 : 4 (*d*) 4 : 3

23. A cell of internal resistance 3 Ω and emf 10 V is connected to a uniform wire of length 500 cm and resistance 3 Ω. The potential gradient in the wire is

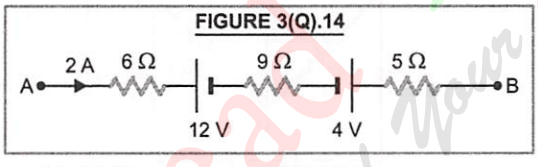
(*a*) 30 mV/cm (*b*) 10 mV/cm
(*c*) 20 mV/cm (*d*) 4 mV/cm

24. When two resistances R_1 and R_2 are connected in series, they consume 12 W power. When they are connected in parallel, they consume 50 W power. What is the ratio of the powers of R_1 and R_2 ?

(*a*) 1/4 (*b*) 4
(*c*) 3/2 (*d*) 3

25. The potential difference between *A* and *B* in the Fig. 3(Q).14 is :

FIGURE 3(Q).14



(*a*) 32 V (*b*) 48 V
(*c*) 24 V (*d*) 14 V

26. In a Wheatston'e bridge, $P = 2\ \Omega$, $Q = 2\ \Omega$, $R = 2\ \Omega$ and $S = 3\ \Omega$. The resistance with which *S* is to be shunted in order that the bridge may be balanced is :

(*a*) 1 Ω (*b*) 2 Ω
(*c*) 4 Ω (*d*) 6 Ω

27. In a potentiometer of wire length *l*, a cell of emf *V* is balanced at a length *l*/3 from the positive end of the wire. For another cell of emf 1·5 V, the balancing length becomes

(*a*) *l*/6 (*b*) *l*/2
(*c*) *l*/3 (*d*) 2 *l*/3

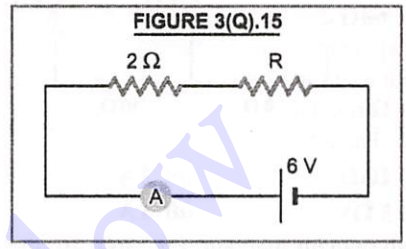
28. Two wires of equal diameters of resistivity ρ_1 and ρ_2 and length l_1 and l_2 respectively, are joined in series. The equivalent resistivity of the combination is :

(*a*) $\dfrac{\rho_1 l_1 + \rho_2 l_2}{(l_1 + l_2)}$ (*b*) $\dfrac{\rho_1 l_2 + \rho_2 l_1}{(l_1 - l_2)}$

(*c*) $\dfrac{\rho_1 l_2 + \rho_2 l_1}{(l_1 + l_2)}$ (*d*) $\dfrac{\rho_1 l_1 - \rho_2 l_2}{(l_1 - l_2)}$

29. If the ammeter in the given circuit shown in the diagram reads 2 A, the resistance *R* is :

FIGURE 3(Q).15



(*a*) 1 Ω (*b*) 2 Ω
(*c*) 3 Ω (*d*) 4 Ω

(CBSE Sample Paper 2020)

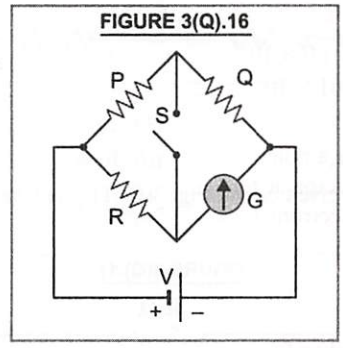
30. The heat produced by 100 W heater in 2 minutes is equal to :

(*a*) 10·5 kJ (*b*) 16·3 kJ
(*c*) 12·0 kJ (*d*) 14·2 kJ

(CBSE Sample Paper 2020)

31. In the circuit shown, $P \neq R$ and the reading of galvanometer is same with switch *S* open or closed. Then

FIGURE 3(Q).16



(*a*) $I_Q = I_R$ (*b*) $I_R = I_G$
(*c*) $I_P = I_G$ (*d*) $I_Q = I_G$

(CBSE 2022)

32. Two wires *A* and *B* of the same material having length in the ratio 1 : 2 and diameter in the ratio 2 : 3 are connected in series with a battery. The ratio of potential differences (V_A/V_B) across the two wires respectively is

(*a*) $\dfrac{1}{3}$ (*b*) $\dfrac{3}{4}$

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(c) $\frac{4}{5}$ (d) $\frac{9}{8}$ (CBSE 2022)

33. A battery of 15 V and negligible internal resistance is connected across a 50 Ω resistor. The amount of energy dissipated as heat in the resistor in one minute is :

(a) 122 J (b) 270 J

(c) 420 J (d) 720 J (CBSE 2022)

34. In a potentiometer experiment, the balancing length with a cell is 120 cm. When the cell is shunted by a 1 Ω resistance, the balancing length becomes 40 cm. The internal resistance of the cell is :

(a) 10 Ω (b) 7 Ω

(c) 3 Ω (d) 2 Ω (CBSE 2022)

35. Two students *A* and *B* calculate the charge flowing through a circuit. *A* concludes that 300 C of charge flows in 1 minute. *B* concludes that $3{\cdot}125 \times 10^{19}$ electrons flow in 1 second. If the current measured in the circuit is 5 A, then the correct calculation is done by

(a) *A* (b) *B*

(c) both *A* and *B* (d) neither *A* or *B*

(CBSE 2022)

36. The resistance of two wires having same length and same area of cross-section are 2 Ω and 8 Ω respectively. If resistivity of 2 Ω wire is $2{\cdot}65 \times 10^{-8}$ Ω-m, then the resistivity of 8 Ω wire is :

(a) $10{\cdot}60 \times 10^{-8}$ Ω m (b) $8{\cdot}32 \times 10^{-8}$ Ω m

(c) $7{\cdot}61 \times 10^{-8}$ Ω m (d) $5{\cdot}45 \times 10^{-8}$ Ω m

(CBSE 2022)

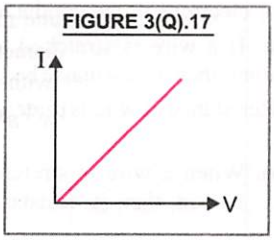
37. When a potential difference *V* is applied across a conductor at temperature *T*, the drift velocity of the electrons is proportional to :

(a) T (b) $\sqrt{T}$

(c) V (d) $\sqrt{V}$ (CBSE 2022)

38. The given Fig. 3(Q).17 shows *I-V* graph of a copper wire whose length and area of cross-section are *L* and *A* respectively. The slope of this curve becomes :

FIGURE 3(Q).17



(a) less if the length of the wire is increased

(b) more if the length of the wire is increased

(c) more if a wire of steel of same dimensions is used

(d) more if the temperature of the wire is increased

(CBSE 2022)

39. The electric power consumed by a 220 V-100 W bulb when operated at 110 V is :

(a) 25 W (b) 30 W

(c) 35 W (d) 45 W (CBSE 2022)

40. A potential difference of 200 V is maintained across a conductor of resistance 100 Ω. The number of electrons passing through it in 1 s is :

(a) $1{\cdot}25 \times 10^{19}$ (b) $2{\cdot}5 \times 10^{18}$

(c) $1{\cdot}25 \times 10^{18}$ (d) $2{\cdot}5 \times 10^{16}$

(CBSE 2022)

41. A car battery is charged by a 12 V supply and energy stored in it is $7{\cdot}20 \times 10^{5}$ J. The charge passed through the battery is :

(a) $6{\cdot}0 \times 10^{4}$ C (b) $5{\cdot}8 \times 10^{3}$ J

(c) $8{\cdot}64 \times 10^{6}$ J (d) $1{\cdot}6 \times 10^{5}$ J

(CBSE 2022)

42. Two sources of equal emf are connected in series. The combination is, in turn connected to an external resistance *R*. The internal resistance of two sources are r_1 and r_2 ($r_2 > r_1$). If the potential difference across the source of internal resistance r_2 is zero, then *R* equals to :

(a) $\frac{r_1 + r_2}{r_2 - r_1}$ (b) $r_2 - r_1$

(c) $\frac{r_1 r_2}{r_2 - r_1}$ (d) $\frac{r_1 + r_2}{r_1 r_2}$

(CBSE 2022)

43. The potential difference *V* applied across a conductor is increased to 2 V with its temperature kept constant, the drift velocity of the free electrons in a conductor will

(a) remains the same

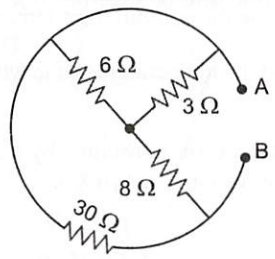
(b) becomes half of its previous value

(c) be doubled of its initial value

(d) becomes zero

44. The equivalent resistance between A and B is :

FIGURE 3(Q).18



(*a*) 3 Ω (*b*) 5·5 Ω
(*c*) 7·5 Ω (*d*) 9·5 Ω

45. A constant voltage is applied between the two ends of a uniform metallic wire, heat H is developed in it. If another wire of the same material, double the radius and twice the length as compared to original wire is used then the heat developed in it will be :

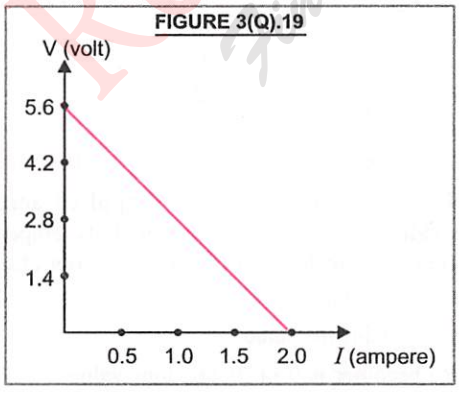
(*a*) $H/2$ (*b*) H
(*c*) $2H$ (*d*) $4H$

46. Three resistors having values R_1, R_2 and R_3 are connected in series to a battery. Suppose R_1 carries a current 2·0 A, R_2 has a resistance 3·0 ohm and R_3 dissipates 6·0 watt of power. Then voltage across R_3 is :

(*a*) 1 V (*b*) 2 V
(*c*) 3 V (*d*) 4 V

47. A straight line plot showing the terminal potential difference (V) of a cell as a function of current (I) drawn from it, is shown in Fig. 3(Q).19. The internal resistance of the cell would be then

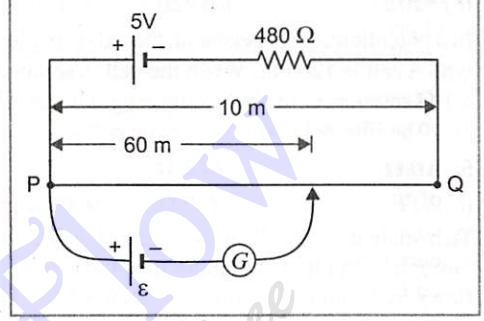
FIGURE 3(Q).19



(*a*) 2·8 Ω (*b*) 1·4 Ω
(*c*) 1·2 Ω (*d*) zero

48. A 10 m long wire of uniform cross-section and 20 Ω resistance is used in a potentiometer. The wire is connected in series with a battery of 5 V along with an external resistance of 480 Ω. If an unknown emf ε is balanced at 6·0 m length of the wire, then the value of unknown emf is :

FIGURE 3(Q).20



(*a*) 1·2 V (*b*) 1·02 V
(*c*) 0·2 V (*d*) 0·12 V

49. A uniform wire of resistance $2R$ is bent in the form of a circle. The effective resistance between the ends of any diameter of the circle is

(*a*) $2R$ (*b*) $R/2$
(*c*) R (*d*) $R/4$ (CBSE 2020)

II. Assertion-Reason Type Questions

Direction. For question numbers 50 to 62, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both A and R are true, and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

50. Assertion. Current flows in a conductor only when there is an electric field within the conductor.

Reason. The electric field decreases the drift velocity of electrons in the conductor.

51. Assertion. If a wire is stretched to increase its length n times, then its resistance becomes n^2 times.

Reason. Resistance of wire is directly proportional to length.

52. Assertion. When a wire is stretched so that its diameter is halved, then its resistance becomes 4 times.

Reason. For the given volume of the wire, resistance of the wire is inversely proportional to square of the diameter.

53. Assertion. When an insulated wire is bent, its resistivity increases.

Reason. On bending drift velocity of electrons in the wire increases.

54. Assertion. Two unequal resistances are connected in parallel across a cell, then the current through smaller resistance is more.

Reason. Current through a resistor is directly proportional to the resistance of resistor.

55. Assertion. Induced emf in two coils made of wire of the same length and same thickness, one of copper and another of aluminium is same. The current in copper coil is more than the aluminium coil.

Reason. Resistance of aluminium coil is more than that of copper coil. **(CBSE 2021)**

56. Assertion. The conductivity of metal decreases with rise in temperature.

Reason. With rise in temperature, the thermal speed of the free electrons and amplitude of a lattice vibration in a metal conductor will increase, resulting increase in the rate of collision between free electrons and lattice.

57. Assertion. A steady current flows in a metallic conductor of non-uniform cross-section.

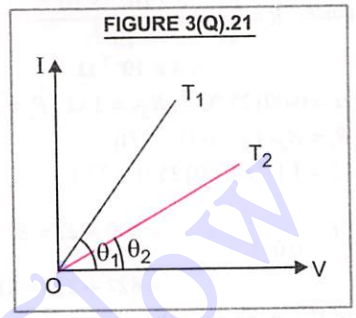
Reason. The current density in a metallic conductor of non-uniform cross-section is also constant.

58. Assertion. A potential difference V is applied to a copper wire of diameter D and length L. On doubling voltage, the drift velocity becomes half.

Reason. $v_d \propto \frac{1}{V}$

59. Assertion. The voltage-current graphs of a conductor for two different temperatures T_1 and T_2 are shown in Fig. 3(Q).21. The resistance of conductor is greater at T_1 than at T_2.

FIGURE 3(Q).21



Reason. Greater is the angle of inclination of V–I graph with voltage axis larger will be its resistance.

60. Assertion. Every metal does not become superconductor at low temperature.

Reason. Resistivity of every metal decreases with decrease in temperature.

61. Assertion. At some enough high temperature, the resistivity of carbon disappears entirely, making it super-conducting.

Reason. Carbon has positive temperature coefficient of resistivity.

62. Assertion. A copper wire is stretched to decrease the radius by 0·1%, the percentage change in resistance is 0·2%.

Reason. $\frac{\Delta R}{R} = -\frac{2\Delta r}{r}$, where R is the resistance and r is the radius of wire.

ANSWERS

I. Multiple Choice Questions

1. (*b*)	**2.** (*b*)	**3.** (*a*)	**4.** (*d*)	**5.** (*d*)	**6.** (*d*)	**7.** (*a*)	**8.** (*d*)	**9.** (*c*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*d*)	**14.** (*a*)	**15.** (*a*)	**16.** (*b*)	**17.** (*c*)	**18.** (*b*)	**19.** (*a*)	**20.** (*b*)
21. (*b*)	**22.** (*b*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*b*)	**28.** (*a*)	**29.** (*a*)	**30.** (*c*)
31. (*b*)	**32.** (*d*)	**33.** (*b*)	**34.** (*d*)	**35.** (*c*)	**36.** (*a*)	**37.** (*c*)	**38.** (*a*)	**39.** (*a*)	**40.** (*a*)
41. (*a*)	**42.** (*b*)	**43.** (*c*)	**44.** (*c*)	**45.** (*c*)	**46.** (*c*)	**47.** (*a*)	**48.** (*d*)	**49.** (*b*)	

II. Assertion-Reason Type Questions

50. (*c*)	**51.** (*b*)	**52.** (*d*)	**53.** (*d*)	**54.** (*c*)	**55.** (*a*)	**56.** (*a*)	**57.** (*c*)	**58.** (*d*)	**59.** (*d*)
60. (*b*)	**61.** (*d*)	**62.** (*d*)							

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. Here, $A = 1\text{ cm} \times 100\text{ cm} = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$;
$l = 1\text{ cm} = 10^{-2}\text{ m}$; $\rho = 3 \times 10^{-7}\ \Omega\text{ m}$

Resistance, $R = \dfrac{\rho l}{A} = \dfrac{3 \times 10^{-7} \times 10^{-2}}{10^{-2}}$

$= \mathbf{3 \times 10^{-7}\ \Omega}$

2. Here, $\alpha = 0{\cdot}00125\ ^\circ\text{C}^{-1}$, $R_{27} = 1\ \Omega$; $R_t = 2\ \Omega$; $t = ?$

$R_t = R_{27}\,[1 + \alpha\,(t - 27)]$

or $\quad 2 = 1\,[1 + 0{\cdot}00125\,(t - 27)]$

$\therefore \quad t = \dfrac{2-1}{0{\cdot}00125} + 27 = 800 + 27 = 827^\circ\text{C}$

$= 827 + 273 = \mathbf{1100\ K}$

3. Slope of V–I graph gives the resistance of a conductor. As slope of graph at temperature T_1 is greater than at temperature T_2, so the resistance of conductor is greater at temperature T_1 than that at temperature T_2. Since resistance of a conductor increases with temperature, so $\boldsymbol{T_1 > T_2}$.

4. $R = \dfrac{\rho l}{A} = \dfrac{\rho l}{V/l} = \dfrac{\rho l^2}{V} = \dfrac{\rho l^2}{m/d}$

where V is volume and d is density of the given copper wire of mass m.

$\therefore \quad R \propto \dfrac{l^2}{m}$;

Hence $R_1 : R_2 : R_3 = \dfrac{(l_1)^2}{m_1} : \dfrac{(l_2)^2}{m_2} : \dfrac{(l_3)^2}{m_3}$

$= \dfrac{5^2}{1} : \dfrac{3^2}{3} : \dfrac{1^2}{5} = 25 : 3 : 1/5$

$= \mathbf{125 : 15 : 1}$

5. Nos. attached to colours red, yellow, green and silver are 2, 4, 6 and ± 10%. The value of resistance

$= \mathbf{24 \times 10^6\ \Omega \pm 10\%}$.

6. Specific resistance of a material is independent of length and area of cross-section of material but depends on the nature of material only.

7. As, $V = \varepsilon - Ir = \varepsilon - \dfrac{V}{R}r$ or $V\left(1 + \dfrac{r}{R}\right) = \varepsilon$

or $\quad V = \dfrac{\varepsilon}{(1 + r/R)}$

Above relation shows that as R increases, V also increases, and when R becomes infinite, V becomes ε. Thus **graph (*a*) is true.**

8. As, $\varepsilon = I\,(R + r) \ \therefore \ \varepsilon = 0{\cdot}9\,(2 + r) = 0{\cdot}3\,(7 + r)$

or $\quad 1{\cdot}8 + 0{\cdot}9\,r = 2{\cdot}1 + 0{\cdot}3\,r$ or $0{\cdot}6\,r = 0{\cdot}3$

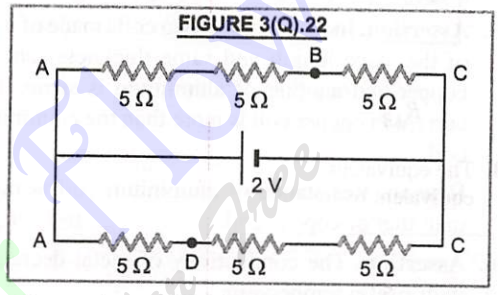
or $\quad r = \mathbf{0{\cdot}5\ \Omega}$

9. $R_{\text{equivalent}} = \dfrac{(30+30)\times 30}{(30+30)+30} = \dfrac{60 \times 30}{90} = 20\ \Omega$

$\therefore$ Current $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{2}{20} = \mathbf{\dfrac{1}{10}}$ **ampere**

10. The equivalent circuit is as shown in Fig. 3(Q).22.

FIGURE 3(Q).22



Current in arm ABC is

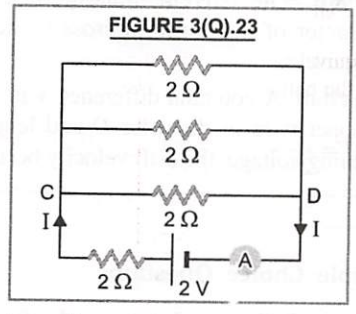
$$I_1 = \frac{2}{5+5+5} = \frac{2}{15}\text{ A}$$

Potential different between A and B is

$$V_{AB} = I_1\,(5+5) = \frac{2}{15}(5+5) = \frac{20}{15} = \mathbf{\frac{4}{3}\ V}$$

11. The equivalent circuit is as shown in Fig. 3(Q).23.

FIGURE 3(Q).23



Effective resistance across C and D is

$$\frac{1}{R_{CD}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \text{ or } R_{CD} = \frac{2}{3}\ \Omega$$

Total resistance of circuit,

$$R = R_{CD} + 2\ \Omega = \frac{2}{3} + 2 = \frac{8}{3}\ \Omega$$

Current, $I = \dfrac{\varepsilon}{R} = \dfrac{2}{8/3} = \dfrac{6}{8} = \mathbf{\dfrac{3}{4}\ A}$

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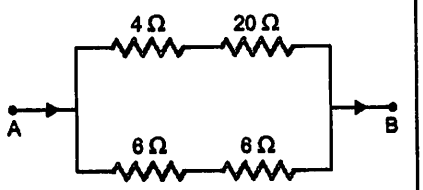
12. In this arrangement of resistances 8 Ω, 16 Ω and 16 Ω are in parallel, their equivalent resistance is 4 Ω. 9 Ω and 18 Ω are also in parallel, their equivalent resistance is 6 Ω. The equivalent circuit is shown in Fig. 3(Q).24. The equivalent resistance between *A* and *B* is

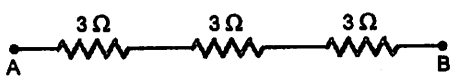
FIGURE 3(Q).24



$$R_{AB} = \frac{(4+20)\times(6+6)}{(4+20)+(6+6)} = \frac{24\times12}{24+12} = 8\ \Omega$$

13. The equivalent circuit is shown in Fig. 3(Q).25. The equivalent resistance between *A* and *B* is

FIGURE 3(Q).25



$$R_{AB} = 3+3+3 = 9\ \Omega$$

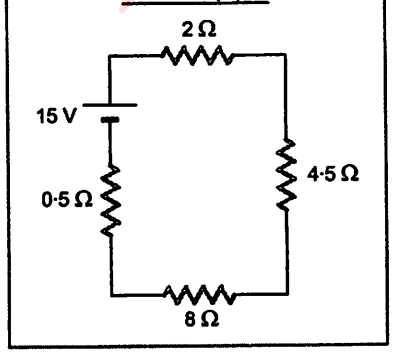
14. In the given circuit 7 Ω, 1 Ω and 10 Ω are in series. They are in parallel to the 6 Ω resistor. Their effective resistance is,

$$R_{eff} = \frac{(7+1+10)\times6}{(7+1+10)+6} = \frac{18\times6}{18+6} = 4{\cdot}5\ \Omega$$

The equivalent circuit is shown in Fig. 3(Q).26. Current from the battery,

$$I = \frac{15}{2+4{\cdot}5+8+0{\cdot}5} = \frac{15}{15} = 1\ \text{A}$$

FIGURE 3(Q).26

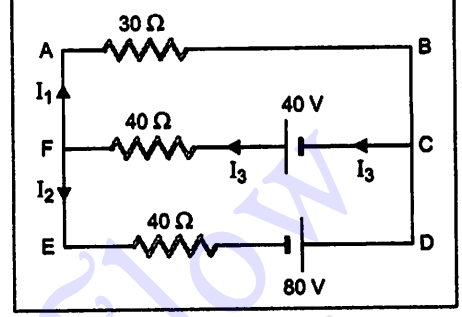


15. Current through branch *BC*

= current through branch *AB* + Current through branch *HB* = 2 + 2 = 4 A

16. Refer to Fig. 3(Q).27, according to Kirchhoff's first rule, at junction *F*,

FIGURE 3(Q).27



$$I_3 = I_1 + I_2 \qquad ...(i)$$

Using Kirchhoff's second rule, for loop *ABCFA*, we have

$$30\,I_1 + 40\,I_3 = 40 \quad \text{or} \quad 30\,I_1 + 40\,(I_1 + I_2) = 40$$

or $\quad 7\,I_1 + 4\,I_2 = 4 \qquad ...(ii)$

For loop *FCDEF*, we have

$$-40\,I_3 - 40\,I_2 = -40 - 80 \quad \text{or} \quad I_3 + I_2 = 3$$

or $\quad (I_1 + I_2) + I_2 = 3 \quad$ or $\quad I_1 + 2\,I_2 = 3$

or $\quad 2\,I_1 + 4\,I_2 = 6 \qquad ...(iii)$

Subtracting (*ii*) from (*iii*), we get

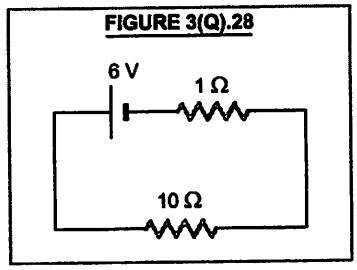
$$-5\,I_1 = 2 \quad \text{or} \quad I_1 = -2/5 = -0{\cdot}4\ \text{A}$$

17. Two similar cells are in parallel, their effective *emf*, ε = 6 V, their effective internal resistance,

$$r = \frac{2\times2}{2+2} = 1\ \Omega$$

The equivalent circuit is shown in Fig. 3(Q).28. Current through 10 Ω resistance is.

FIGURE 3(Q).28



$$I = \frac{6}{1+10} = \frac{6}{11}\ \text{A}$$

18. The given electric network is a balanced Wheatstone bridge. So **no current** flows through *BD*.

19. As, $\frac{P}{Q}=\frac{R}{S}$, so bridge is balanced, hence the resistance of galvanometer can be neglected. Here, P and Q are is series, their resistance = $2R$: Also R and S are in series, their resistance = $2R$

Now, equivalent resistance $=\frac{2R\times 2R}{2R+2R}=R$

20. $\frac{2}{S}=\frac{40}{(100-40)}=\frac{40}{60}$ or $S=3\ \Omega$

21. $r=R\frac{(l_1-l_2)}{l_2}=2\times\left[\frac{50-40}{40}\right]=0{\cdot}5\ \Omega$

22. $\frac{\varepsilon_1}{\varepsilon_2}=\frac{l_1+l_2}{l_1-l_2}=\frac{8+2}{8-2}=\frac{10}{6}=\frac{5}{3}$

23. Potential gradient $=\frac{\varepsilon R}{(R+r)L}=\frac{10\times 3}{(3+3)5}=1$ V/m

$=\frac{1000\text{ mV}}{100\text{ cm}}=$ **10 m V/cm**

24. Let P_1 and P_2 be the powers of resistances R_1 and R_2.

As per question, $\frac{1}{P_1}+\frac{1}{P_2}=\frac{1}{12}$

and $P_1+P_2=50$

On solving, we get ; $P_1=30$ W and $P_2=20$ W

$\therefore\quad \frac{P_1}{P_2}=\frac{30}{20}=\frac{3}{2}$

25. According to Kirchhoff's second rule

$(V_A-V_B)-12+4=2\times 6+2\times 9+2\times 5$

$V_A-V_B=12+18+10+12-4=$ **48 V**

26. Let x be the shunt resistance in parallel with S, for balancing the Wheatstone's bridge. So

$$\frac{P}{Q}=\frac{R}{S\times x/(S+x)}$$

or $$\frac{2}{2}=\frac{2}{(3\times x)/(3+x)}$$

or $$\frac{3x}{3+x}=2$$

or $$x=6\ \Omega$$

27. $\frac{\varepsilon_1}{\varepsilon_2}=\frac{l_1}{l_2}$ or $\frac{V}{1{\cdot}5\,V}=\frac{l/3}{l_2}$ or $l_2=1{\cdot}5\times\frac{l}{3}=\frac{l}{2}$

28. $R_1=\frac{\rho_1 l_1}{A}$ and $R_2=\frac{\rho_2 l_2}{A}$

In series, $R_{eq}=R_1+R_2$

or $\rho_{eq}\times\frac{(l_1+l_2)}{A}=\frac{\rho_1 l_1}{A}+\frac{\rho_2 l_2}{A}$

or $$\rho_{eq}=\frac{\rho_1 l_1+\rho_2 l_2}{l_1+l_2}$$

29. Current, $2=\frac{6}{2+R}$ or $R=1\ \Omega$

30. Heat produced, $H=Pt=100\times(2\times 60)$ W-s

$=12\times 10^3$ J $=$ **12·0 kJ**

31. The reading of galvanometer will not change with switch S open or closed when bridge is balanced. In this state **current through R is same as that through G.**

32. Here, $\frac{l_A}{l_B}=\frac{1}{2}$, $\frac{D_A}{D_B}=\frac{2}{3}$

Since the two wires are in series, they will have the same current in them.

$\therefore\quad \frac{V_A}{V_B}=\frac{IR_A}{IR_B}=\frac{R_A}{R_B}=\frac{\rho l_A/(\pi D_A^2/4)}{\rho l_B/(\pi D_B^2/4)}$

$=\frac{l_A}{l_B}\times\left(\frac{D_B}{D_A}\right)^2=\frac{1}{2}\times\left(\frac{3}{2}\right)^2=\frac{9}{8}$

33. Here, $V=15$ V, $R=50\ \Omega$, $t=60$ s

Heat energy $=\frac{V^2}{R}t=\frac{(15)^2}{50}\times 60=$ **270 J**

34. Here, $l_1=120$ cm, $l_2=40$ cm, $R=1\ \Omega$

$r=\left(\frac{l_1-l_2}{l_2}\right)R=\left(\frac{120-40}{40}\right)\times 1=2\ \Omega$

35. **For student A,** $q=300$ C, $t=60$ s

$\therefore\quad$ Current $I_A=\frac{q}{t}=\frac{300}{60}=$ **5 A**

For student B, no. of electrons,

$n=3{\cdot}125\times 10^{19}$

Time $t=1$ s

Charge flowing,

$q=ne=(3{\cdot}125\times 10^{19})\times(1{\cdot}6\times 10^{-19})$ C

Current $I_B=\frac{q}{t}=\frac{3{\cdot}125\times 1{\cdot}6}{1}=$ **5·0 A**

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36. Here, $l_1 = l_2, A_1 = A_2, R_1 = 2\ \Omega, R_2 = 8\ \Omega$

Then, $\frac{R_1}{R_2} = \frac{\rho_1 l_1 / A_1}{\rho_2 l_2 / A_2} = \frac{\rho_1}{\rho_2}$

or $\rho_2 = \rho_1\left(\frac{R_2}{R_1}\right) = (2{\cdot}65\times10^{-8})\times\left(\frac{8}{2}\right)$

$= \mathbf{10{\cdot}60 \times 10^{-8}\ \Omega\ m}$

37. Drift velocity, $v_d = \frac{eE}{m}\tau = \frac{e}{m}\left(\frac{V}{d}\right)\tau$

$[\because E = V/d]$

$\therefore \quad v_d = \frac{eV}{md}\tau,$ *i.e.*, $v_d \propto V$

38. Slope of *I-V* graph $= \frac{I}{V} = \frac{1}{R} = \frac{A}{\rho L}$

It means if *L* increases slope of graph decreases. If specific resistance ρ increases (when copper wire is replaced with steel wire), then slope of graph decreases. If temperature increases, then resistance *R* increases, so slope of graph decreases.

39. Here, $V = 220$ V, $P = 100$ W

Resistance of bulb, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$

When $V' = 110$ V, then power consumed

$$P' = \frac{V'^2}{R} = \frac{(110)^2}{484} = \mathbf{25\ W}$$

40. Here, $V = 200$ V, $R = 100\ \Omega$

Current, $I = \frac{V}{R} = \frac{200}{100} = 2\ \text{A}$

$$I = \frac{ne}{t}$$

or $n = \frac{It}{e} = \frac{2\times1}{1{\cdot}6\times10^{-19}} = \mathbf{1{\cdot}25 \times 10^{19}}$

41. Here, $V = 12$ V, energy stored, $E = 7{\cdot}20 \times 10^5$ J

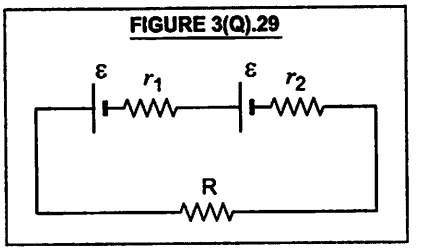
Charge passing through battery,

$$Q = \frac{E}{V} = \frac{7{\cdot}2\times10^5}{12} = \mathbf{6{\cdot}0 \times 10^4\ C}$$

42. Refer to Fig. 3(Q).29, current in circuit

$$I = \frac{\varepsilon+\varepsilon}{r_1+r_2+R} = \frac{2\varepsilon}{r_1+r_2+R} \qquad ...(i)$$

FIGURE 3(Q).29



The potential difference across the source of current of internal resistance r_2 is zero, then

$$\varepsilon - Ir_2 = 0 \quad \text{or} \quad I = \frac{\varepsilon}{r_2} \qquad ...(ii)$$

From (*i*) and (*ii*), $\frac{2\varepsilon}{(r_1+r_2+R)} = \frac{\varepsilon}{r_2}$

On solving, $R = (r_2 - r_1)$

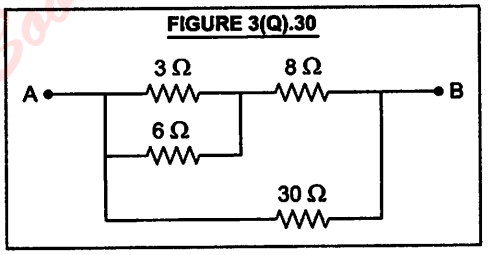
43. $v_d = \frac{eE}{m}\tau = \frac{e}{m}\left(\frac{V}{d}\right)\tau \qquad \left(\because E = \frac{V}{d}\right)$

$\therefore \quad v_d \propto V$

Hence, $\frac{v_{d_2}}{v_{d_1}} = \frac{V_2}{V_1} = \frac{2V}{V} = 2$ or $v_{d_2} = 2v_{d_1}$

44. The equivalent circuit is as shown in Fig. 3(Q).30.

FIGURE 3(Q).30



Here 3 Ω and 6 Ω are in parallel, their effective resistance is

$$R_1 = \frac{3\times6}{3+6} = 2\Omega$$

R_1 and 8 Ω are in series. Their effective resistance is $R_2 = 2 + 8 = 10\ \Omega$

R_2 and 30 Ω are in parallel, the equivalent resistance is

$$R_{eq} = \frac{10\times30}{10+30} = \frac{30}{4} = \mathbf{7{\cdot}5\ \Omega}$$

45. Here, $H_1 = H$, $\frac{r_1}{r_2} = \frac{1}{2}$; $\frac{l_1}{l_2} = \frac{1}{2}$

Heat produced, $H = \frac{V^2}{R} = \frac{V^2}{\rho l/\pi r^2} = \frac{V^2\pi r^2}{\rho \times l}$

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$$i.e.,\quad H \propto \frac{r^2}{l}$$

$$\therefore\quad \frac{H_2}{H_1} = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{l_1}{l_2}\right) = \left(\frac{2}{1}\right)^2 \times \frac{1}{2} = 2$$

or $H_2 = 2\,H_1 = \mathbf{2\ H}$

46. Since three resistors are in series there will be same current I (= 2·0 A) flowing in each resistance. Here P_3 = 6·0 watt,

$$\text{Voltage across } R_3 = \frac{P_3}{I} = \frac{6{\cdot}0}{2{\cdot}0} = \mathbf{3\ V}$$

47. From graph, When, $I = 0$, $V = \varepsilon = 5{\cdot}6$ V ;
When, $V = 0$, $I = 2{\cdot}0$ A

$$\text{Internal resistance, } r = \frac{\varepsilon}{I} = \frac{5{\cdot}6}{2{\cdot}0} = \mathbf{2{\cdot}8\ \Omega}$$

48. Here, $l = 10$ m ; $r = 20\ \Omega$; $R = 480\ \Omega$; $\varepsilon' = 5$ V
Current in potentiometer wire is

$$I = \frac{\varepsilon'}{R+r} = \frac{5}{480+20} = \frac{5}{500} = 0{\cdot}01 \text{ A}$$

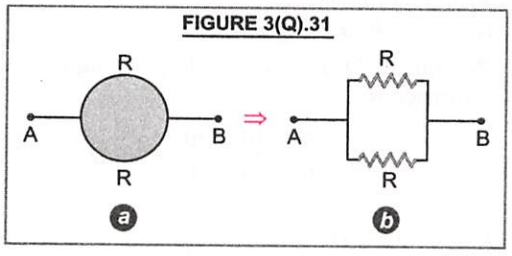
Pot. diff. across potentiometer wire $PQ = Ir$
$= 0{\cdot}01 \times 20$
$= 0{\cdot}2$ V

Thus 10 m wire balances 0·2 V. Therefore 6 m wire will balance the voltage,

$$V' = \frac{0{\cdot}2}{10} \times 6 = \mathbf{0{\cdot}12\ V}$$

49. If a loop is made, the resistances between the ends of any diameter will be in parallel as shown in Fig. 3(Q).31(*a*) and (*b*). Therefore, the effective resistance between A and B is

FIGURE 3(Q).31



$$R_{eff} = \frac{R \times R}{R+R} = \frac{R}{2}$$

II. Assertion-Reason Type Questions

50. As, drift velocity, $v_d = \frac{Ee}{m}\tau$ or $v_d \propto E$. Hence, **Reason is false.** Here **Assertion is true,** because electric field within the conductor force the free electrons to follow a definite path which is responsible for current in the conductor.

51. Here, **Reason is true** and **Assertion is also true but Reason is not the correct explanation of Assertion.**

$$R = \frac{\rho l}{A} = \frac{\rho l}{V/l} = \frac{\rho l^2}{V} \quad [\because \text{ Volume, } V = Al]$$

$$\therefore\quad R' = \frac{\rho l'^2}{V} = \frac{\rho (nl)^2}{V} = n^2 \frac{\rho l^2}{V} = n^2 R$$

52. Resistance, $R = \frac{\rho l}{A} = \frac{\rho (V/A)}{A}$ $(\because V = Al)$

or $R = \frac{\rho V}{A^2} = \frac{\rho V}{(\pi D^2/4)^2} = \frac{16\rho V}{\pi^2 D^4}$, *i.e.*, $R \propto 1/D^4$

When diameter becomes half, *i.e.*, $D' = D/2$, then

$$R' = \frac{16\rho V}{\pi^2 (D/2)^4} = 16R$$

Thus **Assertion is false** and **Reason is also false.**

53. Here, **Assertion is wrong** because resistivity of wire depends on the nature of material of wire and is independent of the bending of wire.
On bending the wire, the area of cross-section of the wire at bend portion increases. So for the given current, the drift velocity of electrons decreases.
Because, $v_d = \frac{I}{nAl}$ or $v_d \propto \frac{1}{A}$. Thus **Reason is false.**

54. When unequal resistances are connected in parallel across a cell, there will be same potential difference across each resistance. Now current $I = \frac{V}{R}$ or $I \propto \frac{1}{R}$ *i.e.*, current through smaller resistance is more than in larger resistance. Hence **Assertion is true but Reason is false.**

55. Resistance of a wire, $R = \frac{\rho l}{A}$ *i.e.*, $R \propto \rho$ (when l and A are constant). ρ is greater for aluminium than for copper, so $R_{Al} > R_{Copper}$. Thus, **Reason is true.**

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Now current $I = \frac{V}{R}$. *i.e.*, $I \propto 1/R$ (when V is constant).

$\therefore$ Current in copper wire is more than the aluminium wire. Thus, **Assertion is true and Reason is the correct explanation of Assertion.**

56. The **Assertion is True and the Reason is the correct explanation of Assertion** because with the rise in temperature, there will be increase in the rate of collision between free electrons and lattice of metal conductor, the mobility of electrons will decrease hence the conductivity of metal will decrease.

57. Here **Assertion is true but Reason is false** because current density $J = \frac{I}{A}$ as A is non-uniform, so J is non-uniform.

58. Both Assertion and Reason are false, because

$$v_d = \frac{eE}{m}\tau = \frac{e}{m}\left(\frac{V}{d}\right)\tau \quad \text{or} \quad v_d \propto V$$

59. From V-I graph, Resistance, $R = \frac{V}{I} = \frac{1}{\tan\theta}$

As, $\theta_1 > \theta_2$ so $R_1 < R_2$ *i.e.* $R_2 > R_1$

Hence, **Both Assertion and Reason are wrong.**

60. Both Assertion and Reason are true but the Reason is not able to explain the Assertion. When temperature of metal is decreased, due to crystal imperfections and impurities, every metal does not become superconductor at low temperature.

61. Both Assertion and Reason are false. The carbon has negative temperature coefficient of resistivity. For carbon, resistivity,

$$\rho = \frac{m}{ne^2\tau} \quad \text{with,} \quad n = n_0 e^{-(E_g/kT)}$$

So, with rise in temperature n will increase, hence ρ will decrease. However as $T \Rightarrow \infty$, $n \Rightarrow n_0$,

so that, $\rho = \frac{m}{n_0 e^2 \tau} = \rho_0 =$ a constant $\neq 0$.

Therefore, carbon at high temperature will behave as metal but not superconductor.

62. Both **Assertion and Reason are false** because

$$\frac{\Delta R}{R} = -\frac{4\Delta r}{r}$$

and % change in resistance $= \frac{\Delta R}{R} \times 100$

$$= \frac{4\Delta r}{r} \times 100$$

$$= 4 \times \frac{0{\cdot}1}{100} \times 100$$

$$= 0{\cdot}4\%$$

CONCEPTUAL PROBLEMS

I. Current and drift velocity

1. A wire is carrying a current. Is it charged ? (Pb. Board 2011)

Sol. No. The current in a wire is due to flow of free electrons in a definite direction. But the number of protons in the wire at any instant is equal to number of electrons and charge on electron is equal and opposite to that of proton. Hence net charge on the wire is zero.

2. It is easier to confine electric current to definite paths (by the use of electric insulators) than to direct heat flow along definite routes using heat insulators. Why ?

Sol. The electrical resistivity of insulators (*e.g.*, glass) differs from metals by a factor of the order of 10^{22}. The thermal conductivity of thermal insulators differs from thermal metals by a factor of the order of 10^3. It is due to this reason, it is easier to send electric currents to definite paths by the use of insulators than to direct heat flow along definite routes using heat insulators.

3. Clarify your elementary notions about current in a metallic conductor by answering the following queries :

(a) The electron drift speed is estimated to be only a few mm s^{-1} for currents in the range of a few amperes. How then is current established almost the instant a circuit is closed ?

(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady drift speed ?

(c) If the electron drift speed is so small, and electron's charge is small, how can we still obtain large amounts of current in a conductor ? (CBSE 2015)

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(d) When electrons drift in a metal from lower to higher potential, does it mean that all the "free" electrons of the metal are moving in the same direction ?

(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field.

(NCERT Solved Example)

Sol. (*a*) When we close the circuit, the electric field is set up in the entire closed circuit instantly with the speed of electromagnetic wave which causes electron drift at every portion of the circuit. Due to it, the current is set up in the entire circuit instantly. The current so set up does not wait for the electrons to flow from one end of the conductor to other end. However, the current does take a little time to reach its steady value.

(*b*) When a potential difference is applied across the ends of a conductor, the electric field is set up inside the conductor. Due to it, each free electron in the conductor experiences a force and is accelerated towards the positive end of the conductor, resulting in the increase in its speed until it collides with a positive ion of the metal. After the collision, the free electron loses its speed but again starts to accelerate by virtue of force on it due to electric field. Its speed increases again only to suffer a collision again and so on. Therefore, on the average, the free electrons acquire only a drift speed in the conductor, under the effect of electric field.

(*c*) In a conductor, we obtain large amount of current even if the electron drift speed is small and electron's charge is small, because the electron number density in the conductor is very large, (of the order of 10^{29} m^{-3}).

(*d*) When a potential difference is applied across the ends of a conductor, the drifting of free electrons takes place from lower to higher potential. These drifting electrons also have thermal velocity. At an instant, each free electron moves in a direction of resultant velocity of its thermal velocity and drift velocity, which is different for various free electrons in a conductor. Hence, **all the free electrons in the metal conductor are not moving in the same direction.**

(*e*) (*i*) *In the absence of electric field,* the paths of electrons are straight lines between successive collisions with positive ions of the metal, since the electrons are moving with their thermal velocity like molecules in a gas.

(*ii*) *In the presence of electric field,* the paths of the electrons between two successive collisions with positive ions are curved as each electron is having two velocities (a) thermal velocity and (b) velocity by virtue of force due to electric field.

4. Current I is flowing through a copper wire of radius r with drift velocity of electrons v_d. If this current is passed through another copper wire of same length and double the radius, what will be the drift velocity of free electrons in it ?

Sol. Drift velocity v_d of the electrons for the current I flowing through a conductor of area of cross-section A, having number density of electrons n and charge on electron e is given by

$$I = n A e v_d \quad \text{or} \quad v_d = \frac{I}{nAe} = \frac{I}{n(\pi r^2)e}$$

where r is the radius of the wire.

Thus, $$v_d \propto \frac{1}{r^2}$$

(Since I, n and e are same for another wire)

$$\therefore \quad \frac{v_d'}{v_d} = \frac{r^2}{(2r)^2} = \frac{1}{4} \quad \text{or} \quad v_d' = \frac{v_d}{4}$$

Thus drift velocity of electrons in another wire **will become one-fourth of the original value.**

5. A potential difference V is aplied across a conductor of length l. How is the drift velocity affected when V is doubled and l is halved ?

(CBSE 2010)

Sol. Drift velocity,

$$v_d = \frac{eE}{m}\tau = \frac{e(V/l)}{m}\tau \quad \text{or} \quad v_d \propto \frac{V}{l}$$

$$\therefore \quad \frac{v_d'}{v_d} = \frac{2V}{V} \times \frac{l}{l/2} = 4 \quad \text{or} \quad v_d' = 4v_d$$

It means drift velocity becomes **four times the original drift velocity.**

6. (*a*) Write the nature of path of free electrons in a conductor in the (*i*) presence of electric field (*ii*) absence of electric field.

(*b*) Between two successive collisions each free electron acquires a velocity from 0 to v where $v = \frac{eE}{m}\tau$. What is the average velocity of a free electron in the presence of an electric field ? Do all electrons have same average velocity.

(*c*) How does this average velocity of free electrons, in the presence of an electric field vary with temperature ? (CBSE 2016)

Sol. (*a*) (*i*) In the presence of electric field, the paths of free electrons in a metal conductor are curved.

(*ii*) In the absence of electric field, the paths of free electrons in a metal conductor are straight lines between two successive collisions.

(*b*) Average velocity of a free electron in the presence of electric field is $v = \frac{eE}{m}\tau$

where the symbols have their usual meanings.

The different free electrons in a metal conductor move with different velocity in the presence of electric field.

(*c*) As the temperature of a conductor increases, the thermal speed of the electrons increases and also the amplitude of vibration of the metal atoms/ions increases. Now the free electrons collide more frequently with the atoms/ions of metal. As a result of it, the value of relaxation time τ decreases. Since average velocity $v \propto \tau$, so the average velocity of free electrons decreases.

7. A potential difference V is applied to a conductor of length L, diameter D. How are the electric field E, the drift velocity v_d and the resistance R affected when (i) V is doubled (ii) L is doubled (iii) D is doubled ?

Sol. We know, Electric field, $E = \frac{V}{L}$;

Drift velocity, $v_d = \frac{eE}{m}\tau = \frac{eV}{mL}\tau$

Resistance, $R = \rho\frac{L}{A} = \frac{4\rho L}{\pi D^2}$

(*i*) When V is doubled, E becomes double, v_d becomes double and R remains unchanged.

(*ii*) When L is doubled, E becomes half, v_d becomes half and R becomes double.

(*iii*) When D is doubled, E remains unchanged, v_d is also unchanged and R becomes one-fourth.

8. A wire whose cross-sectional area is increasing linearly from its one end to the other, is connected across a battery of V volts. Which of the following quantities remain constant in the wire ?

(*a*) Drift speed (*b*) Current density (*c*) Electric current (*d*) Electric field. **(CBSE 2017)**

Sol. Drift speed, $v_d = \frac{I}{nAe}$, *i.e.*, $v_d \propto \frac{1}{A}$

Current density, $J = \frac{I}{A} = \frac{nAev_d}{A} = nev_d$

$= a$ constant

Electric field $E = \frac{J}{l} = \frac{IR}{l} = \frac{I}{l}\frac{Rl}{A} = \frac{IR}{A}$,

i.e., $E \propto \frac{1}{A}$

Electric current, $I = nAlv_d$, *i.e.*, $I \propto A$

Thus **current density remains constant in the wire, whose cross-sectional area is increasing linearly from its one end to the other.**

II. Resistance and effect of temperature

9. Answer the following questions :

(a) A steady current flows in a metallic conductor of non-uniform cross-section. Explain which of these quantities is constant along the conductor : current, current density, electric field and drift speed ?

(b) Is Ohm's law universally applicable for all conducting elements ? If not, give examples of elements which do not obey Ohm's law.

(c) A low voltage supply from which one needs high currents must have low internal resistance. Why ?

(d) A high tension (HT) supply of say 6 kV must have a very large internal resistance. Why ?

Sol. (*a*) Only current through the conductor of non-uniform area of cross-section is constant as the remaining quantities vary inversely with the area of cross-section of the conductor.

(*b*) Ohm's law is not applicable for non-ohmic elements. For example; vacuum tubes, semi-conducting diode, liquid electrolyte etc.

(*c*) As, I_{max} = *e.m.f.*/internal resistance, so for maximum current, internal resistance should be least.

(*d*) A high tension supply must have a large internal resistance otherwise, if accidently the circuit is shorted, the current drawn will exceed safety limit and will cause damage to circuit.

10. A wire of length L_0 has a resistance R_0. It is gradually stretched till its length becomes $2L_0$.

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(a) Plot a graph showing variation of its resistance R with its length l during stretching.

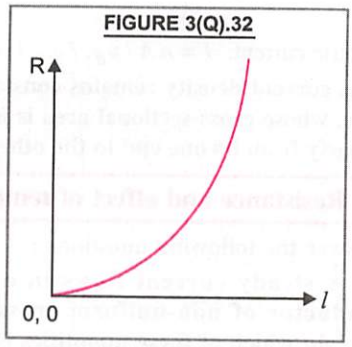
(b) What will be its resistance when its length becomes $2\,L_0$? **(CBSE 2020)**

Sol. (a) $R = \rho\, l/A = \dfrac{\rho\, l}{V/l}$ [$\because$ Volume $V = A\,l$ or $A = V/l$]

$\therefore \quad R = \dfrac{\rho\, l^2}{V}$, *i.e.*, $R \propto l^2$

Thus graph between R and l will be as shown in Fig. 3(Q).32.

FIGURE 3(Q).32



(b) $R_0 = \rho\, l_0/A = \rho\, L_0^2/V$;

When l_{new} becomes $2\,L_0$,

then, $R_{new} = \dfrac{\rho\,(2\,L_0)^2}{V} = 4\dfrac{\rho\, L_0}{V} = 4\,R_0$

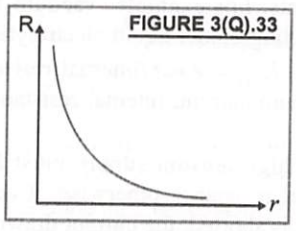
11. Plot a graph showing the variation of resistance of a conducting wire as function of its radius, keeping the length of a wire and its temperature as constant. **(CBSE 2013)**

Sol. Resistance of a conducting wire of length l, radius r and resistivity ρ is given by

$$R = \frac{\rho\, l}{\pi r^2} \quad \text{So} \quad R \propto \frac{1}{r^2}$$

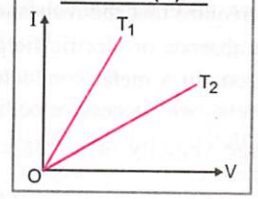
Hence the variation of R versus r is of the type as shown in Fig. 3(Q).33.

FIGURE 3(Q).33



12. V-I graph for a metallic wire at two different temperatures T_1 and T_2 is shown in the Fig. 3(Q).34. Which of the two temperatures is higher and why ? **(CBSE 2015)**

FIGURE 3(Q).34



Sol. We know that resistance, $R = \dfrac{V}{I}$

From Fig. 3.64, for given V, $I_1 > I_2$; therefore, $R_1 < R_2$. As $R \propto T_1$ so $T_1 < T_2$, *i.e.*,

T_2 is greater than T_1.

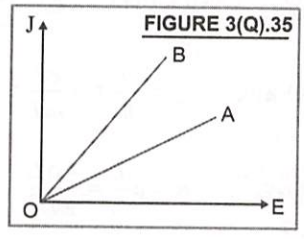
13. Plot a graph showing the variation of current density (J) versus the electric field (E) for two conductors of different materials. What information from this plot regarding the properties of the conducting material, can be obtained which can be used to select suitable materials for use in making (*i*) standard resistance and (*ii*) connecting wires in electric circuit ? **(CBSE 2015)**

Sol. We know that $J = \sigma E$ or $\sigma = J/E$

where σ is the electrical conductivity of the conductor.

The variation of current density (J) versus the electric field (E) for two conductors A and B is shown in Fig. 3(Q).35.

FIGURE 3(Q).35



Here, the slope of each st. line tells the conductivity of that material.

From graph, $\sigma_B > \sigma_A$.

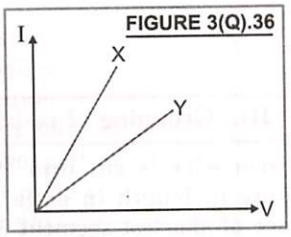
We know that electrical resistivity (ρ) is reciprocal of electrical conductivity.

$$\therefore \quad \frac{\rho_A}{\rho_B} = \frac{\sigma_B}{\sigma_A} > 1 \quad \text{or} \quad \rho_A > \rho_B$$

The material to be used in making a standard resistance, we require that the resistance of the given conducting wire is large. It will be so if its resistivity is large and its conductivity is small. Therefore, the **material A is used in making the standard resistance.**

The material to be used in making a connecting wire, we require that the resistance of the given connecting wire is small. It will be so if its resistivity is small and the conductivity is large. Therefore the **material *B* is used in making a connecting wire in the circuit.**

14. The voltage current variation of two metallic wires X and Y at constant temperature are shown in Fig. 3(Q).36. Assuming that the wires have the same length and same diameter, explain which of the two wires will have larger resistivity. (CBSE 2003)



Sol. Slope of *I-V* line for a wire represents the conductance of the wire. From graph.

Slope of *I-V* line for wire *X* > slope of I-V line for wire *Y*.

∴ Conductance of wire *X* > conductance of wire *Y*.

As, resistance $= \frac{1}{\text{conductance}}$, so

Resistance of wire *X* < resistance of wire *Y*.

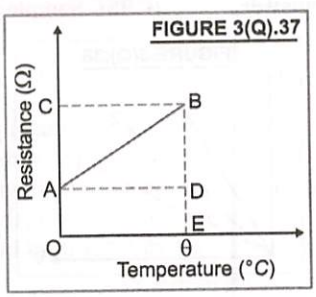
$$\rho_X \frac{l}{A} < \rho_Y \frac{l}{A} \quad \text{or} \quad \rho_X < \rho_Y$$

Thus, **wire *Y* has larger resistivity.**

15. The variation of resistance of a metallic conductor with temperature is shown in Fig. 3(Q).37.

(*i*) Calculate the temperature coefficient of resistance from the graph.

(*ii*) State why the resistance of the conductor increases with the rise in temperature.



Sol. (*i*) From graph, corresponding to point *A*, let the resistance be R_0 and corresponding to point *B*, the resistance be *R*. When temperature changes from 0 to θ°C, the resistance changes from R_0 to *R*.

Thus change in temperature = θ – 0 = θ°C

change in resistance = $R - R_0$

Temperature coefficient of resistance,

$$\alpha = \frac{\text{change in resistance}}{\text{original resistance} \times \text{change in temp.}}$$

$$= \frac{R - R_0}{R_0 \times \theta} = \frac{OC - OA}{OA \times OE}$$

(*ii*) With the rise of temperature of conductor, the resistance of a conductor increases because the frequency of collision of electrons with ions/atoms of the conductor increases, resulting decrease in relaxation time (τ) of electrons.

As, $R \propto 1/\tau$, so *R* increases as τ decreases.

16. Is current density a vector or a scalar quantity? Deduce the relation between current density and potential difference across a current carrying conductor of length *l*, area of cross-section *A*, and number density of free electrons *n*. How does the current density, in a conductor vary with

(*a*) increase in potential gradient ?

(*b*) increase in temperature ?

(*c*) increase in length ?

(*d*) increase in area of cross-section ?

(Assume that the other factors remain constant in each case). (CBSE Sample Paper 2011)

Sol. Current density is a vector quantity. Its direction is same as that of motion of positive charge.

Current, $I = n A e v_d$ and $v_d = \frac{eE}{m}\tau$

Current density, $J = \frac{I}{A} = n e v_d = ne \times \frac{eE}{m}\tau$

$$= \frac{ne^2\tau}{m}\left(\frac{V}{l}\right)$$

(*a*) With increase in potential gradient (*V*/*l*), *J* increases.

(*b*) With increase in temperature, τ decreases, so *J* decreases.

(*c*) With increase in length *J* decrases.

(*d*) With increase in area, *J* remains unchanged as *J* is independent of *A*.

17. Explain why bending a wire does not affect its electrical resistance ?

Sol. Free electrons in a wire have small value of drift velocity and hence low value of inertia of motion. Due to it, they are able to go around the bends easily. That is why, the electrical resistance is not affected on bending the wire till the area of cross-section remains the same at the bend.

18. A copper wire of resistance R_0 is stretched till its length is increased to n times of its original length. What will be its new resistance ?

Sol. Original resistance, $R_0 = \rho\, l_0/A_0$

New resistance, $R = \rho\, l/A$.

As the volume of wire remains constant,

so $l_0 A_0 = l A = (n\, l_0) A$ or $A = A_0/n$

Now $R = \dfrac{\rho l}{A} = \rho \dfrac{n l_0}{A_0/n} = n^2\left(\dfrac{\rho l}{A}\right) = n^2 R_0$

19. A copper wire of length l and radius r is nickel plated till its final radius is 2 r. If the resistivity of the copper and nickel are ρ_c and ρ_n, then find the equivalent resistance of the wire.

Sol. Let R_c and R_n be the resistances of copper wire and nickel plated portion of wire respectively.

Then $R_c = \dfrac{\rho_c l}{\pi r^2}$; and $R_n = \dfrac{\rho_n l}{\pi (2r)^2 - \pi r^2}$

Here, R_c and R_n are in parallel, the equivalent resistance of the wire is

$$R = \frac{R_c R_n}{R_c + R_n} = \frac{\rho_c l/(\pi r^2) \times \rho_n l/[\pi (2r)^2 - \pi r^2]}{\dfrac{\rho_c l}{\pi r^2} + \dfrac{\rho_n l}{[\pi (2r)^2 - \pi r^2]}}$$

$$= \frac{\rho_c \rho_n l}{\pi r^2 (\rho_n + 3\rho_c)}$$

20. Give relation between drift velocity and electric field.

Sol. We know, $I = n e A v_d$

or $I/A = n e v_d$ $[\because I/A = J]$

or $J = n e v_d$ $\left[\because \sigma = \dfrac{\text{current density } (J)}{\text{electric field } (E)}\right]$

or $\sigma E = n e v_d$ or $E = \dfrac{n e v_d}{\sigma} = n e \rho v_d$

$\left[\because \dfrac{1}{\sigma} = \rho = \text{resistivity}\right]$

21. A steady current is flowing in a cylindrical conductor. Is there any electric field within the conductor ? If yes, what is its relation with current density ?

Sol. Yes, the current flows through a conductor only when the electric field established within the conductor exerts force on the free electrons, due to which they move in a definite direction, accounting for current in the conductor. The electric field E inside the conductor is given by

$$E = \frac{\text{potential difference}}{\text{length}} = \frac{V}{l} = \frac{IR}{l} = \frac{I}{l}\frac{\rho l}{A} = \rho J.$$

$$\left(\because \frac{I}{A} = J\right)$$

III. Grouping of resistors

22. A uniform wire is cut into 10 segments increasing in length in equal steps. The resistance of shortest segment is R and the resistance of the other segments increases in steps of 4 Ω. If the resistance of the longest segment is 2 R, find the value of R and the resistance of original wire.

Sol. Resistance of first segment = R Ω

Resistance of second segment = $R + 4 \times 1$ Ω

Resistance of third segment = $R + 4 \times 2$ Ω

Resistance of tenth segment = $R + 4 \times 9$

$= R + 36$ Ω

As per question, $R + 36 = 2R$ or $R = \mathbf{36\ \Omega}$

Resistance of the origin wire

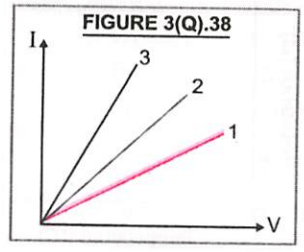
$= R + (R + 4) + (R + 8) + (R + 12) + \ldots + (R + 36)$

$= 10R + 4\,(1 + 2 + 3 + \ldots + 9)$

$= 10 \times 36 + 4 \times 45 = \mathbf{540\ \Omega}$

23. The V-I graphs of two resistors and their series combination are shown in Fig. 3(Q).38. Which one of these graphs represents the series combination of other two ? Give reason for your answer. **(CBSE Sample Paper 2003)**

FIGURE 3(Q).38



Sol. Let for the given voltage V, the values of currents be I_1, I_2 and I_3 for resistors 1, 2 and 3.

$$\therefore \quad R_1 = \frac{V}{I_1}, \quad R_2 = \frac{V}{I_2} \text{ and } R_3 = \frac{V}{I_3}$$

From graph, we note that $I_3 > I_2 > I_1$

$\therefore \quad R_3 < R_2 < R_1$

Hence graph 1 represents the series combination of other two resistances.

24. V-I graphs for parallel and series combination of two metallic resistors are shown in Fig. 3(Q).39. Which graph represents parallel combination ? Justify your answer.

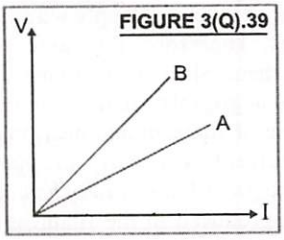


FIGURE 3(Q).39

Sol. As slope of V-I graph $= \frac{V}{I}$ = resistance R.

From the given graphs, the slope of B is greater than that of A. So the resistance of B is greater than that of A. As resistance in series combination is more and in parallel combination is less, so B represents series combination and A **represents parallel combination of resistors.**

25. When we switch on the lights one after the other, what is the effect on the resistance of the electric circuit of the house ? On the current flowing in the main circuit ?

Sol. In a house, all lights are in parallel circuits, to a fixed supply voltage 220 V. When we switch on lights one after the other, the effective resistance of the circuit goes on decreasing. As a result of it, the main current goes on increasing.

26. Two metallic rods, each of length L, area of cross-section A_1 and A_2 having resistivity ρ_1 and ρ_2 are connected in parallel across a d.c. battery. Obtain the expression for the effective resistivity of this combination. **(CBSE 2020)**

Sol. Resistance of first metallic rod, $R_1 = \frac{\rho_1 L}{A_1}$

Resistance of second metallic rod, $R_2 = \frac{\rho_2 L}{A_2}$

Effective resistance when connected in parallel

$$R_{\text{eff}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(\rho_1 L/A_1)(\rho_2 L/A_2)}{\rho_1 L/A_1 + \rho_2 L/A_2}$$

$$= \frac{\rho_1 \rho_2 L^2/A_1 A_2}{\frac{L[\rho_1 A_2 + \rho_2 A_1]}{A_1 A_2}}$$

$$= \frac{\rho_1 \rho_2 L}{\rho_1 A_2 + \rho_2 A_1} \quad \text{...(i)}$$

$$R_{\text{eff}} = \rho_{\text{eff}} \times \frac{L}{A_1 + A_2} \quad \text{...(ii)}$$

From (*i*) and (*ii*), we have

$$\rho_{\text{eff}} \times \frac{L}{(A_1 + A_2)} = \frac{\rho_1 \rho_2 L}{\rho_1 A_2 + \rho_2 A_1}$$

or $$\rho_{\text{eff}} = \frac{\rho_1 \rho_2 (A_1 + A_2)}{\rho_1 A_2 + \rho_2 A_1}$$

IV. Internal resistance and Grouping of cells

27. A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing the variation of terminal potential V with resistance R. Predict from the graph the condition under which V becomes equal of E. **(CBSE 2009)**

Sol. $V = E - Ir = E - \frac{V}{R} r \qquad (\because I = V/R)$

or $$V\left(1 + \frac{r}{R}\right) = E \quad \text{or} \quad V = \frac{E}{(1 + r/R)}$$

As R increases V also increases.

On plotting a graph between V and R, we get a curve of the type as shown in Fig. 3(Q).40.

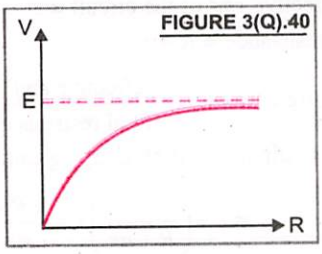


FIGURE 3(Q).40

When graph becomes parallel to resistance axis, then, $V = E$ if $R = \infty$.

28. A battery has an emf E and internal resistance r. A variable resistance R is connected across

the terminals of the battery. Find the value of R such that (a) the current in the circuit is maximum (b) the potential difference across the terminals is maximum.

Sol. (*a*) Current in the circuit is, $I = \frac{E}{r+R}$; I will be maximum when, R = min. = 0.

$\therefore \quad I_{max.} = \frac{E}{r}$

(*b*) Potential difference across the terminals,

$$V = IR = \frac{ER}{r+R} = \frac{E}{(r/R+1)}$$

V will be maximum when $\frac{r}{R}+1$ = minimum,

i.e., $\frac{r}{R}$ = zero, *i.e.*, $R \approx \infty$

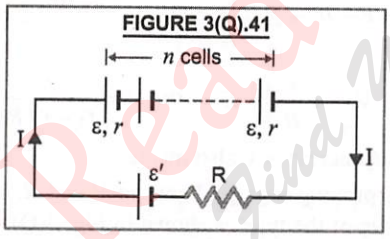
In this situation, $I = 0$ and $V = E$ = max.

29. A number of identical cells, *n*, each of *emf* ε, internal resistance *r*, connected in series are charged by a d.c. source of *emf* ε′, using a resistor *R*. (*i*) Draw the circuit arrangement. (*ii*) Deduce the expression for (*a*) the charging current and (*b*) the potential difference across the combination of the cells.

[CBSE 2003 (C), 2008]

Sol. (*i*) The circuit arrangement for charging the cells is shows in Fig. 3(Q).41.

FIGURE 3(Q).41



(*ii*) Effective *emf* of the circuit = ε′ − nε

Total resistance = $R + nr$

Charging current $I = \frac{\text{effective } emf}{\text{total resistance}} = \frac{\varepsilon' - n\varepsilon}{R+nr}$

(*iii*) Pot. diff. across each charging cell = $\varepsilon + Ir$

$$= \varepsilon + \left(\frac{\varepsilon' - n\varepsilon}{R+nr}\right) r = \frac{\varepsilon R + \varepsilon' r}{R+nr}$$

Pot. diff. across the series combination of n cells

$$V = n(\varepsilon + Ir) = n\left(\frac{\varepsilon R + \varepsilon' r}{R+nr}\right)$$

30. In which respect, does a nearly discharged lead acid secondary cell differ mainly from a freshly charged cell in its emf or in its internal resistance ?

Sol. A discharged lead acid secondary cell differs from a freshly charged cell mainly in internal resistance and partly in its *emf* as a nearly discharged lead acid secondary cell acquires very high internal resistance.

V. Joule's law of heating

31. A wire connected to a bulb does not glow, whereas the filament of the bulb glows when same current flows through them. Why ?

Sol. Filament of bulb and supply wires are connected in series. Therefore, the same current flows through them. Since the resistance of connecting wires is negligibly small as compared to the resistance of filament and heat produced due to given current is directly proportional to its resistance (*from Joule's law of heating*), therefore, the heat produced in the filament is vey large. Hence the bulb glows, but the connecting wires remain practically unheated.

32. An electric motor runs on a d.c. source of emf ε and internal resistance *r*. Show that the power output of the source is maximum when the current drawn by the motor is ε/2 *r*. (NCERT)

Sol. Let I be the current drawn by motor. Then power output of the source, $P = \varepsilon I - I^2 r$

P will be maximum, when

$$\frac{dP}{dI} = 0 \quad \text{or} \quad \frac{d}{dt}(\varepsilon I - I^2 r) = 0$$

$$\text{or} \quad \varepsilon - 2Ir = 0 \quad \text{or} \quad I = \frac{\varepsilon}{2r}$$

33. In the above question, show that power output of electric motor is maximum when the back emf is one-half of the source emf, provided the resistance of the winding of the motor is negligible. (NCERT)

Sol. Here, emf of d.c. source = ε,

resistance of motor $R = 0$

Internal resistance of source = r

Let ε′ be the back emf of motor. Therefore,

current in the circuit, $I = \frac{\varepsilon - \varepsilon'}{r+R} = \frac{\varepsilon - \varepsilon'}{r}$

Power output of the motor,

P = power output of source = $\varepsilon I - I^2 r$

P is maximum when $\frac{dP}{dI} = 0$

or $\frac{d}{dt}(\varepsilon I - I^2 r) = 0$ or $\varepsilon = 2Ir$

or $I = \frac{\varepsilon}{2r}$ or $\frac{\varepsilon - \varepsilon'}{r} = \frac{\varepsilon}{2r}$ or $\varepsilon' = \frac{\varepsilon}{2}$

34. A nichrome heating element across 230 V supply consumes 1·5 kW power and heats upto a temperature of 750°C. A tungsten bulb across the same supply operates at a much higher temperature of 1600°C in order to be able to emit light. Does it mean that tungsten bulb necessarily consumes greater power ?

(NCERT)

Sol. No, the steady temperature acquired by a resistor (*i.e.*, heating element) depends not only on the power consumed but also on the power loss due to radiation, which inturn depends on the characteristics of a resistor such as surface area, emissivity, etc.

VI. Electric power and Electric energy

35. When current is passed through a heater, the heat is generated continuously in it but its temperature becomes constant after sometime. Why ?

Sol. When the temperature of heater becomes more than the temperature of surroundings, it starts lossing heat to surroundings. Finally a stage is reached at which the rate of production of heat becomes equal to the rate of loss of heat. At this stage, the temperature becomes constant.

36. Prove that in parallel combination of elec-trical appliances, total power consumption is equal to the sum of the powers of the individual appliances.

(CBSE 2011)

Sol. Consider electrical appliances of powers, P_1, P_2, P_3 having resistances R_1, R_2, R_3 connected in parallel. Suppose they are operated at main voltage V. Let P be the total power and R be the total resistance of the combination of all appliances. Then according to law of resistances in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Multiplying both sides by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

or $P = P_1 + P_2 + P_3$ $\left[\because \frac{V^2}{R} = P\right]$

Hence total electric power consumed is equal to the sum of the powers of the individual appliances.

37. What is the power transferred per unit volume into joule heat in a resistor ?

Sol. Power, $P = I^2R = I^2 \rho l/A$; $(\because R = \rho l/A)$

Volume of resistor, $V = A\,l$

$\therefore$ Power transferred per unit volume

$$= \frac{P}{V} = \frac{I^2 \rho l / A}{Al} = \left(\frac{I}{A}\right)^2 \rho = J^2 \rho$$

where, $I/A = J$ = current density.

38. Prove that in series combination of electrical appliances, the reciprocal of total power consumption is equal to the sum of the reciprocal of the powers of the individual appliances.

Sol. Let P_1, P_2, P_3 be the powers of electrical appliances at V volt. If R_1, R_2, R_3 are their resistances, then

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3}$$

When the electrical appliances are connected in series, then effective resistance, R is

$$R = R_1 + R_2 + R_2$$

or $$\frac{R}{V^2} = \frac{R_1}{V^2} + \frac{R_2}{V^2} + \frac{R_3}{V^2}$$

or $$\frac{1}{V^2/R} = \frac{1}{V^2/R_1} + \frac{1}{V^2/R_2} + \frac{1}{V^2/R_3}$$

or $$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

39. A battery has an emf E and internal resis- tance r. A variable resistor R is connected across the terminals of the battery. Find the value of R such that the power delivered to the resistor is maximum.

Sol. Current in the circuit, $I = \frac{E}{R + r}$

Power delivered to the resistance R is,

$$P = I^2 \times R = \frac{E^2 R}{(R + r)^2}$$

Power delivered to the load will be maximum when $\frac{R}{(R+r)^2}$ is maximum, *i.e.*,

$$\frac{d}{dR}[R(R+r)^{-2}] = 0$$

or $[(R+r)^{-2} + R(-2)(R+r)^{-3}] = 0$

or $(R+r)^{-3}[r-R] = 0$

As $(R+r)^{-3} \neq 0$ for finite value of R,

so $r - R = 0$ or $R = r$

When $R = r$, the power transferred to the load is

$$= \frac{E^2 r}{(r+r)^2} = \frac{E^2}{4r} = \text{max.}$$

40. For what value of load resistance, the power transfer is maximum when two identical cells each of emf E and internal resistance r are connected (a) in series (b) in parallel.

Sol. (*a*) When cells are connected in series to the external load resistance R, then effective *emf*

$$= E + E = 2E.$$

Total resistance of the circuit $= R + r + r$

$$= R + 2r.$$

Current in load, $I = \frac{2E}{R+2r}$

Power delivered to the external load is,

$$P = I^2R = \left(\frac{2E}{R+2r}\right)^2 \cdot R$$

Power delivered to the external load will be maximum if external resistance = internal resistance of cells, *i.e.*, $R = 2r$. Then maximum power, $P_{max.} = \left(\frac{2E}{2r+2r}\right)^2 \times 2r = \frac{E^2}{2r}$

(*b*) When cells are connected in parallel to the external load resistance R, then effective *emf* $= E$. Total resistance of the circuit

$$= R + \frac{r \times r}{r+r} = \left(R + \frac{r}{2}\right)$$

Current in load, $I = \frac{E}{(R + r/2)}$

Power delivered to the external load,

$$P = I^2R = \left(\frac{E}{R+r/2}\right)^2 R$$

Power is maximum when $R = r/2$. Then

$$P_{max.} = \left(\frac{E}{r/2+r/2}\right)^2 r/2 = \frac{E^2}{2r}$$

Note that in both the cases, maximum power delivered to the load is the same.

41. Long distance power transmission is carried on high voltage lines. Why ?

Sol. When current I is transmitted through a power line of resistance R, Power loss $= I^2R$.

If the power P is transmitted at voltage V, then $P = VI$ or $I = (P/V)$ $\therefore$ Power loss $= \frac{P^2}{V^2}R$

For a given power and given line, P and R are constant. Hence, power loss $\propto (1/V^2)$

It means if power is transmitted at high voltage, power loss will be small and vice-versa.

42. Two tungsten filaments with resistance R_1 and R_2 respectively are connected first in series and then in parallel in a lighting circuit of negligible internal resistance. If $R_1 >> R_2$;

(a) Which lamp will glow more brightly when they are connected in series ?

(b) If the lamp of resistance R_2 now burns out and the lamp R_1 alone is plugged in, will net illumination increase or decrease ?

(c) Which lamp will glow more brightly when they are connected in parallel ?

(d) If the lamp of resistance R_1 now burns out, how will the net illumination produced change ?

Sol. (*a*) When the bulbs are in series, current I = constant. As $P = I^2R$, so $P \propto R$. Hence the ***lamp with higher resistance (i.e. R_1) will glow more brightly.***

(*b*) When the two bulbs are operating in series, total illumination, $P = \frac{V^2}{(R_1 + R_2)}$

When the lamp of resistance R_2 gets burned and bulb of resistance R_1 alone is operated, then illumination, $P' = \frac{V^2}{R_1}$, *i.e.*, $P' > P$

So ***the net illumination will increase.***

(*c*) When the bulbs are in parallel, voltage across each bulb is same. As, $P = \frac{V^2}{R}$, so $P \propto \frac{1}{R}$

Hence, ***the lamp with lesser resistance, i.e., R_2 will glow more brightly.***

(*d*) When both the bulbs are operating in parallel, then total illumination,

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2}$$

When lamp of resistance R_1 burns out, only lamp of resistance R_2 will give light. Then illumination is, $P' = \frac{V^2}{R_2}$, *i.e.*, $P' < P$

So ***the net illumination will decrease.***

VII. Aspects of heating effect of current

43. How does the use of fuse wire save the electrical installations ?

Sol. A fuse wire is one which has high resistance and low melting point. This is connected in series with electrical installations. When the supply voltage exceeds the safe limit, more heat is produced in the fuse wire. ($\because$ rate of heat production $P = V^2/R$), and it melts. The circuit breaks and the damage to the electrical installations is saved.

44. Why the brightness of light emitted by a bulb decreases gradually with its period of use ?

Sol. When the bulb is used, the evaporation of the metal from the filament of bulb takes place with time which deposits on the inner side of the glass wall as black substance. Due to it, the filament of the bulb becomes thinner and thinner with use. This in turn increases the resistance of the bulb. ($\because R \propto 1/\pi r^2$). As $P = V^2/R$, so brightness of bulb decreases gradually with its period of use.

45. If one wants more light at large distances, assuming the wattage is the same, will one use a point source or a cylindrical tube light ?

Sol. ***For a point source,*** the intensity $\propto 1/r^2$ and for a ***cylindrical source,*** the intensity $\propto 1/r$. Thus, the cylindrical tube light is better than the point source of light.

46. A line having a total resistance of 0·4 Ω delivers 20 kW at 220 V to a small factory. Calculate the efficiency of the transmission.

Sol. Power loss in the transmission line in the form of heat

$$P = I^2 R = \left(\frac{P}{V}\right)^2 R = \left(\frac{20{,}000}{220}\right)^2 \times 0{\cdot}4$$
$$= 3306 \text{ W} \simeq 3{\cdot}31 \text{ kW}$$

Efficiency of transmission line

$$\eta = \frac{\text{power delivered by line}}{\text{power supplied to line}}$$
$$= \frac{\text{power delivered}}{\text{power delivered + power loss}}$$
$$= \frac{20}{20 + 3{\cdot}31} = 0{\cdot}858 = \mathbf{85{\cdot}8\%}$$

VIII. Kirchhoff's laws and Wheatstone bridge

47. Can meter bridge be used for finding the resistance of (i) moderate values (ii) high values (iii) low values ? Explain.

Sol. The meter-bridge is most sensitive when the resistance of all the four arms of the bridge are of the same order. It is so for moderate resistances only. The meter bridge becomes insensitive for too high or too low resistances. Moreover, while measuring low resistances, the resistances of the copper strips and connecting wires become comparable to the unknown low resistance and hence cannot be neglected.

48. Using meter bridge, it is advised to obtain the null point in the middle of bridge wire. Why ? **[CBSE 2011 (C), 2013]**

Sol. The error in the measured value of unknown resistance S using meterbridge will be minimum, when the null point is obtained at the middle of meterbridge wire. In this situation, the end error of the bridge will become ineffective. To understand it, let α be the end resistance of left and right copper strips in meterbridge. Let unknown resistance S be in the right gap and known resistance R be in the left gap of meter bridge. If null point is obtained at the middle of bridge wire (*i.e.*, $l = 50$ cm) then

$$\frac{S + \alpha}{R + \alpha} = \frac{100 - 50}{50} = 1 \quad \text{or} \quad S = R$$

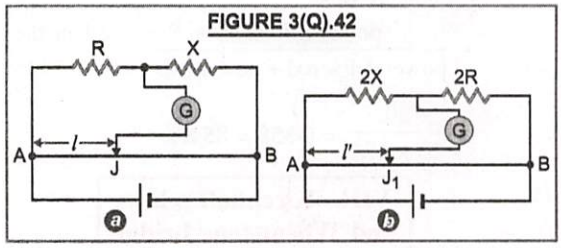
It means the effect of α gets cancelled. In this situation the bridge is most accurate and sensitive. If the null point is not at the centre, there will be some error because S will then not be equal to R.

49. In the meter bridge experiment, a student observes a balance point at the point J, where $AJ = l$, Fig. 3(Q).42. The value of R and X are both doubled and then interchanged. What will be the new position of balance point. If in this set up, the galvanometer and battery are

interchanged at the balance point position, how will the balance point get affected ?

(CBSE 2011)

FIGURE 3(Q).42



Sol. Refer to Fig. 3(Q).43(*a*), $\frac{R}{X} = \frac{l}{(100-l)}$ ...(*i*)

When $R = 2R$ and $X = 2X$ are connected as shown in Fig. 3(Q).43(*b*), let the balance point be at J_1, where $AJ_1 = l'$.

Then $\frac{2X}{2R} = \frac{l'}{(100-l')}$ or $\frac{R}{X} = \frac{100-l'}{l'}$ ...(*ii*)

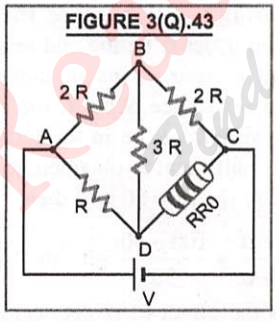
From (*i*) and (*ii*), $\frac{l}{100-l} = \frac{100-l'}{l'}$

On solving, we get $l' = \mathbf{100 - l}$

When galvanometer and battery are interchanged, there is **no change in the balance point position.**

50. In the given Wheatstone bridge, the current in the resistor 3 *R* is zero. Find the value of *R*, if the carbon resistor, connected in one arm of the bridge, has the colour sequence of red, red and orange. Fig. 3(Q).43.

FIGURE 3(Q).43



The resistances of *BC* and *CD* arms are now interchanged and another carbon resistance is connected in place of *R* so that the current through the arm *BD* is again zero. Write the sequence of colour bands of this carbon resistor. Also find the nature of current through it.

(CBSE Sample Paper 2011)

Sol. Let *S* be the resistance of carbon resistor in arm *DC*. Its value is $S = 22 \times 10^3\ \Omega$.

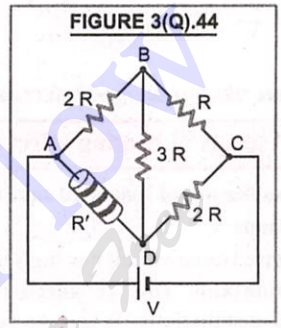
Since the current in *BD* arm is zero, so the Wheatstone bridge is balanced. Therefore,

$$\frac{2R}{2R} = \frac{R}{S}$$

So, $R = S = 22 \times 10^3\ \Omega = \mathbf{22\ k\Omega}$

When the resistances in arms *BC* and *CD* are interchanged and another carbon resistor *R'* is connected in place of *R*, Fig. 3(Q).44. The condition for balanced bridge will be

FIGURE 3(Q).44



$$\frac{2R}{R} = \frac{R'}{2R}$$

or $R' = 4R = 4 \times 22\ \text{k}\Omega = 88\ \text{k}\Omega$

$= 88 \times 10^3\ \Omega$

The sequence of colours will be **grey, grey and orange.**

IX. Potentiometer

51. What do you understand by sensitiveness of a potentiometer and how can you increase the sensitiveness of a potentiometer ?

[CBSE 2011 (C)]

Sol. The sensitiveness of a potentiometer means the smallest potential difference that can be measured with its help and for a small change in pot. diff. to be measured it shows a large change in balancing length. The sensitiveness of a potentiometer can be increased by decreasing its potential gradient. This can be achieved,

(*a*) by increasing the length of potentiometer wire.

(*b*) If the potentiometer wire is of fixed length, the potential gradient can be decreased by reducing the current in the potentiometer wire circuit by increasing the resistance with the help of rheostat.

52. On what factors, does the potential gradient of the potentiometer wire depend ?

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Sol. The fall of potential across a potentiometer wire of length l is given by $V = IR = I\rho\, l/A$

$\therefore$ Potential gradient, $K = \frac{V}{l} = \frac{I\rho}{A}$

Thus potential gradient of a wire depends upon the following factors :

(*i*) $K \propto I$ (*i.e.* current passing through the potentiometer wire (*ii*) $K \propto \rho$ (*i.e.* specific resistance of the material of the potentiometer wire

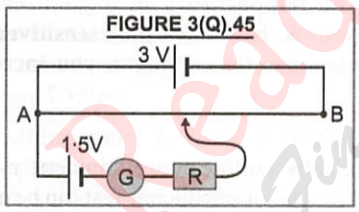
(*iii*) $K \propto \frac{1}{A}$, where A is the area of cross-section of the potentiometer wire.

53. What does the no deflection position in the galvanometer of potentiometer experiment tell us about the flow of current ?

Sol. When the balance point is obtained on the potentiometer wire, no current flows in the galvanometer circuit because the fall of potential across the potentiometer wire between zero end and balance point position becomes just equal to the emf of the cell, whereas the current in the potentiometer wire continues flowing.

54. A potentiometer wire of length 1 m is connected to a driver cell of emf 3 V as shown in Fig. 3(Q).45. When a cell of emf 1·5 V is used in the secondary circuit, the balance point is found to be 60 cm. On replacing this cell with a cell of unknown emf, the balance point shifts to 80 cm.

FIGURE 3(Q).45



(*i*) Calculate unknown emf of the cell.

(*ii*) Explain with reason, whether the circuit works, if the driver cell is replaced with a cell of emf 1 V.

(*iii*) Does the high resistance R, used in the secondary circuit affect the balance point ? Justify your answer. **(CBSE 2008)**

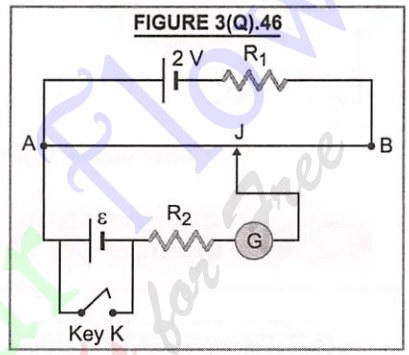
Sol. (*i*) $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$ or $\varepsilon_2 = \varepsilon_1 \times \frac{l_2}{l_1} = 1{\cdot}5 \times \frac{80}{60} = \mathbf{2{\cdot}0\ V}$

(*ii*) When the driver cell of emf 2 V is replaced by a cell of emf 1 V, the circuit will not work because the fall of potential on the entire potentiometer wire will become less than the emf of cell 1·5 V to be balanced on the wire. Due to it, balance point cannot be obtained on potentiometer wire.

(*iii*) When the high resistance R is used in the secondary circuit, the position of balance point on the potentiometer wire will not be affected because no current flows through the cell at the balance point.

55. Fig. 3(Q).46 shows the circuit diagram of a potentiometer for determining the emf ε of a cell of negligible internal resistance.

FIGURE 3(Q).46



(*i*) What is the purpose of using high resistance R_2 ?

(*ii*) How does the position of balance point (J) change when the resistance R_1 is increased ? **(CBSE Sample Paper 2020)**

(*iii*) Why cannot the point be obtained,

(*a*) When the emf ε is greater than 2 V, and

(*b*) When the key K is closed. **(CBSE 2009)**

Sol. (*i*) The purpose of high resistance R_2 is to protect the galvanometer by decreasing the curent through it for positions, which are far away from the balance point.

(*ii*) When resistance R_1 is increased, the current through the potentiometer wire circuit decreases. Due to it, the potential gradient across potentiometer wire decreases. As a result of it, the balance point shifts towards the end B.

(*iii*) (*a*) When emf ε is greater than 2 V, the balance point cannot be obtained on potentiometer wire because the potential drop across the potentiometer wire will be less than 2 V.

(*b*) When the key K is closed, the cell emf ε gets short circuited. Due to it, the balance point is not obtained on the potentiometer wire.

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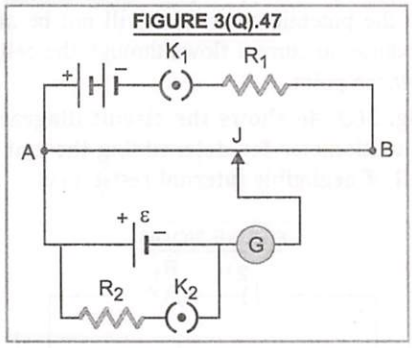
56. For the circuit shown in the Fig. 3(Q).47, would the balancing length increase, decrease or remain the same if (*i*) R_1 is decreased (*ii*) R_2 is increased, without any change (in each case) in the rest of the circuit ? Justify your answer in each case.

FIGURE 3(Q).47



(CBSE Sample Paper 2013)

Sol. (*i*) When R_1 is decreased, the **balancing length decreases.** It is because as R_1 decreases, the current in the potentiometer circuit increases. Due to it, the potential gradient of potentiometer wire increases. As a result of it the same emf will be balanced on smaller length of potentiometer wire.

(*ii*) When R_2 is increased, the **balancing length increases.** It is because as R_2 increases, the current $I\left(=\frac{\varepsilon}{r+R_2}\right)$ decreases. This increases the terminal pot. diff. $V(=\varepsilon - Ir)$ across the unknown cell. As a result of it, the terminal potential difference of cell ε will be balanced on larger length of potentiometer wire.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Current and drift velocity

1. Why do the free electrons in a metal wire, flowing by themselves, not cause any current flow in the wire ? [CBSE (D) 2016 (C)]

Ans. In a metal wire, the free electrons due to thermal motion move at random. There average velocity is zero in one direction. Due to it, there is no current flow in the wire.

2. Is electric current a vector or scalar quantity ? Explain.

Ans. Electric current is a scalar quantity, because it does not follow the laws of vectors addition, *i.e.*, the angle between the wires carrying the current does not affect the total current in the circuit.

3. If the electric current is passed through a nerve, the man is excited, why ?

Ans. The man is excited due to transfer of electric energy to his body.

4. A steady current is flowing in a cylindrical conductor. Is there any electric field within the conductor ?

Ans. Yes. A steady current can flow in a cylindrical conductor if electric force is acting on free electrons which makes the electrons to move in a particular direction. It is possible due to electric field within the conductor.

5. How can you keep a constant current inside a conductor ?

Ans. A constant current can be kept inside a conductor by maintaining a constant potential difference across the two ends of the conductor.

6. If the temperature of a good conductor increases, how does the relaxation time of electrons in the conductor change ?

(CBSE 2002)

Ans. With the increase in temperature, the free electrons collide more frequently with the ions/atoms of conductor, resulting decrease in relaxation time.

7. Two different wires *X* and *Y* of same diameter but different materials are joined in series across a battery. If the number density of electrons in *X* is twice that in *Y*, find the ratio of drift velocity of electrons in the two wires.

(CBSE 2010)

Ans. Drift velocity,

$$v_d = \frac{I}{nAe} = \frac{I}{n(\pi D^2/4)e}, \quad i.e., \quad v_d \propto \frac{1}{n}$$

Since the wires are connected in series, current I through both is same. Therefore,

$$\frac{(v_d)_x}{(v_d)_y} = \frac{n_y}{n_x} = \frac{1}{2}$$

8. The connecting wires are of copper. Why ?

Ans. The electrical conductivity of copper is next only to silver which is costly. Therefore, it conducts the current without offering much resistance. The copper being diamagnetic* material does not get magnetised due to current through it and hence does not disturb the current in the circuit.

9. Show variation of resistivity of silicon with temperature in a graph. **(CBSE 2014)**

Ans. The variation of resistivity of silicon with temperature is shown graphically in Fig. 3.13(d).

10. Two conducting wires X and Y of diameter ratio 2 : 1 but different materials are joined in series across a battery. If number density of electrons in X is twice that in Y, find the ratio of drift velocity of electrons in the two wires.

Ans. In series, the current through the two wires will be same. So $I_X = I_Y$ or $n_x A_x e v_x = n_y A_y e v_y$

or $$\frac{v_x}{v_y} = \frac{n_y}{n_x} \times \frac{A_y}{A_x} = \frac{n_y}{n_x} \times \frac{\pi D_y^2}{\pi D_x^2} = \frac{1}{2} \times \frac{1}{2^2} = \frac{1}{8}$$

11. How does the random motion of free electrons in a conductor get affected when a potential difference is applied across its ends ? **[CBSE 2014 (C)]**

Ans. When a potential difference is applied across the two ends of a conductor, the random motion of free electrons is partially directed towards higher potential side, *i.e.*, positive end of the conductor.

II. Resistance and effect of temperature

12. What should be the properties of standard resistances ?

Ans. The properties of standard resistances should be :

(*i*) Its value should not change with time.

(*ii*) It should show negligible variation in resistance with temperature.

(*iii*) It should have proper capacity for carrying current without overheating.

(*iv*) It should have low inductance.

(*v*) It should have robust construction and convenient size.

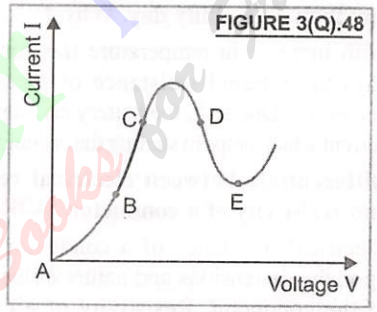
13. Specific resistance of copper, silver and constantan are ($1{\cdot}78 \times 10^{-6}\ \Omega$ cm, $10^{-6}\ \Omega$ cm and $48 \times 10^{-6}\ \Omega$ cm) respectively. Which is the best conductor and why ?

Ans. The best conductor is silver because electrical conductivity is inversely proportional to the resistivity and resistivity of silver is least.

14. Is the formula $V = IR$ true for non ohmic devices also ?

Ans. Yes, the formula defines resistance $R = V/I$ and not Ohm's law, for which, $\frac{V}{I} = R$ = constant.

15. Graph showing the variation of current versus voltage for a material GaAs is shown in Fig. 3(Q).48. Identify the region of



(*i*) negative resistance

(*ii*) where Ohm's law is obeyed.

(CBSE 2015)

Ans. (*i*) *DE* is the region of negative resistance because the slope of curve in this region is negative.

(*ii*) *BC* is the region where Ohm's law is obeyed because in this part the current varies linearly with the voltage.

16. A 4 Ω non insulated resistance wire is bent in the middle by 180° and both the halves are twisted with each other. What will be its new resistance ? **[CBSE 2010 (C)]**

Ans. Here, $R = \frac{\rho l}{A} = 4$;

$$R' = \frac{\rho (l/2)}{2A} = \frac{\rho l}{4A} = \frac{R}{4} = \frac{4}{4} = 1\Omega$$

17. Two wires of equal length one of copper and other of manganin have the same resistance. Which wire is thicker ? **(CBSE 2012)**

Ans. $R = \rho\, l/A$ or $A \propto \rho$ if l and R are constant. Since ρ is greater for manganin than for copper, hence **manganin wire is thicker than copper wire.**

18. The current flowing through a conductor is 2 mA at 50 V and 3 mA at 60 V. Is it an ohmic or nonohmic conductor ?

Ans. The resistance of first case,

$$R_1 = \frac{V_1}{I_1} = \frac{50\,\text{V}}{2\times10^{-3}\,\text{A}} = 25{,}000\ \Omega$$

The resistance of second case,

$$R_2 = \frac{V_2}{I_2} = \frac{60\,\text{V}}{3\times10^{-3}} = 20{,}000\ \Omega$$

As resistance changes with current, so the given **conductor is non-ohmic.**

19. It is easier to start a car engine on a warm day than on a chilly day. Why ?

Ans. With increase in temperature (*i.e.*, on a warm day) the internal resistance of a car battery decreases. Due to it, the battery can supply large current which helps in starting the car engine easily.

20. Differentiate between electrical resistance and resistivity of a conductor. (CBSE 2020)

Ans. Electrical resistance of a conductor depends upon the dimensions and nature of the material of the conductor. Resistivity of a conductor depends upon the nature of material of conductor but is independent of the dimensions of the conductor.

21. If potential difference V applied across a conductor is increased by 2 V, how will the drift velocity of the electrons change ?

Ans. Let R be the resistance of conductor. In first case, current in conductor, $I = V/R$.

In second case current in conductor,

$$I' = \frac{2V+V}{R} = \frac{3V}{R} = 3I$$

As, $I = n\,A\,e\,v_d$ or $v_d \propto I$

($\because n, A, e$ are constant)

$$\therefore \frac{v_d'}{v_d} = \frac{I'}{I} = \frac{3I}{I} = 3 \quad \text{or} \quad v_d' = 3v_d$$

It means **drift velocity becomes tripled.**

22. Alloys of metals have greater resistivity than that of their constituent metals. Why ?

Ans. In an alloy like nichrome (made of nickel and chromium), the free electron finds a disordered arrangement of nickel ions and cromium ions. Due to it, the electron is scattered by them randomly and very frequently. As a result of it, the value of relaxation time of electron decreases and hence resistivity increases because $\rho \propto 1/\tau$.

23. Define the term 'mobility' of charge carriers. Write its S.I. unit. (CBSE 2015)

Ans. Mobility of charge carriers (μ), responsible for currents is defined as the magnitude of drift velocity of charge per unit electric field applied

$$i.e., \quad \mu = \frac{\text{drift velocity}}{\text{electric field}} = \frac{v_d}{E} = \frac{v_d}{V/l} = \frac{v_d}{F/q}$$

S.I. unit of mobility = $m^2\,V^{-1}\,s^{-1} = ms^{-1}\,N^{-1}C$

24. What is the resistance of carbon resistor on which the colour of rings in sequence is black, brown, black and gold.

Ans. Value of carbon resistance = $01 \times 10^0\ \Omega \pm 5\%$

$= 1\ \Omega \pm 5\%$.

25. What will be the bands of colours in sequence on carbon resistor, if its resistance is $0{\cdot}1\ \Omega \pm 5\%$.

Ans. Resistance = $0{\cdot}1\ \Omega \pm 5\% = 01 \times 10^{-1} \pm 5\%$. Thus, the colour of bands in sequence is black, brown and gold. Tolerance of $\pm 5\%$ is indicated by gold ring.

26. What is the colour code for a resistor of resistance $3{\cdot}5\ k\ \Omega$ with 5% tolerance ?

Ans. Given, resistance = $3{\cdot}5\ k\ \Omega \pm 5\%$

$= 35 \times 10^2\ \Omega \pm 5\%$

$\therefore$ Colour code of given resistor is **orange, green, red** and **gold.**

27. What is the most probable cause of super-conductivity ?

Ans. The cause of super-conductivity is that, the free electrons in super-conductor are no longer independent but become mutually dependent and coherent when critical temperature is reached. The ionic vibrations which could deflect the free electrons in metals are unable to deflect this coherent or cooperative cloud of electrons in super-conductors. That is why, there is no resistance offered by the super-conductor to the flow of electrons.

28. The given graph shows the variation of resistance of mercury in the temperature range $0 < T < 4$ K. Name the phenomeon shown by the graph. (CBSE 2003)

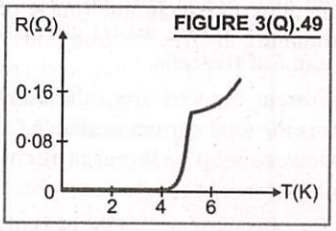


FIGURE 3(Q).49

Ans. Superconductivity.

29. Two square metal plates are of same thickness and material. The side of B is twice that of A. These are connected in series, Fig. 3(Q).50. If the resistance of A and B are denoted by R_A and R_B, find R_A/R_B.

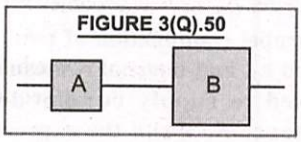


FIGURE 3(Q).50

Ans. Let l, d length and thickness of square metal plate. Then

$$R_A = \frac{\rho l}{A} = \frac{\rho l}{l \times d} = \frac{\rho}{d} \text{ and } R_B = \frac{\rho \times 2l}{2l \times d} = \frac{\rho}{d}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{\rho/d}{\rho/d} = 1$$

III. Grouping of resistance

30. Two identical slabs of given metal are joined together in two different ways as shown in Fig. 3(Q).51(a) and (b). What is the ratio of the resistances of these two combinations ?

[CBSE 2010 (C)]

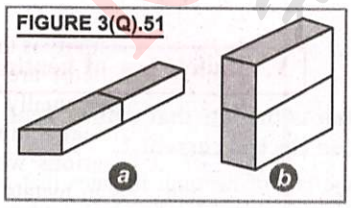


FIGURE 3(Q).51

Ans. Let l and A be the length and area of cross-section of the each of the two slabs. Then $R = \rho\, l/A$

In Fig. 3(Q).50(a), $R_1 = \dfrac{\rho\, 2l}{A} = 2R$

In Fig. 3(Q).50(b), $R_2 = \dfrac{\rho l}{2A} = \dfrac{R}{2}$

$$\therefore \quad \frac{R_1}{R_2} = \frac{2R}{R/2} = 4 \quad \text{or} \quad R_1 : R_2 = 4 : 1$$

31. Explain why resistance becomes more in series combination.

Ans. In series combination of resistors, the effective length of the conductor increases. As $R \propto l$, therefore, resistance increases.

32. Explain why resistance becomes less in parallel combination.

Ans. In parallel combination of resistors, the effective area of cross-section of the conductor increases. As, $R \propto 1/A$, therefore, resistance decreases.

33. Join three resistances of 2 Ω each such that the total resistance of the circuit is 3 Ω.

Ans. Two resistances in parallel and the third resistance is connected in series with them.

IV. Internal resistance and Grouping of cells

34. What is the difference between electromotive force and terminal voltage of a cell ? How are they related with each other.

(Raj. Board 2011)

Ans. Electromotive force is the maximum potential difference between two electrodes of a cell in the open circuit and terminal voltage of a cell is the potential difference between two electrodes of a cell in closed circuit. In a closed circuit, when cell supplies a current in the circuit, then electromotive force = terminal voltage + voltage drop across the internal resistance of the cell.

35. What is the internal resistance of a cell due to ? **(Pb. Board 2011)**

Ans. Internal resistance of a cell depends upon :

(i) the nature, concentration and temperature of electrolyte,

(ii) the nature of electrodes,

(iii) the distance between the electrodes and

(iv) area of the electrodes immersed in the electrolyte.

36. On increasing the current drawn from a cell, the potential difference of its terminals is lowered. Why ?

Ans. This is due to internal resistance r of the cell. We know that terminal potential difference $V = \varepsilon - I\,r$. If I is increased, V will be lowered.

37. Can the terminal potential difference of a cell exceed its *e.m.f.* ?

Ans. Yes, when cell itself is being charged, because terminal potential difference,

$$V = \varepsilon - (-I r) = \varepsilon + I r.$$

38. A (*i*) series (*ii*) parallel combination of two given resistors is connected one by one across a cell. In which case will the terminal potential difference across the cell have higher value ? [CBSE 2008 (C)]

Ans. Terminal potential difference is equal to the voltage drop across the external resistor connected to cell. As resistance of two resistors in series is more than in parallel, so the terminal potential difference will have higher value in series combination of two resistors.

39. The plot of the variation of potential difference across a combination of three identical cells in series, versus current is as shown in Fig. 3(Q).52. What is the *emf* and internal resistance of each cell ?

(CBSE 2016, 2008)

Ans. Total *emf* of three cells in series (3 ε)

= pot. diff. corresponding to zero current

$$\therefore\ 3\varepsilon = 6\ \text{V} \quad \text{or} \quad \varepsilon = \frac{6}{3} = 2\ \text{V}$$

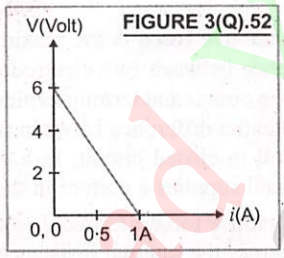


FIGURE 3(Q).52

The internal resistance of each cell

$$r = \frac{\varepsilon}{I_{\max}} = \frac{2}{1} = 2\Omega$$

40. Write any two factors on which internal resistance of a cell depends. (CBSE 2010)

Ans. Internal resistance of a cell depends on (*i*) the nature of electrolyte (*ii*) the concentration of electrolyte.

41. The car-battery is of 12 volt. 8 simple cells connected in series can give 12 volt. But such cells are not used in starting a car ; why ?

Ans. To start a car, high current is required which cannot be obtained from the series combination of 8 simple cells, because their internal resistance is of the order of 10 Ω, while the resistance of the car battery is only of the order of 0·1 Ω.

42. When cells are connected in parallel, what will be the effect on (*i*) current capacity (*ii*) e.m.f of the cells.

Ans. (*i*) Current capacity from the combination of cells is the total current available from the cell. Current capacity increases for the parallel combination of cells.

(*ii*) The effective e.m.f. of the cells of equal e.m.f. in parallel will be equal to e.m.f. of one cell.

43. Is it possible that there is no potential difference between the plates of a cell ? If yes, under what condition ?

Ans. When cell is short circuited, it gets totally discharged. The potential difference between the two plates of the cell becomes zero.

44. A parallel combination of two cells of *emfs* ε_1 and ε_2, and internal resistances r_1 and r_2 is used to supply current to a load of resistance *R*. Write the expression for the current through the load in terms of ε_1, ε_2, r_1 and r_2. (CBSE Sample Paper 2011)

Ans. Equivalent resistance of two cells in parallel

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

Equivalent *emf* of two cells in parallel

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Current in circuit,

$$I = \frac{\varepsilon_{eq}}{R + r_{eq}} = \frac{\dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}{R + \dfrac{r_1 r_2}{(r_1 + r_2)}} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{R(r_1 + r_2) + r_1 r_2}$$

V. Joule's law of heating

45. What is the law that defines heat produced by an electric current ?

Ans. Joule law of heating. It states that the amount of heat produced in a conductor when a constant current flows through it is directly proportional to (*i*) the square of the current, (*ii*) the resistance of the conductor and (*iii*) the time for which current is passed.

46. Name the physical quantity which has its unit joule coulomb^{-1}. Is it a scalar or vector quantity ?

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Ans. A physical quantity with unit, joule coulomb^{-1} = workdone/charge = potential difference. It is a scalar quantity.

47. An ammeter reads a current of 30 A when it is connected across the terminals of a cell of emf 2 V. Neglecting the meter resistance, find the amount of heat (in calories) in cell in 20 seconds.

Ans. Here, $I = 30$ A, $V = 2$ V, $t = 20$ s. Let r be the internal resistance of a cell. Then

$$r = \frac{\varepsilon}{I} = \frac{2{\cdot}0}{30} = \frac{1}{15}\Omega$$

Heat produced in cell, $H = \dfrac{I^2 rt}{J}$

$$= \frac{(30)^2 \times (1/15) \times 20}{4{\cdot}2} = \mathbf{285{\cdot}7\ cals}$$

VI. Electric power and Electric energy

48. What do you understand by maximum power rating of a resistor ?

Ans. Maximum power rating of a resistor is the maximum power which it can tolerate in the form of heat without being melt.

49. Which has greater resistance : 1 kW electro-heater or a 100 W filament bulb, both marked for 220 V ? **(CBSE 2001)**

Ans. Resistance of heater, $R_1 = \dfrac{V^2}{P_1} = \dfrac{(220)^2}{1000}\Omega$

Resistance of bulb, $R_2 = \dfrac{V^2}{P_2} = \dfrac{(220)^2}{100}\Omega$

$$\frac{R_1}{R_2} = \frac{100}{1000} = \frac{1}{10} \quad \text{or} \quad R_2 = 10\,R_1$$

Thus **resistance of bulb is greater than heater.**

50. Distinguish between kilowatt and kilowatt hour.

Ans. Kilowatt is the unit of power. 1 kilowatt = 1000 watt. Kilowatt hour is the unit of electric energy, where

1 kilowatt hour = 1000 watt × 1 hour

= 1000 watt × 60 × 60 second = $3{\cdot}6 \times 10^6$ J

51. A heating element is marked 210 V, 630 W. What is the value of the current drawn by the element when connected to a 210 V dc source ? **(CBSE 2013)**

Ans. Here, $V = 210$ V, $P = 630$ W ;

Current $I = \dfrac{P}{V} = \dfrac{630}{210} = \mathbf{3\,A}$

52. Write an expression for the heat produced when an electric current is passed through it.

Ans. $H = VIt = I^2Rt = \dfrac{V^2}{R}t$ in energy units (joules).

or $H = \dfrac{VIt}{J} = \dfrac{I^2Rt}{J} = \dfrac{V^2t}{JR}$ in heat units (cal.)

53. What is the meaning of 1 unit electric energy in domestic use ?

Ans. 1 unit of electric energy = 1 kWh. If an electrical appliance of power 1000 watt is operated on mains for 1 hour, it will consume 1 unit of electricity.

54. How many joules of energy is equivalent to 1 k Wh ?

Ans. Energy = 1 kWh = 1000 W × (60 × 60) s

$= \mathbf{3{\cdot}6 \times 10^6\ J}$

55. What is the no. of kWh in 1 joule ?

Ans. As $3{\cdot}6 \times 10^6$ J = 1 kWh,

therefore, $1\text{J} = \dfrac{1}{3{\cdot}6 \times 10^6}\text{kWh} = 2{\cdot}77 \times 10^{-7}$ kWh

56. If the current in the electric bulb changes by 1%, then by what percentage will the power change ?

Ans. $P = I^2 R$ or $\log P = 2 \log I + \log R$;

Differentiating it, we get

$$\frac{\Delta P}{P} = \frac{2\Delta I}{I} + 0 \qquad (\because R \text{ is constant})\ ;$$

% change in power

$$= \frac{\Delta P}{P} \times 100 = \frac{2\Delta I}{I} \times 100 = 2\frac{1}{100} \times 100 = \mathbf{2\%}$$

57. A current in a circuit having constant resistance is tripled. How does this affect the power dissipation ?

Ans. The power dissipated in a circuit is $P = I^2 R$. Thus, $P \propto I^2$, when R is constant.

When I becomes 3 times, P becomes 9 times, *i.e.*, **Power dissipation becomes 9 times.**

58. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more ? Justify your answer. **(CBSE 2017)**

Ans. In series combination current I is same in nichrome and copper wires.

Heat produced, $H = I^2Rt = I^2 \frac{\rho l}{A} t$ or $H \propto \rho$

(as I, l, A, t are constant). Then

$$\frac{H_{Nich}}{H_{Cu}} = \frac{\rho_{Nich}}{\rho_{Cu}} > 1 \quad (\because \rho_{Nich} > \rho_{Cu})$$

So $H_{Nich} > H_{Cu}$

Thus **nichrome wire will produce more heat.**

VII. Aspects of heating effect of current

59. Three bulbs 40 W, 60 W and 100 W are connected in series to 220 V 4mains. Which bulb will glow brightly ? (H.P. Board 2001)

Ans. In series combination, the same current flows through each bulb. As resistance

$R = \frac{V^2}{P}$ or $R \propto \frac{1}{P}$. Therefore, the resistance of 40 W bulb is highest for the given three bulbs. Heat produced $H = I^2 Rt$ or $H \propto R$, so maximum heat is produced in 40 W lamp. As a result of it 40 W lamp will glow more brightly than the other two bulbs.

60. What is the composition of materials used in the fuse wire ?

Ans. 63% tin + 37% lead.

61. State the characteristics of fuse wire.

Ans. A fuse wire should have (*i*) high resistivity (*ii*) low melting point and (*iii*) of suitable current rating corresponding to the load in the circuit.

62. What do you mean by specification of a bulb or other electric appliances ?

Ans. When the specification of a bulb or any electric appliance is given (say 220 V, 100 W), then its resistance R and maximum current allowed can be estimated. In the above example,

the resistance of bulb, $R = V^2/P = (220)^2/100 = 484\ \Omega$

and maximum current, $I = P/V = 100/220 = 5/11$ A

VIII. Kirchhoff's laws and Wheatstone bridge

63. Are Kirchhoff's rules applicable to both a.c. and d.c. ?

Ans. Yes, Kirchhoff's rules are equally applicable to a.c. as well as d.c. circuits.

64. State the fundamental concepts on which two Kirchhoff's rules are based.

Ans. Kirchhoff's first rule is based on the fact that the charges are not accumulated at a junction, *i.e.*, the law of conservation of charge.

Kirchhoff's second rule supports the law of conservation of energy.

65. What is the principle of working of meter-bridge ?

Ans. Wheatstone bridge principle.

66. What happens to the balance point if the position of the cell and the galvanometer are inter-changed in balanced Wheatstone bridge ?

Ans. There will be no deflection in the galvanometer as the condition of balanced bridge will still hold good.

67. At what position of the jockey on slide wire bridge, the results are most accurate ?

Ans. The results are most accurate, when balance point is obtained at the mid point of bridge wire. In this case $R = S$. The end resistances of copper strips produce least effect and the bridge is most sensitive.

68. Why is a meter bridge also called a slide wire bridge ?

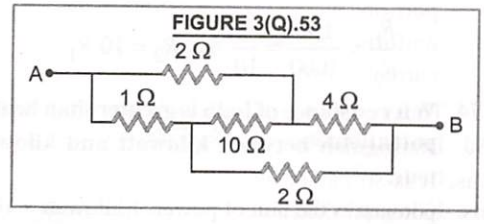
Ans. This is because during the experiment, the jockey is to be slided over the bridge wire.

69. When is Wheatstone Bridge most sensitive ?

Ans. Wheatstone Bridge is most sensitive when the value of resistances of the four arms of the bridge P, Q, R and S are of the same magnitude.

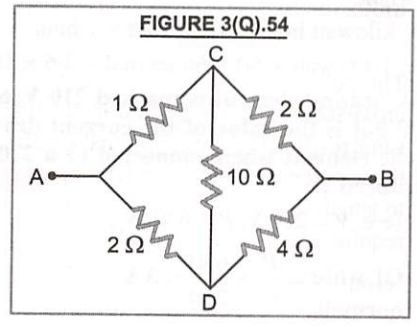
70. Calculate the equivalent resistance between points A and B of the network shown in Fig. 3(Q).53.

FIGURE 3(Q).53



Ans. The equivalent circuit is shown in Fig. 3(Q).54.

FIGURE 3(Q).54



As $\frac{1}{2}=\frac{2}{4}$, so the given circuit is a balanced Wheatstone bridge. The resistance 10 Ω in arm *CD* is ineffective in circuit. We have (1 Ω + 2 Ω) and (2 Ω + 4 Ω) parallel resistances. Thus equivalent resistance between *A* and *B*

$$R_{eq}=\frac{3\times 6}{3+6}=2\Omega$$

71. What do you mean by sentiveness of a Wheatstone bridge ?

Ans. A Wheatstone bridge is said to be sensitive if it shows large deflection in the galvanometer when ever a small change in resistance is taking place in resistance arm.

72. Why is meter bridge method not suitable for measuring high resistances ?

Ans. For measuring high resistance, the resistances of all other arms of the bridge should also be high in order to ensure the sensitivity of the bridge. But this will reduce the current through the galvanometer. As a result of it, the galvanometer becomes insensitive.

IX. Potentiometer

73. State the principle of working of a potentiometer. [CBSE 2009, 2014 (C)]

Ans. Fall of potential across any portion of the wire is directly proportional to the length of that portion of the wire provided the wire is of uniform area of cross-section and a constant current is flowing through it.

74. Why is a potentiometer named as potentiometer ?

Ans. It is so named because it is used to measure potential difference.

75. Why should the potentiometer wire be of uniform cross-section and composition ? [CBSE 2014 (C)]

Ans. The potentiometer wire should be of uniform cross-section and composition so that there is same resistance per unit length throughout. Only then, potential difference will be proportional to length of the potentiometer wire, which is the requirement for the principle of potentiometer.

76. Of which material is a potentiometer wire normally made and why ? [CBSE 2011 (C)]

Ans. Normally, the potentiometer wire is made of an alloy such as nichrome or manganin. Because those alloys have high resistivity and low temperature coefficient of resistance.

77. The emf of the driving cell used in the main circuit of the potentiometer should be more than the potential difference to be measured. Why ? [CBSE 2014 (C)]

Ans. If it is not so, then the balance point will not be obtained on the potentiometer wire.

78. Some times balance point may not be obtained on the potentiometer wire. Why ?

Ans. It is possible only if the fall of potential across the potentiometer wire is less than the potential difference to be balanced by the potentiometer wire.

79. Whether electric field inside potentiometer wire is constant or variable ?

Ans. Electric field intensity $E = V/l$ = a constant.

80. Why do we prefer a potentiometer to measure emf of a cell rather than a voltmeter ? [CBSE 2016 (C)]

Ans. At null point, a potentiometer does not draw any current from the cell whose emf is to be determined, whereas a voltmeter always draws some little current. Therefore, emf measured by voltmeter is slightly less than actual value of emf of the cell.

81. How can you make a potentiometer of given wire length more sensitive using a resistance box ?

Ans. This can be achieved by using more resistance from the resistance box in the potentiometer wire circuit. Which in turn will decrease the current in the potentiometer wire and consequently, the potential gradient of wire will decrease. As a result of it, the sensitivity of the potentiometer will increase.

82. Why do we prefer a potentiometer with a longer bridge wire ? [CBSE 2014 (C)]

Ans. A potentiometer with longer bridge wire will have a small potential gradient. Due to it, it becomes more sensitive. Hence it is preferred.

83. What should be the properties of the material for the selection of potentiometer wire ?

Ans. The material must have high resistivity and low temperature coefficient of resistance.

84. A resistance R is connected across a cell of emf ε and internal resistance r. A potentiometer now measures the potential difference between the terminals of the cell is V. Write the expression for r in terms of ε, V and R. (CBSE 2011)

Ans. $\varepsilon = I(R + r)$ and $V = IR$

$$\therefore \quad \frac{\varepsilon}{V} = \frac{R + r}{R} = 1 + \frac{r}{R}$$

$$\text{or} \quad \frac{r}{R} = \left(\frac{\varepsilon}{V} - 1\right) \quad \text{or} \quad r = \left(\frac{\varepsilon}{V} - 1\right) R$$

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Current and drift velocity

1. How is the current conducted in metals ? Explain.

Ans. Every metal conductor has large number of free electrons which move at random at room temperature. Their average thermal velocity at any instant is zero. When a pot. diff. is applied across the ends of the conductor, an electric field is set up in the conductor. Due to it, the free electrons of the conductor experience force due to electric field and drift towards the positive end of the conductor, causing the electric current (*i.e.* conduction current) in the conductor. The direction of conventional current is opposite to the direction of motion of the free electrons in the conductor.

2. A conductor of length L is connected to a dc source of emf V. If this conductor is replaced by another conductor of same material and same area of cross-section but of length 3 L, how will the drift velocity change ? (CBSE 2011)

Ans. Drift velocity,

$$v_d = \frac{I}{neA} = \frac{V/R}{neA} = \frac{V/(\rho L/A)}{neA} \quad (\because R = \rho L/A)$$

$$\text{or} \quad v_d = \frac{V}{n\rho eL} \quad \text{or} \quad v_d \propto \frac{1}{L}$$

$$\therefore \quad \frac{v_d'}{v_d} = \frac{L}{3L} = \frac{1}{3} \quad \text{or} \quad v_d' = \frac{v_d}{3}$$

3. If the current flowing in a copper wire be allowed to flow in another copper wire of same length but of double the radius, then what will be the effect on the drift velocity of the electrons. If the same current be allowed to flow in an iron wire of the same thickness, then ?

Ans. We know that if two wires are in series, there will be same current in each wire. The current I through the wire is related with drift velocity v_d by the relation

$$I = nAev_d \quad \text{or} \quad Av_d = \text{a constant}$$

$$\text{or} \quad v_d \propto \frac{1}{A} \quad \text{or} \quad v_d \propto \frac{1}{\pi r^2}$$

Since in the second wire, the radius of wire becomes 2 times hence the **drift velocity will become one fourth.** In case of iron wire, the number density of free electrons will be less in comparison to copper wire and hence the **drift velocity will be more in comparison to that in copper wire.**

4. If the resistance of our body is so large ($\approx$ 10 kΩ) why does one experience a strong shock when one accidently touches the line wire, say a 240 volt supply ?

Ans. When a person accidently touches the line wire of a 240 V supply, a current I = 240/10000 = 0·024 A = 24 mA, flows through the body of the person. This current interferes with the nerve process related to our heart beating, which is basically electrical in nature, this in turn accounts for a strong shock.

5. There is an impression among many people that a person touching a high power line gets stuck with the line. Is that true ? Explain.

Ans. This impression is misleading. Infact there is no special attractive force that keeps a person stuck with a high power line while touching that wire, whereas a current of few milliampere is enough to disorganize our nervous system. As a result of it, the affected person loses temporarily his ability to exercise his nervous control to get himself free from the high power line.

6. Currents of the order of 0·1 A through the human body are fatal. What causes the death : heating of the body due to electric current or something else ?

Ans. The cause of death is not heating due to current passing through a person. Though he may receive burns if the currents are too large. The cause of death is the interference of the external

currents with our highly sensitive nervous system which is basically electrical in nature, which inturn affect the heart beating. Beyond a certain point, this interference becomes fatal.

II. Resistance and effect of temperature

7. While making a standard resistance, the coil is made of manganin. The coil is double folded and is wound over non-conducting frame. Why ?

Ans. For manganin, the temperature coefficient of resistance is very low. Due to it, the resistance of manganin wire remains almost unchanged with change in temperature. The resistivity of manganin is high. Therefore, for making a standard resistance of the given value, the smaller length of wire is needed. It is due to these facts, the wire of manganin is used for making standard resistance coil. The coil is double folded on itself to avoid the inductive effect and it is wound over the non-conducting frame in order to avoid the conductive effect and the leakage of current.

8. The V–I graph for a conductor makes angle θ with V-axis. Here V denotes voltage and I denotes current. What is the resistance of this conductor ?

Ans. *V–I* graph for a conductor is a st. line, inclined to voltage axis, according to Ohm's law. If θ is the angle which *V–I* graph makes with *V*-axis, then slope of the graph,

$$\tan\theta = \frac{I}{V} = \frac{1}{R} \quad \text{or} \quad R = \frac{1}{\tan\theta} = \cot\theta\,.$$

9. Define the units of conductance and conductivity. Give their dimensional formulae.

Ans. The units of conductance and conductivity are simen (denoted by *S*) and simen metre^{-1} (denoted by S m^{-1}) respectively. We know that conductance = 1/resistance.

∴ 1 simen is the conductance of a conductor of resistance 1 ohm.

We know, electrical conductivity, $\sigma = \frac{1}{\rho} = \frac{l}{RA}$

If l = 1 m, R = 1 Ω, A = 1 m^2 then σ = 1 S m^{-1}

Thus 1 S m^{-1} is the electrical conductivity of a conductor of length 1 m, area of cross-section 1 m^2 and resistance 1 Ω or conductance 1*S*.

Dimensional formula of conductance,

$$G = \frac{1}{R} = \frac{I}{V} = \frac{[A]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^3A^2]$$

Dimensional formula of conductivity,

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = [M^{-1}L^{-2}T^3A^2] \times \frac{L}{L^2}$$

$$= [M^{-1}L^{-3}T^3A^2]$$

10. Three materials *A*, *B* and *C* have electrical conductivities σ, 2 σ and 2 σ respectively. Their number densities of free electrons are 2 *n*, *n* and 2 *n* respectively. For which material is a average collision time of free electrons maximum ?

Ans. Conductivity, $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

Relaxation time, $\tau = \frac{m\sigma}{ne^2}$, *i.e.*, $\tau \propto \frac{\sigma}{n}$

$$\therefore\ \tau_A : \tau_B : \tau_C = \frac{\sigma}{2n} : \frac{2\sigma}{n} : \frac{2\sigma}{2n} = \frac{\sigma}{2n} : \frac{2\sigma}{n} : \frac{\sigma}{n}$$

Thus, $\tau_B > \tau_C > \tau_A$. So **average collision time for material *B* is maximum.**

11. Explain, with the help of a graph, the variation of conductivity with temperature for a metallic conductor ? **(CBSE 2004)**

Ans. Conductivity of a metallic conductor is

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

For a metal, the number density of electrons n is fixed. When temperature of metal increases, the amplitude of vibrations of the atoms or ions of the metal conductor increases. As a result of it, the collision of electrons with atoms/ions become more frequent, consequently, the relaxation time τ decreases. It means, conductivity of a metallic conductor decreases with the increase of temperature, as shown in Fig. 3(Q).55.

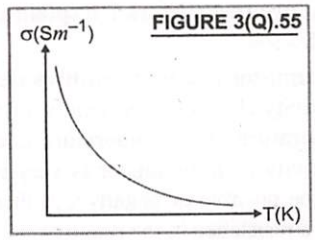


FIGURE 3(Q).55

Read Your Flow Find Yourself in Free

12. Explain how electron mobility changes for a good conductor, when (i) the temperature of the conductor is decreased at constant potential difference and (ii) applied potential difference is doubled at constant temperature. (CBSE 2019)

Ans. Electron mobility in a conductor is given by

$$\mu = \frac{v_d}{E} = \frac{\frac{eE}{m}\tau}{E} = \frac{e\tau}{m}.$$

(*i*) When temperature of the conductor decreases, the relaxation time τ of the electrons in the conductor increases, so mobility μ increases.

(*ii*) Mobility μ is independent of applied potential difference.

13. A wire is drawn into double its length and half its original cross-section. What will be increase in its (*i*) resistance and (*ii*) resistivity ? (HP Board 2013)

Ans. $R_1 = \rho\,(2\,l)/(A/2) = 4\,\rho\,l/A = 4\,R$; so increase in resistance $= R_1 - R = 4\,R - R = 3\,R$. Resistivity will not change as it is independent of the dimensions of the wire but depends upon the nature of the wire and also on the temperature of the wire.

14. Two students *A* and *B* were asked to pick a resistor of 15 kΩ from a collection of carbon resistors. *A* picked a resistor with bands of colours : brown, green, orange while *B* choose a resistor with bands black, green, red. Who picked the correct resistor ? Explain. [CBSE 2013 (C)]

Ans. *For student A,* the numbers attached to brown, green and orange are 1, 5 and 3. So the value of resistor is, $R_A = 15 \times 10^3\ \Omega = 15\ \text{k}\Omega$.

For student B, the numbers attached to black, green and red are 0, 5 and 2 respectively. So the value of resistor is, $R_B = 05 \times 10^2\ \Omega = 500\ \Omega$.

Thus, **student *A* picked up the correct resistor of 15 kΩ.**

15. What are thermistors ? Explain their use in brief.

Ans. A thermistor is a heat sensitive device whose resistivity changes very rapidly with change of temperature. The temperature coefficient of resistivity of a thermistor is very high, which may be positive or negative. A thermistor can have a resistance in the range of 0·1 Ω to 10^7 Ω

Thermistors are used (*i*) to detect small temperature changes (*ii*) to safe guard the filament of picture tube of a television set against the variation of current (*iii*) in temperature control units of industry (*iv*) for voltage stabilisation and remote sensing.

16. Thermistors differ from ordinary resistors. Explain.

Ans. A thermistor differs from an ordinary resistance in the following ways.

(*i*) The resistivity and hence the resistance of thermistor changes very rapidly with change of temperature.

(*ii*) The temperature coefficient of resistivity of thermistor is very high.

(*iii*) The temperature coefficient of resistivity of thermistor can be both, positive and negative.

17. What are super-conductors ? Write their two applications.

Ans. As the temperature of certain metals and alloys decreases, their resistance also decreases. When the temperature reaches a certain critical value called critical temperature, the resistance of material completely disappears, *i.e.*, it becomes zero. Then the material behaves as a super-conductor.

Thus *super-conductors are those material conductors whose resistances disappear at critical temperature.* The critical temperature is different for different materials.

Super conductors are used (*i*) in power transmission (*ii*) to produce very high speed computers.

18. A carbon filament has a resistance of 100 Ω at 0°C. What must be the resistance of copper filament placed in series with carbon so that the combination has the same resistance at all temperatures ? Temperature coefficient of resistance of carbon = – 0·0007 °C^{-1} and that of copper is 0·004 °C^{-1}.

Ans. If R_0, R_t be the resistance of a wire at 0°C and t °C then temperature coefficient of resistance is

$$\alpha = \frac{R_t - R_0}{R_0\,t}$$

or change in resistance $= R_t - R_0 = R_0\,\alpha t$

Let R be the resistance of copper placed in series with carbon at 0°C. If the combination has same resistance at all temperatures, then at temperature t °C,

increase in resistance of copper = decrease in resistance of carbon

$\therefore \quad \alpha_{Cu} R t = \alpha_c R_0 t$

or $0{\cdot}004 \times R \times t = {\cdot}0007 \times 100 \times t$

or $R = \dfrac{0{\cdot}0007 \times 100}{0{\cdot}004} = 17{\cdot}5\ \Omega$

19. A uniform wire is cut into four segments. Each segment is twice as long as the earlier segment. If the shortest segment has a resistance of 2 Ω, find the resistance of original wire.

Ans. Let l be the length of shortest segment. Then lengths of four segments will be l, $2\,l$, $4\,l$, $8\,l$.

Their corresponding resistances will be R, $2\,R$, $4\,R$ and $8\,R$. Given $R = 2\ \Omega$.

Resistance of the original wire

$= R + 2R + 4R + 8R$

$= 15\,R = 15 \times 2 = 30\ \Omega$

20. Why is it unsafe to turn a light switch on or off while taking bath ?

Ans. While taking a bath if we play with a switch of light, the water comes in between current wire and our body. Due to it, the current flows through water to our body, resulting in a shock.

21. Lights of a car become dim when the starter is operated. Why ?

Ans. When the motor starter of a car is operated, it draws more current from the battery for the operation of car. Therefore, the voltage across the light bulb is lowered, hence the light of a car is dimmed.

III. Grouping of resistance

22. To reduce the brightness of a light bulb, should an auxiliary resistance be connected in series with it or in parallel ?

Ans. To reduce the brightness of a light bulb, we should decrease the current flowing through the bulb, which is possible when an auxiliary resistance is connected in series with the bulb.

23. An electric current is passed through a circuit containing two wires of the same material connected in paralle. If the length and radii of the wires are in the ratio 2/3 and 4/3, then find the ratio of the currents passing through the wires.

Ans. $\dfrac{R_1}{R_2} = \dfrac{\rho l_1/\pi r_1^2}{\rho l_2/\pi r_2^2} = \dfrac{l_1}{l_2} \times \dfrac{r_2^2}{r_1^2} = \dfrac{2}{3} \times \left(\dfrac{3}{4}\right)^2 = \dfrac{3}{8}$

For wires connected in parallel

$V_1 = V_2$ or $I_1 R_1 = I_2 R_2$ or $\dfrac{I_1}{I_2} = \dfrac{R_2}{R_1} = \dfrac{8}{3}$

24. At 0°C, the resistance of a conductor B is n times that of conductor A. The temperature coefficients of resistance of A and B are α_1 and α_2 respectively. For the series combination of the two conductors find (a) the resistance at 0°C (b) the temperature coefficient of resistance.

Ans. Let R_0 be the resistance of the conductor at 0°C. Then resistance of conductor B at 0°C = $n\,R_0$.

Resistance of conductor A at θ°C,

$R_1 = R_0 (1 + \alpha_1 \theta)$

Resistance of conductor B at θ°C,

$R_2 = n R_0 (1 + \alpha_2 \theta)$

Thus the total resistance of the series combination at θ°C is

$R_s = R_1 + R_2 = R_0 (1 + \alpha_1 \theta) + n R_0 (1 + \alpha_2 \theta)$

$= R_0 [(1 + n) + (\alpha_1 + n\alpha_2)\theta]$

$= (1+n) R_0 \left[1 + \dfrac{\alpha_1 + n\alpha_2}{1+n} \theta\right]$ ...(*i*)

Comparing this relation with

$R_s = R_{s0} [1 + \alpha_s \theta]$

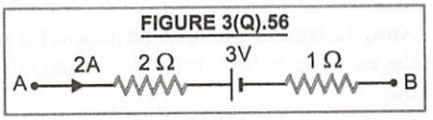
We have, resistance of series combination at 0°C

$R_{s0} = (1 + n) R_0$

Temperature coefficient of resistance of the series combination is $\alpha_s = \dfrac{\alpha_1 + n\alpha_2}{1+n}$

25. Fig. 3(Q).56 represents a part of closed circuit. What is the potential difference between points A and B ?

FIGURE 3(Q).56



Ans. Potential difference across 2 Ω = 2 × 2 = 4 V ; Potential difference across 1 Ω = 1 × 2 = 2 V. Given the e.m.f. of the cell is 3 V. Let the positive electrode of cell be + 3 V and –ve electrode of cell at 0 V. Therefore potential of $A = 4 + 3 = 7$ V :

Potential of $B = -2$ V, as current is flowing from negative terminal of cell to point B, hence B is at lower potential. Therefore potential difference between A and $B = 7 - (-2) = \mathbf{9\ V}$.

Alternative method

According to Kirchhoff's second rule ;

$(V_A - V_B) - 3 = 2 \times 2 + 2 \times 1 = 6$

or $V_A - V_B = 6 + 3 = \mathbf{9\ V}$

IV. Internal resistance and Grouping of cells

26. What is terminal potential difference of a cell ? Can its value be greater than the emf of a cell ? Explain. (CBSE 2013)

Ans. Terminal potential difference of a cell is defined as the potential difference between the two electrodes of a cell in a closed circuit. The value of terminal potential difference of a cell is less than the emf of a cell, when current is drawn from the cell (*i.e.* during discharging of cell). The value of terminal potential difference of a cell becomes greater than the emf of the cell during charging of the cell, *i.e.*, when the positive electrode of the cell is connected to positive terminal of battery charger and negative electrode of the cell is connected to negative terminal of battery charger.

27. A car has a fresh storage battery of emf 12 V and internal resistance $5{\cdot}0 \times 10^{-2}\ \Omega$. If the starter motor draws a current of 90 A, what is the terminal voltage of the battery when the starter is on ?

Ans. $V = \varepsilon - Ir = 12 - 90 \times (5{\cdot}0 \times 10^{-2}) = \mathbf{7{\cdot}5\ V}$

28. After long use of car, as given in question 27, the internal resistance of the storage battery increases to 500 Ω. What maximum current can be drawn from the battery ? Assume the emf of the battery to remain unchanged.

Ans. The maximum current can be drawn from a battery by short circuiting it. Then $V = 0$ and

$I_{max} = \varepsilon/r = 12/500 = 2{\cdot}4 \times 10^{-2}$ A $= 24$ m A

Now, the battery will become useless for starting the car. The battery must be recharged before use.

29. In Ques. 28, if the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater or smaller than its emf 12 V ?

Ans. During charging of the battery, the current flows from the positive to negative terminal inside the cells of battery. Then terminal potential difference is given by $V = \varepsilon + I r$; Hence V must be greater than 12 V, during charging.

30. Three identical cells each of emf 2 V and unknown internal resistance are connected in parallel. This combination is connected to a 5 ohm resistor. If the terminal voltage across the cells is 1·5 volt, what is the internal resistance of each cell ?

Ans. Effective emf of three cells in parallel. $\varepsilon = 2$ V ; Effective internal resistance of three cells in parallel, (when internal resistance of each cell is r) $\approx r/3$; Total resistance of circuit

$= \frac{r}{3} + R$; Current, $I = \frac{\varepsilon}{(r/3) + R}$

Terminal voltage, $V = IR$ or $1{\cdot}5 = \frac{2 \times 5}{(r/3) + 5}$

or $r = \mathbf{5\ \Omega}$

31. A battery of emf ε and internal resistance r sends currents I_1 and I_2, when connected to external resistances R_1 and R_2 respectively. Find the emf and internal resistance of the battery.

Ans. *In first case,* current is

$$I_1 = \frac{\varepsilon}{R_1 + r} \quad \text{or} \quad \varepsilon = I_1 (R_1 + r) \qquad ...(i)$$

In second case, current, $I_2 = \frac{\varepsilon}{R_2 + r}$

or $\varepsilon = I_2 (R_2 + r)$ ...(*ii*)

From (*i*) and (*ii*), we have

$I_1 (R_1 + r) = I_2 (R_2 + r)$

or $r(I_1 - I_2) = I_2 R_2 - I_1 R_1$ or $r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}$

Putting this value of r in equation (*i*), we get

$$\varepsilon = I_1 \left[R_1 + \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2} \right]$$

$$= I_1 \left[\frac{I_1 R_1 - I_2 R_1 + I_2 R_2 - I_1 R_1}{I_1 - I_2} \right]$$

$$= \frac{I_1 I_2 (R_2 - R_1)}{I_1 - I_2}$$

32. Under what conditions will the strength of current in a wire will be the same for connections in series and in parallel of n identical cells ?

Ans. Let ε, r be the emf and internal resistance of each cell. When n cells be connected in series, then the current in the circuit having external resistance R is

$$I_s = \frac{n\varepsilon}{R + nr}$$

When n cells are connected in parallel, then the current in the circuit is

$$I_p = \frac{\varepsilon}{R + r/n} = \frac{n\varepsilon}{nR + r}$$

As, $I_s = I_p$; so, $\frac{n\varepsilon}{R + nr} = \frac{n\varepsilon}{nR + r}$

or $R + nr = nR + r$

or $R(1 - n) = r(1 - n)$ or $\boldsymbol{R = r}$

33. A cell of emf ε and internal resistance r is connected across a variable resistor R. Plot a graph showing variation of terminal voltage V of the cell versus the current I. Using the plot, show how the emf of the cell and its internal resistance can be determined.

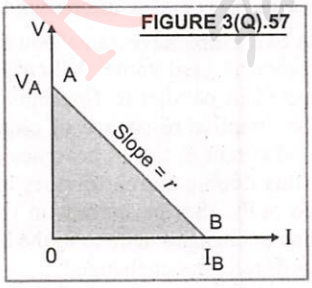
(CBSE 2014)

Ans. Terminal pot. diff. is, $V = \varepsilon - Ir$ or $V = -rI + \varepsilon$. This equation is similar to a straight line equation $y = mx + c$. Hence the graph between V and I is a straight line AB with a negative slope $(= -r)$ as shown in Fig. 3(Q).57.

For point A, $I = 0$, and $V = V_A$, so

$V_A = \varepsilon =$ **intercept on Y-axis.**

FIGURE 3(Q).57



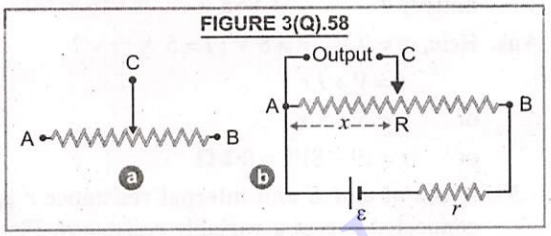
For point B, $V = 0$, $I = I_B$

$$0 = \varepsilon - I_B r \quad \text{or} \quad r = \frac{\varepsilon}{I_B} = \frac{V_A}{I_B}$$

= negative of the slope of V–I graph

34. A schematic rheostat is shown in Fig. 3(Q).58(a). Connect a battery to it so that it acts as a potential divider. Also show the output terminals.

FIGURE 3(Q).58



Ans. The circuit for rheostat used as potential divider is shown in Fig. 3(Q).58(b). The out put voltage is obtained across A and C. In this case, the

output voltage is $V = \left(\frac{\varepsilon}{R + r}\right)\frac{R}{l}x$

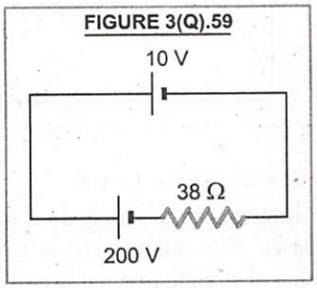
where l is the length AB of resistance R and x is the length of resistance tapped for output. ε is the emf of battery used and r is the internal resistance of the cell.

35. For what basic purpose the cells are connected (i) in series (ii) in parallel and (iii) in mixed grouping ?

Ans. The cells are connected (i) in series, to get maximum voltage, (ii) in parallel, to get maximum current and (iii) in mixed grouping, to get maximum power.

36. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance 38 Ω as shown in Fig. 3(Q).59. Find the value of current in the circuit. **(CBSE 2018)**

FIGURE 3(Q).59



Ans. Effective voltage of the circuit,

$V = 200 - 10 = 190$ V ; $R = 38$ Ω.

Current, $I = \frac{V}{R} = \frac{190}{38} = \mathbf{5\ A}$

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37. A 9 V battery is connected in series with a resistor. The terminal voltage is found to be 8 V. Current through the circuit is measured as 5 A. What is the internal resistance of the battery ? **(CBSE Sample Paper 2018)**

Ans. Here, $\varepsilon = 9$ V ; $V = 8$ V ; $I = 5$ A ; $r = ?$

$\varepsilon = V + Ir$

or $9 = 8 + 5 \times r$

or $r = (9 - 8)/5 = \mathbf{0{\cdot}2\ \Omega}$

38. A cell of emf E and internal resistance r is connected across a variable resistor R. Plot the shape of graphs showing variation of terminal voltage V with (*i*) R and (*ii*) circuit current I. **(CBSE 2020)**

Ans. When a cell of EMF E internal resistance r is connected to external resistor R, then the current in the circuit, $I = \dfrac{E}{R+r}$

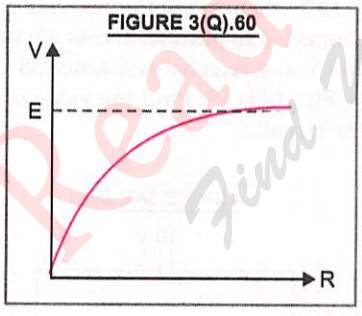
Terminal voltage of cell, $V = IR$

or $V = \dfrac{ER}{R+r} = \dfrac{E}{(1 + r/R)}$ ...(*i*)

From above relation we note that as R increases, V also increases.

When $R = \infty$, $V = E$. Thus the variation between V and R will be a curve as shown in Fig. 3(Q).60.

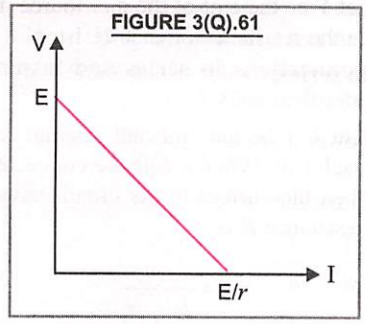
FIGURE 3(Q).60



As, $V = E - Ir = -Ir + E$ ...(*ii*)

This relation shows that as I increases, V decreases. The relation (*ii*) is similar to the equation of straight line, $y = mx + C$. Therefore, the graph between V and I is a straight line, Fig. 3(Q).61, whose slope ($= -r$) tells the internal resistance of the cell and intercept is E (*i.e.*, emf of the cell)

FIGURE 3(Q).61



V. Joule's law of heating

39. The temperature of the filament of an electric bulb is 2700°C when it glows. It is not burnt up at such a high temperature. Why ?

Ans. It is so because the filament of the electric bulb has high melting point and it remains safe in an atmosphere of inert gases which protect its oxidation.

40. Why an electric bulb becomes dim when an electric heater in parallel circuit is switched on ? Why dimness decreases after some time ?

Ans. The electric heater have more power and less resistance than electric bulb. So electric heater when put in parallel circuit of glowing electric bulb, it draws higher current. Some current from the bulb is diverted into heater. Due to it, bulb becomes dim. After sometime, heater element becomes hot, its resistance increases. Due to it, it takes less current. Now the current through the bulb increases and its dimness decreases.

41. The two electric bulbs of same power are connected in parallel circuit. If one bulb is glowing and then another bulb is switched on, then brightness of the first bulb increases, decreases or remains unchanged. Explain.

Ans. When two bulbs have same power, then their resistance is also same. When second bulb connected in parallel to first bulb is switched on, the effective resistance of circuit becomes half and curent in circuit becomes double. Due to it, this double current divides itself equally in two bulbs. So the current in the first bulb remains unchanged. Due to it, the brightness of this bulb remains unchanged.

42. Three identical resistors, each of resistance R, when connected in series with a d.c. source, dissipate power X. If the resistors are connected in parallel to the same d.c. source, how much power will be dissipated ?

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Ans. Let V be the emf of the d.c. source in volt. $R\ \Omega$ be the resistance of each resistor.

In series, total resistance $= R + R + R = 3R$

$$\text{Power dissipated} = \frac{V^2}{3R} = X \quad \text{or} \quad \frac{V^2}{R} = 3X$$

In parallel, total resistance, R' is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R} \quad \text{or} \quad R' = R/3$$

$$\text{Power dissipated} = \frac{V^2}{R'} = \frac{V^2}{R/3} = \frac{3V^2}{R}$$

$$= 3 \times 3X = 9\,\mathbf{X}$$

43. The rate of Joule heat is given by $P = V \times I$ where V is the potential difference across the ends of a conductor and I is the current flowing through it. Does this relation hold for a conductor that does not obey Ohm's law ?

Ans. Yes, because rate of Joule heat is equal to work done in maintaining the current through a conductor in a unit time [*i.e.* $P = VI$] which is independent, whether $V \propto I$ or not. It means the rate of Joule heat will also be applicable to the devices which may not obey Ohm's law.

44. Current is allowed to flow in a metallic wire at a constant potential difference. When the wire becomes hot, cold water is poured on half portion of the wire. By doing so, its other portion becomes still more hot. Explain its reason.

Ans. When cold water is poured on half portion of hot wire, it gets cooled there. Its temperature falls and hence its resistance decreases, due to it, the current in the wire increases. As heat produced, $H \propto I^2$, therefore, the other portion of the wire becomes more hot.

45. An electric kettle has two coils. When one of these is switched on, the water in the kettle boils in 6 minutes. When the other coil is switched on, the water boils in 3 minutes. If the two coils are connected in series, find the time taken to boil the water in the kettle.

Ans. Heat produced, $H = V^2t/JR$ or $R \propto t$ so $R_1 \propto 6$ and $R_2 \propto 3$, therefore, $R_1/R_2 = 6/3 = 2$

or $R_1 = 2R_2$

When coils are connected in series, then net resistance,

$$R' = R_1 + R_2 = 3R_2 \quad \text{so} \quad \frac{R'}{R_2} = \frac{t'}{3}$$

$$\text{or} \quad t' = \frac{3R'}{R_2} = \frac{3 \times 3R_2}{R_2} = \mathbf{9\ minutes}$$

46. In question no. 44, if the two coils are connected in parallel, find the time taken to boil the water in the kettle.

Ans. When the coils are connected in parallel, the effective resistance is,

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{2R_2 \times R_2}{2R_2 + R_2} = \frac{2}{3}R_2$$

$$\therefore \quad \frac{(2/3)R_2}{R_2} = \frac{t'}{3} \quad \text{or} \quad t' = \mathbf{2\ minutes}$$

VI. Electric power and Electric energy

47. Two bulbs are rated (P_1, V) and (P_2, V). If they are connected (*i*) in series and (*ii*) in parallel across a supply V. Find the power dissipated in the two combinations in terms of P_1 and P_2.

(CBSE 2019)

Ans. Let R_1, R_2 be the resistances of the two bulbs of power P_1 and P_2 respectively. Then,

$$R_1 = \frac{V^2}{P_1} \quad \text{and} \quad R_2 = \frac{V^2}{P_2}$$

(*i*) **When bulbs are connected in series,** then total resistance is R_s and total power is P, given by

$$R_s = R_1 + R_2 \quad \text{or} \quad \frac{V^2}{P} = \frac{V^2}{R_1} + \frac{V^2}{R_2} \quad \text{or} \quad \frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

(*ii*) **When bulbs are connected in parallel,** then total resistance is R_p and total power P, is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad \frac{V^2}{R_p} = \frac{V^2}{R_1} + \frac{V^2}{R_2} \quad \text{or} \quad \mathbf{P = P_1 + P_2}$$

48. A boy has two wires of iron and copper of equal length and diameter. He first joins the two wires in series and passes electric current through this combination which increases gradually with time. After that he joins them in parallel and repeat the process of passing the current in this arrangement also. Which wire will glow first in each case and why ?

Ans. We know that resistivity of iron is more than that of copper.

In series combination, the same current I flows through iron and copper wires.

$$\text{Heat produced, } H = I^2Rt = I^2\frac{\rho l}{A}t \quad \textit{i.e.,}\ H \propto \rho$$

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As $\rho_{iron} > \rho_{cu}$ so $H_{iron} > H_{cu}$. Therefore, **iron will start glowing first in series combination.**

In parallel combination, the voltage V is same across iron and copper wires. So

Heat produced, $H = \frac{V^2}{R}t = \frac{V^2 t}{\rho l/A}$ *i.e.*, $H \propto \frac{1}{\rho}$

As $\rho_{iron} > \rho_{cu}$ so $H_{iron} < H_{cu}$

Therefore, **copper will start glowing first in parallel combination of wires.**

49. In an electric kettle, water boils in 10 minutes after the kettle is switched on. With the same supply voltage if the water is to be boiled in 8 minutes, should the length of the heating element be increased or decreased ? Explain.

Ans. For a given supply voltage, heat produced in time t is

$$H = \frac{V^2}{R}t = \frac{V^2}{\rho l/A}t = \frac{V^2 At}{\rho l} \text{ i.e., } H \propto \frac{t}{l}$$

$$\therefore \frac{H_1}{H_2} = \frac{t_1}{t_2} \times \frac{l_2}{l_1} \text{ As } H_1 = H_2, \text{ so } 1 = \frac{t_1}{t_2} \times \frac{l_2}{l_1}$$

$$\text{or } \frac{l_2}{l_1} = \frac{t_2}{t_1} = \frac{8}{10} < 1 \text{ or } l_2 < l_1$$

It means the **length of the heating wire should be decreased.**

50. Two bulbs of resistances 40 Ω and 400 Ω are in series in a circuit fed with supply current. Which one will glow more ? If one of these bulbs be switched off, will the light in the room increase or decrease ?

Ans. Since the bulbs are connected in series, each will carry the same current. We know that light of a bulb is directly proportional to the heat produced by it and the heat produced, $H = I^2 Rt$, *i.e.*, $H \propto R$. Therefore 400 Ω bulb will glow more. When one of these bulbs is switched off, the effective resistance of the circuit decreases and hence the current in the circuit increases. Since $H \propto I^2$, therefore, the light in the room will increase.

51. Two electric bulbs of 50W and 100W are given. When they are (*i*) connected in series (*ii*) connected in parallel, which bulb will glow more ?

Ans. Resistance of a bulb $R = V^2/P$ or $R \propto 1/P$. Thus the resistance of 50W bulb is more than that of 100W bulb. As glow ∝ power of a bulb.

(*i*) *In series connection*, the current is same, so power $= I^2R$, *i.e.*, $P \propto R$.

∴ 50W bulb will glow more than 100W bulb.

(*ii*) *In parallel connection*, power $(P) = V^2/R$, *i.e.*, $P \propto 1/R$. Thus **100W bulb will glow more than that of 50W bulb.**

52. Two electric bulbs *P* and *Q* have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipated in these bulbs.

(CBSE 2018)

Ans. Here, $R_P/R_Q = 1/2$; As the bulbs are connected in series, current (I) in them is same. Ratio of power losses will be

$$\therefore \frac{P_P}{P_Q} = \frac{I^2 R_P}{I^2 R_Q} = \frac{R_P}{R_Q} = \frac{1}{2}$$

VII. Aspects of heating effect of current

53. What is the difference between heating wire and fuse wire ?

Ans. The heating wire must be of high resistivity and of high melting point. Fuse wire must be of high resistivity and low melting point. Heating wire is generally made of nichrome. The fuse wire is made from tin-lead alloy (63% tin + 37% lead).

54. A house hold circuit has a fuse of 5 A rating. Calculate the maximum number of bulbs of rating 60 W – 220 V each which can be connected in this house hold circuit.

(CBSE Sample Paper 2003)

Ans. Current drawn by one bulb,

$$I = \frac{P}{V} = \frac{60}{220} = \frac{3}{11} \text{A}$$

No. of bulbs which can be safely used with 5 A fuse $= \frac{5}{3/11} = \frac{55}{3} = 18{\cdot}33$

Thus, **18 bulbs can be used safely with 5 A fuse.**

VIII. Kirchhoff's laws and Wheatstone bridge

55. Kirchhoff's first rule obeys law of conservation of charge. Explain.

Ans. According to Kirchhoff's first rule, the current (I_1) entering a given junction of the circuit in a certain time (t) = current (I_2) leaving the junction during the same time (t). So ; $I_1 t = I_2 t$ or $q_1 = q_2$, *i.e.*, charge entering the junction is equal to the charge leaving that junction.

56. What is the end error in meter bridge ? How do you remove it ? [CBSE 2017 (C)]

Ans. The end error in the meter bridge is due to the following reasons ;

(*i*) The zero mark of the scale provided along the bridge wire may not start from the position where the bridge wire leaves the copper strip and 100 cm mark of the scale may not be at position, where the bridge wire just touches the other copper strip.

(*ii*) The resistances of the connecting wires and copper strips of meter bridge have not been taken into account.

The end error can be removed by repeating the experiment by interchanging the known and unknown resistances and taking the mean of the resistances determined.

57. Why are the connecting resistors in a metre bridge made of thick copper strips ? (CBSE 2013)

Ans. The thick copper strips are used for connecting resistors as they offer minimum resistance and hence avoid error due to end resistance. The same (*i.e.*, error due to end resistance) has not been taken into account in the bridge formula.

58. Why is the meter bridge method considered unsuitable for the measurement of very low resistances ?

Ans. While measuring a very low resistance using meter bridge, all other resistances used should have low values in order to ensure sensitivity of the bridge. This requires the galvanometer should also be of very low resistance and a very sensitive. Then the end resistances and resistances of the connecting wires used in metre bridge become comparable with the resistance to be measured. This would introduce error in the result. Due to it, the measurement will not be accurate.

59. What are the advantages of a Wheatstone bridge method of measuring resistance over other methods ?

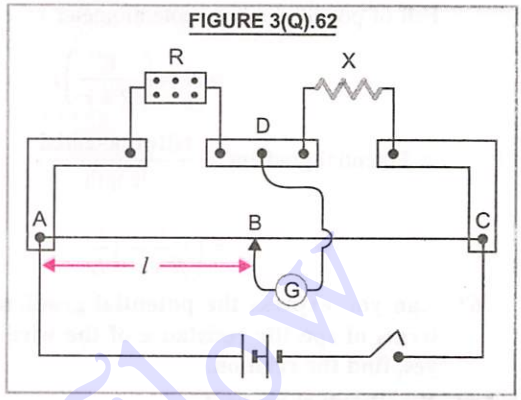
Ans. Following are the advantages of Wheatstone bridge method over other methods :

(*i*) *It is a null method.* Therefore, the result is free from the effect of cell resistance of the circuit.

(*ii*) In null method, it is easier to detect a small change in deflection in galvanometer than to read a deflection in galvanometer correctly.

60. Following circuit was set up in a meter bridge experiment to determine the value of X of an unknown resistance.

FIGURE 3(Q).62



(*a*) Write the formula to be used for finding X from the observations.

(*b*) If the resistance R is increased, what will happen to balancing length ?

(CBSE Sample Paper 2018)

Ans. (*a*) $\frac{R}{X} = \frac{l}{(100-l)}$ or $X = \frac{R\,(100-l)}{l}$

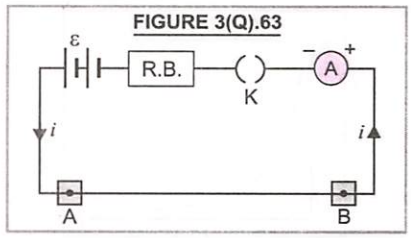
(*b*) When R is increased, then l will have to be increased to get the balance point on meter bridge. Therefore, the jockey point B is to be shifted towards end C of meter bridge.

IX. Potentiometer

61. What is potential gradient ? How is it measured ? Explain.

Ans. The potential gradient is defined as the fall of potential per unit length of the potentiometer wire. Complete the circuit as shown in Fig. 3(Q).63 and note the current from ammeter A.

FIGURE 3(Q).63



Let L = length of potentiometer wire AB,

ε = emf of the battery used in the potentiometer wire circuit, R = resistance from resistance box used in circuit, r = resistance of the wire AB.

Total resistance of the circuit = $R + r$

Current in potentiometer wire, $i = \dfrac{\varepsilon}{R+r}$

Fall of potential across potentiometer wire

$$= ir = \left(\frac{\varepsilon}{R+r}\right) r$$

$\therefore$ Potential gradient $= \dfrac{\text{fall of potential}}{\text{length}}$

$$= \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L}$$

62. Can you express the potential gradient in terms of specific resistance of the wire ? If yes, find the relation.

Ans. Yes. Potential gradient,

$$K = \frac{\text{potential difference applied } (V)}{\text{length of wire } (l)},$$

i.e., $K = \dfrac{V}{l} = \dfrac{(IR)}{l} = \dfrac{I}{l}(\rho\, l/A) = \dfrac{I\rho}{A}$

where ρ is the specific resistance of the material of the wire.

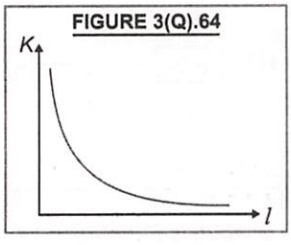
63. In a potentiometer experiment, if the area of cross-section of the wire increases uniformly from one end to another, draw a graph showing how potential gradient would vary as the length of the wire increases from one end. [CBSE 2014 (C)]

Ans. Potential gradient,

$$K = \frac{V}{l} = \frac{IR}{l} = \frac{I}{l}\frac{\rho l}{A} = \frac{I\rho}{A} \quad \text{or} \quad K \propto \frac{1}{A}$$

As per question A increases with l, so, the variation of K versus l when A increases from one end to another is of the type as shown in Fig. 3(Q).64.

FIGURE 3(Q).64



64. If the length of the wire be (*i*) doubled and (*ii*) halved, what will be effect on the position of zero deflection in a potentiometer ? Explain.

Ans. (*i*) When length of the wire is doubled, the potential gradient across the potentiometer wire will decrease. Due to it, the position of zero deflection will occur at longer length. (*ii*) The reverse will be true when length is halved.

65. If the current flowing in the wire of the potentiometer be decreased, what will be effect on the position of zero deflection in potentiometer ? Explain.

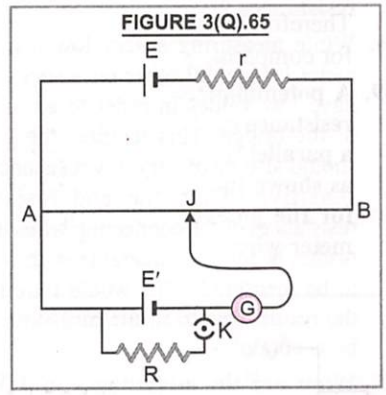
Ans. If the current in the wire of potentiometer is decreased, the potential gradient will decrease and hence the position of zero deflection will occur at longer length.

66. How can you make a potentiometer of a given length more sensitive by using a resistance box ?

Ans. The sensitivity of a potentiometer is the smallest potential difference it can measure. It can be increased by reducing the potential gradient, *i.e.*, potential drop per unit length of potentiometer wire. The same is possible by decreasing the current flowing in the potentiometer wire, which in turn can be achieved by using more resistance from resistance box in series with the potentiometer wire.

67. Fig. 3(Q).65 shows use of potentiometer to measure the internal resistance of a cell.

FIGURE 3(Q).65



(*i*) When the key K is open, how does the balance point change, if the current from the driver cell decreases ?

(*ii*) When the key K is closed, how does the balance point change if R is increased, keeping the current from the driver cell constant ?

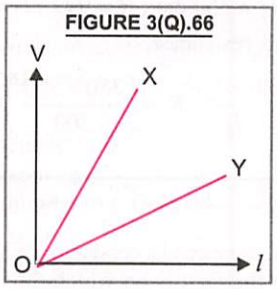
Ans. (*i*) When the current from the driver cell decreases in potentiometer wire, the potential gradient across the wire decreases. Due to it,

the balance point is obtained on longer length of potentiometer wire.

(*ii*) When key K is closed and the value of R is increased, the lesser current will be drawn from the cell E'. Terminal voltage of cell E' will increase, therefore, the balance point will be obtained on longer length of potentiometer wire.

68. The variation of potential difference V with length l in case of two potentiometers X and Y is as shown in the Fig. 3(Q).66. Which one of these two will you prefer for comparing emfs of the two cells and why ? (CBSE 2006)

FIGURE 3(Q).66



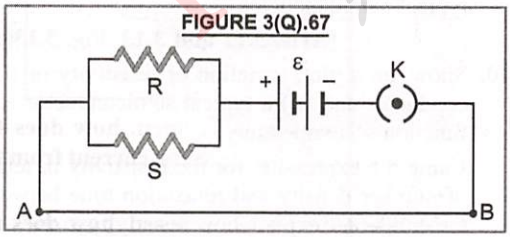
Ans. A potentiometer is said to be sensitive if fall of potential per unit length, *i.e.*, potential gradient (dV/dl) is small. For given length l, $V_y < V_x$

$$\therefore \quad \left(\frac{dV}{dl}\right)_y < \left(\frac{dV}{dl}\right)_x .$$

Therefore, potentiometer Y will be preferred for comparing emfs of the two cells.

69. A potentiometer wire has a length L and a resistance R_0. It is connected to a battery and a parallel resistance combination of R and S as shown in Fig. 3(Q).67. Find an exprssion for the potential gradient of the potentiometer wire.

FIGURE 3(Q).67



What is the maximum emf of a 'test cell' for which one can get a balance point on this potentiometer wire ? What precaution should one take, while connecting this test cell in the circuit. [CBSE 2004 (C)]

Ans. Total resistance of potentiometer circuit

$$= R_0 + \frac{RS}{R+S}$$

Current in the circuit, $I = \dfrac{\varepsilon}{R_0 + RS/(R+S)}$

Potential difference across the wire AB

$$V = IR_0 = \frac{\varepsilon}{R_0 + RS/(R+S)} \times R_0$$

Potential gradient, $K = \dfrac{V}{L} = \dfrac{\varepsilon R_0}{L\left(R_0 + \dfrac{RS}{R+S}\right)}$

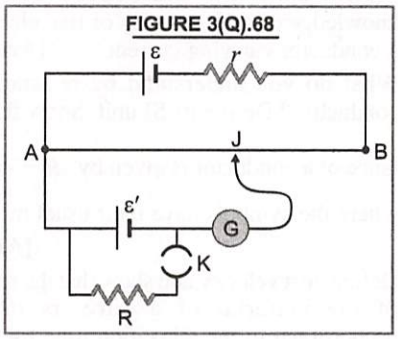
The emf of the test cell must be less than the total potential drop (V) across the wire AB. The positive terminal of test cell must be connected to end A of the wire.

70. The following circuit shows the use of potentiometer to measure the internal resistance of a cell :

(*i*) When the key K is open, how does the balance point change, if the current from the driver cell increases ?

(*ii*) When the key K is closed, how does the balance point change if R is decreased, keeping the current from the driver cell constant ?

FIGURE 3(Q).68



Ans. (*i*) When the key K is open and current from the driver cell increases, the potential gradient across the potentiometer wire increases. Due to it, the balance point shifts towards the end A of potentiometer wire.

(*ii*) When the key K is closed and R is decreased, keeping the current from the driver cell constant, the terminal potential difference of the cell decreases. Due to it, the balance point shifts towards the end A of potentiometer wire.

71. A potentiometer wire of length 1 m has a resistance of 10 Ω. It is connected to a 6 V battery in series with a resistance of 5 Ω. Determine the emf of the primary cell which gives a balance point at 40 cm. (CBSE 2014)

Ans. Here, $L = 1$ m $= 100$ cm ; $R_0 = 10\ \Omega$, $V = 6$ V, $R = 5\ \Omega$, $l = 40$ cm, $\varepsilon = ?$

Current in potentiometer wire, $I = \dfrac{V}{R_0 + R}$

Potential difference across the potentiometer wire $= IR_0 = \dfrac{V}{R_0 + R} \times R_0$

Potential gradient, $K = \dfrac{VR_0}{(R_0 + R)} \dfrac{1}{L}$

EMF of the primary cell, $\varepsilon = \dfrac{VR_0\, l}{(R_0 + R)\, L}$

$$= \frac{6 \times 10 \times 40}{(10+5) \times 100} = \mathbf{1{\cdot}6\ V}$$

72. In a potentiometer arrangement for determining the emf of a cell, the balance point of the cell in open circuit is 350 cm. When a resistance of 9 Ω is used in the external circuit of the cell, the balance point shifts to 300 cm. Determine the internal resistance of the cell. (CBSE 2018)

Ans. Here, $l_1 = 350$ cm ; $R = 9\ \Omega$; $l_2 = 300$ cm

Internal resistance,

$$r = \frac{(l_1 - l_2)}{l_2} R = \left(\frac{350 - 300}{300}\right) \times 9 = \mathbf{1{\cdot}5\ \Omega}$$

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. What do you understand by electric current ? Define its S.I. unit and mention the direction of electric current in the circuit. Also explain if current is a scalar or vector quantity. **[Art. 3.5]**
2. What is drift velocity ? Establish a relation between current and drift velocity. **(CBSE 2016 S, Uttarakhand Board 2012, MP Board 2011, Jharkhand 2011)** **[Art. 3.7, 3.9]**
3. State Ohm's law and deduce it from the knowledge of drift velocity of free electrons in a conductor carrying current. **[Arts. 3.10]**
4. What do you understand by resistance of a conductor ? Define its SI unit. Show that resistance of a conductor is given by $R = \dfrac{ml}{ne^2 \tau A}$, where the symbols have their usual meanings. **[Art. 3.11]**
5. Define drift velocity and show that the resistivity of the material of a wire, is inversely proportional to the relaxation time for the free electrons in the metal. **(CBSE 2016 S)**

   Hint. Resistance of a conductor, $R = \dfrac{ml}{ne^2 \tau A}$.

   Also, $R = \dfrac{\rho l}{A}$;

   So, $\dfrac{\rho l}{A} = \dfrac{ml}{ne^2 \tau A}$ or $\rho = \dfrac{m}{ne^2 \tau}$

   Hence, $\boldsymbol{\rho \propto 1/\tau}$
6. Define resistance of a conductor. What is its cause ? Explain the factors on which the resistance of a conductor depends. **[Arts. 3.11 and 12]**
7. Explain current density, conductance and electrical conductivity. State their SI units. **[Art. 3.13]**
8. (*a*) Define the term conductivity of a metallic wire. Write its SI unit.

   (*b*) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence obtain the relation between current density and the applied electric field *E*. **(CBSE 2018)** **[Art. 3.13]**
9. Define the term 'resistivity' and 'conductivity' and state their SI units. Draw a graph showing the variation of resistivity with temperature for a typical semiconductor. **[Arts. 3.12 and 3.13, Fig. 3.13(*d*)]**
10. Show, on a plot, variation of resistivity of (*i*) a conductor and (*ii*) a typical semiconductor as a function of temperature.

    Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature. **(CBSE 2019)**

Ans. A graph showing the variation of resistivity (ρ) and temperature (T) for a conductor and typical semiconductor is shown in the Fig. 3(Q).69.

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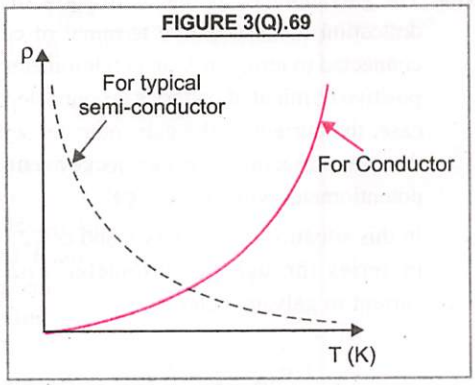
FIGURE 3(Q).69



The expression for the resistivity is

$$\rho = \frac{m}{ne^2\tau}$$

where n is the number density of free electrons and τ is the average relaxation time of free electrons.

In metal conductors, n is not dependent on temperature upto an appreciable extent, but the value of τ decreases with the rise in temperature.

As $\rho \propto \frac{1}{\tau}$, so ρ increases with the rise in temperature.

For semiconductor, n increases with temperature and τ decreases. But increase in n is much more than decrease in τ. Therefore, ρ decreases with rise in temperature.

11. What are ohmic and non-ohmic conductors ? Explain the super-conductivity.

[Arts. 3.17, 3.19]

12. Define the term 'temperature coefficient of resistivity'. Write its S.I. unit. Plot a graph showing the variation of resistivity of copper with temperature. **[Arts. 3.15 and Fig. 3.13(*a*)]**

13. Write the mathematical relation for the resistivity of a material in terms of relaxation time, number density and mass and charge of charge carriers in it.

Explain, using this relation, why the resistivity of the metal increases and that of a semiconductor decreases with rise in temperature.

(CBSE 2007) [Art. 3.16]

14. Explain colour code for carbon resistors with illustrations. **[Art. 3.21]**

15. What do you understand by internal resistance and terminal potential difference of a cell ? On what factors do they depend ? **[Art. 3.24]**

16. Establish a relation between E.M.F. and potential difference of a cell.

(HP Board 2011) [Art. 3.24]

17. Two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 respectively are connected in parallel. Obtain expressions for the equivalent (*i*) internal resistance and (*ii*) emf of the combination. **[CBSE 2018 (C)] [Art. 3.26(*b*)]**

18. State Joule's law for heating effect of electric current. What is the cause of it ? Is it reversible or not ? **[Art. 3.30]**

19. Explain electric power. State its different relations and define SI unit of electric power. **[Art. 3.31]**

20. Explain electric energy. Give the various relations of electric energy and define the commercial unit of electric energy. **[Art. 3.32]**

21. State the two Kirchhoff's rules used in electric networks. How are these rules justified ?

(CBSE 2015) [Art. 3.35]

22. State Wheatstone bridge principle and deduce it using Kirchhoff's rules.

(Uttarakhand Board 2012, Karnataka Board 2012, Raj. Board 2011) [Art. 3.36]

23. State and prove Wheat stone bridge principle. Discuss the determination of unknown temperature with its help.

(Tamil Nadu Board 2012, Pb. Board 2011) [Art. 3.37]

24. Draw a circuit diagram of a meter bridge used to determine the unknown resistance R of a given wire. Hence derive the expression for R in terms of the known resistance S.

[CBSE (OD) 2017 (C)] [Art. 3.38]

25. Explain the principle of potentiometer. How is a potentiometer superior to that of a voltmeter ?

(MP Board 2011) [Arts. 3.39 and 3.44]

26. Discuss the principle of potentiometer and explain the determination of potential difference across a conductor using a potentiometer.

[Arts. 3.39, 3.40]

27. State the principle of potentiometer. With the help of circuit diagram, describe a method to find the internal resistance of a primary cell.

(CBSE 2017 (C), Uttarakhand Board 2012, Raj. Board 2012, Pb. Board 2011) [Arts. 3.39, 3.42]

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28. State the principle of potentiometer. Draw a circuit diagram used to compare the *emfs* of two primary cells. Write the formula used. How can the sensitivity of a potentiometer be increased.

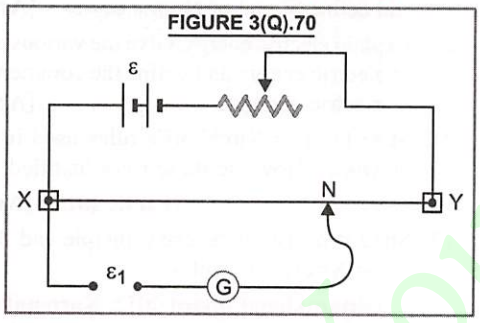
(Kerala Board 2012)

[Arts. 3.39, 3.41 and 3.43]

29. The diagram shows a potentiometer set up. On touching the jockey near to the end X of the potentiometer wire, the galvanometer pointer deflects to left. On touching the jockey near to end Y of the potentiometer, the galvanometer pointer again deflects to left but now by a larger amount. Identify the fault in the circuit and explain, using appropriate equations or otherwise, how it leads to such a one-sided deflection.

(CBSE Sample Paper 2018)

FIGURE 3(Q).70



Ans. The galvanometer pointer will show one sided deflection if the negative terminal of cell ε_1 is connected to terminal X on potentiometer, where positive terminal of battery ε is connected. In this case, the current in the galvanometer will be in the same direction, whether jockey touches the potentiometer wire at X or at Y.

In this situation, the battery ε and cell ε_1 will be in series through potentiometer wire. Then current in galvanometer is

$$I = \frac{\varepsilon + \varepsilon_1}{G + R} = K\theta$$

where G is the resistance of galvanometer and R is the resistance of potentiometer wire in circuit, with reference to jockey point and θ is the deflection in the galvanometer.

When jockey touches the potentiometer wire near the point X, the value of R in the circuit is maximum, so current I is minimum. Due to it, the deflection θ in galvanometer is small. When jockey touches the potentiometer wire near the point Y, the value of R in the circuit is minimum. Due to it, the deflection θ is large.

LONG ANSWER QUESTIONS Carrying 5 or more marks

1. Define relaxation time of the free electrons drifting in a conductor. Establish a relation between drift velocity and time of relaxation. Use this relation to deduce the expression for the electrical resistivity of the material.

(Karnataka Board 2012 ; CBSE 2012)

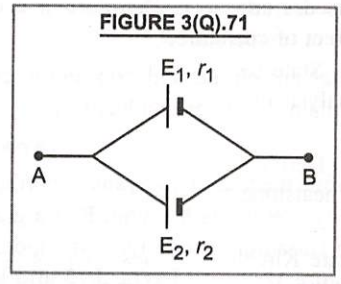
[Arts. 3.7 and 3.12]

2. Derive an expression for drift velocity of electrons in a conductor. Hence deduce ohm's law. (CBSE 2017) **[Arts. 3.7 and 3.10]**

3. (*a*) Explain the term drift velocity of electrons in a conductor. Hence obtain the expression for the current through a conductor in terms of drift velocity. **[Arts. 3.7 and 3.9]**

(*b*) Two cells of emfs E_1 and E_2 and internal resistances r_1 and r_2 respectively are connected in parallel as shown in Fig. 3(Q).71. Deduce the expression for the

FIGURE 3(Q).71



(*i*) equivalent emf of the combination

(*ii*) equivalent internal resistance of the combination

(*iii*) potential difference between points A and B.

(CBSE Sample Paper 2022-23) **[Art. 3.26 (*b*)]**

4. Derive the expression for the current density of a conductor in terms of the conductivity and applied electric field. Explain with reason how the mobility of electrons in a conductor changes when the

potential difference applied is doubled, keeping the temperature of the conductor constant.

[CBSE 2017 (C)] **[Arts. 3.13 and 3.8]**

5. Define resistivity of the material. State its SI unit and discuss its variation with temperature in case of (*i*) metals (*ii*) semiconductors and (*iii*) insulators. **[Arts. 3.12 and 3.16]**
6. Find the total resistance when the various resistors are connected (*i*) in series and (*ii*) in parallel.

(Pb. Board 2011) **[Arts. 3.22 and 23]**

7. What is the *e.m.f.* of a cell ? State the factors on which its value depends. Derive a relation between e.m.f. *E*, terminal potential difference *V*, internal resistance *r* of a cell and external resistance *R*. Prove that e.m.f. is more than potential difference. **[Arts. 3.3 and 3.24]**
8. Discuss the grouping of two unidentical cells in (*i*) series and (*ii*) parallel and find their equivalent emf and internal resistance. **[Art. 3.26]**
9. Explain the various types of grouping of identical cells and find the condition for the maximum current in the external resistor connected to the combination of cells (*ii*) in series (*ii*) in parallel and (*iii*) in mixed grouping. **[Art. 3.27]**
10. What do you understand by heating effect of current ? Explain its cause and find the relation for the heat produced in a conductor for electric current. **[Art. 3.30]**
11. Explain electric power, electric energy and define their units and give their relations.

[Art. 3.31, 3.32]

12. Discuss atleast 3 important aspects of heating effect of current. **[Art. 3.33]**
13. (*a*) State the two Kirchhoff's rules used in the analysis of electric circuit and explain them.

[Art. 3.35]

(*b*) Derive the equation of the balanced state in a Wheatstone bridge using Kirchhoff's rules.

(CBSE Sample Paper 2022-23) **[Art. 3.36]**

14. State Kirchhoff's rules for an electric network. Using Kirchhoff's rules obtain the balance condition in terms of the resistances of four arms of Wheatstone bridge.

(CBSE 2013 ; Pb. Board 2011) **[Arts. 3.35 and 3.36]**

15. State the working principle of a potentiometer. With the help of the circuit diagram, explain how a potentiometer is used to compare the emf's of two primary cells. Obtain the required expression used for comparing the emfs.

Write two possible causes for one sided deflection in a potentiometer experiment.

(CBSE 2013) **[Arts. 3.39 and 3.41]**

16. Write the principle of potentiometer. How will you compare *emfs* of two cells using a potentiometer. Explain with a circuit diagram. Write two precautions to be taken during this experiment.

(Tamil Nadu Board 2012, Pb. Board 2011, CBSE 2010) **[Arts. 3.39 and 3.41]**

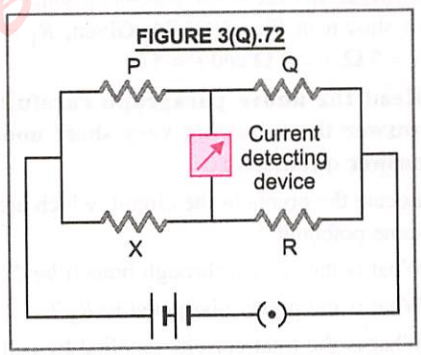
17. (*a*) Obtain the condition under which the current flowing in the current detecting device used in the circuit shown in Fig. 3(Q).72, becomes zero.

[Art. 3.36]

(*b*) Describe briefly the device, based on the above condition. Draw a circuit diagram for this device and discuss, in brief, how it is used for finding as unknown resistance.

[CBSE 2016 (C)]

FIGURE 3(Q).72



[Art. 3.38]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. The resistance of a conductor is the obstruction offered by it to the flow of current through it. The resistance of a conductor depends upon the dimensions (length and area of cross-section) and nature of the material of the conductor. The resistance of a conductor varies with the temperature of the conductor. With the increase in temperature, the resistance of a metal conductors (alloys) increases. The resistance of semiconductors decreases.

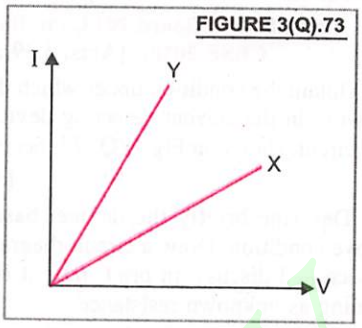
The resistivity of a conductor depends upon the nature of material of the conductor. Its value is low for the conductors, moderate for semiconductors and very high for insulators.

The temperature coefficient of resistivity is very low for manganin than for copper aluminium.

Read the above paragraph carefully and answer the following very short and short answer questions :

1. What is the resistance of a conductor if its length and area of cross-section are doubled ?
2. Why is the resistance of copper wire low and of manganin wire high, when both the wires are of same dimensions ?
3. Why are the manganin wires, used in making the standard resistances ?
4. The voltage-current variation of two metallic wires X and Y at constant temperature are shown in Fig. 3(Q).73. Assuming that the wires have the same length and same diameter, explain which of the two wires have larger resistivity ?

FIGURE 3(Q).73

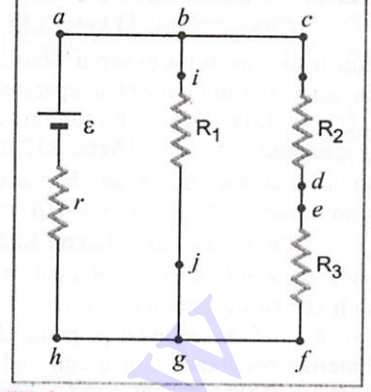


CASE 2. An experiment was set up with the circuit diagram shown in Fig. 3(Q).74. Given, $R_1 = 10\ \Omega$, $R_2 = R_3 = 5\ \Omega$, $r = 0\ \Omega$ and $\varepsilon = 5$ V.

Read the above paragraph carefully and answer the following very short and short answer questions :

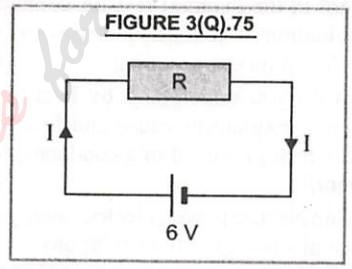
5. Locate the points in the circuit, which are at the same potential ?
6. What is the current through branch bg ?
7. What is the power dissipated in R_1 ?
8. What is the total current supplied by cell in the circuit ?

FIGURE 3(Q).74



CASE 3. A conductor of length 10 cm, cross-sectional area 1 $(\text{mm})^2$ having number of electrons per unit volume $10^{29}\ \text{m}^{-3}$ and of resistivity $6 \times 10^{-5}\ \Omega$ m is connected to the battery of emf 6 V. The electrons in the conductor give up energy at the rate of $I^2 R$ per second to the thermal energy, where I is the current and R is the resistance of circuit as shown in Fig. 3(Q).75.

FIGURE 3(Q).75



Read the above paragraph carefully and answer the following very short and short answer questions :

9. What is the resistance in the circuit ?
10. What is the drift velocity of electrons in the conductor ?
11. How much energy is absorbed by electrons per second from the initial state of no current (ignore thermal motion) to the state of drift velocity ?
12. What is the total kinetic energy of all the electrons while drifting in the conductor ?

ANSWERS

1. The resistance of a conductor of length l, area of cross-section A and resistivity ρ is given by

$$R = \frac{\rho l}{A}$$

When $l' = 2\,l$ and $A' = 2\,A$, then

$$R' = \frac{\rho l'}{A'} = \frac{\rho(2l)}{(2A)} = \frac{\rho l}{A} = R$$

It means, **the resistance of the conductor remains unchanged.**

2. The resistivity of the material of copper wire is low and of manganin wire is high, *i.e.*,

$$\rho_{cu} < \rho_{manganin}.$$

We know, $R = \dfrac{\rho l}{A}$; so $R \propto \rho$ (when A and l are constant). So $\quad R_{cu} < R_{manganin}$.

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3. The temperature coefficient of resistivity of manganin is very low. Consequently, the manganin wire shows a negligible variation of resistance with change in temperature. Due to it, the resistance of manganin wire remains almost same, with change in temperature.

4. In the given graph, the slope of the *I-V* line for a wire represents the conductance of the wire. From graph, slope of *I-V* line for wire *Y* > slope of *I-V* line for wire *X*.

$\therefore$ Conductance of wire *Y* > Conductance of wire *X*.

As, resistance $= \dfrac{1}{\text{Conductance}}$

$\therefore$ Resistance of wire *Y* < resistance of wire *X*

or $\dfrac{\rho_Y l}{A} < \dfrac{\rho_X l}{A}$, *i.e.*, $\rho_Y < \rho_X$

i.e., wire *X* has larger resistivity than wire *Y*.

5. In the given circuit, the points *a*, *b*, *c* and *i* are at the same potential. Also the points *f*, *g*, *j* and *h* are at other same potential.

6. Current through branch *bg* of the circuit
= current through resistance R_1

$$= \frac{\varepsilon}{R_1} = \frac{5}{10} = \frac{1}{2}\text{A}$$

7. Power dissipated in $R_1 = I^2 R_1 = \left(\dfrac{1}{2}\right)^2 \times 10$

$$= \mathbf{2{\cdot}5\ W}$$

8. Total resistance of the circuit

$$R_t = \frac{R_1 \times (R_2 + R_3)}{R_1 + (R_2 + R_3)} + r = \frac{10 \times (5+5)}{10 + (5+5)} + 0$$

$$= 5\ \Omega$$

Current in the circuit, $I = \dfrac{\varepsilon}{R_t} = \dfrac{5}{5} = \mathbf{1\,A}$

9. Resistance of conductor

$$R = \frac{\rho l}{A} = \frac{(6 \times 10^{-5}) \times (0{\cdot}1)}{(10^{-6})} = \mathbf{6\,\Omega}$$

10. Current in the circuit, $I = \dfrac{\varepsilon}{R} = \dfrac{6}{6} = 1\,\text{A}$

Drift velocity,

$$v_d = \frac{I}{nAl} = \frac{1}{(10^{29}) \times (10^{-6}) \times (1{\cdot}6 \times 10^{-19})}$$

$$= \frac{10^{-4}}{1{\cdot}6} = \mathbf{0{\cdot}625 \times 10^{-4}\ ms^{-1}}$$

11. Energy absorbed by electrons per second
= power loss = $I^2 R = 1^2 \times 6 = 6\ \text{Js}^{-1} = \mathbf{6\ W}$

12. No. of electrons in the conductor = $n\,A\,l$

KE of all the electrons in the conductor, when electrons are moving with drift velocity

$$= \frac{1}{2} m v_d^2 \times n A l$$

$$= \frac{1}{2}(9{\cdot}1 \times 10^{-31}) \times \left(\frac{1}{1{\cdot}6} \times 10^{-4}\right)^2 \times 10^{29} \times (10^{-6}) \times 0{\cdot}1$$

$$= \mathbf{2 \times 10^{-17}\ J}$$

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. Electrochemical cell is a device in which the chemical energy is converted into electrical energy. Due to chemical reaction taking place in the cell, a difference of potential is maintained between the two electrodes of a cell. As a result of it, the current can be drawn from the cell.

EMF (ε) of a cell is maximum potential difference between the two electrodes of the cell, when the cell is in the open circuit.

Terminal potential difference (V) of a cell is defined as the potential difference between two electrodes of a cell when cell is in the closed circuit.

If a cell of emf ε and internal resistance r is connected to external resistance R, then current in the circuit is

$$I = \frac{\varepsilon}{R+r} \quad \text{or} \quad \varepsilon = I(R+r) = IR + Ir = V + Ir$$

where V is the terminal potential difference.

When current I is drawn from the cell. Thus

$$r = \frac{\varepsilon - V}{I} = \frac{\varepsilon - V}{V/R} = \left(\frac{\varepsilon - V}{V}\right) R \qquad [\because V = IR]$$

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Based on the above paragraph, answer questions no. 1 to 4 :

1. Terminal potential difference of a cell as compared to emf of the cell is always

   (*a*) lesser

   (*b*) greater

   (*c*) depends upon the condition of circuit for the cell

   (*d*) none of the above

2. The emf of a battery is 12·0 V and internal resistance is 3 Ω. Its potential difference is measured by a voltmeter of resistance 1000 Ω. The percentage error in the reading of emf shown by voltmeter is

   (*a*) 0·2% (*b*) 0·3%

   (*c*) 0·4% (*d*) 0·5%

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false and R is also false.

3. **Assertion.** The emf of the cell does not depend upon the size and shape of the cell.

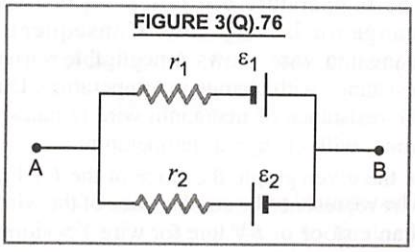
   Reason. The emf of the cell depends upon the nature of electrodes and the concentration of the electrolyte.

4. **Assertion.** An external resistance 990 Ω is connected across a cell of emf 5 V and internal resistance 10 Ω. The terminal potential difference across the cell is 4·95 V.

   Reason. $V = I r$, where V is the terminal potential difference across the cell of internal resistance r and I is the current drawn from the cell.

CASE 2. A battery is a combination of two or more cells. In the following figures, a single battery is represented in which two cells of emf ε_1 and ε_2 and internal resistance r_1 and r_2 respectively are connected.

FIGURE 3(Q).76



Based on the above paragraph, answer questions no. 5 to 8 :

5. The equivalent emf of this combination is

   (*a*) $\dfrac{\varepsilon_1 r_1 + \varepsilon_2 r_2}{r_1 + r_2}$ (*b*) $\dfrac{\varepsilon_1 r_1 - \varepsilon_2 r_2}{r_1 + r_2}$

   (*c*) $\dfrac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{r_1 + r_2}$ (*d*) $\varepsilon_1 - \varepsilon_2$

   (CBSE 2022)

6. For terminal B to be negative

   (*a*) $\varepsilon_1 r_2 > \varepsilon_2 r_1$ (*b*) $\varepsilon_1 r_2 < \varepsilon_2 r_1$

   (*c*) $\varepsilon_1 r_1 > \varepsilon_2 r_2$ (*d*) $\varepsilon_2 r_2 = \varepsilon_1 r_1$

   (CBSE 2022)

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false and R is also false.

7. **Assertion.** For the given circuit, the current I is $\dfrac{\varepsilon_2 + \varepsilon_1}{r_1 + r_2}$.

   Reason. In internal circuit, two cells are in series, so total emf $= \varepsilon_1 + \varepsilon_2$ and total internal resistance is $(r_1 + r_2)$.

8. **Assertion.** If $\varepsilon_1 = \varepsilon_2 = 1{\cdot}5$ V, $r_1 = r_2 = 1$ Ω and the cells are connected in parallel to the external resistance of $R = 7$ Ω, then total resistance of the circuit is 9 Ω.

   Reason. Total resistance $= r_1 + r_2 + R$.

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ANSWERS

1. (*c*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*c*) **6.** (*b*) **7.** (*a*) **8.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

1. The terminal potential difference of a cell is less than emf of a cell when current is drawn from the cell and is greater than the emf of the cell if the cell is charged by external battery.

2. Here, $\varepsilon = 12{\cdot}0$ V ; $r = 3{\cdot}0\ \Omega$

Resistance of voltmeter, $R = 1000\ \Omega$

Current in the circuit, $I = \dfrac{\varepsilon}{R+r} = \dfrac{12{\cdot}0}{1000+3}$

$= 11{\cdot}964 \times 10^{-3}$ A

Potential difference across voltmeter

$V = IR = (11{\cdot}964 \times 10^{-3}) \times 1000 = 11{\cdot}964$

$$\% \text{ error} = \left(\frac{\varepsilon - V}{V}\right) \times 100$$

$$= \left(\frac{12 - 11{\cdot}964}{11{\cdot}964}\right) \times 100 = \mathbf{0{\cdot}3\%}$$

3. Both Assertion and Reason are true and Reason is not the correct explanation of Assertion.

4. Here, $R = 990\ \Omega$, $\varepsilon = 5$ V, $r = 10\ \Omega$

$$I = \frac{\varepsilon}{R+r} = \frac{5}{990+10} = 5 \times 10^{-3} \text{ A}$$

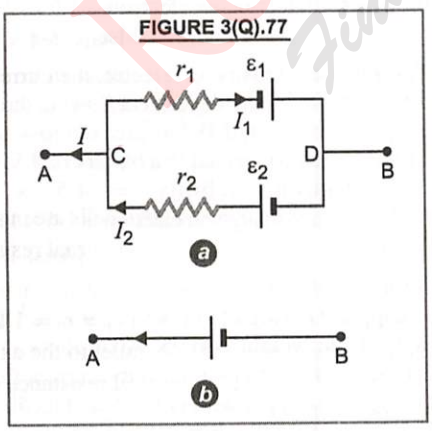
Terminal potential difference across the cell is

$V = IR = (5 \times 10^{-3}) \times 990 = 4{\cdot}950$ V

Thus **Assertion is true but Reason is false.**

5. Refer to Fig. 3(Q).77(*a*), at junction *C*,

FIGURE 3(Q).77



$$I + I_1 = I_2 \quad \text{or} \quad I = (I_2 - I_1) \qquad \text{...(1)}$$

Let V_C and V_D be the potentials at *C* and *D* respectively and *V* be the potential difference between *C* and *D*.

Here, potential difference across the terminals of first cell is equal to the potential difference across the terminals of second less. So, for the first cell

$$V = V_C - V_D = -\varepsilon_1 + I_1 r_1 \quad \text{or} \quad I_1 = \frac{V + \varepsilon_1}{r_1}$$

For the second cell,

$$V = V_C - V_D = \varepsilon_2 - I_2 r_2 \quad \text{or} \quad I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Putting values in (1), we get

$$I = \frac{\varepsilon_2 - V}{r_2} - \frac{V + \varepsilon_1}{r_1} = \left(\frac{\varepsilon_2}{r_2} - \frac{\varepsilon_1}{r_1}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{r_1 r_2} - V\left(\frac{r_2 + r_1}{r_1 r_2}\right)$$

or $r_1 r_2 I = \varepsilon_2 r_1 - \varepsilon_1 r_2 - V(r_1 + r_2)$

or $V = \dfrac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{r_1 + r_2} - I\left(\dfrac{r_1 r_2}{r_1 + r_2}\right)$ ...(2)

If the combination of cells is replaced by a single cell between *C* and *D* of emf ε_{eq} and internal resistance r_{eq}, Fig. 3(Q).78(*b*), then

$$V = \varepsilon_{eq} - I r_{eq} \qquad \text{...(3)}$$

$$\varepsilon_{eq} = \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{r_1 + r_2}$$

Thus, **option (*c*) is true.**

6. The terminal *B* to be negative, the direction of current should be as shown in Fig. 3(Q).78(*b*). It will be so if $\varepsilon_1 r_2 < \varepsilon_2 r_1$. Thus, **option (*b*) is true.**

7. Current $= \dfrac{\text{total emf}}{\text{total internal resistance}} = \dfrac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}$

Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

8. Both Assertion and Reason are wrong. As total resistance will be $= \dfrac{r_1 r_2}{r_1 + r_2} + R = \dfrac{1 \times 1}{1+1} + 7$

$= 7{\cdot}5\ \Omega$

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PROBLEMS FOR PRACTICE

TYPE I. ELECTRIC CURRENT

1. In Bohr model of hydrogen atom, the electron revolves around the nucleus in a circular orbit of radius $5{\cdot}1 \times 10^{-11}$ m at a frequency of $6{\cdot}8 \times 10^{15}$ revolutions per second. Find the equivalent current at any point on the orbit of the electron.

[Ans. $1{\cdot}088 \times 10^{-3}$ A]

2. A solution of sodium chloride discharges $6{\cdot}0 \times 10^{16}$ Na^+ ions and $4{\cdot}5 \times 10^{16}$ Cl^{-1} ions in 2 seconds. What is the current passing through the solution ? **[Ans. 8·4 mA]**

3. In hydrogen atom, the electron moves in an orbit of radius $5{\cdot}0 \times 10^{-11}$ m with a speed of $2{\cdot}2 \times 10^6$ ms^{-1}. Find the equivalent current. Electronic charge = $1{\cdot}6 \times 10^{-19}$ C. **[Ans. 1·12 mA]**

4. If 0·6 mole of eletrons flow through a wire in 50 minutes. What is (*a*) the total charge that passes through the wire, and (*b*) the magnitude of the current ? Avogadro's number = 6×10^{23} per mol.

[Ans. (*a*) $5{\cdot}76 \times 10^4$ C (*b*) 19·2 A]

5. The charge flowing through a conductor varies with time as $q = 8t - 3t^2 + 5t^3$

Find (*i*) the initial current (*ii*) time after which the current reaches a maximum value (*iii*) the maximum or minimum value of current.

[Ans. (*i*) 8 A (*ii*) 0·2 s (*iii*) 7·4 A]

6. Fig. 3(Q).78 shows a plot of current I (in ampere) through the cross-section of the wire over a time interval of 14 s. Find the amount of charge that

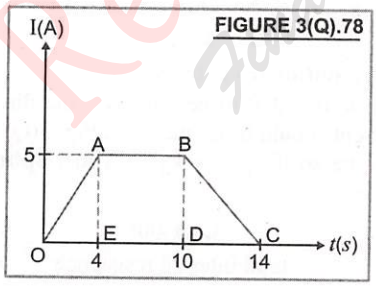


FIGURE 3(Q).78

passes through the wire (*i*) during 10 s (*ii*) during 14 s. **[Ans. (*i*) 40 C (*ii*) 50 C]**

7. An electric current of 20 μA appears in a discharge tube. If the discharge current is due to flow of equal number of electrons and protons, then how many electrons flow across a cross-section of the tube in 2 minutes. **[Ans. $7{\cdot}5 \times 10^{15}$]**

TYPE II. DRIFT VELOCITY AND ELECTRON MOBILITY

8. A conductor of length L is connected to a d.c. source of *emf* ε. The drift velocity of electron is v_d. If the length of conductor is tripled by stretched it, keeping ε constant, find its drift velocity. **(CBSE 2015)** **[Ans. $v_d/3$]**

9. The number of free electrons per 5 cm of ordinary copper wire is 2×10^{21}. The average drift speed of electrons is 0·25 mm/s. What is the current flowing ? **[Ans. 1·6 A]**

10. What is the drift velocity of electrons in a silver wire of length 1 m, having cross-sectional area $3{\cdot}14 \times 10^{-6}$ m^2 and carrying a current of 10 A. Given atomic weight of silver = 108, density of silver $10{\cdot}5 \times 10^3$ kg/m^3, charge of electron $1{\cdot}6 \times 10^{-19}$ C, Avogadro's number $= 6{\cdot}023 \times 10^{26}$ per kg. atom.

[Ans. $3{\cdot}4 \times 10^{-4}$ ms^{-1}]

11. Protons in cosmic-rays strike the earth's upper atmosphere at a rate, average over the earth's surface of 10 protons $m^{-2}s^{-1}$. What does the total current the earth receive from beyond its atmosphere in the form of incident cosmic ray protons ? The earth's radius is $6{\cdot}4 \times 10^6$ m.

[Ans. 0·82 mA]

12. A uniform copper wire of length 1 m and cross sectional area 5×10^{-7} m^2 carries a current of 1 A. Assuming that there are 8×10^{28} free electrons per m^3 in copper, how long will an electron take to drift from one end of the wire to the other. Charge on an electron = $1{\cdot}6 \times 10^{-19}$ C.

[Ans. $6{\cdot}4 \times 10^3$ s]

13. The number density of electrons in copper is $8{\cdot}5 \times 10^{28}$ m^{-3}. Find the current flowing through a copper wire of length 0·2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that electron mobility = $4{\cdot}5 \times 10^{-6}$ $m^2 V^{-1} s^{-1}$ and charge on electron is $1{\cdot}6 \times 10^{-19}$ C.

[Ans. 1·22 A]

14. Assuming that there is one free electron per atom in copper. Density of copper is $8{\cdot}9 \times 10^3$ kg m^{-3} and atomic weight is 63·5. Avogadro's number is $6{\cdot}02 \times 10^{26}$ per kg-atom. If the current flowing through the copper wire is 1·5 A and its diameter is 1·2 mm, find the drift velocity of electron.

[Ans. 10^{-4} ms^{-1}]

TYPE III. OHM'S LAW, RESISTANCE, RESISTIVITY

15. An arc lamp operates at 80 V, 10 A. Suggest a method to use it with a 240 V d.c. source. Calculate the value of the electric component required for this purpose.

[Ans. Resistor of value 16 Ω]

16. A wire 50 cm long and 0·12 mm diameter has a resistance of 4·0 Ω. Find the resistance of another wire of the same material whose length is 1·5 m and diameter is 0·15 mm. **[Ans. 1·92 Ω]**

17. A uniform wire of length l and radius r has resistance 100 Ω. It is recast into a thin wire of (*i*) length 2 l (*ii*) radius $r/2$. Calculate the resistance of new wire in each case.

[Ans. (*i*) 400 Ω (*ii*) 1600 Ω]

18. There are two wires of copper and iron of the same length but different radii. When equal potential difference is applied between the ends of each wire, the same current flows in them. What the ratio of their radii. Specific resistances of copper and iron are $1·6 \times 10^{-8}$ Ωm and $1·0 \times 10^{-7}$ Ωm respectively. **[Ans. 2/5]**

19. A wire of mass 10 g, radius 1 mm is compressed to its length by 10%. Calculate the percentage change in its resistance. **[Ans. 19%]**

20. A wire is stretched to increase its length by 5%. Calculate percentage change in its resistance.

(CBSE 2007) [Ans. 10·25%]

21. A hollow cylinder of length l and of radii a and b is filled with a material of resistivity ρ and is connected to a battery of emf E through an ammeter, as shown in Fig. 3(Q).79. Find the current through ammeter.

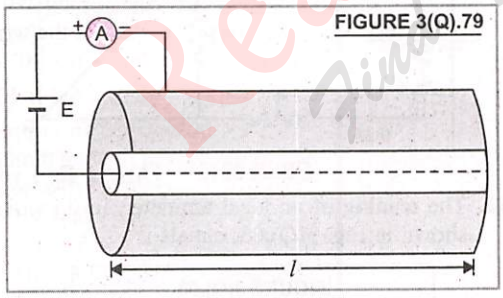


FIGURE 3(Q).79

$$\left[\text{Ans. } \frac{E\,2\pi l}{\rho \log_e (b/a)}\right]$$

22. Calculate the mass of copper required to draw a wire 5 km long having resistance of 5 Ω. The density of copper is $8·9 \times 10^3$ kg m^{-3} and resistivity of copper is $1·7 \times 10^{-8}$ Ωm.

[Ans. 756·5 kg]

23. Two wires A and B of the same material have their lengths in the ratio 5 : 3 and diameter in the ratio 2 : 3. If the resistance of wire A is 15 Ω, find the resistance of wire B. **[Ans. 4 Ω]**

24. A wire of 15 Ω resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3·0 volt battery. Find the current drawn from the battery.

(CBSE 2009) [Ans. 0·2 A]

25. Find the time of relaxation between collision and mean free path of electrons in copper at room temperature. Given resistivity of copper $= 1·7 \times 10^{-8}$ Ω m ; number density of electron in copper $= 8·5 \times 10^{28}$ m^{-3}.

Charge on electron $= 1·6 \times 10^{-19}$ C,

mass of electron $= 9·1 \times 10^{-31}$ kg.

Drift velocity of free electron $= 1·6 \times 10^{-4}$ ms^{-1}

[Ans. $2·5 \times 10^{-14}$ s ; $4·0 \times 10^{-18}$ m]

TYPE IV. CURRENT DENSITY, CONDUCTANCE AND CONDUCTIVITY

26. A rheostat has 1000 turns of a wire of radius 0·4 mm, having resistivity 49×10^{-8} Ω m. The diameter of each turn is 4 cm. What are the maximum values of conductance and conductivity of rheostat wire.

[Ans. $8·2 \times 10^{-3}$ S ; $2·04 \times 10^6$ S m^{-1}]

27. Current flows through a constriction conductor as shown in Fig. 3(Q).80. The diameter and current density to the left of constriction are 2·0 mm and $1·3 \times 10^6$ Am^{-2}. (*i*) How much current flows through the constriction ? (*ii*) If the current density is tripled as it emerges from the right side of the constriction, what is the diameter of the right hand side of constriction ?

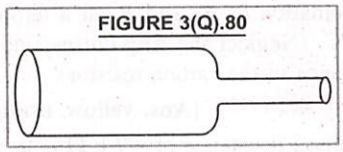


FIGURE 3(Q).80

[Ans. (*i*) 4·09 A ; (*ii*) 1·15 mm]

28. A wire carries a current of 2·0 A, when a potential difference of 3·0 V is applied across it. What is its conductance ? If the wire is of length 3 m and area of cross-section $5·4 \times 10^{-6}$ m^2, calculate its conductivity. **[Ans. 0·67 S ; $3·7 \times 10^5$ S m^{-1}]**

29. Calculate the radius of the wire of conductance 10 Ω^{-1} and length 10 cm whose electrical conductivity is 10^5 Sm^{-1}. **[Ans. 1·78 mm]**

TYPE V. VARIATION OF RESISTANCE WITH TEMPERATURE

30. The resistance of a tungsten filament at 160°C is 132 Ω. What will be its resistance at 400°C ? The temperature coefficient of resistance of tungsten is $4{\cdot}5 \times 10^{-3}$ °C^{-1}. **[Ans. 215 Ω]**

31. The temperature coefficient of a resistance wire is 0·00125°C^{-1}. At 300 K its resistance is 1 Ω. At what temperature the resistance of wire will be 2 Ω ? **[Ans. 1127 K]**

32. A metal wire of diameter 2 mm and length 50 cm has a resistance 0·31 Ω at 25°C and 0·51 Ω at 125°C. Find (*i*) the temperature coefficient of resistance (*ii*), resistance at 0°C and (*iii*) resistivity at 0°C and 25°C.

[Ans. (*i*) $6{\cdot}45 \times 10^{-3}$ °C^{-1} (*ii*) 0·267 Ω, (*iii*) $1{\cdot}68 \times 10^{-6}$ Ωm, $1{\cdot}948 \times 10^{-6}$ Ωm]

33. (*a*) At what temperature would the resistance of a copper conductor be double of its value at 0°C ?

(*b*) Does this same temperature hold for all copper conductors, regardless of the size and shape ? Temperature coefficient of resistance of copper is $4{\cdot}0 \times 10^{-3}$/°C.

[Ans. (*a*) 250°C (*b*) yes]

TYPE VI. COLOUR CODES OF CARBON RESISTORS

34. The temperature coefficient of resistivity for two materials *A* and *B* are 0·0031°C^{-1} and 0·0068°C^{-1} respectively. Two resistors R_1 and R_2 made from materials *A* and *B*, respectively, have resistances of 200 Ω and 100 Ω at 0°C. Show on a diagram, the colour code of a carbon resistance, that would have a resistance equal to the series combination of R_1 and R_2 at a temperature of 100°C. (Neglect the ring corresponding to the tolerance of the carbon resistor).

[CBSE 2016 (C)] [Ans. yellow, orange, black]

35. A carbon resistance of 4·7 k Ω is to be marked with strips or bands of different colours for its identification. Write the sequence of colours.

[Ans. yellow, violet and red]

36. A current of 5 mA is passed through a colour coded carbon resistor with first, second and third rings of black, brown and red. What is the voltage drop across the resistor ? **[Ans. 0·5 V]**

37. What is the colour of the third band of a coded resistor of resistance 0·34 Ω ? **[Ans. silver]**

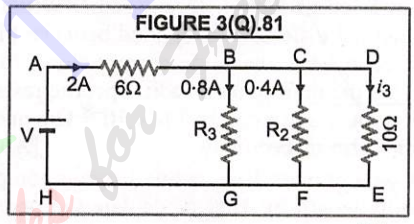
TYPE VII. COMBINATION OF RESISTANCES IN SERIES AND PARALLEL

38. A wire of uniform cross-section and length *l* has a resistance of 16 Ω. It is cut into four equal parts. Each part is stretched uniformly to length *l* and all the four stretched parts are connected in parallel, calculate the total resistance of the combination so formed. Assume that stretching of wire does not cause any change in the density of its material. **[Ans. 16 Ω]**

39. Two wires X, Y have the same resistivity, but their cross-sectional areas are in the ratio 2 : 3 and lengths in the ratio 1 : 2. They are first connected in series and then in parallel to a d.c. source. Find out the ratio of drift speeds of the electrons in the two wires for the two cases. **(CBSE 2008) [Ans. 3 : 2 ; 2 : 1]**

40. What is the equivalent resistance between A and H of resistances as shown in Fig. 3(Q).81.

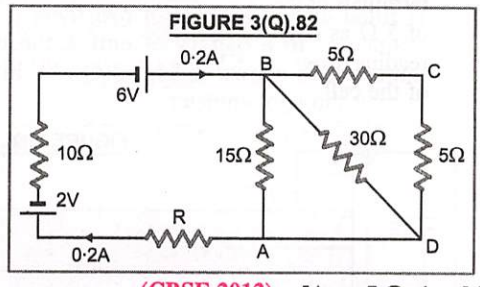
FIGURE 3(Q).81



[Ans. 10 Ω]

41. Calculate the value of the resistance *R* in the circuit shown in the Fig. 3(Q).82, so that the current in the circuit is 0·2A. What would be the potential difference between points *A* and *B* ?

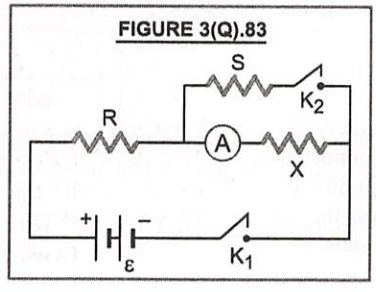
FIGURE 3(Q).82



(CBSE 2012) [Ans. 5 Ω, 1 volt]

42. The reading of an ideal ammeter, in the circuit shown in Fig. 3(Q).83, equals :

FIGURE 3(Q).83

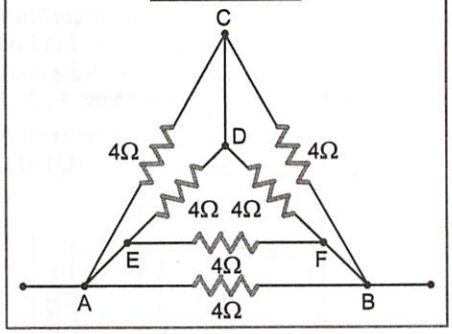


(*i*) I when key K_1 is closed and key K_2 is open
(*ii*) $I/2$ when both keys K_1 and K_2 are closed.
Find the expression for the resistance of X in terms of the resistances of R and S.

[CBSE 2016 (C)] [Ans. $\frac{RS}{R-S}$]

43. Find the equivalent resistance of the circuit shown in Fig. 3(Q).84, between the points A and B.

FIGURE 3(Q).84

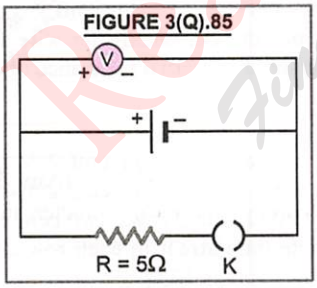


[Ans. (4/3) Ω]

TYPE VIII. EMF, TERMINAL P.D. AND INTERNAL RESISTANCE OF A CELL

44. The reading on a high resistance voltmeter, when a cell is connected across it, is 2·2 V. When the terminals of the cell are connected to a resistance of 5 Ω as shown in Fig. 3(Q).85, the voltmeter reading drops to 1·8 V. Find the internal resistance of the cell.

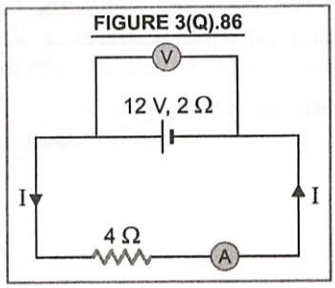
FIGURE 3(Q).85



(CBSE 2010) [Ans. $\frac{10}{9}$ Ω]

45. A battery of *emf* 12 V and internal resistance 2 Ω is connected to a 4 Ω resistor as shown in Fig. 3(Q).86. Show that a voltmeter when placed across the cell and across the resistor, in turn gives the same reading.

FIGURE 3(Q).86



(CBSE 2016) [Ans. 8 V, yes]

46. A cell of *emf* ε and internal resistance r is connected across a variable load resistance R. It is found that when $R = 4$ Ω, the current is 1 A and when R is increased to 9 Ω, the current reduces to 0·5 A. Find the values of the emf ε and internal resistance r. (CBSE 2015) [Ans. 5 V, 1 Ω]

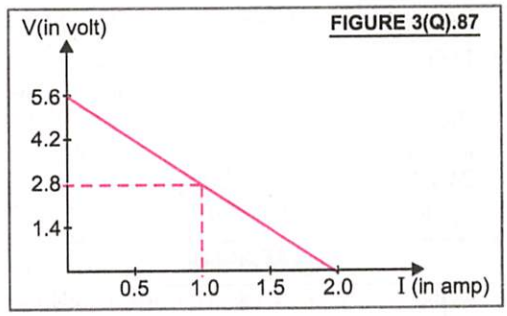
47. A voltmeter of resistance 990 Ω is connected across a cell of emf 3 V and internal resistance 3 Ω. Find the terminal potential difference of the cell and percentage error in the reading of voltmeter. [Ans. 2·99 V, 0·33%]

48. A battery of emf ε, and internal resistance r, gives a current of 0·5 A with an external resistor of 12 Ω and a current of 0·25 A with an external resistor of 25 Ω. Calculate (*i*) internal resistance of the cell and (*ii*) emf of the cell.

(CBSE 2002, 2013 S) [Ans. 1 Ω, 6·5 V]

49. 4 cells each of identical emf E, internal resistance r, are connected in series to a variable resistor. Fig. 3(Q).87 shows the variation of terminal voltage of the combination with the current output :

FIGURE 3(Q).87



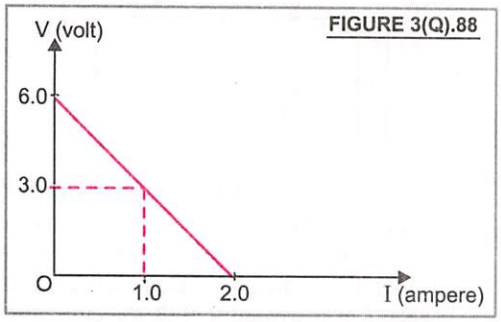
(*a*) What is the emf of each cell used ?
(*b*) For what current from the cells, does maximum power dissipation occur in the circuit.
(*c*) Calculate the internal resistance of each cell.

[CBSE 2006 (C)]

[Ans. (*a*) 1·4 V (*b*) 1 A (*c*) 0·7 Ω]

50. The following graph shows the variation of terminal potential difference V, across a combination of three cells in series to a resistor, versus the current I.

FIGURE 3(Q).88



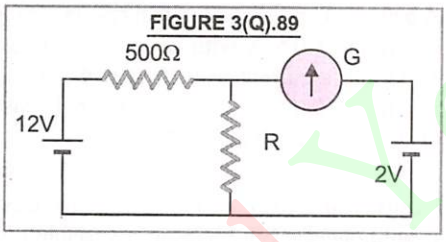
(a) Calculate the emf of each cell.

(b) For what current I, will the power dissipation of the circuit be maximum. **(CBSE 2008)**

[Ans. (a) 2 V (b) 1 A]

51. The galvanometer G in the circuit, Fig. 3(Q).89 reads zero. Find the value of R. Batteries are resistance less.

FIGURE 3(Q).89

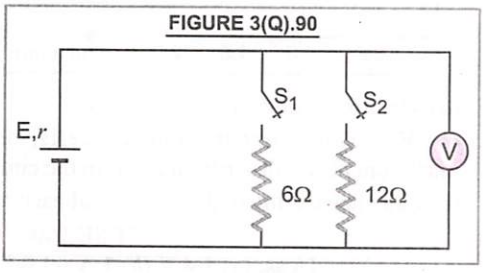


[Ans. 100 Ω]

52. A voltmeter with resistance 500 Ω is used to measure the emf of a cell of internal resistance 4 Ω. What will be the percentage error in the reading of the voltmeter. **[Ans. 0·8%]**

53. In the circuit shown in Fig. 3(Q).90, when switch S_1 is closed and S_2 is open, the ideal voltmeter shows a reading 18 V. When switch S_2 is closed and S_1 is open, the reading of the voltmeter is 24 V. What will be the reading of voltmeter when S_1 and S_2, both are closed ?

FIGURE 3(Q).90

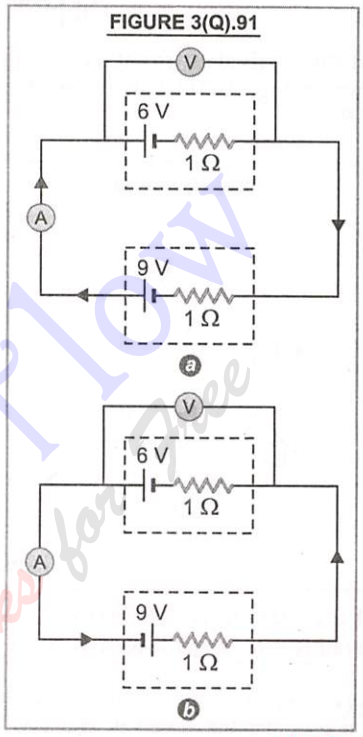


[Ans. 14·4 V]

TYPE IX. GROUPING OF CELLS

54. In the two electric circuit shown in Fig.3(Q).91, determine the readings of ideal ammeter (A) and ideal voltmeter (V).

FIGURE 3(Q).91



[CBSE 2015 (C)]

[Ans. (a) 7·5 V, 1·5 A (b) – 1·5 V, 7·5 A]

55. A cell of emf 1·1 V and internal resistance 0·5 Ω is connected to a wire of resistance 0·5 Ω. Another cell of the same emf is connected in series but the current in the wire remains the same. Find the internal resistance of second cell.

[Ans. 1 Ω]

56. A set of 4 cells each of emf 2 V and internal resistance 1·5 Ω are connected across an external load of 10 Ω with 2 rows, two cells in each row. Calculate the current in each row and potential difference across 10 Ω.

[Ans. 0·175 A, 0·175 A, 3·5 V]

57. Find the minimum number of cells required to produce an electric current of 1·5 A through a resistance of 30 Ω. Given that the emf and internal resistance of each cell are 1·5 V and 1·0 Ω respectively. **[Ans. 120]**

58. n identical cells are joined in series with two cells A and B with reverse polarities. EMF of each cell

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is E and internal resistance r. What is the potential difference across the cell A or B ?

[Ans. $2E\left(1-\frac{2}{n}\right)$]

59. A set of 6 cells, each of emf 2 V and internal resistance 1·5 Ω, are connected across an external load of 10 Ω with 2 rows, 3 cells in each branch. Calculate current in each branch and potential difference across 10 Ω.

[Ans. 0·245 A, 0·245 A, 4·9 V]

60. A student connects a cell, of *emf* ε_2 and internal resistance r_2, with a cell of *emf* ε_1 and internal resistance r_1, such that their combination has a net internal resistance less than r_1. This combination is then connected across a resistance R. Draw a circuit of the 'set up' and obtain an expression for the current flowing through the resistance R. **(CBSE 2016)**

$$\left[\text{Ans. } \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 \times r_2) + R(r_1 + r_2)}\right]$$

TYPE X. ELECTRIC POWER

61. A generator is supplying power to a factory by cables of resistance 20 Ω. If the generator is generating 50 kW power at 5000 V, what is the power received by factory ? **[Ans. 48 kW]**

62. Calculate the amount of heat produced per second (in calories) when a bulb of 100 W, 220 V glows, assuming that only 20% of electric energy is converted into light. $J = 4{\cdot}2\ \text{J cal}^{-1}$.

(Hr. Board 2001) [Ans. 19·05 cal]

63. An electric motor operates on a 50 V supply and draws a current of 15 A. If the motor yields a mechanical power of 150 W, estimate the power dissipated across its windings. Also find the efficiency of the motor ? **[Ans. 600 W, 20%]**

64. Find the resistance of 240 V–200 watt electric bulb when glowing. If this resistance is 10 times the resistance at 0°C and the temperature of the glowing filament is 2000°C, then find the temperature coefficient of resistance of the filament. **[Ans. 288 Ω ; $4{\cdot}5 \times 10^{-3}\ °C^{-1}$]**

65. Three equal resistances connected in series across a source of e.m.f. consume 20 watt. If the same resistors are connected in parallel across the same source of e.m.f., what would be the power dissipated ? **[Ans. 180 watt]**

66. In a house having 220 V line, the following appliances are working (*i*) a 60 W bulb (*ii*) a 1000 W heater (*iii*) a 40 W radio. Calculate (*a*) the current drawn by heater and (*b*) the current passing through the fuse line. **[Ans. (*a*) (50/11) A (*b*) 5 A]**

67. A series battery of 10 lead accumulators each of *emf* 2 V and internal resistor 0·25 ohm is charged by a 220 V d.c. mains. To limit the charging current, a resistance of 47·5 Ω is used in series in the charging circuit. What is (*a*) the power supplied by the mains and (*b*) power dissipated as heat ? (*c*) Account for the difference of power in (*a*) and (*b*).

[Ans. (*a*) 880 W (*b*) 800 W (*c*) 80 W stored in battery as chemical energy]

TYPE XI. ELECTRIC ENERGY AND HEATING EFFECT OF CURRENT

68. A dry cell of *emf* 1·6 V and internal resistance of 0·10 Ω is connected to a resistor of resistance R Ω. If the current drawn from the cell is 2 A, then (*i*) What is the voltage drop across R ? (*ii*) What is the rate of energy dissipation in the resistor ? **[Ans. (*i*) 1·4 V (*ii*) 2·8 W]**

69. Two wires A and B of same material and mass, have their lengths in the ratio 1 : 3. On connecting them, one at a time to the same source of *emf*, the rate of heat dissipated in B is 10 W. What is the rate of heat dissipated in A ? **[Ans. 90 W]**

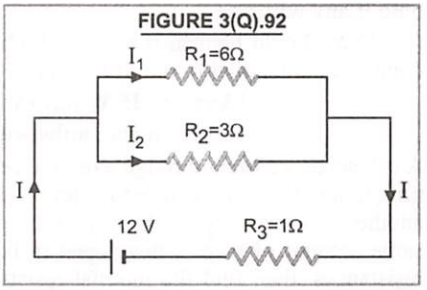
70. An electric bulb is marked to 100 W, 230 V. If the supply voltage drops to 115 V, what is the heat and light energy produced by the bulb in 20 minutes. Calculate the current flowing through it. **(NCERT) [Ans. 30 kJ, (5/23) A]**

71. An electric kettle used to prepare tea, takes 2 minutes to boil 4 cups of water (1 cup contains 200 c.c. of water), and the room temperature is 25°C. (*a*) If the cost of electricity is Rs. 3·00 per unit of electricity, calculate the cost of boiling 4 cups of water. (*b*) What will be the corresponding cost if the room temperature drops to 5°C ?

[Ans. (*a*) 21 paise (*b*) 27 paise]

72. Find the heat developed per minute in each of the three resistors R_1, R_2 and R_3 shown in Fig. 3(Q).92.

FIGURE 3(Q).92



[Ans. 640 J; 1280 J; 960 J]

73. A motor operating at 110 V draws a current of 2A. If the heat is developed in the motor at the rate of 10 cal s^{-1}, what is its efficiency ?

[Ans. 80·9%]

74. A room is lighted by 200 W, 124 V incandescent lamps fed by a generator whose output voltage is 130 V. The conducting wires from the generator to the user are made of aluminium wire of total length 150 m and cross-sectional area 15 mm^2. How many such lamps can be installed ? What is the total power consumed by the user ? sp. resistance of aluminium is $2{\cdot}9 \times 10^{-8}\ \Omega\ m$.

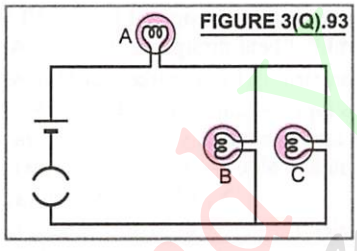
[Ans. 12, 2·4 kW]

75. Twenty one electric bulbs are connected in series with the mains of a 220 V supply. After one bulb is fused, the remaining 20 bulbs are again connected in series across the same mains. By what percentage will the illumination of (*i*) a bulb change (*ii*) all the bulbs change ?

[Ans. (*i*) 10·25% (*ii*) 5%]

76. Three 60 W, 120 V light bulbs are connected across a 120 V power line as shown. Fig. 3(Q).93. Find (*a*) the voltage across each bulb
(*b*) the total power developed across three bulbs.

FIGURE 3(Q).93



[Ans. (*a*) $V_A = 80$ V ; $V_B = V_C = 40$ V ; (*b*) 40 W]

77. A house wiring, supplied with a 220 V supply line is protected by a 9 ampere fuse. Find the maximum number of 60 W bulbs in parallel that can be turned on. **[Ans. 33]**

78. (*a*) Two bulbs rated 25 W – 220 V and 100 W – 220V are connected in series to a 440 V supply. Show with necessary calculations which bulb if any will fuse ?
(*b*) What should happen if the two bulbs were connected in parallel to the same supply ?

**[Ans. (*a*) 25 W bulb will fuse
(*b*) Both the bulbs will fuse]**

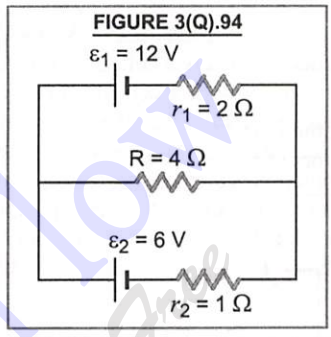
79. A cell sends a current through a resistance R_1 for time *t*, next the same cell sends current through another resistance R_2 for the same time *t*. If the same amount of heat is developed in both the resistances, then find the internal resistance of the cell. **[Ans. $\sqrt{R_1 R_2}$]**

TYPE XII. KIRCHHOFF'S LAWS

80. Find the potential difference across each cell and the rate of energy dissipated in *R*, Fig. 3(Q).94. (CBSE Sample Paper 2011)

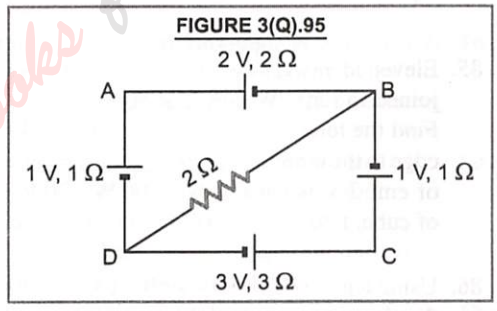
$\left[\text{Ans. } \frac{48}{7}\text{ V}, \frac{48}{7}\text{ V}, 11{\cdot}75\text{ W}\right]$

FIGURE 3(Q).94



81. Using Kirchhoff's rules, calculate the potential difference between *B* and *D* in the circuit as shown in the Fig. 3(Q).95. [CBSE 2018 (C)]

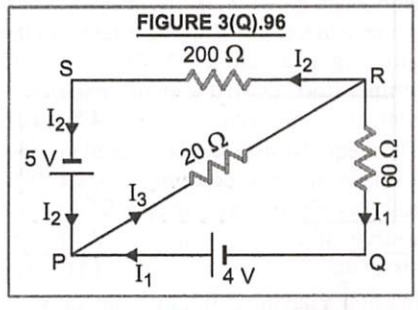
FIGURE 3(Q).95



$\left[\text{Ans. } \frac{2}{13}\text{ V}\right]$

82. Apply Kirchhoff's rules to the loops *PRSP* and *PRQP* to write the expressions for the currents I_1, I_2 and I_3 in the circuit shown in Fig. 3(Q).96.

FIGURE 3(Q).96



(CBSE 2010) $\left[\text{Ans. } \frac{39}{860}\text{ A}; \frac{4}{215}\text{ A}; \frac{11}{172}\text{ A}\right]$

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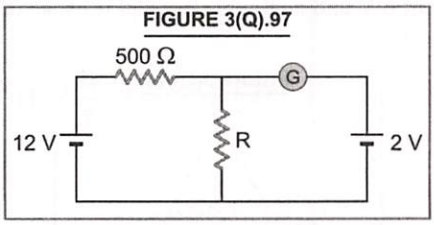
83. In the circuit shown in Fig. 3(Q).97 the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance find the value of the resistor R.

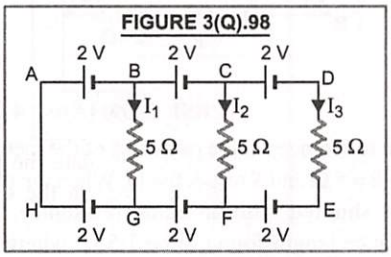
FIGURE 3(Q).97



(CBSE F 2013) **[Ans. 100 Ω]**

84. Find the currents I_1, I_2 and I_3 through the three resistors of the circuit as shown in Fig. 3(Q).98.

FIGURE 3(Q).98



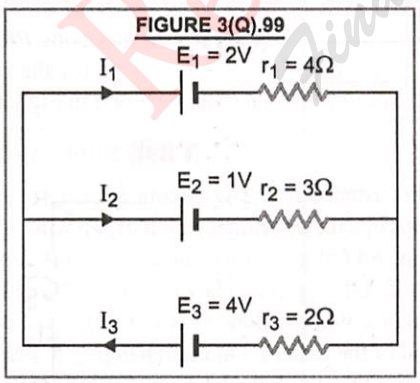
[Ans. Zero in each resistor]

85. Eleven identical wires each of resistance 5 Ω are joined to form the edges of an incomplete cube. Find the total resistance from one end of vacant edge to the other end of the same cube. If a battery of emf 2 V is connected across the vacant edge of cube, find the current supplied by the battery.

[Ans. 7 Ω, 1/7 A]

86. Using Kirchhoff's rules, write the expressions for the currents I_1, I_2 and I_3 in the circuit diagram shown, Fig. 3(Q).99. (CBSE 2010)

FIGURE 3(Q).99

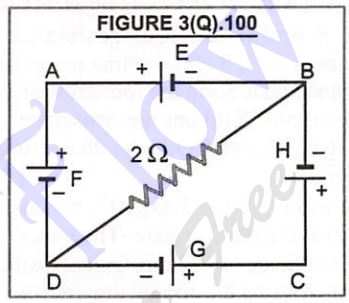


$$\left[\text{Ans. } I_1 = \frac{2}{13}A, I_2 = \frac{7}{13}A, I_3 = \frac{9}{13}A\right]$$

87. Twelve identical wires each of resistance 6 Ω are arranged to form a skeleton cube. A current of 40 m A is led into cube at one corner and out at the diagonally opposite corner. Calculate the potential difference developed across these corners and the effective resistance of the network. **[Ans. 0·2 V ; 5 Ω]**

88. In the circuit Fig 3(Q).100. E, F, G and H are cells of emf 1, 2, 2 and 4 volt and their internal resistances are 1, 2, 2 and 1 Ω respectively. Calculate (*i*) the potential difference between D and B, (*ii*) the potential difference across the terminals and each of the cells G and H.

FIGURE 3(Q).100

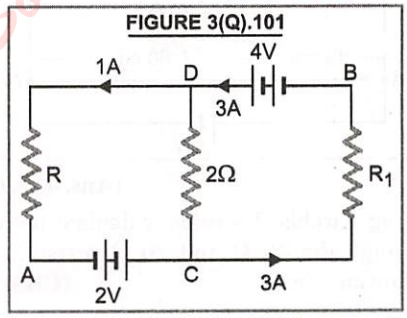


[Ans. (*i*) 0·29 V (*ii*) 3·14 V ; 3·43 V]

89. In the given circuit Fig. 3(Q).101, assuming point A to be at zero potential, use Kirchhoff's rules to determine the potential at point B.

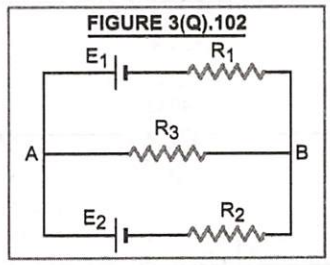
(CBSE 2011) **[Ans. 2 V]**

FIGURE 3(Q).101



90. Find the potential difference across R_3 in the circuit shown in Fig. 3(Q).102.

FIGURE 3(Q).102



$$\left[\text{Ans. } \frac{(E_1R_2 + E_2R_1)R_3}{R_1R_2 + R_1R_3 + R_2R_3}\right]$$

TYPE XIII. WHEATSTONE BRIDGE PRINCIPLE AND SLIDE WIRE BRIDGE

91. (*i*) In a meter bridge, the balance point is found to be at 30 cm from the end *A* when the resistance *R* in left gap of bridge is of 12 Ω. Find resistance *S* in the right gap of bridge.

(*ii*) If the cell and the galvanometer are interchanged at the balance point, would it effect the flow of current through the galvanometer.

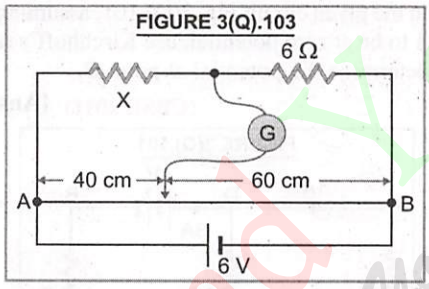
(*iii*) Calculate the balance point of the bridge if *R* and *S* are interchanged.

[Ans. (*i*) 28 Ω (*ii*) no effect (*iii*) 70 cm]

92. *P*, *Q*, *R* and *S* are four resistance wires of resistances 3, 3, 3 and 4 ohms respectively. They are connected to form the four arms of Wheatstone bridge circuit. Find out the resistance with which *S* must be shunted in order that bridge may be balanced. **[Ans. 12 Ω]**

93. In the circuit Fig. 3(Q).103, a meter bridge is shown in its balanced state. The meter bridge wire has a resistance of 1 Ω/cm. Calculate the unknown resistance *X* and the current drawn from the battery of negligible internal resistance.

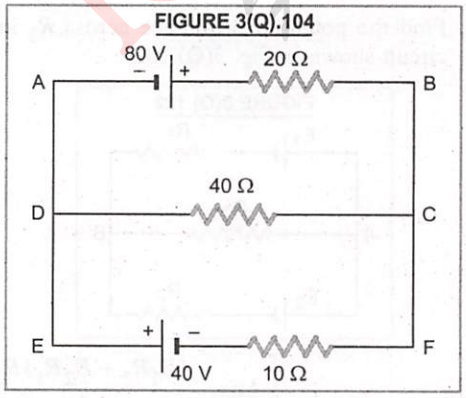
FIGURE 3(Q).103



[Ans. 4 Ω, 0·66 A]

94. Using Kirchhoff's rules, calculate the current through the 40 Ω and 20 Ω resistors in the following circuit. **(CBSE 2019)**

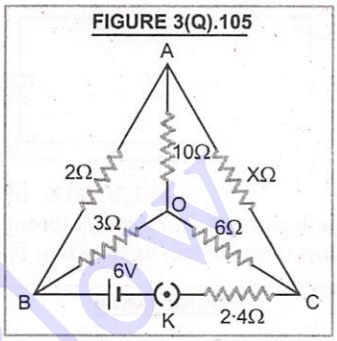
FIGURE 3(Q).104



[Ans. 0 A, 4 A]

95. Find the value of the unknown resistance X, in the following circuit, if no current flows through the section AO, Fig. 3(Q).105. Also, calculate the current drawn by the circuit from the battery of emf 6 V and negligible internal resistance.

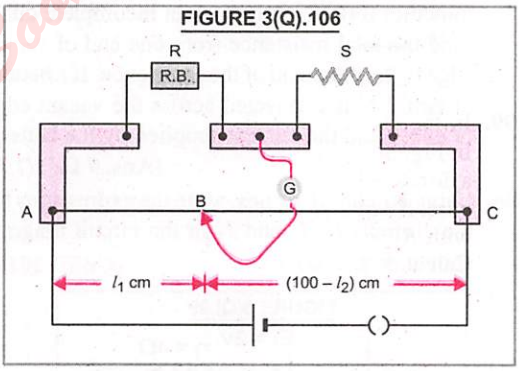
FIGURE 3(Q).105



(CBSE 2009) [Ans. 4 Ω ; 1 A]

96. The resistances in the two arms of the meter bridge are $R = 5\ \Omega$ and *S* respectively. When the resistance *S* is shunted with an equal resistance, the new balance length found to be 1·5 l_1, where l_1 is the initial balancing length. Calculate the value of *S*.

FIGURE 3(Q).106



(CBSE 2019) [Ans. 10 Ω]

97. A resistance $R = 2\ \Omega$ is connected to one of the gaps in a meter bridge, which uses a wire of length 1 m. An unknown resistance $X > 2\ \Omega$ is connected to the other gap as shown in Fig. 3(Q).107. The balance point is noticed at *l* cm from the positive end of the battery. On interchanging *R* and *X*, it is found that the balance point further shifts by 20 cm (away from end *A*). Neglecting the end correction, calculate the value of unknown resistance *X* used.

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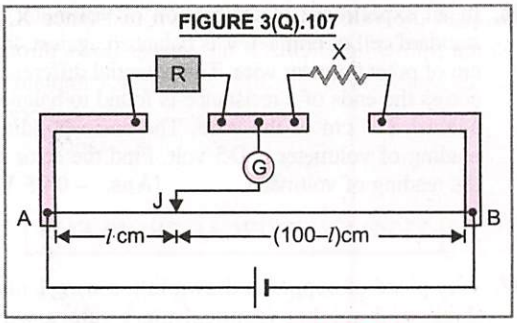
FIGURE 3(Q).107

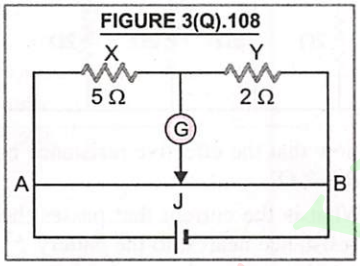


(CBSE 2008) **[Ans. 3 Ω]**

98. In the simple Wheatstone bridge circuit, where the length AB of bridge wire is 1 m the resistors X and Y have values 5 Ω and 2 Ω respectively, Fig. 3(Q).108. When X is shunted by a length of a wire, the balance point is found to be 0·625 m from A. What is the resistance of the shunt ?

If the shunt wire is 0·75 m long and 0·25 mm in diameter, what is the resistivity of the material of the wire ?

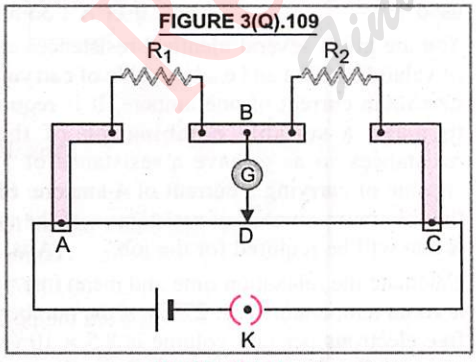
FIGURE 3(Q).108



[Ans. 10 Ω ; 6·54 × 10^{-7} Ω m]

99. In the meterbridge experimental set up, shown in Fig. 3(Q).109, the null point D is obtained at a distance of 40 cm from end A of the meter bridge wire. If a resistance of 10 Ω is connected in series with R_1, null point is obtained at AD = 60 cm. Calculate the values of R_1 and R_2.

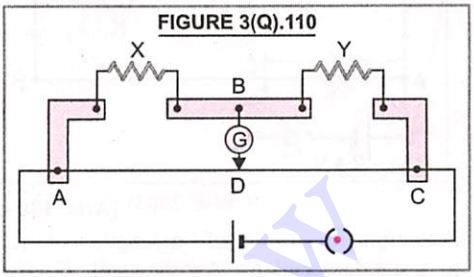
FIGURE 3(Q).109



(CBSE 2013) **[Ans. 8 Ω and 12 Ω]**

100. Fig. 3(Q).110 shows experimental set up of a meter bridge. When the two unknown resistances X and Y are inserted, the null point D is obtained 40 cm from the end A. When a resistance of 30 Ω is connected in series with X, the null point shifts by 10 cm. Find the position of the null point when the 30 Ω resistance is connected in series with resistance Y instead of X. Determine the values of resistances X and Y.

FIGURE 3(Q).110



(CBSE 2009) **[Ans. 33·3 cm ; 60 Ω ; 90 Ω]**

TYPE XIV. POTENTIOMETER

101. In a potentiometer, a standard cell of emf 5 V and of negligible resistance maintains a steady current through the galvanometer wire of length 5 m. Two primary cells of emfs ε_1 and ε_2 are joined in series with (*i*) same polarity and (*ii*) opposite polarity. The combination is connected through a galvanometer and a jockey to the potentiometer. The balancing lengths in the two cases are found to be 350 cm and 50 cm respectively.

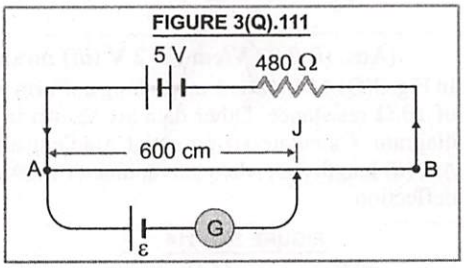
(*i*) Draw the necessary circuit diagram.

(*ii*) Find the value of emfs of the two cells.

[CBSE 2004 (C)] **[Ans. 2·0 V ; 1·50 V]**

102. A 10 metre long wire of uniform cross-section of 20 Ω resistance is used as a potentiometer wire. This wire is connected in series with a battery of

FIGURE 3(Q).111



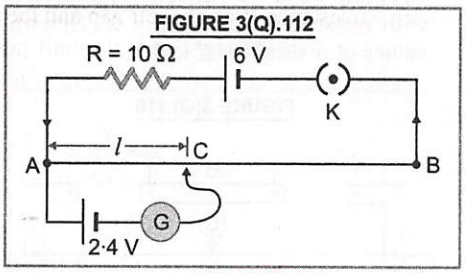
5 V, along with an external resistance of 480 Ω. If an unknown emf ε is balanced at 600 cm of this wire, calculate (*i*) the potential gradient of the potentiometer wire and (*ii*) the value of the unknown emf ε. (CBSE 2006)

[Ans. (*i*) 2 × 10^{-4} V/cm ; (*ii*) 0·12 V]

103. The potentiometer wire of length 200 cm has a resistance of 20 Ω. It is connected in series with a resistance 10 Ω and an accumulator of emf

6 V having negligible internal resistance. A source of 2·4 V is balanced against length l of the potentiometer wire. Find the length l.

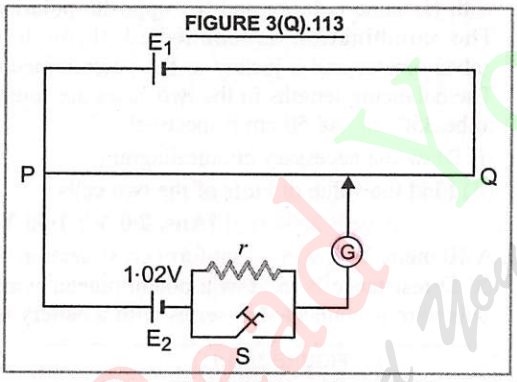
FIGURE 3(Q).112



(CBSE 2003) **[Ans. 120 cm]**

104. Potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell, E_2 of emf 1·02 V is connected as shown in Fig. 3(Q).113 with a resistance r and a switch S. With switch, S open, null position is obtained at a distance of 51 cm from P. Calculate (*i*) potential gradient of the potentiometer wire and (*ii*) emf of the cell E_1. (*iii*) when switch S is closed, will null point move towards P or towards Q ? Give reason for your answer ? (CBSE 2004)

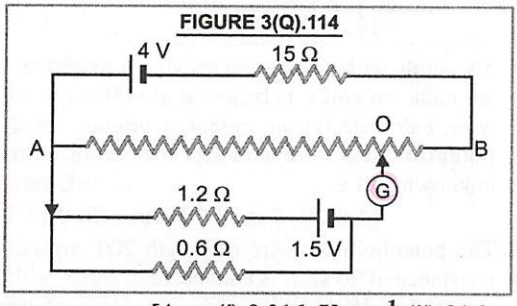
FIGURE 3(Q).113



[Ans. (*i*) 0·02 V/cm (*ii*) 2 V (*iii*) no shift]

105. In Fig. 3(Q).114, AB is 1 metre long uniform wire of 10 Ω resistance. Other data are shown in the diagram. Calculate (*i*) potential gradient along AB, (*ii*) length AO, when galvanometer shows no deflection.

FIGURE 3(Q).114



[Ans. (*i*) 0·016 V cm^{-1} (*ii*) 31·2 cm]

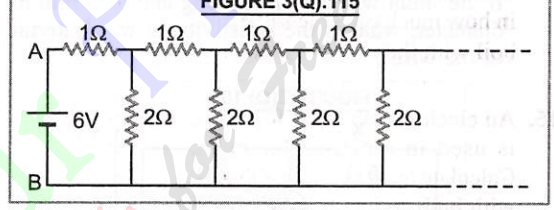
106. In an experiment of calibration of voltmeter, a standard cell of emf 1·1 V is balanced against 440 cm of potentiometer wire. The potential difference across the ends of a resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0·5 volt. Find the error in the reading of voltmeter. **[Ans. – 0·05 V]**

TYPE XV. TYPICAL PROBLEMS

107. 1 kg piece of copper is drawn into a wire 1 mm thick, and another identical piece into a wire 2 mm thick. Compare the resistance of these wires. **[Ans. 16 : 1]**

108. An infinite ladder net work of resistance is constructed with 1 Ω and 2 Ω resistances, as shown in Fig. 3(Q). 115. The 6 V battery between A and B has negligible internal resistance.

FIGURE 3(Q).115



(*i*) Show that the effective resistance between A and B is 2 Ω.

(*ii*) What is the current that passes through the 2 Ω resistance nearest to the battery ?

[Ans. (*ii*) 1·5 A]

109. A cell of emf 2 V and internal resistance 0·1 Ω supplies a current through a coil of resistance 11·9 Ω. The current is being measured by an ammeter whose resistance is 6 Ω. What reading does it give ? What is the percentage difference from the actual current, when the meter is not used ? **[Ans. 0·11 A ; 33·3%]**

110. You are given several identical resistances each of value $R = 10$ Ω and each capable of carrying a maximum current of one ampere. It is required to make a suitable combination of these resistances so as to have a resistance of 5 Ω capable of carrying a current of 4 ampere. Find the minimum number of resistances of the type R that will be required for the job. **[Ans. 8]**

111. Calculate the relaxation time and mean free path at room temperature (*i.e.* 27°C), if the number of free electrons per unit volume is $8{\cdot}5 \times 10^{28}/\text{m}^3$ and resistivity $\rho = 1{\cdot}7 \times 10^{-8}$ Ω-m. Given that mass of electron $= 9{\cdot}1 \times 10^{-31}$ kg, $e = 1{\cdot}6 \times 10^{-19}$ C and $k = 1{\cdot}38 \times 10^{-23}$ JK^{-1}.

[Ans. $2{\cdot}46 \times 10^{-14}$ s ; 28·7 Å]

112. Twelve cells each having the same e.m.f. are connected in series and are kept in a closed box. Some of the cells are connected in reverse order. The battery is connected in series with an ammeter, an external resistance R and two cells of the same type as in the battery. The current when they aid each other is 3 ampere and current is 2 ampere when the two oppose each other. How many cells are connected in reverse order ?

[Ans. 1]

113. A battery of emf E is connected with three resistances R, 2 R and 3 R in series. The voltage across 2 R is measured with a voltmeter whose resistance is 10 R. What is the percentage error ?

[Ans. 11·76%]

114. Water boils in an electric kettle in 15 minutes after switching on. If the length of the heating wire is decreased to 2/3 of its initial value, then in how much time the same amount of water will boil with the same supply voltage.

[Ans. 10 minutes]

115. An electric bulb rated for 500 watt and 100 volt is used in a circuit having a 250 V supply. Calculate the resistance R that may be put in series with bulb so that the bulb delivers 500 watt.

[Ans. 30 Ω]

116. The same mass of copper is drawn into two wires 1 mm and 2 mm thick. These two wires are connected in series to the source of current. What is the ratio of the heat produced in the wires ?

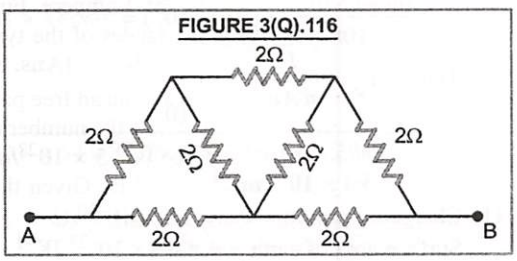
[Ans. 16 : 1]

117. (*i*) A storage battery of emf 8 V, internal resistance 1 Ω is being charged by a 120 V d.c. source, using a 15 Ω resistor in series in the circuit. Calculate the current in the circuit. (*ii*) terminal voltage across the battery during charging and (*iii*) chemical energy stored in the battery in 5 minutes. **(CBSE 2001, 2008)**

[Ans. (*i*) 7 A (*ii*) 15 V (*iii*) 16800 J]

118. In the net work shown in Fig. 3(Q).116 each resistance is 2 Ω. Find the effective resistance between A and B.

FIGURE 3(Q).116



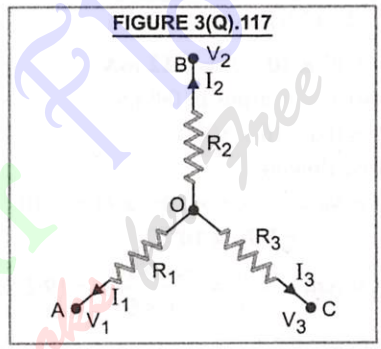
[Ans. 16/7 Ω]

119. In a meter-bridge experiment with a resistance R_1 in left gap and a resistance X in a right gap, null point is obtained at 40 cm from the left end. With a resistance R_2 in the left gap and the same resistance X in the right gap, the null point is obtained at 50 cm from left hand. Find the position of the null point if the right gap is containing X and the left gap is containing R_1 and R_2 (*i*) in series and (*ii*) in parallel.

[Ans. (*i*) 62·5 cm (*ii*) 28·6 cm]

120. A circuit has a section ABC as shown in Fig. 3(Q).117. If the potentials at points A, B and C are V_1, V_2 and V_3 respectively, calculate the potential at point O.

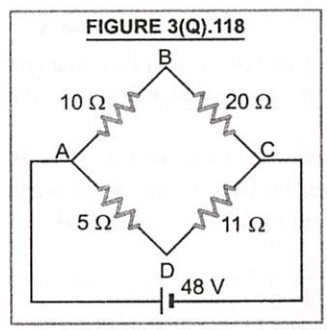
FIGURE 3(Q).117



$$\left[\text{Ans. } \left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1}\right]$$

121. A Wheatstone bride is almost balanced [Fig. 3(Q).118]., with point C grounded. Calculate (*a*) the potential of point B, (*b*) the potential of point D. (*c*) If a galvanometer is connected between B and D, what is the direction of current through it ? (*d*) For what value of the resistance BC, would the bridge be in balanced state ?

FIGURE 3(Q).118



[Ans. (*a*) 32 V, (*b*) 33 V, (*c*) D to B (*d*) 22 Ω]

HINTS/EXPLANATIONS

For Difficult Questions

1. $I = ev = (1{\cdot}6 \times 10^{-19}) \times (6{\cdot}8 \times 10^{15})$

$= \mathbf{1{\cdot}088 \times 10^{-3}\ A}$

2. $I = I_{Na} + I_{Cl} = (n_{Na} + n_{Cl})\, e/t$

$$= \frac{(6{\cdot}0\times10^{16} + 4{\cdot}5\times10^{16})\times1{\cdot}6\times10^{-19}}{2}$$

$= 8{\cdot}4 \times 10^{-3}$ A $= \mathbf{8{\cdot}4\ mA}$

3. $I = \dfrac{e}{T} = \dfrac{e}{2\pi r/v} = \dfrac{ev}{2\pi r}$

$$= \frac{(1{\cdot}6\times10^{-19})\times(2{\cdot}2\times10^{6})}{2\times(22/7)\times(5{\cdot}0\times10^{-11})}$$

$= 1{\cdot}12 \times 10^{-3}$ A $= \mathbf{1{\cdot}12\ mA}$

4. (*a*) No. of electrons in 0·6 mol,

$N = 0{\cdot}6 \times (6 \times 10^{23})$

Charge flowing,

$q = Ne = 0{\cdot}6 \times 6 \times 10^{23} \times (1{\cdot}6 \times 10^{-19})$ C

$= \mathbf{5{\cdot}76 \times 10^{4}\ C}$

(*b*) Current $I = \dfrac{q}{t} = \dfrac{5{\cdot}76\times10^{4}}{50\times60} = \mathbf{19{\cdot}2\ A}$

5. Given, $q = 8t - 3t^2 + 5t^3$

(*i*) Current $I = \dfrac{dq}{dt} = 8 - 6t + 15t^2$ ...(*i*)

When $t = 0$, $I = \mathbf{8A}$

(*ii*) $\dfrac{dI}{dt} = -6 + 30t$

For I to be maximum or minimum

$\dfrac{dI}{dt} = 0$ or $-6 + 30\,t = 0$ or $t = \mathbf{0{\cdot}2\ s}$

(*iii*) Putting this value of t in (*i*), we get

$I = 8 - 6 \times 0{\cdot}2 + 15\,(0{\cdot}2)^2 = 8 - 1{\cdot}2 + 0{\cdot}6$

$= \mathbf{7{\cdot}4\ A}$

As this value of I is less than that at $t = 0$, it will be minimum value. So minimum value of current $= \mathbf{7{\cdot}4\ A}$

6. (*i*) Amount of charge that flows in 10 s

= area (*OABD*) under the *I*-*t* graph

= Area of ΔOAE + area *ABDE*

$$= \frac{5\times4}{2} + 5\times(10-4) = 10 + 30 = \mathbf{40\ C}$$

(*ii*) Amount of charge that flows in 14 s

= area *OABC* = Area *OABD* + area *DBC*

$= 40 + 5 \times (14 - 10)/2 = \mathbf{50\ C}$

7. Total charge flowing in two minutes,

$q = It = (20 \times 10^{-6}\ A) \times (2 \times 60\ s) = 24 \times 10^{-4}$ C

Charge flowing due to electrons

$$= \frac{24\times10^{-4}}{2} = 12\times10^{-4}\ \text{C}$$

No. of electrons flowing across a cross-section of a tube in 2 minutes is

$$n = \frac{12\times10^{-4}}{1{\cdot}6\times10^{-19}} = \mathbf{7{\cdot}5 \times 10^{15}}$$

8. $I = nAev_d$ or $\dfrac{\varepsilon}{R} = nAev_d$ $\quad(\because I = \varepsilon/R)$

or $\dfrac{\varepsilon}{(\rho l/A)} = nAev_d$ $\quad\left[\because R = \dfrac{\rho l}{A}\right]$

or $\dfrac{\varepsilon}{\rho l} = nev_d$ or $v_d = \dfrac{\varepsilon}{ne\rho l}$ *i.e.*, $v_d \propto \dfrac{1}{l}$

$\therefore \dfrac{v_d'}{v_d} = \dfrac{l}{3l}$ or $v_d' = \dfrac{v_d}{\mathbf{3}}$

9. If A is the area of cross-section of the wire, then number of electrons per unit volume of copper wire is

$$n = \frac{2\times10^{21}}{A\times(5\times10^{-2})}\ \text{m}^{-3}$$

$I = nAev_d$

$$= \frac{2\times10^{21}}{A\times(5\times10^{-2})} \times A \times (1{\cdot}6 \times 10^{-19}) \times (0{\cdot}25 \times 10^{-3})$$

$= \mathbf{1{\cdot}6\ A}$

10. No. of electrons per unit volume of silver,

$$n = \frac{N}{M/d} = \frac{Nd}{M}$$

$$= \frac{6{\cdot}023\times10^{26}}{108} \times (10{\cdot}5\times10^{3}) = 5{\cdot}8557 \times 10^{28}$$

Then, $v_d = \dfrac{I}{nAe}$

$$= \frac{10}{5{\cdot}8557\times10^{28}\times3{\cdot}14\times10^{-6}\times1{\cdot}6\times10^{-19}}$$

$= \mathbf{3{\cdot}4 \times 10^{-4}\ ms^{-1}}$

11. Charge on proton $= +e = 1{\cdot}6 \times 10^{-19}$ C.

Surface area of earth $= 4\pi R^2$

$= 4 \times 3{\cdot}14 \times (6{\cdot}4 \times 10^{6})^2$

$= 5{\cdot}14 \times 10^{14}\ \text{m}^2$

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Charge received by earth per second per m^2

$= n e = 10 \times 1{\cdot}6 \times 10^{-19}$ C

So, current = rate of flow of charge

$= n e \times 4\pi R^2$

$= (10 \times 1{\cdot}6 \times 10^{-19}) \times 5{\cdot}14 \times 10^{14}$

$= \mathbf{0{\cdot}82 \times 10^{-3}}$ **A**

12. $t = \dfrac{l}{v_d} = \dfrac{l}{I/nAe} = \dfrac{lnAe}{I}$

$= \dfrac{1 \times (8 \times 10^{28}) \times (5 \times 10^{-7}) \times (1{\cdot}6 \times 10^{-19})}{1}$

$= \mathbf{6{\cdot}4 \times 10^{3}}$ **s**

13. $I = nAe v_d = nAe\mu E = nAe\mu \dfrac{V}{l}$

$= (8{\cdot}5 \times 10^{28}) \times (10^{-3})^2 \times (1{\cdot}6 \times 10^{-19}) \times (4{\cdot}5 \times 10^{-6}) \times \dfrac{4}{0{\cdot}2}$

$= \mathbf{1{\cdot}22}$ **A**

14. Here, density, $d = 8{\cdot}9 \times 10^3$ kg m^{-3} ; $M = 63{\cdot}5$ kg ;

$N = 6{\cdot}02 \times 10^{26}$ per kg atom ; $I = 1{\cdot}5$ A,

$D = 1{\cdot}2$ mm $= 1{\cdot}2 \times 10^{-3}$ m.

Mass of 1 m^3 of copper $= 8{\cdot}9 \times 10^3$ kg

Number of atoms per m^3 of copper $= \dfrac{N}{M/d} = \dfrac{Nd}{M}$

As 1 atom contributes one conduction electron, therefore number of conduction electrons per m^3 of copper is

$n = \dfrac{Nd}{M} = \dfrac{(6{\cdot}02 \times 10^{26}) \times (8{\cdot}9 \times 10^3)}{63{\cdot}5}$

$= 8{\cdot}44 \times 10^{28}$ m^{-3}

$v_d = \dfrac{I}{nAe}$

$= \dfrac{1{\cdot}5}{(8{\cdot}44 \times 10^{28}) \times \left[\dfrac{22}{7} \times \dfrac{(1{\cdot}2 \times 10^{-3})^2}{4}\right] \times (1{\cdot}6 \times 10^{-19})}$

$= 0{\cdot}98 \times 10^{-4}$ $ms^{-1} \approx \mathbf{10^{-4}}$ $\mathbf{ms^{-1}}$

15. Resistance of lamp, $R = \dfrac{V}{I} = \dfrac{80}{10} = 8\Omega$

In order to use an arc lamp with operating voltage 240 V, a resistance R' is to be connected in series with it so that the current in the circuit should not exceed 10 A. Then

$V = I(R + R')$ or $240 = 10(8 + R')$

or $R' = 24 - 8 = \mathbf{16\ \Omega}$

16. Here, $l = 0{\cdot}50$ m ; $D = 0{\cdot}12 \times 10^{-3}$ m, $R = 4{\cdot}0\ \Omega$

$l_1 = 1{\cdot}5$ m ; $D_1 = 0{\cdot}15 \times 10^{-3}$ m, $R_1 = ?$

$R = \dfrac{\rho l}{\pi D^2/4}$ or $R \propto \dfrac{l}{D^2}$

$\therefore \dfrac{R_1}{R} = \dfrac{l_1}{l} \times \dfrac{D^2}{D_1^2} = \dfrac{1{\cdot}5}{0{\cdot}5} \times \dfrac{(0{\cdot}12 \times 10^{-3})^2}{(0{\cdot}15 \times 10^{-3})^2} = \mathbf{1{\cdot}92\ \Omega}$

17. (*i*) Let A, l be the original area of cross section and length of the given wire and A_1, l_1 be the respective new values when the wire is drawn to twice its original length. Then $l_1 = 2\,l$. Using the relation $A\,l = A_1\,l_1$, we have $A_1 = A\,l/l_1$

$= A\,l/2\,l = A/2$. Therefore,

$R_1 = \rho\,l_1/A_1 = \rho\,2\,l/(A/2)$

$= 4\,\rho\,l/A = 4R = 4 \times 100 = \mathbf{400\ \Omega}$

(*ii*) When wire is drawn to half its radius, new area becomes 1/4th. Length would become 4 times. $\therefore R_2 = 16R = 16 \times 100 = \mathbf{1600}$ **ohm**

18. As, under the same potential difference across the two wires, the same current flows in them so the resistance of two wires must be same.

Now, $R = \dfrac{\rho l}{\pi r^2}$;

So $R_{Cu} = \dfrac{\rho_{Cu}\,l}{\pi r_{Cu}^2}$ and $R_I = \dfrac{\rho_I\,l}{\pi r_I^2}$

As $R_{Cu} = R_I$ so $\dfrac{\rho_{Cu}\,l}{\pi r_{Cu}^2} = \dfrac{\rho_I\,l}{\pi r_I^2}$ or $\dfrac{\rho_{Cu}}{\rho_I} = \dfrac{r_{Cu}^2}{r_I^2}$

or $\dfrac{r_{Cu}}{r_I} = \sqrt{\dfrac{\rho_{Cu}}{\rho_I}} = \sqrt{\dfrac{1{\cdot}6 \times 10^{-8}}{1{\cdot}0 \times 10^{-7}}} = 0{\cdot}4 = \dfrac{2}{5}$

19. Let l, A and d be the length, area of cross-section and density of wire of mass m. Then

$m = A\,l\,d$ or $A = m/ld$

Resistance $R = \dfrac{\rho l}{A} = \dfrac{\rho l}{m/ld} = \dfrac{\rho l^2 d}{m}$ *i.e.*, $R \propto l^2$

$\therefore \dfrac{R'}{R} = \dfrac{l'^2}{l^2} = \dfrac{\left(l - \dfrac{10}{100}l\right)^2}{l^2} = \left(1 - \dfrac{1}{10}\right)^2 = \dfrac{81}{100} = 0{\cdot}81$

$R' = 0{\cdot}81\,R$

% decrease in resistance $= \left(\dfrac{R - R'}{R}\right) \times 100$

$= \left(\dfrac{R - 0{\cdot}81R}{R}\right) \times 100 = \mathbf{19\%}$

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20. When a wire is stretched, its volume remains constant, hence

$$l_1 A_1 = l_2 A_2 = V, \text{ the volume}$$

Now, $R_1 = \frac{\rho l_1}{A_1} = \frac{\rho l_1 \times l_1}{l_1 A_1} = \frac{\rho l_1^2}{V}$, *i.e.*, $R_1 \propto l_1^2$

Hence, $\frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \frac{\left(l_1 + \frac{5}{100} l_1\right)^2}{l_1^2} = 1{\cdot}1025$

or $R_2 = 1{\cdot}1025\, R_1$

$\therefore$ % change in resistance

$$= \left(\frac{R_2 - R_1}{R_1}\right) \times 100 = \left(\frac{R_2}{R_1} - 1\right) \times 100$$

$$= (1{\cdot}1025 - 1) \times 100 = \mathbf{10{\cdot}25\%}$$

21. Consider an imaginary cylinder of radius x and thickness dx of the given cylinder. Its resistance is

$$dR = \frac{\rho\, dx}{2\pi x l}$$

Total resistance, $R = \int_a^b \frac{\rho\, dx}{2\pi x l} = \frac{\rho \log_e \left(\frac{b}{a}\right)}{2\pi l}$

Current, $I = \frac{E}{R} = \frac{E \times 2\pi l}{\rho \log_e \left(\frac{b}{a}\right)}$

22. $R = \frac{\rho l}{A}$ or $A = \frac{\rho l}{R} = \frac{(1{\cdot}7 \times 10^{-8}) \times (5000)}{5}$

$$= 1{\cdot}7 \times 10^{-5}\ \text{m}^2$$

Mass of copper required,

m = volume × density = $A\, l \times d$

$= (1{\cdot}7 \times 10^{-5}) \times 5000 \times (8{\cdot}9 \times 10^3) = \mathbf{756{\cdot}5\ kg}$

23. $R = \frac{4\rho l}{\pi D^2}$ or $R \propto \frac{l}{D^2}$

$$\therefore \quad \frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{D_1^2}{D_2^2} = \frac{3}{5} \times \left(\frac{2}{3}\right)^2 = \frac{4}{15}$$

or $R_2 = \frac{4}{15} R_1 = \frac{4}{15} \times 15 = \mathbf{4\ \Omega}$

24. When a resistor is stretched to double its original length, the new resistance becomes four times its original resistance because $R \propto l^2$.

$\therefore$ New resistance = $4R = 4 \times 15 = 60\ \Omega$

Resistance of each part = $30\ \Omega$

i.e. $R_1 = 30\ \Omega$ and $R_2 = 30\ \Omega$

When they are connected in parallel, the effective resistance, $R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{30 \times 30}{30 + 30} = 15\Omega$

Current, $I = \frac{V}{R_p} = \frac{3}{15} = \mathbf{0{\cdot}2\ A}$

25. $\rho = \frac{m}{ne^2 \tau}$ or $\tau = \frac{m}{\rho e^2 n} = \mathbf{2{\cdot}5 \times 10^{-14}\ s}$

Mean free path of free electron = $v_d \tau$

$= 1{\cdot}6 \times 10^{-4} \times 2{\cdot}5 \times 10^{-14} = \mathbf{4{\cdot}0 \times 10^{-18}\ m}$

26. Length of the wire used, $l = 1000\, \pi D$

Conductance $G = \frac{1}{R} = \frac{\pi r^2}{\rho l} = \frac{\pi r^2}{\rho \times 1000 \pi D}$

$$= \frac{r^2}{1000 \rho D} = \frac{(0{\cdot}4 \times 10^{-3})^2}{1000 \times (49 \times 10^{-8}) \times (4 \times 10^{-2})}$$

$$= \mathbf{8{\cdot}2 \times 10^{-3}\ S}$$

Conductivity $\sigma = \frac{1}{\rho} = \frac{1}{49 \times 10^{-8}}$

$$= \mathbf{2{\cdot}04 \times 10^6\ S\ m^{-1}}$$

27. Let D_1 and D_2 be the diameter on the left hand side and right hand side of constriction. Then

$D_1 = 2{\cdot}0\ \text{mm} = 2 \times 10^{-3}\ \text{m}$; $J_1 = 1{\cdot}3 \times 10^6\ \text{Am}^{-2}$

(*i*) Current flowing through the constriction

$$I_1 = J_1 A_1 = J_1 \frac{\pi D_1^2}{4} = (1{\cdot}3 \times 10^6) \times \frac{22}{7} \times \frac{(2 \times 10^{-3})^2}{4}$$

$$= \mathbf{4{\cdot}09\ A}$$

(*ii*) For a steady flow of current, $I_1 = I_2$

or $J_1 A_1 = J_2 A_2$

or $J_1 \frac{\pi D_1^2}{4} = J_2 \frac{\pi D_2^2}{4} = 3 J_1 \frac{\pi D_2^2}{4}$

or $D_2 = \frac{D_1}{\sqrt{3}} = \frac{2{\cdot}0\ \text{mm}}{\sqrt{3}} = \mathbf{1{\cdot}15\ mm}$

28. $G = \frac{1}{R} = \frac{I}{V} = \frac{2{\cdot}0}{3{\cdot}0} = \mathbf{0{\cdot}67\ S}$

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{l}{(V/I)A}$$

$$= \frac{Il}{VA} = \frac{2{\cdot}0 \times 3}{3{\cdot}0 \times 5{\cdot}4 \times 10^{-6}} = \mathbf{3{\cdot}7 \times 10^5\ Sm^{-1}}$$

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29. Here, $G = 10\ \Omega^{-1}$, $l = 10\ \text{cm} = 0{\cdot}10\ \text{m}$,
$\sigma = 10^5\ \Omega\ \text{m}^{-1}$

$$\sigma = \frac{1}{\rho} = \frac{l}{\pi r^2 R} = \frac{lG}{\pi r^2} \text{ or } r = \sqrt{\frac{lG}{\pi\sigma}}$$

$$\therefore\ r = \sqrt{\frac{0{\cdot}10\times 10}{3{\cdot}14\times 10^5}} = 1{\cdot}78\times 10^{-3}\ \text{m} = \mathbf{1{\cdot}78\ mm}$$

30. Here, $R_{160} = 132\ \Omega$, $R_{400} = ?$,
$\alpha = 4{\cdot}5\times 10^{-3}\ ^\circ\text{C}^{-1}$

Now $R_t = R_0\ (1 + \alpha\, t)$

$\therefore\ R_{160} = R_0\ [1 + (4{\cdot}5\times 10^{-3})\times 160]$

or $132 = R_0\ [1 + 4{\cdot}5\times 10^{-3}\times 160]$ ...(*i*)

and $R_{400} = R_0\ [1 + 4{\cdot}5\times 10^{-3}\times 400]$ ...(*ii*)

Dividing (*ii*) by (*i*), we get

$$\frac{R_{400}}{132} = \frac{1+4{\cdot}5\times 10^{-3}\times 400}{1+4{\cdot}5\times 10^{-3}\times 160}$$

$$= \frac{1+1{\cdot}80}{1+0{\cdot}72} = \frac{2{\cdot}80}{1{\cdot}72} = 1{\cdot}63$$

$$R_{400} = 132\times 1{\cdot}63 = \mathbf{215\ \Omega}$$

31. Here $\alpha = 0{\cdot}00125^\circ\text{C}^{-1}$,

$t_1 = 300\ \text{K} = 300 - 273 = 27^\circ\text{C}$

$R_{27} = 1\ \Omega$ and $R_t = 2\ \Omega$

As $R_{27} = R_0\ (1 + \alpha\times 27)$

and $R_t = R_0\ (1 + \alpha\times t)$

or $1 = R_0\ (1 + \alpha\times 27)$ and $2 = R_0\ (1 + \alpha\, t)$

$$\therefore\ \frac{2}{1} = \frac{1+\alpha t}{1+\alpha\times 27} \text{ or } 1 + \alpha t = 2 + 54\alpha$$

$$\text{or } t = \frac{1+54\alpha}{\alpha} = \frac{1+54\times 0{\cdot}00125}{0{\cdot}00125} = 854^\circ\text{C}$$

$$= 854 + 273 = \mathbf{1127\ K}$$

32. Here, $r = 1\ \text{mm} = 10^{-3}\ \text{m}$; $l = 50\ \text{cm} = 0{\cdot}50\ \text{m}$;
$R_1 = 0{\cdot}31\ \Omega$, $t_1 = 25^\circ\text{C}$, $R_2 = 0{\cdot}51\ \Omega$, $t_2 = 125^\circ\text{C}$

(*i*) Temperature coefficient of resistance

$$\alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)} = \frac{0{\cdot}51 - 0{\cdot}31}{0{\cdot}31(125 - 25)}$$

$$= \frac{0{\cdot}20}{0{\cdot}31\times 100} = \mathbf{6{\cdot}45\times 10^{-3}\ ^\circ\text{C}^{-1}}$$

(*ii*) Resistance at 0°C is

$$R_0 = \frac{R_1}{1+\alpha t_1} = \frac{0{\cdot}31}{1+(6{\cdot}4\times 10^{-3})\times 25} = \mathbf{0{\cdot}267\ \Omega}$$

(*iii*) Resistivity at 0°C

$$\rho_0 = \frac{R_0 A}{l} = \frac{R_0\times \pi r^2}{l} = \frac{0{\cdot}267\times 3{\cdot}142\times (10^{-3})^2}{0{\cdot}50}$$

$$= \mathbf{1{\cdot}68\times 10^{-6}\ \Omega\ m}$$

Resistivity at 25°C, $\rho_{25} = \rho_0\ [1 + \alpha\times 25]$

$= 1{\cdot}68\times 10^{-6}\ [1 + 6{\cdot}4\times 10^{-3}\times 25]$

$= \mathbf{1{\cdot}948\times 10^{-6}\ \Omega\ m}$

33. (*a*) $R_t = 2\,R_0 = R_0\ (1 + \alpha\, t)$

$$\text{or } 2 = 1 + \alpha\, t \text{ or } t = \frac{1}{\alpha} = \frac{1}{4\times 10^{-3}} = \mathbf{250^\circ C}$$

(*b*) As $t = \dfrac{1}{\alpha}$, which is independent of dimensions of conductor, so the temperature t (= 250°*C*) is valid for all copper conductors whatsoever may be their shape and size.

34. At 100°C, for material *A*,

$R_1' = R_1\ [1 + \alpha_1\,(100 - 0)]$

$= 200\ [1 + 0{\cdot}0031\times 100]$

$= 200\times 1{\cdot}31 = 262\ \Omega$

At 100°C, for material *B*,

$R_2' = R_2\ [1 + \alpha_2\,(100 - 0)]$

$= 100\ [1 + 0{\cdot}0068\times 100]$

$= 100\times 1{\cdot}68 = 168\ \Omega$

Effective resistance in series combination of resistances

$$R = R_1' + R_2' = 262 + 168 = 430\ \Omega$$

The number attached to 4, 3, 0 are **yellow, orange** and **black.** The given carbon resistor is shown in Fig. 3(Q).119.

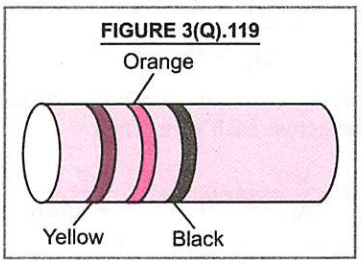

FIGURE 3(Q).119

35. Resistance = $4{\cdot}7\ k\,\Omega = 47\times 10^2\ \Omega$. For numbers, 4, 7 and 2, the colours are **yellow, violet** and **red.**

36. $I = 5\ \text{mA} = 5\times 10^{-3}\ \text{A}$; Resistance of carbon resistor, $R = 01\times 10^2 = 100\ \Omega$

Voltage, $V = IR = 5\times 10^{-3}\times 100 = \mathbf{0{\cdot}5\ V}$

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37. Resistance of resistor = 0·34 Ω = 34×10^{-2} Ω

The colour of the third band of a coded resistor is due to multiplier 10^{-2} which is for **silver.**

38. Resistance of each of the four parts of length $l/4$ is $R_1 = \frac{16}{4} = 4\Omega$.

When each part is stretched to length l, its volume remains constant. So

$$V = A_1 l_1 = A_2 l_2$$

or $$\frac{A_2}{A_1} = \frac{l_1}{l_2} = \frac{l/4}{l} = \frac{1}{4}$$

Now $R_2 = \rho\, l/A$ or $R \propto l/A$

$$\therefore \quad \frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{l}{l/4} \times 4 = 16$$

or $R_2 = 16\, R_1 = 16 \times 4 = 64\ \Omega$

Thus, resistance of each stretched part = 64 Ω.

When these four parts are connected in parallel, the effective resistance R_{eff} of the combination is given by

$$\frac{1}{R_{eff}} = \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} = \frac{4}{64} = \frac{1}{16}$$

or $R_{eff} = \mathbf{16\ \Omega}$

39. Given, $\frac{A_1}{A_2} = \frac{2}{3}$ and $\frac{l_1}{l_2} = \frac{1}{2}$

When two wires are in series, the current through two wires is the same. As

$$I = nAe\, v_d \quad \text{so,} \quad v_d = \frac{I}{nAe} \quad \text{or} \quad v_d \propto \frac{1}{A}$$

$$\therefore \quad \frac{v_{d_1}}{v_{d_2}} = \frac{A_2}{A_1} = \frac{3}{2}$$

When two wires are in parallel, the pot. diff. V is same across each wire. Then

$$I = \frac{V}{R} = nAe\, v_d \quad \text{or} \quad v_d = \frac{V}{nAeR} = \frac{V}{nAe\rho\, l/A}$$

or $$v_d = \frac{V}{ne\rho l} \quad \text{or} \quad v_d \propto \frac{1}{l}$$

$$\therefore \quad \frac{v_{d_1}}{v_{d_2}} = \frac{l_2}{l_1} = \frac{2}{1}$$

40. Here, $i_3 = 2 - (0{\cdot}8 + 0{\cdot}4) = 0{\cdot}8$ A

Potential difference across D and E

$= 10 \times i_3 = 10 \times 0{\cdot}8 = 8$ V

Potential difference across B and G = potential difference across C and F = Potential difference across D and E

$$\therefore \quad 0{\cdot}8\, R_3 = 0{\cdot}4\, R_2 = 8$$

or $$R_3 = \frac{8}{0{\cdot}8} = 10\ \Omega$$

and $$R_2 = \frac{8}{0{\cdot}4} = 20\ \Omega$$

Equivalent resistance between B and G, R' is given

$$\frac{1}{R'} = \frac{1}{10} + \frac{1}{20} + \frac{1}{10} = \frac{5}{20} = \frac{1}{4} \quad \text{or} \quad R' = 4\ \Omega$$

Total resistance between A and H

$= 6 + 4 = \mathbf{10\ \Omega}$

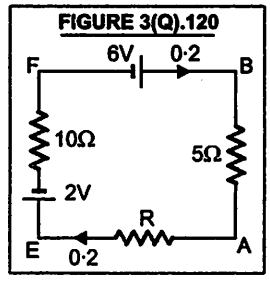
41. Here, resistors 5Ω and 5Ω are in series. They together are in parallel to each of the resistors 30Ω and 15Ω. Thus the effective resistance (R_{eff}) between A and B is

$$\frac{1}{R_{eff}} = \frac{1}{15} + \frac{1}{30} + \frac{1}{(5+5)}$$

$$= \frac{18}{90} = \frac{1}{5} \quad \text{or} \quad R_{eff} = 5\Omega.$$

The equivalent circuit is as shown in Fig. 3(Q).120. Effective e.m.f. of the circuit = 6 – 2 = 4V

FIGURE 3(Q).120



Current $I = \frac{4}{5 + R + 10} = 0{\cdot}2$ A

or $4 = 3 + 0{\cdot}2\, R$ or $R = (4 - 3)/0{\cdot}2 = 5\Omega$

Pot. diff. between B and $A = 5 \times 0{\cdot}2 = \mathbf{1{\cdot}0\ V}$

42. When key K_1 is closed and K_2 is open, resistance of the circuit = $R + X$. So current

$$I_1 = I = \frac{\varepsilon}{R + X} \qquad \text{...(i)}$$

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When keys K_1 and K_2 both are closed, S and X will be in parallel, then their effective resistance is

$$R' = \frac{SX}{S+X}$$

Total resistance of circuit = $R + R'$

$$= R + \frac{SX}{S+X}$$

Total current, $I_2 = \dfrac{\varepsilon}{R + \dfrac{SX}{S+X}}$

Current through resistance X will be

$$I_2' = I_2 \times \frac{S}{S+X} = \frac{\varepsilon}{R + \dfrac{SX}{S+X}} \times \frac{S}{(S+X)}$$

$$= \frac{\varepsilon S}{(S+X)R + SX} \qquad ...(ii)$$

As $I_2' = \dfrac{I}{2}$

So $\dfrac{I}{2} = \dfrac{\varepsilon S}{(S+X)R + SX}$ [From (i) and (ii)]

or $\dfrac{\varepsilon}{(R+X)2} = \dfrac{\varepsilon S}{(S+X)R + SX}$

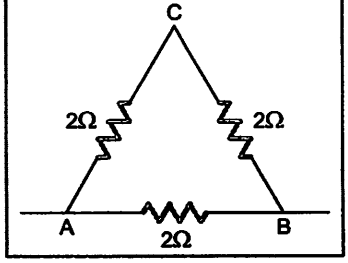
On simplifying, $X = \dfrac{RS}{R-S}$

43. Here, two resistances along each side of triangle are in parallel, their equivalent resistance is

$$= \frac{4\times4}{4+4} = 2\Omega$$

The given network of resistances will be reduced to the equivalent circuit as shown in Fig. 3(Q).121.

FIGURE 3(Q).121



Now resistances in arm AC and CB are in series, their equivalent resistance = 2 Ω + 2 Ω = 4 Ω, which is in parallel with resistance of arm AB (= 2 Ω). Therefore equivalent resistance between A and B is

$$= \frac{4\times2}{4+2} = \frac{8}{6} = \frac{4}{3}\ \Omega$$

44. Here, $\varepsilon = 2{\cdot}2$ V, $V = 1{\cdot}8$ V ; $R = 5\ \Omega$; $r = ?$

As $r = \left(\dfrac{\varepsilon - V}{V}\right)R = \dfrac{(2{\cdot}2 - 1{\cdot}8)}{1{\cdot}8} \times 5$

$$= \frac{10}{9}\ \Omega.$$

45. Here $\varepsilon = 12$ V, $r = 2\ \Omega$, $R = 4\ \Omega$

Total resistance of circuit = $r + R$

$$= 2 + 4 = 6\ \Omega$$

Current in circuit, $I = \dfrac{\varepsilon}{r+R} = \dfrac{12}{6} = 2A$

When voltmeter is placed across the cell, then terminal pot. diff.,

$$V = \varepsilon - Ir = 12 - 2\times2 = 8\ \text{V}$$

When voltmeter is placed across the resistor R, then pot. diff. across R,

$$V' = IR = 2\times4 = 8\ \text{V}.$$

As $V = V'$, hence proved.

46. Current in circuit, $I = \dfrac{\varepsilon}{R+r}$

Case (i), $1 = \dfrac{\varepsilon}{4+r}$ ...(i)

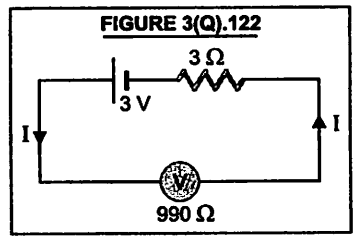
Case (ii), $0{\cdot}5 = \dfrac{\varepsilon}{9+r}$ ...(ii)

Solving (i) and (ii), we get

$$r = 1\ \Omega \quad \text{and} \quad \varepsilon = 5\ \text{V}$$

47. Here, $\varepsilon = 3$ V, $r = 3\ \Omega$,

FIGURE 3(Q).122



Resistance of voltmeter $R = 990\ \Omega$

Refer to Fig. 3(Q).122, current in the circuit,

$$I = \frac{\varepsilon}{R+r} = \frac{3V}{(990+3)\Omega} = \frac{3}{993} = 3{\cdot}02 \times 10^{-3}\ \text{A}$$

Terminal pot. diff. across the cell = pot. diff. across the voltmeter = $V = IR$

$= (3{\cdot}02 \times 10^{-3}) \times 990 = \mathbf{2{\cdot}99\ V}$

Voltmeter used to measure emf of a cell will record 2·99 V instead of 3 V. Hence the percentage error is

$$= \frac{\varepsilon - V}{\varepsilon} \times 100 = \left(1 - \frac{V}{\varepsilon}\right) \times 100 = \left(1 - \frac{2{\cdot}99}{3{\cdot}0}\right) \times 100$$

$= (1 - 0{\cdot}9967) \times 100 = \mathbf{0{\cdot}33\%}$

48. EMF of the cell, $\varepsilon = I(R + r)$

In first case, $\varepsilon = 0{\cdot}5\ (12 + r)$

In second case, $\varepsilon = 0{\cdot}25\ (25 + r)$

$\therefore\ 0{\cdot}5\ (12 + r) = 0{\cdot}25\ (25 + r)$

On solving, $r = \mathbf{1\ \Omega}$

So $\varepsilon = 0{\cdot}5\ (12 + 1) = \mathbf{6{\cdot}5\ V}$

49. (*a*) When $I = 0$, total emf = terminal voltage

$\therefore\ 4\varepsilon = 5{\cdot}6\ \text{V}$

or $\varepsilon = \mathbf{1{\cdot}4\ V}$

From the graph when

$$I = 1{\cdot}0\ \text{A},\ V = \frac{2{\cdot}8}{4} = 0{\cdot}7\ \text{V}\ \text{(for each cell)}$$

(*c*) Internal resistance of one cell,

$$r = \frac{\varepsilon}{I_{max}} = \frac{1{\cdot}4}{2{\cdot}0} = \mathbf{0{\cdot}7\ \Omega}$$

(*b*) The output power is maximum, when external resistance = total internal resistance $= 4r$

$$I_{max} = \frac{\text{total emf}}{\text{total resistance}} = \frac{4\varepsilon}{4r + 4r} = \frac{\varepsilon}{2r}$$

$$= \frac{1{\cdot}4}{2 \times 0{\cdot}7} = \mathbf{1\ A}$$

50. (*a*) When $I = 0$, total emf = terminal voltage

$\therefore\ 3\varepsilon = 6{\cdot}0\ \text{V}$ or $\varepsilon = \mathbf{2{\cdot}0\ V}$

(*b*) When $I = 1{\cdot}0$ A,

$$V = \frac{3{\cdot}0}{3} = 1{\cdot}0\ \text{V}\ \text{(for each cell)}$$

If r is the internal resistance of a cell, then

$$r = \frac{\varepsilon}{I_{max}} = \frac{2{\cdot}0}{2{\cdot}0} = 1{\cdot}0\ \Omega$$

The output power is maximum, when external resistance = internal resistance = $3r$

$$I_{max} = \frac{3\varepsilon}{3r + 3r} = \frac{\varepsilon}{2r} = \frac{2}{2 \times 1} = \mathbf{1\ A}$$

51. Since galvanometer shows no deflection, potential difference across $R = 2$ V. Therefore, potential difference across $500\ \Omega = 12 - 2 = 10$ V

Current through $500\ \Omega = \frac{10}{500} = \frac{1}{50}\ \text{A}$

As same current flows through R,

$$\therefore\ R = \frac{\text{pot.diff.}}{\text{current}} = \frac{2}{(1/50)} = \mathbf{100\ \Omega}$$

52. Here, $R = 500\ \Omega$, $r = 4\ \Omega$,

$$\therefore\ \Delta\varepsilon = \varepsilon - V = Ir = \frac{\varepsilon}{R+r} \times r$$

$$\text{Percentage error} = \frac{\Delta\varepsilon}{\varepsilon} \times 100 = \left(\frac{r}{R+r}\right) \times 100$$

$$= \left(\frac{4}{500+4}\right) \times 100 \approx \mathbf{0{\cdot}8\%}$$

53. As, $V = \varepsilon - Ir$ and $I = \frac{\varepsilon}{R+r}$

$$\therefore\ V = IR = \frac{\varepsilon R}{R+r}\ \text{or}\ 18 = \frac{\varepsilon \times 6}{6+r} \qquad \text{...(i)}$$

and $$24 = \frac{\varepsilon \times 12}{12+r} \qquad \text{...(ii)}$$

Solving (*i*) and (*ii*), we get

$r = 6\ \Omega$ and $\varepsilon = 36$ V

When both the switches S_1 and S_2 are closed, then effective external resistance in the circuit will be

$$R = \frac{6 \times 12}{6 + 12} = 4\ \Omega$$

$$\therefore\ V = \frac{\varepsilon R}{R+r} = \frac{36 \times 4}{4+6} = \mathbf{14{\cdot}4\ V}$$

54. In circuit (a).

Here two cells are connected in opposition, so Effective *emf* of the circuit, $\varepsilon = 9 - 6 = 3$ V.

total resistance of circuit, $R = 1 + 1 = 2\ \Omega$

current recorded by ideal ammeter,

$$I = \frac{\varepsilon}{R} = \frac{3}{2} = \mathbf{1{\cdot}5\ A}$$

As the current I flows from positive to negative terminal inside the cell of 6 V, *i.e.*, the cell of 6 V is being charged, so terminal potential difference of the cell is $V = \varepsilon + Ir = 6 + 1{\cdot}5 \times 1 = 7{\cdot}5$ V

Reading of voltmeter = **7·5 V**

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In circuit (*b*)

Effective *emf* of the circuit, $\varepsilon = 9 + 6 = 15$ V

Total resistance of circuit, $R = 1 + 1 = 2\ \Omega$

Current recorded by ideal ammeter,

$$I = \frac{\varepsilon}{R} = \frac{15}{2} = 7{\cdot}5\ \text{A}$$

As the current I flows from –ve terminal to +ve terminal inside the cell of 6 V, *i.e.*, the cell of 6 V is sending the current. The terminal potential difference of the cell,

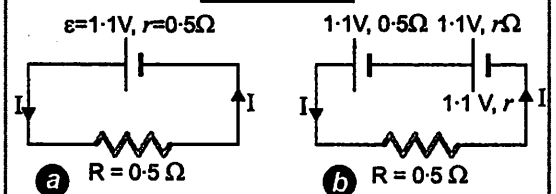
$$V = \varepsilon - Ir = 6 - 7{\cdot}5 \times 1 = -1{\cdot}5\ \text{V}$$

Reading of voltmeter = **– 1·5 V**

Reading of ammeter = **7·5 A**

55. Refer to Fig. 3(Q).123(*a*),

FIGURE 3(Q).123



$$I = \frac{\varepsilon}{R + r} = \frac{1{\cdot}1}{0{\cdot}5 + 0{\cdot}5} = 1{\cdot}1\ \text{A}$$

Refer to Fig. 3(Q).123(*b*)

$$I = \frac{1{\cdot}1 + 1{\cdot}1}{0{\cdot}5 + 0{\cdot}5 + r} = \frac{2{\cdot}2}{1 + r}$$

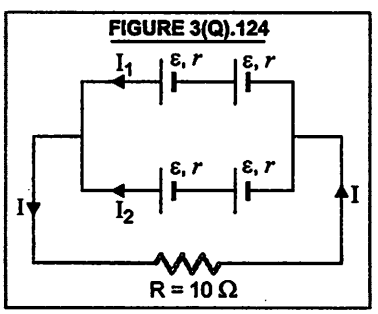
$$\therefore \quad 1{\cdot}1 = \frac{2{\cdot}2}{1 + r} \quad \text{or} \quad 1 + r = 2 \quad \text{or} \quad r = 1\ \Omega$$

56. Here $\varepsilon = 2$ V, $r = 1{\cdot}5\ \Omega$

Refer to Fig. 3(Q).124, $n = 2$, $m = 2$

$$I = \frac{mn\varepsilon}{mR + nr} = \frac{2 \times 2 \times 2}{2 \times 10 + 2 \times 1{\cdot}5} = \frac{8}{23} = 0{\cdot}35\ \text{A}$$

FIGURE 3(Q).124



As two branches are identical, current in each branch is, $I_1 = I_2 = I/2 = 0{\cdot}35/2 = 0{\cdot}175$ **A**

Potential difference across R

$$= IR = 0{\cdot}35 \times 10 = 3{\cdot}5\ \text{V}$$

57. Here, $\varepsilon = 1{\cdot}5$ V, $r = 1{\cdot}0\ \Omega$, $R = 30\ \Omega$; $I = 1{\cdot}5$ A

As r is comparable to R, so cells are to be connected in mixed grouping for maximum current. Let there be n cells in series in each row and m such rows in parallel.

Current in the mixed grouping of cells is

$$I = \frac{mn\varepsilon}{mR + nr} \quad \therefore \quad 1{\cdot}5 = \frac{mn \times 1{\cdot}5}{m \times 30 + n \times 1}$$

or $45\,m + 1{\cdot}5\,n = 1{\cdot}5\,mn$ ...(*i*)

For maximum current, $R = \frac{nr}{m}$

so, $30 = \frac{n \times 1}{m}$ or $n = 30$ m ...(*ii*)

Putting this value in (*i*), we have

$45\,m + 1{\cdot}5 \times 30\,m = 1{\cdot}5\,m \times 30\,m$

or $45\,m + 45\,m = 45\,m^2$

or $90 = 45\,m$ or $m = 2$

From (*ii*), $n = 30 \times 2 = 60$

$\therefore$ Total no. of cells required

$$= mn = 2 \times 60 = 120$$

58. In a series combination of identical cells, each of emf E, if one cell is connected with reverse polarity, it will reduce the effective emf of all cells by 2 E, but total internal resistance of cells will remain unchanged.

As two cells are connected with reverse polarity in the combination of cells, their effective emf

$$= nE - 4E = (n - 4)\,E.$$

Total internal resistance of the cells = nr.

Current in the circuit, $I = \frac{(n-4)E}{nr}$

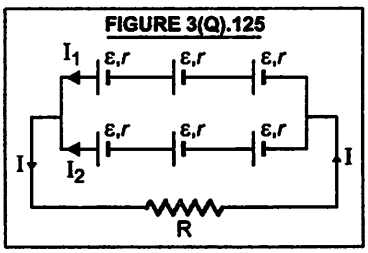
Potential difference across A or B is

$$V = E + Ir = E + \frac{(n-4)Er}{nr} = 2E\left(1 - \frac{2}{n}\right)$$

59. Here, $\varepsilon = 2$ V, $r = 1{\cdot}5\ \Omega$, $m = 2$, $n = 3$, $R = 10$

$$I = \frac{mn\varepsilon}{mR + nr} = \frac{2 \times 3 \times 2}{2 \times 10 + 3 \times 1{\cdot}5} = \frac{12}{24{\cdot}5} = 0{\cdot}49\ \text{A}$$

FIGURE 3(Q).125



As two branches of cells are identical, the current through each branch is

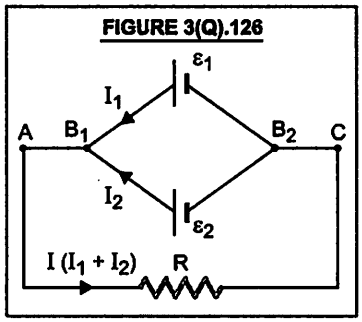
$I_1 = I_2 = I/2 = 0{\cdot}49/2 = \mathbf{0{\cdot}245\ A}$

Potential difference across $R = IR = 0{\cdot}49 \times 10$

$= \mathbf{4{\cdot}9\ V}$

60. As the effective internal resistance of two cells is less than the internal resistance r_1 of one cell, so the cells are connected in parallel between the two points B_1 and B_2. The circuit is shown in Fig. 3(Q).126. Let V be the potential difference between points B_1 and B_2. The potential difference between the terminals of first cell is

FIGURE 3(Q).126



$$V = V_{B_1} - V_{B_2} = \varepsilon_1 - I_1 r_1 \quad \text{or} \quad I_1 = \frac{\varepsilon_1 - V}{r_1}$$

The potential difference between the terminals of second cell is

$$V = V_{B_1} - V_{B_2} = \varepsilon_2 - I_2 r_2 \quad \text{or} \quad I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Hence current in external resistance

$$I = I_1 + I_2 = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or $$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad [\because V = IR]$$

or $$I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right] = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right)$$

or $$I\left[\frac{r_1 r_2 + R\,(r_2 + r_1)}{r_1 r_2}\right] = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}$$

or $$I = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 \times r_2) + R\,(r_2 + r_1)}$$

61. Here, $R = 20\ \Omega$, $P = 50\ \text{kW} = 50 \times 1000\ \text{W}$

$= 5 \times 10^4\ \text{W}$, $V = 5000\ \text{V}$

Current supplied by generator,

$$I = \frac{P}{V} = \frac{5 \times 10^4}{5000} = 10\ \text{A}$$

Power wasted as heat during transmission by cable

$P' = I^2 R = (10)^2 \times 20 = 2000\ \text{W} = 2\ \text{kW}$

Power received by factory $= P - P' = 50 - 2$

$= \mathbf{48\ kW}$

62. Electric energy consumed per second = 100 J

Heat produced per second = 80% $= \frac{80}{100} \times 100$

$= 80\ \text{J} = \frac{80}{4{\cdot}2} = \mathbf{19{\cdot}05\ cal}$

63. Total power of motor, $P = VI = 50 \times 15 = 750\ \text{W}$

Mechanical power of motor, $P' = 150\ \text{W}$

Power dissipated across the winding of motor

$= 750 - 150 = \mathbf{600\ W}$

Efficiency of motor $= \frac{P'}{P} \times 100 = \frac{150}{750} \times 100$

$= \mathbf{20\%}$

64. $R_{2000} = V^2/P = (240)^2/200 = \mathbf{288\ \Omega}$

Given $R_{2000} = 10\ R_0$

then $\alpha = (R_{2000} - R_0)/R_0 \times 2000$

65. In series ; total resistance,

$R_s = R + R + R = 3\ R.$

Given $V^2/(3\ R) = 20$ or $\frac{V^2}{R} = 60$

In parallel, total resistance $R_p = R/3$.

$\therefore$ Power consumed $= V^2/R_p = 3\ V^2/R$

$= 3 \times 60 = \mathbf{180\ watt}$

66. (*a*) Current drawn by heater,

$$I_1 = \frac{P_1}{V} = \frac{1000}{220} = \frac{\mathbf{50}}{\mathbf{11}}\ \text{A}$$

(*b*) Current drawn by bulb, $I_2 = \frac{P_2}{V} = \frac{60}{220} = \frac{3}{11}\ \text{A}$

Current drawn by radio, $I_3 = \frac{P_3}{V} = \frac{40}{220} = \frac{2}{11}\ \text{A}$

Current passing through the fuse line

$= I_1 + I_2 + I_3 = \frac{50}{11} + \frac{3}{11} + \frac{2}{11} = \mathbf{5\ A}$

67. Effective emf of the battery,

$\varepsilon = 10 \times 2 = 20\ \text{V}$

Total internal resistance of battery,

$r = 10 \times 0{\cdot}25 = 2{\cdot}5\ \Omega$

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Total resistance of circuit = $r + R = 2{\cdot}5 + 47{\cdot}5$
$= 50{\cdot}0\ \Omega$

As the battery emf opposes the charging voltage, so effective emf
$= V - \varepsilon = 220 - 20 = 200$ V

Charging current, $I = \dfrac{\text{Effective emf}}{\text{total resistance}} = \dfrac{V-\varepsilon}{r+R}$
$= \dfrac{200}{50} = 4\,A$

(*a*) Power supplied by the mains
$P_1 = VI = 220 \times 4 = \mathbf{880\ W}$

(*b*) Power dissipated as heat
$P_2 = I^2 (R + r) = 4^2 \times 50 = \mathbf{800\ W}$

(*c*) The difference of power = $P_1 - P_2$
$= 880 - 800 = \mathbf{80\ W}$

It is stored in the battery in the form of chemical energy.

68. Here, $\varepsilon = 1{\cdot}6$ V, $r = 0{\cdot}10\ \Omega$, $I = 2$ A

As $I = \dfrac{\varepsilon}{R+r}$ or $R + r = \dfrac{\varepsilon}{I} = \dfrac{1{\cdot}6}{2} = 0{\cdot}8\,\Omega$

or $R = 0{\cdot}8 - 0{\cdot}1 = 0{\cdot}70\ \Omega$

(*i*) Voltage drop across R,
$V = IR = 2 \times 0{\cdot}70 = \mathbf{1{\cdot}4\ V}$

(*ii*) Rate of energy dissipation inside the resistor
$= VI = 1{\cdot}4 \times 2{\cdot}0 = \mathbf{2{\cdot}8\ W}$

69. Let a_1, l_1 be the area of cross-section and length of wire A and a_2, l_2 be the area of cross-section and length of wire B. As the two wires are of same material and mass, so their volume is same. Hence

$$a_1 l_1 = a_2 l_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{l_2}{l_1} = \frac{3l}{l} = 3 \quad \text{or} \quad a_1 = 3a_2$$

If ε is the *emf* of the source, then the rate of dissipation of heat in wire B is

$$H_B = \frac{\varepsilon^2}{R_2} = 10 \quad \text{or} \quad \frac{\varepsilon^2}{\rho l_2 / a_2} = 10$$

$$\text{or} \quad \frac{\varepsilon^2}{\rho 3l / a_2} = 10 \quad \text{or} \quad \frac{\varepsilon^2 a_2}{\rho l} = 30 \qquad [\because\ l_2 = 3\,l]$$

Rate of heat dissipation in wire A is

$$H_A = \frac{\varepsilon^2}{R_1} = \frac{\varepsilon^2}{\rho l / a_1} = \frac{\varepsilon^2}{\rho l}(3a_2)$$
$$= 3 \times 30 = \mathbf{90\ W}$$

70. Resistance of the bulb $R = \dfrac{(230)^2}{100} = 529\,\Omega$

When the voltage drops to $V' = 115$ V, the total heat and light energy produced by the bulb in 20 min will be

$$H = \frac{V'^2}{R} t = \frac{(115)^2}{529} \times (20 \times 60) = 30{,}000\ \text{J}$$
$$= \mathbf{30\ kJ}$$

Current, $I = \dfrac{V'}{R} = \dfrac{115}{529} = \dfrac{5}{23}$ A

71. Volume of water in 4 cups,
$v = 4 \times 200 = 800$ c.c.

mass of water, m = volume × density
$= 800 \times 1 = 800$ gram.

(*a*) Energy spent in boiling the water = $ms\,d\theta$
$= 800 \times 1 \times (100 - 25)$ cal
$= 800 \times 75 \times 4{\cdot}2$ J
$= \dfrac{800 \times 75 \times 4{\cdot}2}{3{\cdot}6 \times 10^6}$ kWh $= 0{\cdot}07$ kWh.

Cost of boiling water = $0{\cdot}07 \times 3 = 0{\cdot}21$ Rs
$= \mathbf{21\ paise}$

(*b*) Energy spent in boiling the water
$= 800 \times 1 \times (100 - 5)$ cal
$= 800 \times 95 \times 4{\cdot}2$ J
$= \dfrac{800 \times 95 \times 4{\cdot}2}{3{\cdot}6 \times 10^6}$ kWh $= 0{\cdot}09$ kWh

Cost of boiling water = $0{\cdot}09 \times 3 = 0{\cdot}27$ Rs.
$= \mathbf{27\ paise}$

72. Equivalent resistance of 6 Ω and 3 Ω in parallel,

$$R_p = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Total resistance of circuit, $R = 2 + 1 = 3\ \Omega$

Total current, $I = \dfrac{12V}{3\Omega} = 4$ A.

Current through 6 Ω resistor,

$$I_1 = I \times \frac{R_2}{R_1 + R_2} = 4 \times \frac{3}{6+3} = \frac{4}{3}\ \text{A}$$

Heat developed in it per minute

$$= I_1^2 R_1 t = \left(\frac{4}{3}\right)^2 \times 6 \times 60 = \mathbf{640\ J}$$

Current through 3 Ω resistor ,

$$I_2 = I \times \frac{R_1}{R_1 + R_2} = 4 \times \frac{6}{6+3} = \frac{8}{3}\ \text{A}$$

Heat developed in it per minute

$$= I_2^2 R_2 t = \left(\frac{8}{3}\right)^2 \times 3 \times 60 = \mathbf{1280\ J}$$

Heat produced per minute in R_3
$= I^2 R_3 t = (4)^2 \times 1 \times 60 = \mathbf{960\ J}$

73. Here, $V = 110$ V ; $I = 2$A,
Power supplied to motor $= VI = 110 \times 2 = 220$ W
Power loss in the form of heat $= 10$ cal s^{-1}
$= 10 \times 4{\cdot}2\ Js^{-1} = 42\ Js^{-1}$
Power used by motor $= 220 - 42 = 178$ W

$$\text{Efficiency, } \eta = \frac{\text{power used by motor}}{\text{power supplied to motor}} \times 100$$

$$= \frac{178}{220} \times 100 = \mathbf{80{\cdot}9\%}$$

74. Resistance of aluminium wire,

$$R_{Al} = \frac{\rho l}{A} = \frac{2{\cdot}9 \times 10^{-8} \times 150}{15 \times 10^{-6}} = 0{\cdot}29\ \Omega$$

Current through main line,

$$I = \frac{130 - 124}{0{\cdot}29} = 20{\cdot}69\ \text{A}$$

Current through one lamp, $I_1 = \dfrac{200}{124} = 1{\cdot}613$ A

$$\therefore \text{ No. of lamps} = \frac{20{\cdot}69}{1{\cdot}613} = 12{\cdot}83$$

Hence no. of lamps to be used $= \mathbf{12}$
Power consumed $= 12 \times 200 = 2400$ W
$= \mathbf{2{\cdot}4\ kW}$

75. Let R be the resistance of each bulb. Total resistance of all the 21 bulbs in series $= 21\ R$
$\therefore$ current, $I = (220/21R)$;
Illuminating power of each bulb,

$$p = I^2 R = \left(\frac{220}{21R}\right)^2 \times R = \left(\frac{220}{21}\right)^2 \frac{1}{R}$$

Illuminating power of all the bulbs,

$$P = \left(\frac{220}{21R}\right)^2 \times 21R = \frac{(220)^2}{21R}$$

When one bulb is fused, then current,
$I = 220/20\ R$
Now illuminating power of each bulb,

$$p' = \left(\frac{220}{20R}\right)^2 \times R = \left(\frac{220}{20}\right)^2 \frac{1}{R}$$

Illuminating power of all the bulbs,

$$P'' = \left(\frac{220}{20R}\right)^2 \times 20R = \frac{(220)^2}{20R}$$

(*i*) % increase in illuminating power of one bulb

$$= \frac{(p' - p)}{p} \times 100 = \left(\frac{p'}{p} - 1\right) \times 100$$

$$= \left(\frac{(220)^2/(20)^2 R}{(220)^2/(21)^2 R} - 1\right) \times 100$$

$$= \left[\left(\frac{21}{20}\right)^2 - 1\right] \times 100$$

$$= \frac{441 - 400}{400} \times 100 = \mathbf{10{\cdot}25\%}$$

(*ii*) % increase in illumination of all the bulbs

$$= \left(\frac{P'' - P}{P}\right) \times 100 = \left(\frac{P''}{P} - 1\right) \times 100$$

$$= \left(\frac{(220)^2/20\ R}{(220)^2/21\ R} - 1\right) \times 100$$

$$= \left(\frac{21}{20} - 1\right) \times 100 = \mathbf{5\%}$$

76. (*a*) Let R be the resistance of each bulb. Here bulbs B and C are in parallel. Their effective resistance

$$R_P = \frac{R \times R}{R + R} = \frac{R}{2}$$

These bulbs together are in series with bulb A. Since in series combination of resistances, potential divides in proportion to resistance so

$$V_A = \frac{R \times V}{R + R/2} = \frac{2}{3} V = \frac{2}{3} \times 120 = \mathbf{80\ V}$$

$$V_B = V_C = \frac{R/2 \times V}{R + R/2} = \frac{1}{3} V = \frac{1}{3} \times 120 = \mathbf{40\ V}$$

Total power of bulb B and C,
$P' = 60 + 60 = 120$ W
(*b*) Total power of all the three bulbs will be

$$\frac{1}{P} = \frac{1}{P_A} + \frac{1}{P'} = \frac{1}{60} + \frac{1}{120} = \frac{3}{120} = \frac{1}{40}$$

or $P = \mathbf{40\ watt}$

77. Current in each bulb, $I = \dfrac{P}{V} = \dfrac{60}{220} = \dfrac{3}{11}$ A

$$\text{No. of bulbs used, } n = \frac{9}{3/11} = \mathbf{33}$$

78. Let i_1, i_2 be the maximum allowed currents in the two bulbs and R_1, R_2 their respective resistances

$$\therefore \quad R_1 = \frac{(220)^2}{25} = 1936\ \Omega,$$

$$i_1 = \frac{P_1}{V} = \frac{25}{220} = 0{\cdot}114\ \text{A}$$

$$R_2 = \frac{(220)^2}{100} = 484\Omega,$$

$$i_2 = \frac{P_2}{V} = \frac{100}{220} = 0{\cdot}454\ \text{A}$$

Case (a). When bulbs are connected in series, net resistance

$R_s = R_1 + R_2 = 1936 + 484 = 2420\ \Omega$

$\therefore$ Current in the circuit,

$$i = \frac{V'}{R_s} = \frac{440}{2420} = 0{\cdot}18\ \text{A}$$

It means a current of 0·18 A passes through both the bulbs connected in series. By comparing this value of current with the value of i_1 and i_2, we note that $i > i_1$ but less than i_2. It shows **25 W bulb will fuse.**

Case (b). When bulbs are connected in parallel, let the currents through I and II bulbs be i_1' and i_2'. In parallel combination, the voltage across each bulb is the same.

$$\therefore \qquad i_1' = \frac{440}{R_1} = \frac{440}{1936} = 0{\cdot}23\ \text{A}$$

and $$i_2' = \frac{440}{R_2} = \frac{440}{1484} = 0{\cdot}91\ \text{A}$$

As $i_1' > i_1$ and $i_2' > i_2$, hence in this case **both the bulbs will fuse**

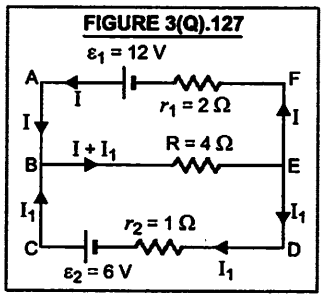
79. As per question ; $\dfrac{\varepsilon^2 R_1}{(R_1 + r)^2} = \dfrac{\varepsilon^2 R_2}{(R_2 + r)^2}$.

Solve it for r in terms of R_1 and R_2, we shall get

$$r = \sqrt{R_1 R_2}$$

80. Refer to Fig. 3(Q).127, applying Kirchhoff's loop law, we get

FIGURE 3(Q).127



For loop *ABEFA*

$4(I + I_1) + 2I = 12$ or $6I + 4I_1 = 12$ ...(*i*)

For loop *BEDCB*

$4(I + I_1) + 1I_1 = 6$ or $4I + 5I_1 = 6$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$$I = \frac{18}{7}\text{A} \quad \text{and} \quad I_2 = -\frac{6}{7}\text{A}.$$

Pot. diff. across R,

$$V = (I + I_1)R = \left(\frac{18}{7} - \frac{6}{7}\right) \times 4 = \frac{48}{7}\text{V}$$

Pot. diff. across each cell = pot. diff. across R

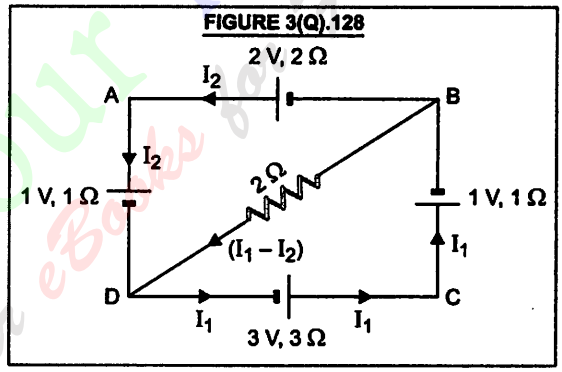
$$= \frac{48}{7}\text{V}$$

Rate of energy dissipated in $R = (I + I_1)^2 R$

$$= \left(\frac{18}{7} - \frac{6}{7}\right)^2 \times 4\,\text{W} = \left(\frac{12}{7}\right)^2 \times 4 = \mathbf{11{\cdot}75\ W}$$

81. Let the currents in the various arms of the circuit be as shown in Fig. 3(Q).128.

FIGURE 3(Q).128



According to Kirchhoff's second rule, in the loop *DCBD*,

$3 - 1 = 3I_1 + 1I_1 + 2(I_1 - I_2)$

or $2 = 6I_1 - 2I_2$ or $1 = 3I_1 - I_2$ ...(*i*)

In the loop *BADB* ;

$2 - 1 = 2I_2 + 1I_2 - 2(I_1 - I_2)$

or $1 = -2I_1 + 5I_2$ ...(*ii*)

From (*i*) and (*ii*), we have

$3I_1 - I_2 = -2I_1 + 5I_2$ or $5I_1 = 6I_2$ or $I_1 = \dfrac{6}{5}I_2$

Putting this value in (*i*), we get

$$1 = 3 \times \frac{6}{5}I_2 - I_2 = \frac{13}{5}I_2 \quad \text{or} \quad I_2 = \frac{5}{13}\text{A}$$

$$\therefore \quad I_1 = \frac{6}{5} \times \frac{5}{13} = \frac{6}{13}\text{A}$$

Pot. diff. between B and $D = (I_1 - I_2)_2$

$$= \left(\frac{6}{13} - \frac{5}{13}\right) \times 2 = \frac{2}{13}\text{V}$$

82. By Kirchhoff's first rule,

$I_3 = I_1 + I_2$ ...(*i*)

According to Kirchhoff's second rule

For loop *PRSP*, $20\, I_3 + 200\, I_2 = 5$ ...(*ii*)

For loop *PRQP*, $20\, I_3 + 60\, I_1 = 4$ ...(*iii*)

Solving (*i*), (*ii*) and (*iii*), we get

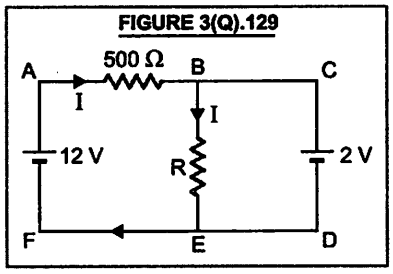
$$I_1 = \frac{39}{860}\text{A} \; ; I_2 = \frac{4}{215}\text{A} \; ; I_3 = \frac{11}{172}\text{A}$$

83. Since galvanometer shows no deflection, hence pot. diff. across $R = 2$ volt. Now the circuit reduces to as shown in Fig. 3(Q).129.

In closed loop *ABCDEFA*, we have

$$500\, I = -2 + 12 = 10 \quad \text{or} \quad I = \frac{10}{500} = \frac{1}{50}\text{A}$$

FIGURE 3(Q).129



In closed loop *ABEFA*, we have

$$500\, I + IR = 12 \quad \text{or} \quad 500 \times \frac{1}{50} + \frac{1}{50} R = 12$$

or $R = \mathbf{100\ \Omega}$

84. According to Kirchhoff's loop rule,

For the loop *ABGHA*, $5\, I_1 = -2 + 2 = 0$ or $I_1 = 0$

For loop, *BCFGB*, $5\, I_2 - 5\, I_1 = -2 + 2 = 0$

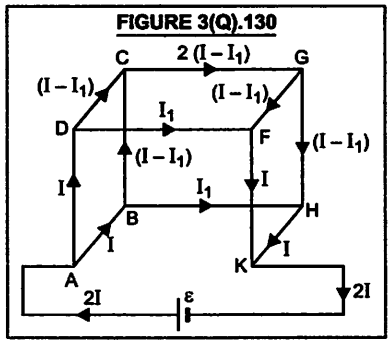
or $5\, I_2 = 5\, I_1 = 5 \times 0 = 0$ or $I_2 = 0$

For loop *CDEFC* $5\, I_3 - 5\, I_2 = -2 + 2 = 0$

$5\, I_3 = 5\, I_2 = 5 \times 0 = 0$ or $I_3 = \mathbf{0}$

85. Let the current through the various branches of network be as shown in the Fig. 3(Q).130. Let R be the equivalent resistance of skelton cube between edges A and K.

FIGURE 3(Q).130



From Ohm's law, $\varepsilon = 2\, IR$ ...(*i*)

Applying Kirchhoff's second rule to loop $\varepsilon ABHK\varepsilon$ we have, $\varepsilon = 5\, I + 5\, I_1 + 5\, I = 10\, I + 5\, I_1$ ...(*ii*)

For loop *DFGCD*,

$5\, I_1 - 5\,(I - I_1) - 5 \times 2\,(I - I_1) - 5\,(I - I_1) = 0$

or $5\, I_1 - 5\, I + 5\, I_1 - 10\, I + 10\, I_1 - 5\, I + 5\, I_1 = 0$

or $25\, I_1 - 20\, I = 0$ or $I_1 = \frac{20}{25} I = \frac{4}{5} I$ ...(*iii*)

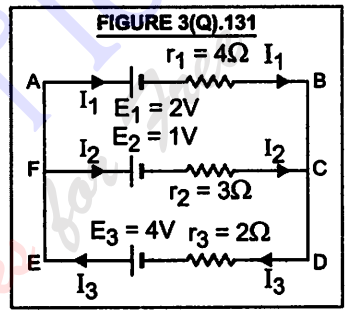
From (*ii*) and (*iii*), $\varepsilon = 10\, I + 5 \times \frac{4}{5} I = 14\, I$ ...(*iv*)

From (*i*) and (*iv*), $2\, IR = 14\, I$ or $R = \mathbf{7\ \Omega}$

From (*i*) $I = \frac{\varepsilon}{2R} = \frac{2}{2 \times 7} = \frac{1}{7}\text{A}$

86. Refer Fig. 3(Q).131, for loop ABCFA

FIGURE 3(Q).131



$I_1 r_1 - I_2 r_2 = -E_1 + E_2$

or $4\, I_1 - 3\, I_2 = -2 + 1 = -1$

or $4\, I_1 - 3\, I_2 = -1$ ...(*i*)

For loop FCDEF, $I_2 r_2 + I_3 r_3 = -E_2 + E_3$

or $3\, I_2 + 2\, I_3 = -1 + 4 = 3$

or $3\, I_2 + 2\, I_3 = 3$ ...(*ii*)

At junction C, $I_3 = I_1 + I_2$ ...(*iii*)

From (*ii*) and (*iii*), we have

$3\, I_2 + 2\,(I_1 + I_2) = 3$ or $2\, I_1 + 5\, I_2 = 3$ ...(*iv*)

Solving (*i*) and (*iv*), we have,

$$I_1 = \frac{2}{13}\text{A} \quad \text{and} \quad I_2 = \frac{7}{13}\text{A}$$

$$I_3 = I_1 + I_2 = \frac{2}{13} + \frac{7}{13} = \frac{9}{13}\text{A}$$

87. Follow example 87, the effective resistance between two points, which are diagonally opposite corners of a skelton cube of 12 wires

$= 5\, r/6 = 5 \times 6/6 = \mathbf{5\ \Omega}$

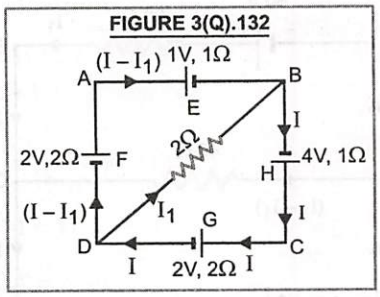
$\therefore$ Pot diff developed = resistance × current

$= 5 \times (40 \times 10^{-3}) = \mathbf{0{\cdot}2\ V}$

88. The distribution of currents in the network, following Kirchhoff's first rule has been shown in Fig. 3(Q).132.

According to Kirchhoff's second rule in the closed loop *ABDA*

FIGURE 3(Q).132



$$-1+2=1\,(I-I_1)-2\,I_1+(I-I_1)\,2$$

or $\quad 1=3\,I-5\,I_1 \qquad ...(i)$

In the closed loop *BCDB*

$4-2=1\times I+2\times I+2\,I_1$ or $2=3\,I+2\,I_1$ ...(*ii*)

Subtracting (*i*) and (*ii*), we get

$$1=7\,I_1 \quad \text{or} \quad I_1=\frac{1}{7}A$$

From (*i*), $1=3\,I-5\times\frac{1}{7}$

or $\quad 3I=1+\frac{5}{7}=\frac{12}{7} \quad$ or $\quad I=\frac{12}{7\times 3}=\frac{4}{7}A$

(*i*) Potential difference between *D* and *B*

$$=I_1\times 2=\frac{1}{7}\times 2=\frac{2}{7}V \quad \mathbf{=0{\cdot}29\ V}$$

(*ii*) Potential difference across the terminals of *G* (receiving current for charging)

$$V_G=\varepsilon+Ir=2+\frac{4}{7}\times 2=\frac{22}{7} \quad \mathbf{=3{\cdot}143\ V}$$

Potential difference across the terminals of *H* (giving current)

$$V_H=\varepsilon'-Ir'=4-\frac{4}{7}\times 1=\frac{24}{7} \quad \mathbf{=3{\cdot}43\ V}$$

89. According to Kirchhoff's first rule, current in arm $DC = 3 - 1 = 2$ A

Pot. diff. across $2\ \Omega = 2\times 2 = 4$ V

As point *A* is at zero potential, so potential of point *C* is $V_C = 2\ V$

Now, $V_D - V_C = 4$ or $V_D - 2 = 4$ or $V_D = 6$ V

Here, $V_D - V_B = 4$ or $V_B = V_D - 4 = 6 - 4 = \mathbf{2\ V}$

90. Let the current through various arms as per Kirchhoff's first rule be as shown in Fig. 3(Q).133.

Using Kirchhoff's voltage rule in the closed circuit *DABCD*

$$(I_1+I_2)\,R_3+I_1\,R_1=E_1$$

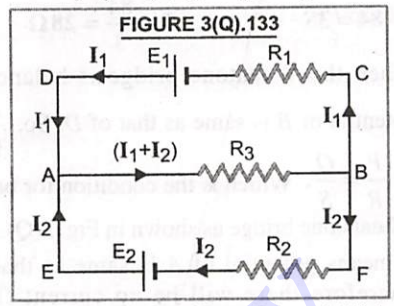
or $\quad I_1\,(R_1+R_3)+I_2\,R_3=E_1 \qquad ...(i)$

In the closed circuit *ABFEA* ;

$$(I_1+I_2)\,R_3+I_2\,R_2=E_2$$

or $\quad I_1\,R_3+I_2\,(R_2+R_3)=E_2 \qquad ...(ii)$

FIGURE 3(Q).133



Multiplying (*i*) by $(R_2 + R_3)$ and (*ii*) by R_3 and subtracting we get

$$I_1=\frac{E_1R_2+E_1R_3-E_2R_3}{R_1R_2+R_2R_3+R_1R_3}.$$

Multiplying (*i*) by R_3 and (*ii*) by $(R_1 + R_3)$ and subtracting, we get

$$I_2=\frac{E_2R_1+E_2R_3-E_1R_3}{R_1R_2+R_2R_3+R_1R_3}$$

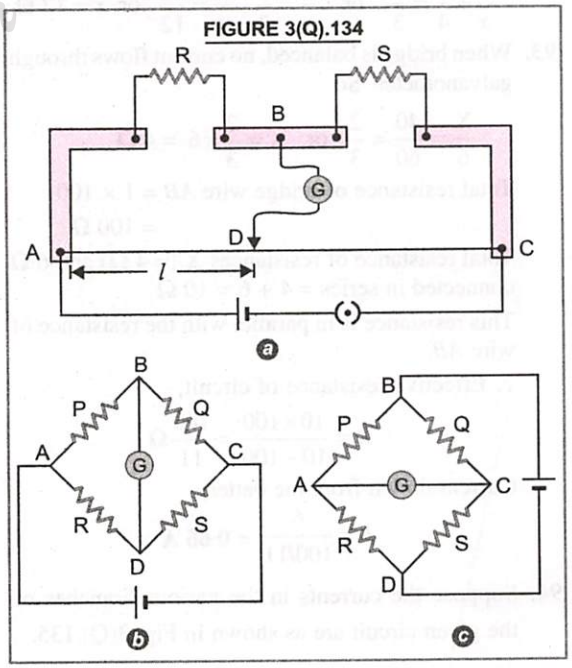
Potential difference across $R_3 = (I_1 + I_2)\,R_3$

$$=\left(\frac{E_1R_2+E_1R_3-E_2R_3}{R_1R_2+R_2R_3+R_1R_3}+\frac{E_2R_1+E_2R_3-E_1R_3}{R_1R_2+R_2R_3+R_1R_3}\right)R_3$$

$$=\frac{(E_1R_2+E_2R_1)R_3}{R_1R_2+R_2R_3+R_1R_3}$$

91. Refer to Fig. 3(Q).134(*a*)

FIGURE 3(Q).134



$$\frac{R}{S}=\frac{l}{(100-l)} \quad \text{or} \quad \frac{12}{S}=\frac{30}{100-30}=\frac{30}{70}=\frac{3}{7}$$

or $84=3S$ or $S=\frac{84}{3}=28\,\Omega$

When the Wheatone bridge is balanced the potential of B is same as that of D. So, $\frac{P}{Q}=\frac{R}{S}$ or $\frac{P}{R}=\frac{Q}{S}$. Which is the condition for balanced Wheatstone bridge as shown in Fig. 3(Q).144(*c*). It means potential of A is same as that of C. Therefore there will be no current flowing through galvanometer.

(*iii*) When R and S are interchanged, then

$$\frac{S}{R}=\frac{l}{(100-l)} \quad \text{or} \quad \frac{28}{12}=\frac{l}{(100-l)}$$

or $\frac{7}{3}=\frac{l}{(100-l)}$ or $l=70$ cm

92. For balanced Wheatstone bridge, $\frac{P}{Q}=\frac{R}{S}$

Here $P=3\,\Omega$, $Q=3\,\Omega$, $R=3\,\Omega$

$\therefore \quad \frac{3}{3}=\frac{3}{S}$ or $S=3\,\Omega$

Therefore, the resistance of 4 Ω must be shunted with a reistance x so that the combined resistance of 4 Ω and x is equal to 3 Ω. So

$$\frac{1}{x}+\frac{1}{4}=\frac{1}{3} \quad \text{or} \quad \frac{1}{x}=\frac{1}{3}-\frac{1}{4}=\frac{1}{12} \quad \text{or} \quad x=12\,\Omega$$

93. When bridge is balanced, no current flows through galvanometer. So

$$\frac{X}{6}=\frac{40}{60}=\frac{2}{3} \quad \text{or} \quad X=\frac{2}{3}\times 6=4\,\Omega$$

Total resistance of bridge wire $AB = 1 \times 100 = 100\,\Omega$

Total resistance of resistances X (= 4 Ω) and 6 Ω connected in series = 4 + 6 = 10 Ω

This resistance is in parallel with the resistance of wire AB.

∴ Effective resistance of circuit,

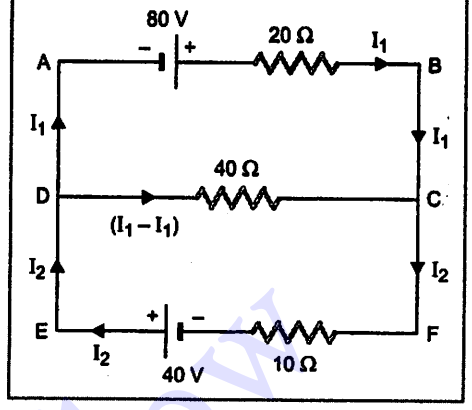
$$R=\frac{10\times 100}{10+100}=\frac{100}{11}\,\Omega$$

Current drawn from the battery

$$I=\frac{6}{100/11}=0{\cdot}66\text{ A}$$

94. Suppose the currents in the various branches of the given circuit are as shown in Fig. 3(Q).135.

FIGURE 3(Q).135



Using Kirchhoff's second rule

for closed mesh $ABCDA$;

$$80=2I_1-40(I_2-I_1)=60I_1-40I_2$$

or $4=3I_1-2I_2$ ...(*i*)

for closed mesh $DCFE$;

$$40=40(I_2-I_1)+10I_2$$
$$=-40I_1+50I_2$$

or $4=-4I_1+5I_2$ ...(*ii*)

From (*i*) and (*ii*), we have ;

$$3I_1-2I_1=-4I_1+5I_2$$

or $=7I_1=7I_2$ or $I_1=I_2$

From (*i*) $4=3I_1-2I_1=I_1$ or $I_1=4$ A and $I_2=4$ A

Current through 40 Ω $=I_2-I_1=4-4=0$ A

Current through 20 Ω $=I_1=4$ A

95. When no current flows through the arm AO, the Wheatstone bridge is balanced. Then

$$\frac{2}{X}=\frac{3}{6} \quad \text{or} \quad X=4\,\Omega$$

Total resistance of circuit,

$$R=\frac{(2+4)\times(3+6)}{(2+4)+(3+6)}+2{\cdot}4=6\,\Omega$$

Current in circuit, $I=\frac{\varepsilon}{R}=\frac{6}{6}=1\text{A}$

96. When bridge is balanced, then $\frac{R}{S}=\frac{l_1}{(100-l_1)}$ ...(*i*)

When resistance S is shunted with resistance S, the effective resistance, $S'=\frac{S\times S}{S+S}=\frac{S}{2}$

Then, $\frac{R}{S/2}=\frac{1{\cdot}5l_1}{(100-1{\cdot}5l_1)}$

or $\frac{R}{S}=\frac{1{\cdot}5l_1}{2(100-1{\cdot}5l_1)}$ ...(*ii*)

From (*i*) and (*ii*), we have,

$$\frac{l_1}{(100-l_1)}=\frac{1{\cdot}5l_1}{2(100-1{\cdot}5l_1)}$$

On solving, $l_1=\frac{100}{3}$ cm.

Putting values in (*i*), we get ;

$$\frac{5}{S}=\frac{(100/3)}{(100-100/3)}=\frac{100}{200}=\frac{1}{2} \text{ or } S=5\times2=\mathbf{10\ \Omega}$$

97. $\frac{2}{X}=\frac{l}{(100-l)}$ ...(*i*)

When R and X are interchanged, then

$$\frac{X}{2}=\frac{(l+20)}{(100-l-20)}=\frac{(l+20)}{(80-l)}$$

$$\frac{2}{X}=\frac{(80-l)}{(l+20)} \quad ...(ii)$$

On solving, from (*i*) and (*ii*), we shall get

$l=40$ cm and $X=\mathbf{3\ \Omega}$

98. Let R be the resistance of shunted wire, the effective resistance of R and 5 Ω in parallel

$=(5\times R)/(5+R)$

At balance point,

$$\frac{5R/(5+R)}{2}=\frac{0{\cdot}625}{1-0{\cdot}625}=\frac{0{\cdot}625}{0{\cdot}375}=\frac{5}{3}$$

On solving we get, $\mathbf{R=10\ \Omega}$

Now, $\rho=\frac{RA}{l}=\frac{R\pi r^2}{l}$

$$=\frac{10\times(22/7)\times(0{\cdot}125\times10^{-3})^2}{0{\cdot}75}$$

$$=\mathbf{6{\cdot}54\times10^{-7}\ \Omega\ m}$$

99. Case (*i*) ; $\frac{R_1}{R_2}=\frac{40}{60}=\frac{2}{3}$ or $R_1=\frac{2}{3}R_2$ ...(*i*)

Case (*ii*) ; $\frac{R_1+10}{R_2}=\frac{60}{40}=\frac{3}{2}$

or $2R_1+20=3R_2$

or $2\times\frac{2}{3}R_2+20=3R_2$ or $4R_2+60=9R_2$

or $R_2=\mathbf{12\ \Omega}$; From (*i*), $R_1=\frac{2}{3}\times12=\mathbf{8\ \Omega}$

100. When null point is at D, the bridge is balanced.

So, $\frac{X}{Y}=\frac{40}{100-40}=\frac{40}{60}=\frac{2}{3}$ or $X=\frac{2}{3}Y$ ...(*i*)

When resistance of 30 Ω is connected in series with X, then at the new null point position

$$\frac{X+30}{Y}=\frac{50}{100-50}=\frac{50}{50}=1 \text{ or } Y=X+30 \quad ...(ii)$$

From (*i*) and (*ii*), $Y=\frac{2}{3}Y+30$

or $Y/3=30$

or $Y=\mathbf{90\ \Omega}$

$\therefore\quad X=\frac{2}{3}\times90=\mathbf{60\ \Omega}$

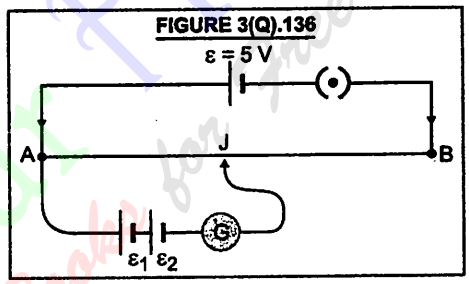
When 30 Ω resistance is connected in series with Y, then $\frac{X}{Y+30}=\frac{l}{(100-l)}$

or $\frac{60}{90+30}=\frac{l}{(100-l)}$ or $\frac{1}{2}=\frac{l}{(100-l)}$

On solving we get, $l=\mathbf{33{\cdot}3\ cm}$

101. (*i*) The circuit diagram is shown in Fig. 3(Q).136.

FIGURE 3(Q).136



(*ii*) Potential gradient of potentiometer wire,

$$K=\frac{5\text{ V}}{5\text{ m}}=\frac{5\text{ V}}{500\text{ cm}}=\frac{1}{100}\text{ V cm}^{-1}$$

In first case, $\varepsilon_1+\varepsilon_2=Kl_1=\frac{1}{100}\times350=3{\cdot}50$ ...(*i*)

In second case, $\varepsilon_1-\varepsilon_2=Kl_2$

$$=\frac{1}{100}\times50=0{\cdot}50 \quad ...(ii)$$

Solving (*i*) and (*ii*), we get

$\varepsilon_1=\mathbf{2{\cdot}0\ V}$ and $\varepsilon_2=\mathbf{1{\cdot}50\ V}$

102. Here, $V=5$ V, $R=480\ \Omega$, resistance of potentiometer wire, $r=20\ \Omega$. Length of potentiometer wire $L=1000$ cm

Potential gradient,

$$K=\left(\frac{V}{R+r}\right)\times\frac{r}{L}$$

$$=\frac{5}{(480+20)}\times\frac{20}{1000}=\mathbf{2\times10^{-4}\ V/cm}$$

Unknown emf ε is balanced against length l = 600 cm. So emf $\varepsilon=Kl=2\times10^{-4}\times600=\mathbf{0{\cdot}12\ V}$

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103. Here, $L = 200$ cm ; $r = 20\ \Omega$, $R = 10\ \Omega$, $V = 6$ V, $\varepsilon = 2{\cdot}4$ V

Potential gradient,

$$K = \left(\frac{V}{R+r}\right)\frac{r}{L} = \left(\frac{6}{20+10}\right)\frac{20}{200}\ \text{V/cm}$$

Balancing length $= \dfrac{\varepsilon}{K} = \dfrac{2{\cdot}4}{0{\cdot}02} = \mathbf{120\ cm}$

104. (*a*) Potential gradient,

$$K = \frac{V}{l} = \frac{1{\cdot}02}{51} = \mathbf{0{\cdot}02\ V/cm}$$

(*b*) emf of the cell

$E_1 = K \times 100 = 0{\cdot}02 \times 100 = \mathbf{2\ V}$

(*c*) When switch S is closed, no current is drawn from the cell E_2, there is no shift in the position of null point as the position of null point depends upon the potential gradient along the potentiometer wire [which depends upon the emf of battery E_1 and resistance of potentiometer wire circuit and length of potentiometer] and emf of the cell E_2 which does not change when switch S is closed.

105. (*i*) Potential gradient along AB

$$= \left(\frac{4}{15+10}\right)\frac{10}{100} = \mathbf{0{\cdot}016\ V\,cm^{-1}}$$

(*ii*) Current through $0{\cdot}6\ \Omega = \dfrac{1{\cdot}5}{1{\cdot}2+0{\cdot}6} = \dfrac{5}{6}$ A ;

Pot. diff. across $0{\cdot}6\ \Omega = \dfrac{5}{6} \times 0{\cdot}6 = 0{\cdot}5$ V

Across length AO, $V = 0{\cdot}3$ V is to be balanced. Let l be the length AO, then

$$0{\cdot}5 = 0{\cdot}016 \times l \quad \text{or} \quad l = \frac{0{\cdot}5}{0{\cdot}016} = \mathbf{31{\cdot}2\ cm}$$

106. Potential gradient, $K = \dfrac{1{\cdot}1}{440} = 0{\cdot}0025$ V/cm.

Potential difference across R

$= 0{\cdot}0025 \times 220 = 0{\cdot}550$ V

Error in voltmeter reading = reading in voltmeter − reading of potentiometer

$= 0{\cdot}5 - 0{\cdot}550 = \mathbf{-0{\cdot}05\ V}$

107. Let d be density of the material of copper wire. Let l_1 and l_2 be lengths of copper wires of diameter 1 mm and 2 mm respectively. As mass = volume × density = $[(\pi D^2/4) \times l]\, d$, so

$$1 = \frac{\pi(10^{-3})^2}{4} l_1 \times d = \frac{\pi(2\times10^{-3})^2}{4} l_2 \times d$$

or $\quad l_1 = 4\, l_2$

$$\therefore \quad R_1 = \frac{\rho l_1}{\pi D_1^2/4} = \frac{\rho 4 l_2}{\pi(10^{-3})^2/4}$$

and $\quad R_2 = \dfrac{\rho l_2}{\pi D_2^2/4} = \dfrac{\rho l_2}{\pi(2\times10^{-3})^2/4}$

so $\quad \dfrac{R_1}{R_2} = 16 \quad$ or $\quad \boldsymbol{R_1 = 16\, R_2}$

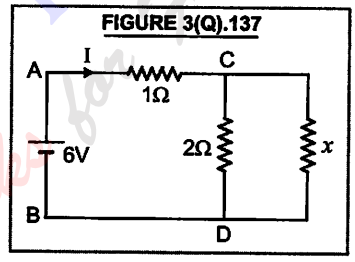
108. Let x be the equivalent resistance of infinite net work. Since the network is infinite, its total resistance x will not change with the addition of one more unit of two resistances. Then the equivalent circuit of net work will be as shown in Fig. 3(Q).137. Here, the total resistance of

net work is $x = 1 + \dfrac{2\times x}{2+x} = \dfrac{2+3x}{2+x}$

or $\quad 2x + x^2 = 2 + 3x \quad$ or $\quad x^2 - x - 2 = 0$

On solving, we get, $\quad \boldsymbol{x = 2\ \Omega}$

FIGURE 3(Q).137



Current in the circuit, $I = \dfrac{\varepsilon}{x} = \dfrac{6}{2} = 3$ A

voltage drop across $1\ \Omega = 3 \times 1 = 3$ V

$\therefore$ Voltage drop across C and $D = 6 - 3 = 3$ V

Hence, current in arm $CD = 3/2 = \mathbf{1{\cdot}5\ A}$

109. When ammeter is in the circuit, total resistance of circuit $= 0{\cdot}1 + 11{\cdot}9 + 6 = 18\ \Omega$. Current in the circuit, $I = 2/18 = 1/9 = \mathbf{0{\cdot}11\ A}$. When ammeter is not in the circuit, total resistance of the circuit $= 0{\cdot}1 + 11{\cdot}9 = 12\ \Omega$.

Current in the circuit, $I' = 2/12 = 1/6$ A

$$\%\ \text{difference in current} = \frac{I' - I}{I'} \times 100$$

$$= \frac{(1/6) - (1/9)}{(1/6)} \times 100 = \mathbf{33{\cdot}3\%}$$

110. Since each path can carry current of one ampere, therefore to pass 4 A current, we need four paths in parallel. Let r be the resistance of each path. The equivalent resistance of 4 parallel paths will be $r/4$. According to given problem,

$r/4 = 5 \quad$ or $\quad r = 5 \times 4 = 20\ \Omega$.

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In order to have 20 Ω resistance in each path, two resistances each of resistance 10 Ω be connected in series. Since there are four paths, the total number of resistances required = 2 × 4 = **8**

111. Relaxation time, $\tau = \dfrac{m}{ne^2\rho}$

$$= \frac{9{\cdot}1\times10^{-31}}{(8{\cdot}5\times10^{28})(1{\cdot}6\times10^{-19})^2\times(1{\cdot}7\times10^{-8})}$$

$$= \mathbf{2{\cdot}46\times10^{-14}\ s}$$

From kinetic theory, $v_{rms} = \sqrt{\dfrac{3kT}{m}}$

$$= \sqrt{\frac{3\times(1{\cdot}38\times10^{-23})\times300}{9{\cdot}1\times10^{-31}}} = 1{\cdot}17\times10^5\ ms^{-1}$$

Therefore, mean free path, $\lambda = v_{rms}\times\tau$

$= 1{\cdot}17\times10^5\times2{\cdot}46\times10^{-14} = \mathbf{2{\cdot}87\times10^{-9}\ m}$

$= \mathbf{28{\cdot}7\ Å}$

112. If one cell is connected in reverse order, then it will destroy one more cell. If n cells are connected in reverse order, then 2 n cells will be destroyed. If E is the emf of each cell, then effective emf. of 12 cells will be = $(12 - 2n)E$.

When this battery if aids two similar cells in series, then it supplies a current of 3 A to resistance R. Therefore,

$$3 = \frac{(12-2n)E + 2E}{R} = \frac{(14-2n)E}{R} \qquad ...(i)$$

When battery and two cells oppose each other, then the current in the external resistance is 2 A. Hence,

$$2 = \frac{(12-2n)E - 2E}{R} = \frac{(10-2n)E}{R} \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{3}{2} = \frac{14-2n}{10-2n} \quad \text{or} \quad n = 1$$

113. Total resistance of the circuit before connecting the voltmeter = $R + 2R + 3R = 6R$.

Hence the current, $I = E/(6R)$.

Voltage across the resistance 2 R is,

$$V = I\times2R = \frac{E}{6R}\times2R = \frac{E}{3}$$

This is the voltage to be measured.

When a voltmeter of resistance 10 R is connected in parallel with 2 R, their effective resistance

$$R_p = \frac{2R\times10R}{2R+10R} = \frac{5}{3}R$$

Now total resistance of the circuit

$$= R + \frac{5R}{3} + 3R = \frac{17}{3}R$$

Hence, current in the circuit,

$$I_1 = \frac{E}{(17/3)R} = \frac{3E}{17R}$$

So, voltage across R_p,

$$V_1 = I_1R_p = \frac{3E}{17R}\times\frac{5}{3}R = \frac{5}{17}E$$

This will be the reading of voltmeter.

$$\therefore \%\ \text{error} = \frac{V - V_1}{V}\times100$$

$$= \left(1 - \frac{V_1}{V}\right)\times100$$

$$= \left[1 - \frac{(5/17)E}{(E/3)}\right]\times100 = \mathbf{11{\cdot}76\%}$$

114. $H = \dfrac{V^2t_1}{R_1} = \dfrac{V^2t_2}{R_2}$ or $\dfrac{R_1}{R_2} = \dfrac{t_1}{t_2}$ ...(*i*)

But $R \propto l$, so $\dfrac{R_1}{R_2} = \dfrac{l_1}{l_2}$ ...(*ii*)

Now, $l_2 = (2/3)\,l_1$

From (*i*) and (*ii*) ;

$$\frac{l_1}{l_2} = \frac{t_1}{t_2} \quad \text{or} \quad \frac{3}{2} = \frac{15}{t_2} \quad \text{or} \quad t_2 = \mathbf{10\ minutes}$$

115. Resistance of bulb, $R_0 = V^2/P$

$= 100\times100/500 = 20\ \Omega$

Current in bulb, $I = V/R_0 = 100/20 = 5$ A

For the same power dissipation in bulb, the current in the circuit must be 5 A. When bulb is operated on 200 V supply, then the **safe resistance** of the circuit (R′) is given by

$$R' = \frac{V}{I} = \frac{250}{5} = 50\ \Omega$$

$\therefore$ Resistance required to be put in series will be

$$R = R' - 20 = 50 - 20 = \mathbf{30\ \Omega}$$

116. $H = I^2R = I^2\dfrac{\rho l}{\pi r^2}$ $(m = \pi r^2 l d)$

$$= I^2\frac{\rho \times m}{(\pi r^2)\times \pi r^2 d} = \frac{I^2 \rho m}{\pi^2 r^4 d}$$

So $H \propto \dfrac{1}{r^4}$ $\therefore$ $\dfrac{H_1}{H_2} = \left(\dfrac{r_2}{r_1}\right)^4 = \left(\dfrac{2}{1}\right)^2$ **= 16 : 1**

117. Effective emf of the circuit, $\varepsilon' = 120 - 8 = 112$ V

Total resistance of circuit, $R = 1 + 15 = 16\ \Omega$

(*i*) Current $I = \dfrac{\varepsilon'}{R} = \dfrac{112}{16}$ **= 7 A**

(*ii*) Terminal voltage during charging

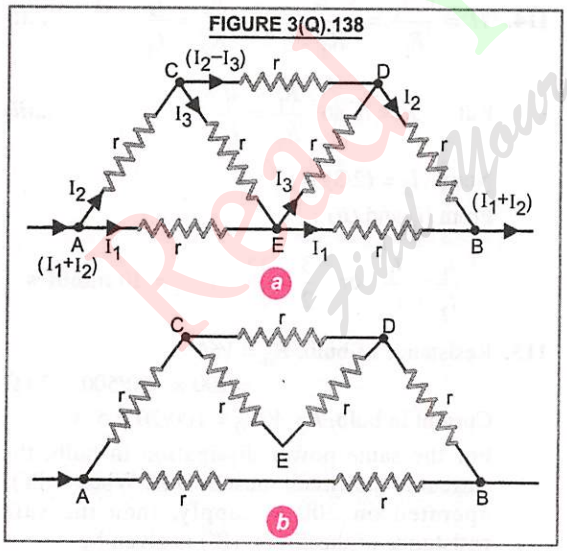
$V = \varepsilon + Ir = 8 + 7 \times 1 =$ **15 V**

(*iii*) Chemical energy stored in the battery in 5 minutes $= \varepsilon It = 8 \times 7 \times (5 \times 60) =$ **16800 J**

118. The distribution of currents in the circuit is shown in the Fig. 3(Q).138(*a*). Due to symmetry, current in arm AE is equal to current in the arm *EB*. Since the current in arm *CE* is equal to the current in the arm *ED*, therefore the resistance of the net work will not change if the wires *CED* and *AEB* are disconnected at *E* as shown in Fig. 3(Q).138(*b*)

Now resistance of path AEB $= r + r = 2r$

FIGURE 3(Q).138



Resistance of path $ACDB = r + \dfrac{(2r)\times r}{(2r)+r} + r$

$$= \frac{8r}{3}$$

The paths *AEB* and *ACDB* are in parallel, therefore, the effective resistance between *A* and *B* will be $\dfrac{1}{R} = \dfrac{1}{2r} + \dfrac{3}{8r} = \dfrac{4+3}{8r} = \dfrac{7}{8r}$ or $R = \dfrac{8r}{7}$

But $r = 2\ \Omega$, therefore, $R = 8 \times 2/7 =$ **16/7 ohm**

119. $\dfrac{R_1}{X} = \dfrac{40}{60} = \dfrac{2}{3}$ or $R_1 = \dfrac{2X}{3}$ ...(*i*)

$\dfrac{R_2}{X} = \dfrac{50}{50} = 1$ or $R_2 = X$ ...(*ii*)

(*i*) when R_1 and R_2 are in series, the effective resistance $= R_1 + R_2 = \dfrac{2X}{3} + X = \dfrac{5X}{3}$

Then ; $\dfrac{5X/3}{X} = \dfrac{l}{(100-l)}$ or $5(100 - l) = 3l$

or $l = 5 \times 100/8 =$ **62·5 cm**

(*ii*) When R_1 and R_2 are in parallel, the effective resistance is

$$R_p = R_1R_2/(R_1 + R_2) = \frac{2X/3 \times X}{(2X/3 + X)} = \frac{2}{5}X$$

Then, $\dfrac{2X/5}{X} = \dfrac{l}{(100-l)}$ or $2(100 - l) = 5l$

or $l = \dfrac{200}{7} =$ **28·6 cm**

120. According to Kirchhoff's first rule at *O*.

$-I_1 - I_2 - I_3 = 0$ or $I_1 + I_2 + I_3 = 0$ ...(*i*)

If V_0 is the electric potential at *O*,

V_1, V_2 and V_3 be the potential at *A*, *B* and *C* respectively, then $V_0 - V_1 = I_1R_1$;

$V_0 - V_2 = I_2R_2$ and $V_0 - V_3 = I_3R_3$

Thus, $I_1 = \dfrac{(V_0 - V_1)}{R_1}$; $I_2 = \dfrac{(V_0 - V_2)}{R_2}$

and $I_3 = \dfrac{(V_0 - V_3)}{R_3}$

Putting values of I_1, I_2 and I_3 in (*i*) we get

$$\frac{V_0 - V_1}{R_1} + \frac{V_0 - V_2}{R_2} + \frac{V_0 - V_3}{R_3} = 0$$

or $V_0\left[\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right] - \left[\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}\right] = 0$

or $V_0 = \left[\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}\right]\left[\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right]^{-1}$

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121. (*a*) Current through arm *ABC*,

$$I = 48/(10 + 20) = 1{\cdot}6 \text{ A}$$

Pot. diff. across *B* and $C = V_B - V_C = 1{\cdot}6 \times 20$

$$= 32 \text{ V}$$

As $V_C = 0$ so $V_B = +32$ V

(*b*) Current through arm *ADC*,

$$I' = 48/(5 + 11) = 3 \text{ A}$$

Pot. diff. across *D* and $C = V_D - V_C = 3 \times 11 = 33$

or $V_D - 0 = 33$ or $V_D = +33$ V

(*c*) As $V_D > V_C$, so direction of current will be *D* to *B*.

(*d*) The bridge will be balanced when

$$\frac{10}{R} = \frac{5}{11}$$

or $$R = \frac{10 \times 11}{5} = 22\ \Omega$$

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NCERT EXERCISES

WITH SOLUTIONS

Q. 1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0·4 Ω, what is the maximum current that can be drawn from the battery ?

Sol. Here, $\varepsilon = 12$ V, $r = 0{\cdot}4\ \Omega$

Maximum current, $I_{max} = \dfrac{\varepsilon}{r} = \dfrac{12}{0{\cdot}4} = \mathbf{30\ A}$

Q. 2. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0·5 A, what is the resistance of the resistor ? What is the terminal voltage of the battery when the circuit is closed ?

Sol. Here, $\varepsilon = 10$ V ; $r = 3\ \Omega$; $I = 0{\cdot}5$ A ; $R = ?$; $V = ?$

$$I = \frac{\varepsilon}{(R+r)} \quad \text{or} \quad (R+r) = \frac{\varepsilon}{I} = \frac{10}{0{\cdot}5} = 20 \quad \text{or} \quad R = 20 - 3 = \mathbf{17\ \Omega}$$

Terminal voltage, $V = IR = 0{\cdot}5 \times 17 = \mathbf{8{\cdot}5\ \Omega}$

Q. 3. At room temperature (27·0°C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature co-efficient of the material of the resistor is $1{\cdot}70 \times 10^{-4}\ °C^{-1}$.

Sol. Here, $R_{27} = 100\ \Omega$; $R_t = 117\ \Omega$; $t = ?$; $\alpha = 1{\cdot}70 \times 10^{-4}\ °C^{-1}$

We know that $\alpha = \dfrac{R_t - R_{27}}{R_{27}(t-27)}$ or $t - 27 = \dfrac{R_t - R_{27}}{R_{27} \times \alpha}$

$$\therefore \quad t = \frac{R_t - R_{27}}{R_{27} \times \alpha} + 27 = \frac{117 - 100}{100 \times 1{\cdot}7 \times 10^{-4}} + 27 = 1000 + 27 = \mathbf{1027°C}$$

Q. 4. A negligibly small current is passed through a wire of length 15 m and uniform cross-section $6{\cdot}0 \times 10^{-7}\ m^2$, and its resistance is measured to be 5·0 Ω. What is the resistivity of the material at the temperature of the experiment ?

Sol. Here, $l = 15$ m ; $A = 6{\cdot}0 \times 10^{-7}\ m^2$; $R = 5{\cdot}0\ \Omega$; $\rho = ?$

$$\rho = \frac{RA}{l} = \frac{5{\cdot}0 \times 6 \times 10^{-7}}{15} = \mathbf{2{\cdot}0 \times 10^{-7}\ \Omega\ m}$$

Q. 5. A silver wire has a resistance of 2·1 Ω at 27·5°C, and a resistance of 2·7 Ω at 100°C, Determine the temperature coefficient of resistivity of silver.

Sol. Here, $R_{27{\cdot}5} = 2{\cdot}1\ \Omega$; $R_{100} = 2{\cdot}7\ \Omega$, $\alpha = ?$

$$\alpha = \frac{R_{100} - R_{27{\cdot}5}}{R_{27{\cdot}5} \times (100 - 27{\cdot}5)} = \frac{2{\cdot}7 - 2{\cdot}1}{2{\cdot}1 \times (100 - 27{\cdot}5)} = \mathbf{0{\cdot}0039°C^{-1}}$$

Q. 6. A heating element using nichrome connected to a 230 V supply draws an initial current of 3·2 A which settles after a few seconds to a steady value of 2·8 A. What is the steady temperature of the heating element if the room temperature is 27·0°C ? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1{\cdot}70 \times 10^{-4} °C^{-1}$.

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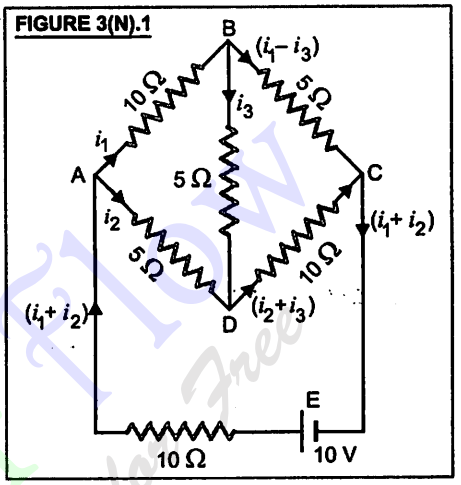
Sol. Here ; $R_{27} = 230/3{\cdot}2 = 2300/32\ \Omega$; $R_t = 230/2{\cdot}8 = 2300/28\ \Omega$; $\alpha = 1{\cdot}7 \times 10^{-4}\ ^\circ C^{-1}$.

$$\alpha = \frac{R_t - R_{27}}{R_{27} \times (t-27)} \qquad \text{or} \qquad t - 27 = \frac{R_t - R_{27}}{R_{27} \times \alpha}$$

or $$t = \frac{R_t - R_{27}}{R_{27} \times \alpha} + 27 = \frac{(2300/28) - (2300/32)}{(2300/32) \times 1{\cdot}7 \times 10^{-4}} + 27 = 840{\cdot}2 + 27 = \mathbf{867{\cdot}2^\circ C.}$$

Q. 7. Determine the current in each branch of the following network : Fig. 3(N).1.



Sol. The currents through the various arms of the circuit have been shown in Fig. 3(N).1.

According to **Kirchhoff's second law;**

In a closed circuit *EABCE* ;

$$10 = 10\,(i_1 + i_2) + 10\,i_1 + 5\,(i_1 - i_3)$$

or $$10 = 25\,i_1 + 10\,i_2 - 5\,i_3$$

or $$2 = 5\,i_1 + 2\,i_2 - i_3 \qquad \text{...(i)}$$

In a closed circuit *ABDA* ;

$$10\,i_1 + 5\,i_3 - 5\,i_2 = 0$$

or $$2\,i_1 + i_3 - i_2 = 0$$

or $$i_2 = 2\,i_1 + i_3 \qquad \text{...(ii)}$$

In a closed circuit *BCDB* ;

$$5\,(i_1 - i_3) - 10\,(i_2 + i_3) - 5\,i_3 = 0$$

or $5\,i_1 - 10\,i_2 - 20\,i_3 = 0$ or $i_1 = 2\,i_2 + 4\,i_3$ ...(iii)

From (*ii*) and (*iii*) ;

$$i_1 = 2\,(2\,i_1 + i_3) + 4\,i_3 = 4\,i_1 + 6\,i_3 \quad \text{or} \quad 3\,i_1 = -6\,i_3 \quad \text{or} \quad i_1 = -2\,i_3 \qquad \text{...(iv)}$$

Putting this value in (*ii*) ; $i_2 = 2\,(-2\,i_3) + i_3 = -3\,i_3$ ...(v)

Putting values in (*i*) ; $2 = 5\,(-2\,i_3) + 2\,(-3\,i_3) - i_3$ or $2 = -17\,i_3$ or $i_3 = \mathbf{-2/17\ A}$

From (*iv*) ; $i_1 = -2\,(-2/17) = \mathbf{4/17\ A}$;

From (*v*), $i_2 = -3\,(-2/17) = \mathbf{(6/17)\ A}$

$\therefore$ $i_1 + i_2 = (4/17) + (6/17) = \mathbf{(10/17)\ A}$

$i_1 - i_3 = 4/17 - (-2/17) = \mathbf{(6/17)\ A}$

$i_2 + i_3 = 6/17 + (-2/17) = \mathbf{4/17\ A}$

Q. 8. A storage battery of emf 8·0 V and internal resistance of 0·5 Ω is being charged by a 120 V d.c. supply using a series resistor of 15·5 Ω. What is the terminal voltage of the battery during charging ? What is the purpose of having a series resistor in the charging circuit ?

Sol. Here, e.m.f. of the battery = 8·0 V ; Voltage of d.c. supply = 120 V

Internal resistance of battery, $r = 0{\cdot}5\ \Omega$; External resistance, $R = 15{\cdot}5\ \Omega$.

Since a storage battery of e.m.f. 8 V is charged with a d.c. supply of 120 V, the effective e.m.f. in the circuit is given by $\varepsilon = 120 - 8 = 112$ V

Total resistance of the circuit $= R + r = 15{\cdot}5 + 0{\cdot}5 = 16{\cdot}0\ \Omega$

$\therefore$ Current in the circuit during charging is given by, $I = \dfrac{\varepsilon}{R+r} = \dfrac{112}{16} = 7{\cdot}0$ A

$\therefore$ Voltage across $R = IR = 7{\cdot}0 \times 15{\cdot}5 = 108{\cdot}5$ V

During charging, the voltage of the d.c. supply in a circuit must be equal to the sum of the voltage drop across R and terminal voltage of the battery

$\therefore$ $120 = 108{\cdot}5 + V$ or $V = 120 - 108{\cdot}5 = \mathbf{11{\cdot}5\ V}$

The series resistor limits the current drawn from the external source of d.c. supply. In its absence the current will be dangerously high.

Q. 9. The number density of free electrons in a copper conductor is estimated at $8{\cdot}5 \times 10^{28}\ m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end ? The area of cross-section of the wire is $2.0 \times 10^{-6}\ m^2$ and it is carrying a current of 3.0 A.

Sol. Here, $n = 8.5 \times 10^{28}\ m^{-3}$; $l = 3.0$ m ; $A = 2.0 \times 10^{-6}\ m^2$; $I = 3.0$ A, $t = ?$

As, $I = n A e v_d \quad \therefore \quad v_d = \dfrac{I}{nAe}$,

Now, $$t = \frac{l}{v_d} = \frac{l n A e}{I} = \frac{3.0 \times 8.5 \times 10^{28} \times 2.0 \times 10^{-6} \times 1.6 \times 10^{-19}}{3.0}$$

$= 2.72 \times 10^4$ s $=$ **7 hour 33 minute.**

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NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. Consider a current carrying wire (current I) in the shape of a circle. Note that as the current progresses along the wire, the direction of $\vec{J}$ (current density) changes in an exact manner, while the current I remains unaffected. The agent that is essentially responsible for is

(*a*) source of emf

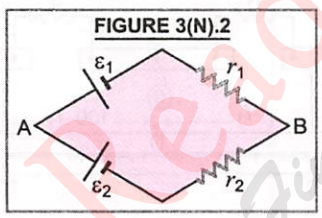
(*b*) electric field produced by charges accumulated on the surface of wire

(*c*) the charges just behind a given segment of wire which push them just the right way by repulsion

(*d*) the charges ahead

2. Two batteries of emf ε_1 and ε_2 ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in Fig. 3(N).2.

FIGURE 3(N).2



(*a*) The equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$

(*b*) The equivalent emf ε_{eq} is smaller than ε_1

(*c*) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always

(*d*) ε_{eq} is independent of internal resistances r_1 and r_2

3. A resistance R is to be measured using a meter bridge. Student chooses the standard resistance S to be 100 Ω. He finds the null point at $l_1 = 2{\cdot}9$ cm. He is told to attempt to improve the accuracy. Which of the following is a useful way ?

(*a*) He should measure l_1 more accurately

(*b*) He should change S to 1000 Ω and repeat the experiment

(*c*) He should change S to 3 Ω and repeat the experiment

(*d*) He should give up hope of a more accurate measurement with a meter bridge

4. Two cells of emfs approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.

(*a*) The battery that runs the potentionmeter should have voltage of 8 V

(*b*) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V

(*c*) The first portion of 50 cm of wire itself should have a potential drop of 10 V

(*d*) Potentiometer is usually used for comparing resistances and not voltages

5. A metal rod of the length 10 cm and a rectangular cross-section of 1 cm × ½ cm is connected to a battery across opposite faces. The resistance will be

(*a*) maximum when the battery is connected across 1 cm × ½ cm faces

(*b*) maximum when the battery is connected across 10 cm × 1 cm faces

(*c*) maximum when the battery is connected across 10 cm × ½ cm faces

(*d*) same irrespective of the three faces

6. Which of the following characteristics of electrons determines the current in a conductor ?

(*a*) Drift velocity alone

(*b*) Thermal velocity alone

(*c*) Both drift velocity and thermal velocity

(*d*) Neither drift nor thermal velocity

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MULTIPLE CHOICE QUESTIONS-II

7. Kirchhoff's junction rule is a reflection of

(*a*) conservation of current density vector

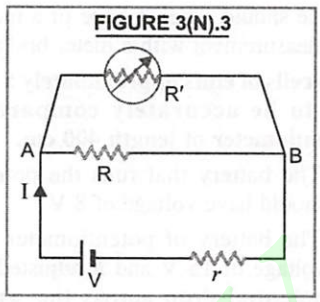
(*b*) conservation of charge

(*c*) the fact that the momentum with which a charged particle approahces a junction is unchanged (as a vector) as the charged particle leaves the junction

(*d*) the fact that there is no accumulation of charges at a junction

8. Consider a simple circuit shown in Fig. 3(N).3. **stands for a variable resis- tance R'. R' can vary from R_0 to infinity. r is internal resistance of the battery ($r << R << R_0$).**

FIGURE 3(N).3



(*a*) Potential drop across AB is nearly constant as R' is varied

(*b*) Current through R' is nearly a constant as R' is varied

(*c*) Current I depends sensitively on R'

(*d*) $I \geq \dfrac{V}{r+R}$ always

9. Temperature dependence of resistivity $\rho(T)$ of semiconductors, insulators and metals is significantly based on the following factors :

(*a*) number of charge carriers can change with temperature T

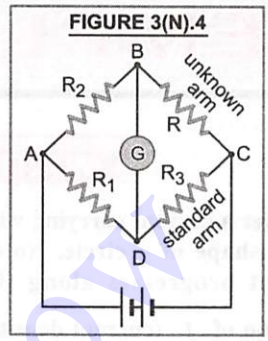
(*b*) time interval between two successive collisions can depend on T

(*c*) length of material can be a function of T

(*d*) mass of carriers is a function of T

10. The measurement of an unknown resistance R is to be carried out using Wheatstone bridge (see Fig. 3(N).4). Two students perform an experiment in two ways. The first student takes R_2 = 10 Ω and R_1 = 5 Ω. The other student takes R_2=1000 Ω and R_1=500 Ω. In the standard arm, both take R_3 = 5 Ω. Both find $R = \dfrac{R_2}{R_1} R_3$ = 10 Ω within errors.

FIGURE 3(N).4



(*a*) The errors of measurement of the two students are the same

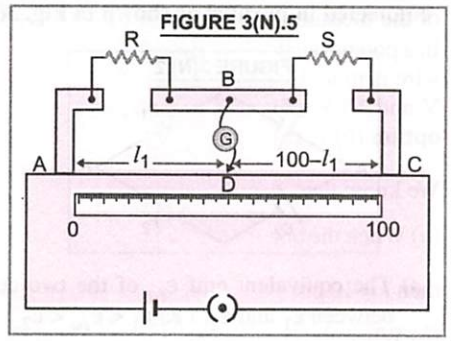
(*b*) Errors of measurement do depend on the accuracy with which R_2 and R_1 can be measured

(*c*) If the student uses large values of R_2 and R_1, the currents through the arms will be feeble. This will make determination of null point accurately more difficult

(*d*) Wheatstone bridge is a very accurate instrument and has no errors of measurement

11. In a meter bridge the point D is a neutral point (Fig. 3(N).5).

FIGURE 3(N).5



(*a*) The meter bridge can have no other neutral point for this set of resistances

(*b*) When the jockey contacts a point on meter wire left of D, current flows to B from the wire

(*c*) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer

(*d*) When R is increased, the neutral point shifts to left

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ANSWERS

1. (*b*) **2.** (*a*) **3.** (*c*) **4.** (*b*) **5.** (*a*) **6.** (*a*) **7.** (*b, d*) **8.** (*a, d*) **9.** (*a, b*) **10.** (*b, c*) **11.** (*a, b, c*)

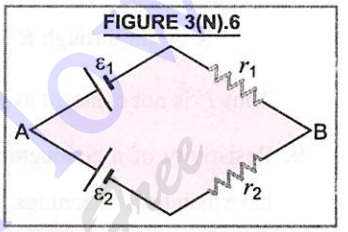
HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. The current density is a vector quantity. Its direction is given by the direction of flow of positive charge in the circuit. The same is possible due to electric field produced by charges accumulated on the surface of wire.

2. Refer Fig. 3(N).6 the equivalent internal resistance of two cells between A and B is

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2} \quad \text{or} \quad r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad ...(i)$$



If ε_{eq} is the equivalent *emf* of the two cells in parallel between A and B, then

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \quad \text{[Refer Art. 3.26(}b\text{)]}$$

$$\therefore \quad \varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \times r_{eq} = \frac{(\varepsilon_1 r_2 + \varepsilon_2 r_1)}{r_1 r_2} \times \frac{r_1 r_2}{(r_1 + r_2)} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 + r_2)}$$

This shows that whatever may be the values of r_1 and r_2, the value of ε_{eq} is in between ε_1 and ε_2. As $\varepsilon_2 > \varepsilon_1$, so $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$.

3. As meter-bridge is balanced, so $\frac{2{\cdot}9}{100 - 2{\cdot}9} = \frac{R}{S} = \frac{R}{100}$ or $R = \frac{100 \times 2{\cdot}9}{97{\cdot}1} \simeq 3\,\Omega$

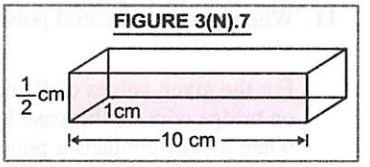
The meter-bridge (*i.e.*, Wheatstone bridge) is most sensitive and accurate when the resistance of all the four arms of the bridge is of the same order. Therefore a student must use $S = 3\ \Omega$ to improve the accuracy of the measurement of resistance R. Hence, **option (*c*) is correct.**

4. In a potentiometer experiment, the *emf* of a cell can be measured if the potential drop along the potentiometer wire is more than the *emf* of the cell to be determined. As values of emfs of two cells are approximately 5 V and 10 V, therefore, the potential drop along the potentiometer wire must be more than 10 V. Hence, **option (*b*) is correct.**

5. We know that $R = \frac{\rho l}{A}$

(*a*) When the battery is connected across 1 cm × ½ cm faces,

then $l = 10$ cm ; $A = 1 \times ½$ cm^2, $R_1 = \frac{\rho \times 10}{1 \times ½} = \mathbf{20}\ \rho\Omega$



(*b*) When the battery is connected across 10 cm × 1 cm faces,

then $l = 1/2$ cm ; $A = 10 \times 1$ cm^2, $R_2 = \frac{\rho \times ½}{10 \times 1} = \frac{\rho}{20}\Omega$

(*c*) When the battery is connected across 10 cm × 1/2 cm faces,

then $l = 1$ cm ; $A = 10 \times 1/2$ cm^2, $R_3 = \frac{\rho \times 1}{(10 \times ½)} = \frac{\rho}{5}\Omega$

6. Current $I = n A e v_d$, *i.e.*, $I \propto v_d$

Therefore current in a conductor is determined by **drift velocity alone.**

Multiple Choice Questions – II

7. Kirchhoff's junction rule or Kirchhoff's current rule is based on the law of conservation of charge and is a reflection of the fact that there is no accumulation of charges at a junction.

8. Current, $I = \dfrac{V}{r + \dfrac{RR'}{R+R'}}$ As $R' >> R$, so $R + R' \geq R'$ Hence, $I = \dfrac{V}{r + RR'/R'} \geq \dfrac{V}{r+R}$

Thus, **option (*d*) is correct.**

Potential difference across A and B, $V_{AB} = I \times \dfrac{RR'}{R+R'}$

$$\therefore \quad V_{AB} = \frac{V}{r + (RR'/R + R')} \times \frac{RR'}{(R+R')} = \frac{V \times RR'}{r(R+R') + RR'} = \frac{V \times RR'}{rR' + RR'} = \frac{VR}{r+R} \qquad [\because R + R' \approx R']$$

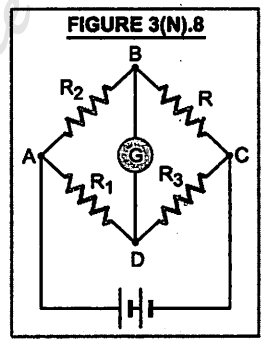
$\therefore V_{AB}$ is constant if R' is varied. Hence, **option (*a*) is correct.**

Current through R', $I' = \dfrac{V_{AB}}{R'} = \dfrac{VR}{(r+R)\,R'} = \dfrac{VR}{rR' + RR'}$

Thus I' is not constant as R' is varied. The current I does not depend on the sensitively on R'.

9. Resistivity of a conductor is given by $\rho = \dfrac{m}{ne^2\tau}$, where the various terms have their usual meanings. As temperature of semiconductor changes, n changes and also τ changes. Hence, ρ changes with temperature. Thus **options (*a*) and (*b*) are true.**

FIGURE 3(N).8



10. Refer Fig. 3(N).8.

For first student ; $R_2 = 10\ \Omega, R_1 = 5\ \Omega, R_3 = 5\ \Omega$

For other student ; $R_2 = 1000\ \Omega, R_1 = 500\ \Omega, R_3 = 5\ \Omega$

According to Wheatstone bridge principle

$$\frac{R_2}{R} = \frac{R_1}{R_3} \quad \text{or} \quad R = R_3 \times \frac{R_2}{R_1}$$

Putting values in above relation, we note that the value of R for both the students is 10 Ω.

The Wheatstone bridge is most sensitive and accurate if the resistance of all the four arms of bridge is of the same value. Therefore, the errors of measurement of the two students depend on the accuracy and sensitivity of the bridge, which inturn depends on the accuracy with which R_2 and R_1 can be measured. When R_2 and R_1 are large, the currents through the arms of bridge will be feeble. This will make the determination of null point accurately more difficult. Thus, **options (*b*) and (*c*) are correct.**

11. When there is a neutral point at D in meter-bridge, then $\dfrac{R}{S} = \dfrac{l_1}{(100 - l_1)}$

For the given values of R and S, there will be only one value of l_1 for which we shall get the neutral point on bridge wire. In this case $V_A - V_B = V_A - V_D$ or $V_B = V_D$. Therefore the galvanometer shows no deflection when jockey contacts a point at D. There is no current in galvanometer arm.

When jockey contacts a point D_1 on meter-bridge wire left of D, the resistance of arm AD_1 becomes smaller than previous value. Due to it, $V_A - V_B > V_A - V_{D_1}$ or $V_{D_1} > V_B$. **Therefore current flows to B from the wire through galvanometer.**

When jockey contacts a point D_2 on meter beidge wire right of D, the resistance of arm AD_2 becomes more than first value. Due to it, $V_A - V_{D_2} > V_A - V_B$ or $V_B > V_{D_2}$

Therefore **the current will flow from *B* to the wire through galvanometer.** When R is increased, the neutral point will shift to the right instead of left on bridge wire. Thus, **options (*a*), (*b*) and (*c*) are correct.**

VERY SHORT ANSWER QUESTIONS

12. Is the motion of a charge across junction momentum conserving ? Why or why not ?

Ans. When motion of charge is taking place across a junction, then momentum of charge is not conserved. Because when a charge (*i.e.*, electron) approaches a junction, in addition to a uniform electric field $\vec{E}$ that it normally faces (which keeps the drift velocity $\vec{v}_d$ fixed), there are accumulation of charges on the surface of wires at the junction. Which also produce electric field, resulting the variation in the direction of momentum of charge.

13. The relaxation time τ is nearly independent of applied electric field *E* whereas it changes significantly with temperature *T*. First fact is (in part) responsible for Ohm's law whereas the second fact leads to variation of ρ with temperature. Elaborate why ?

Ans. The drift velocity of electrons, $v_d = \frac{eE}{m}\tau$, or $\tau = \frac{mv_d}{eE}$ or $\tau \propto v_d/E$

Thus relaxation time depends on velocities of electrons and ions and electric field. The applied electric field affects the velocities of electrons at the order of 1 mm/s, which is very very small and can be neglected. This supports Ohm's law. On the other hand, change in temperature (*TK*) affects velocities of electrons and ions at the order of 10^2 m/s, which affect τ to great extent and hence to ρ, *i.e.*, resistivity as $\rho = \frac{m}{ne^2\tau}$

14. What are the advantages of the null-point method in a Wheatstone bridge ? What additional measurements would be required to calculate $R_{unknown}$ by any other method ?

Ans. In a Wheatstone bridge, the null point method involves a balanced Wheatstone bridge, in which the resistance of galvanometer does not affect the balance point and there is no need to determine the currents in resistances and galvanometer.

The unknown resistance can also be determined by using unbalanced Wheatstone bridge. But in this method we require the additional accurate measurement of all the currents in resistors and galvanometer as well as the internal resistance of galvanometer.

15. What is the advantages of using thick metallic strips to join wires in a potentiometer ?

Ans. In potentiometer the number of straight wires each of length 1 metre are held parallel on a horizontal wooden board and joined by thick metal strips in order to have long effective length of potentiometer wire which is required for accurate measurement. Since the resistance of thick copper strips is very very small, can be ignored ; while measuring the potentiometer length for the null point. The potentiometer length for null deflection can be noted down directly from the metre scale provided on the wire, which leads to accurate measurement.

16. For wiring in the home, one uses Cu wires or Al wires. What considerations are involved in this ?

Ans. For wiring in a house, the two considerations are involved :

(*i*) cost of metal wire used. (*ii*) Good conductivity of metal wire.

The silver wire has the best conductivity but it can not be used due to its high cost. After silver, the copper and aluminium are the next best conductors. Their cost is also low than silver. Therefore copper and aluminium are used for wiring in the house.

Conductivity of copper wire is more than that of aluminium wire and for a given area and length, the mass of copper wire is more than that of aluminium wire. Therefore, if we require better conductivity without considering the cost involved, then copper wire is preferred. If we keep in mind the cost factor involved in the wiring, then aluminium wire is preferred.

17. Why are alloys used for making standard resistance coils ?

Ans. The allows (*i.e.*, manganin or constantan) are used for making the standard resistance coils as they have low value of temperature coefficient of resistance and have high value of resistivity.

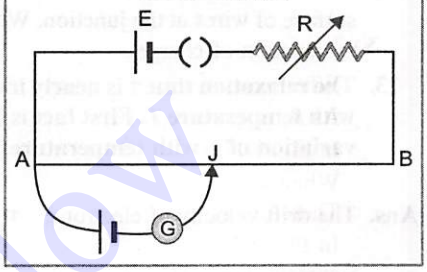
18. Power P is to be delivered to a device via transmission cables having resistance R_c. If V is the voltage across R and I the current through it, find the power wasted and how can it be reduced.

Ans. Power delivered, P = voltage × current = VI or $I = P/V$. If R_C is the resistance of transmission line then power wasted, $P_C = I^2 R_C = \dfrac{P^2}{V^2} R_C$

In order to reduce P_C, V should be large. Therefore the power should be transmitted at high voltage.

19. AB is a potentiometer wire (Fig. 3(N).9). If the value of R is increased, in which direction will the balance point J shift ?

FIGURE 3(N).9



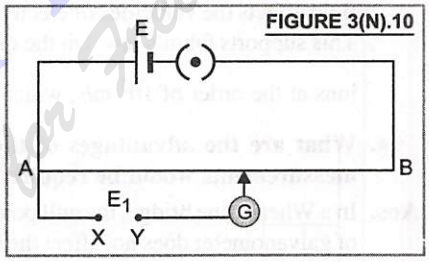
Ans. If R is increased, the current through the potentiometer wire will decrease. Due to it, the potential gradient of potentiometer wire will also decrease. As a result of it, the balancing length will increase, *i.e.*, the position of J **will shift on potentiometer wire towards B.**

20. While doing an experiment with potentiometer (Fig. 3(N).10) it was found that the deflection is one sided and (i) the deflection decreased while moving from one end A of the wire to the end B; (ii) the deflection increased, while the jockey was moved towards the end B.

(a) Which terminal + or –ve of the cell E_1, is connected at X in case (i) and how is E_1 related to E ?

(b) Which terminal of the cell E_1 is connected at X in case (*ii*) ?

FIGURE 3(N).10



Ans. In the given potentiometer experiment, the deflection in the galvanometer will be one sided if the *emf* of cell E_1 is greater than the *emf* of battery E because then *emf* E_1 is greater than fall of potential across the potentiometer wire due to battery E.

(*a*) The deflection in galvanometer will decrease while moving from one end A of the poteniomter wire to the end B, if positive of the cell E_1 is is connected to X and negative of the cell to Y, because in this situation, the current in galvanometer decreases with the increase in length of potentiometer wire.

(*b*) The deflection in galvanometer will increase while moving from end A of the poteniomter wire to the end B if – ve of the cell E_1 is connected to X and + ve to Y. In this situation, the current in galvanometer increases with the increase in the length of potentiometer wire.

21. A cell of emf E and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of P.D. across R, verses R.

Ans. Refer to solution of Conceptual Problem 24, Page 3/93.

SHORT ANSWER QUESTIONS

22. First a set of n equal resistors of R each are connected in series to a battery of emf ε and internal resistance R. A current I is observed to flow. Then the n resistors are connected in parallel to the same battery. It is observed that the current is increased 10 times. What is 'n' ?

Ans. When n resistors are in series, then current in the circuit, $I = \dfrac{\varepsilon}{R + nR} = \dfrac{\varepsilon}{(n+1)R}$ ...(*i*)

When n resistors are in parallel, then effective resistance is $R_{eff} = R/n$.

Current in circuit, $I' = \dfrac{\varepsilon}{R + R/n} = \dfrac{n\varepsilon}{R(n+1)}$

As per question, $I' = 10\, I \quad \therefore \quad \dfrac{n\varepsilon}{R(n+1)} = \dfrac{10\varepsilon}{(n+1)R}$ or $n = \mathbf{10}$

23. Let there be n resistors R_1.....R_n with R_{max} = max (R_1.....R_n) and R_{min} = min {R_1.....R_n}. Show that when they are connected in parallel, the resultant resistance $R_p < R_{min}$ and when they are connected in series, the resultant resistance $R_S > R_{max}$. Interpret the result physically.

Ans. Let R_{max} and R_{min} be the maximum value and minimum value of the given n unequal resistors. When n resistors are connected in parallel, the effective resistance R_p is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n} \qquad \therefore \qquad \frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + ... + \frac{R_{min}}{R_n}$$

As R_p is less than the minimum resistance R_{min}, so $\frac{R_{min}}{R_p} > 1$ or $R_{min} > R_p$ or $R_p < R_{min}$

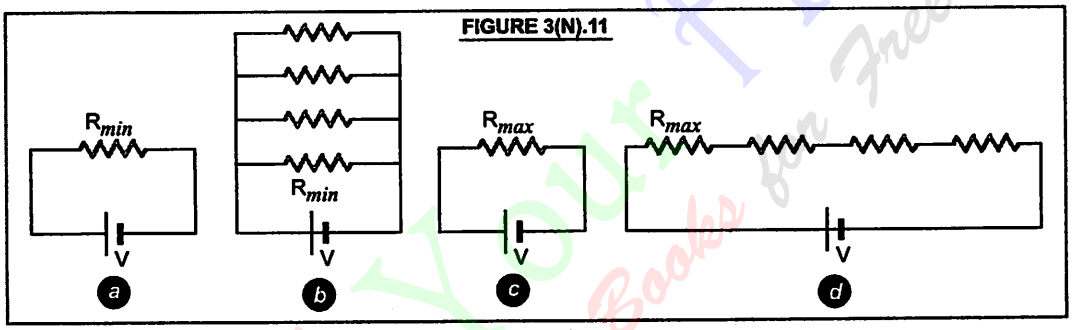
When n resistors are in series, then $R_s = R_1 + R_2 + ... + R_n$

As R_s is greater than the max. resistance, R_{max} $\therefore$ $R_s > R_{max}$

In Fig. 3(N).11(a), due to one resistor of resistance R_{min}, there is only one route for the current in circuit. When n resistors are in parallel, Fig. 3(N).11(b), there are additional $(n-1)$ routes for the current in circuit. Therefore $R_{min} > R_p$ or $R_p < R_{min}$.

In Fig. 3(N).11(c) and (d), due to one resistor of resistance R_{max}, the current in circuit is more than the current due to n resistors in series of effective resistance R_s. So $R_s > R_{max}$.

FIGURE 3(N).11



24. The circuit in Fig. 3(N).12 shows two cells connected in opposition to each other. Cell E_1 is of emf 6 V and internal resistance 2 Ω; the cell E_2 is of emf 4 V sand internal resistance 8 Ω. Find the potential difference between the points A and B.

FIGURE 3(N).12

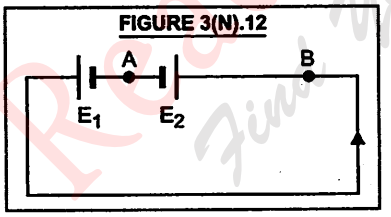
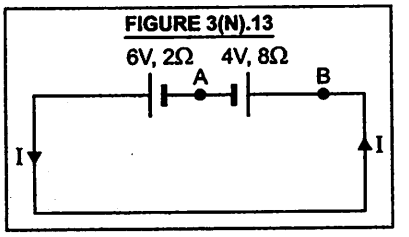


FIGURE 3(N).13



Ans. Here, $E_1 = 6$ V, $r_1 = 2$ Ω, $E_2 = 4$ V, $r_2 = 8$ Ω

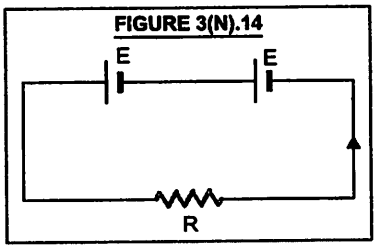
Refer Fig. 3(N).13, current in circuit, $I = \frac{6-4}{2+8} = \frac{1}{5} A$

Potential difference across A and B is the terminal voltage

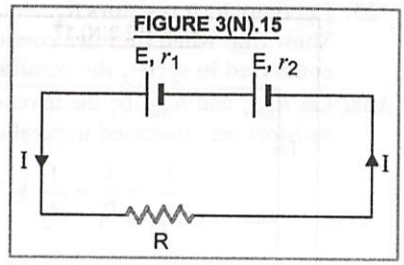
of cell $E_1 = E_1 - Ir_1 = 6 - \frac{1}{5} \times 2 = 5{\cdot}6$ V

25. Two cells of same emf E but internal resistance r_1 and r_2 are connected in series to an external resistor R (Fig. 3(N).14). What should be the value of R so that the potential difference across the therminals of the first cell becomes zero.

FIGURE 3(N).14



Ans. Refer Fig. 3(N).15,



Current in circuit, $I = \dfrac{E+E}{r_1+r_2+R} = \dfrac{2E}{r_1+r_2+R}$

As per question, let for the given value of R, terminal potential difference across first cell be zero, *i.e.*, $E - Ir_1 = 0$

or $$E = I r_1 = \frac{2E}{(r_1+r_2+R)} \cdot r_1$$

or $$1 = \frac{2r_1}{(r_1+r_2+R)} \quad \text{or} \quad r_1+r_2+R = 2r_1 \quad \text{or} \quad R = r_1 - r_2$$

26. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistance R_A to R_B.

Ans. For a solid wire of resistance R_A, $l_1 = l$, $\rho_1 = \rho$, $D_1 = 1$ mm, $A_1 = \pi D_1^2/4 = \pi (1)^2/4$ mm^2

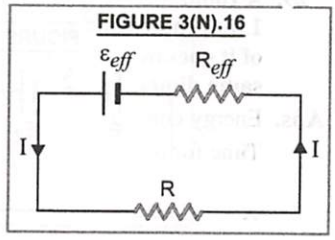
$$R_A = \frac{\rho_1 l_1}{A_1} = \frac{\rho l}{\pi(1)^2/4} = \frac{4\rho l}{\pi}$$

For hollow tube of resistance R_B, $l_2 = l$, $\rho_2 = \rho$, $A_2 = \pi (D_2^2 - D_1^2)/4 = \pi (2^2 - 1^2)/4 = 3\pi/4$ mm^2

$$R_B = \frac{\rho_2 l_2}{A_2} = \frac{\rho l}{(3\pi/4)} = \frac{4\rho l}{3\pi} \qquad \therefore \; R_A / R_B = 3$$

27. Suppose there is a circuit consisting of only resistances and batteries. Suppose one is to double (or increase it to n-times) all voltages and all resistances. Show that currents are unaltered.

Ans. Let there be n batteries in series of the circuit. Let R_{eff} be the effective internal resistance of n resistors in series and ε_{eff} is the effective *emf* of n batteries in series and R is the external resistance in the circuit, Fig. 3(N).16.



Current through R is $$I = \frac{\varepsilon_{eff}}{R_{eff}+R} \qquad ...(i)$$

When all the voltages and resistances are increased n fold, then

$$\varepsilon_{eff}^{new} = n\varepsilon_{eff} \; ; \; R_{eff}^{new} = n R_{eff} \quad \text{and} \quad R^{new} = n R.$$

Current, $$I^{new} = \frac{\varepsilon_{eff}^{new}}{R_{eff}^{new} + R^{new}} = \frac{n\varepsilon_{eff}}{n R_{eff} + nR} = \frac{\varepsilon_{eff}}{R_{eff}+R} = I$$

Thus, **current remains unaltered when all voltages and resistors are increased to n folds.**

LONG ANSWER QUESTIONS

28. Two cells of voltage 10 V and 2 V and internal resistances 10 Ω and 5 Ω respectively, are connected in parallel with the positive end of 10 V battery connected to negative pole of 2 V battery (Fig. 3(N).17). Find the effective voltage and effective resistance of the combination.

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FIGURE 3(N).17

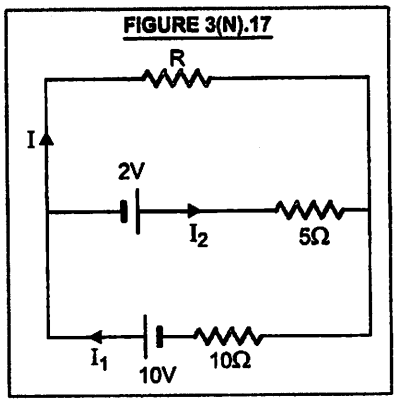
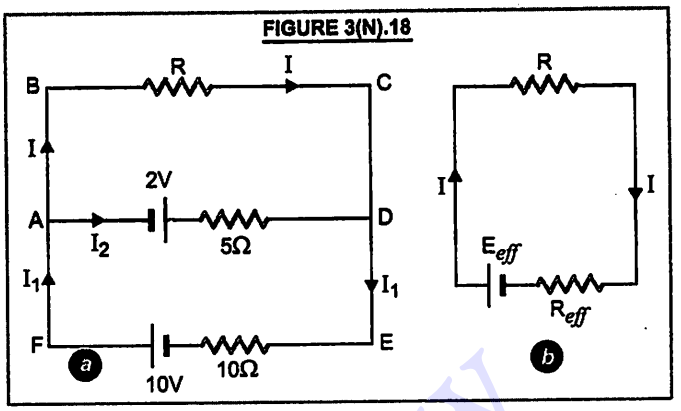


FIGURE 3(N).18



Ans. In Fig. 3(N).18(*a*), according to Kirchhoff's first rule, at junction *A*,

$$I_1 = I + I_2 \quad \text{or} \quad I = I_1 - I_2 \qquad ...(i)$$

According to Kirchhoff's second rule, in closed loop *ABCDA* ; we have

$$IR - I_2 \times 5 = -2 \quad \text{or} \quad I_2 = \frac{IR+2}{5} \qquad ...(ii)$$

In closed loop BCDEFAB, $IR + I_1 \times 10 = 10$ or $I_1 = \frac{10-IR}{10}$ ...(*iii*)

Putting values in (*i*), we get $I = \frac{10-IR}{10} - \frac{IR+2}{5} = \frac{10-IR-2IR-4}{10} = \frac{6-3IR}{10}$

or $10I = 6 - 3IR$ or $I(10+3R) = 6$ or $I = \frac{6}{10+3R} = \frac{6/3}{(10/3)+R}$ ...(*iv*)

If the ε_{eff} and r_{eff} are the effective voltage and effective internal resistance of the combination, then from Fig. 3(N).18(*b*),

$$I = \frac{\varepsilon_{eff}}{r_{eff} + R} \qquad ...(v)$$

Comparing (*iv*) and (*v*), we get, $\varepsilon_{eff} = 6/3 = 2$ V ; $r_{eff} = 10/3\ \Omega$.

29. A room has AC run for 5 hours a day at a voltage of 220 V. The wiring of the room consists of Cu of 1 mm radius and a length of 10 m. Power consumption per day is 10 commercial units. What fraction of it goes in the joule heating in wires ? What would happen if the wiring is made of aluminium of the same dimensions ? [$\rho_{cu} = 1{\cdot}7 \times 10^{-8}\ \Omega$m, $\rho_{Al} = 2{\cdot}7 \times 10^{-8}\ \Omega$m]

Ans. Energy consumed per day = 10 kWh

Time for using the energy per day = 5 h.

$\therefore$ Power consumed (P) $= \frac{10\,\text{kWh}}{5\,\text{h}} = 2\,\text{kW} = 2000\,\text{J/s}$; Current supplied, $I = \frac{P}{V} = \frac{2000\,\text{J/s}}{220\,\text{V}} \approx 9\,\text{A}$

Power loss in copper wire $= R_{cu} I^2 = \frac{\rho_{cu} l}{\pi r^2} \times I^2 = \frac{(1{\cdot}7\times10^{-8})\times10}{(22/7)\times(10^{-3})^2} \times 9^2 \approx 4\,\text{J/s}$

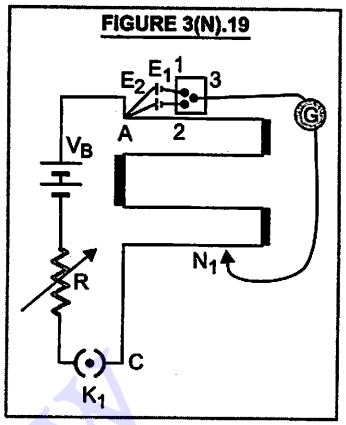
$\therefore$ Fraction of power loss $= \frac{4}{2000} = \frac{2}{1000}$; % of power loss $= \frac{2}{1000} \times 100 = \mathbf{0{\cdot}2\%}$

Power loss in Al wire $= \frac{\rho_{Al} \times l}{\pi r^2} I^2 = \frac{(2{\cdot}7\times10^{-8})\times10\times9^2}{(22/7)\times(10^{-3})^2} = 6{\cdot}4\,\text{J/s}$

% of power loss in Al wire $= \frac{6{\cdot}4}{2000} \times 100 = \mathbf{0{\cdot}32\%}$

30. In an experiment with a potentiometer, $V_B = 10$ V. R is adjusted to be 50 Ω (Fig. 3(N).19). A student wanting to measure voltage E_1 of a battery (approx. 8 V) finds no null point possible. He then diminishes R to 10 Ω and is able to locate the null point on the last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and potential drop per unit length across the wire in the second case.

FIGURE 3(N).19



Ans. Let R' be the resistance of potentiometer wire. When $R = 50$ Ω in series with potentiometer wire circuit, current through potentiometer wire is $I = \dfrac{V_B}{R+R'} = \dfrac{10}{50+R'}$

Potential drop across potentiometer wire $= IR' = \dfrac{10}{(50+R')} \times R'$

As no null point is found on potentiometer wire while measuring voltage E_1 (= 8 V), so

$$\frac{10R'}{(50+R')} < 8 \quad \text{or} \quad 10R' < 400 + 8R' \quad \text{or} \quad R' < 200\ \Omega$$

When $R = 10$ Ω, in series of potentiometer wire, then $\dfrac{10R'}{10+R'} > 8$ or $2R' > 80$ or $R' > 40\ \Omega$

If no deflection in galvanometer is on the last third segment of the potentiometer wire, then

$$\frac{10\times(3/4)R'}{10+R'} < 8 \quad \text{or} \quad 7{\cdot}5R' < 80 + 8R' \quad \text{or} \quad -0{\cdot}5R' < 80 \quad \text{or} \quad R' > 80/0{\cdot}5 \quad \text{or} \quad R' > 160\ \Omega$$

It means R' lies between **160 Ω and 200 Ω**

Therefore, any value of resistance of potentiometer wire between 160 Ω and 200 Ω will help us in achieving the no deflection on fourth wire of potentiometer circuit.

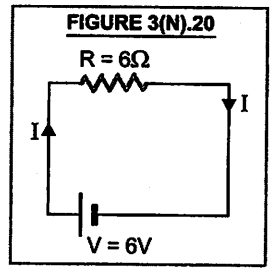
Here, potential drop across 4 m of wire > 8 V, potential drop across 3 m of wire < 8 V.

If ϕ is the potential gradient, then $4\phi > 8$ V ...(*i*) and $3\phi < 8$ V ...(*ii*)

From (*i*), $\phi > 8$ V/4 m = 2 V/m or $\phi > 2$ V/m

From (*ii*), $3\phi < 8$ V or $\phi < \dfrac{8}{3}$ V/m or $\phi < 2\dfrac{2}{3}$ V/m

FIGURE 3(N).20



31. (a) Consider circuit in Fig. 3(N).20. How much energy is absorbed by electrons from the initial state of no current (ignore thermal motion) to the state of drift velocity ?

(b) Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would one associate with energy in problem (a) ? n = no of electron/volume = $10^{29}/m^3$, length of circuit = 10 cm, cross-section, $A = (1\ mm)^2$

Ans. (*a*) Current $I = \dfrac{V}{R} = \dfrac{6\,V}{6\,\Omega} = 1\,A$, Drift velocity, $v_d = \dfrac{I}{nAe} = \dfrac{1}{10^{29}\times(10^{-6})\times(1{\cdot}6\times10^{-19})} = \dfrac{1}{1{\cdot}6}\times10^{-4}$ m/s.

No. of electrons in the wire = nAl

$$\text{KE of all the electrons} = \frac{1}{2}m_e v_d^2 \times nAl = \frac{1}{2}\times(9{\cdot}1\times10^{-31})\times\left(\frac{1}{1{\cdot}6}\times10^{-4}\right)^2\times10^{29}\times(10^{-6})\times10^{-1}$$

$$\approx \mathbf{2\times10^{-17}\ J}$$

(*b*) Power loss = $I^2R = 1^2 \times 6 = 6$ J/s

All the KE of electrons would be lost in time $= \dfrac{2\times10^{-17}\ J}{6\ J/s} = 0{\cdot}33\times10^{-17}\ s \approx \mathbf{10^{-17}\ s}$

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I. Multiple Choice Questions (with One Correct Answer)

I. Electric current and drift velocity

1. The electron of a hydrogen atom revolves around the proton in a circular nth orbit of radius $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$ with a speed $v_n = \frac{e^2}{2\epsilon_0 n h}$. The current due to circulating charge is proportional to

(*a*) e^2 (*b*) e^3
(*c*) e^5 (*d*) e^6

[AMU (Med.) 2012]

2. The mean free path of electrons in a metal is 4×10^{-8} m. The electric field which can give on an average 2 eV energy to an electron in the metal will be in the units V/m,

(*a*) 5×10^{-11} (*b*) 8×10^{-11}
(*c*) 5×10^{7} (*d*) 8×10^{7}

(AIPMT 2009)

3. The current flowing through wire depends on time as, $I = 3t^2 + 2t + 5$.
The charge flowing through the cross-section of the wire in time $t = 0$ to $t = 2$ second is

(*a*) 22 C (*b*) 20 C
(*c*) 18 C (*d*) 5 C **(WB JEE 2009)**

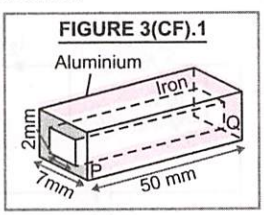
4. A uniform copper wire of length 1 m and cross-sectional area 0·5 mm^2 carries a current of 1 A. Assuming that there are 8×10^{22} free electrons per cm^3 in copper, how long will an electron take to drift from one end of the wire to the other end ?

(*a*) $1{\cdot}6 \times 10^3$ s (*b*) $3{\cdot}2 \times 10^3$ s
(*c*) $6{\cdot}4 \times 10^3$ s (*d*) $8{\cdot}0 \times 10^3$ s

II. Ohm's law and Resistance

5. In an aluminium (Al) bar of square cross-section, a square hole is drilled and is filled with Iron (Fe) as shown in the Fig. 3(CF).1, The electrical resistivities of Al and Fe are $2{\cdot}7 \times 10^{-8}$ Ωm and $1{\cdot}0 \times 10^{-7}$ Ωm, respectively. The electrical resistance between two faces P and Q of the composite bar is

FIGURE 3(CF).1



(*a*) $\frac{2475}{64}\mu\Omega$ (*b*) $\frac{1875}{64}\mu\Omega$
(*c*) $\frac{1875}{49}\mu\Omega$ (*d*) $\frac{2475}{132}\mu\Omega$

(JEE Advanced 2015)

6. In order to quadruple the resistance of a wire of uniform cross-section, a fraction of its length was uniformly stretched till the final length of the entire wire was 1·5 times the initial length. The fraction is

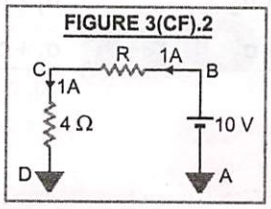
(*a*) 1/2 (*b*) 1/4
(*c*) 1/6 (*d*) 1/8

ANSWERS

1. (*c*) 2. (*c*) 3. (*a*) 4. (*c*) 5. (*b*) 6. (*d*)

7. In the Fig. 3(CF).2, the potential at points B and C, and the value of resistance R are

FIGURE 3(CF).2



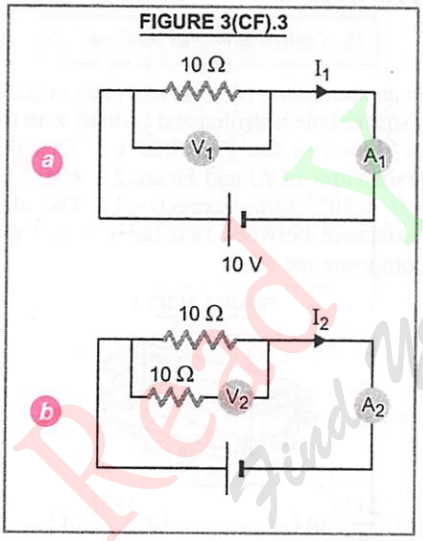
(*a*) 10 V, 4 V, 4 Ω (*b*) 6 V, 4 V, Ω
(*c*) 10 V, 4 V, 6 Ω (*d*) 6 V, 4 V, 4 Ω

8. A carbon resistor of $(47 \pm 4.7)\ k\Omega$ is to be marked with rings of different colours for its identification. The colour code sequence will be
(*a*) Yellow – Green – Violet – Gold
(*b*) Yellow – Violet – Orange – Silver
(*c*) Violet – Yellow – Orange – Silver
(*d*) Green – Orange – Violet – Gold

(NEET 2018)

9. In the circuits shown in Fig. 3(CF).3(*a*) and (*b*), the readings of the voltmeters and the ammeters will be

FIGURE 3(CF).3



(*a*) $V_2 > V_1$ and $I_1 = I_2$ (*b*) $V_1 = V_2$ and $I_1 > I_2$
(*c*) $V_1 = V_2$ and $I_1 = I_2$ (*d*) $V_2 > V_1$ and $I_1 > I_2$

(NEET 2019)

10. A copper wire is stretched to make it 0·5% longer. The percentage change in its electric resistance, if its volume remains unchanged, is :
(*a*) 2·0% (*b*) 2·5%
(*c*) 1·0% (*d*) 0·5%

(JEE Main 2019)

11. A 2 W carbon resistor is colour coded with green, black, red and gold respectively. The maximum current which can be passed through this resistor is
(*a*) 20 mA (*b*) 100 mA
(*c*) 0·4 mA (*d*) ·63 mA

(JEE Main 2019)

III. Resistivity, Electrical conductivity and effect of temperature

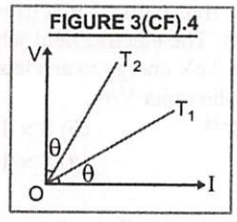
12. In a conductor, if the number of conduction electrons per unit volume is $8{\cdot}5 \times 10^{28}\ m^{-3}$ and mean free path is 25 fs (femto second), its approximate resistivity is :
$[m_e = 9{\cdot}1 \times 10^{-31}\ kg]$
(*a*) $10^{-5}\ \Omega m$ (*b*) $10^{-6}\ \Omega m$
(*c*) $10^{-7}\ \Omega m$ (*d*) $10^{-8}\ \Omega m$

(JEE Main 2019)

13. Copper and iron wires of same length and diameter are in series and connected across a battery. The resistivity of copper is about one sixth of that of iron. If E_1 and E_2 are the electric fields in the copper and iron wires respectively, then which one of the following is correct ?
(*a*) $E_1 < E_2$ (*b*) $E_1 = E_2 \neq 0$
(*c*) $E_1 > E_2$ (*d*) $E_1 = E_2 = 0$

14. V–I graph for a conductor (Platinum wire) at temperatures T_1 and T_2 is as shown in Fig. 3(CF).4. $T_2 - T_1$ is proportional to

FIGURE 3(CF).4



(*a*) cos 2 θ (*b*) sin 2 θ
(*c*) cot 2 θ (*d*) tan 2 θ.

15. When 5 V potential difference is applied across a wire of length 0·1 m, the drift speed of electrons is $2{\cdot}5 \times 10^{-4}\ ms^{-1}$. If the electron density in the wire is $8 \times 10^{28}\ m^{-3}$. The resistivity of the material is close to
(*a*) $1{\cdot}6 \times 10^{-8}\ \Omega m$ (*b*) $1{\cdot}6 \times 10^{-7}\ \Omega m$
(*c*) $1{\cdot}6 \times 10^{-6}\ \Omega m$ (*d*) $1{\cdot}6 \times 10^{-5}\ \Omega m$

(JEE Main 2015)

ANSWERS

7. (*c*) 8. (*b*) 9. (*c*) 10. (*c*) 11. (*a*) 12. (*d*) 13. (*a*) 14. (*c*) 15. (*d*)

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16. At 0°C the electric resistance of a conductor B is n times that of conductor A. The temperature coefficient of resistance for conductors A and B are α_1 and α_2 respectively. The temperature coefficient of resistance of a circuit segment containing A and B in series is

(a) $\frac{n\alpha_1 + n\alpha_2}{1+n}$ (b) $\frac{n\alpha_1 - n\alpha_2}{1+n}$

(c) $\frac{\alpha_1 + n\alpha_2}{1+n}$ (d) $\frac{\alpha_2 + n\alpha_1}{1+n}$

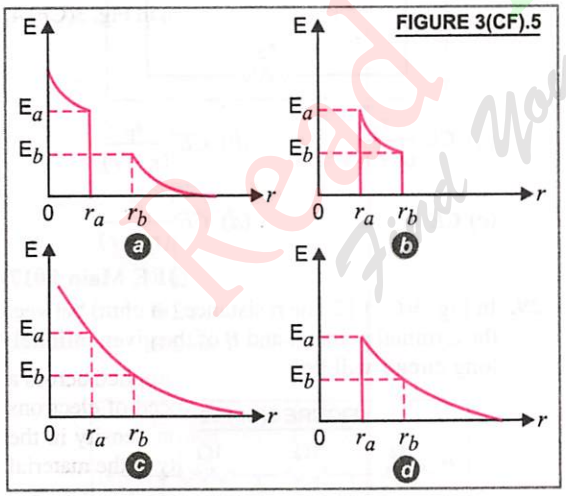
17. Following the above question, the temperature coefficient of resistance of a circuit segment containing A and B in parallel is

(a) $\frac{n\alpha_1 + n\alpha_2}{1+n}$ (b) $\frac{n\alpha_1 - n\alpha_2}{1+n}$

(c) $\frac{\alpha_1 + n\alpha_2}{1+n}$ (d) $\frac{\alpha_2 + n\alpha_1}{1+n}$

18. The region between two concentric spheres of radii r_a and r_b is filled with a conducting material, of resistivity ρ. The potential of inner sphere is V_a and of outer sphere is V_b where $V_a > V_b$. Due to it, there is a current in radially outward direction. The variation of electric field intensity E as a function of distance r from centre of sphere is given by

FIGURE 3(CF).5



19. Two metal wires of identical dimensions are connected in series. If σ_1 and σ_2 are the conductivities of the metal wires respectively, the effective conductivity of the combination is :

(a) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ (b) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

(c) $\frac{\sigma_1+\sigma_2}{2\sigma_1\sigma_2}$ (d) $\frac{\sigma_1+\sigma_2}{\sigma_1\sigma_2}$

(RE-AIPMT 2015)

20. A current of 5 A is flowing through magnesium wire. The current density is making an angle of 60° with area vector. Find the electric field.

(Given area = 2 m^2,

ρ = resistivity of magnesium = 11×10^{-4} SI units)

(a) 55×10^{-4} V/m (b) $\frac{5}{11} \times 10^{-4}$ V/m

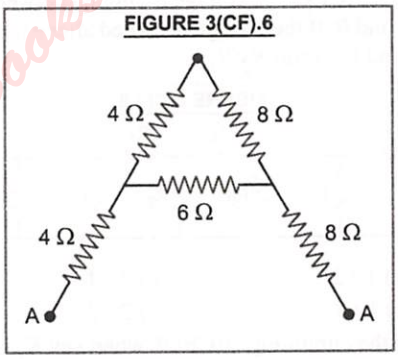
(c) $\frac{11}{5} \times 10^{-4}$ V/m (d) $\frac{55}{2} \times 10^{-4}$ V/m

(JEE Main 2021)

IV. Resistances in series and Parallel

21. The equivalent resistance between A and B for the mesh shown in Fig. 3(CF).6 is :

FIGURE 3(CF).6



(a) 4·8 Ω (b) 7·2 Ω

(c) 16 Ω (d) 30 Ω

(NEET 2020)

22. The effective resistance of a parallel connection that consists of four wires of equal legth, equal area of cross-section and same material is 0·25 Ω. What will be the effective resistance if they are connected in series ?

(a) 4 Ω (b) 0·25 Ω

(c) 0·5 Ω (d) 1 Ω **(NEET 2020)**

ANSWERS

16. (c) 17. (d) 18. (b) 19. (b) 20. (a) 21. (c) 22. (a)

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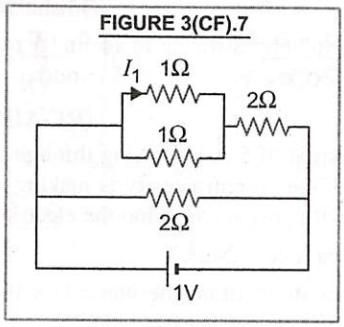
23. The current I_1 (in A) flowing through 1 Ω resistor in the following circuit is

FIGURE 3(CF).7



(*a*) 0·5 (*b*) 0·2 (*c*) 0·25 (*d*) 0·4

(JEE Main 2020)

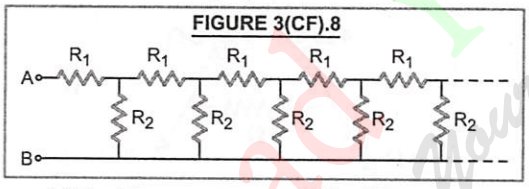
24. A set of 'n' equal resistors, of value 'R' each, are connected in series to a battery of *emf* 'E' and internal resistance 'R'. The current drawn is I. Now, the 'n' resistors are connected in parallel to the same battery. Then the current drawn from battery becomes 10 I. The value of 'n' is

(*a*) 20 (*b*) 11

(*c*) 10 (*d*) 9 **(NEET 2018)**

25. Consider an infinite ladder of network shown in Fig. 3(CF).8. A voltage is applied between points A and B. If the voltage is halved after each section. Find the ratio R_2/R_1.

FIGURE 3(CF).8

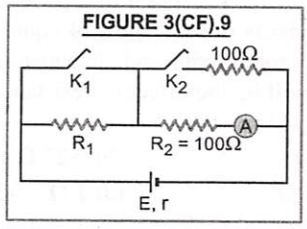


(*a*) 1 : 2 (*b*) 2 : 1

(*c*) 1 : 1 (*d*) 2 : 3

26. In the circuit Fig. 3(CF).9, when key K_1 is closed, the ammeter reads I_0 whether K_2 is open or closed. But when K_1 is open, the ammeter reads $I_0/2$ when K_2 is closed. Assuming that ammeter resistance is much less than R_2, find the values of r and R_1 (in ohms)

FIGURE 3(CF).9

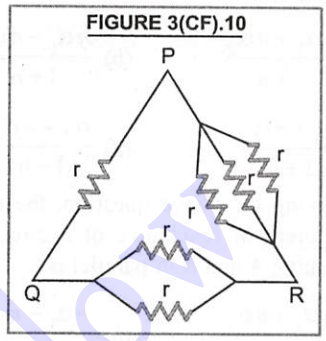


(*a*) 100, 50 (*b*) 50, 100

(*c*) 0, 100 (*d*) 0, 50

27. Six equal resistances are connected between points P, Q and R as shown in Fig. 3(CF).10. Then net resistance will be maximum between

FIGURE 3(CF).10

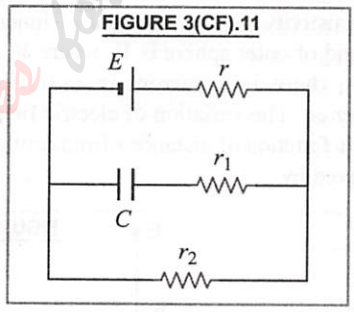


(*a*) P and Q (*b*) Q and R

(*c*) P and R (*d*) any two points

28. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be

FIGURE 3(CF).11



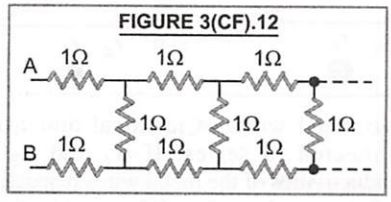
(*a*) $CE\dfrac{r_2}{(r+r_2)}$ (*b*) $CE\dfrac{r_1}{(r_1+r)}$

(*c*) CE (*d*) $CE\dfrac{r_1}{(r_2+r)}$

(JEE Main 2017)

29. In Fig. 3(CF).12, the resistance (in ohm) between the terminal points A and B of the given infinitely long circuit will be

FIGURE 3(CF).12



ANSWERS

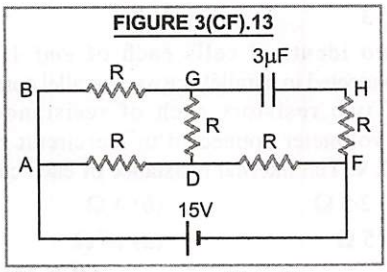
23. (*b*) 24. (*c*) 25. (*b*) 26. (*d*) 27. (*a*) 28. (*a*)

(a) $(\sqrt{3}-1)$ (b) $(1-\sqrt{3})$

(c) $(1+\sqrt{3})$ (d) $(2+\sqrt{3})$.

30. In the circuit shown, the cell is ideal with emf = 15 V, each resistance is of 3 Ω. The potential difference across the capacitor is

FIGURE 3(CF).13



(a) Zero (b) 9 V

(c) 12 V (d) 15 V

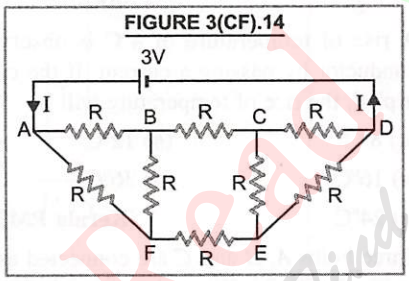
31. An electric current of 5 A is passing through a circuit containing three wires arranged in parallel. If the lengths and radii of the wires are in the ratio 2 : 3 : 4 and 3 : 4 : 5, then the ratio of currents passing through wires should be

(a) 27 : 32 : 35 (b) 54 : 64 : 75

(c) 9 : 16 : 25 (d) 4 : 9 : 16

32. Fig. 3(CF).14 shows a network of eight resistors each of resistance R (= 2 Ω), connected to a 3 V battery of negligible internal resistance. The current I in the circuit is

FIGURE 3(CF).14

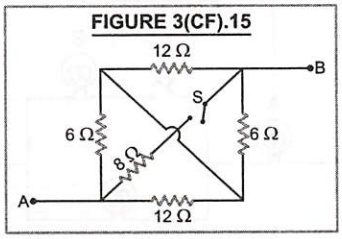


(a) 0·25 A (b) 0·50 A

(c) 0·75 A (d) 1·0 A

33. The equivalent resistance between points A and B with switch S open and closed are respectively

FIGURE 3(CF).15



(a) 4 Ω, 8 Ω (b) 8 Ω, 4 Ω

(c) 6 Ω, 9 Ω (d) 9 Ω, 6 Ω

(Odisha JEE 2010)

34. If equivalent resistance of identical resistors in series combination is S and in parallel combination is P. If $S = n P$, then find the minimum possible value of n ?

(a) 1 (b) 2

(c) 0 (d) 4 **(JEE Main 2021)**

V. Terminal potential difference and internal resistance of a cell

35. The internal resistance of primary cell is 4 Ω. It generates a current of 0·2 A in an external resistance of 21 Ω. The rate at which chemical energy is consumed in providing the current is

(a) 0·42 J/s (b) 0·24 J/s

(c) 5 J/s (d) 1 J/s **(AIIMS 2013)**

36. A student measures the terminal potential difference (V) of a cell (of emf ε and internal resistance r) as a function of current (I) flowing through it. The slope and the intercept, of the graph between V and I, then respectively, equal

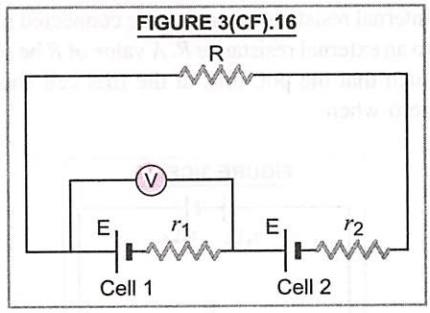
(a) $-r$ and ε (b) r and $-\varepsilon$

(c) $-\varepsilon$ and r (d) ε and $-r$

(AIPMT 2009)

37. In the circuit shown in Fig. 3(CF).16, cells are of equal emf E but of different internal resistances, $r_1 = 6\ \Omega$ and $r_2 = 4\ \Omega$. Reading of the ideal voltmeter connected across cell 1 is zero. The value of the external resistance R in ohm is equal to

FIGURE 3(CF).16



(a) 10 Ω (b) 4 Ω

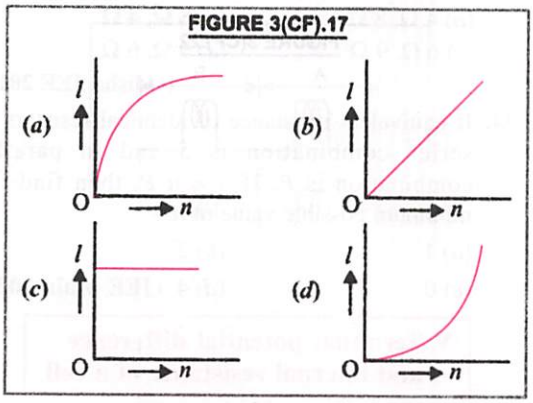
(c) 2 Ω (d) 2·4 Ω

38. A battery consists of a variable number 'n' of identical cells (having internal resistance 'r' each) which are connected in series. The terminals of the battery are short-circuited and the current I is measured. Which of the graphs shows the correct relationship between I and n ?

ANSWERS

29. (c) 30. (c) 31. (b) 32. (d) 33. (b) 34. (d) 35. (d) 36. (a) 37. (c)

FIGURE 3(CF).17

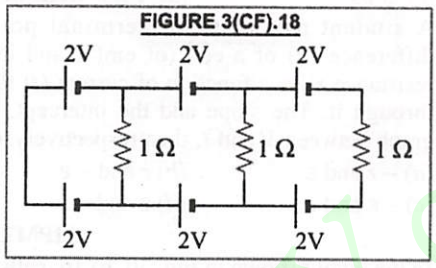


(NEET 2018)

VI. Combination of cells

39. In the following circuit, the current in each resistance is

FIGURE 3(CF).18

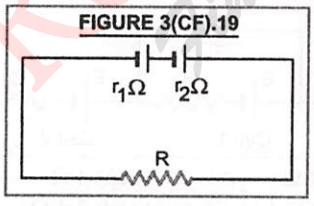


(a) 0·5 A (b) 0 A
(c) 1 A (d) 0·25 A

(JEE Main 2017)

40. Two cells with the same e.m.f. E and different internal resistance r_1 and r_2 are connected in series to an external resistance R. A value of R be selected such that the pot. diff. at the first cell should be zero when

FIGURE 3(CF).19



(a) $R = r_1 + r_2$ (b) $R = r_1 - r_2$
(c) $R = r_1/r_2$ (d) $R = r_1 = r_2$.

41. 12 cells, each having same *emf*, are connected in series and are kept in a closed box. Some of cells are wrongly connected. This battery is connected in series with an ammeter and two cells, identical with the others. The current is 3 A with the cells and battery aid with each other and 2 A when the cells and battery oppose each other. How many cells are wrongly connected.

(a) 1 (b) 2
(c) 3 (d) 4

42. Two identical cells each of *emf* 1·5 V are connected in parallel across a parallel combination of two resistors each of resistance 20 Ω. A voltmeter connected in the circuit measures 1·2 V. The internal resistance of each cell is :

(a) 2·5 Ω (b) 4 Ω
(c) 5 Ω (d) 10 Ω

(JEE Main 2022)

VII. Joule's law of heating, Electric power and Electric energy

43. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$, where a and b are positive constants. The total heat produced in R is :

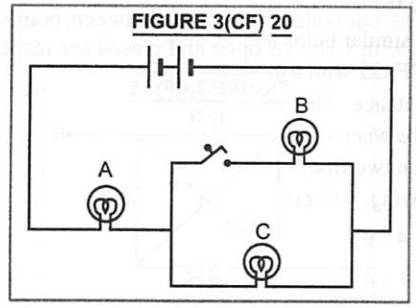
(a) $\frac{a^3R}{3b}$ (b) $\frac{a^3R}{2b}$
(c) $\frac{a^3R}{b}$ (d) $\frac{a^3R}{6b}$ **(NEET 2016)**

44. A rise of temperature of 4°C is observed in a conductor by passing a current. If the current is tripled, the rise of temperature will be

(a) 8°C (b) 12°C
(c) 16°C (d) 36°C
(e) 24°C **(Kerala PMT 2010)**

45. Three bulbs A, B and C are connected as shown in Fig. 3(CF).20. What changes occur in brightness of the bulbs when the switch is closed ?

FIGURE 3(CF) 20



ANSWERS

38. (c) 39. (b) 40. (b) 41. (a) 42. (c) 43. (d) 44. (d)

(*a*) Brightness of *A* increases, but that of *C* decreases.

(*b*) Brightness of *A* remains the same but that of *C* decreases.

(*c*) Brightness of both *A* and *C* decreases.

(*d*) Brightness of *A* increases but that of *C* remains the same.

46. If two bulbs of wattage 25 W and 100 W respectively each rated at 220 V connected in series with the supply of 440 V, which bulb will fuse ?

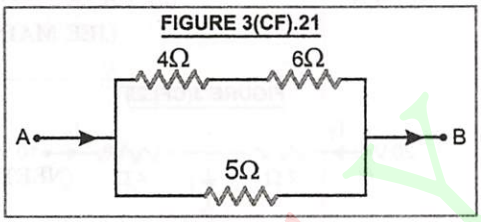
(*a*) 25 W bulb (*b*) 100 W bulb

(*c*) both of them (*d*) none of them.

(AIPMT 2012)

47. In the circuit shown in Fig. 3(CF).21, the heat produced in 5 Ω resistor due to current flowing through it is 10 calories per second. The heat produced in 6 Ω resistor is

FIGURE 3(CF).21



(*a*) 1 calorie/sec (*b*) 2 calorie/sec

(*c*) 3 calorie/sec (*d*) 4 calorie/sec.

48. How many lamps each of 50 W and 100 V can be connected in parallel across a 120 V battery of internal resistance 10 Ω, so that each glows to full power ?

(*a*) 2 (*b*) 4

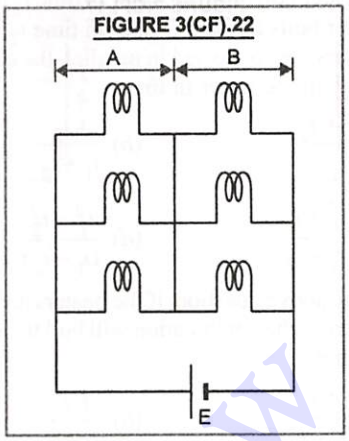
(*c*) 6 (*d*) 8.

49. Six similar bulbs are connected as shown in the Fig. 3(CF).22 with a *dc* source of *emf E* and zero internal resistance. The ratio of power consumption by the bulbs when (*i*) all are glowing and (*ii*) in the situation when two from section *A* and one from section *B* are glowing, will be :

(*a*) 4 : 9 (*b*) 9 : 4

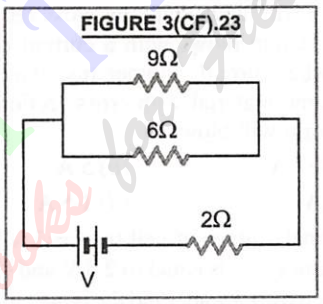
(*c*) 1 : 2 (*d*) 2 : 1 **(NEET 2019)**

FIGURE 3(CF).22



50. If power dissipated in the 9 Ω resistor in the circuit shown in Fig. 3(CF).23, is 36 watt, the potential difference across the 2 Ω resistor is

FIGURE 3(CF).23



(*a*) 4 volt (*b*) 8 volt

(*c*) 10 volt (*d*) 2 volt

[AIPMT (Prelim) 2011]

51. The supply voltage to room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb ?

(*a*) zero volt (*b*) 2·9 volt

(*c*) 13·3 volt (*d*) 10·04 volt

[JEE Main 2012]

52. A d.c. voltage with appreciable ripple expressed as $V = V_1 + V_2 \cos \omega t$ is applied to a resistor *R*. The amount of heat generated per second is given by

(*a*) $\frac{V_1^2 + V_2^2}{2R}$ (*b*) $\frac{2V_1^2 + V_2^2}{2R}$

(*c*) $\frac{V_1^2 + 2V_2^2}{2R}$ (*d*) none of the above.

ANSWERS

45. (*a*) 46. (*a*) 47. (*c*) 48. (*b*) 49. (*b*) 50. (*c*) 51. (*d*) 52. (*b*)

53. A heater boils 1 kg of water in time t_1 and another heater boils the same water in time t_2. If both the heaters are connected in parallel, the combination will boil the water in time

(a) $\frac{t_1 t_2}{t_1 - t_2}$ (b) $\frac{t_1 t_2}{t_1 + t_2}$

(c) $\frac{t_1^2 + t_2^2}{t_1 + t_2}$ (d) $\frac{t_1^2 - t_2^2}{(t_1 - t_2)}$

54. In the above question, if the heaters are connected in series, the combination will boil the same water in time

(a) $\frac{t_1 t_2}{t_1 - t_2}$ (b) $\frac{t_1 t_2}{t_1 + t_2}$

(c) $\frac{t_1^2 + t_2^2}{t_1 + t_2}$ (d) $\frac{t_1^2 - t_2^2}{(t_1 - t_2)}$

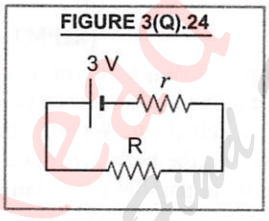
55. A fuse wire with a circular cross-sectional radius of 0·02 mm blows with a current of 5 ampere. For what current, another fuse wire made from the same material with cross-sectional radius of 0·04 mm will blow ?

(a) 14·7 A (b) 5 A

(c) 3 A (d) 1·5 A

56. Terminal voltage of cell (emf = 3 V and internal resistance = r) is equal to 2·5 V and heat loss in R is given by 0·5 watt. Find the power loss in internal resistance.

FIGURE 3(Q).24



(a) 0·3 W (b) 0·5 W

(c) 0·1 W (d) 1 W

(JEE Main 2020)

57. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$$T_t = T_0 (1 + \beta t^{1/4})$$

where β is constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is :

(a) $\frac{4P (T_t - T_0)^3}{\beta^4 T_0^4}$ (b) $\frac{4P (T_t - T_0)}{\beta^4 T_0^2}$

(c) $\frac{4P (T_t - T_0)^4}{\beta^4 T_0^5}$ (d) $\frac{4P (T_t - T_0)^2}{\beta^4 T_0^3}$

(JEE Advanced 2019)

58. Two resistors of resistance, 100 Ω and 200 Ω are connected in parallel in an electric circuit. The ratio of the thermal energy developed in 100 Ω to that in 200 Ω in a given time is

(a) 1 : 2 (b) 2 : 1

(c) 1 : 4 (d) 4 : 1 **(NEET 2022)**

VIII. Kirchhoff's laws

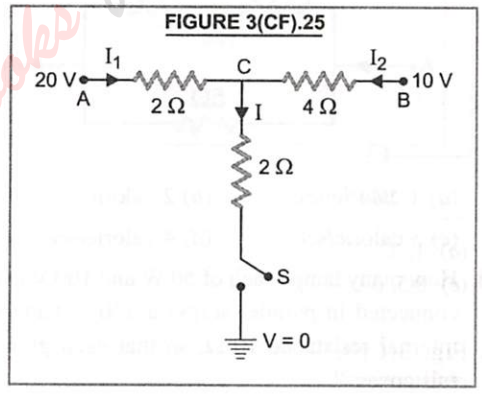
59. When the switch S, in the circuit shown in Fig. 3(CF).25, is closed, then the value of current I will be

(a) 3 A (b) 5 A

(c) 4 A (d) 2 A

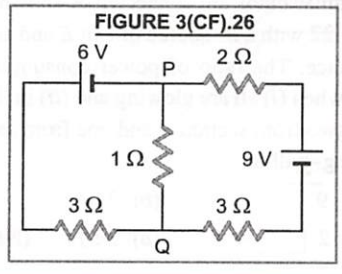
(JEE MAIN 2019)

FIGURE 3(CF).25



60. In the circuit shown, the current in the 1 Ω resistor is [Fig. 3(CF).26]

FIGURE 3(CF).26



ANSWERS

53. (b) 54. (d) 55. (a) 56. (c) 57. (a) 58. (b) 59. (b)

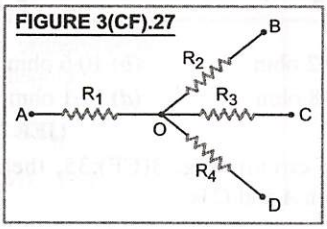
(*a*) 1·3 A from *P* to *Q* (*b*) 0 *A*

(*c*) 0·13 *A* from *Q* to *P*

(*d*) 0·13 *A*, from *P* to *Q* **[JEE Main 2015]**

61. In the Fig. 3(CF).27, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 40\ \Omega$, $R_4 = 80\ \Omega$ and $V_A = 5$ V, $V_B = 10$ V, $V_C = 20$ V, $V_D = 15$ V. The current in the resistance R_1 will be

FIGURE 3(CF).27

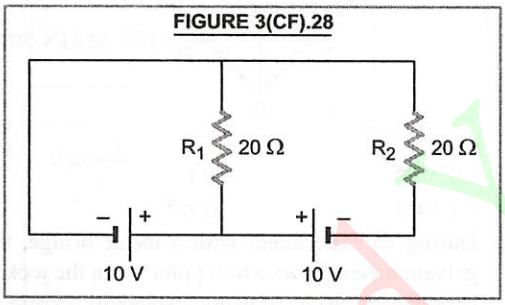


(*a*) 0·4 A towards O (*b*) 0·4 A away from O

(*c*) 0·8 A towards O (*d*) 0·8 A away from O

62. In the given circuit the cells have zero internal resistance. The current (in amperes) passing through resistance R_1 and R_2 respectively, are :

FIGURE 3(CF).28



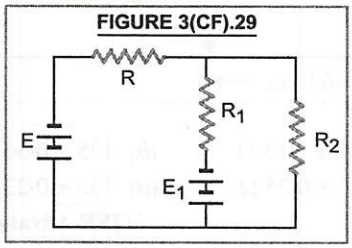
(*a*) 1, 2 (*b*) 2, 2

(*c*) 0·5, 0 (*d*) 0, 1

(JEE Main 2019)

63. Fig. 3(CF).29 shows a circuit with known resistances *R*, R_1 and R_2. Neglect the resistances of the conducting wires and internal resistance of current sources. The magnitude of electromotive force E_1 such that the current *I* through the resistance *R* is zero will be

FIGURE 3(CF).29

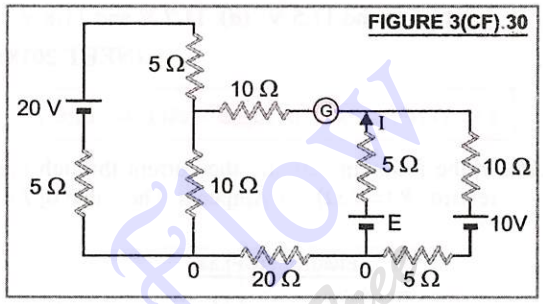


(*a*) $E\left(\frac{R_2}{R_1}\right)$ (*b*) $E\left(\frac{R_1}{R_2}\right)$

(*c*) $E\left(\frac{R_1+R_2}{R_2}\right)$ (*d*) $E\left(\frac{R_1}{R_1+R_2}\right)$.

64. What would be the value of E so that the galvanometer G in the circuit shown in Fig. 3(CF).30 shows no deflection.

FIGURE 3(CF).30

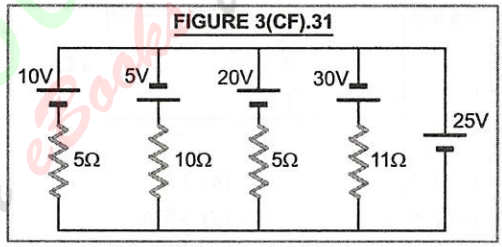


(*a*) 5 V (*b*) 10 V

(*c*) 15 V (*d*) 20 V

65. In the circuit shown in Fig. 3(CF).31, the current flowing through 25 V cell is

FIGURE 3(CF).31

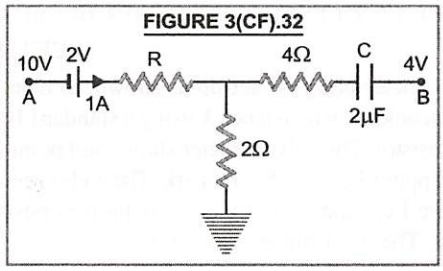


(*a*) 7·2 A (*b*) 6 A

(*c*) 12 A (*d*) 14·2 A

66. A network of resistances, cell and capacitor C (= 2 μF) is shown in Fig. 3(CF).32. In steady state condition, the charge on 2 μF capacitor is *Q*, while *R* is unknown resistance. Values of *Q* and *R* are respectively

FIGURE 3(CF).32



ANSWERS

60. (*c*) 61. (*b*) 62. (*c*) 63. (*c*) 64. (*b*) 65. (*c*)

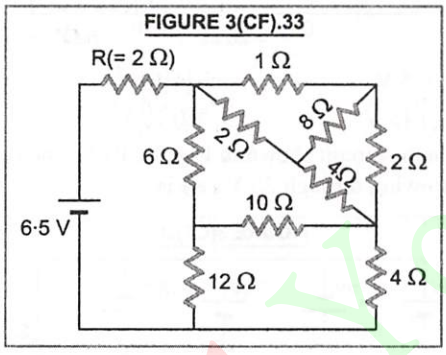
(*a*) 2 μ C and 2 Ω (*b*) 4 μ C and 4 Ω
(*c*) 4 μ C and 10 Ω (*d*) 8 μ C and 4 Ω

67. Two batteries with *emf* 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between
(*a*) 11.6 V and 11.7 V (*b*) 11.5 V and 11.6 V
(*c*) 11.4 V and 11.5 V (*d*) 11.7 V and 11.8 V
(NEET 2018)

IX. Wheatstone bridge/Meter bridge

68. In the following circuit, the current through the resistor R (= 2 Ω) is I Amperes. The value of I is

FIGURE 3(CF).33

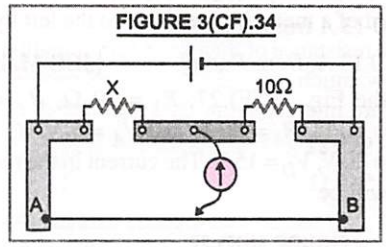


(*a*) 1 A (*b*) 2 A
(*c*) 2·5 A (*d*) 3·5 A
(JEE Advanced 2015)

69. A resistance wire connected in the left gap of a metre bridge balances a 10 Ω resistance in the right gap at a point which divides the bridge wire in the ratio 3 : 2. If the length of the resistance wire is 1·5 m, then the length of 1 Ω of the resistance wire is :
(*a*) $1{\cdot}0 \times 10^{-1}$ m (*b*) $1{\cdot}5 \times 10^{-1}$ m
(*c*) $1{\cdot}5 \times 10^{-2}$ m (*d*) $1{\cdot}0 \times 10^{-2}$ m
(NEET 2020)

70. A meter bridge is set-up as shown, to determine an unknown resistance X using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is :

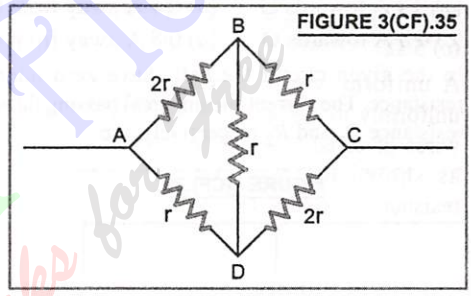
FIGURE 3(CF).34



(*a*) 10·2 ohm (*b*) 10·6 ohm
(*c*) 10·8 ohm (*d*) 11·1 ohm
(JEE Main 2011)

71. In the circuit, Fig. 3(CF).35, the resistance between A and C is

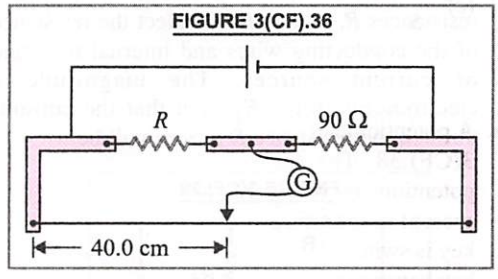
FIGURE 3(CF).35



(*a*) 7 *r*/5 (*b*) *r*
(*c*) 3 *r*/5 (*d*) *r*/5.

72. During an experiment with a meter bridge, the galvanometer shows a null point when the jockey is pressed at 40·0 cm using a standard resistance of 90 Ω as shown in Fig. 3(CF).36. The least count of the scale used in the meter bridge is 1 mm. The unknown resistance R is

FIGURE 3(CF).36



(*a*) 60 ± 0·15 Ω (*b*) 135 ± 0·56 Ω
(*c*) 60 ± 0·25 Ω (*d*) 135 ± 0·23 Ω
(JEE Advanced 2014)

ANSWERS
66. (*c*) 67. (*b*) 68. (*a*) 69. (*a*) 70. (*b*) 71. (*a*) 72. (*c*)

73. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistance ?

(*a*) 990 Ω (*b*) 505 Ω

(*c*) 550 Ω (*d*) 910 Ω

(JEE Main 2018)

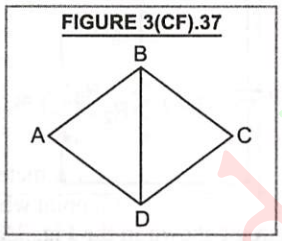
74. A resistance of 2 Ω is connected across one gap of a meter-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance points shifts by 20 cm. Neglecting any correction, the unknown resistance is

(*a*) 3 Ω (*b*) 4 Ω

(*c*) 5 Ω (*d*) 6 Ω **(IIT 2007)**

75. A uniform wire of resistance R is stretched uniformly to n times. It is then cut to form five wires of equal length. These wires are arranged as shown in Fig. 3(CF).37. The effective resistance between A and C is

FIGURE 3(CF).37



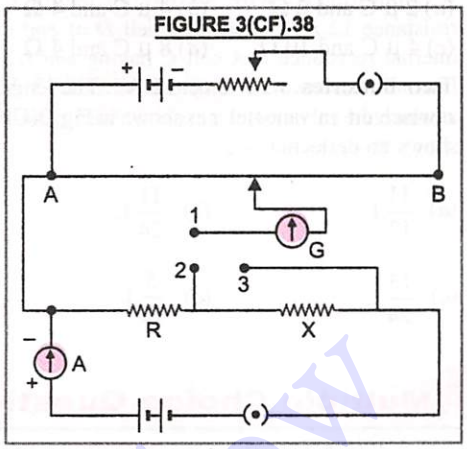
(*a*) $\frac{n^2 R}{3}$ (*b*) $\frac{n^2 R}{5}$

(*c*) $\frac{nR}{5}$ (*d*) $\frac{nR}{3}$

X. Potentiometer

76. A potentiometer circuit is set up as shown in Fig. 3(CF).38. The potential gradient, across the potentiometer wire, is K volt/cm and the ammeter, present in the circuit, reads 1·0 A when two way key is switched off. The balance points, when the key between the terminals (*i*) 1 and 2 (*ii*) 1 and 3, is plugged in, are found to be at lengths l_1 cm and l_2 cm respectively. The magnitudes, of the resistors R and X, in ohms, are then, equal, respectively, to

FIGURE 3(CF).38



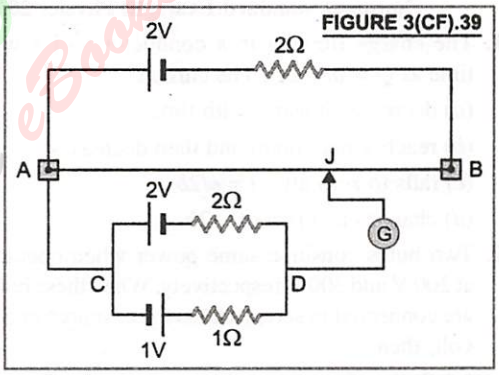
(*a*) $K(l_2 - l_1)$ and $K l_2$ (*b*) $K l_1$ and $K(l_2 - l_1)$

(*c*) $K(l_2 - l_1)$ and $K l_1$ (*d*) $K l_1$ and $K l_2$

(AIPMT 2010)

77. A battery of emf 2 V is connected across a long uniform wire AB of length 100 cm and resistance 0·02 Ω cm^{-1}. Two cells of emfs 1 V and 2 V are connected as shown in Fig. 3(CF).39. If the galvanometer shows no deflection at point J, the length of wire AJ is equal to

FIGURE 3(CF).39



(*a*) 0 (*b*) 25 cm

(*c*) 50 cm (*d*) 100 cm

78. A potentiometer wire is 100 cm long and a constant potential difference is maintained across it. Two cells are connected in series first to support one another and then in opposite direction. The balance points are obtained at 50 cm and 10 cm from the positive end of the wire in the two cases. The ratio of the emf is :

(*a*) 5 : 4 (*b*) 3 : 4

(*c*) 3 : 2 (*d*) 5 : 1 **(NEET 2016)**

ANSWERS

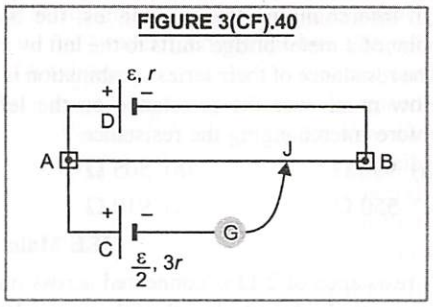
73. (*c*) 74. (*a*) 75. (*b*) 76. (*b*) 77. (*a*) 78. (*c*)

79. A potentiometer wire AB having length L and resistance $12\ r$ is joined to a cell D of *emf* ε and internal resistance r, A cell C having *emf* $\varepsilon/2$ and internal resistance $3\ r$ is connected. The length AJ at which the galvanometer as shown in Fig. 3(CF).40 shows no deflection is :

(*a*) $\frac{11}{12}$ L (*b*) $\frac{11}{24}$ L

(*c*) $\frac{13}{24}$ L (*d*) $\frac{5}{12}$ L

FIGURE 3(CF).40



(JEE Main 2019)

II. Multiple Choice Questions (with One or More than One Correct Answers)

80. A resistance of 4 Ω is connected across a cell. Then it is replaced by another resistance of 1 Ω. It is found that power dissipated in resistance in both the cases is 16 watt. Then

(*a*) internal resistance of the cell is 2 Ω

(*b*) emf of the cell is 12 V

(*c*) maximum power that can be dissipated in the external resistance is 18 watt

(*d*) short circuit current from the cell is infinite

(National Standard Exam. in Physics 2012)

81. The charge flowing in a conductor varies with time as $Q = at - bt^2$, The current :

(*a*) decreases linearly with time

(*b*) reaches maximum and then decreases

(*c*) falls to zero after $t = a/2b$

(*d*) changes at a rate of $-2b$

82. Two bulbs consume same power when operated at 200 V and 300 V respectively. When these bulbs are connected in series across a d.c. source of 500 volt, then

(*a*) Ratio of potential difference across them is 3/2

(*b*) Ratio of potential difference across them is 4/9

(*c*) Ratio of power consumed across them is 4/9

(*d*) Ratio of power consumed across them is 2/3

83. For the circuit shown in the Fig. 3(CF).38

(*a*) the current I through the battery is 7·5 mA

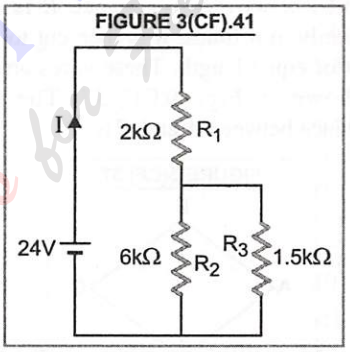
(*b*) the potential difference across R_3 is 18 V

(*c*) ratio of powers dissipated in R_1 and R_2 is 3

(*d*) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_3 will decrease by a factor of 9

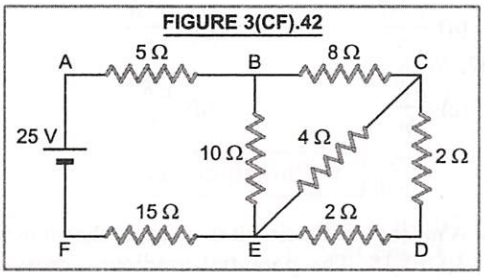
(IIT 2009)

FIGURE 3(CF).41



84. For the circuit shown in the Fig. 3(CF).42

FIGURE 3(CF).42



(*a*) potential difference between points B and E is 5 V

(*b*) potential difference between points A and B is 10 V

(*c*) the current in the 4 Ω resistor is 0·25 A

(*d*) the current in the 4 Ω resistor is 0·5 A

ANSWERS

79. (*c*) 80. (*a,b,c*) 81. (*a,c,d*) 82. (*b,c*) 83. (*a,d*) 84. (*a,c*)

85. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0·5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2 d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K ?

(a) 4 if wires are in parallel

(b) 2 if wires are in series

(c) 1 if wire are in series

(d) 0·5 if wires are in parallel

(JEE Advanced 2014, Paper 1)

86. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true ?

(a) The temperature distribution over the filament is uniform.

(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of frequencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb 2

(JEE Advanced 2016)

87. Which of the following statements is/are correct ?

(a) If n identical cells are connected in series and then the battery thus formed, is short circuited by a conduction wire, current through the wire will be independent of n.

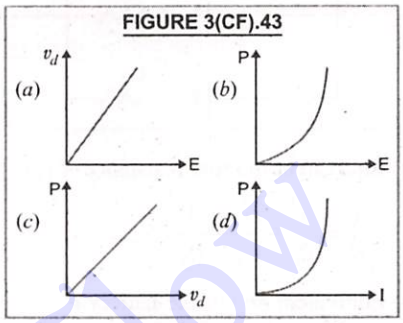
(b) If n identical cells are connected in parallel and then the battery thus formed, is short circuited by conducting wire, the current through the wire will be proportional to n.

(c) If n identical cells are connected in parallel are then the battery thus formed, is short circuited by a wire having a constant resistance, the current through the wire will increase as n increases.

(d) none of these.

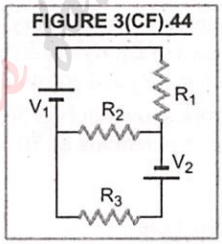
88. If E denotes electric field in a uniform conductor, I corresponding current through it, v_d drift velocity of electrons and P denotes thermal power produced in the conductor, then which of the following graphs is/are correct ?

FIGURE 3(CF).43



89. Two ideal batteries of *emf* V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in Fig. 3(CF).44. The current in resistance R_2 would be zero if

FIGURE 3(CF).44



(a) $V_1 = V_2$ and $R_1 = R_2 = R_3$

(b) $V_1 = V_2$ and $R_1 = 2\,R_2 = R_3$

(c) $V_1 = 2\,V_2$ and $2\,R_1 = 2\,R_2 = R_3$

(d) $2\,V_1 = V_2$ and $2\,R_1 = R_2 = R_3$

(JEE Advanced 2014)

90. A uniform wire of resistance R is shaped into a regular n-sided polygon (n is even). The equivalent resistance between any two corners can have :

(a) the maximum value $R/4$

(b) the maximum value R/n

(c) the minimum value $R\left(\frac{n-1}{n^2}\right)$

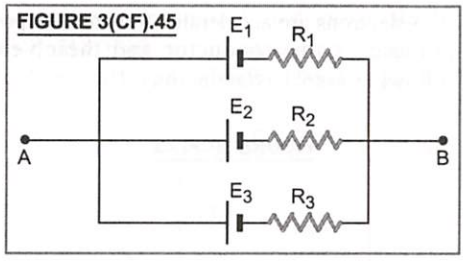
(d) the minimum value R/n

ANSWERS

85. (b,d) **86.** (c,d) **87.** (a,b,c) **88.** (a,b,d) **89.** (a,b,d) **90.** (a,c)

91. For the circuit shown in Fig. 3(CF).45,

FIGURE 3(CF).45



(*a*) Equivalent internal resistance R is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

(*b*) Equivalent *emf* of the battery is

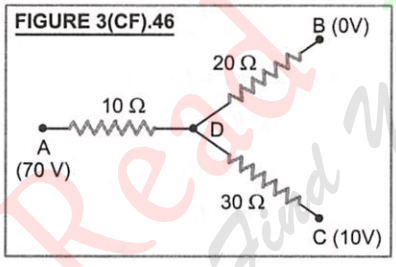
$E = (E_1 + E_2 + E_3)/3$.

(*c*) If $E_3=\left(\frac{E_1R_2+E_2R_1}{R_1+R_2}\right)$ equivalent *e.m.f.* of the battery will be equal to E_3.

(*d*) Equivalent *emf* of the battery not only depends upon values of E_1, E_2 and E_3 but depends upon values of R_1, R_2 and R_3 also.

92. In the net work shown in Fig. 3(CF).46, points *A*, *B* and *C* are at potentials of 70 V, zero and 10 V respectively. Then

FIGURE 3(CF).46



(*a*) the potential of point *D* is 40 V

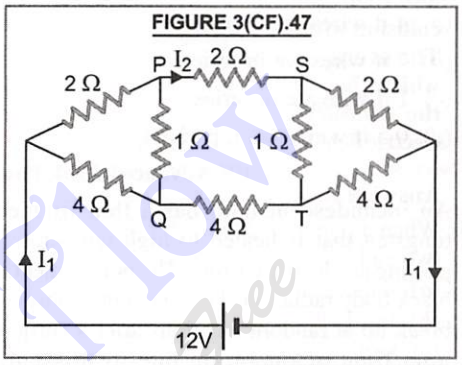
(*b*) the currents in the sections *AD*, *DB*, *DC* are in the ratio 3 : 2 : 1

(*c*) the currents in the sections *AD*, *DB*, *DC* are in the ratio of 1 : 2 : 3

(*d*) the net work draws a total power of 200 W.

93. For the resistance network shown in the Fig. 3(CF).47, choose the correct option(*s*)

FIGURE 3(CF).47



(*a*) The current through *PQ* is zero

(*b*) $I_1 = 3\ A$

(*c*) The potential at *S* is less than that at *Q*

(*d*) $I_2 = 2\ A$ **(IIT 2012)**

94. Two resistors having equal resistances are joined inseries and current passed through the combination. Neglect any variation in resistance as the temperature chagnes. In a given time internval

(*a*) unequal amounts of thermal energy may be produced

(*b*) equal amounts of thermal energy must be produced

(*c*) the temperature may rise equally in the resistances

(*d*) the temperature must rise equally in the resistors.

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1

A cylindrical copper conductor *AB* of length *l*, area of cross-section *a* has large number of free electrons, which at room temperature move at random within the body of the conductor, like the molecules of a gas. The average thermal speed of the free electrons in random motion at room temperature is of the order of $10^5\ ms^{-1}$. When a potential difference *V* is applied across the two ends of a given conductor, the free

ANSWERS

91. (*a,c,d*) **92.** (*a,b,d*) **93.** (*a,b,c,d*) **94.** (*b,c*)

electrons in the conductor experience a force and are accelerated towards the positive end of the conductor. On their way, they suffer frequent collisions with the ions/atoms of the conductor and lose their gained kinetic energy. After each collision, the free electrons are again accelerated due to electric field, towards the positive end of the conductor and lose their gained kinetic energy in the next collision with the ions/atoms of the conductor. The average speed of the free electrons with which they drift towards the positive end of the conductor under the effect of applied electric field is called drift speed of the electrons.

Answers the following questions

95. When a potential difference is applied across the two ends of the conductor, an electric field exists
(*a*) outside the conductor
(*b*) inside the conductor
(*c*) both outside and inside the conductor
(*d*) no where

96. The motion of electrons inbetween two successive collisions with the atoms/ions follows
(*a*) a straight path (*b*) circular path
(*c*) elliptical path (*d*) curved path

97. The drift speed of the electrons depends on
(*a*) dimensions of the conductor
(*b*) number density of free electrons in the conductor
(*c*) both (*a*) and (*b*) (*d*) none of these above

98. Drift speed of electrons in a conductor is small ($= 10^{-4}$ ms^{-1}) when the switch is closed, the bulb at a distance glows immediately. It is so because
(*a*) drift velocity of electrons increases when switch is closed.
(*b*) electrons are accelerated towards the positive end of the conductor and their velocity increases towards the other end of the conductor
(*c*) the drifting of electrons takes place at the entire length of the connecting wire. This electric effect propagates with the speed of light.
(*d*) the electrons move towards the positive end and protons of conductor move towards negative end of the conductor.

Comprehension 2 **A wire of length 12 cm ; resistance 12 Ω and of uniform area of crosssection is cut into twelve equal parts, which are connected to form a skelton cube. A cell of emf 2 V is connected across the two diagonally opposite corners of the cube. Using kirchhoff's laws of junction and loop.**

Answers the following questions

99. The effective resistance of the circuit is
(*a*) 4/5 Ω (*b*) 5/6 Ω
(*c*) 6/7 Ω (*d*) 7/12 Ω

100. The current drawn from the battery is
(*a*) 2·5 A (*b*) 2·4 A
(*c*) 2·3 A (*d*) 3·4 A

101. The maximum current flowing in an arm of network is
(*a*) 0·4 A (*b*) 0·8 A (*c*) 1·2 A (*d*) 2·4 A

102. The minimum potential difference across an arm of network is
(*a*) 0·4 V (*b*) 0·8 V (*c*) 1·2 V (*d*) 2·4 V

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, Match column I and column II and select the correct match out of the four given choices.

103.

Column I		Column II	
(A)	The standard resistance coil	(*p*)	of nickel-chromium alloy
(B)	The best connecting wire	(*q*)	of tin-lead alloy
(C)	The heating wire	(*r*)	of silver metal
(D)	The fuse wire	(*s*)	of copper manganese nickel alloy

(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*r* ; C-*s* ; D-*p* (*c*) A-*r* ; B-*s* ; C-*p* ; D-*q* (*d*) A-*s* ; B-*r* ; C-*p* ; D-*q*

ANSWERS

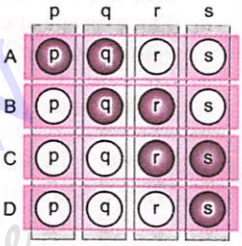
95. (*b*) **96.** (*d*) **97.** (*d*) **98.** (*c*) **99.** (*b*) **100.** (*b*)
101. (*b*) **102.** (*a*) **103.** (*d*)

104. There are physical quantities in column I and dimensional formulae in column II

	Column I		Column II
(A)	Current density	(*p*)	$[ML^3T^{-3}A^{-2}]$
(B)	Conductance	(*q*)	$[M^{-1}L^{-3}T^3A^2]$
(C)	Electrical conductivity	(*r*)	$[M^0L^{-2}T^0A^1]$
(D)	Specific resistance	(*s*)	$[M^{-1}L^{-2}T^3A^2]$

(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*r* ; C-*s* ; D-*p* (*c*) A-*r* ; B-*s* ; C-*p* ; D-*q* (*d*) A-*r* ; B-*s* ; C-*q* ; D-*p*

V. Matrix-Match Type Questions



DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*p*, A-*q* ; B-*q*, B-*r* ; C-*r*, C-*s* and D-*s*, then the correctly bubbled matrix will look like the one shown here.

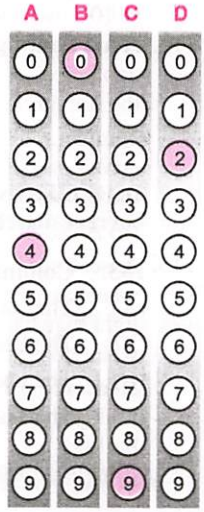
105. There are physical quantities in column I and the statements are in column II :

	Column I		Column II
(A)	Current	(*p*)	Motion of electrons in a closed path
(B)	EMF	(*q*)	Motion of electrons through a conductor without any collision
(C)	Resistance	(*r*)	Motion of electrons in a definite direction
(D)	Superconductor	(*s*)	The collision of electrons while moving through the conductor

106.

	Column I		Column II
(A)	Max. potential difference between two points of a circuit	(*p*)	Voltmeter
(B)	EMF of the cell in a closed circuit	(*q*)	Potentiometer
(C)	The capacity of supplying the current in the circuit	(*r*)	Watt-hour
(D)	The electric energy consumed in the circuit	(*s*)	Ampere hour

VI. Integer Type Questions

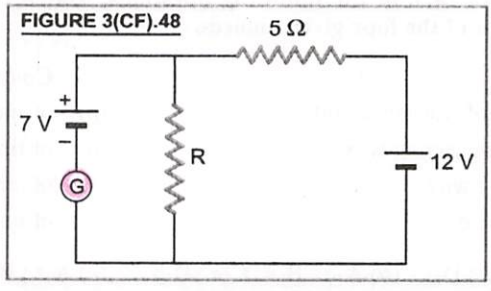


DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

107. The length of a potentiometer wire is 10 cm. A cell of emf *E* is balanced at a length 10/3 cm from the positive end of the wire. If the length of the wire is increased by 5 cm, at what distance (in cm) from positive end will the same cell give a balance point ?

108. Refer Fig. 3(CF).48, for what value of *R* (in ohm) will the current in galvanometer *G* be zero ?

FIGURE 3(CF).48



ANSWERS

104. (*d*) **105.** A-*p*, *r* ; B-*p* ; C-*s* ; D-*q* **106.** A-*p*, *q* ; B-*p*, *q* ; C-*s* ; D-*r*

107. 5 **108.** 7

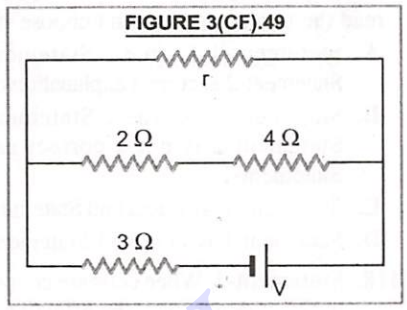
109. Total momentum of electrons in a straight wire of length 1000 m carrying a current of 70 ampere is about .....$\times 10^{-7}$ Ns.

110. A battery of emf E and internal resistance r is connected to an external resistance R. The maximum power in the external circuit is 9 W. The current flowing in the circuit under the conditions is 3 A. What is the value of E in volt ?

111. In the circuit, Fig. 3(CF).49, what should be the value of r in ohm so that power developed in the resistor r will be maximum ?

112. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2{\cdot}25\ J_2$, then the value of R in ohm is : **(JEE 2010)**

FIGURE 3(CF).49



VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. Ineach of the following questions read the two statements and choose if

A. both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. both Assertion and Reason are true, but Reason is not the correct explanation of the Assertion.

C. Assertion is true, but the Reason is false.

D. both Assertion and Reason are false.

113. Assertion. The bending of an insulated wire increases the resistance of wire.

Reason. The drift velocity of electrons in bent wire decreases.

(*a*) A (*b*) B (*c*) C (*d*) D

114. Assertion. In parallel combination of electrical appliances, total power consumption is equal to the sum of the powers of the individual appliances.

Reason. In parallel combination, the voltage across each appliance is the same, as required for the proper working of electrical appliance.

(*a*) A (*b*) B (*c*) C (*d*) D

115. Assertion. Each bulb in a frill of 20 bulbs in series when connected to supply voltage will emit more light than each bulb in a frill of 19 bulbs in series when connected to same supply voltage.

Reason. Each bulb in a frill of 20 bulbs in series will get more voltage than that in a frill of 19 bulbs.

(*a*) A (*b*) B (*c*) C (*d*) D

116. Assertion. In a meter bridge experiment, null point is obtained at a point for an unknown resistance. When an unknown resistance is put inside an enclosure maintained at a higher temperature, the null point can be obtained at the same point as before by decreasing the value of the standard resistance.

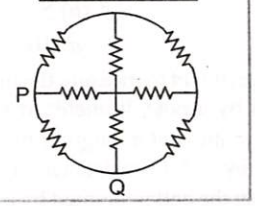
Reason. Resistance of metal increases with decrease in temperature.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

117. Assertion. The effective resistance of the network between P and Q is $\frac{4}{5}r$.

FIGURE 3(CF).50



Reason. Symmetry can be applied to the network with respect to centre.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

ANSWERS

109. 4 110. 6 111. 2 112. 4 113. (*d*) 114. (*a*) 115. (*d*) 116. (*d*) 117. (*d*)

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if

A. Statement-1 is true ; Statement-2 is true ; Statement-2 is correct explanation of Statement-1.

B. Statement-1 is true ; Statement-2 is true ; Statement-2 is not a correct explanation of Statement-1.

C. Statement-1 is correct and Statement-2 is false.

D. Statement-1 is false and Statement-2 is true.

118. Statement-1. When cells are connected in parallel to the external load, the effective emf increases.
Statement-2. All the cells will be sending the current to the external load in the same direction.
(*a*) A (*b*) B (*c*) C (*d*) D

119. Statement-1. Electric appliances with metallic body ; *e.g.* heaters, presses etc, have three pin connections, whereas an electric bulb has a two pin connection.
Statement-2. Three pin connections reduce heating of connecting cables
(*a*) A (*b*) B (*c*) C (*d*) D

120. Statement-1. A laser beam 0·2 W power can drill holes through a metal sheet, whereas 1000 W torch-light cannot.
Statement-2. The frequency of laser light is much higher than that of torch light.
(*a*) A (*b*) B (*c*) C (*d*) D

121. Statement-1. When the cell is in the open circuit, there is no force on a test charge inside the electrolyte of the cell.
Statement-2. There is no field inside the cell, when the cell is in open circuit.
(*a*) A (*b*) B (*c*) C (*d*) D

VIII. Multiple Choice Questions (Based on Experimental Skills)

122. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is
(*a*) 6% (*b*) zero
(*c*) 1% (*d*) 3%

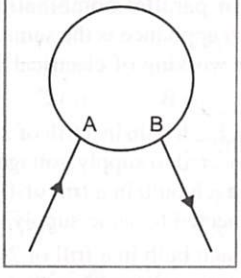
123. Three unequal resistors in parallel are equivalent to a resistance 1 ohm. If two of them are in the ratio of 1 : 2 and if no resistance value is fractional, the largest of three resistances in ohm is
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 10

124. In an experiment to measure the internal resistance of a cell by a potentiometer, it is found that the balance point is at a length 2 m, when the cell is shunted by a 5 Ω resistance and at a length of 3 m, when the cell is shunted by a 10 Ω resistance. The internal resistance of the cell is then
(*a*) 1 Ω (*b*) 1·5 Ω
(*c*) 10 Ω (*d*) 15 Ω

125. An unknown resistance R_1 is connected in series with a resistance of 10 Ω. This combination is connected to one gap of a metre bridge ; while a resistance R_2 is connected in the other gap. The balance point is at 50 cm. Now when 10 Ω resistance is removed, the balance point shifts by 10 cm. The value of R_1 is
(*a*) 10 Ω (*b*) 20 Ω
(*c*) 40 Ω (*d*) 60 Ω

126. A uniform wire of resistance 20 Ω having resistance 1 Ω m^{-1} is bent in the form of a circle as shown in Fig. 3(CF).51. If the equivalent resistance bet- ween *A* and *B* is 1·8 Ω, then length of the shorter section is

FIGURE 3(CF).51



(*a*) 1·8 m (*b*) 2 m
(*c*) 3·6 m (*d*) 4 m

ANSWERS

118. (*d*) 119. (*c*) 120. (*c*) 121. (*c*) 122. (*a*) 123. (*c*) 124. (*c*) 125. (*b*) 126. (*b*)

HINTS/EXPLANATIONS

For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. As, $T = \dfrac{\text{distance}}{\text{speed}} = \dfrac{2\pi r_n}{v_n}$

$$= \frac{2\pi\epsilon_0 n^2 h^2 \times 2\epsilon_0 nh}{\pi me^2 \times e^2} = \frac{4\epsilon_0^2 n^3 nh^3}{me^4}$$

Current $I = \dfrac{e}{T} = \dfrac{e \times me^4}{4\epsilon_0^2 n^3 h^3} = \dfrac{me^5}{4\epsilon_0^2 n^3 h^3}$

So $\boldsymbol{I \propto e^5}$

2. Average energy = workdone = $e\,E\,\lambda$

$\therefore \quad 2\,eV = eE \times 4 \times 10^{-8}$

or $\quad E = \dfrac{2}{4\times10^{-8}}\dfrac{V}{m} = \mathbf{5\times10^7\ V\,m^{-1}}$

3. $I = \dfrac{dQ}{dt} = 3t^2 + 2t + 5$

$dQ = (3\,t^2 + 2\,t + 5)\,dt$

$$\therefore\ Q = \int_{t=0}^{t=2s} (3t^2 + 2t + 5)\,dt = \left(\frac{3t^3}{3} + \frac{2t^2}{2} + 5t\right)_0^2$$

$$= \left(t^3 + t^2 + 5t\right)_0^2 = \mathbf{22\,C}$$

4. Here, $l = 1$ m, $A = 0{\cdot}5 \times 10^{-6}\ m^2 = 5 \times 10^{-7}\ m^2$

$n = 8 \times 10^{22}\ cm^{-3} = 8 \times 10^{28}\ m^{-3}$, $I = 1$ A

$$v_d = \frac{I}{nAe}$$

Time, $t = \dfrac{l}{v_d} = \dfrac{ln\,Ae}{I}$

$$= \frac{1\times(8\times10^{28})\times(5\times10^{-7})\times1{\cdot}6\times10^{-19}}{1}$$

$$= \mathbf{6{\cdot}4 \times 10^3\ s}$$

5. For Aluminium

$$R_{Al} = \frac{\rho_{Al}\,l}{A} = \frac{2{\cdot}7\times10^{-8}\times50\times10^{-3}}{(49-4)\times10^{-6}}$$

$$= 3 \times 10^{-5}\ \Omega$$

For Iron

$$R_{Fe} = \frac{1{\cdot}0\times10^{-7}\times50\times10^{-3}}{4\times10^{-6}} = 125 \times 10^{-5}\ \Omega$$

Both will act as two resistances joined in paralle

$$\therefore\ R_{eq} = \frac{R_{Al}\times R_{Fe}}{R_{Al}+R_{Fe}} = \frac{3\times10^{-5}\times125\times10^{-5}}{3\times10^{-5}+125\times10^{-5}}$$

$$= \frac{1875\times10^{-6}\,\Omega}{64} = \mathbf{\frac{1875}{64}\,\mu\Omega}$$

6. Let l be the original length of the wire. A portion of the length of wire is stretched so that the final length becomes $1{\cdot}5\ l$, Fig. 3(CF).52.

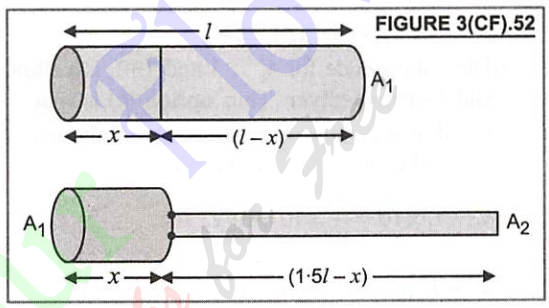


FIGURE 3(CF).52

Let R_1, R_2 be the resistances of the wire before and after stretching. Let A_1 be the area of cross-section of the original wire and A_2 be the area of cross-section of the portion $(1{\cdot}5\,l - x)$ of the wire. On stretching, the volume of the wire remains unchanged, so $l\,A_1 = x\,A_1 + (1{\cdot}5\,l - x)\,A_2$

or $\quad A_2 = \dfrac{(l-x)}{(1{\cdot}5l-x)}A_1$

Also, $\quad R_1 = \dfrac{\rho\,l}{A_1}$ and $R_2 = \dfrac{\rho\,x}{A_1} + \dfrac{\rho(1{\cdot}5l-x)}{A_2}$

As per question $R_2 = 4\,R_1$

$$\therefore\ \frac{4\rho l}{A_1} = \frac{\rho x}{A_1} + \frac{\rho(1{\cdot}5l-x)}{A_2} = \frac{\rho x}{A_1} + \frac{\rho(1{\cdot}5l-x)^2}{(l-x)A_1}$$

On solving it we get, $\dfrac{x}{l} = \dfrac{7}{8}$

$\therefore$ Fraction of wire stretched

$$= \frac{(l-x)}{l} = \left(1 - \frac{x}{l}\right) = 1 - \frac{7}{8} = \mathbf{\frac{1}{8}}$$

7. Let the potential at points A, B, C and D be V_A, V_B, V_C and V_D respectively.

Pot. diff. between the points B and A

$$= V_B - V_A = 10\ V$$

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As, earth potential is taken zero, so $V_A = 0$

$\therefore \quad V_B - 0 = 10\text{ V}$ or $V_B = \mathbf{10\text{ V}}$

Pot. diff. between the points C and D

$= V_C - V_D = 1 \times 4 = 4\text{ V}$

$\therefore \quad V_C - 0 = 4$ or $V_C = \mathbf{4\text{ V}}$

Pot. diff. between the points B and C

$= 10 - 4 = 6\text{ V}$

$\therefore$ Resistance $R = \dfrac{6}{1} = \mathbf{6\,\Omega}$

8. The value of given carbon resistor is

$$R = (47 \pm 4{\cdot}7)\,k\,\Omega = 47\,k\,\Omega \pm \frac{4{\cdot}7}{47} \times 100\%$$

$$= 47 \times 10^3\,\Omega \pm 10\%$$

The colour code for 4, 7, 3 and 10% is **yellow-violet-orange-silver**. Thus **option (*b*) is true.**

9. For ideal voltmeter, the resistance is infinite and for the ideal ammeter, resistance is zero.

$$V_1 = I_1 \times 10 = \frac{10}{10} \times 10 = 10\text{ V}$$

$$V_2 = I_2 \times 10 = \frac{10}{10} \times 10 = 10\text{ V} \quad \therefore V_1 = V_2$$

$$I_1 = \frac{10\text{ V}}{10\,\Omega} = 1\text{ A} \text{ and } I_2 = \frac{10\text{ V}}{10\,\Omega} = 1\text{ A}$$

$\therefore I_1 = I_2$

10. $R = \dfrac{\rho l}{A} = \dfrac{\rho l}{Vl/l} = \dfrac{\rho l^2}{V}$

$$\therefore \frac{dR}{R} = \frac{2dl}{l} = 2 \times 0{\cdot}5 = 1$$

$$\%\text{ change in resistance} = \frac{dR}{R}\% = \mathbf{1\%}$$

11. By colour coding, the resistance of carbon resistor is $R = 50 \times 10^2\,\Omega \pm 5\%$.

As power, $P = I^2 R$ or $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{2}{50 \times 10^2}}$

$= 20 \times 10^{-3}\text{ A} = \mathbf{20\text{ mA}}$

12. Here, $n = 8{\cdot}5 \times 10^{28}\text{ m}^{-3}$;

$\tau = 25\text{ fs} = 25 \times 10^{-15}\text{ s}$

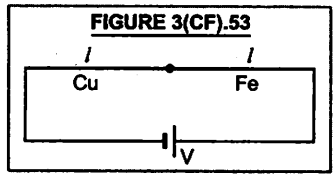
Resistivity, $\rho = \dfrac{m}{ne^2\tau}$

$$= \frac{(9{\cdot}1 \times 10^{-31})}{(8{\cdot}5 \times 10^{28}) \times (1{\cdot}6 \times 10^{-19})^2 \times (25 \times 10^{-15})}$$

$= 1{\cdot}67 \times 10^{-8}\,\Omega\text{m} \approx \mathbf{10^{-8}\,\Omega m}$

13. Let l, A be the length and area of crosssection of each wire in series as shown in Fig. 3(CF).53.

FIGURE 3(CF).53



Given, $\rho_{Cu} = \dfrac{1}{6}\rho_{Fe}$

As $\quad V = IR = I\dfrac{\rho l}{A}$

For copper, $V_{Cu} = I\rho_{Cu}\dfrac{l}{A} = I\dfrac{\rho_{Fe}}{6}\dfrac{l}{A}$;

$$E_1 = \frac{V_{Cu}}{l} = \frac{I\rho_{Fe}}{6A}$$

For iron, $V_{Fe} = I\rho_{Fe}\dfrac{l}{A}$; $E_2 = \dfrac{V_{Fe}}{l} = \dfrac{I\rho_{Fe}}{A}$

$\therefore \quad \dfrac{E_1}{E_2} = \dfrac{1}{6}$; Hence, $\mathbf{E_1 < E_2}$

14. $R_1 = \tan\theta = R_0(1 + \alpha T_1)$

and $R_2 = \cot\theta = R_0(1 + \alpha T_2)$

$\therefore \cot\theta - \tan\theta = R_0(1 + \alpha T_2) - R_0(1 + \alpha T_1)$

$= R_0\alpha(T_2 - T_1)$

or $T_2 - T_1 = \dfrac{1}{\alpha R_0}(\cot\theta - \tan\theta)$

$$= \frac{1}{\alpha R_0}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) = \frac{2\cos 2\theta}{\alpha R_0 \sin 2\theta}$$

$$= \frac{2}{\alpha R_0}\cot 2\theta \text{ or } T_2 - T_1 \propto \mathbf{\cot 2\,\theta}$$

15. According to Ohm's law

$$V = IR = I\rho\frac{l}{A} \quad \text{or} \quad \rho = \frac{VA}{Il}$$

Now as, $I = A\,ne\,v_d$ so, $\rho = \dfrac{V\,A}{l\,Ane\,v_d} = \dfrac{V}{l\,ne\,v_d}$

$$\rho = \frac{5}{0{\cdot}1 \times 8 \times 10^{28} \times (1{\cdot}6 \times 10^{-19}) \times 2{\cdot}5 \times 10^{-4}}$$

$= \mathbf{1{\cdot}6 \times 10^{-5}\,\Omega m}$

16. Let R_1 and R_2 be the resistances of A and B at 0°C and R_1' and R_2' be the resistances of A and B at t °C. Then

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$R_2 = nR_1$; $R_1' = R_1(1+\alpha_1 t)$; $R_2' = R_2(1+\alpha_2 t)$

When resistances are connected in series, then

$$R_s' = R_1' + R_2' = R_1(1+\alpha_1 t) + nR_1(1+\alpha_2 t)$$

or $R_s' = R_1[(1+n) + (\alpha_1 + n\alpha_2)t]$ ...(i)

Let α_s be the temp. coeff. of resistance of combination of two resistances in series.

Standard equation for series combination is

$$R_s' = (R_1 + R_2)(1+\alpha_s t) = (R_1 + nR_1)(1+\alpha_s t)$$
$$= R_1(1+n)(1+\alpha_s t)$$
$$= R_1[(1+n) + (1+n)\alpha_s t] \quad ...(ii)$$

Comparing (i) and (ii), we have

$$(\alpha_1 + n\alpha_2) = (1+n)\alpha_s \text{ or } \alpha_s = \frac{\alpha_1 + n\alpha_2}{1+n}$$

17. In parallel combination of resistances,

$$R_p = \frac{R_1 \times nR_1}{R_1 + nR_1} = \frac{nR_1}{1+n}$$

$$R_p' = \frac{R_1' \times R_2'}{R_1' + R_2'} = \frac{R_1(1+\alpha_1 t) \times nR_1(1+\alpha_2 t)}{R_1(1+\alpha_1 t) + nR_1(1+\alpha_2 t)}$$

$$= \frac{nR_1(1+\alpha_1 t)(1+\alpha_2 t)}{(1+\alpha_1 t) + n(1+\alpha_2 t)} \quad ...(i)$$

If α_p is the temp. coeff. of resistance for parallel combination of resistances, then

$$R_p' = R_p(1+\alpha_p t) = \frac{nR_1}{(1+n)}[1+\alpha_p t] \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{nR_1}{(1+n)}[1+\alpha_p t] = \frac{nR_1(1+\alpha_1 t)(1+\alpha_2 t)}{(1+\alpha_1 t) + n(1+\alpha_2 t)}$$

On solving, we get, $\alpha_p = \dfrac{\alpha_2 + n\alpha_1}{1+n}$

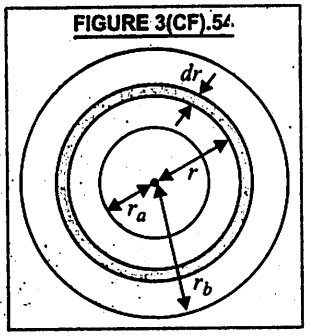
18. Consider a spherical shell of radius r and thickness dr as shown in Fig. 3(CF).54.

Resistance of this shell is, $dR = \dfrac{\rho\, dr}{4\pi r^2}$

Total resistance,

$$R = \int dR = \int_{r_a}^{r_b} \frac{\rho\, dr}{4\pi r^2} = \frac{\rho}{4\pi}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

FIGURE 3(CF).54



Current, $I = \dfrac{V_a - V_b}{R}$

The electric field intensity at the location of shell is $E = \dfrac{J}{\sigma} = \dfrac{I/4\pi r^2}{1/\rho} = \dfrac{\rho I}{4\pi r^2}$

where J is the current density and σ is the electrical conductivity of the conducting material.

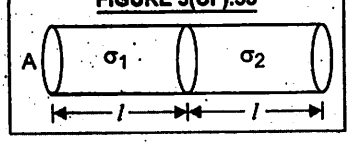
$$\therefore E = \frac{\rho}{4\pi r^2} \times \left(\frac{V_a - V_b}{R}\right) = \frac{\rho}{4\pi r^2} \cdot \frac{(V_a - V_b) \times 4\pi}{\rho\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

$$= \frac{(V_a - V_b)}{r^2\left[\frac{1}{r_a} - \frac{1}{r_b}\right]}$$

As current I is zero for $r < r_a$ and $r > r_b$, so variation of E and r will be represented by the curve shown in **option (b)**.

19. Refer to Fig. 3(CF).55 given below ;

FIGURE 3(CF).55



Effective resistance of two wires in series is

$$R_{eff} = R_1 + R_2 = \frac{l}{\sigma_1 A} + \frac{l}{\sigma_2 A}$$

$$R_{eff} = \frac{2l}{\sigma_{eq} A}$$

$$\therefore \frac{2l}{\sigma_{eq} A} = \frac{l}{A}\left[\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right] = \frac{l(\sigma_1 + \sigma_2)}{A\sigma_1\sigma_2}$$

$$\sigma_{eq} = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$$

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20. Here, $I = 5$ A, $\theta = 60°$, $A = 2$ m^2, $\rho = 11 \times 10^{-4}$ SI units.

Current density, $J = \dfrac{I}{A\cos\theta} = \dfrac{5}{2 \times \cos 60°}$

$= \dfrac{5}{2 \times 1/2} = 5$ A/m^2

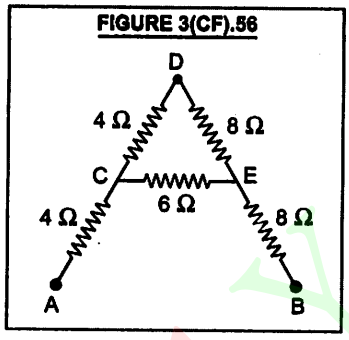
Electric field, $E = J\rho = 5 \times (11 \times 10^{-4})$

$= \mathbf{55 \times 10^{-4}}$ **V/m**

21. Refer to Fig. 3(CF).56, the resistance between C and E is,

$$R_{CE} = \frac{(4+8)\times 6}{(4+8)+6} = 4\Omega$$

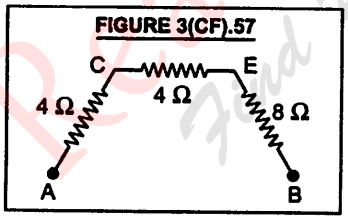
FIGURE 3(CF).56



Refer to Fig. 3(CF).57, the resistance between A and B is,

$R_{AB} = 4 + 4 + 8 = \mathbf{16\ \Omega}$

FIGURE 3(CF).57



22. Since all the four wires are identical and of the same material, so each wire will be of the same resistance (say R).

When these four wires are connected in parallel Fig. 3(CF).58(a), then effective resistance R_p is

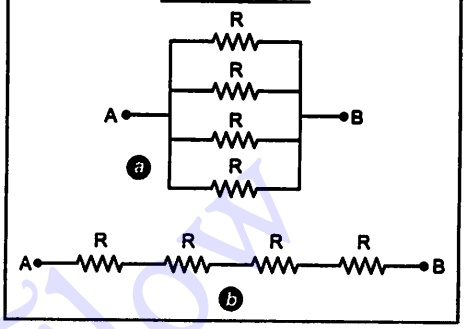
$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{4}{R} \quad \text{or} \quad R = 4R_p$$

Given, $R_p = 0{\cdot}25\ \Omega$, so $R = 4 \times 0{\cdot}25 = 1{\cdot}0\ \Omega$

When these four wires are connected in series as shown in Fig. 3(CF).58(b), then the effective resistance

$R_s = R + R + R + R = 4R = 4 \times 1{\cdot}0 = \mathbf{4{\cdot}0\ \Omega}$

FIGURE 3(Q).58



23. In the upper branch of the circuit, equivalent resistance, $R_{eq} = \dfrac{1\times 1}{1+1} + 2 = 2{\cdot}5\ \Omega$

Voltage across upper branch, $V = 1$ volt

Current in upper branch, $I = \dfrac{V}{R} = \dfrac{1}{2{\cdot}5} = 0{\cdot}4$ A

This current is equally divided between two 1 Ω resistances each, so

$$I_1 = \frac{0{\cdot}4}{2} = \mathbf{0{\cdot}2\ A}$$

24. When n resistors are connected in series to the battery, then current in circuit is

$$I = \frac{E}{nR + R} \qquad ...(i)$$

When n resistors are connected in parallel to the battery, then current is

$$10\,I = \frac{E}{(R/n) + R} \qquad ...(ii)$$

Dividing (ii) by (i), we get

$$10 = \frac{(n+1)\,R}{\left(\dfrac{1}{n}+1\right)R} = \frac{(n+1)}{(1+n)/n} = n$$

So $n = \mathbf{10}$

25. Let R be the equivalent resistance. Since the network is infinite. The addition of one more unit of two resistances R_1 and R_2 will not alter the total resistance of network, *i.e.*, it should be R. Fig. 3(CF).59.

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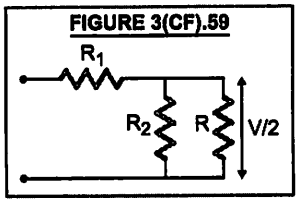
FIGURE 3(CF).59



Now $R = R_1 + \dfrac{R_2 R}{R_2 + R}$ ...(*i*)

Since the voltage is halved after each section,

$\therefore \quad R_1 = \dfrac{R_2 R}{R_2 + R}$ ...(*ii*)

Putting the value of R_1 in (*i*), we get

$R = \dfrac{2R_2 R}{R_2 + R}$ or $R_2 + R = 2R_2$

$R_2 = R$ ...(*iii*)

From equation (*ii*), $R_1 = \dfrac{R^2}{2R} = \dfrac{R}{2}$ ...(*iv*)

$2R_1 = R$

From (*iii*) and (*iv*), $\dfrac{R_2}{R_1} = 2$

26. When K_1 is closed, the resistance R_1 is ineffective.

$\therefore \quad I_0 = \dfrac{E}{100 + r} = \dfrac{E}{2(50 + r)}$

or $r = 0$

When K_1 is open, then

$\dfrac{I_0}{2} = \dfrac{E}{2(R_1 + 50)} = \dfrac{E}{2 \times 100}$

or $R_1 = 50\ \Omega$

27. In the branch PR, the effective resistance = $r/3$.
In the branch QR, the effective resistance = $r/2$.

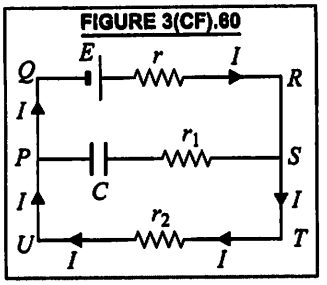
Resistance in the branch $PRQ = \dfrac{r}{3} + \dfrac{r}{2} = \dfrac{5r}{6}$

The resistance of branch PRQ is in parallel to resistance of branch PQ. Thus effective resistance between P and $Q = \dfrac{r \times 5r/6}{r + 5r/6} = \dfrac{5r}{11}$

The equivalent resistance between Q and R or R and P will be less than that between P and Q.

28. At steady state no current flows through capacitor C. Thus the current through the network will be as shown in Fig. 3(CF).60. Then

FIGURE 3(CF).60



$I = \dfrac{E}{r + r_2}$

Potential difference across P and S,

$V_S - V_P = I r_2$

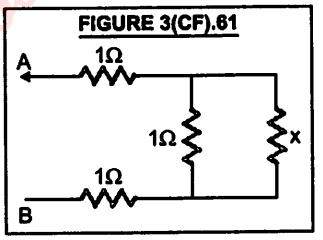
Charge on the capacitor,

$Q = C(V_S - V_P) = C I r_2$

$= C \times \dfrac{E}{(r + r_2)} r_2 = \dfrac{CE r_2}{r + r_2}$

29. Let x be the equivalent resistance of infinite net work. Since the net work is infinite, the addition of one more unit of three resistances each of value 1 Ω across the terminals will not alter, the total resistance of network, *i.e.*, it should be x. Therefore, the network would appear as shown in Fig. 3(CF).61 and its total resistance should remain x. Effective resistance between A and B is

FIGURE 3(CF).61



$x = 1 + \dfrac{1 \times x}{1 + x} + 1 = 2 + \dfrac{x}{1 + x}$

On solving we get, $x = (1 + \sqrt{3})\Omega$

30. A fully charged capacitor draws no current. Therefore, no current flows in arm GHF. So the resistance R of arm HF is ineffective. The total resistance of other resistors in circuit is

$R' = \dfrac{(R + R) \times R}{(R + R) + R} + R = \dfrac{6 \times 3}{6 + 3} + 3 = 5\ \Omega$

Total current, $I = \dfrac{E}{R'} = \dfrac{15}{5} = 3\,\text{A}$

Since in parallel circuit, the current divides in the inverse ratio of resistance so current in arm $ABGD$ = 1 A and current in arm AD = 2 A.

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So Pot. diff. between G and D

$$= V_G - V_D = 1 \times 3 = 3 \text{ V}$$

Pot. diff. between D and F

$$= V_D - V_F = 3 \times 3 = 9 \text{ V}$$

$$\therefore \quad V_G - V_F = (V_G - V_D) + (V_D - V_F)$$
$$= 3 + 9 = 12 \text{ V}$$

Pot. diff. across capacitor $= V_G - V_F = \mathbf{12\ V}$

31. Given, $l_1 : l_2 : l_3 = 2 : 3 : 4 = 2\,l : 3\,l : 4\,l$

$$r_1 : r_2 : r_3 = 3 : 4 : 5 = 3\,r : 4\,r : 5\,r$$

As, resistance of wire, $R = \dfrac{\rho l}{\pi r^2}$

$$\therefore \quad R_1 = \frac{\rho 2l}{\pi (3r)^2} = \frac{2}{9}R \ ; \ R_2 = \frac{\rho 3l}{\pi (4r)^2} = \frac{3}{16}R \ ;$$

$$R_3 = \frac{\rho 4l}{\pi (5r)^2} = \frac{4}{25}R$$

$$I_1 = \frac{V}{R_1} \ ; \ I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

$$\therefore I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} = \frac{9}{2R} : \frac{16}{3R} : \frac{25}{4R}$$

$$= \frac{9}{2} : \frac{16}{3} : \frac{25}{4} = \mathbf{54 : 64 : 75}$$

32. Due to symmetry of network, there will no current in arms BF and CE. Therefore, the effective resistance of arms AB, BC and CD in series,

$$R_1 = 2 + 2 + 2 = 6\ \Omega.$$

The effective resistance of arms AF, FE and ED in series, $R_2 = 2 + 2 + 2 = 6\ \Omega$.

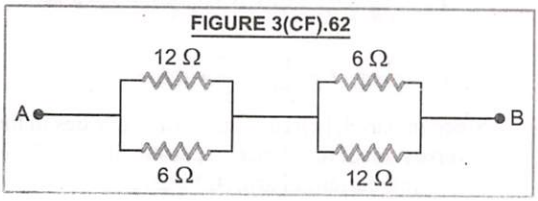
Between points A and D, R_1 and R_2 are in parallel, the effective resistance is

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 6}{6 + 6} = 3\Omega$$

Current, $I = \dfrac{3V}{3\Omega} = \mathbf{1{\cdot}0\ A}$

33. When switch S is opened the equivalent circuit will be as shown in Fig. 3(CF).62.

FIGURE 3(CF).62



The equivalent resistance between A and B is

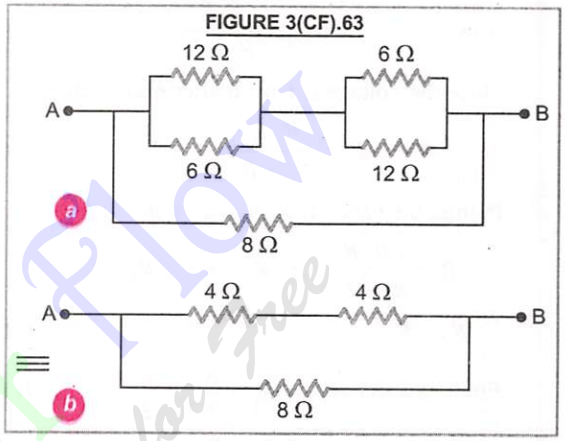
$$R_{eq} = \frac{12 \times 6}{12 + 6} + \frac{6 \times 12}{6 + 12} = 4 + 4 = \mathbf{8\ \Omega}$$

When switch S is closed, the equivalent circuit will be as shown in Fig. 3(CF).63(a) and (b).

The equivalent resistance between A and B is

$$R'_{eq} = \frac{(4+4) \times 8}{(4+4) + 8} = \mathbf{4\ \Omega}$$

FIGURE 3(CF).63



34. Let there be N resistors each of resistance r. When resistors are connected in series, their total resistance is, $S = N\,r$

When resistors are connected in parallel, then total resistance is, $P = r/N$

Given, $S = n\,P \quad \therefore \quad N\,r = n \times r/N$

or $\quad N^2 = n \quad$ or $\quad n = N^2$

Since N is an integer and combination of resistors will not be possible if $N = 1$. So minimum value of $N = 2$, then, $n = 2^2 = \mathbf{4}$

35. Here, $r = 4$ W, $I = 0{\cdot}2$ A, $R = 21$ W

If ε is emf of the cell, then

$$\varepsilon = I\,(R + r) = 0{\cdot}2\,(21 + 4) = 5 \text{ volt.}$$

Rate at which chemical energy is consumed

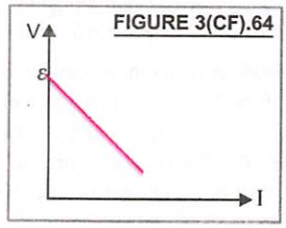
$$= \varepsilon\,I = 5 \times 0{\cdot}2 = \mathbf{1\ J/s}$$

36. Terminal pot. diff. $V = \varepsilon - I\,r = -I\,r + \varepsilon \quad ...(i)$

It shows as I increases, V decreases.

The variation of V and I will be a straight line with negative slope as shown in Fig. 3(CF).64.

FIGURE 3(CF).64



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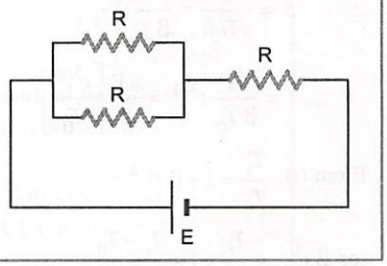
$$R_{eff} = \frac{R}{3} + \frac{R}{3} = \frac{2R}{3}$$

Power consumed, $P_i = \frac{E^2}{R_{eff}} = \frac{3E^2}{2R}$ ...(*i*)

When two bulbs from section *A* and one bulb from section *B* are glowing, Fig. 3(CF).67, then effective resistance of the circuit is

FIGURE 3(CF).67



$$R_{eff} = \frac{R}{2} + R = \frac{3R}{2}$$

Power consumed, $P_f = \frac{E^2}{3R/2} = \frac{2E^2}{3R}$ ...(2)

$$\frac{P_i}{P_f} = \frac{3E^2/2R}{2E^2/3R} = \frac{9}{4}$$

50. As, $P = I^2 R$ or $I = \sqrt{P/R}$

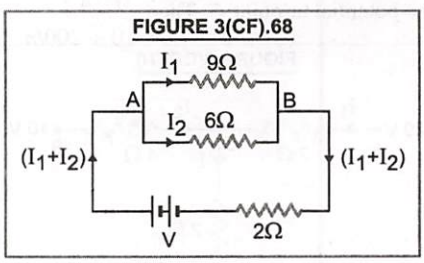
∴ Current through 9 Ω resistor in Fig. 3(CF).68 given below is

$$I_1 = \sqrt{\frac{36}{9}} = 2\,A$$

As resistors 9 Ω and 6 Ω are connected in parallel, therefore, potential difference

$V_A - V_B = 9\,I_1 = 6\,I_2$ or $I_2 = 9\,I_1/6 = \frac{9\times2}{6} = 3\,A$

FIGURE 3(CF).68



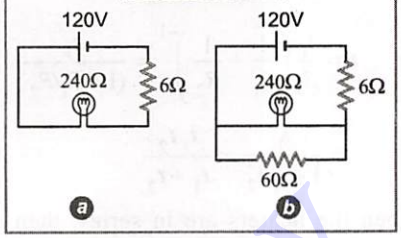
Current drawn from the battery = $I_1 + I_2$
= 2 + 3 = 5 A.

Pot. diff. across 2 Ω resistor = (5 A) (2 Ω) = **10 V**

51. Here, $V = 120$ V ; $P = 60$ W

$$R_{bulb} = \frac{V^2}{P} = \frac{(120)^2}{60} = 240\ \Omega$$

FIGURE 3(CF).69



Refer to Fig. 3(CF). 69(*a*), $R_{eq} = 240 + 6 = 246\ \Omega$

Voltage across bulb, $V_B = \frac{120}{246} \times 240 = 117{\cdot}07$ V

Refer to Fig. 3(CF). 69(*b*),

Resistance of heater, $R_{heater} = \frac{(120)^2}{240} = 60\ \Omega$

Equivalent resistance of circuit is

$$R'_{eq} = 6 + \frac{240\times60}{240+60} = 6 + 48 = 54\ \Omega$$

Voltage across bulb, $V'_B = \frac{120}{54} \times 48 = 106{\cdot}66$ V

Change in voltage = 117·07 – 106·66 = **10·04 V**

52. Heat generated per sec

$$H = \frac{V^2}{R} = \frac{(V_1 + V_2\cos\omega t)^2}{R}$$

$$= \frac{V_1^2 + V_2^2\cos^2\omega t + 2V_1V_2\cos\omega t}{R}$$

Average value of $\cos^2 \omega t$ over a cycle $= \frac{1}{2}$

Average value of $\cos \omega t$ over a cycle = 0

∴ Heat generated per second,

$$H = \frac{V_1^2 + V_2^2\times\frac{1}{2} + 2V_1V_2\times0}{R} = \frac{2V_1^2 + V_2^2}{2R}$$

53. Let R_1, R_2 be the resistance of two heaters and *H* is the heat required to boil 1 kg of water. If *V* is the voltage applied to heater, then

$$H = \frac{V^2}{R_1}t_1 = \frac{V^2}{R_2}t_2 \quad ...(i)$$

or $\frac{t_1}{R_1} = \frac{t_2}{R_2}$ or $\frac{R_1}{R_2} = \frac{t_1}{t_2}$ ...(*ii*)

when two heaters are in parallel and *t* is time taken to boil the given water, then

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$$H = \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right) t = \frac{V^2}{R_1} t_1$$

or $$t = \frac{t_1}{R_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{t_1}{(1 + R_1/R_2)}$$

$$= \frac{t_1}{1 + t_1/t_2} = \frac{t_1 t_2}{t_1 + t_2}$$

54. When the heaters are in series, then current through each heater, $I = V/(R_1 + R_2)$. If t is the time taken to boil the water, then

$$H = \left(\frac{V}{R_1 + R_2}\right)^2 (R_1 + R_2) t = \frac{V^2}{R_1} t_1$$

or $$\frac{t}{R_1 + R_2} = \frac{t_1}{R_1}$$

or $$t = t_1\left(\frac{R_1 + R_2}{R_1}\right) = t_1\left(1 + \frac{R_2}{R_1}\right)$$

or $$t = t_1\left(1 + \frac{t_2}{t_1}\right) = t_1 \frac{(t_1 + t_2)}{t_1} = t_1 + t_2$$

$$= \frac{t_1^2 - t_2^2}{(t_1 - t_2)}$$

55. $$I_2 = I_1\left(\frac{r_2}{r_1}\right)^{3/2} = 5\left(\frac{0{\cdot}04}{0{\cdot}02}\right)^{3/2} = \mathbf{14{\cdot}7\ A}$$

56. Let I be the current in the circuit. As per question,

$$2{\cdot}5 = IR \qquad ...(i)$$

$$3 = I(R + r) \qquad ...(ii)$$

and $$0{\cdot}5 = I^2 R \qquad ...(iii)$$

Dividing (*iii*) by (*i*), we get,

$$I = \frac{0{\cdot}5}{2{\cdot}5} = \frac{1}{5}\text{ A}$$

From (*i*), $$R = \frac{2{\cdot}5}{I} = \frac{2{\cdot}5}{1/5} = 12{\cdot}5\ \Omega$$

From (*ii*), $$r = \frac{3}{I} - R = \frac{3}{1/5} - 12{\cdot}5 = 2{\cdot}5\ \Omega$$

Power loss across internal resistance

$$= I^2 r = \left(\frac{1}{5}\right)^2 \times 2{\cdot}5 = \mathbf{0{\cdot}1\ watt}$$

57. Power, P = rate of flow of heat $= \frac{dQ}{dt}$

$$\therefore \quad P = \frac{dQ}{dt} = \frac{d}{dt}(ms\,T) = ms\,\frac{dT}{dt}$$

Heat capacity, $S = ms$

$$\therefore \quad P = S \times \frac{dT}{dt} \text{ or } S = \frac{P}{dT/dt}$$

Given, $T_t = T_0(1 + \beta\, t^{1/4})$ ...(*i*)

$$\frac{dT}{dt} = T_0\left(0 + \beta \times \frac{1}{4} t^{-3/4}\right) = \frac{\beta T_0}{4} t^{-3/4}$$

$$\therefore \quad S = \frac{P}{dT/dt} = \frac{4P}{\beta T_0 t^{-3/4}}$$

$$= \frac{4P}{\beta T_0} t^{3/4} = \frac{4P}{\beta}\left[\frac{t^{3/4}}{T_0}\right] \qquad ...(ii)$$

From (*i*), $$\frac{T_t}{T_0} = 1 + \beta\, t^{1/4}$$

or $$\beta\, t^{1/4} = \frac{T_t}{T_0} - 1 = \frac{T_t - T_0}{T_0}$$

or $$t^{3/4} = \left[\frac{T_t - T_0}{\beta T_0}\right]^3$$

$$\therefore \quad S = \frac{4P}{T_0 \beta}\left[\frac{T_t - T_0}{\beta T_0}\right]^3 = \frac{4P}{\beta^4 T_0^4}[T_t - T_0]^3$$

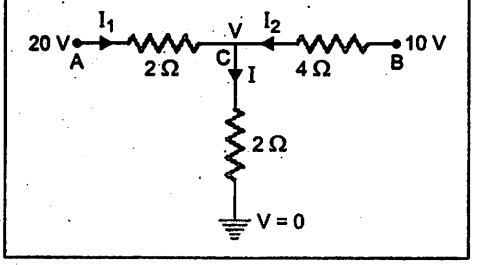
58. Let V be the potential difference applied across the parallel combination of resistors. Then thermal energy developed in a resistor of resistance R for time t is

$$E = \frac{V^2}{R} t$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{V^2 t/R_1}{V^2 t/R_2} = \frac{R_2}{R_1} = \frac{200}{100} = \frac{2}{1}$$

59. Refer to circuit shown in Fig. 3(CF).70. Let V be the potential at point C. Then

FIGURE 3(CF).70



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$$I = I_1 + I_2 \quad \therefore \quad \frac{V}{2} = \frac{20-V}{2} + \frac{10-V}{4}$$

On solving, $V = 10$ V

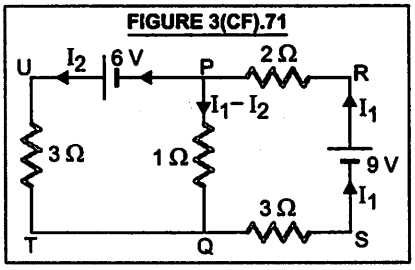
$$\therefore \quad I_1 = \frac{20-10}{2} = \frac{10}{2} = 5 \text{ A}$$

$$I_2 = \frac{10-10}{4} = 0$$

$$\therefore \quad I = 5 + 0 = 5 \text{ A}$$

60. The circuit can be redrawn as

FIGURE 3(CF).71



In closed loop *RPQSR*,

$$2I_1 + 1(I_1 - I_2) + 3I_1 = 9$$

$$6I_1 - I_2 = 9 \qquad ...(i)$$

In closed loop *PUTQP*,

$$3I_2 - 1(I_1 - I_2) = 6$$

$$-I_1 + 4I_2 = 6 \qquad ...(ii)$$

From (*i*) and (*ii*),

$$I_1 - I_2 = -0{\cdot}13 \text{ A}$$

This current is **0·13 A from *Q* to *P*.**

61. Let V_0 be the potential at O and I_1, I_2, I_3 and I_4 are the currents through R_1, R_2, R_3 and R_4 respectively. Then

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$\frac{V_0 - V_A}{R_1} + \frac{V_0 - V_B}{R_2} + \frac{V_0 - V_C}{R_3} + \frac{V_0 - V_D}{R_4} = 0$$

or $$\frac{V_0 - 5}{10} + \frac{V_0 - 10}{20} + \frac{V_0 - 20}{40} + \frac{V_0 - 15}{80} = 0$$

On solving, we shall get, $V_0 = 9$ V

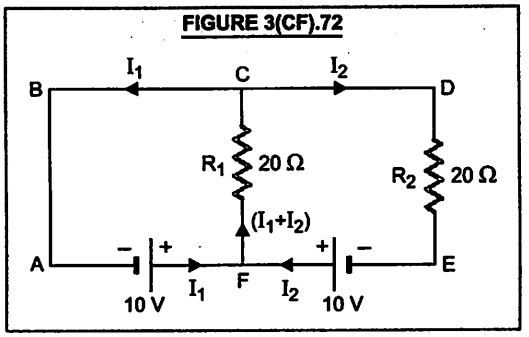
Current in the resistance R_1 is

$$I_1 = \frac{V_0 - V_A}{R_1} = \frac{9-5}{10} = 0{\cdot}4 \text{ A}$$

As $V_0 > V_A$, so the current is flowing from O towards A *i.e.* **away from O.**

62. Refer to circuit Fig. 3(CF).72. According to Kirchhoff's second rule in closed loop *AFCBA*, we have

FIGURE 3(CF).72



$$10 = 20(I_1 + I_2) \qquad ...(i)$$

In closed loop *FCDEF*, we have

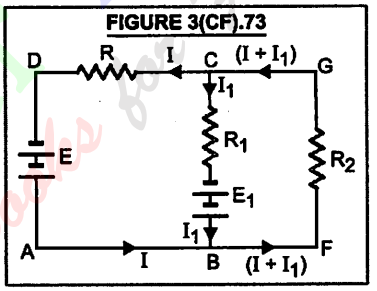
$$10 = 20(I_1 + I_2) + 20I_2 \qquad ...(ii)$$

Solving (*i*) and (*ii*), we get

$$I_1 = 0{\cdot}5 \text{ A} \; ; \; I_2 = 0$$

63. The distribution of current in the various arms as per Kirchhoff's first rule, has been shown in Fig. 3(CF).73.

FIGURE 3(CF).73



Using Kirchhoff's second rule in closed circuits *ABFGCDA* and *BFGCB*, we have

$$E = (I + I_1)R_2 + IR$$

and $$E_1 = (I + I_1)R_2 + I_1 R_1$$

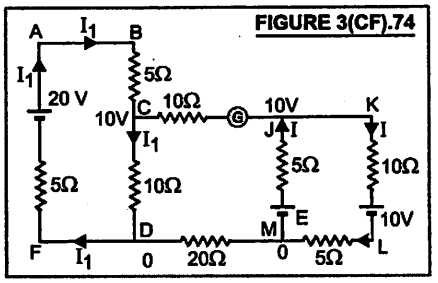
Putting $I = 0$, we have ;

$$E = I_1 R_2 \text{ and } E_1 = I_1(R_1 + R_2)$$

$$\therefore \quad E_1 = \frac{E}{R_2}(R_1 + R_2)$$

64. Refer Fig. 3(CF).74, *G* will show no deflection if points *C* and *J* are at the same potential.

FIGURE 3(CF).74



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In circuit $ABCDFA$, current $I_1 = \dfrac{20}{5+10+5} = 1\text{A}$

Pot. diff. between C and $D = I_1 \times 10 = 1 \times 10$
$= 10$ V

Since point D is a zero potential, therefore pot. of point C = 10 V.

Hence, potential of point J = potential at point C
$= 10$ V

Since point M is at zero potential, pot. diff. between E and $J = (E - 10)$ V.

In circuit $KLMJK$, current I is

$$I = \frac{E-10}{5+10+5} = \frac{E-10}{20}$$

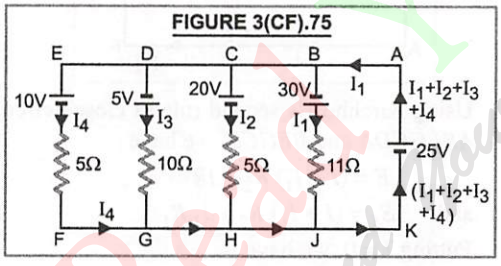
Pot. diff. across 5 Ω between points E and J is

$$E - 10 = \left(\frac{E-10}{20}\right)5 = \frac{E-10}{4}$$

or $4E - 40 = E - 10$ or $3E = 30$ or $E = \mathbf{10\ V}$

65. Refer to Fig. 3(CF).75, using Kirchhoff's voltage rule to closed circuits ABJKA, ACHKA, ADGKA, AEFKA, we have

FIGURE 3(CF).75



$30 + 25 = I_1 \times 11$ ...(*i*)

$-20 + 25 = I_2 \times 5$ ...(*ii*)

$5 + 25 = I_3 \times 10$ ...(*iii*)

$-10 + 25 = I_4 \times 5$ ...(*iv*)

Solving equations (*i*), (*ii*), (*iii*) and (*iv*), we get

$I_1 = 5$ A, $I_2 = 1$ A,

$I_3 = 3$ A, $I_4 = 3$ A

∴ Current through 25 V cell $= I_1 + I_2 + I_3 + I_4$
$= 5 + 1 + 3 + 3 = \mathbf{12\ A}$

66. In steady state, no current will flow through capacitor, Fig. 3(CF).76. Using Kirchhoff's voltage rule to the circuit $ACDA$, we have

$+ 10 + 2 = 1 \times R + 1 \times 2$ or $R = \mathbf{10\ \Omega}$

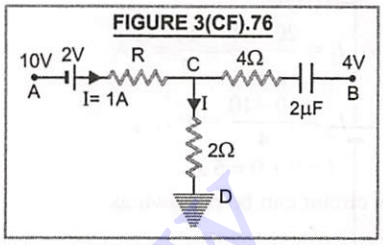
Pot. diff. across C and D, $V_C - V_D = 2 \times 1 = 2$ V

As, $V_D = 0$ V, so $V_C = 2$ V

Pot. diff. across capacitor = 4 – 2 = 2 V

Charge on capacitor, $Q = 2\text{ V} \times 2\ \mu\text{F} = \mathbf{4\ \mu C}$

FIGURE 3(CF).76



67. Refer to circuit given below

Using Kirchhoff's voltage rule in loops $ABCDE_1A$ and $ABCDE_2A$, we have

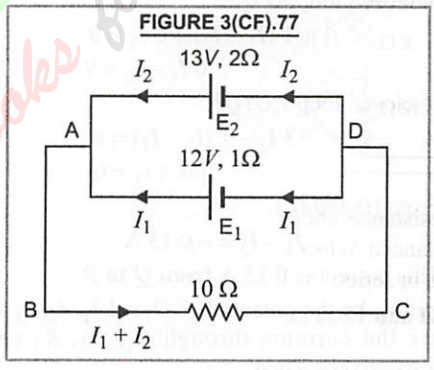
$(I_1 + I_2)\ 10 + I_1 \times 1 = 12$

or $11\ I_1 + 10\ I_2 = 12$ ...(*i*)

and $(I_1 + I_2)\ 10 + I_2 \times 2 = 13$

or $10\ I_1 + 12\ I_2 = 13$ ...(*ii*)

FIGURE 3(CF).77



Solving (*i*) and (*ii*), we get ;

$$I_1 = \frac{7}{16}A\ ;\quad I_2 = \frac{23}{32}A$$

Voltage across load $= (I_1 + I_2) \times 10$

$$= \left(\frac{7}{16} + \frac{23}{32}\right) \times 10 = \mathbf{11{\cdot}56\ V}$$

68. Resistors above and below 8 Ω makes balanced Wheatstone bridge. So circuit reduces to Fig. 3(CF).78

Now 1 Ω and 2 Ω are in series, total Resistance
$= 1 + 2 = 3\ \Omega$

2 Ω and 4 Ω are also in series so, total Resistance
$= 2 + 4 = 6\ \Omega$

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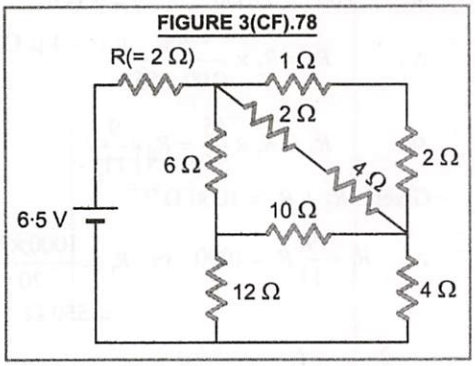
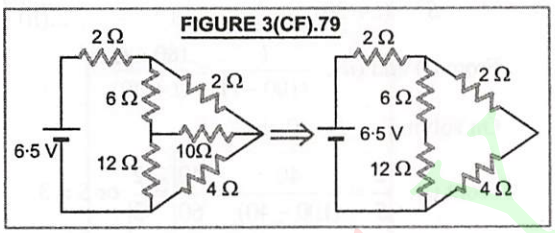


Now this 3 Ω and 6 Ω are in parallel, So their effective resistance $R_P = \frac{6\times3}{6+3} = 2\,\Omega$

Circuit reduces to Fig. 3(CF).79

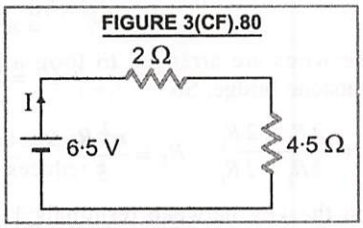


Resistance above and below 10 Ω again forms balanced Wheatstone bridge. Now 2 Ω and 4 Ω are in series $= 2 + 4 = 6\,\Omega$

6 Ω and 12 Ω are also in series $= 6 + 12 = 18\,\Omega$

Effective resistance $= \frac{6\times18}{6+18} = 4{\cdot}5\,\Omega$

Circuit reduces to as shown in Fig. 3(CF).80.



Total Resistance = 4·5 + 2 = 6·5 Ω

Current, $I = \frac{6{\cdot}5}{6{\cdot}5} = \mathbf{1\,A}$

69. As bridge is balanced,

so, $\frac{P}{10} = \frac{3}{2}$ or $P = \frac{3}{2}\times10 = 15\,\Omega$

Here, 15 Ω resistance is of wire of length 1·5 m.

$\therefore$ 1 Ω resistance is of wire of length $= \frac{1{\cdot}5}{15} = 0{\cdot}1\,\text{m}$

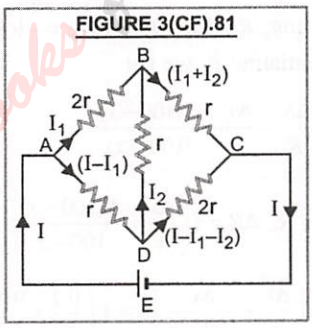
$= \mathbf{1{\cdot}0\times10^{-1}\,m}$

70. Using the condition for balanced Wheatstone bridge, we get

$$\frac{X}{10} = \frac{(52+1)\,\text{cm}}{(100-52+2)\,\text{cm}} = \frac{53}{50}$$

or $X = \frac{53\times10}{50} = \mathbf{10{\cdot}6\,\Omega}$

71. Connect a battery of emf E across A and C. Let the distribution of current in the various branches of net work be as shown in Fig. 3(CF).81. According to Kirchhoff's second law in a closed circuit, $ABDA$ we have



$$2\,r\,I_1 - r\,I_2 - r\,(I - I_1) = 0$$

or $3\,I_1 = I + I_2$ ...(*i*)

In a closed circuit $BCDB$,

$$r\,(I_1 + I_2) - 2\,r\,(I - I_1 - I_2) + I_2\,r = 0$$

or $3\,I_1 = 2\,I - 4\,I_2$ ...(*ii*)

From (*i*) and (*ii*), $2\,I - 4\,I_2 = I + I_2$

or $5\,I_2 = I$ or $I_2 = I/5$

From (*i*), $I_1 = (I + I_2)/3 = [I + (I/5)]/3$

$$= \frac{6I}{15} = \frac{2I}{5}$$

In a closed circuit $EABCEA$,

$$E = 2\,r\,I_1 + r\,(I_1 + I_2)$$

or $E = (3\,I_1 + I_2)\,r = 3\times\frac{2I}{5}r + \frac{I}{5}r = \frac{7}{5}I\,r$ ...(*iii*)

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If R is the resistance of network between A and C then $E = IR$ ...(iv)

From (iii) and (iv), $IR = \frac{7}{5} Ir$ or $R = \frac{7r}{5}$

Alternate method.

Here, $R_1 = r$; $R_2 = 2r$

The effective resistance of net work between A and C is

$$= \frac{R_1(R_1 + 3R_2)}{R_2 + 3R_1} = \frac{r(r + 3\times 2r)}{2r + 3r} = \frac{7r}{5}$$

72. $\frac{R}{90} = \frac{40}{60}$ or $R = \mathbf{60\ \Omega}$

If balancing length of bridge wire from zero end is x, then $\frac{R}{90} = \frac{x}{(100 - x)}$

Taking log of both the sides, we get

$$\log_e R - \log_e 90 = \log_e x - \log_e (100 - x)$$

Differentiating it, we get

$$\frac{\Delta R}{R} = \frac{\Delta x}{x} - \frac{\Delta(100 - x)}{(100 - x)}$$

Error in R, $\Delta R = \pm\left(\frac{\Delta x}{x} + \frac{\Delta(100 - x)}{100 - x}\right)R$

$$= \pm\left(\frac{\Delta x}{x} + \frac{\Delta x}{100 - x}\right)R = \pm\left(\frac{0{\cdot}1}{40} + \frac{0{\cdot}1}{60}\right)\times 60$$

$$= \pm\ 0{\cdot}25\ \Omega$$

$\therefore$ $R = \mathbf{60 \pm 0{\cdot}25\ \Omega}$

73. Let R_1 be the resistance on the left slot and R_2 be the resistance on the right slot of meter bridge before interchanging the resistances. Let l cm be the balancing length of balanced bridge. Then

$$\frac{R_1}{R_2} = \frac{l}{(100 - l)} \quad ...(i)$$

On interchanging the resistances

$$\frac{R_2}{R_1} = \frac{(l - 10)}{(110 - l)} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{110 - l}{l - 10}$$

$$\therefore \quad \frac{l}{(100 - l)} = \frac{110 - l}{l - 10}$$

or $l^2 - 10\,l = (100 - l)(110 - l)$

$$= 11000 - 210\,l + l^2$$

or $200\,l = 11000$ or $l = 55$ cm

$$\therefore \quad R_1 = R_2 \times \frac{55}{(100 - 55)}$$

or $R_2 = R_1 \times \frac{45}{55} = R_1 \times \frac{9}{11}$

Given, $R_1 + R_2 = 1000\ \Omega$

$$\therefore \quad R_1 + \frac{9}{11}R_1 = 1000 \quad \text{or} \quad R_1 = \frac{1000\times 11}{20}$$

$$= \mathbf{550\ \Omega}$$

74. $\frac{2}{S} = \frac{l}{(100 - l)}$ ...(i)

and $\frac{S}{2} = \frac{(l + 20)}{(100 - l) - 20} = \frac{(l + 20)}{(80 - l)}$

or $\frac{2}{S} = \frac{(80 - l)}{(l + 20)}$ ...(ii)

From (i) and (ii), $\frac{l}{(100 - l)} = \frac{(80 - l)}{(l + 20)}$

On solving, $l = 40$ cm.

From (i), $\frac{2}{S} = \frac{40}{(100 - 40)} = \frac{40}{60} = \frac{2}{3}$ or $S = \mathbf{3\ \Omega}$

75. Here, $R = \frac{\rho l_1}{A_1}$ When $l_2 = n\,l_1$

then $A_1\,l_1 = A_1\,l_2$ or $A_2 = \frac{A_1 l_1}{l_2} = \frac{A_1 l_1}{n l_1} = \frac{A_1}{n}$

New resistance of the stretched wire

$$R' = \frac{\rho l_2}{A_2} = \frac{\rho n l_1}{(A_1/n)} = n^2 R$$

Resistance of each part of wire, $R_1 = R'/5$

$$= n^2 R/5$$

These wires are arranged to form a balanced Wheatstone bridge. So

$$R_{AB} = \frac{2R_1 \times 2R_1}{2R_1 + 2R_1} = R_1 = \frac{\mathbf{n^2 R}}{\mathbf{5}}$$

76. When the key between terminals 1 and 2 is plugged in, R is in the circuit.

$V_1 = IR = 1\,R = \mathbf{Kl_1}$ ($\because$ $I = 1$ A)

When the key between terminals 1 and 3 is plugged in, R and X, both are in the circuit

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$$V_2 = I(R + X) = 1(R + X) = Kl_2$$
$$V_2 - V_1 = R + X - R = X = K(l_2 - l_1)$$

77. Resistance of wire $AB = 0{\cdot}02 \times 100 = 2\ \Omega$

Current in wire AB, $I = \dfrac{2}{2+2} = \dfrac{1}{2}\text{A}$

Let x be the resistance of length AJ of wire for which galvanometer shows no deflection, then

$$V_A - V_J = x\,I = x \times 1/2 \quad ...(i)$$

Here, $\varepsilon_1 = 2\text{ V}$; $\varepsilon_2 = 1\text{ V}$; $r_1 = 2\ \Omega$; $r_2 = 1\ \Omega$

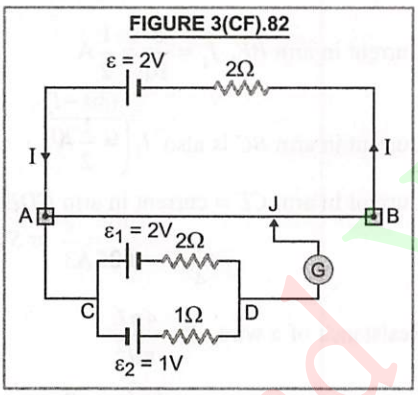
Current in closed circuit having ε_1 and ε_2 in series,

$$I_1 = \frac{2+1}{2+1} = 1\text{A}$$

Pot. diff. across C and D, *i.e.*,

$$V_C - V_D = \varepsilon_1 - I_1 r_1 = 2 - 1 \times 2 = 0.$$

FIGURE 3(CF).82



Here, $V_A - V_J = V_C - V_D$ or $\dfrac{x}{2} = 0$ or $x = 0$

i.e., length of wire $AJ = \mathbf{0}$

78. Let ε_1 and ε_2 be the emfs of two cells and K be the potential gradient of potentiometer wire.

Case (*i*), $\varepsilon_1 + \varepsilon_2 = K \times 50$ ...(*i*)

Case (*ii*), $\varepsilon_1 - \varepsilon_2 = K \times 10$ ...(*ii*)

Dividing (*i*) by (*ii*), we get

$$\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 - \varepsilon_2} = \frac{50}{10} = 5$$

or $\varepsilon_1 + \varepsilon_2 = 5\varepsilon_1 - 5\varepsilon_2$ or $4\varepsilon_1 = 6\varepsilon_2$

or $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{6}{4} = \mathbf{\dfrac{3}{2}}$

79. Let length $AJ = l$, of potentiometer wire. Current flowing through the potentiometer wire is

$$I = \frac{\varepsilon}{12r + r} = \frac{\varepsilon}{13r}$$

Potential gradient of potentiometer wire

$$k = \frac{I \times 12r}{L} = \frac{\varepsilon \times 12r}{13r \times L} = \frac{12\,\varepsilon}{13\,\text{L}}$$

$$V_{AJ} = kl = \frac{12\,\varepsilon}{13\,\text{L}} \times l$$

As galvanometer shows no deflection, so

$$V_{AJ} = \frac{\varepsilon}{2} \quad \text{or} \quad \frac{12\,\varepsilon}{13\,\text{L}}\,l = \frac{\varepsilon}{2} \quad \text{or} \quad l = \mathbf{\frac{13\,L}{24}}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

80. $\left(\dfrac{\varepsilon}{4+r}\right)^2 \times 4 = \left(\dfrac{\varepsilon}{1+r}\right)^2 \times 1$

$$\mathbf{r = 2\ \Omega}$$

Current in circuit, $I = \dfrac{\varepsilon}{4+2}$

$$16 = \left(\frac{\varepsilon}{4+2}\right)^2 \times 4 \quad \text{or} \quad \mathbf{\varepsilon = 12\ V}$$

Power dissipated is maximum when external resistance is equal to internal resistance, *i.e.*,

$R = r = 2\ \Omega$.

$$P_{\max} = \left(\frac{12}{2+2}\right)^2 \times 2 = \mathbf{18\ W}$$

81. (*a*) $I = \dfrac{dQ}{dt} = \dfrac{d}{dt}(at - bt^2) = a - 2\,bt$,

i.e., ***I* decreases with time**

(*c*) For $I = 0$, $a - 2\,bt = 0$ or $t = \mathbf{a/2b}$

(*d*) Rate of change of current

$$\frac{dI}{dt} = \frac{d}{dt}(a - 2bt) = \mathbf{-2b}$$

82. Here $V_1 = 200\text{ V}$, $V_2 = 300\text{ V}$

As $P = \dfrac{V^2}{R}$ so $R = \dfrac{V^2}{P}$

For same power 'P', $R \propto V^2$,

So $\dfrac{R_1}{R_2} = \left(\dfrac{V_1}{V_2}\right)^2 = \left(\dfrac{200}{300}\right)^2 = \dfrac{4}{9}$

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For series, current is same, $P = I^2R$, so $P \propto R$ and $V = IR$ so $V \propto R$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9} \quad \text{and} \quad \frac{P_1}{P_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

83. Total resistance of circuit

$$= R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2 + \frac{6 \times 1{\cdot}5}{6 + 1{\cdot}5}$$

$$= 2 + 1{\cdot}2 = 3{\cdot}2\ k\Omega$$

(*a*) Current through the battery,

$$I = \frac{24\,\text{V}}{3{\cdot}2\,\text{k}\Omega} = \frac{24}{3{\cdot}2 \times 1000} = \mathbf{7{\cdot}5\,mA}$$

(*b*) Pot. diff. across R_2 and R_3,

$V_2 = (7{\cdot}5\ \text{mA}) \times (1{\cdot}2\ \text{k}\Omega) = 9\ \text{V}$.

(*c*) Pot. diff. across R_1,

$V_1 = 7{\cdot}5\ \text{mA} \times 2\ \text{k}\Omega = 15\ \text{V}$

Power dissipated across R_1, $P_1 = \frac{V_1^2}{R_1} = \frac{15^2}{2 \times 10^3}$

Power dissipated across R_2, $P_2 = \frac{V_2^2}{R_2} = \frac{9^2}{6 \times 10^3}$

$$\therefore \quad \frac{P_1}{P_2} = \frac{15^2 \times 6 \times 10^3}{2 \times 10^3 \times 9^2} = 8{\cdot}3$$

Power dissipated across R_3,

$$P_3 = \frac{9^2}{1{\cdot}5 \times 10^3} = 54\ \text{mW}.$$

(*d*) If R_1 and R_2 are interchanged, total resistance of circuit is $= 6 + \frac{2 \times 1{\cdot}5}{2 + 1{\cdot}5} = 6 + \frac{6}{7} = \frac{48}{7}\,\text{k}\Omega$

Total current, $I = \frac{24 \times 7}{48 \times 1000} = 3{\cdot}5\,\text{mA}$

Pot. diff. across $R_3 = (3{\cdot}5\,\text{mA}) \times \frac{6}{7}\,\text{k}\Omega = 3\,\text{V}$

Power dissipated across R_3,

$$P_3' = \frac{3^2}{1{\cdot}5 \times 10^3} = 6\,\text{mW}$$

As $P_3' < P_3$, so power dissipation in R_3 will decrease

$$\therefore \quad \frac{P_3}{P_3'} = \frac{54}{6} = \mathbf{9}\ \textbf{(decrease).}$$

84. The resistances 2 Ω and 2 Ω in series of arm *CDE* are in parallel to resistance 4 Ω in arm *CE*. Their effective resistance $= \frac{4 \times (2+2)}{4 + (2+2)} = 2\ \Omega$. This resistance is in series with 8 Ω of arm *BC*. The effective resistance is 8 + 2 = 10 Ω, which is in parallel across 10 Ω resistance of arm *BE*. The effective resistance between *B* and *E*,

$$R = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

Total resistance of circuit = 5 + 5 + 15 = 25 Ω

Current in circuit, $I = \frac{25\,V}{25\,\Omega} = 1\,\text{A}$

Potential difference across *B* and *E* = *IR*

$= 1 \times 5 = \mathbf{5\ V}$

Potential difference across *A* and *B*

$= 25 - 5 = 20\ \text{V}$

Current in arm *BE*, $I_1 = \frac{5}{10} = \frac{1}{2}\,\text{A}$

Current in arm *BC* is also $I_1 \left(= \frac{1}{2}\,\text{A}\right)$

Current in arm *CE* = current in arm *CDE*

$$= \frac{1}{4}\,\text{A} = \mathbf{0{\cdot}25\,A}$$

85. Resistance of a wire, $R = \frac{4\rho L}{\pi d^2}$

when $d = 2d$, then $R' = \frac{4\rho L}{\pi (2d)^2} = \frac{R}{4}$

When two wires are in series, Then their effective resistance, $R_1 = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$

When two wires are in parallel, their effective resistance, $R_2 = \frac{(R/4)\,(R/4)}{(R/4) + R/4} = \frac{R}{8}$

As per question $H = \frac{V^2}{R} \times 4 = \frac{V^2}{(R/2)} t_1 = \frac{V^2}{(R/8)} t_2$

On solving, $t_1 = \mathbf{2\ min.}$ and $t_2 = \mathbf{0{\cdot}5\ min.}$

86. Since evaporation is given to be non-uniform, hence temperature must be non-uniform. Thus option (*a*) is wrong. Due to evaporation, the cross-sectional area of the filament at section decreases. Hence resistance of the given section increases as $R \propto 1/A$. Thus option (*b*) is wrong.

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The portion of the filament of the bulb having highest evaporation rate would have rapidly reduced area and would become break up area of cross-section. At break up, junction temperature would be highest. Due to it, the light of highest band frequency would be emitted at those cross-sections. Thus **option (c) is true.**

Due to continuous heating of filament, the cross-sectional area of a section of filament decreases. Hence the resistance of the filament on the whole increases with time. As a result of it, the power decreases because $P = V^2/R$ or $P \propto 1/R$ as V is constant. Thus **option (d) is true.**

87. Let E, r be the *emf* and internal resistance of each cell.

In series, current $I = \dfrac{nE}{nr} = \dfrac{E}{r}$; **(independent of n)**

In parallel, current $I = \dfrac{E}{r/n} = \dfrac{nE}{r}$; *(i.e.* $I \propto n$)

When n parallel cells are short circuited by a wire of constant resistance R, the current,

$$I = \frac{E}{(R + r/n)} = \frac{nE}{nR + r}$$

As n increases r/n decreases, so I increases.

88. $v_d = \dfrac{eE}{m}\tau$, *i.e.*, $v_d \propto E$

$$P = \frac{V^2}{R} = \frac{(E/d)^2}{R} = \frac{E^2}{d^2 R}, \text{ i.e., } P \propto E^2$$

But $E \propto v_d$ so $P \propto v_d^2$

$P = VI = I^2 R$, *i.e.*, $P \propto I^2$

89. Refer to Fig. 3(CF).83; using Kirchhoff's voltage law in closed circuit. *ABCDA*, we get

$$V_1 = I R_1 + 0 \times R_2 \quad \text{or} \quad I = \frac{V_1}{R_1} \qquad ...(i)$$

In closed circuit *ABCDEFA*, we get

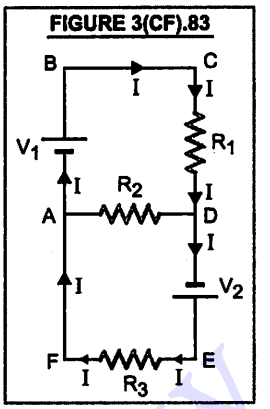
$$V_1 + V_2 = IR_1 + IR_3 \quad \text{or} \quad I = \frac{V_1 + V_2}{R_1 + R_3} \qquad ...(ii)$$

From (*i*) and (*ii*), $\dfrac{V_1}{R_1} = \dfrac{V_1 + V_2}{R_1 + R_3}$

or $V_1 R_1 + V_1 R_3 = V_1 R_1 + V_2 R_1$

or $V_1 R_3 = V_2 R_1 \qquad ...(iii)$

FIGURE 3(CF).83



Since, current in R_2 is zero R_2 can have any value.

From (*iii*), we note that, if $V_1 = V_2$, then

$R_1 = R_3 = R_2$ or $R_1 = 2R_2 = R_3$

If $V_1 = 2V_2$, $R_1 = 2R_3$

If $2V_1 = V_2$, $R_3 = 2R_1 = R_2$

Thus options (***a, b* and *d*) are true.**

90. The resistance of each side of polygon = R/n. For maximum resistance between opposite corners, we have two resistances each of $R/2$ in parallel.

The maximum resistance $= \dfrac{(R/2)\times(R/2)}{(R/2)+(R/2)} = \dfrac{R}{4}$

For resistance between adjacent corners, we have two resistances R/n and $(n-1)\,R/n$ in parallel. Their effective resistance

$$= \frac{(R/n)\times[(n-1)\,R/n]}{(R/n)+(n-1)\,R/n} = \frac{R(n-1)}{n^2}$$

91. As cells are in parallel, the equivalent internal resistance R is $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

On applying Kirchhoff's voltage law in the loops, we can get $E_3 = \dfrac{E_1 R_2 + E_2 R_1}{R_1 + R_2}$

Equivalent *emf* of the battery depends upon the values of E_1, E_2, E_3 and also on R_1, R_2 and R_3.

92. Refer Fig. 3(CF).84, let V be the potential of point D.

Then $70 - V = 10\,I_1 \qquad ...(i)$

and $V - 0 = 20\,I_2$ or $V = 20\,I_2 \qquad ...(ii)$

and $V - 10 = 30\,(I_1 - I_2) \qquad ...(iii)$

From (*i*) and (*ii*),

$70 - 20\,I_2 = 10\,I_1$

or $10\,I_1 + 20\,I_2 = 70 \qquad ...(iv)$

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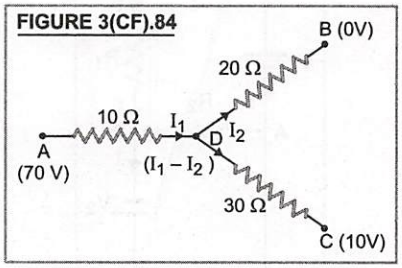
FIGURE 3(CF).84



From (*ii*) and (*iii*),

$$20\,I_2 - 10 = 30\,I_1 - 30\,I_2$$

or $$30\,I_1 - 50\,I_2 = -10 \quad ...(v)$$

On solving, we get

$$I_1 = 3\text{ A}, I_2 = 2\text{ A}, (I_1 - I_2) = 1\text{ A}$$

and $V = 40$ V

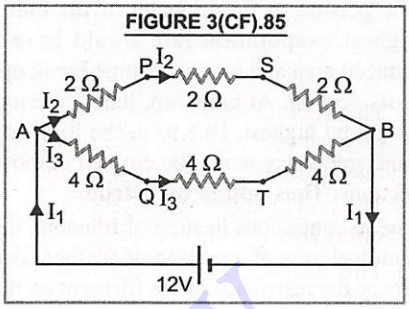
∴ Total power consumed

$= (3)^2 \times 10 + (2)^2 \times 20 + (1)^2 \times 30$

$= 90 + 80 + 30 =$ **200 W**

93. If a plane mirror is placed along a line which is perpendicular bisector of line *AB*, then the left part of the circuit is exact image of right part of the circuit. So current in *AP* = current in *SB* and current in *AQ* = current in *TB*. Therefore, in the given circuit the points *P* and *Q* are at the same potential. Also the points *S* and *T* are also at the same potential. Therefore, **no current flows through arms *PQ* and *ST*.** The equivalent circuit will be as shown in Fig. 3(CF).85.

FIGURE 3(CF).85



The equivalent resistance of the circuit

$$R = \frac{(2+2+2)\times(4+4+4)}{(2+2+2)+(4+4+4)} = 4\ \Omega$$

Pot. diff. across *A* and *B* = 12 V.

$$\therefore\ I_1 = \frac{12}{4} = 3\text{ A}$$

$$I_2 = \frac{12}{2+2+2} = 2\text{ A}$$

Since current is flowing from *P* to *S*, therefore $V_P > V_S$. As $V_P = V_Q$, so $V_Q > V_S$ or $V_S < V_Q$

94. Heat produced, $H = I^2 Rt$. Since *I*, *R* and *t* are same for the two resistors, hence **equal amounts of thermal energy must be produced. Also $H = mc\,\theta$, therefore the temperature may rise equally in the resistors if their sp-heat is same, otherwise not.**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

95. When a potential difference is applied across the two ends of a conductor, the **electric field exists inside the conductor.**

96. The free electrons moving towards the positive end of the conductor have two types of velocities (*i*) thermal velocity (*ii*) velocity due to the acceleration by virtue of force on it due to electric field. As a result of it, the electron follows **a curved path** in between two collisions in a conductor.

97. The drift speed of the electrons, $v_d = \frac{eE}{m}\tau$, which is independent of dimensions and number density of free electrons in the conductor.

98. Knowledge based question.

99. The effective resistance between diagonally opposite corners of a skelton cube is 5 *r*/6, where *r* is the resistance of each arm of skelton cube. Here, $r = 12/12 = 1\ \Omega$/cm.

∴ Resistance between diagonally opposite corners of a skelton cube is $= \frac{5}{6}\times 1 =$ **5/6 Ω**

100. Current drawn from the battery,

$$I = \frac{\text{e.m.f.}}{\text{resistance}} = \frac{2}{5/6} = 2{\cdot}4\text{ A}$$

101. The maximum current in an arm of the skelton cube = $2\,I/6 = 2 \times 2{\cdot}4/6 =$ **0·8 A**

102. The minimum potental difference across an arm of network = $I\,r/6 = 2{\cdot}4 \times 1/6 =$ **0·4 V**

HINTS/EXPLANATIONS
For Difficult Questions

IV. Matching Type Questions

103. (A) The standard resistance coil is made of manganin, *i.e.*, an alloy of copper-magnanese-nickel.

(B) The best connecting wire is of silver, which offers least resistance.

(C) The heating wire is generally made of nichrome, *i.e.*, of nickel-chromium alloy.

(D) The fuse wire is made of tin-lead alloy.

104. (A) Current density $= \frac{I}{a} = [AL^{-2}]$

(B) Conductance $= \frac{1}{R} = \frac{I}{V} = \frac{Iq}{W} = \frac{A \times AT}{ML^2T^{-2}}$

$= [M^{-1}L^{-2}T^3A^2]$

(C) Electrical conductivity

$$\sigma = \frac{l}{Ra} = (M^{-1}L^{-2}T^3A^2)\frac{L}{L^2} = [M^{-1}L^{-3}T^3A^2]$$

(D) Specific resistance, $\rho = \frac{1}{\sigma} = [M^1L^3T^{-3}A^{-2}]$.

V. Matrix-Match Type Questions

105. (A) Current is due to motion of electrons in a closed path and in a definite direction.

(B) EMF of the cell is the workdone in taking the unit charge once around the closed path.

(C) Resistance of a conductor is due to the collisions of the electrons with ions/atoms while moving through the conductor.

(D) Superconductor is that conductor which offers no resistance to the electrons flowing through it, *i.e.*, the electrons suffer no collision with the ions/atoms while passing through the super-conductor.

106. (A) The maximum potential difference between two points of a circuit is measured by voltmeter and potentiometer.

(B) The emf of the cell in the closed circuit (*i.e.*, terminal pot. diff.) is measured by voltmeter and potentiometer.

(C) The capacity of supplying the current in the circuit is in ampere hour.

(D) The electric energy consumed in the circuit is taken in watt-hour.

VI. Integer Type Questions

107. Let E_0 be the pot. diff. applied across the total length l (= 10 cm) of potentiometer wire.

Potential gradient in the first case $= \frac{E_0}{l}$.

As per question, $E = \frac{l}{3}\left(\frac{E_0}{l}\right) = \frac{E_0}{3}$ ...(*i*)

Potential gradient in second case $= \frac{E_0}{3l/2} = \frac{2E_0}{3l}$.

If x is the desired length of potentiometer to balance the emf E of the cell, then

$$E = x \times \frac{2E_0}{3l} \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{E_0}{3} = x \times \frac{2E_0}{3l}$$

or $x = \frac{l}{2} = \frac{10}{2} = \mathbf{5\,cm}$

108. When the current through galvanometer is zero, then pot. diff. across $R\,\Omega = 7$ V

pot. diff. across $5\,\Omega = 12 - 7 = 5$ V

current in $5\,\Omega = \frac{5}{5} = 1$ A

Resistance, $R = \frac{7}{1} = \mathbf{7\,\Omega}$

HINTS/EXPLANATIONS
For Difficult Questions

109. Let a be the area of cross-section of wire and n be the number density of electrons in the wire. If N is the total no. of electrons crossing per second through a cross-section of wire due to current I flowing, then

$$N = \frac{I}{e} = nal \quad \text{or} \quad I = nale \qquad ...(i)$$

But $I = naev_d$

or $v_d = \frac{I}{nae} = \frac{nale}{nae} = l$

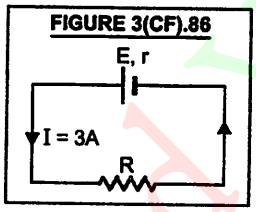
Total momentum of electrons in the straight wire is $= Nmv_d = \frac{I}{e} \times m \times l$

$$= \frac{70 \times (9 \times 10^{-31}) \times 1000}{1{\cdot}6 \times 10^{-19}}$$

$$\approx 4 \times 10^{-7} \text{ Ns}$$

110. Refer Fig. 3(CF).86, there will be maximum power in the external circuit if $R = r$.

FIGURE 3(CF).86



Maximum power consumed,

$$P_{max} = \left(\frac{E}{r+r}\right)^2 \times r = \frac{E^2}{4r} = 9 \qquad ...(i)$$

Current, $I = \frac{E}{r+r} = 3$

or $E = 6r \qquad ...(ii)$

Solving (i) and (ii), we get

$$E = 6 \text{ V}$$

111. The equivalent circuit with the distribution of current is shown in Fig. 3(CF).87.

Using Kirchhoff's second law, in closed circuit ABEFA, we have,

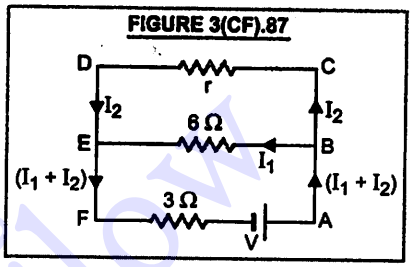
$$6I_1 + 3(I_1 + I_2) - V = 0$$

or $V = 9I_1 + 3I_2 \qquad ...(i)$

Using Kirchhoff's second law, in closed circuit BCDEB, we have

$$rI_2 - 6I_1 = 0 \quad \text{or} \quad I_1 = \frac{I_2}{6} r$$

FIGURE 3(CF).87



From (i),

$$V = 9 \times \frac{I_2}{6} r + 3I_2 = \frac{3}{2} I_2 r + 3I_2 = \frac{3I_2}{2}(r+2)$$

or $I_2 = \frac{2V}{3(r+2)}$

Power developed in resistor r,

$$P = I_2^2 r = \frac{4V^2}{9(r+2)^2} \times r$$

Power developed is maximum, when $(r+2)^2$ is minimum

or $(r+2)^2 = 0$

or $r^2 + 4r + 4 = 0$

or $r^2 - 4r + 4 + 8r = 0$

or $(r-2)^2 + 8r = 0$

or $r = 2\ \Omega$

112. Given, $J_1/J_2 = 2{\cdot}25$

$$\left(\frac{2E}{2r+R}\right)^2 R = J_1 \quad \text{and} \quad \left(\frac{2E}{r+2R}\right)^2 R = J_2$$

$$\therefore \quad \frac{(r+2R)^2}{(2r+R)^2} = \frac{J_1}{J_2} = 2{\cdot}25$$

or $\frac{r+2R}{2r+R} = 1{\cdot}5 = \frac{3}{2}$

On solving, we get,

$$R = 4r = 4 \times 1 = 4\ \Omega$$

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VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

113. The bending of the insulating wire does not alter the resistance of the wire. Hence **assertion is wrong.** Here **reason is also wrong.**

114. In parallel combination of electrical appliances

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

As voltage (V) across each appliance is same so

$$\frac{V^2}{R}=\frac{V^2}{R_1}+\frac{V^2}{R_2}+\frac{V^2}{R_3} \quad \text{or} \quad P=P_1+P_2+P_3$$

115. The resistance of 20 bulbs in series is more than that of 19 bulbs. The current in a frill of 20 bulbs will be less than that of 19 bulbs. Since power $\propto$ (current)^2, so the bulb in the frill of 19 bulbs will glow with more brightness than that of 20 bulbs frill. Thus **both Assertion and Reason are false.**

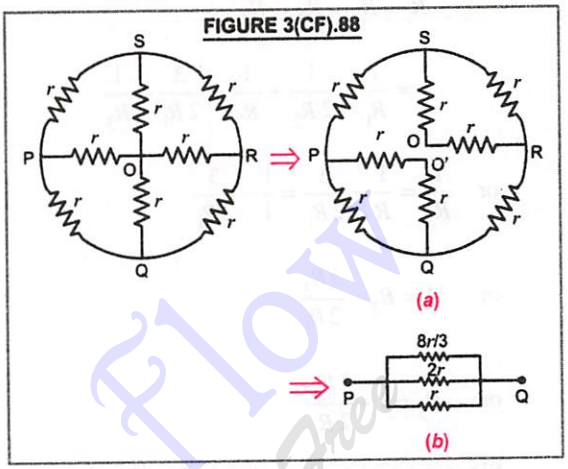
116. Here both Assertion and Reason are wrong. We know that with the increase in temperature, the value of unknown resistance of conductor increases.

In balanced meter bridge, $\frac{R}{X}=\frac{l_1}{l_2}$

where R is standard known resistance and X is unknown resistance. On heating, as X increases, so for the same value of l_1/l_2, R should be increased and not decreased.

117. The equivalent circuit is shown in Fig. 3(CF).88(a) and (b)

FIGURE 3(CF).88



$$\frac{1}{R_{PQ}}=\frac{1}{r}+\frac{1}{2r}+\frac{3}{8r}=\frac{15}{8r} \quad \text{or} \quad R_{PQ}=\frac{8r}{15}$$

Both Assertion and Reason are wrong.

FOR ENGINEERING STUDENTS

118. When the identical cells are connected in parallel, the effective e.m.f. is equal to e.m.f. of one cell. Hence **statement-1 is wrong.** Here **statement-2 is correct.**

119. The electrical appliances with metallic body like heater, press etc. have three pin connections. Two pins are for supply line and third pin is for earth connection for safety purposes. Here **statement-2 is wrong.**

120. A laser beam is a beam of light which is light amplification by stimulated emission of radiation. The energy per unit area of the laser beam is very high as compared to the torch light. Here **statement-2 is wrong.**

121. Here **statement-1 is correct but statement-2 is wrong,** because the electric field is there but the net electric field is zero.

VIII. Multiple Choice Questions (Based on Experimental Skills)

122. From $R=\frac{V}{I}$

$$\frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}=\frac{3}{100}+\frac{3}{100}=\frac{6}{100}$$

$\therefore$ Percentage error in measured value of resistance = **6%**

123. Given, $\frac{R_1}{R_2}=\frac{1}{2}$ or $R_2=2R_1$

HINTS/EXPLANATIONS
For Difficult Questions

The equivalent resistance of parallel combination is $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$

$$=\frac{1}{R_1}+\frac{1}{2R_2}+\frac{1}{R_3}=\frac{3}{2R_1}+\frac{1}{R_3}$$

or $\frac{1}{R_3}=\frac{1}{R}-\frac{3}{2R_1}=\frac{1}{1}-\frac{3}{2R_1}$

or $1=R_3-\frac{3R_3}{2R_1}$

or $R_3=1+\frac{3R_3}{2R_1}$

Since no resistance value is fractional, hence,

minimum value of $\frac{R_3}{R_1}=\frac{2}{3}$

So $R_3=1+\frac{3}{2}\times\frac{2}{3}=1+1=2\Omega$

and $R_1=\frac{3R_3}{2}=\frac{3\times2}{2}=3\Omega$

∴ Maximum resistance value is of

$R_2=2R_1=2\times3=\mathbf{6\ \Omega}$

124. $\frac{ER_1}{(R_1+r)}=kl_1$ and $\frac{ER_2}{(R_2+r)}=kl_2$

$\therefore \frac{ER_1/(R_1+r)}{ER_2/(R_2+r)}=\frac{l_1}{l_2}$ or $\frac{R_1(R_2+r)}{R_2(R_1+r)}=\frac{l_1}{l_2}$

$\therefore \frac{5(10+r)}{10(5+r)}=\frac{2}{3}$

On solving, $r=\mathbf{10\ \Omega}$

125. For first case $\frac{R_1+10}{R_2}=\frac{50}{100-50}=1$

or $R_2=R_1+10$ ...(*i*)

When 10 Ω resistance is removed, the balance point will shift towards lower length.

∴ New balancing length = 50 – 10 = 40 cm.

For second case,

$\frac{R_1}{R_2}=\frac{40}{100-40}=\frac{40}{60}=\frac{2}{3}$ ∴ $R_2=\frac{3}{2}R_1$

From (*i*), $\frac{3}{2}R_1=R_1+10$ or $R_1=\mathbf{20\ \Omega}$

126. Let the resistance of shorter part AB be x. Since the total resistance is 20 Ω, the resistance of longer AB part is $(20-x)$. Between points A and B, the two resistances x and $(20-x)$ are connected in parallel, so their equivalent resistance is

$$R_{eq}=\frac{(20-x)\times x}{(20-x)+x}=1{\cdot}8$$

On solving get, $x=2\ \Omega$

Thus, length of the shorter section = 2 × 1 = 2 m

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UNIT-3

MOVING CHARGES AND MAGNETISM

CHAPTER 4.

MOVING CHARGES AND MAGNETISM

CHAPTER 5.

MAGNETISM AND MATTER

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4

MOVING CHARGES AND MAGNETISM

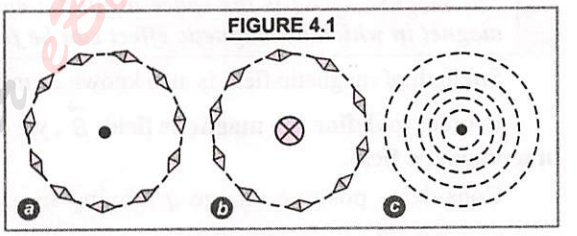
4.1. INTRODUCTION

Both, electricity and magnetism have been known for more than 2000 years. However, their intimate relationship was discovered only 200 years ago. Oersted discovered in the year 1820 that a straight wire carrying current caused a noticeable deflection in a nearby magnetic compass needle. The wire carrying current must be associated with a magnetic field causing deflection in the compass needle.

When a wire carrying current is held perpendicular to the plane of paper, and a ring of compass needles surrounds the wire the orientation of the needles is shown in Fig. 4.1(*a*) when current emerges out of the plane of the paper. However, when current moves into the plane of the paper, the orientation of the needles is shown in Fig. 4.1(*b*).

If we were to sprinkle iron filings around the wire carrying current, they arrange themselves in concentric circles with the wire as the centre, as shown in Fig. 4.1(*c*). Oersted thus concluded that moving charges or currents produce a magnetic field in the surrounding space.

FIGURE 4.1



In this unit, we shall study :

(*i*) How currents produce magnetic fields,

(*ii*) How magnetic fields exert forces on moving charged particles like electrons, protons and current carrying wires,

(*iii*) How charged particles can be accelerated to very high energies in a cyclotron.

(*iv*) How currents and voltages are detected and measured by a galvanometer.

4.2. OERSTED'S EXPERIMENT

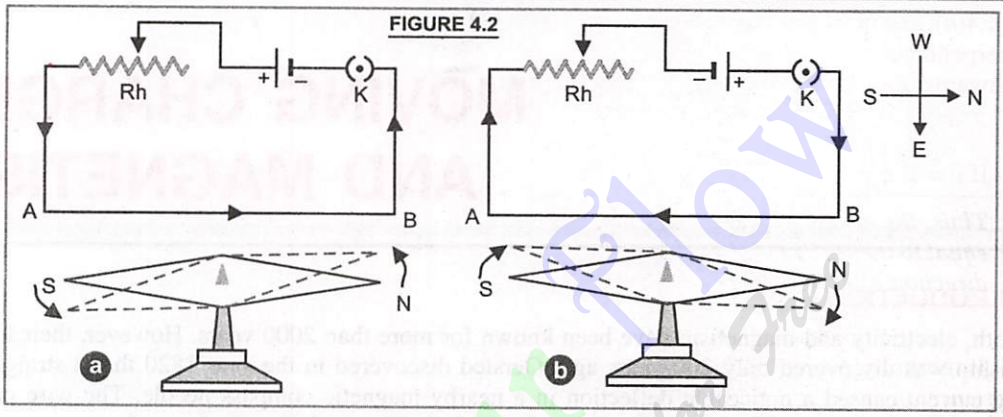
Take a magnetic needle *NS*, which can rotate freely about a vertical axis in a horizontal plane. Hold a conducting wire *AB* over the magnetic needle *NS* parallel to it. Complete the circuit by closing the key *K* such that current flows from *A* to *B*.

It will be found that N-Pole of the magnetic needle gets deflected towards the west, Fig. 4.2(*a*). If the direction of current in the wire is reversed (*i.e.* from *B* to *A*), the N-pole of magnetic needle gets deflected towards east, Fig. 4.2(*b*). Since the magnetic needle can be deflected only by the interaction of another magnetic field, therefore, the current in the wire must be producing a magnetic field in the

surrounding space. The direction of deflection of magnetic needle due to current in the wire is given by ***Ampere's swimming rule.***

> ***Ampere's swimming rule.*** ***According to this rule, if we imagine a man swimming along the wire in the direction of current with his face always turned towards the needle, so that the current enters through his feet and leaves at his head, then the N-pole of the magnetic needle will be deflected towards his left hand. This rule can be recollected with the help of the word SNOW, where each letter has a definite sense. It means, if current flows from South to North, in a wire held Over the magnetic needle, the north pole of the needle will be deflected towards West.***

FIGURE 4.2



4.3. MAGNETIC FIELD

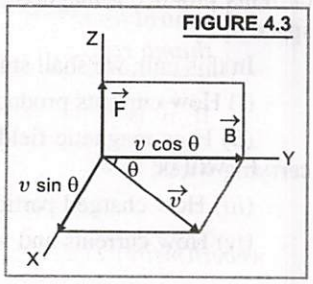
Just as a static charge produces an electric field, a moving charge or current through a conductor produces a magnetic field.

> ***The magnetic field*** ***is the space around a conductor carrying current or the space around a magnet in which its magnetic effect can be felt.***

Strength of magnetic field is also known as magnetic flux density or magnetic induction.

In order **to define the magnetic field** $\vec{B}$ **,** we deduce an expression **for the force on a moving charge in a magnetic field.**

FIGURE 4.3



Consider a positive charge q moving in a uniform magnetic field $\vec{B}$ (along OY) with a velocity $\vec{v}$. Let the angle between $\vec{v}$ and $\vec{B}$ be θ, Fig. 4.3. Due to interaction between the magnetic field produced due to moving charge (*i.e.* current) and magnetic field applied, the charge q experiences a force.

Experimentally, it has been found that the magnitude of force $\vec{F}$ experienced by the moving charge

(*i*) is directly proportional to the magnitude of the charge, *i.e.*, $F \propto q$

(*ii*) is directly proportional to the component of velocity acting perpendicular to the direction of magnetic field, *i.e.* $F \propto v \sin\theta$

(*iii*) is directly proportional to the magnitude of the magnetic field applied, *i.e.*, $F \propto B$

Combining the above factors, we get $F \propto q\, v \sin\theta\, B$ or $F = k\, q\, v\, B \sin\theta$

where k is a constant of proportionality. Its value is found to be one *i.e.* $k = 1$.

$\therefore \quad F = q\, v\, B \sin\theta$ ...(1)

or $|\vec{F}| = q\,|\vec{v} \times \vec{B}|$ or $\vec{F} = q(\vec{v} \times \vec{B})$ ...(2)

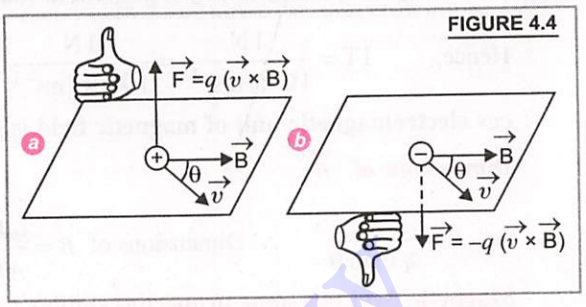
Note : F is called ***Magnetic Lorentz Force.***

Direction of $\vec{F}$

The direction of $\vec{F}$ is given by the **Right-Handed-Screw Rule** or **Right-Hand Rule.**

If $\vec{v}$ and $\vec{B}$ are in the plane of paper, then according to Right-Hand Rule, the direction of $\vec{F}$ on positively charged particle will be perpendicular to the plane of paper directed upward as shown in Fig. 4.4(*a*), and on negatively charged particle will be perpendicular to the plane of paper directed downwards, Fig. 4.4(*b*).

FIGURE 4.4



Definition of $\vec{B}$

If $v = 1$, $q = 1$ and $\sin\theta = 1$ or $\theta = 90°$, then from (1), $F = 1 \times 1 \times B \times 1 = B$

Thus, the magnetic field induction or magnetic flux density at a point in the magnetic field is equal to the force experienced by a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point.

Special Cases. Case (*i*) If $\theta = 0°$ or $180°$, then $\sin\theta = 0$. $\therefore$ From (1), $F = q\,v\,B\,(0) = 0$

It means, *a charged particle is moving along or opposite to the direction of magnetic field, it does not experience any force.*

Case (*ii*) If $\theta = 90°$, then $\sin\theta = 1$ $\therefore$ $F = q\,v\,B\,(1) = q\,v\,B$ (Maximum)

Case (*iii*) If $\theta = 270°$, then $F = q\,v\,B\sin 270° = -q\,v\,B$

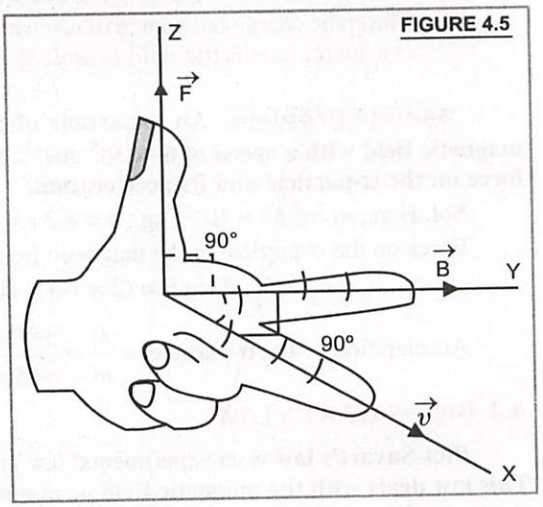
Case (*iv*) If $v = 0$, then, $F = q\,v\,B\sin\theta = 0$

It means, *if a charged particle is at rest in a magnetic field, it experiences no force and if a charged particle is moving along a line perpendicular to the direction of magnetic field, it experiences maximum force*. The direction of this force can also be determined by **Fleming's Left Hand Rule,** which can be stated as follows :

If we stretch the first finger, the central finger and the thumb of left hand in mutually perpendicular directions such that the first finger points to the direction of magnetic field, the central finger points to the direction of electric current (motion of the positive charge), then the thumb represents the direction of force experienced by the charged particle.

If $\vec{v}$ is along X-axis and $\vec{B}$ along Y-axis, then $\vec{F}$ will be along Z-axis as illustrated in Fig. 4.5.

FIGURE 4.5



Unit of $\vec{B}$. SI unit of B is *tesla* (T) or *weber/(metre)*^{2}, *i.e.*, (Wb/m^2) or Ns C^{-1} m^{-1}

From (1), $$B = \frac{F}{q\,v\sin\theta}$$

If $q = 1$ C, $v = 1$ ms^{-1}, $\theta = 90°$ or $\sin\theta = 1$

and $F = 1$ N ; then

$$B = \frac{1}{1\times1\times1} = 1\text{T}$$

Thus, the magnetic field induction or magnetic flux density at a point is said to be one tesla if a charge of one coulomb while moving at right angle to a magnetic field, with a velocity of 1 ms^{-1} experiences a force of 1 newton, at that point.

Hence, $1\,T = \dfrac{1\,N}{1\,C \times 1\,ms^{-1}} = \dfrac{1\,N}{1\,As \times 1\,ms^{-1}} = 1\,N\,A^{-1}\,m^{-1}$. (1 C = 1 ampere × 1 sec.)

***cgs* electromagnetic unit of magnetic field is gauss** (denoted by G), where 1 G = 10^{-4} T.

Dimensions of $\vec{B}$.

As, $B = \dfrac{F}{q v \sin\theta}$ ∴ Dimensions of $B = \dfrac{MLT^{-2}}{AT(LT^{-1})} = [MA^{-1}T^{-2}]$

Magnetic field has some properties similar to electric field

1. Magnetic field is a vector quantity like electric field.

2. Magnetic field has three dimensional relevance like electric field.

3. Magnetic field can depend on time like electric field.

4. Magnetic field depends upon the permeability (μ) of the material medium present in the field, whereas electric field depends upon the permittivity (∈) of the material medium present there. Infact both μ and ∈ are the measure of the extent to which the magnetic field and the electric field can penetrate the material medium.

5. The various magnetic fields $\vec{B}_1, \vec{B}_2, \vec{B}_3$ and so on, at a point due to different sources in a region can be added vectorially by **superposition principle** *i.e.*, $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \ldots$. whereas the various electric fields at a point can be added vectorially by superposition principle *i.e.*, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$.

Retain in Memory

1. No force is experienced by a static charge in a magnetic field.
2. The force is experienced by a moving charge in a magnetic field only if the charge is not moving parallel or antiparallel to the magnetic field.
3. Magnetic force on the charge deflects the path of charge.
4. Strength of magnetic field is generally referred to as simply "magnetic field".
5. Earth's magnetic field at its surface is about $3{\cdot}5 \times 10^{-5}$ T.
6. The static charge is a source of electric field but not of magnetic field whereas the moving charge is a source of electric field as well as of magnetic field.

Sample Problem **An α-particle of mass $6{\cdot}65 \times 10^{-27}$ kg is travelling at right angles to a magnetic field with a speed of 6×10^5 ms^{-1}. The strength of the magnetic field is 0·2 T. Calculate the force on the α-particle and its acceleration.**

Sol. Here, $m = 6{\cdot}65 \times 10^{-27}$ kg ; $q = +2e = +2 \times 1{\cdot}6 \times 10^{-19}$ C ; $v = 6 \times 10^5$ ms^{-1} ; $B = 0{\cdot}2$ T ; $\theta = 90°$

Force on the α-particle in the magnetic field is

$$F = q\,v\,B \sin\theta = (2 \times 1{\cdot}6 \times 10^{-19}) \times (6 \times 10^5) \times 0{\cdot}2 \times \sin 90° = \mathbf{3{\cdot}84 \times 10^{-14}\,N}$$

Acceleration of the α-particle $= \dfrac{F}{m} = \dfrac{3{\cdot}84 \times 10^{-14}}{6{\cdot}65 \times 10^{-27}} = \mathbf{5{\cdot}77 \times 10^{12}\ ms^{-1}}$

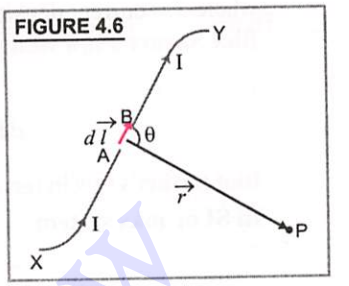
4.4. BIOT-SAVART'S LAW

Biot-Savart's law is an experimental law predicted by Biot and Savart. It is also called **Laplace's law. This law deals with the magnetic field or magnetic induction at a point due to a small current element** (*i.e.*, a small part of any conductor carrying current).

Let us consider a small element AB (of length dl) of the conductor XY carrying a current I. Let $\vec{r}$ be the position vector of the point P from the current element $I\,\vec{dl}$ * and θ be the angle between $\vec{dl}$ and $\vec{r}$, Fig. 4.6.

According to Biot-Savart's law, the magnitude of the magnetic field induction dB (also called magnetic flux density) at a point P due to current element depends upon the following factors as detailed below.

FIGURE 4.6



(*i*) $dB \propto I$ (*ii*) $dB \propto dl$ (*iii*) $dB \propto \sin\theta$ (*iv*) $dB \propto \dfrac{1}{r^2}$

Combining these factors, we get

$$dB \propto \frac{I\,dl\sin\theta}{r^2} \quad \text{or} \quad dB = K\frac{I\,dl\sin\theta}{r^2}$$

where K is a constant of proportionality. Its value depends on the system of units chosen for the measurement of the various quantities and also on the medium between point P and the current element. When ***there is free space between current element and point,*** then

In SI units, $K = \dfrac{\mu_0}{4\pi}$ and **In cgs system,** $K = 1$

where μ_0 is absolute magnetic permeability of free space

and $\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} $m^{-1} = 4\pi \times 10^{-7}$ T A^{-1} m $(\because 1\text{ T} = 1\text{ Wb m}^{-2})$

In SI units, $$dB = \frac{\mu_0}{4\pi} \times \frac{Idl\sin\theta}{r^2} \qquad ...(3)$$

In cgs system, $$dB = \frac{Idl\sin\theta}{r^2}$$

In vector form, we may write $|\vec{dB}| = \dfrac{\mu_0}{4\pi}\dfrac{I|\vec{dl}\times\vec{r}|}{r^3}$ or $\vec{dB} = \dfrac{\mu_0}{4\pi}\dfrac{I(\vec{dl}\times\vec{r})}{r^3}$ ...(4)

Direction of $\vec{dB}$. From (4), the direction of $\vec{dB}$ would obviously be the direction of the cross product vector, $\vec{dl}\times\vec{r}$. It is represented by the **Right handed screw rule** or **Right Hand Rule.** Here $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ and is directed inwards. If the point P is to the left of the current element, $\vec{dB}$ will be perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$, directed outwards.

Magnetic field induction at point P due to current through entire wire is

$$\vec{B} = \int \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} \qquad ...4(a)$$

or $$B = \int \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

*Current element is the product of current (I) and length of very small segment of current carrying wire (dl). The current element $I\,\vec{dl}$ is a vector. Its direction is tangent to the element and is acting in the direction of current flow in the conductor.

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Biot Savart's law in terms of current density J, states that

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{J}\times\vec{r}}{r^3}dV \qquad \left(\because J = \frac{I}{A} = \frac{I\,dl}{A\,dl} = \frac{I\,dl}{dV}\right)$$

where J = current density at any point on the current element, dV = volume of the element.

Biot Savart's law in terms of charge (q) and its velocity (v) is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} \qquad \left[\because I\,d\vec{l} = \frac{q}{dt}.d\vec{l} = q\frac{d\vec{l}}{dt} = q\vec{v}\right]$$

Biot Savart's law in terms of magnetising force or magnetising intensity (H)* of the magnetic field is :

In SI or mks system

$$d\vec{H} = \frac{d\vec{B}}{\mu_0} = \frac{1}{4\pi}\frac{I\,d\vec{l}\times\vec{r}}{r^3} = \frac{1}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{and} \quad dH = \frac{1}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

In cgs em units

$$d\vec{H} = \frac{I\,d\vec{l}\times\vec{r}}{r^3} \quad \text{and} \quad dH = \frac{I\,dl\sin\theta}{r^2}$$

Some important features of Biot Savart's law

1. Biot Savart's law is valid for a symmetrical current distribution.

2. Biot Savart's law is applicable only to very small length conductor carrying current.

3. This law can not be easily verified experimentally as the current carrying conductor of very small length can not be obtained practically.

4. This law is analogous to Coulomb's law in electrostatics.

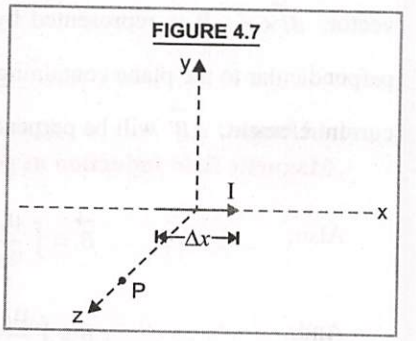
5. The direction of $d\vec{B}$ is perpendicular to both $I\,d\vec{l}$ and $\vec{r}$.

6. If $\theta = 0°$ or $180°$, *i.e.*, the point P lies on the axis of the linear conductor carrying current (or on the wire carrying current) then $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 0°}{r^2} = 0$, which is **minimum.**

It means *there is no magnetic field induction at any point on the thin linear current carrying conductor.*

7. If $\theta = 90°$ *i.e.* the point P lies at a perpendicular position *w.r.t.* current element, then

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$, which is **maximum.**



Sample Problem **An element, $\Delta\vec{l} = \Delta x\,\hat{i}$ is placed at the origin as shown in Fig. 4.7 and carries a current 5 A. Find out the magnetic field at a point P on the z-axis at a distance of 2·0 m due to the element Δx = 1 cm. Give also, the direction of the magnetic field produced.**

Sol. Here, $I\,d\vec{l} = I\,\Delta x\,\hat{i}$, where $I = 5$ A

$\Delta x = 1$ cm $= 10^{-2}$ m ; $r = 2{\cdot}0$ m ; $\vec{r} = 2{\cdot}0\,\hat{k}$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I(\Delta x\,\hat{i})\times(2{\cdot}0\,\hat{k})}{r^3} = 10^{-7}\times\frac{5\times10^{-2}\times2{\cdot}0}{2^3}(-\hat{j})$$

$$= \mathbf{1{\cdot}25\times10^{-9}\ T\ along\ -y\ direction}$$

*For detail of magnetising force or magnetising intensity refer to Art. 5.24.

4.5. SIMILARITIES AND DIS-SIMILARITIES BETWEEN THE BIOT-SAVART'S LAW FOR THE MAGNETIC FIELD AND COULOMB'S LAW FOR ELECTROSTATIC FIELD

Similarities

(*i*) *Both the laws for fields are long range,* since in both the laws, the field at a point varies inversely as the square of the distance from the source to point of observation.

(*ii*) *Both the fields obey superposition principle.*

(*iii*) The magnetic field is linearly related to its source, namely the current element $I\,d\vec{l}$, and the electric field is linearly related to its source, namely the electric charge q.

Dis-similarities

(*i*) The electrostatic field is produced by a scalar source namely, the electric charge q and the magnetic field is produced by a vector source, a current element $I\,d\vec{l}$.

(*ii*) The electrostatic field is acting along the displacement vector, *i.e.*, line joining the source and the field point. The magnetic field is acting perpendicular to the plane containing the current element $I\,d\vec{l}$ and displacement vector $\vec{r}$, *i.e.*, along the direction of $(I\,d\vec{l}\times\vec{r})$.

(*iii*) The electrostatic field at a point due to a charge is independent of angle θ, whereas the magnetic field at a point due to a current element is angle dependent. It means **Coulomb law is independent of angle whereas the Biot-Savart's law is angle dependent.**

4.6. MAGNETIC FIELD DUE TO A STRAIGHT WIRE CARRYING CURRENT

Consider a straight wire conductor XY lying in the plane of paper, carrying current I in the direction X to Y, Fig. 4.8(*a*). Let P be a point at a perpendicular distance a from the straight wire conductor. Clearly, $PC = a$. Let the conductor be made of small current elements. Consider one such small current element $I\,d\vec{l}$ of the straight wire conductor at O. Let $\vec{r}$ be the position vector of P w.r.t. current element and θ be the angle between $I\,d\vec{l}$ and $\vec{r}$. Let $CO = l$.

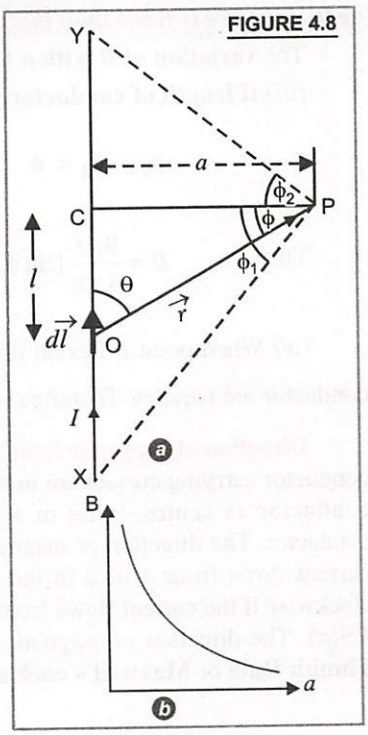


FIGURE 4.8

According to Biot-Savart's law, the magnetic field $d\vec{B}$ (*i.e.*, magnetic flux density or magnetic induction) at point P due to current element $I\,d\vec{l}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I\,d\vec{l}\times\vec{r}}{r^3}$$

or
$$dB = \frac{\mu_0}{4\pi}\times\frac{I\,dl\sin\theta}{r^2} \qquad ...(5)$$

In rt. angled ΔPOC, $\theta + \phi = 90°$ or $\theta = 90° - \phi$

$\therefore$ $\sin\theta = \sin(90° - \phi) = \cos\phi$ ...(6)

Also, $\cos\phi = \frac{a}{r}$ or $r = \frac{a}{\cos\phi}$ ...(7)

And, $\tan\phi = \frac{l}{a}$ or $l = a\tan\phi$

Differentiating it, we get $dl = a\sec^2\phi\,d\phi$ ...(8)

Putting the values in (5) from (6), (7) and (8), we get

$$dB = \frac{\mu_0}{4\pi}\frac{I\,(a\sec^2\phi\,d\phi)\cos\phi}{\left(\frac{a^2}{\cos^2\phi}\right)} = \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\,d\phi \qquad ...(9)$$

The direction of $\vec{dB}$, **according to right hand thumb rule,** will be perpendicular to the plane of paper directed inwards. As all the current elements of the conductor will also produce magnetic field in the same direction, therefore, the total magnetic field at point *P* due to current through the whole straight conductor *XY* can be obtained by integrating Eq. (9) within the limits $-\phi_1$ and $+\phi_2$. Thus

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0}{4\pi}\frac{I}{a}\int_{-\phi_1}^{\phi_2}\cos\phi\,d\phi = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin\phi]_{-\phi_1}^{\phi_2} = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin\phi_2 - \sin(-\phi_1)]$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2) \qquad ...(10)$$

Special cases. (*i*) ***When the conductor XY is of infinite length and the point P lies near the centre of the conductor then*** $\phi_1 = \phi_2 = 90°$

So, $$B = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 90° + \sin 90°] = \frac{\mu_0}{4\pi}\frac{2I}{a}$$ or $$B = \frac{\mu_0 I}{2\pi a} \qquad ...(11)$$

(*ii*) **When the conductor XY is of infinite length but the point *P* lies near the end *Y* (or *X*).** then $\phi_1 = 90°$ and $\phi_2 = 0°$.

So, $$B = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 90° + \sin 0°] = \frac{\mu_0}{4\pi}\frac{I}{a}$$ *i.e.*, $$B = \frac{\mu_0 I}{4\pi a} \qquad ...(11a)$$

Thus, we note that the magnetic field due to an infinite long linear conductor carrying current near its centre is twice than that near one of its ends.

The **variation of *B* with *a*** for linear wire carrying current is shown in Fig. 4.8(b).

(*iii*) **If length of conductor is finite, say L and point *P* lies on right bisector of conductor,** then

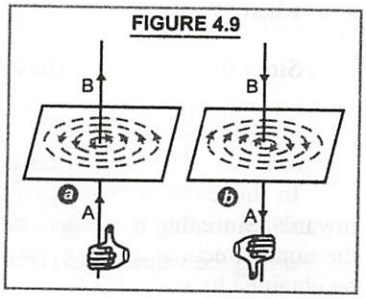
$$\phi_1 = \phi_2 = \phi \quad \text{and} \quad \sin\phi = \frac{L/2}{\sqrt{a^2 + (L/2)^2}} = \frac{L}{\sqrt{4a^2 + L^2}}$$

Then, $$B = \frac{\mu_0 I}{4\pi a}[\sin\phi + \sin\phi] = \frac{\mu_0}{4\pi}\frac{2I}{a}\sin\phi = \frac{\mu_0}{4\pi}\frac{2I}{a}\frac{L}{\sqrt{4a^2 + L^2}}$$

(*iv*) When point *P* lies on the wire conductor, then $\vec{dl}$ and $\vec{r}$ for each element of the straight wire conductor are parallel. Therefore $\vec{dl} \times \vec{r} = 0$. So the magnetic field induction at $P = 0$.

Direction of magnetic field. The magnetic field lines due to straight conductor carrying current are in the form of concentric circles with the conductor as centre, lying in a plane perpendicular to the straight conductor. The direction of magnetic field lines is anticlockwise, if the current flows from *A* to *B* in the straight conductor Fig. 4.9(*a*) and is clockwise if the current flows from *B* to *A* in the straight conductor, Fig. 4.9(*b*). The direction of magnetic field lines is given by **Right Hand Thumb Rule** or **Maxwell's cork screw rule.**

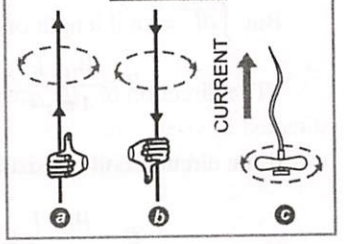
FIGURE 4.9



Right hand thumb rule. ***According to this rule, if we imagine the linear wire conductor to be held in the grip of the right hand so that the thumb points in the direction of current, then the curvature of the fingers around the conductor will represent the direction of magnetic field lines, Fig. 4.10(a) and (b).***

Maxwell's cork screw rule. ***According to this rule, if we imagine a right handed screw placed along the current carrying linear wire conductor, rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force. This rule is illustrated in Fig. 4.10(c)***

FIGURE 4.10



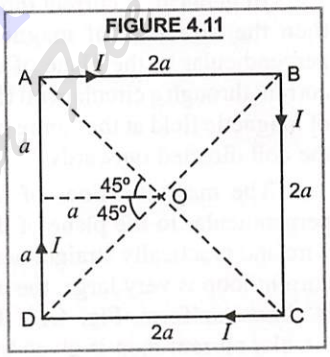
Sample Problem 1 **An infinitely long straight conductor carries a current of 100 A. At what distance from the conductor is the magnetic field caused by the current equal to $0{\cdot}5 \times 10^{-4}$ T ?**

Sol. Here, $I = 100$ A ; $a = ?$; $B = 0{\cdot}5 \times 10^{-4}$ T

As, $B = \dfrac{\mu_0}{4\pi}\dfrac{2I}{a}$ or $a = \dfrac{\mu_0}{4\pi}\dfrac{2I}{B} = \dfrac{10^{-7} \times 2 \times 100}{0{\cdot}5 \times 10^{-4}} = \mathbf{0{\cdot}4\,m}$

Sample Problem 2 **Find an expression for the magnetic field induction at the centre of a coil bent in the form of a square of side 2 *a*, carrying current I, Fig. 4.11.**

FIGURE 4.11



Sol. Here, $\phi_1 = 45°$; $\phi_2 = 45°$

Total magnetic field induction at O due to current through all the sides of a square is four times that due to current through one arm of the square and is given by

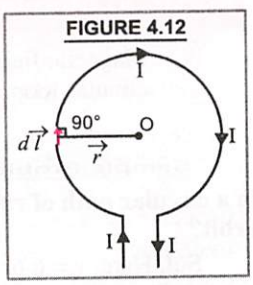
$$B = 4 \times \frac{\mu_0}{4\pi}\frac{I}{a}(\sin 45° + \sin 45°) = 4 \times \frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\,\mu_0 I}{\pi a}$$

4.7. MAGNETIC FIELD AT THE CENTRE OF THE CIRCULAR COIL CARRYING CURRENT

Consider a circular coil of radius r with centre O, lying with its plane in the plane of paper. Let I be the current flowing in the circular coil in the direction shown, Fig. 4.12. Suppose the circular coil is made of a large number of current elements each of length dl.

According to Biot-Savart's law, the magnetic field at the centre of the circular coil due to the current element $I\,\vec{dl}$ is given by

FIGURE 4.12



$$\vec{dB} = \frac{\mu_0}{4\pi} I\left(\frac{\vec{dl} \times \vec{r}}{r^3}\right) \quad \text{or} \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

where $\vec{r}$ is the position vector of point O from the current element.

Since the angle between $\vec{dl}$ and $\vec{r}$ is 90° (*i.e.*, $\theta = 90°$), therefore,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90°}{r^2} \quad \text{or} \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} \qquad \text{...(12)}$$

In this case, the direction of $\vec{dB}$ is perpendicular to the plane of the current loop and is directed inwards. Since the current through all the elements of the circular coil will contribute to the magnetic field in the same direction, therefore, the total magnetic field at point O due to current in the whole circular coil can be obtained by integrating Eq. (12). Thus

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl$$

But $\int dl$ = total length of the circular coil = circumference of the current loop = $2\pi r$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}.2\pi r = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$$

If the circular coil consists of n turns, then $\int dl = 2\pi r n$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}2\pi r n = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r} = \frac{\mu_0}{4\pi}\frac{I}{r}\times 2\pi n \qquad ...(13)$$

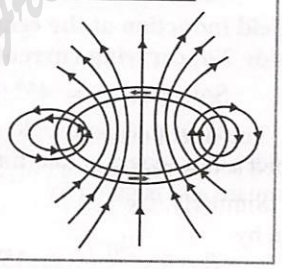
i.e., $B = \frac{\mu_0}{4\pi}\frac{I}{r}\times$ angle subtended by coil at the centre.

Direction of $\vec{B}$

In the above case, the direction of magnetic field induction $\vec{B}$ (*i.e.*, magnetic flux density) at point O, due to current through circular coil is perpendicular to the plane of the circular coil directed inwards.

In general, if current through a circular coil flows in clockwise direction, then the direction of magnetic field at the centre of the circular coil is perpendicular to the plane of the coil, directed inwards. On the other hand, if current through a circular coil flows in anticlockwise direction, then the direction of magnetic field at the centre of the circular coil is perpendicular to the plane of the coil directed outwards.

FIGURE 4.13



The magnetic lines of force due to circular coil carrying current are perpendicular to the plane of the wire loop and are concentric circles near the wire and practically straight near the centre of the wire loop. If the radius of the current loop is very large, the magnetic field near the centre of the current loop is almost uniform, Fig. 4.13. The direction of magnetic field at the centre of circular current loop is given by **Right hand rule.**

Right Hand Rule. ***According to this rule, if we hold the thumb of right hand mutually perpendicular to the grip of the fingers such that the curvature of the fingers represents the direction of current in the wire loop, then the thumb of the right hand will point in the direction of magnetic field near the centre of the current loop.***

Retain in Memory

Magnetic field lines around a current carrying wire are crowded up when the wire is bent into a circular loop.

Sample Problem **In hydrogen atom, the electron is making $6{\cdot}6 \times 10^{15}$ revolutions per second in a circular path of radius 0·53 A°. What is the magnetic field induction produced at the centre of the orbit?**

Sol. Here, $v = 6{\cdot}6 \times 10^{15}$ rps ; $r = 0{\cdot}53 \times 10^{-10}$ m ;

Current due to orbital motion of electron is ;

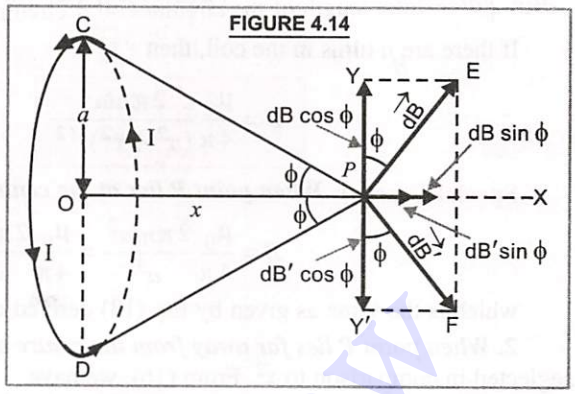
$$I = \frac{e}{T} = ev = (1{\cdot}6 \times 10^{-19}) \times (6{\cdot}6 \times 10^{15}) = 1{\cdot}056 \times 10^{-3}\ \text{A}$$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r} = \frac{10^{-7} \times 2 \times (22/7) \times 1 \times (1{\cdot}056 \times 10^{-3})}{(0{\cdot}53 \times 10^{-10})} = \mathbf{12{\cdot}5\ Wb/m^2}$$

4.8. MAGNETIC FIELD AT A POINT ON THE AXIS OF A CIRCULAR COIL CARRYING CURRENT

Consider a circular coil of radius a with centre O. Let the plane of the coil be perpendicular to the plane of the paper and current I be flowing in the coil in the direction shown in Fig. 4.14. Suppose P is any point on the axis of the circular coil at a distance x from its centre O. Clearly, $OP = x$.

FIGURE 4.14



Consider two small elements of the coil each of length dl, at C and D which are situated at diametrically opposite edges. Then

$$PC = PD = r = \sqrt{a^2 + x^2}$$

Let $\angle CPO = \phi = \angle DPO$

According to Biot Savart's law, the magnitude of magnetic field induction (*i.e.* magnetic flux density) at P due to current element $I\,\vec{dl}$ at C is given by

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90°}{r^2} \qquad (\because\ a \text{ is small, therefore } \theta = 90°)$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad \text{...(14)}$$

The direction of $\vec{dB}$ is in the plane perpendicular to $\vec{dl}$ and $\vec{r}$, and is directed as given by right handed screw rule, *i.e.* acting along $PE \perp$ to CP.

Similarly, the magnitude of magnetic field induction at P due to current element of length dl at D is given by

$$dB' = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin 90°}{r^2} = \frac{\mu_0\,I\,dl}{4\pi(a^2 + x^2)} \qquad \text{...(15)}$$

Its direction is along $PF \perp$ to DP.

From (14) and (15), $$dB = dB' = \frac{\mu_0\,I\,dl}{4\pi(a^2 + x^2)}$$

Here, $\angle YPE = \angle Y'PF = \angle CPO = \angle DPO = \phi$

Resolving $\vec{dB}$ and $\vec{dB'}$ into two rectangular components, we have (*i*) $dB\cos\phi$ acts along PY and $dB\sin\phi$ acts along PX, (*ii*) $dB'\cos\phi$ acts along PY' and $dB'\sin\phi$ acts along PX.

Since the components of the magnetic field induction acting along PY and PY' are equal and opposite, they cancel out each other and the components of the magnetic field acting along PX (*i.e.* along the axis of the coil) being in the same direction are added up. The same is true for all the diametrically opposite pairs of current elements of the circular coil.

Thus total magnetic field induction (*i.e.* magnetic flux density) at P due to current through the whole circular coil is given by

$$B = \int dB\sin\phi = \int \frac{\mu_0\,I\,dl\sin\phi}{4\pi(a^2 + x^2)} = \frac{\mu_0\,I}{4\pi(a^2 + x^2)}\sin\phi\int dl$$

Since, $$\sin\phi = \frac{a}{\sqrt{a^2 + x^2}} \quad \text{and} \quad \int dl = 2\pi a$$

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$$\therefore \quad B = \frac{\mu_0 I}{4\pi(a^2+x^2)}\frac{a}{\sqrt{a^2+x^2}}2\pi a = \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{(a^2+x^2)^{3/2}} \text{ along PX}$$

If there are n turns in the coil, then $\int dl = 2\pi a n$

$$\therefore \quad \boxed{B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}} \qquad ...(16)$$

Special Cases. 1. ***When point P lies at the centre of the circular coil,*** then $x = 0$. From (16), we have

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{a^3} = \frac{\mu_0}{4\pi}\frac{2\pi n I}{a}$$

which is the same as given by Eq. (13) derived earlier.

2. ***When point P lies far away from the centre of the coil,*** then $x >> a$. Now $a^2 + x^2 = x^2$ as a^2 can be neglected in comparison to x^2. From (16), we have

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2nI\,A}{x^3} \qquad ...(17)$$

(where $\pi a^2 = A$ = Area of current loop.)

or $$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3} \qquad ...(18)$$

where, $n\,I\,A = M$ = ***magnetic dipole moment of the current loop.***

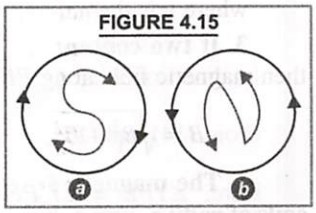
Thus the *current loop can be regarded as a magnetic dipole* which produces its magnetic field and ***magnetic dipole moment of the current loop is equal to the product of ampere turns and area of current loop.***

S.I. unit of magnetic dipole moment is A-m^2.

The polarity of the magnetic dipole due to current loop can be decided as follows :

Look at one face of the coil. If the direction of current through the coil is clockwise, then that face has South polarity. Fig. 4.15(a). If the direction of current is anticlockwise, then that face has North polarity. Fig. 4.15(b).

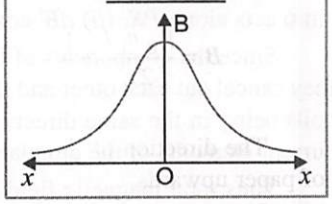
FIGURE 4.15



3. ***When the point lies on the axis of the coil at a distance equal to the radius of the coil i.e. $x = a$,***

then, $$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+a^2)^{3/2}} = \frac{\mu_0 n I}{2^{5/2} a}$$

FIGURE 4.16



4. The variation of magnetic field induction (B) and distance (x) of a point on the axis of a circular coil carrying current is represented by graph as shown in Fig. 4.16.

Sample Problem **An electric current is flowing in a circular coil of radius a. At what distance from the centre on the axis of the coil will the magnetic field be 1/8th of its value at the centre ?**

Sol. Magnetic field induction at a point on the axis at a distance x from the centre of the circular coil carrying current is, $$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}$$

Magnetic field induction at the centre of the circular coil carrying current is, $B_2 = \frac{\mu_0}{4\pi}\frac{2\pi n I}{a}$

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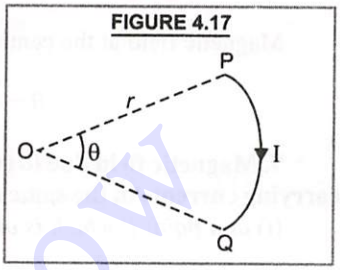
As per question ; $B_1 = \frac{B_2}{8}$ so $\frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+x^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi n I}{a}\times\frac{1}{8}$ or $\frac{a^2}{(a^2+x^2)^{3/2}} = \frac{1}{8a}$

or $8a^3 = (a^2+x^2)^{3/2}$ or $2a = (a^2+x^2)^{1/2}$ or $4a^2 = a^2+x^2$ or $x = \sqrt{3}a$

4.9. MAGNETIC FIELD DUE TO VARIOUS CURRENT CARRYING CONFIGURATIONS

1. The magnetic field at the centre *O* of the circular arc of radius *r* lying in the plane of paper as shown in Fig. 4.17 is

FIGURE 4.17

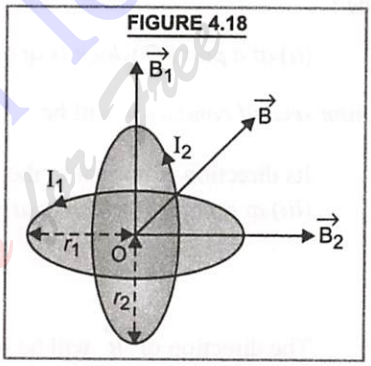


$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta$$

Its direction (according to Right Hand rule) is normal to the plane of paper downwards. If the direction of current in the arc is from *Q* to *P*, then the direction of $\vec{B}$ is normal to the plane of paper upwards.

2. Magnetic field at the centre *O* of two concentric coils inclined at 90° Fig. 4.18 is

FIGURE 4.18



$$B = \sqrt{B_1^2 + B_2^2}$$

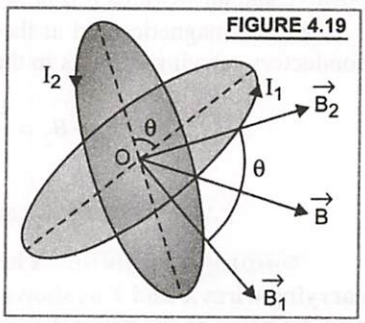
If $I_1 = I_2 = I$ and $r_1 = r_2 = r$

Then $$B = \sqrt{2}\,B_1 = \sqrt{2}\frac{\mu_0 n I}{2r}$$

where *n* is the number of turns in each coil.

3. If two concentric coils are inclined at an angle θ, Fig. 4.19, then magnetic field at *O* is

FIGURE 4.19



$$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$$

4. The magnetic field at the common centre of two concentric coils of radii r_1 and r_2 having turns n_1 and n_2 in which the same current *I* is flowing in anticlockwise direction, Fig. 4.20(*a*), is

$$B = \frac{\mu_0 n_1 I}{2r_1} + \frac{\mu_0 n_2 I}{2r_2} = \frac{\mu_0 I}{2}\left[\frac{n_1}{r_1} + \frac{n_2}{r_2}\right]$$

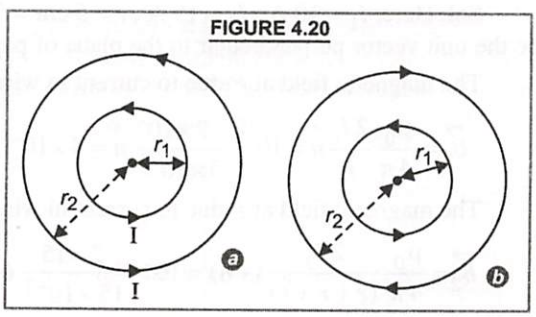
The direction of $\vec{B}$ will be normal to the plane of paper upwards.

If current is flowing in two opposite directions, Fig. 4.20(*b*), then

$$B = \frac{\mu_0 I}{2}\left[\frac{n_1}{r_1} - \frac{n_2}{r_2}\right]$$

Its direction is normal to the plane of paper upwards.

FIGURE 4.20



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5. If a charged ring having charge q, of radius r rotates with a frequency v then current in the ring is, $I = qv$.

Magnetic field at the centre of the ring is, $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}qv$

6. If an electron of charge e is revolving around the nucleus in a circular path of radius r with speed v, then current $I = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$

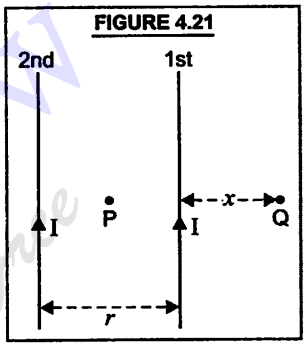
Magnetic field at the centre of that circle, *i.e.*, at the nucleus is,

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\left(\frac{ev}{2\pi r}\right) = \frac{\mu_0 ev}{4\pi r^2}$$

7. Magnetic field due to two parallel very long thin linear conductors carrying currents in the same direction, separated by distance r, Fig. 4.21.

(*i*) *at a point P which is at a distance r/2 from both the conductors* will be : $B = \frac{\mu_0}{4\pi}\frac{2I}{r} - \frac{\mu_0}{4\pi}\frac{2I}{r} = 0$

FIGURE 4.21



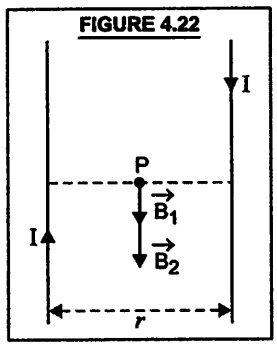
(*ii*) *at a point Q which is at a distance x from first conductor and* $(r + x)$ *from second conductor* will be : $B = \frac{\mu_0}{4\pi}2I\left[\frac{1}{x} + \frac{1}{r+x}\right]$

Its direction is normal to the plane of paper downwards.

(*iii*) *at a point P which is at a distance x from first and* $(r - x)$ *from second conductor* will be :

$$B = \frac{\mu_0}{4\pi}2I\left[\frac{1}{x} - \frac{1}{r-x}\right]$$

FIGURE 4.22



The direction of $\vec{B}$ will be normal to the plane of paper upwards if B is positive and downwards if B is negative.

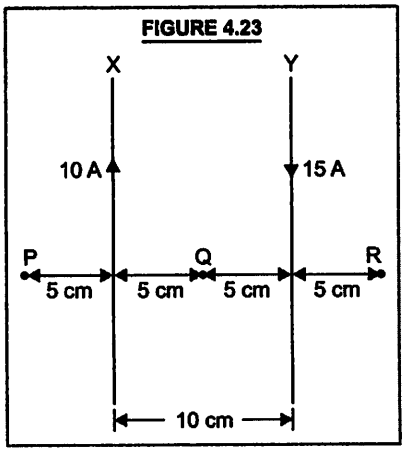
8. The magnetic field at the centre of separation P of two long linear conductors carrying currents in the opposite directions, Fig. 4.22, is :

$$B = B_1 + B_2 = \frac{\mu_0}{4\pi}\frac{2I}{(r/2)} + \frac{\mu_0}{4\pi}\frac{2I}{(r/2)} = \frac{\mu_0}{4\pi}\frac{8I}{r}$$

[$\because$ Direction of $\vec{B_1}$ and $\vec{B_2}$ is perpendicular to the plane of linear conductor directed inwards]

Sample Problem **There are two parallel current carrying wires X and Y as shown in Fig. 4.23. Find the magnitude and direction of the magnetic field at points, P, Q and R.**

FIGURE 4.23



Sol. Here, $I_1 = 10$ A ; $I_2 = 15$ A ; $r = 5$ cm $= 5 \times 10^{-2}$ m. Let $\hat{n}$ be the unit vector perpendicular to the plane of paper unwards.

The magnetic field at P due to current in wire X is

$$\vec{B_1} = \frac{\mu_0}{4\pi}\frac{2I_1}{r}\hat{n} = 10^{-7}\,\frac{2\times10}{5\times10^{-2}}\,\hat{n} = 4\times10^{-5}\,\hat{n}\,T$$

The magnetic field at P due to current in wire Y is

$$\vec{B_2} = \frac{\mu_0}{4\pi}\frac{2I_2}{(r+r+r)}(-\hat{n}) = 10^{-7}\,\frac{2\times15}{15\times10^{-2}}(-\hat{n}) = -2\times10^{-5}\hat{n}\,T$$

The resultant magnetic field at P is

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 = (4\times10^{-5})\hat{n} - (2\times10^{-5})\hat{n} = 2\times10^{-5}\,T\,\hat{n}$$

$$= \mathbf{2 \times 10^{-5}\ T\ acting\ normally\ outwards.}$$

At point Q, both $\vec{B}_1$ and $\vec{B}_2$ will be acting normally inwards. So

$$\vec{B}_Q = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2\times10}{5\times10^{-2}}(-\hat{n}) + \frac{\mu_0}{4\pi}\frac{2\times15}{5\times10^{-2}}(-\hat{n})$$

$$= \left[10^{-7}\times\frac{20}{5\times10^{-2}} + 10^{-7}\times\frac{30}{5\times10^{-2}}\right](-\hat{n}) = (4\times10^{-5} + 6\times10^{-5})(-\hat{n})$$

$$= \mathbf{10^{-4}\ T\ acting\ normally\ inwards.}$$

At point R, $\vec{B}_1$ will be acting normally inwards and $\vec{B}_2$ will be acting normally outwards. So,

$$\vec{B}_R = \vec{B}_1 + \vec{B}_2 = 10^{-7}\times\frac{2\times10}{(5+5+5)\times10^{-2}}(-\hat{n}) + 10^{-7}\times\frac{2\times20}{(5\times10^{-2})}(\hat{n})$$

$$= \left(-\frac{4}{3}\times10^{-5} + 8\times10^{-5}\right)(\hat{n}) = \mathbf{\frac{20}{3}\times10^{-5}\ T;\ acting\ normally\ outwards}$$

4.10. AMPERE'S CIRCUITAL LAW

In electrostatics, we have studied Gauss's Law as an alternative form of Coulomb's Law. Exactly in the same way, Ampere Circuital Law is an alternative form of Biot Savart Law in magnetostatics. Though Ampere's circuital Law is quite general, we use this law to calculate magnetic field in symmetrical situations.

Consider an open surface with a boundary C, Fig. 4.24, and the current I is passing through the surface. Let the boundary C be made of large number of small line elements, each of length dl. The direction of $\vec{dl}$ of small line element under study is acting tangentially to its length dl. Let B_t be the tangential component of the magnetic field induction at this element. Then $\vec{B}_t$ and $\vec{dl}$ are acting in the same direction, angle between them is zero. We take the product of B_t and dl for that element. Then

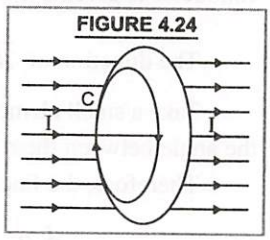


FIGURE 4.24

$$B_t\,dl = \vec{B}.\vec{dl}$$

If length dl is very small and products for all elements of closed boundary are added together, then the sum tends to be an integral around the closed path or loop (*i.e.*, $\oint$). Therefore, sum of $\vec{B}.\vec{dl}$ over all elements on a closed path $= \oint \vec{B}.\vec{dl}$ = Line integral of $\vec{B}$ around the closed path or loop whose boundary coincides with the closed path. *Ampere's circuital Law states that this integral (i.e., line integral of $\vec{B}$ around the closed path) is equal to μ_0 times the total current passing through the surface*

i.e.,

$$\oint \vec{B}.\vec{dl} = \mu_0 I \qquad \text{...(19)}$$

where I is the total current threading the closed path or loop and μ_0 is the absolute permeability of the space. Thus,

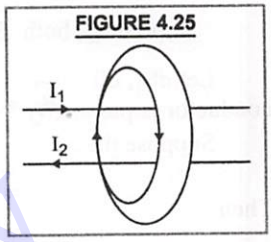
The relation (19) involves a **sign convention ;** given by the **Right Hand Rule.** According to this rule, when fingers of right hand are curled in the sense, the boundary is traversed in the loop integral $\oint \vec{B}.d\vec{l}$, then the direction of the thumb gives the sense in which current I is regarded as positive.

According to sign convention, for the closed path shown in Fig. 4.25, I_1 is positive and I_2 is negative. Then, according to Ampere's circuital law

$$\oint \vec{B}.d\vec{l} = \mu_0 (I_1 - I_2) = \mu_0 I_e$$

where I_e is the total current enclosed by the loop or closed path.

FIGURE 4.25



The relation (19) is independent of the size and shape of the closed path or loop enclosing the current.

Thus Ampere's circuital law states that the line integral of the magnetic field $\vec{B}$ around any closed path in free space is equal to absolute permeability (μ_0) times the net current (I) passing through the surface enclosed by the closed path i.e.,

$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

Proof of Ampere's Circuital Law taking a circular closed path. Consider a long thin straight conductor XY lying in the plane of paper, carrying current I flowing in the direction shown in Fig. 4.26(a). The magnetic field produced around the conductor is shown by concentric circles perpendicular to the plane of paper, with their centre lying on the conductor.

The magnitude of magnetic field induction produced at a point P at a perpendicular distance r from the conductor is given by, $B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$

The direction of $\vec{B}$ is along the tangent to the magnetic field line at that point.

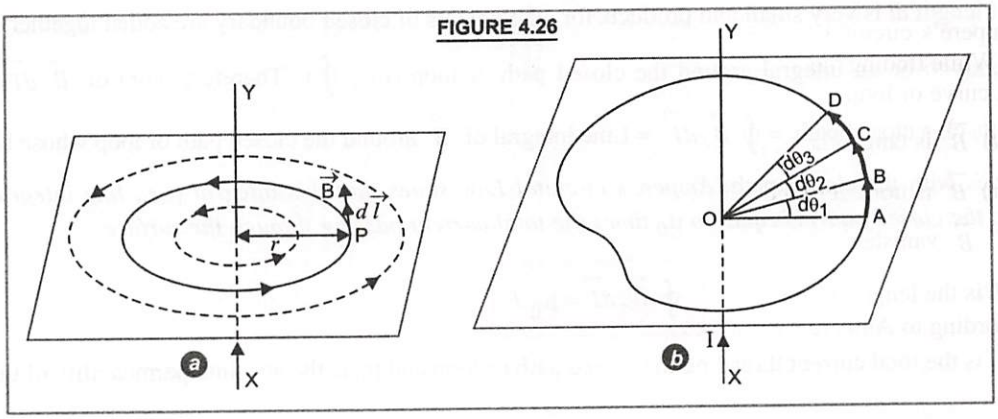
Take a small element of length dl of the closed path at P. The direction of $\vec{B}$ and $d\vec{l}$ is the same, *i.e.*, the angle between them is zero.

Therefore, the line integral of $\vec{B}$ around the closed circular path of radius r is

$$\oint \vec{B}.d\vec{l} = \oint B\,dl\cos 0° = B\oint dl = \frac{\mu_0 I}{2\pi r} \times 2\pi r = \mu_0 I$$

where $\oint dl = 2\pi r$ = circumference of the circular path. This is Ampere circuital law.

FIGURE 4.26



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Proof of Ampere's Circuital law taking any arbitrary Closed Path

Consider any arbitrary closed path perpendicular to the plane of paper around a long straight conductor XY carrying current I from X to Y, lying in the plane of paper, Fig. 4.26(b).

Let the closed path be made of large number of small elements, where

$$\overrightarrow{AB} = d\vec{l}_1 \; ; \; \overrightarrow{BC} = d\vec{l}_2 \; ; \; \overrightarrow{CD} = d\vec{l}_3 .....$$

Let $d\theta_1, d\theta_2, d\theta_3$, ... be the angles subtended by the various small elements at point O through which conductor is passing. Then $d\theta_1 + d\theta_2 + d\theta_3 + ... = 2\pi$

Suppose these small elements AB, BC, CD, ... are small circular arcs of radii r_1, r_2, r_3, ... respectively.

Then $$d\theta_1 = \frac{dl_1}{r_1}; \; d\theta_2 = \frac{dl_2}{r_2}; \; d\theta_3 = \frac{dl_3}{r_3}; .....$$

If $\vec{B}_1, \vec{B}_2, \vec{B}_3$ ..... are the magnetic field induction at a point along the small elements $d\vec{l}_1, d\vec{l}_2, d\vec{l}_3$,.....,

then $$B_1 = \frac{\mu_0}{4\pi}\frac{2I}{r_1}; \; B_2 = \frac{\mu_0}{4\pi}\frac{2I}{r_2}; \; B_3 = \frac{\mu_0}{4\pi}\frac{2I}{r_3}; .....$$

In case of each element, the magnetic field induction $\vec{B}$ and current element vector $d\vec{l}$ are in the same direction.

Line integral of $\vec{B}$ around the arbitrary closed path is

$$\oint \vec{B}.d\vec{l} = \vec{B}_1.d\vec{l}_1 + \vec{B}_2.d\vec{l}_2 + \vec{B}_3.d\vec{l}_3 + ... = B_1(dl_1) + B_2(dl_2) + B_3(dl_3) + ...$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{r_1}dl_1 + \frac{\mu_0}{4\pi}\frac{2I}{r_2}dl_2 + \frac{\mu_0}{4\pi}\frac{2I}{r_3}dl_3 + ... = \frac{\mu_0 2I}{4\pi}\left[\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + ...\right]$$

$$= \frac{\mu_0}{4\pi}2I[d\theta_1 + d\theta_2 + d\theta_3 + ...] = \frac{\mu_0}{4\pi}2I \times 2\pi = \mu_0 I$$, which proves Ampere circuital law.

Obviously, this result is independent of r, *i.e.*, size of the closed path.

Note. If the point P lies inside the conductor carrying current which is at some distance from the axis of conductor, then according to Ampere circuital law $\oint \vec{B}.d\vec{l} = \mu_0 \mu_r I$

where μ_r is the relative magnetic permeability of the material of conductor. For its detail, refer to Art. 5.24.

4.11. APPLICATIONS OF AMPERE'S CIRCUITAL LAW

Ampere's circuital law is a simple tool for evaluating easily the magnetic field at a point due to a current. While finding the magnetic field due to a current, using Ampere circuital law, we shall have to choose a closed curve or loop (called an amperian loop) such that at each point of the loop either.

(*i*) $\vec{B}$ is tangential to the loop or B is non-zero constant or

(*ii*) $\vec{B}$ is normal to the loop or

(*iii*) $\vec{B}$ vanishes.

If L is the length of the loop for which $\vec{B}$ is tangential and I_e is the total current enclosed by the loop, then according to Ampere's circuital law

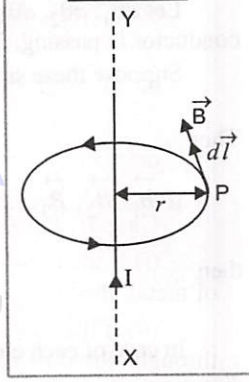
$$\oint \vec{B}.d\vec{l} = BL = \mu_0 I_e$$

4.12. MAGNETIC FIELD DUE TO INFINITE LONG STRAIGHT WIRE CARRYING CURRENT

Consider an infinite long straight wire lying in the plane of paper. Let I be the current flowing through it from X to Y. A magnetic field is produced which has the same magnitude at all points that are at the same distance from the wire, *i.e.* the magnetic field has cylindrical symmetry around the wire.

Let P be a point at a perpendicular distance r from the straight wire and $\vec{B}$ be the magnetic field at P. It will be acting tangentially to the magnetic field line passing through P. Consider an amperian loop as a circle of radius r, perpendicular to the plane of paper with centre on wire such that point P lies on the loop, Fig. 4.27. The magnitude of magnetic field is same at all points on this loop. The magnetic field $\vec{B}$ at P will be tangential to the circumference of the circular loop.

FIGURE 4.27



We shall integrate the amperian path anticlockwise. Then $\vec{B}$ and $d\vec{l}$ are acting in the same direction. The line integral of $\vec{B}$ around the closed loop is

$$\oint \vec{B}.d\vec{l} = \oint B\,dl\cos 0° = B\oint dl = B\,2\pi r$$

As per sign convention, I is positive,

Using Ampere's circuital law,

$$\oint \vec{B}.d\vec{l} = \mu_0 I \qquad \text{or} \qquad B\,2\pi r = \mu_0 I$$

or
$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi}\frac{2I}{r} \qquad ...(21$$

The above result as given by (21) is of great importance from several points of view.

(*i*) It tells that the magnitude of magnetic field at every point on a circle of radius r with wire along th axis at the centre is same. It means the **magnetic field due to current through infinite straight wire has cylindrical symmetry.**

(*ii*) The direction of magnetic field at every point on the circle is tangential to it and lines of constan magnitude of magnetic field form concentric circles. **These circular lines are called magnetic field lines** That is why we say that the magnetic field lines have no starting point or end point but form a closed pat unlike the electrostatic field in which the electrostatic field lines have a starting point at positive charge an end point on negative charge.

(*iii*) The expression for the magnetic field at a point due to current through a straight wire provides theoretical support to Oersted's experiment.

(*iv*) Though the wire carrying current is infinite, the magnetic field due to it at non-zero distance is no infinite. Since $B \propto 1/r$, therefore, the magnitude of magnetic field increases as distance of point from the wir decreases.

4.13. MAGNETIC FIELD DUE TO CURRENT THROUGH A VERY LONG CIRCULAR CYLINDER

Consider an infinite long cylinder of radius R with axis XY. Let I be the current passing through th cylinder. A magnetic field is set up due to current through the cylinder in the form of circular magnetic line of force, with their centres lying on the axis of cylinder. These lines of force are perpendicular to the length c cylinder.

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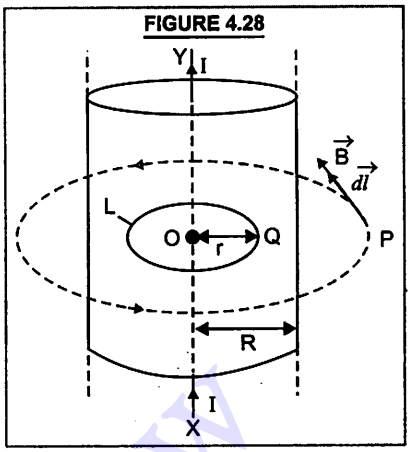
Case I. Point P is lying outside the cylinder. Let r be the perpendicular distance of point P from the axis of cylinder, where $r > R$. Let $\vec{B}$ be the magnetic field induction at P. It is acting tangential to the magnetic line of force at P directed into the paper. Here $\vec{B}$ and $\vec{dl}$ are acting in the same direction.



Applying Ampere circuital law we have

$$\oint \vec{B}.\vec{dl} = \mu_0 I \quad \text{or} \quad \oint B\,dl\cos 0° = \mu_0 I$$

or $$\oint B\,dl = \mu_0 I \quad \text{or} \quad B\,2\pi r = \mu_0 I$$

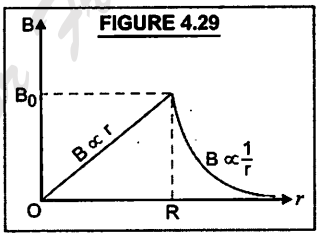
or $$B = \frac{\mu_0 I}{2\pi r}, \quad i.e., \; B \propto 1/r$$

Case II. Point Q is lying inside cylinder. Here $r < R$. We may have two possibilities.

(*i*) If the current is only along the surface of cylinder which is so if the conductor is a cylindrical sheet of metal, then current through the closed path L is zero. Using Ampere circutal law, we have, $B = 0$

(*ii*) If the current is uniformly distributed throughout the cross-section of the conductor, then the current through closed path L is given by

$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I r^2}{R^2}$$



Applying Ampere's circuital law, we have

$$\oint \vec{B}.\vec{dl} = \mu_0 \mu_r I'$$

or $$2\pi r B = \mu_0 \mu_r I' = \frac{\mu_0 \mu_r I r^2}{R^2} \quad \text{or} \quad B = \frac{\mu_0 \mu_r I r}{2\pi R^2} \quad i.e., \; B \propto r.$$

If we plot a graph between magnetic field induction B and distance from the axis of cylinder for a current flowing through a solid cylinder, we get a curve of the type, as shown in Fig. 4.29.

Here we note that the magnetic field induction is maximum for a point on the surface of solid cylinder carrying current and is zero for a point on the axis of cylinder.

Sample Problem **A long straight solid metal rod of radius 4 cm carries a current 2 A, uniformly distributed over its circular cross-section. Find the magnetic field induction at a distance 3 cm from the axis of the wire.**

Sol. The point P is lying inside the rod at a perpendicular distance r (= 3 cm) from the axis of the wire. Draw a circular closed path of radius r with centre on the axis of rod such that the point P lies on this closed path.

Current enclosed by the closed path $I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I r^2}{R^2}$

The magnetic field produced due to current flowing in the rod at every point over the closed path is tangential to it and is equal in magnitude at all the points of the path.

Line integral of $\vec{B}$ over the closed path is $\oint \vec{B}.\vec{dl} = \mu_0 I'$ or $B\,2\pi r = \mu_0 \frac{I r^2}{R^2}$

or $$B = \frac{\mu_0 I r^2}{2\pi r R^2} = \frac{\mu_0}{4\pi}\frac{2 I r}{R^2} = \frac{10^{-7} \times 2 \times 2 \times 3 \times 10^{-2}}{(4 \times 10^{-2})^2} = \mathbf{7{\cdot}5 \times 10^{-6}\ T}$$

Note. As the point lies inside the conductor, the magnetic field produced at that point will be μ_r times the above value obtained, where μ_r is the relative permeability of the material of the conductor.

4.14. THE SOLENOID

A solenoid is a piece of equipment which is used for generating magnetic field. A long solenoid is one whose length is very large, compared to its radius.

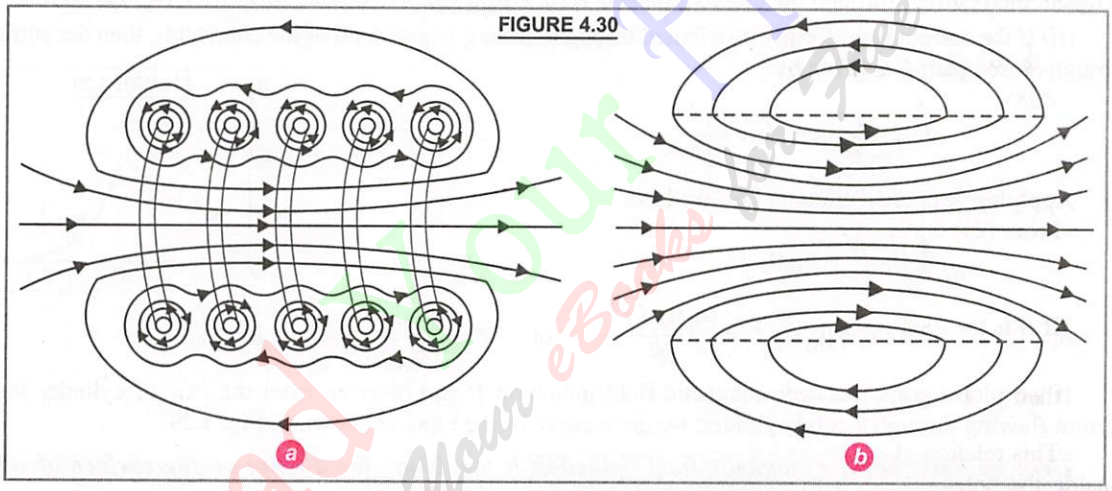
A solenoid consists of a long metallic insulated wire wound in the form of a helix, where the neighbouring, turns are closely spaced. Each turn can be regarded as a circular loop.

A solenoid consists of an insulating long wire closely wound in the form of a helix. Its length is very large as compared to its diameter.

Magnetic field due to a solenoid

When current is passed through the solenoid, then each turn of the solenoid can be regarded as a circular loop carrying current and thus will be producing a magnetic field. The total magnetic field is the vector sum of the magnetic fields due to currents through all the turns in the solenoid. Fig. 4.30(a) represents the magnetic field lines due to a section of the solenoid in an enlarged manner.

FIGURE 4.30

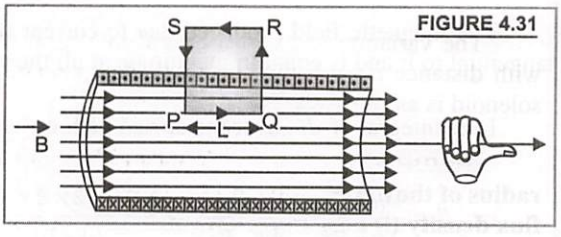


At a point outside the solenoid, the magnetic fields due to neighbouring loops oppose each other and at a point inside the solenoid, the magnetic fields are in the same direction. As a result of it, the effective magnetic field outside the solenoid becomes weak, whereas the magnetic field in the interior of solenoid becomes strong and uniform, acting along the axis of the solenoid, Fig. 4.30(b). When the solenoid is more and more tightly packed, the magnetic field due to current through the solenoid will become zero for a point outside near the middle of solenoid and magnetic field becomes strong and uniform at a point in the interior of solenoid.

Let n be the number of turns per unit length of solenoid and I be the current flowing through the solenoid.

Consider a rectangular amperian loop $PQRS$ near the middle of solenoid as shown in Fig. 4.31, where $PQ = L$. Let the magnetic field along the path PQ be B. This field along RS is zero. As the paths QR and SP are perpendicular to the axis of solenoid, the magnetic field component along these paths is zero. Therefore, the paths QR and SP will not contribute to the line integral of magnetic field $\vec{B}$.

FIGURE 4.31



The total number of turns in length $L = n\,L$.

The line integral of magnetic field induction $\vec{B}$ over the closed path $PQRS$ is

$$\oint_{PQRS} \vec{B}.d\vec{l} = \int_{P}^{Q} \vec{B}.d\vec{l} + \int_{Q}^{R} \vec{B}.d\vec{l} + \int_{R}^{S} \vec{B}.d\vec{l} + \int_{S}^{P} \vec{B}.d\vec{l} \quad \text{...(20)}$$

Here, $\int_{P}^{Q} \vec{B}.d\vec{l} = \int_{P}^{Q} B\,dl\cos 0° = BL$ and $\int_{Q}^{R} \vec{B}.d\vec{l} = \int_{Q}^{R} B\,dl\cos 90° = 0 = \int_{S}^{P} \vec{B}.d\vec{l}$

Also, $\int_{R}^{S} \vec{B}.d\vec{l} = 0$ ($\because$ outside the solenoid, $B = 0$)

$$\oint_{PQRS} \vec{B}.d\vec{l} = BL + 0 + 0 + 0 = BL \quad \text{...(21)}$$

From Ampere's circuital law

$$\oint_{PQRS} \vec{B}.d\vec{l} = \mu_0 \times \text{total current through the rectangle } PQRS$$

$$= \mu_0 \times \text{no. of turns in rectangle} \times \text{current}$$

$$= \mu_0\, n\, LI \quad \text{...(22)}$$

From (21) and (22), we have

$$BL = \mu_0\, n\, LI \quad \text{or} \quad \boxed{B = \mu_0\, n\, I} \quad \text{...(23)}$$

If N is the total number of turns of the solenoid, then $n = N/L$

then $B = \mu_0\, nI = \mu_0 \dfrac{N}{L} I$

This relation gives the magnetic field induction at a point well inside the solenoid. **At a point near the end of a solenoid, the magnetic field induction is found to be $\mu_0\, n\, I/2 = \mu_0 NI/2\, L$, which is half the magnetic field induction at the centre of the solenoid.**

From the above study we find that ***magnetic field lines due to current carrying straight solenoid, is same as is due to a permanent magnet.***

If a magnetic material of permeability μ is filled inside the solenoid, then magnetic field induction well inside the solenoid is

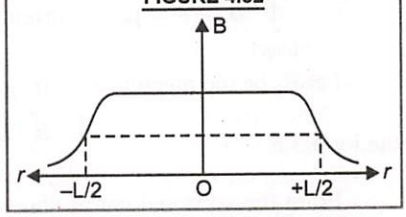
$$B = \mu\, n\, I = \frac{\mu N}{L} I$$

The variation of magnetic field induction (B) of the solenoid with distance r from the centre of the solenoid along the axis of solenoid is as shown in Fig. 4.32.

DO YOU KNOW ?

1. A linear solenoid carrying current is equivalent to a bar magnet.
2. The magnetic field lines due to current carrying solenoid resemble exactly with those of a bar magnet.
3. The magnetic field induction at a point near the middle just outside the curved face of the solenoid carrying current is zero.
4. The net force on the magnetic dipole inside the solenoid carrying current is zero.

FIGURE 4.32



Sample Problem **A solenoid 50 cm long has 4 layers of windings of 350 turns each. The radius of the lowest layer is 1·4 cm. If the current carried is 6·0 A, estimate the magnitude of magnetic flux density (i) near the centre of the solenoid on its axis, (ii) near the ends on its axis, (iii) outside the solenoid near its centre.**

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Sol. Here, $l = 50$ cm $= 0{\cdot}50$ m ; $r = 1{\cdot}4$ cm $= 1{\cdot}4 \times 10^{-2}$ m ; $I = 6{\cdot}0$ A ;

No. of turns per unit length, $n = \dfrac{4 \times 350}{0{\cdot}50} = 2800\ \text{m}^{-1}$

(*i*) The magnitude of $\vec{B}$ near the centre of solenoid on the axis is

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 2800 \times 6{\cdot}0 = \mathbf{2{\cdot}11 \times 10^{-2}\ T}$$

(*ii*) The magnitude of $\vec{B}$ near the end of solenoid on the axis is

$$B = \frac{\mu_0 n I}{2} = \frac{2{\cdot}11 \times 10^{-2}}{2} = \mathbf{1{\cdot}05 \times 10^{-2}\ T}$$

(*iii*) Outside the solenoid, the magnetic field ***is negligibly small as compared to that inside the solenoid.***

4.15. THE TOROID

The toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. In fact, a toroid is an endless solenoid in the form of a ring, Fig. 4.33(*a*).

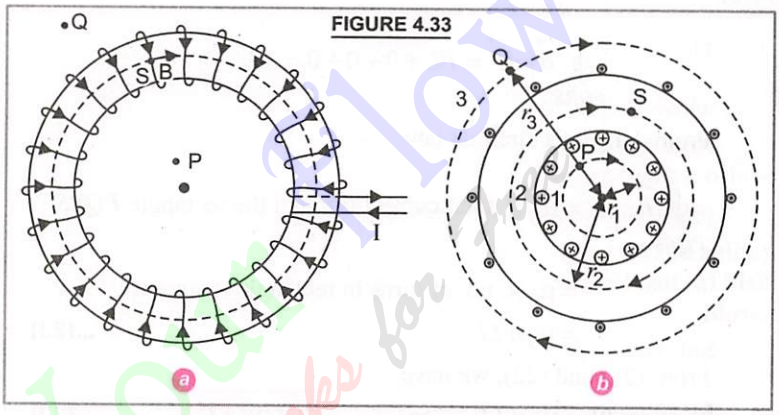

FIGURE 4.33

Magnetic field due to current in ideal toroid

Let n be the number of turns per unit length of toroid and I be the current flowing through it. In case of ideal toroid, the coil turns are circular and closely wound. A magnetic field of constant magnitude is set up inside the turns of toroid in the form of concentric circular magnetic field lines. The direction of the magnetic field at a point is given by the tangent to the magnetic field line at that point. We draw three circular amperian loops, 1, 2 and 3 of radii r_1, r_2 and r_3 to be traversed in clockwise direction as shown by dashed circles in Fig. 4.33(*b*), so that the points P, S and Q may lie on them. The circular area bounded by loops 2 and 3, both cut the toroid. Each turn of current carrying wire is cut once by the loop 2 and twice by the loop 3. Let B_1 be the magnitude of magnetic field along loop 1. Line integral of magnetic field B_1 along the loop 1 is

$$\oint_{loop\,1} \vec{B}_1 . d\vec{l} = \oint_{loop\,1} B_1\, dl \cos 0^\circ = B_1\, 2\pi r_1 \qquad \text{...(i)}$$

Loop 1 encloses no current.

According to Ampere's circuital law

$$\oint_{loop\,1} \vec{B}_1 . d\vec{l} = \mu_0 \times \text{current enclosed by loop 1} = \mu_0 \times 0 = 0 \quad \text{or} \quad B_1\, 2\pi r_1 = 0 \quad \text{or} \quad B_1 = 0$$

Let B_3 be the magnitude of magnetic field along the loop 3. The line integral of magnetic field B_3 along the loop 3 is

$$\oint_{loop\,3} \vec{B}_3 . d\vec{l} = \oint_{loop\,3} B_3\, dl \cos 0^\circ = B_3\, 2\pi r_3$$

From the sectional cut as shown in Fig. 4.33(*b*), we note that the current coming out of the plane of paper is cancelled exactly by the current going into it. Therefore, the total current enclosed by loop 3 is zero.

According to Ampere's circuital law

$$\oint_{loop\,3} \vec{B}_3 . d\vec{l} = \mu_0 \times \text{total current through loop 3} \quad \text{or} \quad B_3\, 2\pi r_3 = \mu_0 \times 0 = 0 \quad \text{or} \quad B_3 = 0$$

Let B the magnitude of magnetic field along the loop 2. Line integral of magnetic field along the loop 2 is $\oint_{loop\,2} \vec{B}.d\vec{l} = B\,2\pi r_2$

Current enclosed by the loop 2 = number of turns × current in each turn = $2\pi r_2 n \times I$

According to Ampere's circuital law

$$\oint_{loop\,2} \vec{B}.d\vec{l} = \mu_0 \times \text{total current} \quad \text{or} \quad B\,2\pi r_2 = \mu_0 \times 2\pi r_2 n I \quad \text{or} \quad \boxed{B = \mu_0 n I}$$

This result is similar to that of solenoid.

The magnetic field at any point inside the empty space surrounded by toroid or outside the toroid is zero.

The magnetic field at any point inside the toroid $= \mu_0 n I = \mu_0 \dfrac{N}{2\pi r} I$

where N is the total number of turns in toroid and r is the mean radius of Toroid.

Note that in reality, the turns of the toroid form a helix and not circle as is the case for ideal toroid. There is also a small magnetic field in the external portion of the toroid.

Sample Problem **A coil wrapped around a toroid has inner radius of 10·0 cm and an outer radius of 12·0 cm. If the wire wrapped makes 1100 turns and carries a current of 5 A, find the magnetic field (*a*) inside the core of the toroid (*b*) outside the toroid at a distance 10 cm from the outer surface of toroid.**

Sol. Here, $r_1 = 10{\cdot}0$ cm ; $r_2 = 12{\cdot}0$ cm ; $I = 5$ A

Mean radius of toroid, $r = \dfrac{10+12}{2} = 11{\cdot}0 \text{ cm} = 11 \times 10^{-2} \text{ m} = 0{\cdot}11 \text{ m}$

Total length of toroid = circumference of toroid = $2\pi r = 2\pi \times 0{\cdot}11 = 0{\cdot}22\,\pi$ m

total number of turns, $N = 1100$

Number of turns per unit length will be, $n = \dfrac{N}{2\pi r} = \dfrac{1100}{0{\cdot}22\pi}$ m^{-1}

(*a*) Magnetic field induction inside the core of the toroid,

$$B = \mu_0 nI = (4\pi \times 10^{-7}) \times \frac{1100}{0{\cdot}22\pi} \times 5 = \mathbf{0{\cdot}01\ T}$$

(*b*) Magnetic field induction outside the toroid is **zero** because the field is only confined within the core of the toroid on which winding has been made.

4.16. COMPARISON BETWEEN AMPERE'S CIRCUITAL LAW AND BIOT SAVART'S LAW

Similarities.

(1) Both relate the magnetic field with current.

(2) Both express the same physical consequences of a steady current.

Differences.

(1) Ampere's law is applicable for symmetrical current distributions whereas the Biot Savart's law is applicable for asymmetrical current distributions.

(2) Ampere's law is for integral form of $\vec{B}$ and Biot Savart's law is for differential form of $\vec{B}$.

(3) Ampere's law is based on the principle of electromagnetism and Biot Savart's law is based on magnetism.

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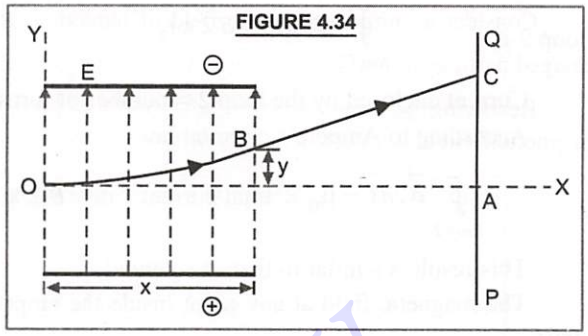
4.17. MOTION OF A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

Suppose a particle of mass m, carrying charge $+q$ is moving with velocity v along OX (*i.e.* along X-axis). In the absence of an electric field, it meets the screen PQ at point A, Fig. 4.34. Let the charged particle be subjected to a uniform electric field of strength E acting along OY (*i.e.* along Y-axis). Force on charged particle due to electric field, $F = qE$



It is directed along $\vec{E}$, *i.e.*, along OY.

$\therefore$ Acceleration of particle along OY is,

$$a = \frac{qE}{m}$$

This acceleration being perpendicular to the motion of charged particle, will not change the speed of the charged particle along OX.

If x is the length of the region of electric field, then the time taken by the charged particle to traverse it is given by

$$t = \frac{x}{v}$$

If y is the displacement of the charged particle along OY direction in the electric field during time t, then using the relation

$$S = ut + \frac{1}{2}at^2$$

We have, $$y = (0)t + \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2}\frac{qE}{m}\left(\frac{x}{v}\right)^2$$

or $$x^2 = \frac{2mv^2}{qE}y$$

or $$x^2 = ky \qquad \left(\because \frac{2mv^2}{qE} = k = a \text{ constant}\right)$$

This is an equation of a parabola. Hence ***inside the electric field, the charged particle moves on a parabolic path OB*** and on leaving the field, it moves along a straight path BC, tangent to the curved path OB at B.

Important points

1. If a positively charged particle having charge q is at rest or moving in the direction of electric field $\vec{E}$, it experiences a force $q\vec{E}$ in the direction of $\vec{E}$ and hence the particle is accelerated in the direction of $\vec{E}$.

2. If a positively charged particle is moving opposite to the direction of electric field $\vec{E}$, if experiences a force $-q\vec{E}$ and hence it is retarded.

3. If the charged particle moves perpendicular to the direction of electric field $\vec{E}$, it describes a parabolic path in the electric field.

DO YOU KNOW ?

If a charged particle is describing a circular path of radius r under the effect of perpendicular electric field, (*e.g.* motion of electron around the nucleus of an atom), then

1. the force on the charged particle is inversely proportional to the square of radius of the circular path *i.e.* $F \propto \frac{1}{r^2}$.

2. the velocity of the charged particle is inversely proportional to the square root of the radius of the circular path *i.e.* $v \propto 1/\sqrt{r}$

3. the square of time period of revolution of the charged particle is directly proportional to the cube of the radius of circular path *i.e.* $T^2 \propto r^3$.

4. A charged particle moving perpendicular to the direction of electric field gets accelerated along or opposite to the direction of electric field which causes the change in speed, velocity, momentum and kinetic energy of particle.

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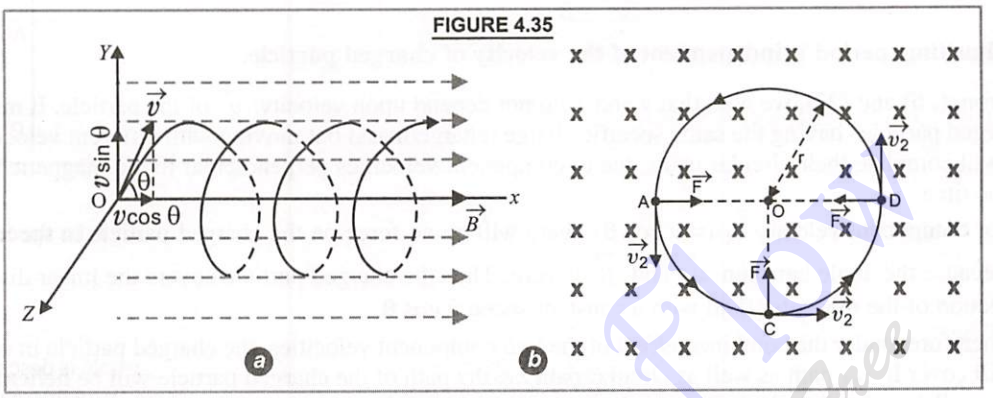
4.18. MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Consider a uniform magnetic field of induction $\vec{B}$ directed along OX in the plane of the paper. Let a charged particle of mass m carrying charge q enter this field with a velocity $\vec{v}$ making an angle θ with OX.

Resolving $\vec{v}$ into two rectangular components, we have : $v \cos\theta\ (= v_1)$ acting in the direction of the magnetic field and $v \sin\theta\ (= v_2)$ acting perpendicular to the direction of magnetic field.

FIGURE 4.35



For component velocity $\vec{v}_2$, the force acting on the charged particle due to magnetic field is

$$\vec{F} = q\,(\vec{v}_2 \times \vec{B})$$

or $$F = q\,|\,\vec{v}_2 \times \vec{B}\,| = q\,v_2 B \sin 90^\circ = q\,(v \sin\theta)\,B \quad \text{...(24)}$$

The direction of this force $\vec{F}$ is given by Right Hand Rule and is perpendicular to the plane containing $\vec{B}$ and $\vec{v}_2$. As this force is perpendicular to $\vec{v}_2$ it cannot change the magnitude of velcity $\vec{v}_2$. It changes only the direction of motion of the particle. Due to this force, the charged particle is made to move on a circular path in the magnetic field, as shown in Fig. 4.35(*b*).

Here, magnetic field is shown perpendicular to the plane of paper directed inwards and particle is moving in the plane of paper. When the particle is at points *A*, *C* and *D* the direction of magnetic force on the particle will be along *AO*, *CO* and *DO* respectively, *i.e.*, directed towards the centre *O* of the circular path.

The force F on the charged particle due to magnetic field provides the required centripetal force $= (mv_2^2 / r)$ necessary for motion along a circular path of radius r.

$$\therefore \qquad B\,q\,v_2 = m\,v_2^2 / r \quad \text{or} \quad v_2 = B\,q\,r/m$$

or $$v \sin\theta = B\,q\,r/m \quad \text{...(25)}$$

The angular velocity of rotation of the particle in magnetic field will be,

$$\omega = \frac{v \sin\theta}{r} = \frac{B\,q\,r}{m\,r} = \frac{B\,q}{m}$$

DO YOU KNOW ?

When a charged particle is projected perpendicular to the magnetic field,

(1) its path is circular in a plane perpendicular to the plane of magnetic field and direction of motion of the charged particle.

(2) the speed and kinetic energy of the particle remain constant.

(3) the velocity of the charged particle changes only in direction. Hence velocity and momentum of particle will charge.

(4) the force acting on the charged particle is independent of the radius of circular path but depends upon the speed of the charged particle *i.e.*, $F \propto r^0$ (but $F \propto v$).

(5) the time period of revolution of a charged particle in the magnetic field is independent of velocity of particle and radius of the circular path *i.e.*, $T \propto r^0\, v^0$. The time period depends upon the ratio of charge/mass.

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The frequency of rotation of the particle in magnetic field will be,

$$\nu = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m} \quad ...(26)$$

The time period of revolution of the particle in the magnetic field will be,

$$T = \frac{1}{\nu} = \frac{2\pi m}{Bq} \quad ...(27)$$

Thus time period is independent of the velocity of charged particle.

From (26) and (27), we note that ν and T do not depend upon velocity $\vec{v}$ of the particle. It means, all the charged particles having the same specific charge (charge/mass) but moving with different velocities at a point, will complete their circular paths due to component velocities perpendicular to the magnetic fields in the same time.

For component velocity v_1 ($= v \cos\theta$), there will be no force on the charged particle in the magnetic field, because the angle between $\vec{v}_1$ and $\vec{B}$ is zero. Thus the charged particle covers the linear distance in the direction of the magnetic field with a constant speed $v \cos\theta$.

Therefore, under the combined effect of the two component velocities, the charged particle in magnetic field will cover linear path as well as circular path *i.e.* the path of the charged particle will be **helical,** whose axis is parallel to the direction of magnetic field, Fig. 4.35(*a*).

The linear distance covered by the charged particle in the magnetic field in time equal to one revolution of its circular path **(known as pitch of helix)** will be, $d = v_1 T = v \cos\theta \dfrac{2\pi m}{Bq}$

Discussion

1. If a charged particle having charge q is at rest in a magnetic field $\vec{B}$, it experiences no force; as $v = 0$ and $F = q\,v\,B \sin\theta = 0$

2. If charged particle is moving parallel to the direction of $\vec{B}$, it also does not experience any force because angle θ between $\vec{v}$ and $\vec{B}$ is 0° or 180° and sin 0° = sin 180° = 0. Therefore, the charged particle in this situation will continue moving along the same path with the same velocity.

3. If charged particle is moving perpendicular to the direction of $\vec{B}$, it experiences maximum force, which acts perpendicular to the direction of $\vec{B}$ as well as $\vec{v}$. Hence this force will provide the required centripetal force and the charged particle will describe a circular path in the magnetic field of radius r, given by

$$\frac{mv^2}{r} = Bqv \quad \text{or} \quad r = \frac{mv}{Bq} = \frac{v}{(q/m)B}$$

where (q/m) represents the **specific charge of charged particle.**

Thus, radius of the circular path is directly proportional to the speed and mass of the charged particle or inversely proportional to specific charge of the charged particle.

4. From above relation it is clear that $r \propto v$, *i.e.*, a slow moving charged particle describes a circular path of small radius and fast moving charged particle describes a circular path of larger radius.

5. As electron and proton have same magnitude of charge but mass of proton is greater than that of electron so specific charge of electron is greater than that of proton.

Therefore, if both electron and proton moving with same velocity enter a uniform magnetic field acting perpendicular to the direction of motion of particles, then the electron describes a circular path of smaller radius in magnetic field than that described by a proton because $r \propto 1/(q/m)$.

6. When two equal and oppositively charged particles (say electron and proton) having same momentum enter a uniform perpendicular magnetic field, then the radii of their circular paths are equally curved.

Sample Problem 1 **A beam of ions with velocity 2×10^5 ms^{-1} enters normally into a uniform magnetic field of 0·04 tesla. If the specific charge of ion is 5×10^7 C kg^{-1}, find the radius of the circular path described.**

Sol. Here, $v = 2 \times 10^5$ ms^{-1} ; $B = 0{\cdot}04$ T ; $q/m = 5 \times 10^7$ C/kg

As $q v B = m v^2/r$ or $r = \dfrac{mv}{qB} = \dfrac{v}{(q/m)\,B} = \dfrac{2\times10^5}{(5\times10^7)\times0{\cdot}04} = 0{\cdot}1$ m

Sample Problem 2 **An electron of mass $0{\cdot}90 \times 10^{-30}$ kg, under the action of magnetic field moves in a circle of radius 2·0 cm at a speed of $3{\cdot}0 \times 10^6$ ms^{-1}. If a proton of mass $1{\cdot}8 \times 10^{-27}$ kg were to move in a circle of the same radius in the same magnetic field, find its speed.**

Sol. Here, $m_e = 9 \times 10^{-31}$ kg ; $r_e = 0{\cdot}02$ m ; $v_e = 3 \times 10^6$ ms^{-1} ; $m_p = 1{\cdot}8 \times 10^{-27}$ kg ; $v_p = ?$; $r_p = 0{\cdot}02$ m.

We know that $q v B = m v^2/r$ or $v = q r B/m$;

So $\dfrac{v_p}{v_e} = \dfrac{q_p r_p B/m_p}{q_e r_e B/m_e} = \dfrac{m_e}{m_p}$ [as $q_p = q_e$; $r_p = r_e$]

$\therefore$ $v_p = v_e \times \dfrac{m_e}{m_p} = \dfrac{3\times10^6\times9\times10^{-31}}{1{\cdot}8\times10^{-27}} = 1{\cdot}5 \times 10^3$ ms^{-1}

4.19. LORENTZ FORCE

The force experienced by a charged particle moving in space where both electric and magnetic fields exist is called Lorentz force.

Force due to electric field. When a charged particle carrying charge $+q$ is subjected to an electric field of strength $\vec{E}$, it experiences a force given by

$$\vec{F}_e = q\vec{E} \qquad ...(28)$$

The direction of $\vec{F}_e$ is the same as that of $\vec{E}$.

Force due to magnetic field. If the charged particle is moving in a magnetic field $\vec{B}$, with a velocity $\vec{v}$ it experiences a force given by $\vec{F}_m = q\,(\vec{v}\times\vec{B})$

The direction of this force is in the direciton of $\vec{v}\times\vec{B}$ *i.e.* perpendicular to the plane containing $\vec{v}$ and $\vec{B}$ as per Right hand screw rule.

Due to both, the electric and magnetic fields, the total force experienced by the charged particle is

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\,(\vec{v}\times\vec{B}) = q\,(\vec{E} + \vec{v}\times\vec{B})$$

$$\boxed{\vec{F} = q\,(\vec{E} + \vec{v}\times\vec{B})} \qquad ...(29)$$

This is called ***Lorentz force.***

Special cases

Case I. When $\vec{v}$, $\vec{E}$ and $\vec{B}$, all the three are collinear. In this situation, the charged particle is moving parallel or antiparallel to the fields, the magnetic force on the charged particle is zero. The electric force on the charged particle will produce acceleration $\vec{a} = \dfrac{q\vec{E}}{m}$,

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along the direction of electric field. As a result of it, there will be change in the speed of charged particle along the direction of the field. In this situation, there will be no change in the direction of motion of the charged particle but, the speed, velocity, momentum and kinetic energy of charged particle will change.

Case II. When $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular to each other. In this situation, if $\vec{E}$ and $\vec{B}$ are such that $\vec{F} = \vec{F}_e + \vec{F}_m = 0$, then acceleration of the particle, $\vec{a} = \frac{\vec{F}}{m} = 0$. It means the particle will pass through the fields without any change in its velocity. Here, $F_e = F_m$ so $qE = q\,v\,B$ or $v = E/B$.

This concept has been used in **velocity-selector** to get a charged beam having a definite velocity.

Sample Problem **An α-particle is moving in a magnetic field of $(3\hat{i} + 2\hat{j})$ tesla with a velocity of $(5 \times 10^5\,\hat{i})\,\text{ms}^{-1}$. What will be the magnetic force acting on the particle ?**

Sol. Here, $\vec{B} = (3\hat{i} + 2\hat{j})$ and $\vec{v} = (5 \times 10^5\,\hat{i})$; $q = 2e$.

$$\vec{F} = q\,(\vec{v} \times \vec{B}) = q[(5 \times 10^5\,\hat{i}) \times (3\hat{i} + 2\hat{j})] = q \times 10^6\,\hat{k} = 2e \times 10^6\,\hat{k} = 2 \times 1{\cdot}6 \times 10^{-19} \times 10^6\,\hat{k}$$

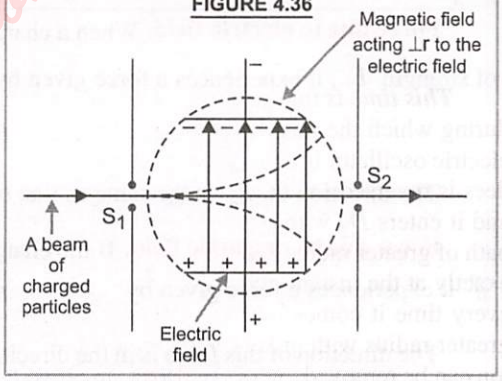
or $|\vec{F}| = \mathbf{3{\cdot}2 \times 10^{-13}\ N}$, **towards positive z-direction.**

4.20. VELOCITY FILTER

***Velocity filter** is an arrangement of cross electric and magnetic fields in a region which helps us to select from a beam, charged particles of the given velocity irrespective of their charge and mass.*

A velocity selector consists of two slits S_1 and S_2 held parallel to each other, with common axis, some distance apart. In the region between the slits, uniform electric and magnetic fields are applied, perpendicular to each other as well as to the axis of slits, as shown in Fig. 4.36. When a beam of charged particles of different charges and masses after passing through slit S_1 enters the region of crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, each particle experiences a force due to these fields. Those particles which are moving with the velocity v, irrespective of their mass and charge, the force due to electric field (qE) is equal and opposite to the force due to magnetic field ($q\,v\,B$), then

FIGURE 4.36



$$q\,E = q\,v\,B \quad \text{or} \quad v = E/B$$

Such particles will go undeviated and filtered out of the region through the slit S_2. Therefore, the particles emerging from slit S_2 will have the same velocity even though their charge and mass may be different.

The velocity filter is used in mass spectrograph which helps to find the mass and specific charge (charge/mass) of the charged particle.

4.21. CYCLOTRON

A cyclotron is an equipment used for accelerating positively charged particles like proton, deutron, alpha particles and heavier positive ions to desired velocities. It was developed by Lawrence and Livingstone.

Principle. *The working of the cyclotron is based on the fact that a positively charged particle can be accelerated to a sufficiently high energy with the help of smaller values of oscillating electric field by making it to cross the same electric field time and again, with the use of strong magnetic field.*

Construction. It consists of two D-shaped hollow evacuated metal chambers D_1 and D_2 called the dees. These dees are placed horizontally with their diametric edges parallel and slightly separated from each other. The dees are connected to high frequency oscillator which can produce a potential difference of the order of 10^4 volts at frequency $\approx 10^7$ Hz. The two dees are enclosed in an evacuated steel box and are well insulated from it. The box is placed in a strong magnetic field produced by two pole pieces N,S of strong electromagnets. The magnetic field is perpendicular to the plane of the dees. The ionic surface or the source of positively charged particles is held at the centre P, as shown in Fig. 4.37.

Working and theory. At any instant, suppose D_1 is at negative potential and D_2 is at positive potential. Therefore, the positively charged ion produced at P, will be accelerated towards D_1. On reaching ***inside D_1, the ion will be in a field free space.*** Hence it moves with a constant speed in D_1 say v. But due to perpendicular magnetic field of strength B, the ion will describe a circular path of radius r (say) in D_1, given by

$$B q v = \frac{m v^2}{r}$$

where m and q are the mass and charge of the ion.

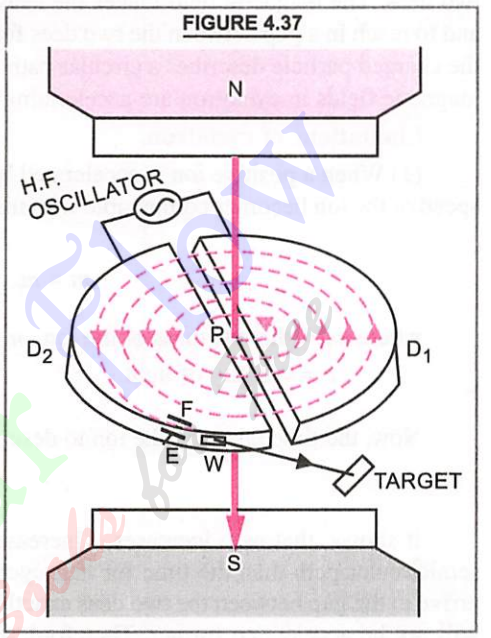
$$\therefore \qquad r = \frac{m v}{Bq}$$

Time taken by ion to describe a semicircular path is given by,

$$t = \frac{\pi r}{v} = \frac{\pi m}{Bq} = \frac{\pi}{B(q/m)}$$

= a constant.

FIGURE 4.37



This time is independent of both the speed of the ion and radius of the circular path. In case the time during which the positive ion describes a semicircular path is equal to the time during which half cycle of electric oscillator is completed, then as the ion arrives in the gap between the two dees, the polarity of the two dees is reversed *i.e.* D_1 becomes positive and D_2 negative. Then, the positive ion is accelerated towards D_2 and it enters D_2 with greater speed which remains constant inside D_2. The ion will describe a semicircular path of greater radius due to perpendicular magnetic field and again will arrive in a gap between the two dees exactly at the instant, the polarity of the two dees is reversed. Thus, the positive ion will go on accelerating every time it comes into the gap between the dees and will go on describing circular path of greater and greater radius with greater and greater speed and finally acquires a sufficiently high energy. The accelerated ion can be removed out of the dees from window W, by applying the electric field across the deflecting plates E and F.

Maximum Energy of positive ion

Let v_0, r_0 = maximum velocity and maximum radius of the circular path followed by the positive ion in cyclotron.

Then, $\dfrac{m v_0^2}{r_0} = Bq v_0$ or $v_0 = \dfrac{Bq r_0}{m}$ $\therefore$ Max.K.E. $= \dfrac{1}{2} m v_0^2 = \dfrac{1}{2} m \left(\dfrac{Bq r_0}{m} \right)^2 = \dfrac{B^2 q^2 r_0^2}{2m}$

Cyclotron Frequency

If T is the time period of oscillating electric field then $T = 2t = 2\pi m/Bq$

The **cyclotron frequency** is given by $\nu = \dfrac{1}{T} = \dfrac{Bq}{2\pi m}$

It is also known as **magnetic resonance frequency.**

The **cyclotron angular frequency** is given by $\omega_c = 2\pi\nu = Bq/m$

Note : In a given magnetic field, the two charged particles having the same cyclotron frequency, will have the same value of specific charge (charge/mass).

Functions of Electric and Magnetic fields in a Cyclotron

In cyclotron, the electric field is used to accelerate the heavy charged particle in a region between the two dees. The magnetic field makes the heavy charged particle to describe the circular path inside the dees and to reach in a gap between the two dees for further acceleration by electric field. Since in a magnetic field, the charged particle describes a circular path so its motion is accelerated. Therefore, both the electric and the magnetic fields in cyclotron are accelerating the charged particle.

Limitations of cyclotron.

(1) When a positive ion is accelerated by the cyclotron, it moves with greater and greater speed. As the speed of the ion becomes comparable with that of light, the mass of the ion increases according to the relation

$$m = m_0 \Big/ \sqrt{1-\frac{v^2}{c^2}}$$

where m_0 = the rest mass of the ion, m = the mass of the ion while moving with velocity v,
c = velocity of light.

Now, the time taken by the ion to describe semicircular path is $t = \frac{\pi m}{Bq} = \frac{\pi}{Bq} \cdot \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

It shows, that as v increases, t increases. It means, the positive ion will take longer time to describe semicircular path than the time for half cycle of oscillating electric field. As a result of it, the ion will not arrive in the gap between the two dees exactly at the instant, the polarity of the two dees is reversed and hence will not be accelerated further. Therefore, ***the ion cannot move with a speed beyond a certain limit in a cyclotron.***

(2) Cyclotron is suitable only for accelerating heavy particles like proton, deutron, α-particle etc.

(3) The uncharged particles (*e.g.* neutrons) cannot be accelerated by cyclotron.

(4) The lighter charged particle like electrons can not be accelerated by cyclotron because the mass of the electron is small and a small increase in energy of the electron makes the electrons move with a very high speed. As a result of it, the electrons go quickly out of step with oscillating electric field.

Uses of cyclotron

1. The high energy particles produced in a cyclotron are used for artificial transmulation in order to produce radio isotopes which are used for medical purposes in hospitals, to diagnose the chronic diseases and for their treatment.

2. The high energy particles produced in a cyclotron are used to bombard the atomic nuclei and then to study the resulting nuclear reaction which helps to investigate the nuclear structure.

3. It is used to add ions into the solids in order to improve the quality of solids.

4. The high energy particles produced in a cyclotron are used to produce other high energy particles (like neutrons) through bombardment process, which are used in atomic reactors.

Sample Problem 1 The magnetic flux density applied in a cyclotron is 3·5 T. What will be the frequency of electric field that must be applied between the dees in order (a) to accelerate protons (b) α-particles ? mass of proton $1{\cdot}67 \times 10^{-27}$ kg.

Sol. *(a)* $\nu = \frac{Bq}{2\pi m} = \frac{3{\cdot}5 \times 1{\cdot}6 \times 10^{-19}}{2 \times (22/7) \times 1{\cdot}67 \times 10^{-27}} = 5{\cdot}35 \times 10^7$ **Hz.**

(b) $v = \dfrac{B \times 2e}{2\pi \times 4m} = \dfrac{3{\cdot}5 \times 2 \times 1{\cdot}6 \times 10^{-19}}{2 \times (22/7) \times 4 \times 1{\cdot}67 \times 10^{-27}} = 2{\cdot}675 \times 10^{7}$ Hz.

Sample Problem 2 **Magnetic field applied on a cyclotron is 0·7 T and radius of its dees is 1·8 m. What will be the energy of the emergent protons in MeV ? Mass of proton = $1{\cdot}67 \times 10^{-27}$ kg.**

Sol. $E = \dfrac{q^2 B^2 r^2}{2m} = \dfrac{(1{\cdot}6 \times 10^{-19})^2 \times (0{\cdot}7)^2 \times (1{\cdot}8)^2}{2 \times 1{\cdot}67 \times 10^{-27}}$ J $= \dfrac{(1{\cdot}6 \times 10^{-19})^2 \times (0{\cdot}7)^2 \times (1{\cdot}8)^2}{2 \times 1{\cdot}67 \times 10^{-27} \times 1{\cdot}6 \times 10^{-13}}$ MeV

$= 76$ MeV

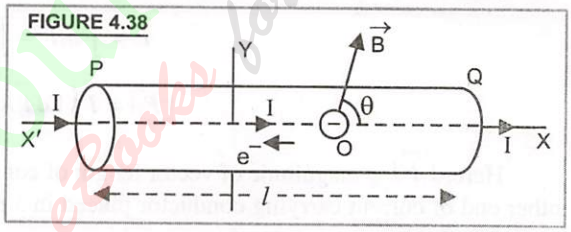
4.22. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD

We know that current through a conductor is due to the motion of free electrons in the conductor in a definite direction. When such a conductor is placed in a magnetic field, each electron moving in the magnetic field experiences a force. Hence the current carrying conductor experiences a force when placed in the magnetic field.

Expression for the force acting on the conductor carrying current placed in a magnetic field.

Consider a straight cylindrical conductor PQ of length l, area of cross-section A, carrying current I placed in a uniform magnetic field of induction, $\vec{B}$. Let the conductor be placed along X-axis and magnetic field be acting in XY plane making an angle θ with X-axis. Suppose the current I flows through the conductor from the end P to Q, Fig. 4.38. Since the current in a conductor is due to motion of electrons, therefore, electrons are moving from the end Q to P (along XX').

FIGURE 4.38



Let, $\vec{v}_d$ drift velocity of electron,

$-e$ = charge on each electron.

Then magnetic Lorentz force on an electron is given by $\vec{f} = -e(\vec{v}_d \times \vec{B})$

If n is the number density of free electrons *i.e.* number of free electrons per unit volume of the conductor, then total number of free electrons in the conductor will be given by $N = n\,(Al) = nAl$

∴ Total force on the conductor is equal to the force acting on all the free electrons inside the conductor while moving in the magnetic field and is given by

$$\vec{F} = N\vec{f} = nAl[-e\,(\vec{v}_d \times \vec{B})] = -nAle(\vec{v}_d \times \vec{B}) \quad \text{...(30)}$$

We know that current through a conductor is related with drift velocity by the relation

$$I = nAev_d \quad \therefore \quad Il = nAev_d \cdot l$$

We represent $I\vec{l}$ as current element vector. It acts in the direction of flow of current *i.e.* along OX. Since $I\vec{l}$ and $\vec{v}_d$ have opposite directions, hence we can write

$$I\vec{l} = -nAle\vec{v}_d \quad \text{...(31)}$$

From (30) and (31), we have $\vec{F} = I\vec{l} \times \vec{B}$ ...(32)

$$|\vec{F}| = I|\vec{l} \times \vec{B}|$$

$$F = IlB\sin\theta \quad \text{...(33)}$$

where θ is the smaller angle between $I\vec{l}$ and $\vec{B}$.

DO YOU KNOW ?

The force on a wire carrying current of any shape in a uniform magnetic field is independent of its shape but depends on the vector length joining the beginning and ending of the current carrying wire.

Special cases

Case I. If $\theta = 0°$ or $180°$, $\sin\theta = 0$,

From (33), $F = I\,lB\,(0) = 0$ (Minimum)

It means a linear conductor carrying a current when placed parallel to the direction of magnetic field, **experiences no force.**

Case II. If $\theta = 90°$, $\sin\theta = 1$;

From (33), $F = IlB \times 1 = IlB$ (Maximum)

It means a linear conductor carrying current when placed perpendicular to the direction of magnetic field, experiences **maximum force,** the direction of which is given by **Right handed screw rule** or **Fleming's Left Hand Rule.** Accordingly, the direction of $\vec{F}$ is perpendicular to the plane of paper directed upwards.

It is to be noted that the relation (33) is valid only if magnetic field $\vec{B}$ is uniform over the whole length of the linear conductor.

Important Information

1. If the conductor carrying current I is not straight one and is placed in a uniform magnetic field $\vec{B}$, then the magnetic force on the small element of this conductor of length dl is

$$d\vec{F} = I\,(d\vec{l} \times \vec{B})$$

Total force experienced by the whole conductor is

$$\vec{F} = \int d\vec{F} = \int I\,(d\vec{l} \times \vec{B})$$

$$|\vec{F}| = I\int |d\vec{l} \times \vec{B}| = I\,|\vec{l}\,|B\sin\theta$$

Here, $|\vec{l}\,|$ = magnitude of vector length of conductor ; which is the least distance between one end and other end of current carrying conductor placed in a magnetic field.

2. When a semicircular coil of radius r, carrying current I is placed in a uniform magnetic field $\vec{B}$, then $|\vec{l}\,| = 2r$.

Total magnetic force on semicircular coil is $F = I\,(2\,r)\,B\sin\theta$.

3. When a circular coil or closed loop of any shape, carrying current I is placed in a uniform magnetic field then $|\vec{l}\,| = 0$.

Total magnetic force on coil is zero as $F = I \times 0 \times B\sin\theta = 0$

In this case, the different elementary parts of coil will experience different forces due to their different orientations with the direction of magnetic field. The resultant force of all the forces on the various elementary parts of coil is zero.

4. When a linear conductor is placed in a non-uniform magnetic field, then different elements of the conductor will experience different forces.

Sample Problem **The horizontal component of earth's magnetic field at a certain place is $3{\cdot}0 \times 10^{-5}$ T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west (b) south to north ?**

(NCERT Solved Example)

Sol. Here, $B = 3{\cdot}0 \times 10^{-5}$ T ; $I = 1\,A$; $l = 1$ m

(*a*) When current is flowing from east to west, $\theta = 90°$

$$F = I\,l\,B \sin\theta = 1 \times 1 \times 3{\cdot}0 \times 10^{-5} \times \sin 90° = \mathbf{3{\cdot}0 \times 10^{-5}\ N\ m^{-1}}$$

(*b*) When current is flowing from south to north, $\theta = 0°$

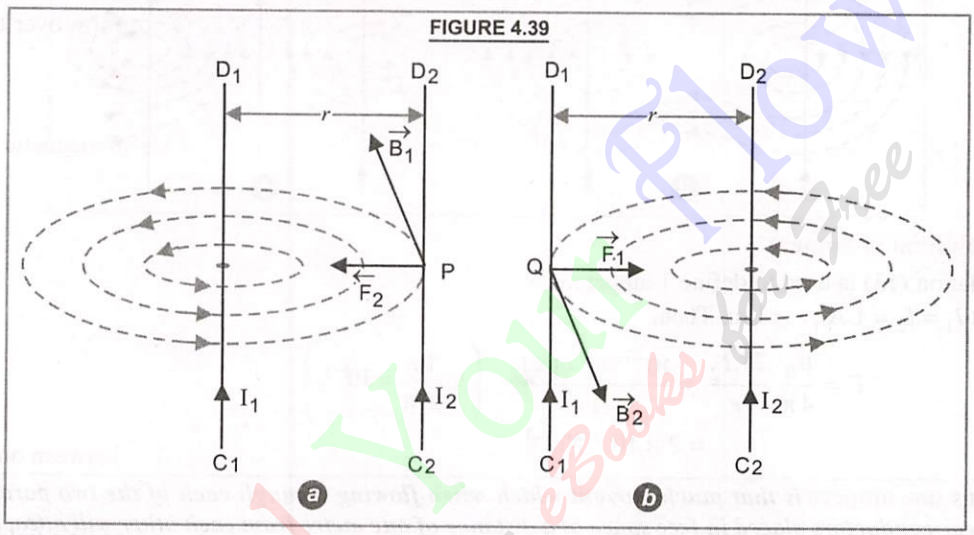
$$\therefore \quad F = I\,l\,B \sin\theta = 1 \times 1 \times 3{\cdot}0 \times 10^{-5} \times \sin 0° = 0$$

Hence **no force is acting per unit length on the conductor.**

4.23. FORCE BETWEEN TWO PARALLEL LINEAR CONDUCTORS CARRYING CURRENT

In Fig. 4.39, $C_1\,D_1$ and $C_2\,D_2$ are two infinite long straight conductors carrying currents I_1 and I_2 in the same direction. They are held parallel to each other at a distance r apart, in the plane of paper. Magnetic field is produced due to current through each conductor. Since each conductor is in the magnetic field produced by the other, therefore, each conductor experiences a force.

FIGURE 4.39



Magnetic field induction at a point P on conductor $C_2\,D_2$ due to current I_1 passing through $C_1\,D_1$ is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{r} \qquad \text{...(34)}$$

According to ***right hand rule***, the direction of magnetic field $\vec{B}_1$ is perpendicular to the plane of paper, directed inwards.

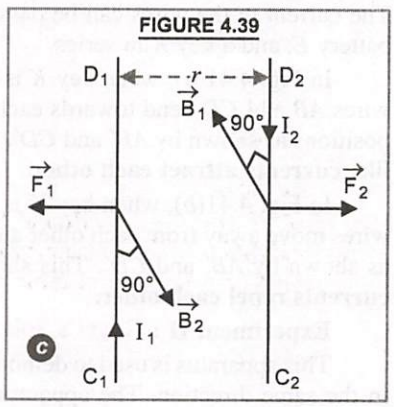
As the current carrying conductor C_2D_2 lies in the magnetic field $\vec{B}_1$ (produced by the current through C_1D_1), therefore, the unit length of $C_2\,D_2$ will experience a force given by

$$F_2 = B_1\,I_2 \times 1 = B_1\,I_2$$

Putting the value of B_1, we have $\boxed{F_2 = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2I_1I_2}{r}}$ ...(35)

According to *Fleming's Left Hand Rule,* force on conductor $C_2\,D_2$ acts in the plane of the paper perpendicular to $C_2\,D_2$, directed towards $C_1\,D_1$. Similarly, it can be shown that the conductor $C_1\,D_1$ also experiences a force given by (35), which acts in the plane of paper, perpendicular to $C_1\,D_1$ and directed towards $C_2\,D_2$. Hence $C_1\,D_1$ and $C_2\,D_2$ attract each other.

FIGURE 4.39



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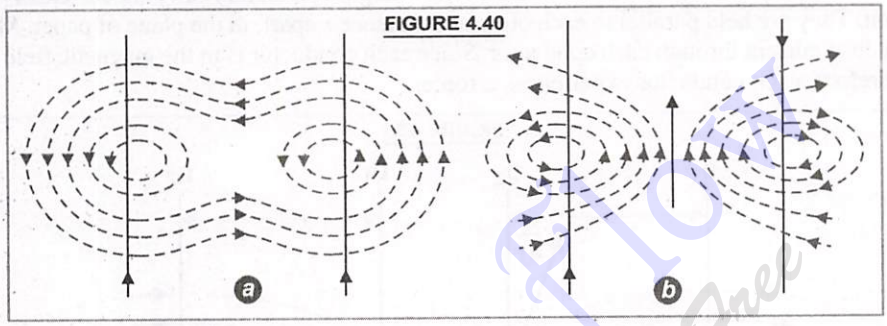
It means the **two linear parallel conductors carrying currents in the same direction attract each other.** The force experienced by the unit length of each conductor is given by Eq. (35).

Further it can be shown that if the currents in ***conductors C_1D_1 and C_2D_2 are in opposite directions*, Fig. 4.39(*c*), they repel each other** with the same force as given by Eq. (35).

The effective magnetic field due to two long linear parallel conductors carrying currents in the same direction is shown in Fig. 4.40(*a*). When currents are in opposite directions, the resultant magnetic field is shown in Fig. 4.40(*b*). The pattern of the magnetic field indicates the force of attraction between two parallel conductors carrying current in the same direction and force of repulsion between two parallel conductors carrying current in opposite directions.

FIGURE 4.40



Definition of ampere

Relation (13) is used to define 1 ampere.

Let $I_1 = I_2 = 1$ A ; $r = 1$ m. Then,

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} = \frac{10^{-7}\times 2\times 1\times 1}{1} \quad \left(\because \frac{\mu_0}{4\pi} = 10^{-7}\right)$$

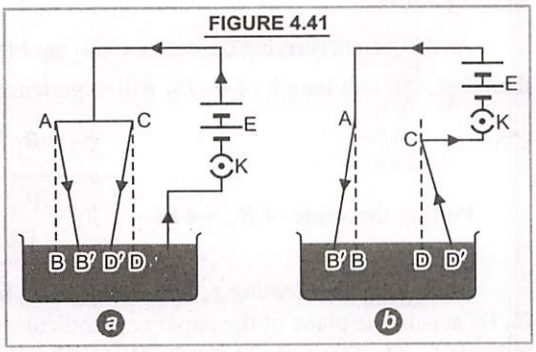
$$= 2\times 10^{-7}\ \text{Nm}^{-1}$$

Thus **one ampere** ***is that much current which when flowing through each of the two parallel linear conductors placed in free space at a distance of one metre from each other will attract or repel each other with a force of 2×10^{-7} N per metre of their length.***

4.24. EXPERIMENTAL DEMONSTRATION OF THE MAGNETIC FORCE BETWEEN PARALLEL CURRENTS

Experiment I. Take two metallic wires *AB* and *CD*. Suspend them vertically parallel to each other. The lower ends of the wires are kept inside mercury in a trough. The current in the wires can be passed with the help of battery *E*, and a key *K* in series.

FIGURE 4.41



In Fig. 4.41(*a*), when key *K* is closed, the vertical wires *AB* and *CD* bend towards each other and take the positions as shown by *AB′* and *CD′*. This shows that the **like currents attract each other.**

In Fig. 4.41(*b*), when key K is closed, the vertical wires move away from each other and take the positions as shown by *AB′* and *CD′*. This shows that the **unlike currents repel each other.**

Experiment II : Roget's Spiral

This apparatus is used to demonstrate the force of attraction between two parallel wires carrying currents in the same direction. The apparatus used is shown in Fig. 4.42, where *S* is a soft metallic spring whose

natural period of oscillation is about 0·5 s to 1 s. It is suspended vertically from a rigid support. The lower end of the spring has a pointed tip *B*. *D* is a dish of brass having some mercury in it. *E* is a source of direct current, *i.e.*, a battery which is connected between *A* and *D* through a key *K*.

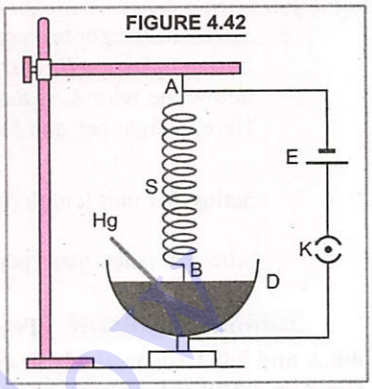


FIGURE 4.42

Working. Adjust the spring so that the tip *B* of the spring just touches the surface of mercury in dish. On closing key *K*, the tip of the spring starts dancing up and down. It can be understood as follows :

When tip of spring is just touching the mercury surface in dish and the key *K* is closed, the circuit is completed. The current begins to flow through the spring. The current through the spring will be equivalent to parallel currents through the turns of spring, flowing in the same direction. The turns of spring attract eachother, and the spring contracts. The tip of the spring moves up and goes out of the surface of mercury in dish. Now the circuit is broken, the current stops flowing. The spring relaxes and tries to comeback to its original position. The tip of spring again touches mercury surface in dish, thereby completing the circuit. The cycle continues with tick, tick, tick ..... and the spring continues dancing.

Note. While doing Roget's experiment, the students are advised to keep their faces away from mercury vapours as they are poisonous. Students are also advised not to perform this experiment for a long time because in that situation they may inhale mercury vapours for long time which is very dangerous for health.

Retain in Memory

1. If two linear current carrying conductors of unequal length are held parallel to each other, then the **force on long conductor is due to magnetic field interaction due to current of short conductor and long conductor.**

   If l, L = length of short and long conductor respectively

   I_1, I_2 = currents through short and long conductors

   r = separation between these two parallel conductors

   (*i*) Then force on long conductor = force on short conductor $= \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{r} l$

   (*ii*) The force on each conductor is same in magnitude and opposite in direction *i.e.* the forces on two conductors obey Newton's third law of motion.

2. When two currents approach a point or they go away from that point, then they experience an attractive force. Fig. 4.43.

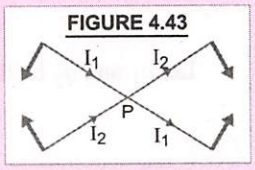


FIGURE 4.43

3. When one current out of the two, approaches a point and another one goes away from that point, then they experience a force of repulsion, Fig. 4.44.

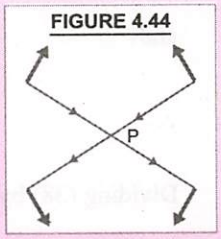


FIGURE 4.44

4. When two charges of values q_1 and q_2 moving with velocities v_1 and v_2 on parallel straight paths distance r apart, then the force acting between them is given by

$$F = \frac{\mu_0}{4\pi} \frac{q_1 q_2 v_1 v_2}{r^2}$$

Proof. When a charge q moving with velocity v for time dt covers a distance dl, it will behave as a current element $I\,dl$, where

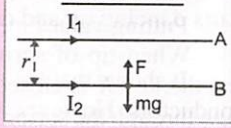
$$I\,dl = \frac{q}{dt} dl = q \frac{dl}{dt} = qv$$

If I_1, I_2 are the currents corresponding to motion of charges q_1 and q_2 with velocity v_1 and v_2, then $I_1\,dl_1 = q_1 v_1$ and $I_2\,dl_2 = q_2 v_2$.

The magnitude of magnetic force acting between these two parallel current elements distance r apart is given by $F_m = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2} dl_1\, dl_2$ or $F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2}{r^2} v_1 v_2$

5. A fixed horizontal wire A carrying current I_1 will support another horizontal wire B, carrying current I_2, hanging in air at a distance r below the wire A, if the direction of currents in the two wires is same. Here, weight per unit length of wire B is equal to the force of attraction acting per unit length on the wire B, *i.e.*, $mg = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$

FIGURE 4.45



where m is the mass per unit length of wire B and g is the acceleration due to gravity.

Sample Problem **Two straight wires A and B of lengths 10 m and 16 m and carrying currents 4·0 A and 5·0 A respectively in opposite directions, lie parallel to each other 4·0 cm apart. Compute the force on a 10 cm long section of the wire B near its centre.**

Sol. The magnitude of repulsive force per unit length on each wire is

$$\frac{F}{l} = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r} = 10^{-7} \times \frac{2 \times 4{\cdot}0 \times 5{\cdot}0}{(4 \times 10^{-2})} = 10^{-4}\ \text{Nm}^{-1}$$

$\therefore$ Force on a 10 cm length of the wire $B = 10^{-4} \times 0{\cdot}1 = \mathbf{10^{-5}\ N}$

4.25. COMPARISON OF ELECTRICAL AND MAGNETIC FORCES

From Coulomb's law, the magnitude of electrostatic force between two charges q_1 and q_2 separated by distance r is given by

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \quad \text{...(36)}$$

Consider two parallel conducting wires of length dl_1 and dl_2 carrying currents I_1 and I_2 separated by distance r.

The magnitude of the magnetic force acting between these two current elements is given by

$$F_m = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2} dl_1\, dl_2 \quad \text{...(37)}$$

Let q_1 and q_2 be the charges flowing for time t in wires for currents I_1 and I_2. Then

$$I_1 dl_1 = \frac{q_1}{t}\cdot dl_1 = q_1 v_1$$

and $$I_2 dl_2 = \frac{q_2}{t}\cdot dl_2 = q_2 v_2$$

$\therefore$ $$F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{r^2} \quad \text{...(38)}$$

Dividing (38) by (36) we get $$\frac{F_m}{F_e} = v_1 v_2 (\mu_0 \epsilon_0) \quad \text{...(39)}$$

Since left hand side is dimensionless, therefore, right hand side of (39) must also be dimensionless. It means the quantity $\mu_0 \epsilon_0$ must have the dimensions of (velocity)$^{-2}$. Here v_1 and v_2 are the drift velocities of electrons in current elements. As drift velocities are of the order of 10^{-4} ms^{-1}, therefore

$$v_1 v_2 = 10^{-4} \times 10^{-4} = 10^{-8}\ \text{m}^2\ \text{s}^{-2}$$

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It is also found that $c^2 = \dfrac{1}{\mu_0 \in_0}$ or $\mu_0 \in_0 = c^{-2}$ ...(39a*)

where c is the velocity of light in vacuum, *i.e.*, $c = 3 \times 10^8$ ms^{-1}

Putting values in (39), we get $\dfrac{F_m}{F_e} = \dfrac{10^{-8}}{(3\times10^8)^2} = 10^{-25}$

It shows that the magnetic forces are very much smaller than the electrostatic forces in current carrying conductors. However, the electric forces never dominate the magnetic forces in current carrying conductors because

(*i*) the matter is electrically neutral to very high degree of accuracy. The flowing charges do not disturb the neutrality of the matter. Thus the Coulomb's electric forces are very rarely in evidence.

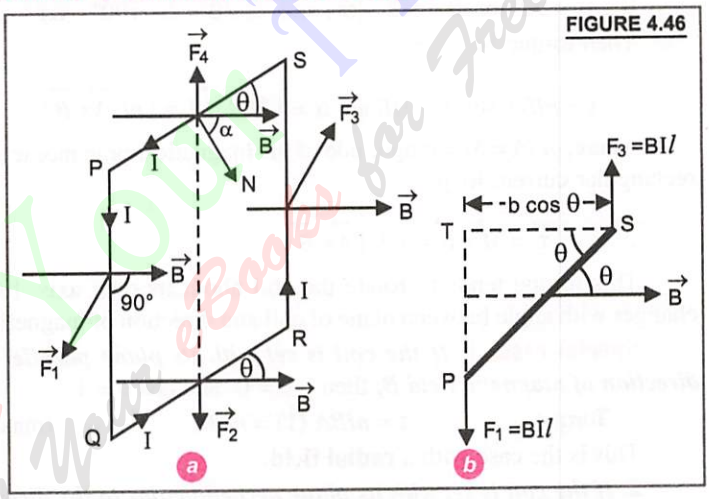
(*ii*) In an electric current, the large number of electrons are drifting towards the same direction. This adds up weak magnetic fields which become evident.

Note that if v_1 and v_2 were comparable to the speed of light, the magnetic and electric forces will be comparable.

4.26. TORQUE ON A CURRENT CARRYING COIL IN A MAGNETIC FIELD

Consider a rectangular coil $PQRS$ suspended in a uniform magnetic field of induction $\vec{B}$. Let $PQ = RS = l$ and $QR = SP = b$. Let I be the current flowing through the coil in the direction $PQ\,RS$ and θ be the angle which plane of the coil makes with the direction of magnetic field Fig. 4.46(*a*).

FIGURE 4.46



Let $\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ and $\vec{F_4}$ be the forces acting on the four current carrying arms PQ, QR, RS and SP of the coil, respectively.

The force on arm SP is given by,

$$\vec{F_4} = I\,(\overrightarrow{SP} \times \vec{B})$$

or $F_4 = I\,(SP)\,B \sin(180° - \theta) = Ib\,B \sin\theta$

The direction of this force is in the direction of $(\overrightarrow{SP} \times \vec{B})$ *i.e.* in the plane of coil directed upwards.

The force on the arm QR is given by, $\vec{F_2} = I\,(\overrightarrow{QR} \times \vec{B})$ or $F_2 = I\,(QR)\,B \sin\theta = I\,b\,B \sin\theta$

The direction of this force is in the plane of the coil directed downwards.

Since the forces $\vec{F_2}$ and $\vec{F_4}$ are equal in magnitude and acting in opposite directions along the same straight line, they cancel out each other *i.e.* their resultant effect on the coil is zero.

Now, the force on the arm PQ is given by

$$\vec{F_1} = I\,(\overrightarrow{PQ} \times \vec{B}) \quad \text{or} \quad F_1 = I\,(PQ)\,B \sin 90° = I\,l\,B \qquad (\because \overrightarrow{PQ} \perp \vec{B})$$

According to Fleming's Left Hand Rule, the direction of this force is perpendicular to the plane of the coil directed towards the reader.

---

* $\mu_0 \in_0 = \dfrac{\mu_0}{4\pi} \times 4\pi\in_0 = 10^{-7} \times \dfrac{1}{9\times10^9} = \dfrac{1}{9\times10^{16}} = \dfrac{1}{(3\times10^8)^2} = \dfrac{1}{c^2} = c^{-2}$ where c is the velocity of light.

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And, force on the arm RS is given by

$$\vec{F}_3 = I(\overrightarrow{RS} \times \vec{B}) \quad \text{or} \quad F_3 = I(PQ)B\sin 90° = I l B \qquad (\because \overrightarrow{RS} \perp \vec{B})$$

The direction of this force, according to Fleming's Left Hand Rule, is perpendicular to the plane of coil and directed away from the reader.

The forces acting on the arms PQ and RS are equal, parallel and acting in opposite directions having different lines of action. These forces form a ***couple,*** the effect of which is to rotate the coil in the anticlockwise direction about the dotted line as axis.

The torque on the coil (equal to moment of couple) is given by τ = either force × arm of the couple.

The forces F_1 and F_3 acting on the arms PQ and RS will be as shown in Fig. 4.46(b) when seen from the top. Arm of couple = $ST = PS\cos\theta = b\cos\theta$.

$\therefore \quad \tau = I\,lB \times b\cos\theta = IBA\cos\theta$

$(\because\ l \times b = A$ = area of coil $PQRS)$

If the rectangular coil has n turns, then $\tau = n\,I\,B\,A\cos\theta$.

Note that if the normal drawn on the plane of the coil makes an angle α with the direction of magnetic field, then

$\theta + \alpha = 90°$ or $\theta = 90° - \alpha$; and $\cos\theta = \cos(90° - \alpha) = \sin\alpha$

Then torque becomes,

$$\tau = nIBA\sin\alpha = MB\sin\alpha = |\vec{M} \times \vec{B}| = |nI\,\vec{A} \times \vec{B}|$$

where, $n\,IA = M$ = magnitude of the magnetic dipole moment of the rectangular current loop

$$\therefore \quad \vec{\tau} = \vec{M} \times \vec{B} = n\,I\,(\vec{A} \times \vec{B})$$

This torque tends to rotate the coil about its own axis. Its value changes with angle between plane of coil and direction of magnetic field.

Special cases 1. ***If the coil is set with its plane parallel to the direction of magnetic field B,*** then $\theta = 0°$ and $\cos\theta = 1$

$\therefore$ Torque, $\tau = nIBA\,(1) = nIBA$ (maximum)

This is the case with a **radial field.**

2. ***If the coil is set with its plane perpendicular to the direction of magnetic field B,*** then

$\theta = 90°$ and $\cos\theta = 0$

$\therefore$ Torque, $\tau = nIBA\,(0) = 0$ (Minimum)

Sample Problem **A circular coil of 20 turns and radius 10 cm carries a current of 5 A. It is placed in a uniform magnetic field of 0·10 T. Find the torque acting on the coil when the magnetic field is applied (a) normal to the plane of the coil (b) in the plane of coil. Also find out the total force acting on the coil.**

Sol. Here; $n = 20$; $r = 0{\cdot}10$ m ; $I = 5$ A ; $B = 0{\cdot}10$ T

Area of each turn of coil, $A = \pi r^2 = \frac{22}{7} \times (0{\cdot}1)^2 = 0{\cdot}0314\ \text{m}^2$

Torque acting on the coil, $\tau = n\,I\,AB\sin\alpha$

(a) Here, $\alpha = 0°$ so $\tau = 20 \times 5 \times (0{\cdot}0314) \times 0{\cdot}1 \times \sin 0° = 0$

(b) Here, $\alpha = 90°$ so $\tau = 20 \times 5 \times (0{\cdot}0314) \times 0{\cdot}1 \times \sin 90° = \mathbf{0{\cdot}314\ N\text{-}m.}$

Coil carrying current acts as a magnetic dipole. The force acting on magnetic dipole in a uniform magnetic field is **zero.**

DO YOU KNOW ?

1. The net magnetic force on a current loop in a uniform magnetic field is zero but torque may or may not be zero.
2. The net magnetic force on a current loop in a non-uniform magnetic field is not zero but torque may or may not be zero.
3. When a current loop is in the position of equilibrium in a uniform magnetic field, then the net magnetic force as well as torque on current loop are zero.
4. Torque on the coil is maximum, when coil is suspended in a radial magnetic field.

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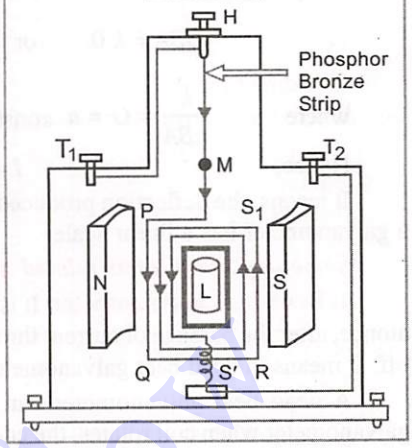
4.27. MOVING COIL GALVANOMETER

Moving coil galvanometer is an instrument used for the detection and measurement of small electric currents. It is also called a **dead beat galvanometer.**

Principle. Its working is based on the fact that when a current carrying coil is placed in a uniform magnetic field, it experiences a torque.

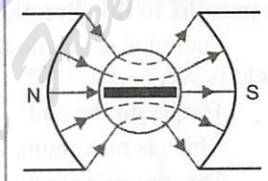
Construction. It consists of a coil $PQRS_1$ having large number of turns of insulated copper wire, Fig. 4.47. The coil is wound over a non-magnetic metallic frame (usually brass) which may be rectangular or circular in shape. The coil is suspended from a movable torsion head H by means of a suspension fibre of phosphor bronze strip in a uniform magnetic field produced by two strong cylindrical magnetic pole pieces N and S.

FIGURE 4.47



The lower end of the coil is connected to one end of a hair spring S' of quartz or phosphor bronze. The other end of this highly elastic spring* S' is connected to a terminal T_2. L is a soft iron core which may be spherical if the coil is circular and cylindrical, if the coil is rectangular. It is so held within the coil, that the coil can rotate freely without touching the iron core and pole pieces. This makes the magnetic field linked with coil to be **radial field** *i.e.* the plane of the coil in all positions remains parallel to the direction of magnetic field, Fig. 4.48. M is concave mirror attached to the phosphor bronze strip. This helps us to note the deflection of the coil using lamp and scale arrangement. The whole arrangement is enclosed in a non-metallic case to avoid disturbance due to air, etc. The case is provided with levelling screws at the base. The torsion head is connected to terminal T_1. The galvanometer can be connected to the circuit through terminals T_1 and T_2.

FIGURE 4.48



Radial Field (Top View of The Galvanometer)

Theory.

Let, l = length PQ or RS_1 of the coil, b = breadth QR or S_1P of the coil, n = number of turns in the coil.

Area of each turn of the coil, $A = l \times b$.

Let , B = strength of the magnetic field in which coil is suspended.

I = current passing through the coil in the direction $PQRS_1$ as shown in Fig. 4.47.

Let at any instant, α be the angle which the normal drawn on the plane of the coil makes with the direction of magnetic field.

As already discussed, the rectangular coil carrying current when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = nIBA \sin \alpha$.

If the magnetic field is radial *i.e.* the plane of the coil is parallel to the direction of the magnetic field then $\alpha = 90°$* and $\sin \alpha = 1$. $\therefore$ $\tau = nIBA$

Due to this torque, the coil rotates. The phosphor bronze strip gets twisted. As a result of it, a restoring torque comes into play in the phosphor bronze strip, which would try to restore the coil back to its original position.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze strip, then total restoring torque produced = $k\,\theta$

*The spring S' does three jobs for us : (*i*) It provides passage of current for the coil $PQRS_1$ (*ii*) It keeps the coil in position and (*iii*) generates the restoring torque on the twisted coil.

In equilibrium position of the coil, deflecting torque = restoring torque

$$\therefore \quad nIBA = k\,\theta \quad \text{or} \quad I = \frac{k}{nBA}\,\theta \quad \text{or} \quad I = G\,\theta$$

where $\frac{k}{nBA} = G = a$ constant for a galvanometer. It is known as **galvanometer constant.**

Hence, $I \propto \theta$

It means, the deflection produced is proportional to the current flowing through the galvanometer. Such a galvanometer has a linear scale.

Some important terms related to Moving Coil Galvanometer

1. Dead beat galvanometer. It is that galvanometer in which the coil in the deflected state comes to rest atonce, after the passage of current through it and attains its initial position of rest atonce if current is switched off. It means in dead beat galvanometer, the deflection can be noted in no time.

A dead beat galvanometer can be achieved by winding the coil on a conducting frame. In such galvanometer when coil rotates, the eddy currents are set up in metallic frame which produces damping effect on the coil and coil comes to rest soon.

2. Radial magnetic field. It is that magnetic field in which the plane of the coil in all the positions is parallel to the direction of magnetic field.

For radial magnetic field, the angle between the normal to the plane of the coil and direction of magnetic field is 90°, *i.e.*, $\alpha = 90°$.

Torque on the coil, $\tau = nIBA \sin 90° = nIBA$

which is maximum. In this situation, $\tau \propto I$.

Thus by using radial magnetic field in galvanometer, the deflection of the coil is proportional to the current flowing through it. As a result of it, a linear scale can be used to determine the deflection of the coil in galvanometer.

A **radial magnetic field can be achieved in galvanometer** by (*i*) properly cutting the magnetic pole pieces in the shape of concave faces (*ii*) using the soft iron core within the coil.

3. Properties of Suspension fibre used in galvanometer

(*i*) It should have high tensile strength.

(*ii*) It should have high electrical conductivity.

(*iii*) It should be rustless with time.

(*iv*) Its temperature coefficient of resistance should be small.

(*v*) Its restoring torque per unit twist should be small.

4. Sensitivity of galvanometer

(*a*) **Current sensitivity** *of a galvanometer is defined as the deflection produced in the galvanometer when a unit current flows through it.*

If θ is the deflection in the galvanometer when current I is passed through it, then

Current sensitivity, $I_s = \frac{\theta}{I} = \frac{nBA}{k}$ $\left(\because I = \frac{k}{nBA}\,\theta\right)$

The unit of current sensitivity is rad. A^{-1} or div. A^{-1}.

(*b*) **Voltage sensitivity** *of a galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.*

Let, V = voltage applied across the two terminals of the galvanometer,

θ = deflection produced in the galvanometer.

Then, voltage sensitivity, $V_s = \theta/V$

*θ = 0°, θ + α = 90° implies α = 90°.

If R = resistance of the galvanometer, I = current through it.

Then $V = IR$

$\therefore$ Voltage sensitivity, $$V_s = \frac{\theta}{IR} = \frac{nBA}{kR} = \frac{I_s}{R}$$

The unit of V_s is rad V^{-1} or div. V^{-1}.

Conditions for a sensitive galvanometer.

A galvanometer is said to be very sensitive if it shows large deflection even when a small current is passed through it.

From the theory of galvanometer, $$\theta = \frac{nBA}{k} I$$

For a given value of I, θ will be large if nBA/k is large. It is so if

(a) n is large (b) B is large (c) A is large and (d) k is small.

(a) ***The value of n can not be increased beyond a certain limit*** because it results in an increase of the resistance of the galvanometer and also makes the galvanometer bulky. This tends to decrease the sensitivity. Hence n can not be increased beyond a limit.

(b) ***The value of B can be increased*** by using a strong horse shoe magnet.

(c) ***The value of A can not be increased beyond a limit*** because in that case the coil will not be in a uniform magnetic field. Moreover, it will make the galvanometer bulky and unmanageable.

(d) ***The value of k can be decreased.*** The value of k depends upon the nature of the material used as suspension strip. The value of k is very small for quartz or phosphor bronze. That is why, in sensitive galvanometer, quartz or phosphor bronze strip is used as a suspension strip.

Uses of moving coil galvanometer

1. It is used to detect and measure a very small current.

2. It can be converted into ammeter by connecting a low resistance shunt in parallel to it.

3. It can be converted into voltmeter by connecting a high resistance in series of it.

4. It can be converted into Avometer (*i.e.*, ammeter, voltmeter and ohm meter) by using special arrangement of resistance in it.

Sample Problem **A current of 300 μA deflects the coil of a moving coil galvanometer through 60°. What should be the current to cause rotation through π/5 rad ? Determine the sensitivity of this galvanometer.**

Sol. Here, $I_1 = 300\ \mu A$; $\theta_1 = 60°$; $\theta_2 = \frac{\pi}{5}$ rad $= \frac{180°}{5} = 36°$; $I_2 = ?$

As $$I = \frac{k}{n\,BA}\theta \quad \text{or} \quad I \propto \theta \qquad \left[\because \frac{k}{n\,BA} = \text{a constant}\right]$$

so $$\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1} \quad \text{or} \quad I_2 = \frac{\theta_2}{\theta_1} = I_1 = \frac{36}{60} \times 300 = \mathbf{180\ \mu A}$$

Current sensitivity, $$I_s = \frac{\theta_2}{I_2} = \frac{36°}{180\ \mu A} = 0{\cdot}2°/\mu A$$

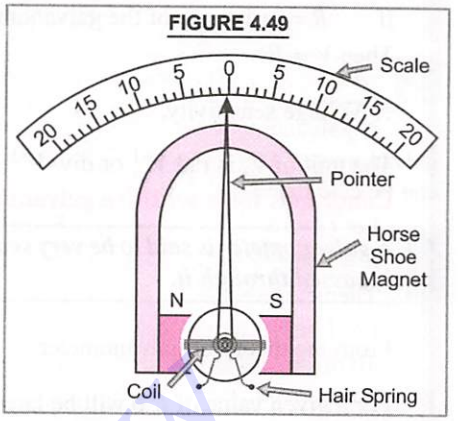
4.28. WESTON TYPE GALVANOMETER

It is an electrical instrument which is used for the detection and measurement of current flowing through the circuit. A weston type galvanometer is most commonly used in laboratory and is essentially **a moving coil, pivoted type dead beat galvanometer.** It consists of a coil of many turns of fine insulated copper wire wound over a metallic frame. The coil is mounted on two jewelled pivots and is symmetrically

placed between the cylindrical pole pieces of a strong permanent horse-shoe magnet. The motion of the coil is controlled by two hair springs, one above and other below the coil, which are soldered to the ends of the coil. These hair springs also serve as leads for the current. An aluminium pointer is attached to its moving part which moves on a scale with zero at its centre and equal divisions (generally 25 or 30) on either side of central zero. Fig. 4.49.

When current is passed through the galvanometer, its coil experiences a torque and gets rotated. As a result of it, the hair springs get twisted, consequently the pointer shows a reading on the scale. This galvanometer is very sensitive and gives full scale deflection for a small current.

FIGURE 4.49



4.29. SHUNT

***Shunt** is a low resistance connected in parallel with the galvanometer or ammeter. It protects the galvanometer or ammeter from the strong currents.*

If the current flowing in a circuit is strong, a galvanometer or a low range ammeter can not be put directly in series with the circuit because that will damage the instrument. To overcome this difficulty, a low resistance (*i.e.*, shunt) is connected in parallel with this instrument. Then a major portion of the current passes through this low resistance (*i.e.*, shunt) and only a small portion of current passes through the instrument. Due to it, the galvanometer or an ammeter remains safe.

To find the current through the galvanometer circuit, let

I = total current in the circuit, G = resistance of the galvanometer, S = resistance of the shunt, I_g, I_s = currents through galvanometer and shunt respectively. It is clear that

$$I = I_g + I_s \quad \text{or} \quad I_s = I - I_g \quad \text{or} \quad I_g = I - I_s \qquad \text{...(40)}$$

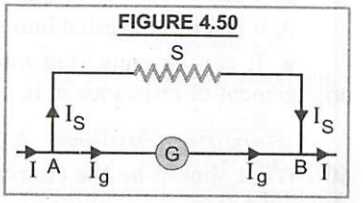
According to ohm's law,

Potential difference between points A and $B = I_g G = I_s S$ ...(41)

or $I_g G = (I - I_g) S$

or $I_g (G + S) = IS$ or $\boxed{I_g = I\left(\frac{S}{G+S}\right)}$

FIGURE 4.50



Putting the value of I_g from (18) in (19), we get

$$(I - I_s) G = I_s S \quad \text{or} \quad IG = I_s (G + S) \quad \text{or} \quad \boxed{I_s = I\left(\frac{G}{G+S}\right)}$$

Uses of shunt. 1. A shunt is used to protect the galvanometer from the strong currents.

2. A shunt is used for converting a galvanometer into an ammeter.

3. A shunt may be used for increasing the range of ammeter.

Sample Problem **A galvanometer has a resistance of 50 Ω. A resistance of 5 Ω is connected across its terminals. What part of the total current will flow through the galvanometer ?**

(Hr. Board 2001)

Sol. Here, $G = 50\ \Omega$; $S = 5\ \Omega$

As $S = \frac{I_g G}{I - I_g}$ or $\frac{I_g}{I} = \frac{S}{G+S} = \frac{5}{50+5} = \frac{1}{11}$

Fraction of total current through galvanometer is $= \frac{I_g}{I} = \frac{1}{11}$

4.30. AMMETER

An ammeter is a low resistance galvanometer. It is used to measure the current in a circuit.

A galvanometer can be converted into an ammeter by using a low resistance wire in parallel with the galvanometer. The resistance of this wire (called the shunt wire) depends upon the range of the ammeter and can be calculated as follows :

Let G = resistance of galvanometer, n = number of scale divisions in the galvanometer,

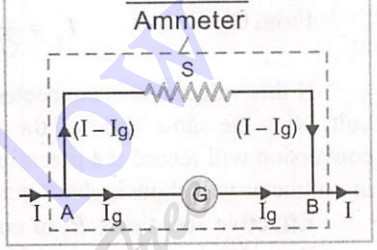
K = **figure of merit** or current for one scale deflection in the galvanometer.

Then current which produces full scale deflection in the galvanometer, $I_g = nK$.

Let I be the maximum current to be measured by galvanometer.

To do so, a shunt of resistance S is connected in parallel with the galvanometer so that out of the total current I, a part I_g should pass through the galvanometer and the remaining part $(I - I_g)$ flows through the shunt, Fig. 4.51.

FIGURE 4.51
Ammeter



$$V_A - V_B = I_g G = (I - I_g)\, S \quad \text{or} \quad S = \left(\frac{I_g}{I - I_g}\right) G \qquad ...(42)$$

Thus S can be calculated.

If this value of shunt resistance S is connected in parallel with galvanometer, it works as an ammeter for the range 0 to I ampere. Now the same scale of the galvanometer which was recording the maximum current I_g before conversion into ammeter will record the maximum current I, after conversion into ammeter. It means each division of the scale in ammeter will be showing higher* current than that of galvanometer.

The effective resistance R_p of ammeter (*i.e.* shunted galvanometer) will be

$$\frac{1}{R_p} = \frac{1}{G} + \frac{1}{S} = \frac{S+G}{GS} \quad \text{or} \quad R_p = \frac{GS}{G+S}$$

As the shunt resistance is low, the combined resistance of the galvanometer and the shunt is very low and hence ammeter has a much lower resistance than galvanometer. **An ideal ammeter has zero resistance.**

Sample Problem **An ammeter of resistance 0·80 Ω can measure current upto 1·0 A.**

(*a*) What must be the value of shunt resistance to enable the ammeter to measure current upto 5·0 A ?

(*b*) What is the combined resistance of the ammeter and the shunt ? **(CBSE 2013)**

Sol. Here, $G = 0{\cdot}80\ \Omega$; $I_g = 1{\cdot}0$ A ; $I = 5{\cdot}0$ A ; S.

(*a*) $$S = \frac{I_g G}{I - I_g} = \frac{1 \times 0{\cdot}8}{5 - 1} = 0{\cdot}2\ \Omega$$

(*b*) Combined resistance of shunt and ammeter $= \dfrac{GS}{G+S} = \dfrac{0{\cdot}8 \times 0{\cdot}2}{0{\cdot}8 + 0{\cdot}2} = 0{\cdot}16\ \Omega$

4.31. VOLTMETER

A voltmeter is a high resistance galvanometer. It is used to measure the potential difference between any two points of a circuit.

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with the galvanometer. The value of the resistance depends upon the range of voltmeter and can be calculated as follows :

*Initial reading of each division of galvanometer to be used as ammeter is I_g/n and the reading of the same one division after conversion into ammeter is I/n.

Let, G = resistance of galvanometer, n = number of scale divisions in the galvanometer,

K = figure of merit of galvanometer, *i.e.*, current for one scale deflection of the galvanometer.

$\therefore$ Current which produces full scale deflection in the galvanometer,

$$I_g = nK.$$

Let V be the potential difference to be measured by galvanometer.

To do so, a resistance R of such a value is connected in series with the galvanometer so that if a potential difference V is applied across the terminals A and B, a current I_g flows through the galvanometer. Fig. 4.52.

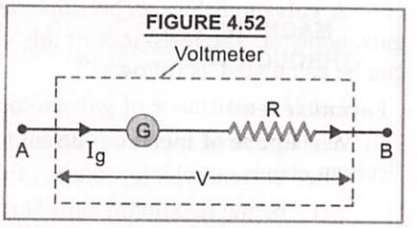


FIGURE 4.52

Now, total resistance of voltmeter = $G + R$

From Ohm's law, $I_g = \dfrac{V}{G+R}$ or $G+R = \dfrac{V}{I_g}$ or $R = \dfrac{V}{I_g} - G$ ...(43)

If this value of R is connected in series with galvanometer, it works as a voltmeter of the range 0 to V volt. Now the same scale of the galvanometer which was recording the maximum potential $I_g G$ before conversion will record the potential V after conversion into voltmeter. It means each division of the scale in voltmeter will show higher* potential than that of the galvanometer.

Effective resistance R_s of converted galvanometer into voltmeter is $R_s = G + R$

For voltmeter, a high resistance R is connected in series with the galvanometer, therefore, the resistance of voltmeter is very large as compared to that of galvanometer. **The resistance of an ideal voltmeter is infinity.**

Retain in Memory

1. The ammeter is connected in series and voltmeter in parallel in the circuit.
2. The range of ammeter can be increased but can not be decreased.
3. The ammeter of lower range has higher resistance than the ammeter of higher range.
4. If the ammeter is connected in parallel to the circuit, the circuit resistance will reduce. The current in the circuit will increase. Now ammeter will draw more amount of current due to its small resistance. As a result of it may get damaged.
5. If the ammeter range is to be increased n times its original range, the value of shunt resistance should be $S = \dfrac{G}{n-1}$.
6. The range of voltmeter can be increased or decreased.
7. The voltmeter of lower range has lower resistance than the voltmeter of higher range.
8. If voltmeter is connected in series to the circuit, the resistance of circuit will become high and current in circuit will reduce. Now the voltmeter will measure the *emf* of battery connected in circuit.
9. If the voltmeter range is to be increased to n times its original value, the series resistance to be connected should be, $R = (n - 1)\, G$.

Sample Problem **A galvanometer with a coil of resistance 12 Ω shows a full scale deflection for a current of 2·5 mA. How will you convert it into a voltmeter of range 7·5 V ? Also, find the total resistance of voltmeter formed.**

Sol. A galvanometer can be converted into a voltmeter of given range by connecting a suitable resistance R in series with galvanometer, which is given by, $R = \dfrac{V}{I_g} - G = \dfrac{7{\cdot}5}{2{\cdot}5 \times 10^{-3}} - 12 = \mathbf{2988\ \Omega}$

Total resistance of voltmeter formed = $R + G = 2988 + 12 = \mathbf{3000\ \Omega}$

*Initial reading of each division of galvanometer to be used as voltmeter is $I_g\, G/n$ and the reading of same each division after conversion into voltmeter is $I_g\, (G + R)/n$.

SOLVED EXAMPLES

TYPE I. BIOT-SAVART'S LAW AND MAGNETIC FIELD DUE TO CURRENT THROUGH THE STRAIGHT CONDUCTOR

Formulae used

(*i*) Magnetic field induction dB at a point due to current element is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

where $I\,dl$ is the current element and r is the distance of point from the current element. θ is the angle between $\vec{dl}$ and $\vec{r}$.

(*ii*) Magnetic field induction at a point due to linear conductor carrying current I is,

$$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2)$$

where a is the perpendicular distance of point from the linear conductor. ϕ_1, ϕ_2 are the angles shown in Fig. 4.8.

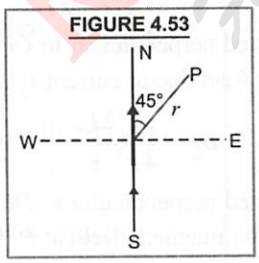
(*iii*) In case of infinite long conductor, $B = \frac{\mu_0}{4\pi}\frac{2I}{a}$

Units used. B is in tesla, I in ampere, a or r in metre.

Example 1 **A current of 5 A is flowing from south to north in a straight wire. Find the magnetic field due to a 1 cm piece of wire at a point 1 m north east from the piece of wire.**

Solution. Refer to Fig. 4.53, P is the point at which magnetic field is to be determined.

FIGURE 4.53



Here, $I = 5$ A, $dl = 1$ cm $= 0{\cdot}01$ m ;
$r = 1$ m, $\theta = 45°$

As, $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

$$\therefore \quad dB = \frac{10^{-7}\times 5\times(0{\cdot}01)\times\sin 45°}{(1)^2}$$

$$= \mathbf{3{\cdot}54\times 10^{-9}\ T}$$

Its direction is **vertically downwards.**

Example 2 **An element $\vec{dl} = dx\,\hat{i}$ (where $dx = 1$ cm) is placed at the origin and carries a large current $i = 10$ A. What is the magnetic field on the Y-axis at a distance of 0·5 m ?**

(NCERT Solved Example)

Solution. Here, $dl = dx = 1$ cm $= 10^{-2}$ m ;

$$I = 10\text{ A} ;\ r = 0{\cdot}5\text{ m} ;\ \vec{r} = r\,\hat{j}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,dx}{r^2}(\hat{i}\times\hat{j})$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dx}{r^2}\hat{k}$$

$$\therefore \quad \vec{dB} = \frac{10^{-7}\times 10\times 10^{-2}}{(0{\cdot}5)^2}\hat{k} = \mathbf{4\times 10^{-8}\ T\,\hat{k}}$$

Example 3 **A current element 3 dl is at (0, 0, 0) along y-axis. If $dl = 1$ cm, find the magnetic field at a distance 20 cm on the x-axis.**

Solution. Here, $dl = 1$ cm $= 10^{-2}$ m,

$$r = (20\text{ cm}) = (0{\cdot}20\text{ m}) \text{ and } \vec{r} = 0{\cdot}2\,\hat{i}$$

$$I\,\vec{dl} = (3\times 10^{-2})\,\hat{j}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3}$$

$$= \frac{10^{-7}\times(3\times 10^{-2}\,\hat{j})\times(0{\cdot}20\,\hat{i})}{(0{\cdot}20)^3}$$

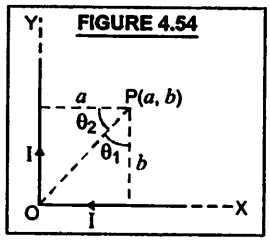
$$= 7{\cdot}5\times 10^{-8}(\hat{j}\times\hat{i}) = \mathbf{7{\cdot}5\times 10^{-8}(-\hat{k})}$$

$$= \mathbf{-(7{\cdot}5\times 10^{-8}\ T)\hat{k}}$$

Thus magnitude of magnetic field is **$7{\cdot}5\times 10^{-8}$ T** and its direction is along **negative z-axis.**

Example 4 **Two semi infinitely long straight current carrying conductors are held in the form as shown in Fig. 4.54. One common end**

of them is at the origin. If both the conductors carry same current I, find the value of the magnetic field induction at a point (a, b).



Solution. Magnetic field induction at point P due to current through the conductor along X-axis is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{b}\left[\sin\theta_1 + \sin\frac{\pi}{2}\right]$$

acting along negative Z-axis.

$$= \frac{\mu_0}{4\pi}\frac{I}{b}\left[\frac{a}{\sqrt{a^2+b^2}} + 1\right]$$

Magnetic field induction at point P due to current through the conductor along Y-axis is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin\theta_2 + \sin\frac{\pi}{2}\right]$$

acting along negative Z-axis.

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{b}{\sqrt{a^2+b^2}} + 1\right]$$

The resultant magnetic field induction at P is $B = B_1 + B_2$ acting along negative Z-axis.

$$= \frac{\mu_0}{4\pi}\frac{I}{b}\left[\frac{a}{\sqrt{a^2+b^2}} + 1\right] + \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{b}{\sqrt{a^2+b^2}} + 1\right]$$

$$= \frac{\mu_0 I}{4\pi}\left[\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{1}{\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right)\right]$$

or $$B = \frac{\mu_0 I}{4\pi}\left[\frac{(a+b)}{ab} + \frac{\sqrt{a^2+b^2}}{ab}\right]$$

$$= \frac{\mu_0 I}{4\pi ab}\left[(a+b) + \sqrt{a^2+b^2}\right]$$

Example 5 Same current I is flowing in three infinitely long wires along x, y and z directions. What is the magnetic field at point $(0, 0, -a)$?

Solution. The point $(0, 0, -a)$ lies on z-axis. Therefore the magnetic field induction at the given point due to current along z-axis is zero.

The magnetic field induction due to current along x-axis at the given point is $\vec{B}_x = \frac{\mu_0}{4\pi}\frac{I}{a}\hat{j}$

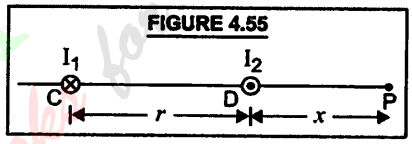
The magnetic field induction due to current along y-axis at the given point is

$$\vec{B}_y = \frac{\mu_0}{4\pi}\frac{I}{a}(-\hat{i})$$

$\therefore$ Total magnetic field induction,

$$\vec{B} = \vec{B}_x + \vec{B}_y = \frac{\mu_0}{4\pi}\frac{I}{a}(\hat{j} - \hat{i})$$

Example 6 Two parallel wires C and D placed perpendicular to the plane of paper at a separation of r (= 8 cm), carry electric currents I_1 = 6 A and I_2 = 3 A in opposite directions as shown in Fig. 4.55. Find the point on the line CD where the resultant magnetic field is zero.



Solution. The required point on the line CD where the resultant magnetic field will be zero when the magnetic field at that point due to one wire must be equal and opposite to the magnetic field due to another wire. This will be possible when the required point lies on the right side of wire D. Let this point be P at distance x from D.

Magnetic field due to current I_1 at point P is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{(r+x)}$$

It is directed perpendicular to CP downwards.

Magnetic field due to current I_2 at point P is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2I_2}{x}$$

It is directed perpendicular to DP upwards.

As resultant magnetic field at P is zero so

$$B_1 = B_2 \quad \text{or} \quad \frac{\mu_0}{4\pi}\frac{2I_1}{(r+x)} = \frac{\mu_0}{4\pi}\frac{2I_2}{x}$$

or $$\frac{I_1}{(r+x)} = \frac{I_2}{x}$$

$$\text{or } x = \frac{I_2 r}{(I_1 - I_2)} = \frac{3\times(8\times10^{-2})}{6-3}$$

$$= 8 \times 10^{-2} \text{ m} = 8 \text{ cm}$$

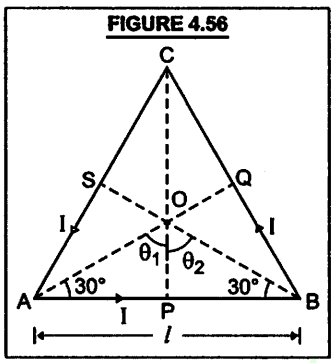
Example 7 **A current of 1·0 A flowing in the sides of an equilateral triangle of side $4{\cdot}5 \times 10^{-2}$ m. Find the magnetic field at the centroid of the triangle.**

Solution. Here, $I = 1{\cdot}0$ A, $l = 4{\cdot}5 \times 10^{-2}$ m

Fig. 4.56, $OP = OQ = OS$

$$= \frac{l}{2}\tan 30° = \frac{l}{2\sqrt{3}} = \frac{4{\cdot}5\times10^{-2}}{2\sqrt{3}} \text{ m}$$

FIGURE 4.56



Magnetic field at the centroid O of the triangle due to current through arm AB is given by

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin\theta_1 + \sin\theta_2]$$

It is directed normally outwards the plane of paper

Where, $a = OP = \dfrac{4{\cdot}5\times10^{-2}}{2\sqrt{3}}$ m

and $\theta_1 = \theta_2 = 60°$

The magnetic field due to currents in the arms AB, BC and CD is the same, in magnitude and direction, so the resultant magnetic field at O is

$$B = 3B_1 = 3\times\frac{\mu_0}{4\pi}\frac{I}{a}[\sin\theta_1 + \sin\theta_2]$$

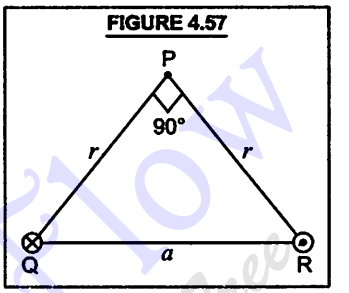
$$= 3\times10^{-7}\times\frac{1\times2\sqrt{3}}{4{\cdot}5\times10^{-2}}[\sin 60° + \sin 60°]$$

$$= \frac{6\sqrt{3}}{4{\cdot}5}\times10^{-5}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right] = 4\times10^{-5}\text{ T}$$

It is directed normally outwards, the plane of paper.

Example 8 **Fig. 4.57 shows a right-angled isosceles ΔPQR having its base equal to a. A current of I ampere is passing downwards along a thin straight wire cutting the plane of paper normally as shown at Q. Likewise a similar wire carries an equal current passing normally upwards at R. Find the magnitude and direction of the magnetic field induction B at P. Assume the wires to be infinity long.**

FIGURE 4.57



Solution. Here, $PQ = PR = r$. In rt angled ΔPQR, we have,

$$a^2 = r^2 + r^2 = 2r^2$$

$$\text{or } r = a/\sqrt{2}$$

Magnetic field at point P due to current through wire at Q is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(a/\sqrt{2})} = \frac{\mu_0 I}{\sqrt{2}\pi a}$$

It is acting along PR.

Magnetic field at point P due to current through wire at R is

$$B_2 = \frac{\mu_0 I}{\sqrt{2}\pi a}, \text{ acting along } PQ$$

As $\vec{B_1}$ and $\vec{B_2}$ are acting perpendicular to each other, so the resultant magnetic field at P is

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I}{\sqrt{2}\pi a}\right)^2 + \left(\frac{\mu_0 I}{\sqrt{2}\pi a}\right)^2}$$

$$= \sqrt{2}\frac{\mu_0 I}{\sqrt{2}\pi a}$$

$$= \frac{\mu_0 I}{\pi a}$$

It acts towards the mid-point of QR.

TYPE II. MAGNETIC FIELD DUE TO CURRENT THROUGH CIRCULAR COIL

Formulae used. (*i*) Magnetic field induction B at the centre of circular coil carrying current I is,

$$B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$$ where n = no. of turns in the coil

(*ii*) Magnetic field induction at a point on the axis of the circular coil of radius r carrying current I is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi nIr^2}{(r^2+x^2)^{3/2}}$$

where x is the distance of point on the axis of circular coil from the centre of the circular coil.

(*iii*) Magnetic field induction at the centre of an arc of radius r subtending an angle θ, carrying current I is

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta$$

Units used. B in tesla, I in ampere, r and x in metre.

Example 9 **A straight wire carrying a current of 12 A is bent into a semicircular arc of radius 2·0 cm as shown in Fig. 4.58(*a*). Consider the magnetic field $\vec{B}$ at the centre of the arc.**

(*a*) What is the magnetic field due to the straight segments ?

(*b*) In what way the contribution to $\vec{B}$ from the semicircle differs from that of a circular loop and in what way does it resemble ?

(*c*) Would your answer be different if the wire were bent into a semicircle arc of the same radius but in the opposite way as shown in Fig. 4.58(*b*)?

(NCERT Solved Example)

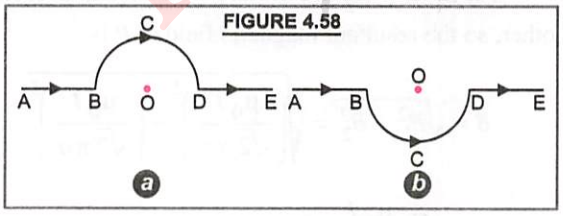


FIGURE 4.58

Solution. (*a*) For a point O, $\vec{dl}$ and $\vec{r}$ for each element of the straight segments AB and DE are parallel. Therefore, $\vec{dl}\times\vec{r} = 0$. Hence, magnetic field at O due to current through straight segments AB and DE is

$$\vec{B} = \sum \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} = 0$$

(*b*) The magnetic field induction at O due to semicircular arc is

$$B = \frac{\mu_0}{4\pi}\frac{I\pi}{r} = \frac{\mu_0}{4}\frac{I}{r} = \frac{4\pi\times10^{-7}\times12}{4\times2\times10^{-2}}$$

$$= 1{\cdot}9\times10^{-4}\ \text{T}$$

The **direction of $\vec{B}$**, according to Right hand rule is **normal to the plane of paper directed inwards.** The magnetic field induction at the centre of a circular loop of radius r carrying current I is,

$$B' = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r} = \frac{4\pi\times10^{-7}\times12}{2\times2\times10^{-2}}$$

$$= 3{\cdot}8\times10^{-4}\ \text{T}$$

It is double than that due to semi-circular arc. The direction of magnetic field induction at the centre of circular arc is the same as that due to semicircular arc if current in them is same and in same direction (*i.e.*, either clockwise or anticlockwise) and the direction of magnetic field induction at the centre of circular arc is different from that due to semicircular arc if current in them is in opposite directions.

(*c*) When the wire is in the form shown in Fig. 4.58(b), the magnetic field induction at O will remain same in magnitude ($= 1{\cdot}9\times10^{-4}$ T) but **its direction will be normal to the plane of paper directed outwards.**

Example 10 **A tightly wound 100 turn coil of radius 10 cm is carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil ?** **(NCERT Solved Example)**

Solution. Here, $n = 100$, $r = 10$ cm $= 0{\cdot}10$ m ; $I = 1$ A.

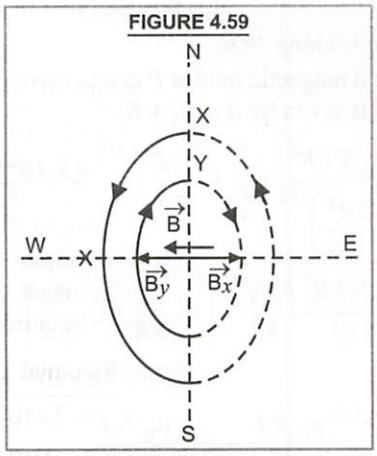
$$B = \frac{\mu_0}{4\pi}\times\frac{2\pi nI}{r} = \frac{10^{-7}\times2\pi\times100\times1}{0{\cdot}10}$$

$$= 2\pi\times10^{-4} = 6{\cdot}28\times10^{-4}\ \text{T}$$

Example 11 **Two concentric coils X and Y of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the north- south direction. Coil X has 20 turns and carries a current of 16 A ; coil Y has 25 turns and carries a current of 18 A. The sense of current in X is anti-clockwise and in Y, clockwise, for an observer looking at the**

coil facing west, Fig. 4.59. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

FIGURE 4.59



Solution. ***For coil X.***

$r = 16$ cm $= 0{\cdot}16$ m, $n = 20$, $I = 16$ A

Magnetic field induction at the centre of the coil X is given by

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r} = \frac{10^{-7}\times 2\pi\times 20\times 16}{0{\cdot}16}$$

$$= 4\pi\times 10^{-4}\ \text{T}$$

The direction of magnetic field induction $\vec{B}_x$ is towards **east.**

For coil Y.

$I = 18$ A, $n = 25$, $r = 10$ cm $= 0.10$ m

Magnetic field induction at the centre of coil Y is given by

$$B_y = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r} = \frac{10^{-7}\times 2\pi\times 25\times 18}{0.10}$$

$$= 9\pi\times 10^{-4}\ \text{T}$$

The direction of magnetic field induction B_y is towards **west**

$\therefore$ Net magnetic field $= B_y - B_x$

$= 9\pi\times 10^{-4} - 4\pi\times 10^{-4}$

$= \mathbf{5\pi\times 10^{-4} = 1{\cdot}6\times 10^{-3}\ T}$ **(Towards West)**

Example 12 **The electron in a hydrogen atom circles around the proton with a speed of $2{\cdot}18\times 10^6$ ms^{-1} in an orbit of radius $5{\cdot}3\times 10^{-11}$ m. Calculate (a) the equivalent current (b) magnetic field produced at the proton. Given charge on electron is $1{\cdot}6\times 10^{-19}$ C and $\mu_0 = 4\pi\times 10^{-7}$ TmA^{-1}.**

Solution. Here, $v = 2{\cdot}18\times 10^6$ ms^{-1}, $r = 5{\cdot}3\times 10^{-11}$ m, $e = 1{\cdot}6\times 10^{-19}$ C.

(a) Time period of revolution of electron is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi\times 5{\cdot}3\times 10^{-11}}{2{\cdot}18\times 10^6} = 1{\cdot}528\times 10^{-16}\ \text{s}$$

Equivalent current, $I = \dfrac{\text{charge}}{\text{time}} = \dfrac{e}{\text{T}}$

$$= \frac{1{\cdot}6\times 10^{-19}}{1{\cdot}528\times 10^{-16}} = \mathbf{1{\cdot}05\times 10^{-3}\ A}$$

(b) $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I}{r} = \dfrac{10^{-7}\times 2\pi\times 1{\cdot}05\times 10^{-3}}{5{\cdot}3\times 10^{-11}}$

$$= \mathbf{12{\cdot}4\ T}$$

Example 13 **For a circular coil of radius R and N turns carrying current I, the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by**

$$B = \frac{\mu_0 I R^2 N}{2(x^2+R^2)^{3/2}}$$

(*a*) Show that this reduces to the familiar result for field at the centre of the coil.

(*b*) Consider two parallel coaxial circular coils of equal radius R, and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R and is given by

$$B = 0{\cdot}72\frac{\mu_0 NI}{R}\ \textbf{approximately.}$$

[Such as arrangement to produce a nearly uniform magnetic field over a small region is known as Helmholtz coils.]

Solution. *(a)* Given, $B = \dfrac{\mu_0 I R^2 N}{2(x^2+R^2)^{3/2}}$

When point is at the centre of circular coil, $x = 0$.

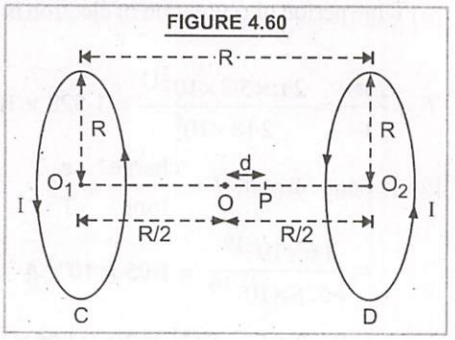
Then, $B = \dfrac{\mu_0 I R^2 N}{2R^3} = \dfrac{\mu_0 I N}{2R}$

Which is the same as the standard result.

(b) Here, $a = R$, $n = N$. Let d be the distance of point P from the mid-point O lying on the axis of the

two circular coils C and D, where $d << R$. In Fig. 4.60,

FIGURE 4.60



For coil C, $O_1P = x_1 = \frac{R}{2} + d$

For coil D, $O_2P = x_2 = \frac{R}{2} - d$

We know that magnetic field at a point on the axis of the circular coil of radius a, having n turns and carrying current I, which is at a distance x from the centre of the coil is given by

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(x^2+a^2)^{3/2}} = \frac{\mu_0 n I a^2}{2(x^2+a^2)^{3/2}}$$

Due to coil C, magnetic field at P is

$$B_1 = \frac{\mu_0 N I R^2}{2(x_1^2+R^2)^{3/2}} = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2}+d\right)^2+R^2\right]^{3/2}}$$

$$= \frac{\mu_0 N I R^2}{2}\left[\frac{1}{\left(\frac{R^2}{4}+d^2+Rd+R^2\right)^{3/2}}\right]$$

As d^2 can be neglected in comparison to R^2, therefore,

$$B_1 = \frac{\mu_0 N I R^2}{2}\times\frac{1}{\left(\frac{5R^2}{4}\right)^{3/2}}\left[\left(1+\frac{4d}{5R}\right)^{-3/2}\right]$$

It acts along PO_2.

Similarly, **due to coil D,** magnetic field at P is given by

$$B_2 = \frac{\mu_0 N I R^2}{2(x_2^2+R^2)^{3/2}} = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2}-d\right)^2+R^2\right]^{3/2}}$$

$$= \frac{\mu_0 N I R^2}{2}\times\frac{1}{\left(\frac{5R^2}{4}\right)^{3/2}}\left[\left(1-\frac{4d}{5R}\right)^{-3/2}\right]$$

It acts along PO_2.

Total magnetic field at P due to current through the coils is given by $B = B_1 + B_2$

$$= \frac{\mu_0 N I R^2}{2\left(\frac{5R^2}{4}\right)^{3/2}}\times\left[\left(1+\frac{4d}{5R}\right)^{-3/2}+\left(1-\frac{4d}{5R}\right)^{-3/2}\right]$$

$$= \frac{\mu_0 N I R^2}{2R^3}\left(\frac{4}{5}\right)^{3/2}\left[1-\frac{6d}{5R}+1+\frac{6d}{5R}\right]$$

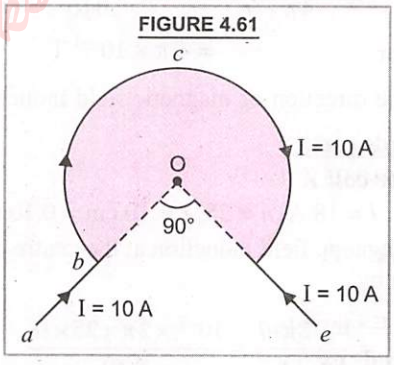
(From Binomial Theorem)

$$= \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 N I}{R} = 0{\cdot}72\frac{\mu_0 N I}{R}$$

Example 14 **The wire shown in Fig. 4.61. carries a current of 10 A. Determine the magnitude of the magnetic field at the centre O. Given radius of the bent coil is 3 cm.**

(Pb. Board 2001; AIIMS 2013)

FIGURE 4.61



Solution. Let $\hat{n}$ be the unit vector perpendicular to the plane of wire loop directed outwards.

Here, $I = 10$ A ; $r = 3$ cm $= 3 \times 10^{-2}$ m. Angle subtended by circular arc at O, $\theta = 270° = (3\pi/2)$ rad. Magnetic field at O is

$$\vec{B}_0 = \vec{B}_{ab} + \vec{B}_{bcd} + \vec{B}_{de} = 0 + \frac{\mu_0}{4\pi}\frac{I}{r}\theta(-\hat{n}) + 0$$

$\vec{B}_{ab}$ and $\vec{B}_{de}$ are zero because point O lies on the wires ab and de.

$$\therefore\ B_0 = \frac{\mu_0}{4\pi}\times\frac{I}{r}\theta = 10^{-7}\times\frac{10}{3\times10^{-2}}\times\frac{3}{2}\pi$$

$$= 1{\cdot}57 \times 10^{-4}\ \text{T}$$

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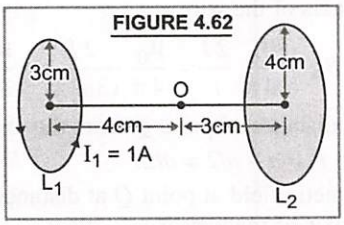
It is acting perpendicular to the plane of circular arc directed downwards.

Example 15 **Two coaxial circular loops L_1 and L_2 of radii 3 cm and 4 cm are placed as shown in Fig. 4.62. What should be the magnitude and direction of the current in the loop L_2 so that the net magnetic field at the point O be zero ?** (CBSE 2008)

FIGURE 4.62



Solution. The resultant magnetic field at the point O will be zero only **if the direction of current in loop L_2 is opposite to that in loop L_1.** Then magnetic field at O due to current I_1 in loop L_1 = magnetic field at O due to current I_2 in loop L_2

$$\text{or } \frac{\mu_0}{4\pi}\frac{2\pi I_1 a_1^2}{(a_1^2+x_1^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi I_2 a_2^2}{(a_2^2+x_2^2)^{3/2}}$$

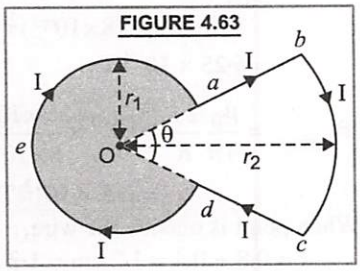
$$\text{or } \frac{I_1 a_1^2}{(a_1^2+x_1^2)^{3/2}} = \frac{I_2 a_2^2}{(a_2^2+x_2^2)^{3/2}}$$

$$\text{or } \frac{1\times(0{\cdot}03)^2}{[(0{\cdot}03)^2+(0{\cdot}04)^2]^{3/2}} = \frac{I_2\times(0{\cdot}04)^2}{[(0{\cdot}04)^2+(0{\cdot}03)^2]^{3/2}}$$

$$\text{or } I_2 = \frac{1\times(0{\cdot}03)^2}{(0{\cdot}04)^2} = \frac{9}{16} = 0{\cdot}56 \text{ A}$$

Example 16 **The current loop *abcde* formed by two circular segments of radii r_1 (= 4 cm) and r_2 (= 6 cm) with common centre at *O*, carries a current I (= 2 A) as shown in Fig. 4.63. What is the magnetic field at the common centre *O* ? What will be the value of magnetic field at *O* when $\theta = 90°$?**

FIGURE 4.63



Solution. Magnetic field at the centre *O* due to current through the whole current loop is

$B = B_{ab} + B_{bc} + B_{cd} + B_{dea}$ directed normally downwards

$$= 0 + \frac{\mu_0}{4\pi}\frac{I}{r_2}\theta + 0 + \frac{\mu_0}{4\pi}\frac{I}{r_1}(2\pi-\theta)$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{\theta}{r_2} + \frac{2\pi-\theta}{r_1}\right]$$

directed normally downwards

If $\theta = 90° = (\pi/2)$ rad, then

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{\pi/2}{r_2} + \frac{(2\pi-\pi/2)}{r_1}\right] = \frac{\mu_0 I}{8}\left[\frac{1}{r_2} + \frac{3}{r_1}\right]$$

$$= \frac{(4\pi\times10^{-7})\times2}{8}\left[\frac{100}{6} + \frac{300}{4}\right]$$

$$= 2{\cdot}88 \times 10^{-5} \text{ T}$$

TYPE III. AMPERE'S CIRCUITAL LAW

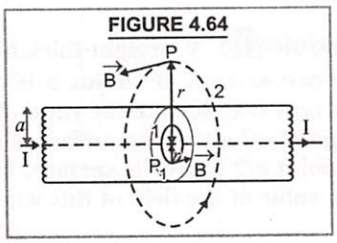
Formulae used. $\oint \vec{B}.\vec{dl} = \mu_0 I$

where $\oint$ represents the line integral over the closed path, B is the magentic field induction at a point on the closed path and I is the current threading the closed path.

Units used. B in tesla, I in ampere and r in metre.

Example 17 **Fig. 4.64 shows a long straight wire of circular crosssection (radius a) carrying steady current I. The current I is uniformly distributed across this crosssection. Calculate the magnetic field in the region $r < a$ and $r > a$.** (NCERT Solved Example)

FIGURE 4.64



Solution. (*a*) Taking the case, when $r < a$. Let the point P_1 be inside the wire at a distance r from the axis of wire. Consider an Amperian loop labelled 1, which is a circle of radius r concentric with crosssection such that point P_1 lies on this loop.

Current enclosed by the loop,

$$I' = \frac{I}{\pi a^2} \times \pi r^2 = \frac{I r^2}{a^2}$$

Let B be the magnetic field induction at P_1. Line integral of $\vec{B}$ over the Amperian loop is

$$= \oint \vec{B} . d\vec{l} = B\,(2\pi r)$$

According to Ampere's circuital law,

$\oint \vec{B} . d\vec{l} = \mu_0 \times$ current threading the closed path

$$= \frac{\mu_0 I r^2}{a^2}$$

$$\therefore \quad B 2\pi r = \mu_0 \times \frac{I r^2}{a^2} \quad \text{or} \quad B = \frac{\mu_0 I r}{2\pi a^2}$$

It means, $B \propto r$ (When $r < a$)

(b) Taking the case when $r > a$. Take the point P outside the wire at a distance r from the axis of wire. Consider an Amperian loop labelled 2 which is a circle concentric with crosssection of wire such that point P lies on this loop. Current enclosed by this loop is I.

Let B be the magnetic field induction at P. Line integral of $\vec{B}$ over the Amperian loop

$$= \oint \vec{B} . d\vec{l} = B\,(2\pi r)$$

According to Ampere's circuital law

$\oint \vec{B} . d\vec{l} = \mu_0 \times$ current threading the closed loop $= \mu_0 I$

$$\therefore \quad B\,2\pi r = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r}$$

It means, $B \propto \frac{1}{r}$ (When $r > a$)

Example 18 **A straight thick long wire of uniform cross-section of radius a is carrying a steady current I. Calculate the ratio of magnetic field at a point $a/2$ above the surface of the wire to that at a point $a/2$ below its surface. What is the maximum value of the field of this wire ?**

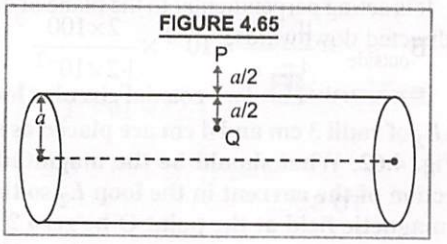
(CBSE 2010)

Solution. Suppose the point P lies at distance $a/2$ above the surface of the wire and point Q lies at distance $a/2$ below the surface of wire. Fig. 4.65.

The distance of point P from the axis of wire

$$= a + a/2 = 3a/2.$$

FIGURE 4.65



Magnetic field at point P at distance $r\,(= 3\,a/2)$ from the axis of the wire is

$$B_P = \frac{\mu_0}{4\pi} \frac{2I}{r} = \frac{\mu_0}{4\pi} \frac{2I}{(3a/2)} = \frac{\mu_0 I}{3\pi a}$$

The distance of point Q from the axis of wire

$$r' = a - a/2 = a/2.$$

Magnetic field at point Q at distance $r'\,(= a/2)$ from the axis of the wire is

$$B_Q = \frac{\mu_0}{2\pi} \frac{I r'}{a^2} = \frac{\mu_0}{2\pi} \frac{I}{a^2} \times \frac{a}{2} = \frac{\mu_0}{4\pi} \frac{I}{a}$$

$$\frac{B_P}{B_Q} = \frac{\mu_0 I}{3\pi a} \times \frac{4\pi a}{\mu_0 I} = \frac{4}{3}$$

Magnetic field is maximum on the surface of wire, where $r = a$. Hence,

$$B_{max} = \frac{\mu_0}{4\pi} \frac{2I}{r} = \frac{\mu_0 I}{2\pi a}$$

Example 19 **A wire of radius 0·8 cm carries a current of 100 A which is uniformly distributed over its cross-section. Find the magnetic field (*a*) at 0·2 cm from the axis of the wire (*b*) at the surface of the wire and (*c*) at a point outside the wire 0·4 cm from the surface of the wire. Neglect the permeability of the material of wire.**

Solution. Here, $R = 0{\cdot}8$ cm $= 8 \times 10^{-3}$ m ; $I = 100$ A

(a) $B_{inside} = \frac{\mu_0}{4\pi} \frac{2Ir}{R^2}$

$$= 10^{-7} \times \frac{2 \times 100 \times (0{\cdot}2 \times 10^{-2})}{(8 \times 10^{-3})^2}$$

$$= \mathbf{6{\cdot}25 \times 10^{-4}\ T}$$

(b) $B_{surface} = \frac{\mu_0}{4\pi} \frac{2I}{R} = 10^{-7} \times \frac{2 \times 100}{8 \times 10^{-3}}$

$$= \mathbf{2{\cdot}5 \times 10^{-3}\ T}$$

(c) When point is outside the wire,

$$r = 0{\cdot}8 + 0{\cdot}4 = 1{\cdot}2 \text{ cm} = 1{\cdot}2 \times 10^{-2} \text{ m}.$$

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$$B_{outside} = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2\times100}{1{\cdot}2\times10^{-2}}$$
$$= 1{\cdot}67 \times 10^{-3}\ \text{T}$$

TYPE IV. MAGNETIC FIELD DUE TO SOLENOID AND TOROID

Formulae used. (*i*) Magnetic field induction B at a point well inside the long solenoid carrying current I is,

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I$$

where n = no. of turns per unit length of solenoid, N = no. of turns in the solenoid of length l.

(*ii*) Magnetic field induction B at a point at one end on the axis of a long solenoid carrying current I is

$$B = \frac{\mu_0 nI}{2} = \frac{\mu_0 NI}{2l}$$

(*iii*) Magnetic field induction B at a point inside the toroid carrying current I having n turns per unit length is $B = \mu_0 nI$

(*iv*) Magnetic field induction B at a point outside the toroid is **zero.**

Units used. B in tesla and I in ampere.

Example 20 **A solenoid of length 0·5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid ?**

(NCERT Solved Example)

Solution. Here, $N = 500$; $l = 0{\cdot}5$ m ; $I = 5$ A ;
$r = 1$ cm $= 10^{-2}$ m.

As $\frac{l}{r} = \frac{0{\cdot}5}{10^{-2}} = 50$ so $l >> r$.

Hence, we can use the formula of magnetic field induction at a point inside the long solenoid

$$B = \mu_0 \frac{NI}{l} = 4\pi\times10^{-7} \times \frac{500}{0{\cdot}5} \times 5$$
$$= 6{\cdot}28 \times 10^{-3}\ \text{T}$$

Example 21 **A solenoid of length 50 cm, having 100 turns carries a current of 2·5 A. Find the magnetic field, (a) in the interior of the solenoid, (b) at one end of the solenoid.**

Given, $\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}.

Solution. Here, $I = 2{\cdot}5$ A,

$$n = 100/0{\cdot}50 = 200\ \text{m}^{-1}$$

(*a*) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 200 \times 2{\cdot}5$
$$= 6{\cdot}28 \times 10^{-4}\ \text{T}$$

(*b*) $B = \frac{\mu_0 nI}{2} = \frac{4\pi\times10^{-7}\times200\times2{\cdot}5}{2}$
$$= 3{\cdot}14 \times 10^{-4}\ \text{T}$$

Example 22 **A copper wire having a resistance of 0·015 Ω per metre is used to wind a 500 turns solenoid of radius 1·0 cm and length 22 cm. If the e.m.f. of the battery used is 6 V, then find the magnetic field near the centre of the solenoid.**

Solution. Resistance of wire per metre
$$= 0{\cdot}015\ \Omega\ \text{m}^{-1}$$
No. of turns in solenoid, $N = 500$;
radius, $r = 1 \times 10^{-2}$ m,
length of solenoid $L = 22$ cm $= 22 \times 10^{-2}$ m,
e.m.f., $\varepsilon = 6$ V
Length of wire used in solenoid

$$l = 2\pi r \times N = 2\times\frac{22}{7}\times(1\times10^{-2})\times500\ \text{m}$$
$$= 31{\cdot}4\ \text{m}$$

Total resistance of wire used in solenoid,
R = resistance per metre × l
$$= 0{\cdot}015 \times 31{\cdot}4\ \Omega$$
Current in solenoid,

$$I = \frac{\varepsilon}{R} = \frac{6}{0{\cdot}015\times31{\cdot}4} = 12{\cdot}73\ \text{A}$$

Magnetic field near the centre of solenoid

$$B = \mu_0 \frac{NI}{L} = (4\pi\times10^{-7})\times\frac{500\times12{\cdot}73}{(22\times10^{-2})}$$
$$= 3{\cdot}64 \times 10^{-2}\ \text{T}$$

Example 23 **A toroid has a core of inner radius 20 cm and outer radius 22 cm around which 4200 turns of a wire are wound. If the current in the wire is 10 A, what is the magnetic field (a) inside the core of toroid (b) outside the toroid (c) in the empty space surrounded by toroid ?**

Solution. Here, inner radius, $r_1 = 20$ cm.
Outer radius, $r_2 = 22$ cm ; $I = 10$ A
∴ Mean radius of toroid,

$$r = \frac{r_1 + r_2}{2} = \frac{20+22}{2} = 21\ \text{cm} = 0{\cdot}21\ \text{m}$$

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Total length of toroid = circumference of toroid
$= 2\pi r = 2\pi \times 0{\cdot}21$
$= 0{\cdot}42\pi$ m

Total number of turns, $N = 4200$

$\therefore$ Number of turns per unit length will be,

$$n = \frac{4200}{0{\cdot}42\pi} = \frac{10000}{\pi}\ \text{m}^{-1}$$

(*a*) Magnetic field induction inside the core of toroid, $B = \mu_0 nI = 4\pi\times10^{-7}\times\frac{10000}{\pi}\times10 = \mathbf{0{\cdot}04\ T}$

(*b*) Magnetic field induction outside the toroid is **zero,** since the field is only confined inside the core of the toroid on which winding has been made.

(*c*) Magnetic field induction in the empty space surrounded by toroid is also **zero.**

TYPE V. MOTION OF CHARGED PARTICLE IN (*i*) ELECTRIC FIELD AND (*ii*) MAGNETIC FIELD

Formulae used

(*i*) Force on a charged particle of charge q in an electric field of intensity E is $\vec{F} = q\vec{E}$

(*ii*) Force on a charged particle of charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$\vec{F} = q(\vec{v}\times\vec{B})$ or $F = qvB\sin\theta$

where θ is the smaller angle between $\vec{v}$ and $\vec{B}$.

(*iii*) When magnetic field is acting perpendicular to the direction of motion of charged particle then

$F = qvB\sin 90° = \frac{mv^2}{r}$ or $r = \frac{mv}{qB}$

(*iv*) Time period of revolution, $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$.

Units used. F in *newton* ; q in *coulomb* ; E in *V/m* or *N/C*, B in *tesla*, v in *m/s* and r in *metre*.

Example 24 **A cathode ray tube contains a pair of parallel metal plates 1·0 cm apart and 3·0 cm long. A narrow horizontal beam of electrons with velocity of 3×10^7 ms^{-1} is passed down the tube mid way between the two plates. When the potential difference of 550 V is maintained across the plates, it is found that the electron beam is so deflected that it just strikes the end of one of the plates. Find the specific charge of an electron.**

Solution. Here, $y = \frac{1}{2}$ cm $= 0{\cdot}5\times10^{-2}$ m ;

$l = 3$ cm $= 3\times10^{-2}$ m ; $v = 3\times10^7$ ms^{-1} ;

$d = 1{\cdot}0$ cm $= 10^{-2}$ m ; $V = 550$ V

Time to cross the tube,

$$t = \frac{l}{v} = \frac{3\times10^{-2}}{3\times10^7} = 10^{-9}\ \text{s}$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{Ee}{m}\right)t^2 = \frac{1}{2}\frac{(V/d)e}{m}t^2$$

or $\frac{e}{m} = \frac{2yd}{Vt^2} = \frac{2\times(0{\cdot}5\times10^{-2})\times10^{-2}}{550\times(10^{-9})^2}$

$= \mathbf{1{\cdot}8\times10^{11}\ C\ kg^{-1}}$

Example 25 **A charged particle of charge 2·0 μC moving along x-axis with a speed of 3×10^6 ms^{-1} enters a magnetic field, $\vec{B} = (0{\cdot}3\hat{j} + 0{\cdot}4\hat{k})$ tesla acting in a space. What is the magnitude of magnetic force on the charged particle ?**

Solution. Here, $q = 2{\cdot}0\ \mu\text{C} = 2\times10^{-6}$ C ;

$\vec{v} = (3\times10^6\,\hat{i})$ ms^{-1},

$B = (0{\cdot}3\hat{j} + 0{\cdot}4\hat{k})$ T

Magnetic force on the charged particle is

$\vec{F} = q(\vec{v}\times\vec{B})$

$= (2\times10^{-6})[(3\times10^6)\hat{i}\times(0{\cdot}3\hat{j} + 0{\cdot}4\hat{j})]$

$= (2\times10^{-6})\times10^6[3\hat{i}\times(0{\cdot}3\hat{j} + 0{\cdot}4\hat{k})]$

$= 2[(3\times0{\cdot}3)(\hat{i}\times\hat{j}) + 3\times0{\cdot}4(\hat{i}\times\hat{k})]$

$= 2[0{\cdot}9\hat{k} - 1{\cdot}2\hat{j}] = [1{\cdot}8\hat{k} - 2{\cdot}4\hat{j}]$

$\therefore$ **Magnitude of magnetic field is**

$|\vec{F}| = \sqrt{(1{\cdot}8)^2 + (-2{\cdot}4)^2}$

$= \sqrt{3{\cdot}24 + 5{\cdot}76} = \mathbf{3{\cdot}0\ N}$

Example 26 **A copper has $8{\cdot}0\times10^{28}$ electrons per cubic metre. A copper wire of length 1 m and cross-sectional area $8{\cdot}0\times10^{-6}$ m^2 carrying a current and lying at right angle to a magnetic field of strength 5×10^{-3} T experiences a force of $8{\cdot}0\times10^{-2}$ N. Calculate the drift velocity of free electrons in wire.**

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Solution. Here, $n = 8{\cdot}0 \times 10^{28}$ m^{-3} ; $l = 1$ m ;
$A = 8 \times 10^{-6}$ m^2, $e = 1{\cdot}6 \times 10^{-19}$ C,
$B = 5 \times 10^{-3}$ T, $F = 8{\cdot}0 \times 10^{-2}$ N, $\theta = 90°$

Total charge contained in the wire is

q = Volume of wire $\times\ ne = Alne$

$= (8 \times 10^{-6}) \times 1 \times (8 \times 10^{28}) \times (1{\cdot}6 \times 10^{-19})$

$= 1{\cdot}024 \times 10^5$ C

If v_d is the drift velocity of electrons, then

$$F = q\, v_d\, B \sin\theta$$

or $$v_d = \frac{F}{qB\sin\theta}$$

$$= \frac{(8{\cdot}0\times10^{-2})}{(1{\cdot}024\times10^5)\times(5\times10^{-3})\times\sin 90°}$$

$$= \mathbf{1{\cdot}56 \times 10^{-4}\ ms^{-1}}$$

Example 27 **What is the radius of the path of an electron (mass 9×10^{-31} kg and charge $1{\cdot}6 \times 10^{-19}$ C) moving at a speed of 3×10^7 ms^{-1} in a magnetic field of 6×10^{-4} T perpendicular to it ? What is its frequency ? Calculate its energy in *keV*.** (NCERT Solved Example)

Solution. Here, $m = 9 \times 10^{-31}$ kg ;
$q = 1{\cdot}6 \times 10^{-19}$ C ; $v = 3 \times 10^7$ ms^{-1} ;
$B = 6 \times 10^{-4}$ T

$$r = \frac{mv}{qB} = \frac{(9\times10^{-31})\times(3\times10^7)}{(1{\cdot}6\times10^{-19})\ (6\times10^{-4})} = \mathbf{0{\cdot}28\ m}$$

$$\nu = \frac{v}{2\pi r} = \frac{Bq}{2\pi m} = \frac{(6\times10^{-4})\times(1{\cdot}6\times10^{-19})}{2\times(22/7)\times9\times10^{-31}}$$

$$= \mathbf{1{\cdot}7 \times 10^7\ Hz}$$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\times(9\times10^{-31})\times(3\times10^7)^2\ \text{J}$$

$$= 40{\cdot}5 \times 10^{-17}\ \text{J} = \frac{40{\cdot}5\times10^{-17}}{1{\cdot}6\times10^{-16}}\ \text{keV}$$

$$= \mathbf{2{\cdot}53\ keV}$$

Example 28 **An electron emitted by a heated cathode and accelerated through a potential difference of 2·0 kV enters a region with a uniform magnetic field of 0·15 T. Determine the trajectory of the electron if the field (*a*) is transverse to its initial velocity (*b*) makes an angle of 30° with the initial velocity.**

Solution. Here, $V = 2{\cdot}0$ kV $= 2{\cdot}0 \times 10^3$ V ;

$B = 0{\cdot}15$ T. Let e, m be the charge and mass of the electron. If v is the velocity gained by electron when accelerated under a potential difference V, then

$$eV = \frac{1}{2}mv^2$$

or $$v = \left(\frac{2eV}{m}\right)^{1/2}$$

$$= \left(\frac{2\times1{\cdot}6\times10^{-19}\times2{\cdot}0\times10^3}{9{\cdot}0\times10^{-31}}\right)^{1/2}$$

$$= \frac{8\times10^7}{3}\ \text{m/s}$$

(*a*) Force on electron due to transverse magnetic field is = Bev, which is perpendicular to $\vec{B}$ as well as $\vec{v}$. It will provide the required centripetal force to the electron for its circular motion. Therefore, the trajectory of electron in magnetic field **is circular.** Its radius r can be given by

$$Bev = \frac{mv^2}{r}$$

or $$r = \frac{mv}{Be} = \frac{9\times10^{-31}\times(8\times10^7/3)}{0{\cdot}15\times1{\cdot}6\times10^{-19}}$$

$$= 10^{-3}\ \text{m} = \mathbf{1\ mm}$$

(*b*) When electron makes an angle θ with the direction of magnetic field, its component velocity perpendicular to the field,

$$v_1 = v\ \sin 30° = \frac{8}{3}\times10^7\times\frac{1}{2} = \frac{4}{3}\times10^7\ \text{m/s}\ .$$

Due to this velocity, the force acting on the electron due to magnetic field will be providing the required centripetal force, hence the electron will describe a circular path of radius r_1. The component velocity along the magnetic field = $v \cos 30°$

$$= \frac{8}{3}\times10^7\times\frac{\sqrt{3}}{2} = \frac{4\sqrt{3}}{3}\times10^7\ \text{m/s}\ .$$

Due to this velocity, no force acts on the electron in magnetic field, and electron moves without change in its speed along the magnetic field. Therefore, due to two perpendicular component velocities, the electron describes a helical path.

The radius of the helical path,

$$r = \frac{mv_1}{Be} = \frac{9\times10^{-31}\times(4/3)\times10^7}{0{\cdot}15\times1{\cdot}6\times10^{-19}}$$

$$= 0{\cdot}5 \times 10^{-3}\ \text{m} = \mathbf{0{\cdot}5\ mm}$$

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Example 29 **A proton, a deutron and α-particle, whose kinetic energies are same, enter perpendicularly a uniform magnetic field. Compare the radii of their circular paths.**

Solution. Here, mass of proton, $m_p = m$; mass of deutron, $m_d = 2$ m and mass of α-particle $m_\alpha = 4$ m ; charge on proton, $q_p = e$; charge on deutron, $q_d = e$ and charge on α-particle, $q_\alpha = 2e$.

K.E. of charged particle,

$$E = \frac{1}{2} mv^2 \quad \text{or} \quad v = \sqrt{2E/m}$$

Radius of the circular path of a charged particle in the magnetic field is given by

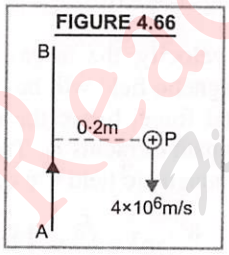
$$r = \frac{mv}{Bq} = \frac{m}{Bq}\sqrt{\frac{2E}{m}} = \frac{\sqrt{2mE}}{Bq} \; ; \; \text{so } r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \; r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

Example 30 **A long straight wire *AB* carries a current of 4 A. A proton P travels at 4×10^6 ms^{-1} parallel to the wire, 0·2 m from it and in a direction opposite to the current as shown in Fig. 4.66. Calculate the force which the magnetic field of current exerts on the proton. Also specify the direction of the force.** (CBSE 2002)

FIGURE 4.66



Solution. Here, $I = 4$ A ; $v = 4 \times 10^6$ ms^{-1} ; $r = 0{\cdot}2$ m.

Magnetic field induction at *P* is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{10^{-7} \times 2 \times 4}{0{\cdot}2} = 4 \times 10^{-6} \text{ T}$$

The direction of $\vec{B}$, according to Right Hand Thumb rule, is perpendicular to the plane of paper directed inwards.

Since proton is moving in opposite direction to the current carrying straight wire, hence the proton is moving perpendicular to the direction of magnetic field due to current through straight wire. The force on moving proton of charge *q* due to magnetic field is $F = q v B \sin 90°$

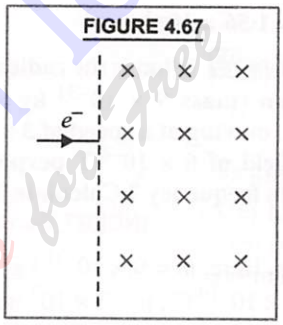
$$= (1{\cdot}6 \times 10^{-19}) \times (4 \times 10^6) \times (4 \times 10^{-6})$$

$$= \mathbf{2{\cdot}56 \times 10^{-18}\ N}$$

According to Fleming's Left Hand Rule, the force acts in the **plane of paper towards right.**

Example 31 **An electron moving horizontally with a velocity of 4×10^4 ms^{-1} enters a region of uniform magnetic field of 10^{-5} T, acting vertically downwards as shown in Fig. 4.67. Draw its trajectory and find out the time it takes to come out of the region of magnetic field.** (CBSE (F) 2015)

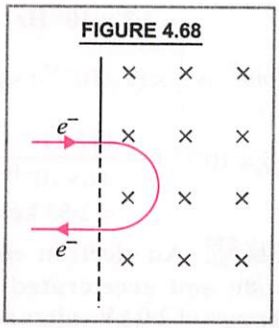
FIGURE 4.67



Solution. Here, $v = 4 \times 10^4$ ms^{-1} ; $B = 10^{-5}$ T

When a moving electron is subjected to a perpendicular magnetic field, it will discribe a semicircular trajectory inside the magnetic field and comes out as shown in Fig. 4.68. Radius *r*, of the circular path of electron is given by

FIGURE 4.68



$$\frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

$$\therefore \quad r = \frac{(9{\cdot}1 \times 10^{-31}) \times (4 \times 10^4)}{(1{\cdot}6 \times 10^{-19}) \times 10^{-5}}$$

$$= 22{\cdot}75 \times 10^{-3} \text{ m}$$

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Time taken to come out of the region of magnetic field is

$$t = \frac{\pi r}{v} = \frac{(22/7)\times(22{\cdot}75\times10^{-3})}{4\times10^{4}}$$
$$= \mathbf{1{\cdot}8 \times 10^{-6}\ s}$$

TYPE VI. (*i*) MOTION OF CHARGED PARTICLE, BOTH IN ELECTRIC AND MAGNETIC FIELDS (*ii*) CYCLOTRON

Formulae used. (*i*) Force, $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

(*ii*) Cyclotron frequency, $\nu = \frac{1}{T} = \frac{qB}{2\pi m}$

(*iii*) Maximum K.E. of accelerated charged particle is, $K_{max.} = \frac{B^2q^2r_0^2}{2m}$

Unit used. q in C, E in N/C, B in tesla, r_0 in m and mass m in kg.

Example 32 **In a chamber of a uniform magnetic field of 8·0 G [1 G = 10^{-4} T] is maintained. An electron with a speed of $4{\cdot}0 \times 10^6$ ms^{-1} enters the chamber in a direction normal to the field**

(*i*) Describe the path of electron.

(*ii*) What is the frequency of revolution of electron ?

(*iii*) What happens to the path of the electron if it progressively loses its energy due to collisions with the atoms or molecules of the environment ? **(NCERT)**

Solution. (*i*) Here, $B = 8{\cdot}0$ G $= 8{\cdot}0 \times 10^{-4}$ T, $v = 4{\cdot}0 \times 10^6$ ms^{-1}, $\theta = 90°$.

If r is the radius of circular path, then

$$Bev\sin 90° = \frac{mv^2}{r} \text{ or } r = \frac{mv}{Be}$$

$$\therefore\quad r = \frac{(9{\cdot}1\times10^{-31})\times(4{\cdot}0\times10^{6})}{(8{\cdot}0\times10^{-4})\times(1{\cdot}6\times10^{-19})}$$
$$= 2{\cdot}8 \times 10^{-2}\ \text{m} = \mathbf{2{\cdot}8\ cm}$$

The sense of rotation of electron in a circular path can be predicted from the direction of centripetal force, $\vec{F} = -e(\vec{v}\times\vec{B})$. If we see along the direction of $\vec{B}$, the electron will be revolving in **clockwise direction.**

(*ii*) The frequency of revolution of electron

$$\nu = \frac{eB}{2\pi m} = \frac{(1{\cdot}6\times10^{-19})\times(8{\cdot}0\times10^{-4})}{2\times3{\cdot}14\times(9{\cdot}1\times10^{-31})}$$
$$= 22 \times 10^6\ \text{Hz} = \mathbf{22\ MHz}$$

(*iii*) If the electron loses its energy in successive collisions, then electron loses its speed progressively. If after collision, the velocity of electron remains in the same plane of the initial circular orbit, the radius of the circular orbit will decrease in proportion to the decreased speed. If it is not so then the path of the electron will be helical between two collisions.

Example 33 **A beam of electrons enters a uniform magnetic field of 0·3 T with a velocity of 4×10^5 ms^{-1} at an angle of 60° to the field. Find the radius of the helical path taken by the beam. Also find the pitch of the helix (distance travelled by a proton parallel to the magnetic field during one period of rotation). Mass of proton is $1{\cdot}67 \times 10^{-27}$ kg.**

Solution. Here, $B = 0{\cdot}3$ T, $v = 4 \times 10^5$ ms^{-1}, $\theta = 60°$.

$q = 1{\cdot}6 \times 10^{-19}$ C, $m = 1{\cdot}67 \times 10^{-27}$ kg

Components of the velocity of proton parallel and perpendicular to the direction of magnetic field are

$$v_{\parallel} = v\cos\theta = 4\times10^5\times\cos 60°$$
$$= (4\times10^5)\times(1/2) = 2\times10^5\ \text{ms}^{-1}$$
$$v_{\perp} = v\sin\theta = (4\times10^5)\times\sin 60°$$
$$= 4\times10^5\times(\sqrt{3}/2) = 3{\cdot}464\times10^5\ \text{m/s}.$$

The proton moving with component velocity $v_{\parallel}$ will move along the magnetic field and with component velocity $v_{\perp}$ will describe a circular path in the magnetic field.

Hence, the path of proton in the magnetic field is helical path.

The radius of the helical path is

$$qv_{\perp}B = \frac{mv_{\perp}^2}{r}$$

$$\text{or } r = \frac{mv_{\perp}}{qB} = \frac{(1{\cdot}67\times10^{-27})\times(3{\cdot}464\times10^5)}{(1{\cdot}6\times10^{-19})\times0{\cdot}3}$$
$$= 1{\cdot}2 \times 10^{-2}\ \text{m} = \mathbf{1{\cdot}2\ cm}$$

Period of revolution of electron is

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2\times3{\cdot}14\times(1{\cdot}2\times10^{-2})}{(3{\cdot}464\times10^5)}$$
$$= 2{\cdot}175 \times 10^{-7}\ \text{s}$$

Pitch of the helix is

$p = v_{\parallel} \times T = (2 \times 10^5) \times (2{\cdot}175 \times 10^{-7})$

$= 4{\cdot}35 \times 10^{-2}$ cm = **4·35 cm**

Example 34 **A beam of alpha particle passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of the beam. If the magnitude of the electric and magnetic fields are 120 kV m^{-1} and 60 mT respectively, calculate (*i*) velocity of the beam (*ii*) force with which it strikes a target on a screen, if the alpha particle beam current is equal to 1 mA ? Mass of proton = mass of neutron = 1·675 × 10^{-27} kg.**

Solution. Here, $E = 120$ kV m^{-1}

$= 120 \times 10^3$ Vm^{-1} ;

$B = 60$ mT $= 60 \times 10^{-3}$ T

(*i*) For undeflected beam of alpha particle

$q\,v\,B = qE$

or $v = \dfrac{E}{B} = \dfrac{120 \times 10^3}{60 \times 10^{-3}} = \mathbf{2 \times 10^6\ ms^{-1}}$

(*ii*) Current carried by alpha particle beam

$I = 1$ mA $= 10^{-3}$ A

Number of alpha particles striking the screen per second

$$n = \frac{I}{q} = \frac{I}{2e} = \frac{10^{-3}}{2 \times 1{\cdot}6 \times 10^{-19}}$$

$= 3{\cdot}125 \times 10^{15}$ s^{-1}

Mass of alpha particle,

$m_\alpha = 2\,m_p + 2\,m_n = 4\,m_p$

$= 4 \times 1{\cdot}675 \times 10^{-27}$ kg

Force with which alpha particle beam strikes a target on the screen

$F = m_\alpha\, n\, v = (4 \times 1{\cdot}675 \times 10^{-27}) \times (3{\cdot}125 \times 10^{15}) \times 2 \times 10^6$

$= \mathbf{4{\cdot}19 \times 10^{-5}\ N}$

Example 35 **A cyclotron oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons ? If the radius of its dees is 60 cm, what is the kinetic energy of the proton beam produced by the accelerator in MeV ?**

($e = 1{\cdot}6 \times 10^{-19}$ C, $m_p = 1{\cdot}67 \times 10^{-27}$ kg ; 1 MeV = 1·6 × 10^{-13} J) **(CBSE 2005)**

(NCERT Solved Example)

Solution. $B = \dfrac{2\pi\nu m}{q}$

$= 2 \times \dfrac{22}{7} \times \dfrac{(10 \times 10^6) \times (1{\cdot}67 \times 10^{-27})}{1{\cdot}6 \times 10^{-19}} \approx \mathbf{0{\cdot}66\ T}$

$K_{max.} = \dfrac{B^2 q^2 r_0^2}{2m}$

$= \dfrac{(0{\cdot}66)^2 \times (1{\cdot}6 \times 10^{-19})^2 \times (0{\cdot}60)^2}{2 \times 1{\cdot}67 \times 10^{-27}}$ J

$= \dfrac{(0{\cdot}66)^2 \times (1{\cdot}6 \times 10^{-19})^2 \times (0{\cdot}60)^2}{2 \times (1{\cdot}67 \times 10^{-27}) \times 1{\cdot}6 \times 10^{-13}}$ MeV

$= \mathbf{7{\cdot}5\ MeV}$

Example 36 **If the maximum value of accelerating potential provided by a radio frequency oscillator be 25 kV, find the number of revolutions made by a proton in a cyclotron to achieve one sixth of the speed of light. Mass of proton = 1·67 × 10^{-27} kg.**

Solution. In a cyclotron, when a proton crosses a region of potential difference V, the energy gained is eV. In cyclotron, in one revolution, the proton crosses the gap twice. So the energy gained by proton in each revolution = 2 eV.

Let the proton make n revolutions before emerging from the dees. The gain in its kinetic energy is

$$E_k = \frac{1}{2}\,mv^2 = n \times 2eV$$

or $n = \dfrac{mv^2}{4eV} = \dfrac{m(c/6)^2}{4eV}$

or $n = \dfrac{mc^2}{4 \times 36 \times eV}$

$= \dfrac{1{\cdot}67 \times 10^{-27} \times (3 \times 10^8)^2}{4 \times 36 \times (1{\cdot}6 \times 10^{-19}) \times (25 \times 10^3)}$

$\approx \mathbf{261\ revolutions}$

TYPE VII. FORCE ON CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

Formulae used.

(*i*) $F = I\,lB \sin\theta$ and (*ii*) $\vec{F} = I(\vec{l} \times \vec{B})$

Units used. F in newton, I in ampere, l in metre and B in tesla.

Example 37 **The horizontal component of the earth's magnetic field at a certain place is $3{\cdot}0 \times 10^{-5}$ T and the direction of the field is from geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of current is (*a*) east to west (*b*) south to north ?** (NCERT)

Solution. Here, $B = 3{\cdot}0 \times 10^{-5}$ T, $I = 1$A.

Force on current carrying conductor in magnetic field is $F = BIl \sin\theta$

Force per unit length, $f = \dfrac{F}{l} = BI \sin\theta$

(*a*) When current flows from east to west, then $\theta = 90°$

$\therefore \quad f = BI \sin 90° = (3{\cdot}0 \times 10^{-5}) \times 1$

$= \mathbf{3 \times 10^{-5}\ Nm^{-1}}$

(*b*) When current flows from south to north, then $\theta = 0°$

$\therefore \quad f = BI \sin 0° = \mathbf{0}$

Example 38 **A straight wire of mass 200 g and length 1·5 m carries a current of 2 *A*. It is suspended in mid air by a uniform horizontal magnetic field $\vec{B}$. What is the magnitude of the magnetic field ?** (NCERT Solved Example)

Solution. Here, $m = 200$ g $= 0{\cdot}200$ kg,

$I = 2$ A ; $l = 1{\cdot}5$ m.

For mid air suspension, $mg = BI\,l$

or $\quad B = \dfrac{mg}{Il} = \dfrac{0{\cdot}200 \times 9{\cdot}8}{2 \times 1{\cdot}5} = \mathbf{0{\cdot}65\ T}$

Example 39 **A long straight conductor *C* carrying a current of 3 A is placed parallel to a short conductor D of length 5 cm, carrying a current 4 A. The two conductors are 10 cm apart. Find (*i*) the magnetic field due to *C* at *D*. (*ii*) The approximate force on *D*.**

Solution. (*i*) Magnetic field due to *C* at *D* is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 3}{0{\cdot}10} = \mathbf{6 \times 10^{-6}\ T}$$

(*ii*) Force on *D*, $F = BI_1 l \sin\theta$

$= (6 \times 10^{-6}) \times 4 \times (5 \times 10^{-2}) \times \sin 90°$

$= \mathbf{1{\cdot}2 \times 10^{-6}\ N}$

Example 40 **A uniform magnetic field of 1·5 T is in a cylindrical region of radius 10·0 cm with its direction parallel to the axis along east to west. A wire carrying current of 7·0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if (a) the wire intersects the axes, (b) the wire is turned from N-S to north east-south west direction, (c) the wire in the N-S direction is lowered from the axis by a distance 6·0 cm?**

Solution. Here, $B = 1{\cdot}5$ T, $I = 7{\cdot}0$ A

(*a*) Length of the wire in a cylindrical magnetic field, is equal to the diameter of the cylinderical region *i.e.* $l = 20$ cm $= 0{\cdot}20$ m. Angle between current and magnetic field $\theta = 90°$

$\therefore$ Force on the wire,

$F = BIl \sin\theta = 1{\cdot}5 \times 7{\cdot}0 \times 0{\cdot}20 \times \sin 90° = \mathbf{2{\cdot}1\ N}$

The direction of this force is **vertically downwards,** using Flemings' left hand rule.

(*b*) When the wire is turned from *N-S* to northeast-northwest direction, the length of the wire in the cylindrical region of magnetic field is l_1 and $\theta = 45°$

$\therefore$ Force on the wire, $F_1 = BIl_1 \sin 45° = BIl$

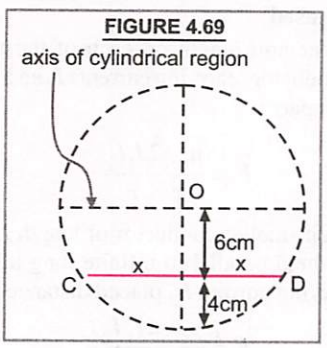
$[\because\ l_1 \sin 45° = l]$

or $\quad F_1 = 1{\cdot}5 \times 7{\cdot}0 \times 0{\cdot}20 = \mathbf{2{\cdot}1\ N}$

The direction of this force is **vertically downwards,** using Fleming's Left hand Rule.

(*c*) When the wire is lowered by 6·0 cm *i.e.* at position *CD* in Fig. 4.69 (front view is taken), then length of wire in magnetic field, $l_2 = 2x$

FIGURE 4.69



where, $x \times x = 4 \times (10 + 6)$ or $x = 8$ cm.

$\therefore \quad l_2 = 2 \times 8$ cm $= 16$ cm $= 0{\cdot}1\,6$ m.

Force on wire,

$F_2 = BIl_2 = 1{\cdot}5 \times 7 \times 0{\cdot}16$

$= \mathbf{1{\cdot}68\ N}$ **vertically downwards.**

Example 41 **A straight horizontal conducting rod of length 0·60 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5·0 A is set up in the rod through the wire. (*a*) What magnetic field should be set up**

normal to the conductor in order that the tension in the wire is zero ?

(*b*) What will be total tension in the wires if the direction of current is reversed, keeping the magnetic field same as before (Ignore mass of the wire), $g = 10\ ms^{-2}$.

Solution. Here, $l = 0{\cdot}60$ m ;

$m = 60$ g $= 60 \times 10^{-3}$ kg ; $I = 5{\cdot}0$ A

(*a*) Tension in the wire is zero if the force on the wire carrying current due to magnetic field is equal and opposite to the weight of wire, *i.e.*,

$$BIl = mg$$

$$\text{or} \quad B = \frac{mg}{Il} = \frac{(60\times10^{-3})\times10}{5{\cdot}0\times0{\cdot}60} = \mathbf{0{\cdot}20\ T}$$

The force on the conductor due to magnetic field will be upwards if the direction of magnetic field is horizntal and normal to the conductor.

(*b*) When direction of current is reversed, BIl and mg will act vertically downwards, the effective tension in the wires,

$$T = BIl + mg$$
$$= 0{\cdot}2 \times 5{\cdot}0 \times 0{\cdot}60 + (60 \times 10^{-3}) \times 10$$
$$= \mathbf{1{\cdot}2\ N}$$

TYPE VIII. FORCE BETWEEN TWO PARALLEL STRAIGHT CONDUCTORS CARRYING CURRENTS

Formulae used

(*i*) Force per unit length on each of the two long parallel conductors carrying currents I_1 and I_2 placed distance r apart is

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$

(*ii*) Force on smaller conductor of length l carrying current I_1, held parallel to infinite long linear conductor carrying current I_2, placed distance r apart is

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2l}{r}$$

(*iii*) Force on long linear conductor carrying current I_1, placed parallel to a small conductor of length l, carrying current I_2 is

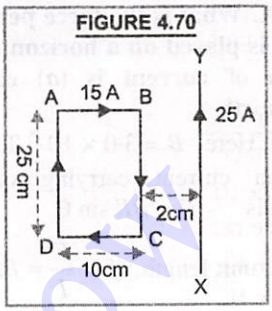
$$F' = \frac{\mu_0}{4\pi}\frac{2I_1I_2l}{r}$$

The direction of F' is opposite to F given in point (*ii*).

Units used. I_1, I_2 in ampere, l and r in metre, and F in newton.

Example 42 **Fig. 4.70 shows a rectangular current-carrying loop placed 2 cm away from a long, straight, current carrying conductor. What is the direction and magnitude of the net force acting on the loop ?** (CBSE 2005)

FIGURE 4.70



Solution. Here, $I_1 = 15$ A ; $I_2 = 25$ A ;

$r_1 = 2 \times 10^{-2}$ m ; $r_2 = (2 + 10) \times 10^{-2}$ m

Force on BC, $F_1 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_1} \times$ length BC

$$= 10^{-7} \times \frac{2\times15\times25}{(2\times10^{-2})} \times (25\times10^{-2})$$

$= 9{\cdot}375 \times 10^{-4}\ N$ (repulsive, away from XY)

Force on DA, $F_2 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_2} \times$ length DA

$$= 10^{-7} \times \frac{2\times15\times25}{(2+10)\times10^{-2}} \times 25 \times 10^{-2}$$

$= 1{\cdot}5625 \times 10^{-4}$ N (attractive towards XY)

Net force on the loop $F = F_1 - F_2$

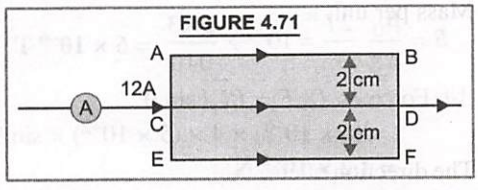
$= (9{\cdot}375 - 1{\cdot}5625) \times 10^{-4}$

$= \mathbf{7{\cdot}8175 \times 10^{-4}\ N}$

(respulsive, away from XY)

Example 43 **In Fig.4.71, the three parallel wires *AB, CD* and *EF* are long and have identical resistance. The separation between two neighbouring wires is 2·0 cm. The wires *AE* and *BF* have negligible resistance and ammeter reads 12 A. Calculate the magnetic force per unit length of *AB* and *CD*.**

FIGURE 4.71



Solution. Due to symmetry, the current through each of wires *AB, CD* and *EF* is 4 A.

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Force per unit length on AB due to current in CD is,

$$F_1 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} = 10^{-7}\times\frac{2\times4\times4}{(2\times10^{-2})}$$

$= 16\times10^{-5}$ Nm^{-1}, directed downward

Force per unit length on AB due to current in EF is,

$$F_2 = \frac{\mu_0}{4\pi}\frac{2I_1I_3}{(r+r)} = 10^{-7}\times\frac{2\times4\times4}{(4\times10^{-2})}$$

$= 8\times10^{-5}$ Nm^{-1} directed downward

Total force per unit length on AB is

$F = F_1 + F_2 = 16\times10^{-5} + 8\times10^{-5}$

$= \mathbf{24\times10^{-5}\ Nm^{-1}}$ **directed downwards**

Force per unit length on CD due to current in AB is,

$$F_1' = 10^{-7}\times\frac{2\times4\times4}{(2\times10^{-2})}$$

$= 16\times10^{-5}$ Nm^{-1}, directed upwards

Force per unit length on CD due to current in EF is,

$$F_2' = 10^{-7}\times\frac{2\times4\times4}{2\times10^{-2}}$$

$= 16\times10^{-5}$ Nm^{-1} directed downwards

Total force per unit length of $CD = F_1' + F_2' = 0$

Example 44 **A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying current 5 A is held vertically above AB at a height of 1 mm. Find the mass per unit length of the wire CD so that it remains suspended at the position when left free. Give the direction of the current flowing in CD with respect to that in AB. $g = 10$ ms^{-2}.** (CBSE 2013)

Solution. Weight per unit length of upper wire = magnetic force per unit length

$$\frac{mg}{l} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$

Mass per unit length

$$= \frac{m}{l} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{rg} = \frac{10^{-7}\times2\times12\times5}{(10^{-3})\times10}$$

$= \mathbf{1{\cdot}2\times10^{-3}\ kg\ m^{-1}}$

The direction of current in CD must be opposite to that of current in AB so that the force between the two wires is repulsive.

TYPE IX. TORQUE ON A CURRENT LOOP

Formulae used.

$$\tau = |\vec{M}\times\vec{B}| = MB\sin\alpha = (nIA)\,B\sin\alpha.$$

where M = magnetic dipole moment of the current loop (in Am^2) :

n = number of turns in the coil,

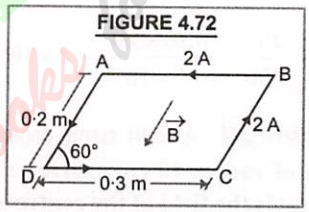
I = current in the coil,

α = angle which a normal drawn on the plane of coil makes with the direction of magnetic field.

Units used. I in ampere, A in m^2, B in tesla, α in degrees

Example 45 **A parallelogram shaped coil $ABCD$ of sides 0·3 m and 0·2 m carries a current of 2·0 A as shown in Fig. 4.72. It is placed in a uniform magnetic field $\vec{B} = 50T$ parallel to AD. Find (*i*) forces on the sides of the coil and (*ii*) torque on the coil.**

FIGURE 4.72



Solution. Here, $l = 0{\cdot}3$ m, $b = 0{\cdot}2$ m ;

$I = 2{\cdot}0$ A, $B = 50$ T

(*i*) Since magnetic field $\vec{B}$ is parallel to sides AD and BC, hence no force acts on these sides.

Force on the side AB is

$F = IlB\sin\theta = 2{\cdot}0\times0{\cdot}3\times50\times\sin60°$

$= 2\times0{\cdot}3\times50\times\sqrt{3}/2 = 25{\cdot}98\ \text{N} \approx \mathbf{26\ N}$

According to Flemings Left Hand rule the direction of the force is **normally upwards.** Similarly force on the side DC will also be $\approx$ **26 N.**

Its direction is **normally downwards.**

(*ii*) Since force on sides AB and DC are equal, parallel and opposite, they form a couple which exerts a torque. Therefore, torque on coil is

$$\tau = F\times AD\sin60° = 26\times0{\cdot}2\times\sqrt{3}/2$$

$= \mathbf{4{\cdot}5\ Nm}$

Example 46 **A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the**

current in the coil is 5.0 A, what is the

(*a*) total torque on the coil,

(*b*) total force on the coil,

(*c*) average force on each electron in the coil due to magnetic field ? The coil is made of copper wire of cross-sectional area 10^{-5} m^2 and the free electron density in copper is given to be about 10^{29} m^{-3}.

Solution. Here, $n = 20$, $r = 10$ cm $= 0.10$ m,
$B = 0.10$ T, $\alpha = 0°$, $I = 5.0$ A

Area of the coil, $A = \pi r^2 = \frac{22}{7} \times (0.10)^2$ m^2

(*a*) Now, $\tau = n\,I\,B\,A \sin\alpha = n\,I\,B\,A \sin 0° = \mathbf{0}$

(*b*) Total force on a planar current loop in a magnetic field is always **zero.**

(*c*) Given, $N = 10^{29}$ m^{-3}, $A = 10^{-5}$ m^2

Force on an electron of charge e, moving with drift velocity v_d in the magnetic field is given by

$$F = Bev_d = Be\frac{I}{NeA} \qquad (\because I = NeAv_d)$$

$$F = \frac{BI}{NA} = \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = \mathbf{5 \times 10^{-25}\ N}$$

Example 47 **A 100 turn closely wound circular coil of radius 10 cm carries a current of 3·2 A. (*i*) What is the field at the centre of the coil ? (*ii*) What is the magnetic moment of this arrangement ? The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2 T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field. (*iii*) What are the magnitudes of the torques on the coil in the initial and final positions ? (*iv*) What is the angular speed acquired by the coil when it has rotated by 90° ? The moment of inertia of the coil is 0·1 kg m^2.** (NCERT Solved Example)

Solution. (*i*) Here, $n = 100$, $r = 0{\cdot}10$ m,
$i = 3{\cdot}2$ A, $B = 2$ T, $I = 0{\cdot}1$ kg m^2

$$B = \frac{\mu_0}{4\pi}\frac{2\pi ni}{r}$$

$$= 10^{-7} \times 2 \times \frac{22}{7} \times \frac{100 \times 3{\cdot}2}{0{\cdot}10} = \mathbf{2 \times 10^{-3}\ T}$$

(*ii*) $M = niA = ni\,\pi r^2$

$$= 100 \times (3{\cdot}2) \times \left(\frac{22}{7}\right) \times (0{\cdot}10)^2$$

$$= \mathbf{10\ Am^2}$$

(*iii*) $\tau = |\vec{M} \times \vec{B}| = MB \sin\theta$ where θ is the angle between $\vec{M}$ and $\vec{B}$ or between $\vec{A}$ and $\vec{B}$.

Initially, $\theta = 0°$, $\tau = MB \sin 0° = 0$.

Finally, $\theta = 90°$, $\tau = MB \sin 90° = MB$
$= 10 \times 2 = \mathbf{20\ Nm}$

(*iv*) $\tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta} \times \frac{d\theta}{dt}$

$$= I\frac{d\omega}{d\theta} \times \omega = MB \sin\theta.$$

$\therefore \quad I\omega\, d\omega = MB \sin\theta\, d\theta.$

Integrating it within the given conditions,

$$I\int_0^{\omega} \omega\, d\omega = \int_0^{\pi/2} MB \sin\theta\, d\theta$$

$$I\frac{\omega^2}{2} = MB\,(-\cos\theta)_0^{\pi/2}$$

$$= -MB\left[\cos\frac{\pi}{2} - \cos 0°\right] = MB$$

or $$\omega = \left(\frac{2MB}{I}\right)^{1/2} = \left(\frac{2 \times 20}{0{\cdot}1}\right)^{1/2}$$
$$= \mathbf{20\ rad/s}$$

Second Method for (iv)

Change in KE of rotation = work done in rotation

$$\frac{1}{2}I\omega^2 = MB\,(\cos\theta_1 - \cos\theta_2)$$

where $\theta_1 = 0°$; $\theta_2 = 90°$, $I = 0{\cdot}1$ kg m^2 ;
$MB = 20$ Nm.

$$\therefore \quad \omega = \left[\frac{2MB\,(\cos\theta_1 - \cos\theta_2)}{I}\right]^{1/2}$$

$$= \left[\frac{2 \times 20 \times (\cos 0° - \cos 90°}{0{\cdot}1}\right]^{1/2}$$

$$= \mathbf{20\ rad/s}$$

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TYPE X. MOVING COIL GALVANOMETER

Formulae used. (*i*) $I = \frac{k}{nBA}\theta$

where, k = restoring torque per unit twist,
B = magnetic field in which coil of galvanometer is suspended,
A = area of each turn of coil, θ = deflection in galvanometer.

(*ii*) Current sensitivity, $I_s = \frac{\theta}{I} = \frac{nBA}{k}$ (degree/A)

(*iii*) Voltage sensitivity,

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{nBA}{kR} = \frac{I_s}{R} \quad \text{(degree/V)}$$

Here, R is the resistance of coil.

Units used. I in ampere, A in m^2, B in tesla, θ in degrees, V in volt.

Example 48 **Compare the current sensitivity and voltage sensitivity of the following moving coil galvanometers :**

Meter A : $n = 30$, $A = 1{\cdot}5 \times 10^{-3}\ m^2$, $B = 0{\cdot}25$ T, $R = 20\ \Omega$

Meter B : $n = 35$, $A = 2{\cdot}0 \times 10^{-3}\ m^2$, $B = 0{\cdot}25$ T, $R = 30\ \Omega$.

You are given that the springs in the two metres have the same torsional constants.

Solution. For metre A, $n_1 = 30$, $A_1 = 1{\cdot}5 \times 10^{-3}\ m^2$, $B_1 = 0{\cdot}25$ T, $R_1 = 20\ \Omega$.

For metre B, $n_2 = 35$, $A_2 = 2{\cdot}0 \times 10^{-3}\ m^2$, $B_2 = 0{\cdot}25$ T, $R_2 = 30\ \Omega$.

$$\text{Current sensitivity of a meter} = \frac{nBA}{k}$$

$$\therefore \frac{\text{Current sensitivity of meter A}}{\text{Current sensitivity of meter B}} = \frac{n_1B_1A_1}{k_1} \times \frac{k_2}{n_2B_2A_2} = \frac{n_1B_1A_1}{n_2B_2A_2} \quad (\because k_1 = k_2)$$

$$= \frac{30 \times 0{\cdot}25 \times 1{\cdot}5 \times 10^{-3}}{35 \times 0{\cdot}25 \times 2{\cdot}0 \times 10^{-3}}$$

$$= \frac{45}{70} = \frac{\mathbf{9}}{\mathbf{14}}$$

Now, voltage sensitivity of a meter $= \frac{nBA}{kR}$

$$\therefore \frac{\text{Voltage sensitivity of meter A}}{\text{Voltage sensitivity of meter B}} = (\text{Ratio of current sensitivity}) \times \frac{R_2}{R_1}$$

$$= \frac{9}{14} \times \frac{30}{20} = \frac{\mathbf{27}}{\mathbf{28}}$$

Example 49 **A current of 300 μA deflects a coil of a moving coil galvanometer through 30°. What should be the current to cause the rotation through π/5 radians ? What is the current sensitivity of the galvanometer ? If resistance of galvanometer is 50 Ω, find its voltage sensitivity.**

Solution. Here, $I_1 = 300\ \mu A$, $\theta_1 = 30°$,

$$\theta_2 = \frac{\pi}{5} \text{ rad} = 36°,\ I_2 = ?$$

For galvanometer,

$$I_1 = \frac{k}{nBA}\theta_1 \text{ and } I_2 = \frac{k}{nBA}\theta_2$$

$$\frac{I_2}{I_1} = \frac{\theta_2}{\theta_1}$$

or $I_2 = \frac{\theta_2}{\theta_1} \times I_1 = \frac{36°}{30°} \times 300\,\mu A = \mathbf{360\ \mu A}$

Current sensitivity,

$$I_s = \frac{\theta_1}{I_1} = \frac{30°}{300\,\mu A} = \mathbf{0{\cdot}1\ degree/\mu A}$$

Voltage sensitivity,

$$V_s = \frac{I_s}{R} = \frac{0{\cdot}1 \text{ degree } (\mu A)^{-1}}{50\,\Omega}$$

$$= \frac{0{\cdot}1^0 \times (10^{-6})^{-1} A}{50\,\Omega} = \mathbf{2000\ degree/volt}$$

Example 50 **The current sensitivity of a moving coil galvanometer increases by 20% when its resistance is increased by a factor 2. Calculate by what factor does the voltage sensitivity change ?**

Solution. Given, $I_s' = I_s + \frac{20}{100}I_s = \frac{120}{100}I_s$;

$R' = 2R$

Then, initial voltage sensitivity, $V_s = \frac{I_s}{R}$

New voltage sensitivity,

$$V_s' = \frac{I_s'}{R'} = \left(\frac{120}{100}I_s\right) \times \frac{1}{2R} = \frac{3}{5}V_s$$

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∴ % decrease in voltage sensitivity

$$= \frac{V_s - V_s'}{V_s} \times 100 = \frac{V_s - \frac{3}{5}V_s}{V_s} \times 100 = \mathbf{40\%}$$

Example 51 **A current of 500 μA deflects the coil of a moving coil galvanometer through 60°. What should be the current to cause the rotation through π/5 radian ? What is the current sensitivity of galvanometer ?**

Solution. Here, $I_1 = 500\ \mu A = 500 \times 10^{-6}$ A,

$$\theta_1 = 60°,\ I_2 = ?,\ \theta_2 = \frac{\pi}{5}\text{rad.} = \frac{180°}{5} = 36°$$

$$I_1 = \frac{k}{nBA}\theta_1 \text{ and } I_2 = \frac{k}{nBA}\theta_2$$

$$\therefore \frac{I_2}{I_1} = \frac{\theta_2}{\theta_1} \text{ or}$$

$$I_2 = \frac{\theta_2}{\theta_1} I_1 = \frac{36}{60} \times 500 \times 10^{-6} = \mathbf{300 \times 10^{-6}\ A}$$

$$\text{Current sensitivity} = \frac{\theta_2}{I_2} = \frac{36°}{300 \times 10^{-6}}$$

$$= 0{\cdot}12 \times 10^6 \text{ degree/A} = \mathbf{0{\cdot}12\ degree/\mu A}$$

TYPE XI. CONVERSION OF GALVANOMETER INTO (*a*) AMMETER AND (*b*) VOLTMETER

Formulae used. (*i*) $I_g = nK$

(*ii*) $S = \frac{I_g G}{(I - I_g)}$ (*iii*) $R = \frac{V}{I_g} - G$

n = number of scale divisions in galvanometer,
K = figure of merit of galvanometer,
I_g = current for full scale deflection,
I = total current to be measured,
S = shunt resistance, G = galvanometer resistance,
V = potential difference to be measured,
R = resistance to be connected in series

Units used. I, I_g in A, V in volt, G, R in ohm.

Example 52 **A galvanometer with a scale divided into 100 equal divisions has a current sensitivity of 10 divisions per mA and a voltage sensitivity of 2 divisions per mV. What adoptions are required to read (*i*) 5A for full scale and (*ii*) 1 division per volt ?**

Solution. In the given galvanometer, 10 divisions deflection is observed by 1 mA. So current for full scale deflection is

$$I_g = \frac{1}{10} \times 100\text{ mA} = 10\text{ mA} = 10 \times 10^{-3}\text{ A}$$
$$= 0{\cdot}01\text{ A}$$

As 2 divisions deflection is observed by 1 mV, so voltage for full scale deflection is

$$V = \frac{1}{2} \times 100\text{ mV} = 50\text{ mV} = 50 \times 10^{-3}\text{ V}$$
$$= 0{\cdot}05\text{ V}$$

∴ Galvanometer resistance,

$$G = \frac{V}{I_g} = \frac{0{\cdot}05}{0{\cdot}01} = 5\,\Omega$$

(*i*) **For conversion into ammeter,** $I = 5$ A,

$$S = \frac{I_g}{I - I_g} G = \frac{0{\cdot}01}{5 - 0{\cdot}01} \times 5 = \mathbf{\frac{5}{499}\ \Omega}$$

Therefore, a shunt of (5/499) Ω should be connected across the galvanometer to read 5 A for full scale deflection.

(*ii*) **For conversion into voltmeter.** For reading 1 div per volt, and there are 100 divisions, so the voltage range should be 100 V.

$$R = \frac{V}{I_g} - G = \frac{100}{0{\cdot}01} - 5 = 10000 - 5 = \mathbf{9995\ \Omega}$$

Therefore, a resistance of 9995 Ω should be connected in series of the galvanometer to read 1 div/volt.

Example 53 **A resistance of 1980 Ω is connected in series with a voltmeter, after which the scale division becomes 100 times larger. Find the resistance of voltmeter.**

Solution. Let R be the resistance of voltmeter. Let n be the number of divisions in the voltmeter. The voltage (V) recorded by each division of voltmeter when current i_g flows through it is

$$i_g R/n = V \quad \text{...(i)}$$

When resistance is connected in series with voltmeter, then

$$i_g (R + 1980)/n = 100\text{ V} \quad \text{...(ii)}$$

Dividing (*ii*) by (*i*), we get

$$R + 1980 = 100\text{ R}$$

or $R = 1980/99 = \mathbf{20\ \Omega}$

Example 54 **A galvanometer reads 5·0 V at full scale deflection and is graded according to its resistance per volt at full scale deflection as 5000 Ω V^{-1}. (*i*) How will you convert it into a voltmeter that reads 20 V at full scale deflection? (*ii*) Will it still be graded 5000 Ω V^{-1} ? (*iii*) Will you prefer this voltmeter to one that is graded 2000 Ω V^{-1} ?** [CBSE 2001 (C)]

Solution. Here, resistance per volt = 5000 Ω V^{-1}. It means for 1 volt pot. difference, the resistance of galvanometer is 5000 Ω. So current for full scale deflection,

$$I_g = \frac{1}{5000}\text{A} = 0{\cdot}2 \times 10^{-3}\text{ A}$$

(*i*) In order to convert it into a voltmeter of range 0 to 20 V, let a resistance R be connected in series with it. Then on applying an extra potential difference = 20 V – 5 V = 15 V, the current through it for full scale deflection is again $0{\cdot}2 \times 10^{-3}$ A *i.e.*, $I_g = 0{\cdot}2 \times 10^{-3}$ A.

$$\therefore R\,I_g = 15 \text{ or } R = \frac{15}{I_g} = \frac{15}{0{\cdot}2\times10^{-3}} = \mathbf{75000\ \Omega}$$

Thus, to convert a given voltmeter (of range 0 to 5 V) into a voltmeter (of range 0 to 20 V) a resistance of 75000 Ω should be connected in series, with the given meter.

(*ii*) Original resistance of voltmeter

$$= 5000\ \Omega\ \text{V}^{-1} \times 5\ \text{V} = 25000\ \Omega$$

Total resistance after conversion

$$= 25000 + 75000 = 100{,}000\ \Omega$$

Resistance per volt of new meter

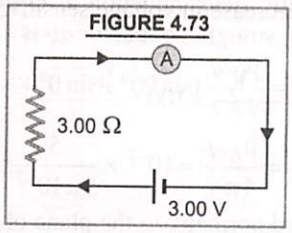
$$= \frac{100{,}000}{20} = 5000\ \Omega\ \text{V}^{-1}$$

i.e. **it has the same grading as before.**

(*iii*) If greater is the resistance per volt of a meter, the lesser will be the current drawn from the circuit by it. Due to it, it works better. That is why **this meter (graded 5000 Ω V^{-1}) is more accurate that the one graded as 2000 Ω V^{-1}.**

Example 55 **In the circuit shown in Fig. 4.73, the current is to be measured. What is the value of the current if the ammeter shown (*i*) is a galvanometer with a resistance R_G = 60·00 Ω. (*ii*) is a galvanometer described in (*i*) but converted to an ammeter by a shunt resistance r_s = 0·02 Ω ; (*iii*) is an ideal ammeter with zero resistance ?** (NCERT Solved Example)

FIGURE 4.73



Solution. (*i*) Let G be the resistance of ammeter (*i.e.* galvanometer). Then current in the circuit,

$$I = \frac{\varepsilon}{R+G} = \frac{3{\cdot}00}{3{\cdot}00 + 60{\cdot}00} = \mathbf{0{\cdot}048\ A.}$$

(*ii*) When the ammeter (*i.e.*, galvanometer) is shunted with resistance S, its effective resistance,

$$R_p = \frac{GS}{G+S} = \frac{60\times0{\cdot}02}{60+0{\cdot}02} \approx 0{\cdot}02\,\Omega$$

Current in the circuit,

$$I = \frac{\varepsilon}{R+R_p} = \frac{3{\cdot}00}{3{\cdot}00+{\cdot}02} = \mathbf{0{\cdot}99\ A}$$

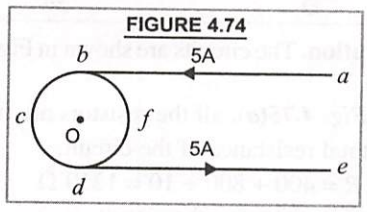
(*iii*) For the ideal ammeter with zero resistance,

current, $I = \dfrac{3{\cdot}00}{3{\cdot}00} = \mathbf{1{\cdot}00\ A}$

TYPE XII. TYPICAL EXAMPLES

Example 56 **In Fig. 4.74 bcdf is a circular coil of non-insulated thin uniform conductors, ab and de are very long straight parallel conductors, tangential to the coil at the points b and d. If the current 5 A enters the coil from a to b, find the magnetic field induction at O, the centre of the coil. The diameter of the coil is 10 cm.**

FIGURE 4.74



Solution. The current in the part *bcd* of the coil is equal to the current in the part *bfd* of the coil, which is equal to 2·5 A. Here,

$$r = Ob = Oc = Od = Of = 5\text{ cm} = 5 \times 10^{-2}\text{ m.}$$

Magnetic field induction at O due to current through circular coil will be zero because the magnetic field induction at O due to current through segment *bcd* of the coil is equal and opposite to that due to current through segment *bfd* of the coil.

Magnetic field induction at O due to current through long straight conductor ab is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r} = 10^{-7} \times \frac{5}{5\times10^{-2}} = 10^{-5}\ \text{T}$$

outward normally to the plane of paper.

Magnetic field induction at O due to current through long straight conductor de is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r} = 10^{-7} \times \frac{5}{5\times10^{-2}} = 10^{-5}\ \text{T}$$

outwards normally to the plane of paper

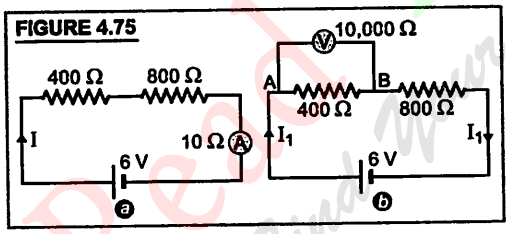
$\therefore$ Total magnetic field induction at O

$B = B_1 + B_2 = 10^{-5} + 10^{-5} = 2 \times 10^{-5}$ T

normal to plane of paper outwards.

Example 57 **Two resistors 400 Ω and 800 Ω are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10 Ω resistance is used for this purpose Fig. 4.75. What will be the reading in the ammeter ? Similarly, if a voltmeter of 10,000 Ω resistance is used to measure the potential difference across the 400 Ω resistor, what will be the reading in the voltmeter ?**

FIGURE 4.75



Solution. The circuits are shown in Figs. 4.75(a) and (b).

In Fig. 4.75(a), all the resistors are in series,

$\therefore$ total resistance of the circuit,

$R = 400 + 800 + 10 = 1210\ \Omega$

Hence, current in the circuit,

$$I = \frac{V}{R} = \frac{6}{1210} = 0{\cdot}00496\ \text{A}$$

Reading in ammeter is **0·00496 A**

In Fig. 4.75(b), the resistors of 400 Ω and 10,000 Ω are in parallel, their effective resistance R_p will be

$$R_p = \frac{400\times10,000}{400+10,000} = \frac{5000}{13}\ \Omega$$

$\therefore$ Total resistance of the circuit

$$= \frac{5000}{13} + 800 = \frac{15400}{13}\ \Omega$$

Current in the circuit, $I_1 = \dfrac{6}{15400/13} = \dfrac{39}{7700}$ A

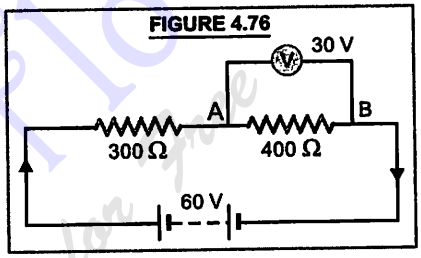
Potential difference across A and B

$$= I_1 R_p = \frac{39}{7700} \times \frac{5000}{13} = \frac{150}{77} = 1{\cdot}95\ \text{V}$$

$\therefore$ Reading of voltmeter is **1·95 V**

Example 58 **In the circuit shown in Fig. 4.76, a voltmeter reads 30 V when it is connected across 400 Ω resistance. Calculate what the same voltmeter will read when connected across the 300 Ω resistance.**

FIGURE 4.76



Solution. The circuit is shown in Fig. 4.29.

Let G be the resistance of voltmeter, connected in parallel with 400 Ω, their effective resistance becomes, $R_p = \dfrac{400\times G}{G+400}$

$\therefore$ Current in the circuit,

$$I = \frac{60}{300 + 400G/(G+400)}$$

$$= \frac{60\ (G+400)}{300\ (G+400) + 400G}$$

Pot. diff. across A and $B = IR_p = 30$ V

or $$\frac{60\ (G+400)}{300\ (G+400)+400G} \times \frac{400G}{(G+400)} = 30$$

On solving, $G = 1200\ \Omega$

When this voltmeter is connected in parallel with the resistance of 300 Ω, the effective resistance becomes, $R_p' = \dfrac{1200\times300}{1200+300} = 240\ \Omega$

Total resistance of the circuit

$= 240 + 400 = 640\ \Omega$

$\therefore$ Current, $I' = \dfrac{60}{640} = \dfrac{3}{32}$ A

$\therefore$ Reading of voltmeter = pot. diff. across

$$R_p' = I' R_p' = \frac{3}{32} \times 240 = 22{\cdot}5\ \text{V}$$

REVISION AT A GLANCE

1. **Magnetic effect of current.** It is a phenomenon by virtue of which a current produces, a magnetic field. A straight current (*i.e.* linear conductor carrying current) produces a circular magnetic field and a circular current (*i.e.* circular coil carrying current) produces a straight magnetic field at the centre of the circular coil.

2. **Magnetic force** on a charge q, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by

$$|\vec{F}| = q|\vec{v} \times \vec{B}| = qvB\sin\theta$$

If $q = 1, v = 1$, $\sin\theta = 1$ or $\theta = 90°$, then $F = B$.

Thus strength of magnetic field at a point in the field is equal to the force acting on a unit charge moving with a unit velocity, perpendicular to the direction of the magnetic field at that point.

Magnetic force on the charged particle in a magnetic field is maximum if the charged particle is moving perpendicular to the direction of magnetic field (*i.e.* $\theta = 90°$) and is minimum, if the charged particle is moving along the direction of the magnetic field (*i.e.* $\theta = 0°$).

3. **SI Unit of magnetic field strength is tesla.** If a charge of 1 coulomb, while moving with a velocity of 1 ms^{-1} at a point, perpendicular to the direction of magnetic field, experiences a force of 1 newton, the strength of magnetic field at that point is 1 tesla.

$$1 \text{ tesla} = 1 \text{ N } A^{-1} m^{-1}$$

4. **Dimensions of B** = $[MA^{-1}T^{-2}]$.

5. **Biot-Savart's law.** According to this law, the magnitude of magnetic field induction at a point due to a current element of length dl, carrying current I, at a distance r from the element is given by

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I|d\vec{l} \times \vec{r}|}{r^3} \quad \textbf{(in SI Units)}$$

The direction of $d\vec{B}$ is given by Right Handed Screw Rule or Right Hand Rule.

6. **Magnetic field induction at the centre of a circular coil of radius r carrying current I is given by**

$$B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r} = \frac{\mu_0 nI}{2r} \quad \textbf{(in SI Units)}$$

The direction of this magnetic field induction is given by ***Right Hand palm rule.***

7. **Magnetic field induction at a point, at perpendicular distance 'a' from the axis of long linear conductor carrying current is given by,**

$$B = \frac{\mu_0}{4\pi}\frac{2I}{a} \quad \textbf{(in SI Units)}$$

The direction of the magnetic field lines is given by ***Right Hand thumb rule***

8. **Magnetic field induction at a point on the axis of a circular coil carrying current is given by**

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n Ia^2}{(a^2 + x^2)^{3/2}} \quad \textbf{(in SI Units)}$$

where a is the radius of the coil, x is the distance of the point on the axis, from the centre of the coil, n is the number of turns in the coil.

9. **Magnetic dipole moment of the current loop** is given by

$$M = nIA = nI\pi a^2.$$

where n is the number of turns in the loop, carrying current I and a is the radius of the loop. The SI unit of M is ampere (metre)^2 [A-m^2]

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10. **Ampere's circuital law.** It states that the line integral of magnetic field induction $\vec{B}$ around any closed path in vacuum is equal to μ_0 times the total current threading the closed path *i.e.*

$$\oint \vec{B}.d\vec{l} = \mu_0 I .$$

11. **A solenoid** consists of an insulated long wire closely wound in the form of a helix. Its length is very large as compared to its diameter.

Magnetic field induction at a point well inside the solenoid of length l, having N turns carrying current I is given by

$$B = \mu_0 NI/l.$$

Magnetic field induction at a point on one end of the solenoid carrying current is $B = \mu_0 NI/2\,l$

12. **Force on a charged particle in an electric field** is

$$F = qE.$$

This force acts in the direction of electric field. The charged particle describes a parabolic path if electric field is acting perpendicular or at some angle with the initial direction of motion of the particle.

13. **Magnetic force on a charged particle in a magnetic field** is

$$|\vec{F}| = q\,|\vec{v}\times\vec{B}| = q\,v\;B\sin\theta.$$

This force acts perpendicular to the plane containing $\vec{v}$ and $\vec{B}$, and is directed as given by Right Handed screw rule. If the moving charged particle is subjected to a perpendicular magnetic field, it describes a circular path and if moving charged particle is subjected to a magnetic field, making some angle with its initial direction of motion, it will describe a helical path.

14. **Lorentz force.** The force experienced by a charged particle moving in space where both electric and magnetic fields exist is called Lorentz force. *i.e.*

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\,\vec{E} + q(\vec{v}\times\vec{B}) .$$

15. **Cyclotron frequency.** When the frequency of oscillations of a charged particle in cyclotron becomes equal to the frequency of the oscillating electric field applied, then the frequency of the particle is called cyclotron frequency, which is given by

$$\nu = \frac{Bq}{2\pi m}$$

16. **Force on a current carrying conductor in a magnetic field** is given by

$$|\vec{F}| = I\,|\vec{l}\times\vec{B}| = IlB\sin\theta$$

This force is maximum if $\sin\theta = 1$ or $\theta = 90°$ and is minimum if $\sin\theta = 0$ or $\theta = 0°$ or $180°$.

17. **Two long parallel conductors carrying currents in the same direction attract each other and carrying curents in the opposite direction, repel each other.** The force acting per unit length of each conductor will be

$$F = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$$

where I_1, I_2 = currents in the two linear parallel conductors, placed distance r apart.

18. **One ampere of current** is that much current which when flowing through each of the two parallel long linear conductors placed in free space at a distance of one metre from each other will attract or repel each other with a force of 2×10^{-7} N per metre of their length.

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19. **Torque on a coil of area A,** having n turns, carrying current I, when suspended in a magnetic field of strength B is given by

$$\tau = nIBA \sin \alpha$$

where α is the angle which a normal drawn on the plane of the coil makes with the direction of magnetic field.

Torque is minimum if the plane of the coil is perpendicular to the direction of magnetic field, because $\alpha = 0°$.

Torque is maximum if the plane of the coil is parallel to the direction of magnetic field because $\alpha = 90°$.

20. **Radial magnetic field.** It is that magnetic field, in which the plane of the coil in all positions remains parallel to the direction of the magnetic field.

21. **Current sensitivity of a galvanometer.** It is defined as the deflection produced in the galvanometer, when unit current flows through it.

Current sensitivity, $$I_s = \frac{\theta}{I} = \frac{nBA}{k}$$

The unit of I_s is rad. A^{-1} or div. A^{-1}

22. **Voltage sensitivity of a galvanometer.** It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

Voltage sensitivity, $$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{nBA}{kR} = \frac{I_s}{R}$$

Unit of voltage sensitivity is rad. V^{-1} or div. V^{-1}.

23. **Shunt.** It is a low resistance wire connected in parallel with the galvanometer or ammeter. It protects the galvanometer or ammeter from the strong currents.

24. **Conversion of galvanometer into ammeter**

A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S in parallel to the given galvanometer, whose value is given by :

$$S = \left(\frac{I_g}{I - I_g}\right) G$$

where I_g is the current for full scale deflection of galvanometer, I is the current to be measured by the galvanometer and G is the resistance of galvanometer.

An ideal ammeter has zero resistance.

25. **Conversion of galvanometer into voltmeter**

A galvanometer can be converted into voltmeter of given range by connectintg a suitable resistance R in series with the galvanometer, whose value is given by

$$R = \frac{V}{I_g} - G$$

where V is the voltage to be measured, I_g is the current for full scale deflection of galvanometer and G is the resistance of galvanometer.

The resistance of an ideal voltmeter is infinity.

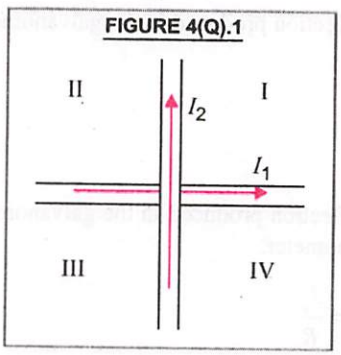
OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. Two wires carrying currents I_1 and I_2 lie, one slightly above the other, in a horizontal plane as shown in Fig. 4(Q).1. The region of vertically upward strongest magnetic field.

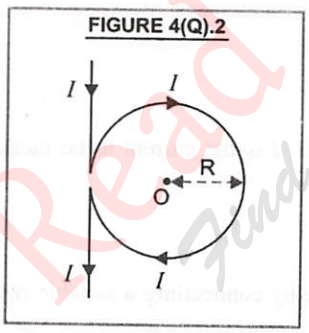
FIGURE 4(Q).1



(*a*) I (*b*) II
(*c*) III (*d*) IV **(CBSE 2022)**

2. A current I flows through a long straight conductor which is bent into a circular loop of radius R in the middle as shown in the Fig. 4(Q).2. The magnitude of the net magnetic field at point O will be :

FIGURE 4(Q).2



(*a*) zero (*b*) $\frac{\mu_0 I}{2R}(1+\pi)$

(*c*) $\frac{\mu_0 I}{4\pi R}$ (*d*) $\frac{\mu_0 i}{2R}\left(1-\frac{1}{\pi}\right)$

(CBSE 2020)

3. A current 10 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due north on the table is :

(*a*) 2×10^{-5} T, acting downwards
(*b*) 2×10^{-5} T, acting upwards
(*c*) 4×10^{-5} T, acting downwards
(*d*) 4×10^{-5} T, acting upwards **(CSBE 2020)**

4. An electron and a proton are moving along the same direction with the same kinetic energy. They enter a uniform magnetic field acting perpendicular to their velocities. The dependence of radius of their paths on their masses is :

(*a*) $r \propto m$ (*b*) $r \propto \sqrt{m}$

(*c*) $r \propto \frac{1}{m}$ (*d*) $r \propto \frac{1}{\sqrt{m}}$

(CBSE 2020)

5. A current of 5 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due south on the table is :

(*a*) 1×10^{-5} T acting downwards
(*b*) 1×10^{-5} T acting upwards
(*c*) 2×10^{-5} T acting downwards
(*d*) 2×10^{-5} T acting upwards **(CBSE 2020)**

6. There are uniform electric and magnetic fields in a region pointing along X-axis. An α-particle is projected along Y-axis with a velocity v. The shape of the trajectory will be :

(*a*) circular in XZ plane
(*b*) circular in YZ plane
(*c*) helical with its axis parallel to X-axis
(*d*) helical with its axis parallel to Y-axis

(CBSE 2020)

7. A charged particle after being accelerated through a potential difference V enters in a uniform magnetic field and moves in a circle of radius r. If V is doubled, the radius of the circle will becomes :

(*a*) $2r$ (*b*) $\sqrt{2}r$

(*c*) $4r$ (*d*) $r/\sqrt{2}$ **(CBSE 2020)**

8. Two concentric and coplanar circular loops P and Q have their radii in the ratio 2 : 3. Loop Q carries a current 9 A in the anticlockwise direction. For

the magnetic field to be zero at the common centre, loop P must carry

(a) 3 A in clockwise direction

(b) 9 A in clockwise direction

(c) 6 A in anticlockwise direction

(d) 6 A in the clockwise direction

(CBSE Sample Paper 2022-23)

9. A long straight wire of circular cross-section of radius a carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnitudes of magnetic field at a point $a/2$ above the surface of the wire to that at a point distant $a/2$ below its surface is :

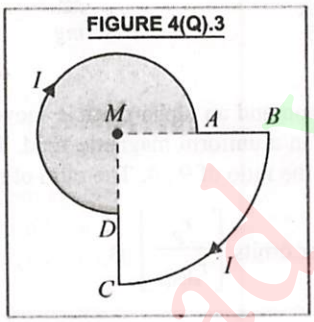
(a) 4 : 1 (b) 1 : 1

(c) 4 : 3 (d) 3 : 4

(CBSE Sample Paper 2022-23)

10. A current I is flowing through the loop as shown in Fig. 4(Q).3 ($MA = R$, $MB = 2R$). The magnetic field at the centre of the loop is $\frac{\mu_0 I}{R}$ times :

FIGURE 4(Q).3



(a) $\frac{5}{16}$, into the plane of the paper

(b) $\frac{5}{16}$, out of the plane of the paper

(c) $\frac{7}{16}$, out of the plane of the paper

(d) $\frac{7}{16}$, into the plane of the paper

(CBSE 2022)

11. A long straight wire in the horizontal plane carries a current of 15 A in north to south direction. The magnitude and direction of magnetic field at a point 2·5 m east of the wire respectively are :

(a) 1·2 μT, vertically upward

(b) 1·2 μT vertically downward

(c) 0·6 μT, vertically upward

(d) 0·6 μT vertical downward **(CBSE 2022)**

12. An electron is projected with velocity $\vec{v}$ along the axis of a current carrying long solenoid. Which of the following statements is true ?

(a) The path of the electron will be circular about the axis

(b) The electron will be accelerated along the axis

(c) The path of the electron will be helical

(d) The electron will continue to move with the same velocity $\vec{v}$ along the axis of the solenoid **(CBSE 2022)**

13. If the speed v of a charged particle moving in a magnetic field $\vec{B}$ ($\vec{v}$ is perpendicular to $\vec{B}$) is halved, then the radius of its path will :

(a) not change (b) become two times

(c) become one-fourth (d) become half

(CBSE 2022)

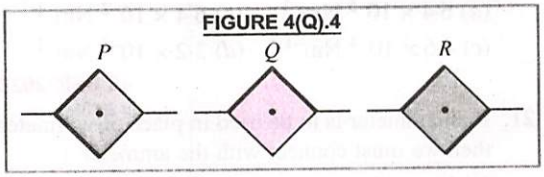
14. A test charge of $1{\cdot}6 \times 10^{-19}$ C is moving with a velocity $\vec{v} = (4\hat{i} + 3\hat{k})$ ms^{-1} in a magnetic field $\vec{B} = (3\hat{k} + 4\hat{i})$ T. The force on this test charge is :

(a) $24\hat{j}$ N (b) $-24\hat{i}$ N

(c) $24\hat{k}$ N (d) 0

15. Two identical thick wires and two identical thin wires, all of the same material and the same length form a square in three different ways P, Q and R as shown in Fig. 4(Q).4. Due to the current on these loops, the magnetic field at the centre of the loop will be zero in case of :

FIGURE 4(Q).4



(a) P and R only (b) Q and R only

(c) P and Q only (d) P, Q and R

(CBSE 2022)

16. A circular current carrying coil produces a magnetic field B_0 at its centre. The coil is rewound so as to have three turns and the same current is

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passed through it, the new magnetic field at the centre is :

(a) $3\,B_0$ (b) $\frac{B_0}{3}$

(c) $\frac{B_0}{3}$ (d) $9\,B_0$ (CBSE 2022)

17. A long solenoid carrying current produces a magnetic field B along its axis. If the number of turns in the solenoid is halved and current in it is doubled the new magnetic field will be :

(a) $\frac{B}{2}$ (b) B

(c) $2\,B$ (d) $4\,B$ (CBSE 2022)

18. Two moving coil galvanometers G_1 and G_2 have the following particulars respectively.

$N_1 = 30, A_1 = 3{\cdot}6 \times 10^{-3}\ m^2, B_1 = 0{\cdot}25\ T$

$N_2 = 42, A_2 = 1{\cdot}8 \times 10^{-3}\ m\ ; B_2 = 0{\cdot}50\ T$

The spring constant is same for both the galvanometers. The ratio of current sensitivities of G_1 and G_2 is :

(a) 5 : 7 (b) 7 : 5

(c) 1 : 4 (d) 1 : 1 (CBSE 2022)

19. A current carrying square loop is suspended in a uniform magnetic field acting in the plane of the loop. If the force on one arm of the loop is $\vec{F}$, the net force on the remaining three arms of the loop will be

(a) $3\vec{F}$ (b) $-3\vec{F}$

(c) $\vec{F}$ (d) $-\vec{F}$ (CBSE 2022)

20. Two parallel conductors carrying current of 4·0 A and 10·0 A are placed 2·5 cm apart in vacuum. The force per unit length between them is

(a) $6{\cdot}4 \times 10^{-5}\ Nm^{-1}$ (b) $6{\cdot}4 \times 10^{-2}\ Nm^{-1}$

(c) $4{\cdot}6 \times 10^{-4}\ Nm^{-1}$ (d) $3{\cdot}2 \times 10^{-4}\ Nm^{-1}$

(CBSE 2022)

21. If an ammeter is to be used in place of voltmeter, then we must connect with the ammeter a :

(a) low resistance in parallel

(b) low resistance in series

(c) high resistance in parallel

(d) high resistance in series (CBSE 2022)

22. The magnetic field at the centre of a current carrying circular loop of radius R is B_1. The magnetic field at a point on its axis at a distance R from the centre of loop is B_2. Then the ratio (B_1/B_2) is :

(a) $2\sqrt{2}$ (b) $1\sqrt{2}$

(c) $\sqrt{2}$ (d) 2 (CBSE 2022)

23. A current carrying wire kept in a uniform magnetic field will experience a maximum force when it is

(a) perpendicular to the magnetic field

(b) parallel to the magnetic field

(c) at an angle of 45° to the magnetic field

(d) at an angle of 60° to the magnetic field

(CBSE 2022)

24. A straight conducting rod of length l and mass m is suspended in a horizontal plane by a pair of flexible strings in a magnetic field of magnitude B. To remove the tension in the supporting strings, the magnitude of current in the wire is :

(a) $\frac{mgB}{l}$ (b) $\frac{mgl}{B}$

(c) $\frac{mg}{lB}$ (d) $\frac{lB}{mg}$

(CBSE 2022)

25. A proton and an alpha particle move in circular orbits in a uniform magnetic field. Their speeds are in the ratio of 9 : 4. The ratio of radii of their circular orbits $\left[\frac{r_p}{r_{alpha}}\right]$ is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{8}{9}$ (d) $\frac{9}{8}$

(CBSE 2022)

26. The current sensitivity of a galvanometer increases by 20%. If its resistance also increases by 25%, the voltage sensitivity will

(a) decrease by 1%

(b) increase by 5%

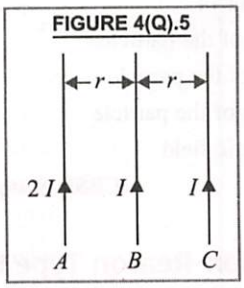
(c) increase by 10%

(d) decrease by 4%

27. Three infinity long parallel straight current carrying wires A, B and C are kept at equal distance from each other as shown in Fig. 4.(Q).5. The wire C experiences net force F. The net force

on wire C, when the current in wire A is reversed will be

FIGURE 4(Q).5



(a) zero (b) $F/2$
(c) F (d) $2F$

28. In a certain region of space, electric field $\vec{E}$ and magnetic field $\vec{B}$ are perpendicular to each other. An electron enters perpendicularly to both the field and moves undeflected. The velocity of electron is

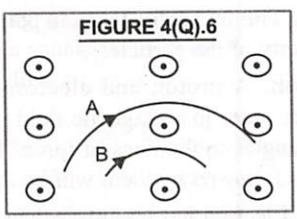
(a) $\frac{E}{B}$ (b) $\frac{B}{E}$
(c) $\vec{E} \times \vec{B}$ (d) $\vec{E} \cdot \vec{B}$

29. A deuteron of kinetic energy 50 keV is describing a circular orbits of radius 0·5 m in a plane perpendicular to the magnetic field $\vec{B}$. The kinetic energy of the proton that describes a circular orbit of same radius and inside same $\vec{B}$ is

(a) 25 kev (b) 50 kev
(c) 200 kev (d) 100 keV

30. Two particles A and B with same charges and different masses (m_A and m_B respectively) are moving in a plane inside uniform magnetic field which is perpendicular to the plane. The speed of the particles are V_A and V_B respectively and the trajectories are as shown in Fig. 4(Q).6. Then

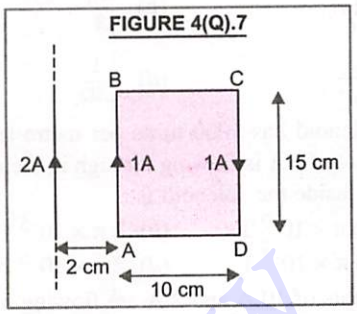
FIGURE 4(Q).6



(a) $m_A V_A < m_B V_B$ (b) $m_A V_A > m_B V_B$
(c) $m_A < m_B$ and $V_A < V_B$
(d) $m_A = m_B$ and $V_A < V_B$

31. A rectangular coil $ABCD$ is placed near a long straight current carrying straight wire as shown. What is the net force on the rectangular coil ?

FIGURE 4(Q).7



(a) 25×10^{-7} N towards the wire
(b) 25×10^{-7} N Away from the wire
(c) 35×10^{-7} N, towards the wire
(d) 35×10^{-7} N away from the wire

32. Two wires of same length are shaped into a square and a circle if they carry same current, ratio of magnetic moment is :

(a) $2 : \pi$ (b) $\pi : 2$
(c) $\pi : 4$ (d) $4 : \pi$

33. Current sensitivity of a galvanometer can be increased by decreasing :

(a) Magnetic field B
(b) number of turns n
(c) torsional constant k
(d) Area A

34. An electric current passes through a long straight copper wire. At a perpendicular distance 5 cm from the straight wire, the magnetic field is B. The magnetic field at perpendicular distance 20 cm from the straight wire would be

(a) $\frac{B}{6}$ (b) $\frac{B}{4}$
(c) $\frac{B}{3}$ (d) $\frac{B}{2}$

35. A wire in the form of a circular loop, of one turn carrying a current, produces magnetic induction B at the centre. If the same wire is looped into a coil of two turns and carries the same current, the new value of magnetic induction at the centre is

(a) B (b) 2 B
(c) 4 B (d) 8 B

36. A circular coil of radius a carries an electric current. The magnetic field due the coil at a point on the

axis of the coil located at a distance r from centre of the coil, such that $r >> a$, varies

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$

(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^{3/2}}$

37. A solenoid has 1000 turns per metre length. If a current of 5 A is flowing through it, then magnetic field inside the solenoid is

(a) $2\pi \times 10^{-3}$ T (b) $2\pi \times 10^{-5}$ T

(c) $4\pi \times 10^{-3}$ T (d) $2\pi \times 10^{-5}$ T

38. Currents of 10 A and 2 A are flowing in opposite directions through two parallel wires A and B respectively. If the wire A is infinitely long and wire B is 2 m long, then force on wire B which is situated at 10 cm from A, is

(a) 8×10^{-5} N (b) 6×10^{-5} N

(c) 4×10^{-5} N (d) 2×10^{-5} N

39. If distance between two current-carrying wires is doubled, then force between them is

(a) halved (b) doubled

(c) tripled (d) quadrupled

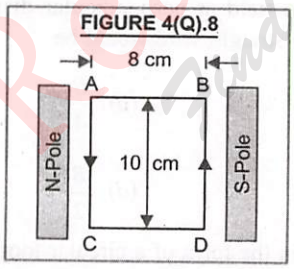
40. The ratio of voltage sensitivity (V_s) and current sensitivity (I_s) of a moving coil galvanometer is

(a) $\frac{1}{G}$ (b) $\frac{1}{G^2}$

(c) G (d) G^2

41. A 100 turns coil shown in the Fig. 4(Q).8 carries a current of 2 A in a magnetic field of 0·2 Wb – m^{-2}. The torque acting on the coil is

FIGURE 4(Q).8



(a) 0·32 N-m tending to rotate the side AC into the page

(b) 0·32 N-m tending to rotate the side AC out of the page

(c) 0·64 N-m tending to rotate the side AC into the page

(d) 0·64 N-m tending to rotate the side AC out of the page

42. Time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of

(a) speed of the particle

(b) mass of the particle

(c) charge of the particle

(d) magnetic field

(CBSE Sample Paper 2020)

II. Assertion-Reason Type Questions

Direction. For question numbers 43 to 60, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below :

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false, and R is also false.

43. Assertion. When radius of a current carrying loop is doubled, its magnetic moment becomes four times.

Reason. The magnetic moment of a current carrying loop is directly proportional to the area of the loop. (CBSE 2022)

44. Assertion. Higher the range, lower is the resistance of an ammeter.

Reason. To increase the range of ammeter, additional shunt is added in series to it.

(CBSE 2022)

45. Assertion. When a charged particle moves with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ ($\vec{v} \perp \vec{B}$), the force on the particle does not work.

Reason. The magnetic force is perpendicular to the velocity of the particle.

46. Assertion. A proton and electron, with same momenta, enter in a magnetic field in a direction at right angles to the lines of force. The radius of the paths followed by them will be the same.

Reason. Electron has less mass than the proton.

47. Assertion. A vertical wire carrying a current in the upward direction is placed in a horizontal magnetic field directed towards north. The wire will experience a force directed towards east.

Reason. The direction of force on a wire carrying current in a perpendicular magnetic field will be given by Fleming's Right Hand Rule.

48. Assertion. On increasing the current sensitivity of a galvanometer by increasing the number of turns, may not necessarily increase its voltage sensitivity.

Reason. The resistance of the coil of the galvanometer increases on increasing the number of turns.

49. Assertion. To increase the range of ammeter, we must connect a suitable low resistance in parallel to it.

Reason. The ammeter with increased range should have high resistance.

50. Assertion. A magnetic field is not associated with a charge in uniform motion.

Reason. A charge in a uniform motion produces electric field only.

51. Assertion. If electron velocity is $(2\hat{i}+3\hat{j})$ and is subjected to a magnetic field $(2\hat{i}+3\hat{j})$, then the magnitude of force on the electron is zero.

Reason. Force on the moving charged particle in magnetic field is, $\vec{F} = q(\vec{v} \times \vec{B})$.

52. Assertion. The maximum energy of deutron coming out of a cyclotron accelerator is 20 MeV. The maximum energy of a proton that can be obtained from this accelerator is 10 MeV.

Reason. Maximum energy of a charged particle accelerated by cyclotron is directly proportional to mass of the particle.

53. Assertion. Electrons can not be accelerated to a very high energy by means of cyclotron.

Reason. The electron while accelerated in cyclotron goes quickly out of step with accelerating electric field.

54. Assertion. A straight wire of diameter 0·5 mm carrying a current of 1 A is replaced by another wire of 1 mm diameter carrying the same current. The strength of the magnetic field at distance 5 cm from the axis of wire becomes one quarter of the earlier value.

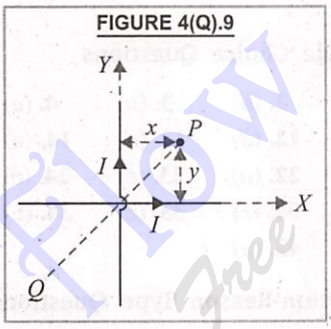
Reason. Magnetic field at a point due to straight current wire is inversely proportional to square of the distance of point from the axis of wire.

55. Assertion. A current of 1 A is passed through a straight wire of length 2·0 m. The magnetic field at a point in air at a distance of 4 m from one end of the wire lying on the axis of wire will be $\frac{\mu_0}{8\pi}$ tesla.

Reason. Magnetic field at a point on the axis of wire at a distance r from one end is $B = \frac{\mu_0}{4\pi}\frac{2I}{r}$.

56. Assertion. Two infinitely long, thin, insulated, straight wires lie in the x-y plane along the x and y-axes respectively. Each wire carries a current I in the positive x-direction and positive y-direction, Fig. 4(Q).9. The magnetic field will be zero at all points on the straight line, $y = x$.

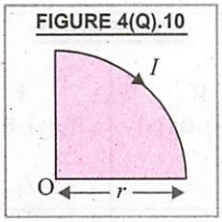
FIGURE 4(Q).9



Reason. The magnetic field at a point distance r from a long linear current carrying wire is given by $\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{n}$ where $\hat{n}$ is a unit vector $\perp r$ to the plane containing conductor and the point.

57. Assertion. A wire is bent in the form of a quadrant of radius r as shown in Fig. 4(Q).10 The magnetic field at the centre O due to current I is $\frac{\mu_0 I}{8r}$.

FIGURE 4(Q).10



Reason. The magnetic field at the centre of an arc of radius r, subtending an angle θ at centre is

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta.$$

58. Assertion. The magnetic field at the end of a very long current carrying solenoid is half of that at the centre.

Reason. For a sufficiently long solenoid carrying current, the magnetic field within the solenoid is uniform.

59. Assertion. A circular loop of radius R carrying a current I is placed in a uniform magnetic field with its plane parallel to magnetic field $\vec{B}$. The torque on the loop is zero.

Reason. Torque on the current loop in the magnetic field is $BAI \sin\theta$, where θ is the angle between plane of loop and magnetic field.

60. Assertion. If an electron and a proton are projected at right angles to a uniform magnetic field with the same linear momentum, the electron trajectory will be less curved than the proton trajectory.

Reason. Radius of the curved path of the charged particle moving in a perpendicular magnetic field is directly proportional to the mass of the particle.

ANSWERS

I. Multiple Choice Questions

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*a*)	**5.** (*b*)	**6.** (*c*)	**7.** (*b*)	**8.** (*d*)	**9.** (*c*)	**10.** (*d*)
11. (*a*)	**12.** (*d*)	**13.** (*d*)	**14.** (*d*)	**15.** (*a*)	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*d*)	**20.** (*d*)
21. (*d*)	**22.** (*a*)	**23.** (*c*)	**24.** (*c*)	**25.** (*d*)	**26.** (*d*)	**27.** (*a*)	**28.** (*a*)	**29.** (*d*)	**30.** (*b*)
31. (*a*)	**32.** (*c*)	**33.** (*c*)	**34.** (*b*)	**35.** (*c*)	**36.** (*c*)	**37.** (*a*)	**38.** (*a*)	**39.** (*a*)	**40.** (*a*)
41. (*b*)	**42.** (*a*)								

II. Assertion-Reason Type Questions

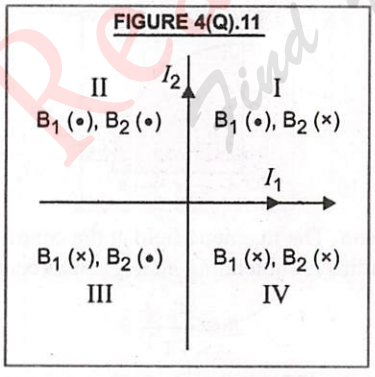
43. (*a*)	**44.** (*c*)	**45.** (*a*)	**46.** (*b*)	**47.** (*d*)	**48.** (*a*)	**49.** (*c*)	**50.** (*d*)	**51.** (*a*)	**52.** (*d*)
53. (*a*)	**54.** (*d*)	**55.** (*d*)	**56.** (*a*)	**57.** (*a*)	**58.** (*b*)	**59.** (*d*)	**60.** (*d*)		

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. Let B_1 and B_2 be the magnetic field due to the currents I_1 and I_2 respectively. Then the magnetic fields in all the four regions is shown in the Fig.4(Q).11.

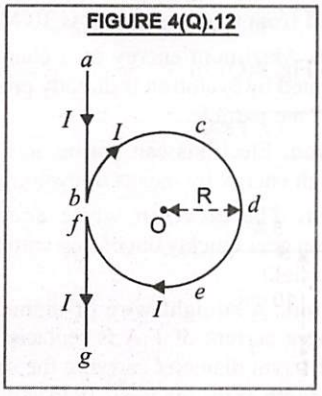
FIGURE 4(Q).11



The region of vertically upward strongest magnetic field will be in **region** II.

2. Let $\hat{n}$ be the unit vector perpendicular to the plane of paper downwards. The magnetic field $\vec{B}$ at O due to current through the entire structure is

FIGURE 4(Q).12



$$\vec{B} = \vec{B}_{bcdef} + \vec{B}_{abfg} = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}\hat{n} + \frac{\mu_0}{4\pi}\frac{2I}{R}(-\hat{n})$$

$$= \frac{\mu_0 I}{2R}\left[1 - \frac{1}{\pi}\right]\hat{n}$$

$$\therefore \quad |\vec{B}| = \frac{\mu_0 I}{2R}\left[1 - \frac{1}{\pi}\right]$$

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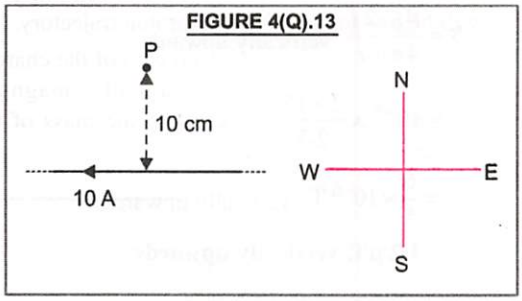
3. Refer to Fig. 4(Q).13, the magnetic field at P due to current in long straight wire

FIGURE 4(Q).13



$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7}) \times 10}{2\pi \times (10 \times 10^{-2})} = 2 \times 10^{-5}\ \text{T}$$

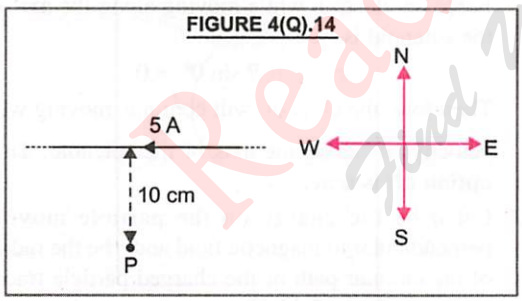
According to Right Hand Thumb Rule, the direction of $\vec{B}$ is **acting downwards.**

4. A charged particle of charge q, mass m while moving with speed v enters a uniform magnetic field B, then, force on charged particle due to magnetic field provides the required centripetal force. If r is the radius of the circular path traced by charged particle in the uniform magnetic field, then

$$Bqv = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv^2}{Bq} \quad i.e., \ r \propto m$$

5. Refer to Fig. 4(Q).14, $I = 5$ A, $r = 0{\cdot}10$ m

FIGURE 4(Q).14



Magnetic field at P due to current in long straight wire is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 5}{0{\cdot}10}$$

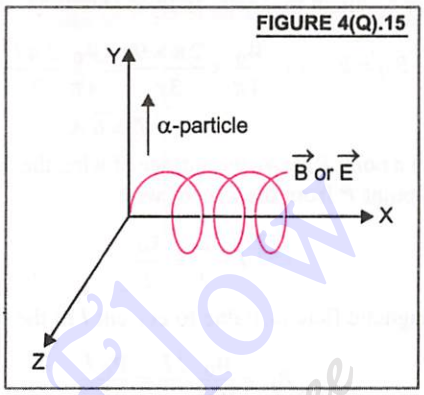
$$= 1 \times 10^{-5}\ \text{T, acting upwards,}$$

according to Right Hand Thumb Rule.

6. Force on charged particle due to electric field, $= q\,E$, acting along X-axis.

Force on charged particle due to magnetic field $= q\,v\,B$, acting along-Z-axis. Under the effect of these forces, the **charged particle will describe a helical path with its axis parallel to X-axis Fig. 4(Q).15.**

FIGURE 4(Q).15



7. Since, $qV = \frac{1}{2}mv^2$

$\therefore$ velocity acquired, $v = \sqrt{\frac{2qV}{m}}$

Now, radius of the circular path in magnetic field,

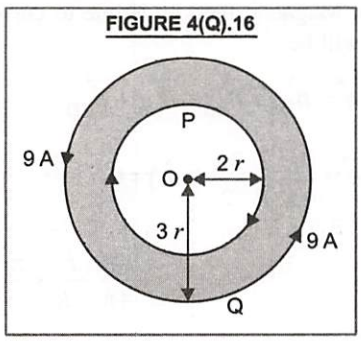
$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} \quad i.e., \ r \propto \sqrt{V}$$

When V is doubled, then $\frac{r'}{r} = \frac{\sqrt{2V}}{V} = \sqrt{2}$

or $r' = 2\sqrt{2}$

8. Refer to Fig. 4(Q).16, magnetic field at O due to current in loop Q

FIGURE 4(Q).16



$B_Q = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0}{4\pi} \times \frac{2\pi \times 9}{3r}$ acting vertically upwards.

The magnetic field at O due to current in loop P will neutralise the magnetic field at O due to current

in loop Q if the direction of current in P is **clockwise**. If I' is the current in loop P, then

$$B_P = \frac{\mu_0}{4\pi}\frac{2\pi I'}{2r} \text{ acting vertically downwards}$$

The resultant magnetic field at O will be zero

if $B_Q = B_P$ or $\frac{\mu_0}{4\pi}\times\frac{2\pi\times 9}{3r} = \frac{\mu_0}{4\pi}\frac{2\pi I'}{2r}$

or $I' = \mathbf{6\ A}$

9. For a point P above the surface of wire, the distance of point P from the axis of wire,

$$r = a + \frac{a}{2} = \frac{3a}{2}$$

Magnetic field at P due to current I in the wire,

$$B_P = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

$$= \frac{\mu_0 I}{2\pi(3a/2)} = \frac{\mu_0 I}{3\pi a}$$

For a point Q, below the surface of wire, the distance of point Q, from the axis of wire

$r' = a - a/2 = a/2$

Magnetic field at Q due to current I in the wire,

$$B_Q = \frac{\mu_0}{4\pi}\frac{2Ir'}{a^2} = \frac{\mu_0 I(a/2)}{2\pi a^2} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \quad \frac{B_P}{B_Q} = \frac{\mu_0 I/3\pi a}{\mu_0 I/4\pi a} = \frac{4}{3}$$

10. Let $\hat{i}$ be the unit vector out of the plane of the paper. Magnetic field at M due to current in the loop will be

$$\vec{B} = \vec{B}_{DA} + \vec{B}_{AB} + \vec{B}_{BC} + \vec{B}_{CD}$$

$$\therefore \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{I}{R}\frac{3\pi}{2}(-\hat{i}) + 0$$

$$+\frac{\mu_0}{4\pi}\frac{I}{2R}\times\frac{\pi}{2}(-\hat{i}) + 0$$

$$= \frac{\mu_0 I}{R}\left[\frac{3}{8}+\frac{1}{16}\right](-\hat{i}) = \frac{\mu_0 I}{R}\times\frac{7}{16}(-\hat{i})$$

$$= \frac{\mu_0 I}{R}\times\frac{7}{16}, \textbf{ into the plane of the paper.}$$

11. Refer to Fig. 4(Q).17, $I = 15$ A, $r = 2{\cdot}5$ m.

The magnetic field at point P is

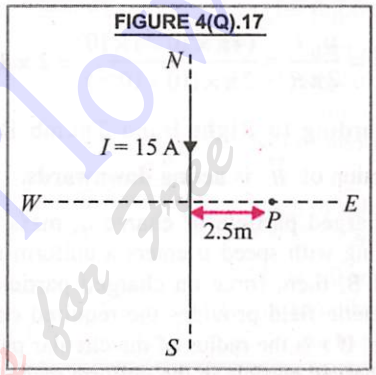
$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} \text{ vertically upwards}$$

$$= 10^{-7}\times\frac{2\times 15}{2{\cdot}5}$$

$$= \frac{6}{5}\times 10^{-6}\text{ T, vertically upwards}$$

$$= \mathbf{1{\cdot}2\ \mu T, \text{ vertically upwards}}$$

FIGURE 4(Q).17



12. In a current carrying solenoid, the magnetic field B is along the axis of solenoid, *i.e.*, in the direction of motion of electron, *i.e.*, angle between $\vec{B}$ and $\vec{v}$ is zero.

Force on electron while moving along the axis of the solenoid is

$$F = q\,v\,B\sin 0^\circ = 0$$

Therefore, the electron will continue moving with velocity $\vec{v}$ along the axis of the solenoid. Thus **option (*d*) is true.**

13. Let q be the charge on the particle moving perpendicular to magnetic field and r be the radius of the circular path of the charged particle traced in the magnetic field. Then

$$B\,q\,v\sin 90^\circ = \frac{mv^2}{r} \qquad ...(1)$$

When speed of particle becomes $v/2$, then

$$B\,q\frac{v}{2}\sin 90^\circ = \frac{m(v/2)^2}{r'} \qquad ...(2)$$

Dividing (1) by (2) we get

$$2 = 4r'/r \quad \text{or} \quad r' = \mathbf{r/2}$$

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14. Force, $\vec{F} = q(\vec{v} \times \vec{B})$

$$= 1{\cdot}6 \times 10^{-19}\,[(4\hat{i} + 3\hat{k}) \times (3\hat{k} + 4\hat{i})]$$

$$= 1{\cdot}6 \times 10^{-19}\,[4 \times 3(\hat{i} \times \hat{k}) + 4 \times 4(\hat{i} \times \hat{i}) + 3 \times 3(\hat{k} \times \hat{k}) + 3 \times 4(\hat{k} \times \hat{i})]$$

$$= 1{\cdot}6 \times 10^{-19}\,[12(-\hat{j}) + 16 \times 0 + 9 \times 0 + 12(\hat{j})]$$

$$= 1{\cdot}6 \times 10^{-19} \times 0 = \mathbf{0}$$

15. The current will be divided equally in two halves of a loop if there is equal resistance of two halves of a loop. It will be so in case of loop P and R. Now magnetic field at the centre of loop will be zero as the magnetic field at the centre of loop due to current in upper half of loop will be equal and opposite to the magnetic field at the centre of the loop due to lower half of the loop. Thus **option (*a*) is true.**

16. Let l be the total length of the circular coil and r be the radius of circular coil. Magnetic field at the centre of circular coil due to current I is

$$B_0 = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0}{4\pi}\frac{2\pi I}{l/2\pi} \quad ...(1)$$

$[\because l = 2\pi r]$

When the coil is rewound to have three turns let, r_1 be the new radius of circular coil,

then, $l = 3 \times 2\pi r_1$ or $r_1 = l/6\pi$

Magnetic field at the centre of current carrying this coil is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I \times 3}{r_1} = \frac{\mu_0}{4\pi}\frac{2\pi I \times 3}{l/6\pi}$$

$$= 9 \times \frac{\mu_0}{4\pi}\frac{2\pi I}{l/2\pi} = 9B_0$$

17. Magnetic field at a point on the axis well inside the solenoid carrying current is

$$B = \frac{\mu_0 NI}{l}$$

where N = no. of turns of the solenoid and l is the length of solenoid.

When, $N_1 = N/2$ and $I_1 = 2I$, then

$$B_1 = \frac{\mu_0 N_1 I_1}{l} = \frac{\mu_0 (N/2) \times 2I}{l}$$

$$= \frac{\mu_0 NI}{l} = B$$

18. Current sensitivity, $I_s = \frac{\theta}{I} = \frac{NBA}{k}$

For galvanomter G_1,

$$I_{s_1} = \frac{N_1 B_1 A_1}{k} = \frac{30 \times 0{\cdot}25 \times (3{\cdot}6 \times 10^{-3})}{k}$$

For galvanometer G_2,

$$I_{s_2} = \frac{N_2 B_2 A_2}{k} = \frac{42 \times 0{\cdot}50 \times (1{\cdot}8 \times 10^{-3})}{k}$$

$$\frac{I_{s_1}}{I_{s_2}} = \frac{30 \times 0{\cdot}25 \times 3{\cdot}6 \times 10^{-3}}{42 \times 0{\cdot}50 \times 1{\cdot}8 \times 10^{-3}} = \frac{30}{42} = \frac{5}{7}$$

19. When a current loop is suspended in a uniform magnetic field, it will experience a torque but the resultant force on the loop is zero. The loop will come to rest with its plane perpendicular to magnetic field, when the force on the remaining three arms of the loop is $-\vec{F}$.

20. Here, $I_1 = 4{\cdot}0$ A ; $I_2 = 10{\cdot}0$ A ;

$r = 2{\cdot}5$ cm $= 2{\cdot}5 \times 10^{-2}$ m.

Force per unit length, $F = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$

$$= \frac{10^{-7} \times 2 \times 4{\cdot}0 \times 10{\cdot}0}{2{\cdot}5 \times 10^{-2}} = \mathbf{3{\cdot}2 \times 10^{-4}\ Nm^{-1}}$$

21. Ammeter can be converted into a voltmeter by using a high resistance in series of ammeter.

22. Magnetic field at the centre of current carrying circular loop is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} \quad ...(i)$$

Magnetic field at a point on the axis of a circular loop is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi I R^2}{(R^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi IR^2}{R^3 \times (2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi I}{R \times (2)^{3/2}} \quad ...(ii)$$

$$\therefore \quad \frac{B_1}{B_2} = (2)^{3/2} = 2\sqrt{2}$$

23. Force experienced by a current carrying wire when placed in magnetic field is

$$F = BIl \sin\theta$$

where θ is the angle which the wire makes with the direction of magnetic field. The force will be maximum if $\sin\theta = 1$ or $\theta = 90°$. *i.e.*, the **wire is kept perpendicular to the magnetic field.**

24. There will be no tension in the strings, if the force on current carrying conducting rod due to magnetic field is equal and opposite to the weight of the rod.

i.e., $BIl = mg$ or $I = \dfrac{mg}{lB}$

25. Here $v_p / v_\alpha = \dfrac{9}{4}$; $r_p / r_\alpha = ?$

$$B q_p v_p = m_p v_p^2 / r_p \quad \text{or} \quad r_p = m_p v_p / B q_p$$

and $B q_\alpha v_\alpha = m_\alpha v_\alpha^2 / r_\alpha$ or $r_\alpha = m_\alpha v_\alpha / B q_\alpha$

$$\therefore \quad \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{v_p}{v_\alpha} \times \frac{q_\alpha}{q_p}$$

$$= \frac{1}{4} \times \frac{9}{4} \times \frac{2}{1} = \frac{9}{8}$$

26. New current sensitivity, $I_s' = I_s + \dfrac{20}{100} I_s = 1{\cdot}2 I_s$

When resistance of galvanometer is increased by 25%, then

$$G' = G + \frac{25}{100} G = 1{\cdot}25 G$$

New voltage sensitivity,

$$V_s' = \frac{I_s'}{G'} = \frac{1{\cdot}2 I_s}{1{\cdot}25 G} = \frac{120}{125} V_s = \frac{24}{25} V_s$$

$$\% \text{ change} = \frac{(V_s' - V_s)}{V_s} \times 100$$

$$= \frac{\left(\frac{24}{25} V_s - V_s\right) \times 100}{V_s}$$

$$= \left(\frac{24 - 25}{25}\right) \times 100$$

$$= -\frac{1}{25} \times 100 = -4\%$$

27. Refer to three infinite long parallel straight wires carrying current shown in Fig. 4(Q).18(*a*)

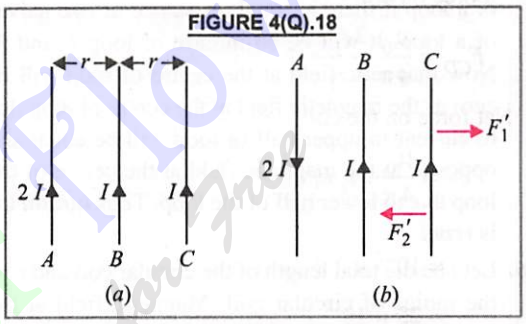
The force per unit length on wire *C* due to current in wire *A* is

$$F_1 = \frac{\mu_0}{4\pi} \frac{2I \times I}{2r} \text{ towards left hand side}$$

The force per unit length on wire *C* due to current in wire *B* is

$$F_2 = \frac{\mu_0}{4\pi} \frac{I \times I}{r} \text{ towards left hand side}$$

FIGURE 4(Q).18



Net force on *C* per unit length

$$= F_1 + F_2 = \frac{\mu_0}{4\pi} \frac{I^2}{r} (1+1)$$

$$= 2 \times \frac{\mu_0}{4\pi} \frac{I^2}{r} = F \quad \text{(Given)}$$

Refer to Fig. 4(Q).18(*b*),

Repulsive force per unit length on wire *C* due to wire *A* is

$$F_1' = \frac{\mu_0}{4\pi} \frac{2I \times I}{2r} = \frac{\mu_0}{4\pi} \frac{I^2}{r}$$

Attractive force per unit length on wire *C* due to wire *B* is

$$F_2' = \frac{\mu_0}{4\pi} \frac{I \times I}{r} = \frac{\mu_0}{4\pi} \frac{I^2}{r}$$

Net force on wire $C = F_1' - F_2'$

$$= \frac{\mu_0}{4\pi} \frac{I^2}{r} - \frac{\mu_0}{4\pi} \frac{I^2}{r} = 0$$

28. $e E = e v B$ or $v = E/B$

29. Kinetic energy, $K = \dfrac{q^2 B^2 r^2}{2m}$ or $K \propto \dfrac{1}{m}$

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$\therefore \quad \frac{K_p}{K_d} = \frac{m_d}{m_p} = \frac{2m_p}{m_p} = 2$

or $K_p = 2\,K_d = 2 \times 50 = 100$ keV

30. $q\,v\,B = \frac{mv^2}{r}$ or $mv = q\,v\,r$ or $mv \propto r$

As $r_A > r_B$, so $m_A\,v_A > m_B\,v_B$

31. Here, $I_1 = 2$ A ; $I_2 = 1$ A, $l = 0{\cdot}15$ m ;
$r_1 = 0{\cdot}02$ m ; $r_2 = 0{\cdot}02 + 0{\cdot}10 = 0{\cdot}12$ m

Force on arm AB of coil, $F_{AB} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_1}\,l$, towards the wire

Force on arm CD of coil,

$F_{CD} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_2}\,l$, away from the wire

Net force on the coil = $F_{AB} - F_{CD}$, towards the wire

$= \frac{\mu_0}{4\pi} 2I_1I_2\,l\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= 10^{-7} \times 2 \times 2 \times 1 \times 0{\cdot}15\left[\frac{1}{0{\cdot}02} - \frac{1}{0{\cdot}12}\right]$

$\approx 25 \times 10^{-7}$ **N towards the wire**

32. $L = 4\,l = 2\pi r$ or $l = \pi r/2$

$\frac{M_1}{M_2} = \frac{l^2 \times I}{\pi r^2 \times I} = \frac{l^2}{\pi r^2} = \frac{\pi^2 r^2/4}{\pi r^2} = \frac{\pi}{4}$

33. Current sensitivity, $I_s = \frac{nBA}{k}$ *i.e.*, $I_s \propto \frac{1}{k}$

As **torsional constant k decrease, I_s increases.**

34. $B = \frac{\mu_0}{4\pi}\frac{2I}{r}$ or $B \propto \frac{1}{r}$

$\therefore \quad \frac{B_2}{B_1} = \frac{r_1}{r_2} = \frac{5}{20} = \frac{1}{4}$

or $B_2 = \frac{B_1}{4} = \frac{B}{4}$

35. Let r_1, r_2 be the radius of the circular loop of one turn and 2 turns of the given wire. Then

$2\pi r_1 = 2 \times 2\pi r_2$ or $r_1 = 2r_2$

$B = \frac{\mu_0}{4\pi}\frac{2\pi I n}{r}$ or $B \propto \frac{n}{r}$

$\therefore \quad \frac{B_2}{B_1} = \frac{n_2}{n_1} \times \frac{r_1}{r_2} = \frac{2}{1} \times \frac{2r_2}{r_2} = 4$

or $B_2 = 4\,B_1 = 4\,B$

36. $B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{(a^2 + r^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2\pi nI}{r^3}$

[As $a^2 + r^2 \approx r^2$]

$\therefore \quad B \propto \frac{1}{r^3}$

37. Here, $n = 1000$ m^{-1} ; $I = 5$ A ;

$B = \mu_0\,n\,I = (4\pi \times 10^{-7}) \times 1000 \times 5$

$= 2\pi \times 10^{-3}$ **T**

38. Here, $I_1 = 10$ A ; $I_2 = 2$ A, $l_2 = 2$ m ; $r = 0{\cdot}10$ m

$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2l_2}{r} = 10^{-7} \times \frac{2 \times 10 \times 2 \times 2}{0{\cdot}10}$

$= 8 \times 10^{-5}$ **N**

39. $B = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$ *i.e.*, $B \propto \frac{1}{r}$

$\therefore \quad \frac{B_2}{B_1} = \frac{r_1}{r_2} = \frac{r}{2r} = \frac{1}{2}$ or $B_2 = \frac{B_1}{2}$

40. Voltage sensitivity, $V_s = \frac{nBA}{kG}$

Current sensitivity, $I_s = \frac{nBA}{k}$ $\therefore \quad \frac{V_s}{I_s} = \frac{1}{G}$

41. Here, $n = 100$, $I = 2$ A ;

$B = 0{\cdot}2$ Wb/m^2, $A = 0{\cdot}08 \times 0{\cdot}10$ m^2

$\tau = n\,I\,AB \sin 90° = 100 \times 2 \times (0{\cdot}08 \times 0{\cdot}10) \times 0{\cdot}2 \times 1$

$= 0{\cdot}32$ **Nm**

The direction of force on arm AC is perpendicular to the arm AC upwards. The direction of force on arm BD is perpendicular to the arm BD downwards. As a result of it, the coil will tend to rotate with side **AC moving out of the page.**

42. Time period, $T = \frac{2\pi m}{Bq}$; it is independent of speed of the particle.

II. Assertion-Reason Type Questions

43. Magnetic moment, $M = I\,A = I\,\pi\,r^2$ where r is the radius of the current carrying loop, *i.e.*, $M \propto r^2$. When $r' = 2\,r$, then

$M' \propto (2\,r)^2 \therefore \frac{M'}{M} = 4$ or $M' = 4\,M$

Thus both, **Assertion and Reason are true and Reason is the correct explanation of Assertion.**

44. An ammeter of higher range should have very low resistance. For an ideal ammeter the resistance must be zero. But in practice, it is impossible to have

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zero resistance. Therefore, to increase the range of ammeter, the additional shunt is added in parallel to it, which reduces the resistance of ammeter. Hence **Assertion is true but Reason is false.**

45. Force on a particle, $\vec{F} = q(\vec{v} \times \vec{B})$, where $\vec{F}$ is perpendicular to both $\vec{v}$ as well as $\vec{B}$.

Workdone for this force $= \vec{F} \cdot \vec{s} = F s \cos 90° = 0$.

Hence, **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

46. When a moving charged particle enters a perpendicular magnetic field, then the force on particle will make the particle to move on a circular path of radius r, given by

$$B q v = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{Bq} = \frac{p}{Bq}$$

[$\because mv = p$ = momentum of particle]

Since p and q are same for proton and electron so r is same, *i.e.*, **Assertion is true.** Here **Reason is also true but Reason is not the correct explanation of Assertion.**

47. The direction of force on wire carrying current in a perpendicular magnetic field will be towards west, according to Fleming's left hand Rule, which will be applicable here. Thus, **both Assertion and Reason are false.**

48. Current sensitivity, $I_s = \frac{nBA}{k}$ *i.e.*, $I_s \propto n$

Voltage sensitivity, $V_s = \frac{nBA}{kR} = \frac{I_s}{R}$

When we increase current sensitivity by increasing the number of turns, then the resistance of galvanometer coil also increases. But n/R increases by different amounts depending upon the values of n and R. Due to it, voltage sensitivity may increase or decrease. **Thus Assertion is true.** Here **Reason is also true and is correct explanation of Assertion.**

49. Here, **Assertion is true but Reason is false,** because ammeter with increased range has low resistance since a low resistance shunt is used in parallel to increase the range of ammeter.

50. A charge in a uniform motion produces a steady current, to which both the electric and magnetic fields are associated. Thus, **both Assertion and Reason are false.**

51. Force, $\vec{F} = q(\vec{v} \times \vec{B})$. As angle between $\vec{v}$ and $\vec{B}$ is zero, so $|\vec{F}| = 0$. Thus, **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

52. **Both Assertion and Reason are wrong** as energy of a charged particle accelerated by cyclotron is inversely proportional to mass of the particle.

53. **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

54. Magnetic field at a point due to linear wire carrying current at a distance r is, $B = \frac{\mu_0}{2\pi}\frac{I}{r}$, *i.e.*, $B \propto 1/r$.

When r is constant, B remains unchanged. Thus **Both Assertion and Reason are wrong.**

55. **Both Assertion and Reason are false** as the magnetic field at a point on the axis of current carrying wire is zero.

56. Magnetic field at point P due to currents in wires along x and y axes respectively will be

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2I}{y}\hat{k} \quad \text{and} \quad \vec{B}_2 = \frac{\mu_0}{4\pi}\frac{2I}{x}(-\hat{k})$$

The resultant magnetic field at P will be zero if

$$\vec{B}_1 + \vec{B}_2 = 0, \text{ i.e., } \frac{\mu_0}{4\pi}\frac{2I}{y}\left[\frac{1}{y} - \frac{1}{x}\right] = 0$$

This is possible if $\frac{1}{y} - \frac{1}{x} = 0$ or $y = x$

Thus **both Assertion and Reason are correct and the Reason is the correct explanation of Assertion.**

57. For a quadrant of radius r, $\theta = \pi/2$ so $B = \frac{\mu_0}{4\pi}\frac{I}{r} \times \frac{\pi}{2} = \frac{\mu_0 I}{8r}$. Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

58. **Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.**

59. **Both Assertion and Reason are wrong** because torque on the current loop in the magnetic field $= BAI \sin\theta$, where θ is the angle between $\vec{B}$ and $\vec{A}$. In the given Assertion, $\theta = 90°$, so torque on the coil is $= B\pi R^2 I \sin 90° = B\pi R^2 I$ and not zero.

60. Force on particle in magnetic field

$$F = Bqv = \frac{mv^2}{R} \quad \text{or} \quad R = \frac{mv}{Bq} = \frac{p}{Bq},$$

So, radius R is constant as p, B, q are same for both electron and proton. Hence **both Assertion and Reason are wrong.**

CONCEPTUAL PROBLEMS

I. Biot Savart's law and magnetic field due to a linear current carrying wire

1. How is a magnetic field produced ?

Sol. A magnetic field can be produced by any of the following ways : (*i*) by a magnet (*ii*) by a current carrying conductor (*iii*) by a moving charge and (*iv*) by changing electric field.

2. How will you identify whether the magnetic field at a point is due to the earth or due to some current carrying conductor ?

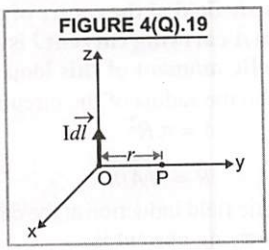
Sol. We know that a freely suspended magnetic needle always points to north-south direction due to earth's magnetic field. If a freely suspended magnetic needle always rests in north-south direction, the magnetic field is due to earth, and if this magnetic needle turns to some other direction and returns back to north-south when the current is switched off, then the magnetic field is due to a current carrying conductor.

3. The net charge in a current-carrying conductor is zero, even then it experiences a force in a magnetic field. Why ?

Sol. We know that a magnetic force acts on a charge moving in a magnetic field. In a conductor carrying current, the free electrons move towards the positive end of the conductor with some drift velocity, hence magnetic force acts on them in a magnetic field. The positive ions of the conductor being stationary, do not experience any magnetic force.

4. A current *I* flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element $\vec{dl}$ at point *P* situated at a distance $\vec{r}$ from the element as shown in the Fig. 4(Q).19. (CBSE 2009)

FIGURE 4(Q).19



Sol. Here current element $I\,\vec{dl}$ is along *z*-direction and position vector $\vec{r}$ is along *y*-direction. So $I\,\vec{dl} = I\,dl\,\hat{k}$ and $\vec{r} = r\,\hat{j}$

According to Biot Savart's law, the magnetic field at point *P* due to current element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\frac{[dl\,\hat{k}\times r\,\hat{j}]}{r^3}$$

$$= \frac{\mu_0 I}{4\pi}\frac{dl}{r^2}(-\hat{i})$$

Hence, magnetic field at point *P* is directed **along negative *x*-direction.**

5. Compare the electrostatic field given by Coulomb's law and magnetic field given by Biot Savart's law.

Sol. 1. Both the fields obey inverse square law, hence both are of long range.

2. The electrostatic field is produced by an electric charge which is a scalar. The magnetic field is produced by a current element $I\,\vec{dl}$, which is a vector.

3. The electrostatic field due to a charge acts along or opposite to the line joining the source of charge and the field point, depending upon whether the field is due to positive charge or negative charge.

The magnetic field at a point due to current element $I\,\vec{dl}$ is perpendicular to the plane containing the displacement vector $\vec{r}$ and the current element $I\,\vec{dl}$.

4. The electrostatic field is linearly related with the source of charge and magnetic field is linearly related with source of magnetic field *i.e.* $I\,\vec{dl}$, hence both the fields obey superposition principle.

5. The electrostatic field at a point due to a point charge is angle independent whereas the magnetic field at a point due to current element depends on the angle between $I\,\vec{dl}$ and $\vec{r}$.

6. Electrostatic field at a point due to source charge decreases when a medium is inserted between the source charge and given point ; as

$$E \propto \frac{1}{\epsilon_0 \epsilon_r}$$

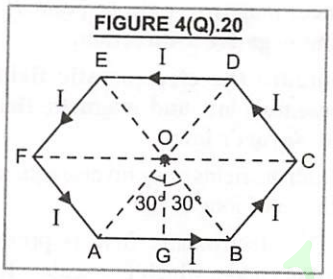
The magnetic field at a point due to current element increases when a medium is inserted between the current element and the given point ; as $B \propto \mu_0 \mu_r$.

6. A closed circuit is in the form of a regular hexagon of side *r*. If the circuit carries current *I*, what is the magnetic field induction at the centre of the hexagon ?

Sol. Refer to Fig. 4(Q).20, $AG = GB = r/2$

$$OG = AG \cot 30° = \frac{r}{2} \times \sqrt{3}$$

FIGURE 4(Q).20



Magnetic field induction at *O* due to current in arm *AB* is

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{(r\sqrt{3}/2)} [\sin 30° + \sin 30°]$$

$$= \frac{\mu_0 I}{2\pi r\sqrt{3}},$$ acting perpendicular to the plane of closed circuit upwards.

The magnetic field induction at *O* due to currents in arms *BC, CD, DE, EF* and *FA* is also equal to B_1 and acting in same direction. Therefore, total magnetic field induction at *O* due to current in closed current is

$$B = 6B_1 = 6 \times \frac{\mu_0 I}{2\pi r\sqrt{3}} = \frac{\sqrt{3}\mu_0 I}{\pi r}$$

II. Magnetic field due to a circular coil carrying current

7. What do you understand by magnetic dipole moment of the current loop.

Sol. We know that magnetic field induction at any point on the axis of a circular coil of *n* turns each of area *A*, carrying current *I* is given by

$$B = \frac{\mu_0}{4\pi} \frac{2nIA}{x^3} = \frac{\mu_0}{4\pi} \frac{2M}{x^3} \quad \text{[From (18)]}$$

where, $nIA = M$ = Magnetic dipole moment of the current loop.

Thus the **current loop can be regarded as a magnetic dipole** which produces its magnetic field and **magnetic dipole moment of the current loop is equal to the product of ampere turns and area of current loop.** SI unit of magnetic dipole moment is A-m^2.

8. What is the magnetic moment of an electron orbiting in a circular orbit of radius *r* with a speed *v* ?

Sol. We know that orbiting electron behaves as a current loop. The current due to orbital motion of electron is

$$I = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

Area of the current loop = πr^2

∴ Magnetic moment of current loop

$$= IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

9. A long wire carries a steady current, *I*. First it is bent into a circular coil of one turn when the magnetic field induction at the centre is B_0. Then the same wire is bent to form a circular coil of smaller radius but n turns then find the magnetic field induction at the centre of this circular coil.

Sol. Let *R* be the radius of one turn circular coil.

Then, $$B_0 = \frac{\mu_0}{4\pi} \frac{2\pi I}{R}.$$

If *r* is the radius of circular coil of *n* turns, then

$2\pi r n = 2\pi R$ or $r = R/n$

So, $$B = \frac{\mu_0}{4\pi} \frac{2\pi nI}{r} = \frac{\mu_0}{4\pi} \frac{2\pi nI}{(R/n)}$$

$$= \frac{\mu_0}{4\pi} \frac{2\pi I}{R} n^2$$

$$= n^2 B_0$$

10. Magnetic field at the centre of a circular loop of area *A* carrying current *I* is *B*. What is the magnetic moment of this loop ?

Sol. Let *R* be the radius of the circular loop. Then,

$$A = \pi R^2$$

or $$R = \sqrt{A/\pi}$$

Magnetic field induction at the centre of circular loop carrying current is

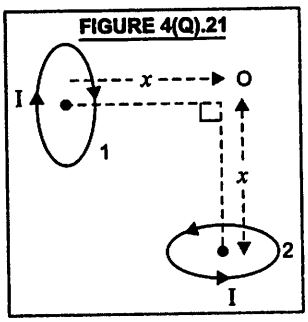
$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}$$

or $$I = \frac{2BR}{\mu_0}$$

$$M = IA = \frac{2BR}{\mu_0}A = \frac{2BA}{\mu_0}\sqrt{\frac{A}{\pi}}$$

11. Two small circular loops, marked (1) and (2), carrying equal currents are placed with the geometrical axes perpendicular to each other as shown in Fig. 4(Q).21. Find the magnitude and direction of the net magnetic field produced at the point *O*.

FIGURE 4(Q).21



Sol. Magnetic field induction at *O* due to current loop 1 is

$$B_1 = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}},$$

acting towards left.

Magnetic field induction at *O* due to current loop 2 is

$$B_2 = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

acting vertically upwards.

Resultant magnetic field induction at *O* will be

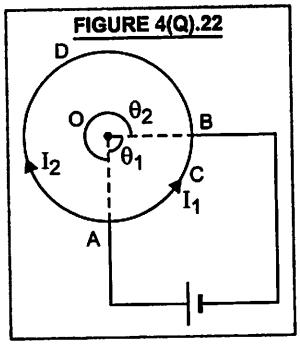
$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{2}\,B_1 \quad (\because B_1 = B_2)$$

$$= \sqrt{2}\times\frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 I R^2}{\sqrt{2}(x^2+R^2)^{3/2}}$$

12. As shown in Fig. 4(Q).22 a cell is connected across two points *A* and *B* of a uniform circular conductor of radius *r*. Prove that the magnetic field induction at its centre *O* will be zero.

FIGURE 4(Q).22



Sol. Let l_1 and l_2 be the lengths of the two circular arcs *ACB* and *ADB* respectively. ρ be the resistance per unit length of the circular conductor.

Resistance of circular arc *ACB*, $R_1 = l_1\rho$

Resistance of circular arc *ADB*, $R_2 = l_2\rho$

Let I_1, I_2 be the currents in arms *ACB* and *ADB* respectively. As these two arcs are connected in parallel, so the potential difference across them must be equal *i.e.*,

$$I_1R_1 = I_2R_2$$

or $$I_1 l_1\rho = I_2 l_2\rho$$

or $$I_1 l_1 = I_2 l_2 \qquad ...(i)$$

Magnetic field induction at O due to current in arc *ACB* is,

$$B_1 = \frac{\mu_0}{4\pi}\frac{I_1}{r}\theta_1 = \frac{\mu_0}{4\pi}\frac{I_1}{r}\times\frac{l_1}{r} = \frac{\mu_0}{4\pi}\frac{I_1 l_1}{r^2}$$

It is acting upwards, perpendicular to the plane of circular conductor.

Magnetic field induction at *O* due to current in arc *ADB* is,

$$B_2 = \frac{\mu_0}{4\pi}\frac{I_2}{r}\theta_2 = \frac{\mu_0}{4\pi}\frac{I_2 l_2}{r^2}$$

It is acting downwards, perpendicular to the plane of circular conductor.

As $I_1 l_1 = I_2 l_2$,

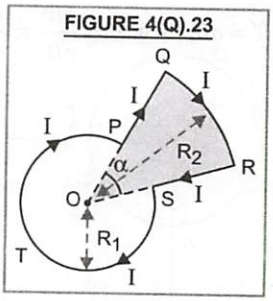
so $B_1 = B_2$ and $\vec{B_1} = -\vec{B_2}$

∴ Resultant magnetic field at O $= \vec{B_1} + \vec{B_2}$

$= (-\vec{B_1}) + \vec{B_2} = 0$

13. The circular loop *PQRSTP* formed by two circular segments of radii R_1 and R_2, carries a current *I*. What is the magnetic field induction at the centre *O* ? What will be the magnetic field induction at O if α = 90° ?

FIGURE 4(Q).23



Sol. Let $\hat{n}$ be the unit vector perpendicular to the plane of loop downwards. Total magnetic field induction at O due to current through circular loop is

$$\vec{B} = \vec{B}_{PQ} + \vec{B}_{QR} + \vec{B}_{RS} + \vec{B}_{STP}$$

$$= 0 + \frac{\mu_0}{4\pi}\frac{I}{R_2}\alpha\hat{n} + 0 + \frac{\mu_0}{4\pi}\frac{I}{R_1}(2\pi-\alpha)\hat{n}$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{\alpha}{R_2} + \frac{(2\pi-\alpha)}{R_1}\right]\hat{n}$$

or $$B = \frac{\mu_0 I}{4\pi}\left[\frac{\alpha}{R_2} + \frac{(2\pi-\alpha)}{R_1}\right]$$

Directed downwards

If $\alpha = 90° = \pi/2$ rad., then

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{\pi/2}{R_2} + \frac{3\pi/2}{R_1}\right] = \frac{\mu_0 I}{8}\left[\frac{1}{R_2} + \frac{3}{R_1}\right]$$

III. Ampere's circuital law, solenoid and magnetic field due to current in conducting rod

14. Distinguish between Biot Savart's law and Ampere's circuital law.

Sol.

S. No.	Biot-Savart's law	Ampere's circuital law
1.	This law is based on the principle of magnetism.	1. This law is based on the principle of electromagnetism.
2.	This law is valid for asymmetrical current distribution.	2. This law is valid for symmetrical current distributions.
3.	This law is the differential form of magnetic field induction $\vec{B}$ or magnetising force $\vec{H}$.	3. This law is the integral form of $\vec{B}$ or $\vec{H}$.

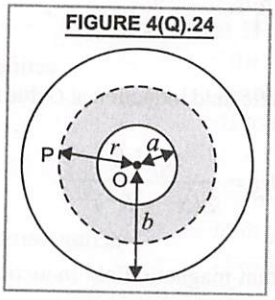
15. The magnetic field at a point near the centre but outside a current carrying long solenoid is zero. Explain why ?

Sol. When current is passed through a long straight solenoid, then each turn of the solenoid can be regarded as a circular loop carrying current and thus will be producing a magnetic field. The total magnetic field at a point is the vector sum of the magnetic fields due to currents through all the turns in the solenoid. At a point near the centre outside the solenoid, the magnetic fields due to neighbouring loops are equal in magnitude and opposite in direction. Hence the resultant magnetic field at a point near the centre outside the solenoid is zero.

16. A hollow cylindrical conductor of inner radius a and outer radius b carries a current I uniformly spread over its cross-section. Find the magnetic field induction at a point inside the body of the conductor at a distance r [where $a < r < b$] from the axis of the cylinder.

Sol. The Fig. 4(Q).24, shows the transverse cross-section of hollow cylindrical conductor. Let P be a point at distance r from the axis of the conductor well inside the conductor. Draw an imaginary cylinder of radius r whose axis coincing with the axis of hollow conductor.

FIGURE 4(Q).24



Current passing through the cylindrical shell of inner radius a and outer radius r is

$$I' = \frac{I}{\pi(b^2-a^2)} \times \pi(r^2-a^2) = \frac{I(r^2-a^2)}{(b^2-a^2)}$$

Let $\vec{B}$ be the magnetic field induction at P. Using Ampere's circuital law.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I'$$

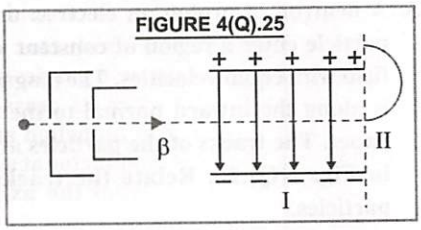
or $$B\,2\pi r = \mu_0 \frac{I(r^2-a^2)}{(b^2-a^2)}$$

or $$B = \frac{\mu_0 I (r^2 - a^2)}{2\pi (b^2 - a^2) r}$$

IV. Force on charged particle due to electric and magnetic fields and cyclotron

17. A fine pencil of β-particles, moving with a speed v, enters a region (region I), where a uniform electric field and a uniform magnetic field are both present. These β-particles then move into region II where only the magnetic field, (out of the two fields present in region I) exists. The path of the β-particles, in the two regions is as shown in the Fig. 4(Q).25 ;

FIGURE 4(Q).25



(*i*) state the direction of magnetic field.

(*ii*) state the relation between E and B in region I.

(*iii*) Drive the expression for the radius of the circular path of the β-particle in region II.

(*iv*) If the magnitude of magnetic field, in region II is changed to n times its earlier value, (without changing the magnetic field in region I) find the factor by which the radius of this circular path would change.

(CBSE Sample Paper 2013)

Sol. (*i*) As the β-particle goes undeviated while passing through the electric and magnetic fields, so the force on moving β-particle due to electric field $(-e\vec{E})$ must be equal and opposite to the force due to magnetic field $[-e(\vec{v}\times\vec{B})]$. Due to it, the **direction of $\vec{B}$ must be perpendicular to plane of paper downwards.**

(*ii*) In region I as β-particle goes undeviated

$$-e\vec{E} = -e(\vec{v}\times\vec{B}) \text{ or } eE = evB \text{ or } v = E/B$$

(*iii*) If r is the radius of circular path of the β-particle in region II, then

$$evB = mv^2/r \quad \text{or} \quad r = \frac{mv}{eB}$$

(*iv*) If $B' = nB$, then $r' = \frac{mv}{eB'} = \frac{mv}{neB}$

$$\therefore \quad \frac{r'}{r} = \frac{1}{n} \quad \text{or} \quad r' = \frac{r}{n}$$

18. Describe qualitatively the path of a charged particle moving in a region with uniform electrosatic and magnetic fields parallel to each other, with initial velocity (*i*) parallel (*ii*) perpendicular (*iii*) at an arbitrary angle with the common direction of the fields.

Sol. (*i*) When a charged particle moves parallel to both electrostatic and magnetic fields then the force on charged particle due to magnetic field is zero but due to electric field, the force on the charged particle will act along the direction of field and the charged particle will continue to move along the original straight line path.

(*ii*) When a charged particle moves perpendicular to both electric and magnetic fields, then due to electric field, the charged particle describes a parabolic path and due to magnetic field, it will describe a circular path. As a result of it, the particle will follow a resultant path in electric and magnetic fields.

If electric and magnetic fields are perpendicular to each other as well as perpendicular to the motion of charged particle, the particle will continue moving along the same path if force on particle due to electric field is equal and opposite to the force on it due to magnetic field.

(*iii*) If we resolve $\vec{v}$ into two rectangular components ; along $\vec{B}$ and perpendicular to $\vec{B}$. Then due to component velocity perpendicular to $\vec{B}$, the particle will describe a circular path and due to component velocity along $\vec{B}$ or $\vec{E}$, it gets accelerated or retarded due to $\vec{E}$, along the same path.

19. A charged particle q moving in a straight line is accelerated by a potential difference V. It enters a uniform magnetic field B perpendicular to its path. Deduce in terms of V an expression for the radius of the circular path in which it travels. (CBSE 2000)

Sol. When charged particle is accelerated through a potential diff. V, then work done = gain in K.E.

or $qV = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2qV}{m}}$

The force on charged particle due to perpendicular magnetic field ($= q\,v\,B$) provides the centripetal force ($= mv^2/r$) so

$$qvB = \frac{mv^2}{r}$$

or $r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2mV}{qB^2}}$

20. Both, the electric and magnetic fields can deflect a moving electron. What is the difference between these deflections ?

Sol. In a perpendicular electric field, the force acts on the electron in the opposite direction of electric field. Hence the deflection is in the plane of electric field. In this case, the speed of the electron changes and hence kinetic energy of the moving electron changes in the electric field.

In a perpendicular magnetic field, the direction of force exerted on a moving electron is perpendicular to the plane of its direction of motion as well as of magnetic field. Hence the deflection of the electron beam is perpendicular to the plane of the magnetic field. In this case, the work done by the force due to magnetic field acting on the moving electron is zero. Therefore, the kinetic energy of the moving electron does not change in the magnetic field.

21. An electron beam is deflected in a given field. How will you detect whether the given field is a uniform magnetic field or a uniform electric field ?

Sol. We can detect the nature of field by observing the trajectory and measuring the kinetic energy of electron. In a uniform electric field, the path of electron beam moving perpendicular to field will be a parabola and the kinetic energy (and hence the speed) of the electron changes. In a uniform magnetic field, the path of electron beam perpendicular to field will be circular and kinetic energy (and hence the speed) of the electron does not change.

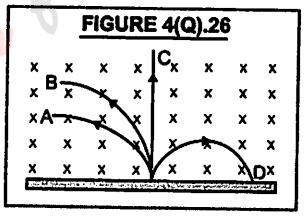
22. In what ways electric and magnetic fields are different ?

Sol. (*i*) Electric field is due to charges at rest as well as in motion, whereas magnetic field is due to a magnet or current through a conductor or by a moving charge.

(*ii*) The strength of electric field at a point decreases with the dielectric medium but the strength of magnetic field increases when a permeable medium is inserted there.

(*iii*) The electric lines of force representing the electric field do not form closed paths whereas the magnetic lines of force form closed paths.

23. A neutron, a proton an electron and an α-particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of paper. The tracks of the particles are shown in Fig. 4(Q).26. Relate the tracks to the particles.



Sol. We know that force on a charged particle in the magnetic field is

$$|\vec{F}| = q\,|\vec{v} \times \vec{B}| \quad \text{or} \quad F = q\,v\,B\sin\theta, \text{ so}$$

(*i*) For neutral particle *i.e.*, neutron, $q = 0$, hence $F = 0$. It means neutron will go undeflected *i.e.* **track C corresponds to neutron.**

(*ii*) For negatively charged particle *i.e.* electron, the direction of force, according to Fleming's Left hand rule will be towards right. **So track D corresponds to electron.**

(*iii*) For positively charged particle, the direction of force, according to Fleming's left hand rule will be towards left. So both tracks *A* and *B* correspond to positively charged particles (*i.e.* protons and α-particles).

When a moving charged particle is subjected to a perpendicular magnetic field, it describes a circular path of radius r given by

$$r = \frac{mv}{Bq} \quad \text{or} \quad r \propto \frac{m}{q}$$

$$\frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \left(\frac{m}{4m}\right) \times \left(\frac{2e}{e}\right) = \frac{1}{2}$$

or $r_\alpha = 2\, r_p$ *i.e.*, $r_\alpha > r_p$.

i.e., **track B corresponds to α-particle and track A to proton.**

24. Differentiate between electric and magnetic forces.

Sol.

S.No.	Electric force	Magnetic force
1.	Electric force acts on a charged particle at rest or in motion in any direction in electric field.	1. No magnetic force acts on a charged particle at rest in a magnetic field. Magnetic force acts on a moving charged particle only if it is not moving parallel to the direction of magnetic field.
2.	Electric force always acts on the charged particle in the direction of electric field.	2. Magnetic force always acts in a direction perpendicular to the direction of magnetic field as well as the direction of motion of charged particle.
3.	The electric force on a charged particle is, $\vec{F} = q\vec{E}$, It depends upon the charge on the particle and is independent of velocity of particle.	3. The magnetic force on a charged particle is, $\vec{F} = q(\vec{v} \times \vec{B})$. It depends upon the charge and velocity of the particle.
4.	Work is required to be done in displacing the charged particle in an electric field.	4. No work is done by the magnetic field in displacing the charged particle because the force and the displacement are perpendicular to each other.

25. A charged particle enters a (non-uniform) magnetic field varying from point to point, both in magnitude and direction, with a certain initial velocity. What do you say about the final velocity of the particle when it leaves the field ?

Sol. The force on the moving charged particle in magnetic field is $\vec{F} = q(\vec{v} \times \vec{B})$. So $\vec{F}$ is always perpendicular to the particle velocity $\vec{v}$. Hence the magnitude of velocity remains unchanged, only the direction changes. Thus the particle will leave the varying magnetic field with its initial velocity, but in a different direction.

V. Force on current carrying wire in magnetic field and force between two parallel conductors carrying currents

26. A straight horizontal conducting rod of length l and mass m is suspended by two vertical wires at its ends. A current I is set up in the rod through the wires.

(*i*) How much magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ?

(*ii*) What will be the total tension in the wires if the direction of current is reversed, keeping the magnetic field same as before ?

Sol. (*i*) Tension in the supporting wires will be zero if the weight of the rod acting vertically downwards is balanced by the upward force on rod due to magnetic field, *i.e.*,

$$mg = IlB \quad \text{or} \quad B = \frac{mg}{Il}$$

(*ii*) If the direction of current is reversed, the magnetic force on rod will act in the vertical downward direction. Now total tension in the wires will be

$$T = mg + IlB = mg + mg = 2\,mg = 2\,IlB$$

27. Two parallel wires carrying current in the same direction attract each other while two beams of electrons travelling in the same direction repel each other. Why ?

Sol. Two parallel wires carrying currents in the same direction attract each other due to magnetic interaction between two wires carrying currents because the current in a wire produces a magnetic field and the magnetic interaction is of attractive nature when current in the two parallel wires is in the same direction.

The two beams of electrons travelling in the same direction will be a source of both an electric and magnetic fields. Due to magnetic interaction, there will be force of attraction between the two moving electrons but due to electrostatic interaction, there will be a force of repulsion between them. If the beams of electrons are moving slowly, the eletrostatic force of repulsion between the electrons dominates the magnetic attraction between them.

28. A loosely wound helix made of stiff metal wire is mounted vertically with the lower end just touching the mercury in a dish. When the current from a battery is started in the metal wire through the mercury, the wire executes oscillatory motion with the lower end jumping out of and into the mercury. Explain.

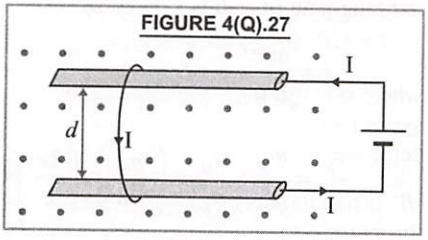
Sol. The windings of the metal helix carry currents in the same direction, therefore they experience attractive forces, pulling the lower end of metal wire out of mercury. When it happens, the circuit breaks, the current disappears and hence the force of attraction disappears. As a result of it, the wire helix regains its original position, completing the circuit again. The process is repeated time and again. Due to it, the oscillatory motion is set up.

29. Why should a solenoid tend to contract when a current passes through it ?

Sol. We know that two parallel conductors carrying currents, in the same direction attract each other and in opposite directions repel each other. Therefore, when current is passed through the coil of a solenoid, the parallel currents in the various turns of solenoid flow in the same direction. As a result of it, the various turns start attracting one another and solenoid tends to contract.

30. A metal wire of mass m slides without friction on two horizontal rails spaced distance d-apart as shown in Fig.4(Q).27. The rails are situated in a uniform magnetic field B, directed vertically upwards, and a battery is sending a current I through them. Find the velocity of the wire as a function of time, assuming it to be at rest initially.

FIGURE 4(Q).27



Sol. The magnetic force on the wire of length d due to magnetic field is $F = I\,Bd \sin 90° = IBd$

This force, according to Left Hand Rule, acts perpendicular to the wire in the plane of paper towards left hand side. Due to it, the wire moves towards left with the acceleration.

$$a = \frac{F}{m} = \frac{I\,Bd}{m}$$

Velocity of the wire at a time t will be

$$v = u + at = 0 + \frac{I\,Bd}{m}\,t$$

or $v = \dfrac{IBd}{m}\,t$ **towards left**

VI. Torque on coil in magnetic field and galvanometer

31. A current carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (*i.e.*, turns about the vertical axis). (CBSE 2013)

Sol. **No**, because the loop can turn around itself if the torque τ is acting in the vertical direction.

Since, torque, $\vec{\tau} = (\vec{M} \times \vec{B}) = I\,(\vec{A} \times \vec{B})$ and $\vec{M}$ or $\vec{A}$ of the horizontal loop is in the vertical direction, so the direction of $\vec{\tau}$ will be in the plane of the loop for any direction of $\vec{B}$.

32. A current carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium ? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

(CBSE 2013) (NCERT Solved Example)

Sol. The current carrying circular loop behaves as a magnetic dipole of magnetic moment $\vec{M}$ acting

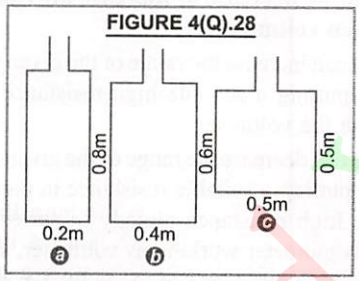
perpendicular to its plane. The torque on the current loop of magnetic dipole moment M in the magnetic field B is

$$\tau = MB \sin \alpha = IA \times B \sin \alpha, \quad (\because M = AI)$$

where α is the angle between $\vec{M}$ and $\vec{B}$. The system will be in stable equilibrium if torque is zero, which is so if $\alpha = 0°$. This is possible if $\vec{B}$ is parallel to $\vec{A}$ *i.e.* $\vec{B}$ is perpendicular to the plane of the loop. In this orientation, the magnetic field produced by the loop is in the same direction as that of external field, both normal to the plane of loop. It is due to this fact, the magnetic flux due to total field is maximum.

33. Three wires each of length 2·0 m are bent into different rectangular loops and then suspended in a magnetic field, Fig. 4(Q).28. If the current in each of them be the same, which loop shall be acted upon by largest torque ? If any of the wires be bent into circular loop, then ?

FIGURE 4(Q).28



Sol. Torque (τ) on a current loop suspended in a uniform magnetic field is given by $\tau = I\,AB \sin \alpha$ *i.e.* $\tau \propto A$. *Since the area of loop* (c) = 0·5 m × 0·5 m is maximum; hence the largest torque will be acting on loop (c). When any wire is bent into a circular loop, the torque will be even more because for a given perimeter the area of the circle is maximum.

34. A length L of a wire carries a current I. Prove that if the wire is formed into a circular loop, the maximum torque in a given magnetic field will be developed if the coil has one turn only. Find the value of the maximum torque acting on circular coil.

Sol. Let the circular coil formed has n turns, each of radius r. Then $L = 2\pi r n$ or $r = L/2\pi n$

Area of the coil, $A = \pi r^2 = \dfrac{\pi L^2}{4\pi^2 n^2} = \dfrac{L^2}{4\pi n^2}$

Torque on coil in magnetic field is

$$\tau = nIBA \sin \alpha = nIB \times \frac{L^2}{4\pi n^2} \sin \alpha$$

$$= \frac{L^2 IB}{4\pi n} \sin \alpha$$

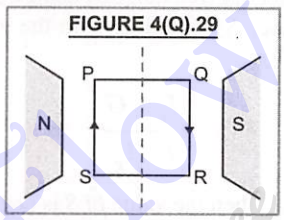
Torque will be maximum if n is least (= 1) and $\sin \alpha$ has maximum value *i.e.*, $\sin \alpha = 1$

Thus torque will be maximum when $n = 1$

Maximum value of Torque, $\tau_{max.} = \dfrac{L^2 IB}{4\pi}$

35. A rectangular current carrying loop *PQRS* is kept in a uniform magnetic field as shown in the Fig. 4(Q).29.

FIGURE 4(Q).29



(*a*) What is the direction of magnetic moment of the current loop ?

(*b*) When is the torque acting on the loop (*i*) maximum, (*ii*) zero ? (CBSE 2005, 2009)

Sol. (*a*) Magnetic moment of the current loop, $\vec{M} = I\vec{A}$. Its direction is normal to the loop inwards the plane of coil.

(*b*) Torque on the loop, $\tau = MB \sin \alpha$

(*i*) Torque is maximum if $\alpha = 90°$, *i.e.*, the plane of coil is parallel to the direction of magnetic field.

(*ii*) Torque is zero if $\alpha = 0°$, *i.e.*, when plane of the loop is perpendicular to the magnetic field.

VII. Conversion of galvanometer into ammeter and voltmeter

36. While converting galvanometer into an ammeter, how does the parallel low resistance (shunt) bring the required changes in the galvanometer ?

Sol. When a galvanometer is to be converted into an ammeter of a given range, a suitable low resistance $S = I_g G/(I - I_g)$ is connected in parallel with the galvanometer. In doing so, the limited current of circuit flows through galvanometer and remaining current of the circuit flows through shunt. Due to it, a galvanometer works as an ammeter of given range.

37. A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape ? (NCERT Solved Example)

Sol. A loop of irregular shape carrying current when placed in an external magnetic field, it assumes circular shape with its plane normal to the field in order to maximise magnetic flux linked with loop. This is because for a given perimeter, a circle encloses maximum area than any other shape.

38. A galvanometer gives full scale deflection with current I_g. Can it be converted into an ammeter of range $I < I_g$?

Sol. A galvanometer can be converted into an ammeter of given range by using a suitable shunt resistance S in parallel with the galvanometer, given by

$$S = \frac{I_g \times G}{I - I_g}$$

If $I < I_g$, then the value of S is negative. Hence, a galvanometer can not be converted into an ammeter of range $I < I_g$.

39. Can we increase or decrease the range of a given ammeter ?

Sol. We can increase the range of the given ammeter by using a suitable shunt in parallel with the ammeter but we can not decrease the range of the ammeter for the reason explained in the above question.

40. Write two reasons, why a galvanometer cannot be used as such to measure current in a given circuit. (CBSE 2011)

Sol. A galvanometer as such cannot be used to measure the current in a circuit due to following reasons :

(*i*) Galvanometer is quite a sensitive device. It shows full scale deflection with a current of few microampere, hence may get damaged if strong current is passed through it.

(*ii*) While measuring current, the galvanometer whose resistance is large than ammeter has to be connected in series of the circuit. Due to it, there will be decrease in current of the circuit. Thus, measurement of current will not be true.

41. When a voltmeter is put across a part of the circuit, does it record slightly less or more than the actual voltage across that part ? Explain.

Sol. When a voltmeter is put across a part of the circuit, it draws some current from the circuit. Due to it, the potential difference across that part becomes less than actual. As a result of it, voltmeter will record slightly less than the actual potential difference across that part.

42. Why should the resistance of an ideal voltmeter be infinite and of ideal ammeter be zero ?

Sol. A voltmeter is used to measure the potential difference across the two points of a circuit. For it, the voltmeter is connected in parallel across those two points. The potential difference measured by voltmeter will be accurate if no current is drawn by voltmeter. It will be so if the resistance of voltmeter is infinite. Then the voltmeter will be ideal voltmeter. Thus, ideal voltmeter has infinite resistance.

An ammeter is used to measure the current in the circuit. For it, the ammeter is connected in series of the circuit. The current measured by ammeter will be accurate if there is no change in current of the circuit by the use of ammeter in series of circuit. It is possible if resistance of ammeter is zero. Then, the ammeter is called ideal ammeter. Thus ideal ammeter has zero resistance.

43. Can we decrease or increase the range of the given voltmeter ?

Sol. We can increase the range of the given voltmeter by putting a suitable high resistance in series with the voltmeter.

We can decrease the range of the given voltmeter by putting a suitable resistance in parallel with the high resistance already in series with the galvanometer working as voltmeter, so that the effective resistance of the voltmeter multiplied with the current (I_g) gives the required potential difference to be measured now.

44. Why is ammeter connected in series and voltmeter in parallel in the circuit ? (CBSE 2016)

Sol. An ammeter is a low resistance galvanometer. It is used to measure the current in ampere. To measure the current of a circuit, the ammeter is connected in series in the circuit so that the current to be measured must pass through it. Since, the resistance of ammeter is low, so its inclusion in series in the circuit does not change the resistance and hence the main current in the circuit.

A voltmeter is a high resistance galvanometer. It is used to measure potential difference between any two points of the circuit. To measure the potential difference between the two points of a circuit, the voltmeter is connected in parallel in the circuit. The voltmeter resistance being high, it draws

minimum current from the main circuit and the potential difference to be measured is not affected materially.

45. By mistake, a voltmeter is connected in series and an ammeter is connected in parallel, with a resistance in an electrical circuit. What will happen to the instruments ?

Sol. Voltmeter resistance being very high, when used in series with a resistance in an electrical circuit, the effective resistance of the circuit becomes very high. Due to it, the current in the circuit becomes very low. As voltmeter measures potential difference between the two points, it will show the reading but will not be damaged. The ammeter resistance is very low. When it is connected in parallel with a resistance, the weak current flowing through the circuit will pass through the ammeter. Due to it, the ammeter will not show appreciable deflection. Therefore, the ammeter also will not be damaged.

46. A galvanometer of resistance G can be converted into a voltmeter of range $0 - V$ volts by connecting a resistance R in series with it. How much resistance will be required to change its range from 0 to $V/2$?

[CBSE 2015 (C)]

Sol. In first case, $R = \dfrac{V}{I_g} - G$ or $I_g = \dfrac{V}{R+G}$

Let R' be the required resistance to change the range from 0 to $V/2$.

In second case, $I_g = \dfrac{V/2}{R'+G}$

$\therefore \quad \dfrac{V}{R+G} = \dfrac{V/2}{R'+G}$

or $2R' + 2G = R + G$ or $R' = \dfrac{R-G}{2}$

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Biot Savart's law and magnetic field due to a linear current carrying wire

1. The north pole of a magnet is brought near a stationary negatively charged conductor. Will the pole experience any force ?

Ans. No, a stationary charge does not produce magnetic field.

2. What is the unit of magnetic field strength in cgs system and SI ? State the relation between them.

Ans. Unit of magnetic field in cgs system is gauss and in SI is tesla or N A^{-1} m^{-1} or weber m^{-2}.

1 tesla = 10^4 gauss.

3. Explain, how moving charge is a source of magnetic field ?

Ans. A moving charge produces an electric current which in turn causes the magnetic field.

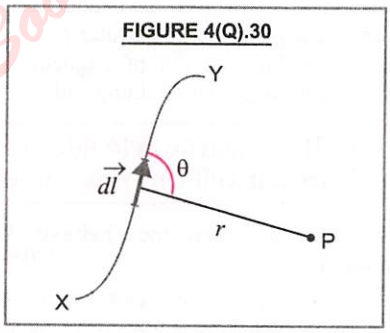
4. What are the dimensions of $\mu_0/4\pi$?

Ans. $[M^1 L^1 T^{-2} A^{-2}]$.

5. State Biot-Savart's law and express this law in the vector form. **(CBSE 2017)**

Ans. Biot-Savart's law states that the magnetic field induction at a point due to current I in element of length dl, at distance r from the element is

FIGURE 4(Q).30

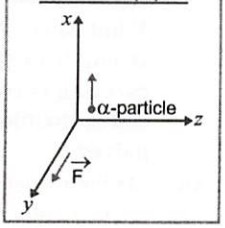


$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

6. A beam of α particles projected along + x-axis, experiences a force due to a magnetic field along the + y-axis Fig. 4(Q).31. What is the direction of magnetic field. **(CBSE 2010)**

FIGURE 4(Q).31



Ans. According to Fleming's Left Hand rule, the direction of magnetic field is along $-z$ axis.

7. State the rule that is used to find the direction of magnetic field acting at a point near a current carrying straight conductor.

Ans. The direction of magnetic field acting at a point near a current carrying straight conductor can be given by **Right Hand thumb rule.** According to this rule, if we imagine the linear wire conductor to be held in the grip of the right hand so that the thumb points in the direction of current, then the curvature of the fingers around the conductor will represent the direction of magnetic field lines.

8. Where is the magnetic field of a current element (*i*) minimum and (*ii*) maximum ?

Ans. (*i*) The magnetic field of a current element is minimum (which is zero) along the axis of a current element. (*ii*) The magnetic field of a current element is maximum in a plane passing through the element and perpendicular to its axis.

9. What is the nature of the magnetic field associated with the current in a straight conductor ?

Ans. The magnetic field lines due to straight conductor carrying current are in the form of concentric circles with the conductor as centre, lying in a plane perpendicular to the straight conductor. The direction of magnetic field lines is given by Right Hand thumb rule.

II. Magnetic field due to a circular coil carrying current

10. How does a current loop behave like a bar magnet ? (CBSE 2011)

Ans. A current loop behaves as a bar magnet because (*i*) one face of current loop behaves as a south pole and the other face as north pole.

(*ii*) it possesses a magnetic dipole moment ($M = IA$) and

(*iii*) it experiences a torque in an external magnetic field, which tends to align the axis of the loop along the direction of magnetic field as bar magnet does.

11. What will be the change in the magnetic field strength at the centre of the circular coil carrying current, if the current through the coil is doubled and the radius of the coil is halved ?

Ans. Magnetic field induction at the centre of the circular coil carrying current is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r} \quad i.e., \quad B \propto \frac{n}{r}$$

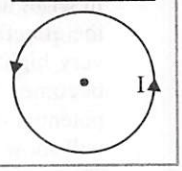
$$\therefore \quad \frac{B'}{B} = \frac{2n}{(r/2)} \times \frac{r}{n} = 4 \quad \text{or} \quad B' = 4\,B$$

Change in magnetic field $= B' - B = 4B - B$

$= 3\,B$

12. In Fig. 4(Q).32 is shown a circular loop carrying current *I*. Show the direction of the magnetic field with the help of lines of force. (CBSE 2004)

FIGURE 4(Q).32



Ans. The magnetic field lines of a circular loop carrying current *I* are shown in Fig. 4.13.

III. Ampere's circuital law, solenoid and magnetic field due to current in conducting rod

13. A current is set up in a long copper pipe. Is there a magnetic field (*a*) inside (*b*) outside the pipe ?

Ans. Inside the pipe, the magnetic field is zero, but there is a finite value of the magnetic field outside the pipe.

14. Compare Gauss's law and Ampere's law.

Ans. Gauss's law corelates surface integral of electric field $\vec{E}$ with charge q over a surface [*i.e.* $\oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$], whereas Ampere's law corelates the line integral of magnetic field $\vec{B}$ with current over a closed path [*i.e.* $\oint \vec{B}.\vec{dl} = \mu_0 I$].

15. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why ? (CBSE 2009)

Ans. It is so because the magnetic field induction outside the toroid is zero.

16. How is the magnetic field inside a given solenoid made strong ? (CBSE 2011)

Ans. It is done by inserting laminated iron core inside the solenoid.

17. What is the difference between solenoid and toroid ? (CBSE 2019)

Ans. A solenoid consists of an insulating long wire closely wound in the form of helix. Its length is very large as compared to its diameter.

A toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. Infact, toroid is an endless solenoid.

18. In a solenoid carrying current, where is the magnetic field (*i*) maximum (*ii*) minimum and (*iii*) half of the maximum value ?

Ans. A solenoid carrying current has (*i*) maximum value of magnetic field at a point will inside the solenoid lying on the axis of solenoid.

(*ii*) There is a minimum value of magnetic field at a point outside the solenoid near the middle of solenoid and (*iii*) half of the maximum value at a point near the end of the solenoid.

IV. Force on charged particle due to electric and magnetic fields and cyclotron

19. What is the value of absolute permeability of free space ? Give its units.

Ans. $\mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$ or $Wb\ m^{-1}\ A^{-1}$

20. Is the source of magnetic field analogue to the source of electric current.

Ans. No, because the source of magnetic field is a magnetic dipole, but the source of electric field is an electric charge.

21. When a charged particle moving with a velocity $\vec{v}$ is subjected to a magnetic field $\vec{B}$, the force acting on it is non zero. Would the particle gain any energy ? (CBSE 2013)

Ans. The magnetic force on a moving charged particle in a magnetic field is, $\vec{F} = q(\vec{v} \times \vec{B})$. Its direction is perpendicular to direction of motion of particle as well as perpendicular to the direction of magnetic field. Due to it, no work is done by the magnetic force on the charged particle. Therefore, the particle does not gain any kinetic energy.

22. Show that the kinetic energy of the particle moving in a magnetic field remains constant. (CBSE 2017)

Ans. A charged particle moving in a perpendicular magnetic field describes a circular path, on which its speed v does not change. So its kinetic energy $K\left(=\frac{1}{2}mv^2\right)$ remains unchanged.

23. An electron is moving vertically downwards. If it passes through a magnetic field which is directed from south to north in a horizontal plane, then in which direction the electron would be deflected ?

Ans. Towards west

24. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency ? (CBSE 2018)

Ans. Frequency of revolution of charged particle moving in a perpendicular uniform magnetic field is

$$v = \frac{Bq}{2\pi m} \quad i.e., \quad v \propto \frac{1}{m}$$

As $m_e < m_p$, so $\frac{v_e}{v_p} = \frac{m_p}{m_e} > 1$ or $v_e > v_p$

25. Two protons of equal kinetic energies enter a region of uniform magnetic field. The first proton enters normal to the field direction while the second enters at 30° to the field direction. Name the trajectories followed by them. [CBSE 2018 (C)]

Ans. The first proton will describe a circular path in a plane perpendicular to the direction of magnetic field. The second proton will describe a helical path with axis parallel to the direction of magnetic field.

26. Derive an expression for the velocity v_c of a positive ion passing undeflected through a region where crossed and uniform electric field E and magnetic field B are simultaneously present. (CBSE Sample Paper 2019)

Ans. The positive ion of charge q moving with velocity v will go undeflected while passing through a region where two crossed uniform electric and magnetic fields are present if the force on ion due to electric field is equal and opposite to the force on ion due to magnetic field *i.e.*,

$$Eq = qvB \quad \text{or} \quad v = E/B$$

27. A charged particle moving in a uniform magnetic field penetrates a layer of lead and thereby loses one half of its kinetic energy. How does the time period of revolution of particle change.

Ans. The radius of the circular path, $r = mv/qB$. If E_K is the kinetic energy of the particle, then

$$E_K = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{2E_K/m}$$

$$\therefore \text{ radius, } r = \frac{m\sqrt{2E_K/m}}{qB} = \frac{\sqrt{2mE_K}}{qB};$$

$$\text{Time period} = \frac{2\pi r}{v} = \frac{2\pi\sqrt{2mE_K}}{qB\sqrt{2K_K/m}} = \frac{2\pi m}{qB}$$

It means time period of revolution is independent of K.E. This shows that if KE is halved, there is no change in time period of revolution.

28. Establish analytically that the gain in kinetic energy of the charged particle moving in a magnetic field is zero.

Ans. Total gain in K.E. = work done

$$= \int \vec{F}_m \cdot d\vec{r} = \int_{t_1}^{t_2} \vec{F}_m \cdot \frac{d\vec{r}}{dt} dt = \int_{t_1}^{t_2} \vec{F}_m \cdot \vec{v}\, dt$$

$$= \int_{t_1}^{t_2} q(\vec{v} \times \vec{B}) \cdot \vec{v}\, dt = \int_{t_1}^{t_2} q(\vec{v} \times \vec{v}) \cdot \vec{B}\, dt = 0$$

29. A charged particle moving in a uniform magnetic field penetrates a layer of lead and there by loses one-half of its kinetic energy. How does the radius of curvature of its path change ? **(CBSE 2010)**

Ans. Kinetic energy, $K = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2K}{m}}$.

Radius of curvature,

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{qB}$$

Therefore, $r \propto \sqrt{K}$,

So $\frac{r'}{r} = \sqrt{\frac{K/2}{K}} = \frac{1}{\sqrt{2}}$ or $r' = \frac{r}{\sqrt{2}}$

30. State the principle of working of a cyclotron. **(Pb. Board 2001)**

Ans. The working of the cyclotron is based on the fact that a positively charged particle can be accelerated to a sufficiently high energy with the help of smaller values of oscillating electric field by making it to cross the same electric field time and again with the use of strong magnetic field.

31. An electron moves with velocity $\vec{v} = a\hat{k}$ in a magnetic field, $\vec{B} = b\hat{i} + c\hat{j}$. Find the magnetic force on the electron.

Ans. The force on electron in the magnetic field is

$$\vec{F} = -e(\vec{v} \times \vec{B}) = -e[a\hat{k} \times (b\hat{i} + c\hat{j})]$$

$$= -e[ab\hat{j} + ac(-\hat{i})] = ea[c\hat{i} - b\hat{j}]$$

32. A uniform magnetic field and a uniform electric field are produced, pointing in the same direction. An electron is projected with its velocity pointed in the same direction. What will be the effect on electron?

Ans. The electron velocity will decrease as it will experience force due to electric field opposite to the direction of electric field but no force will be acting on electron due to magnetic field.

33. What is meant by cyclotron frequency ?

Ans. It is the frequency of oscillation of a charged particle inbetween two dees of cyclotron, which is equal to the frequency of high frequency oscillator, creating electric field between two dees of cyclotron. Cyclotron frequency, $\nu = Bq/2\pi m$, which is independent of the radius of the circular path and velocity of the charged particle in the two dees of cyclotron.

34. Looking downward, an electron appears moving anticlock-wise on a horizontal circle under a magnetic field. What is the direction of the magnetic field ?

Ans. The motion of electron in anticlock wise direction causes the current in clockwise direction. The magnetic force on the electron is along the radius towards the centre of circle. It will be so if the direction of magnetic field is **vertically upwards.**

35. Which physical quantity has the unit Wb/m^2 ? Is it a scalar or a vector quantity ?

Ans. Wb/m^2 is the *SI* unit of magnetic field induction *B*, which is a vector quantity.

V. Force on current carrying wire in magnetic field and force between two parallel conductors carrying currents

36. Write expression for the force between two short parallel wires carrying the currents when they are :

(*a*) much separated

(*b*) very close

Ans. (*a*) $F = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2 dl_1 dl_2}{r^2}$ (*b*) $F = \frac{\mu_0}{4\pi} \frac{2I_1 I_2 dl}{r}$

37. What is the force that a conductor $d\vec{l}$ carrying a current *I* experiences, when placed in a magnetic field $\vec{B}$, when its length is making an angle of 30° with the direction of field ?

Ans. $|d\vec{F}| = I\,|d\vec{l} \times \vec{B}| = I\,dl\,B \sin 30° = \mathbf{I\,dl\,B/2}$

38. If the distance between two parallel current carrying wires is doubled, what is the force between them ?

Ans. The force acting on one wire due to currents through two wires is inversely proportional to the distance between them. Thus the force

becomes **1/2 times** if the distance between the wires is doubled.

39. In which orientation is the force experienced by a current-carrying conductor placed in a magnetic field (*i*) minimum (*ii*) maximum ?

Ans. Force on a current carrying conductor in magnetic field, $F = IlB \sin\theta$.

(*i*) Force F will be minimum if $\sin\theta = 0$ or $\theta = 0°$ or $180°$, *i.e.*, the linear conductor carrying current is parallel or antiparallel to the direction of magnetic field.

(*ii*) Force F will be maximum, if $\sin\theta = 1$ or $\theta = 90°$, *i.e.*, the linear conductor carrying current is perpendicular to the direction of uniform magnetic field.

40. Two current elements are placed a certain distance apart but not parallel to each other. Do they exert equal and opposite forces on each other ?

Ans. Two current elements, placed a certain distance apart but not parallel to each other also exert forces on each other which are equal and opposite according to Newton's third law of action and reaction.

41. Wires that carry equal but opposite currents are often twisted together to reduce their magnetic effect at distant points. Why is this effective ?

Ans. It is so because the magnetic field $\vec{B}$ at a point is a function of distance and direction of current. Twisting of wires eliminates the magnetic effects at a distant point because of the following reasons: (*i*) average distance is practically the same and (*ii*) current flows in the two wires in opposite directions.

42. What is the ratio of electric and magnetic forces between two moving charges ?

Ans. $F_e / F_m = 10^{25}$. [Refer to Art. 4.25]

43. A horizontal wire placed perpendicular to a magnetic field carries a current from left to right. The magnetic field is horizontal, directed towards you. What is the direction of magnetic force on the wire ?

Ans. According to Fleming's left hand rule, it is vertically downwards.

VI. Torque on coil in magnetic field and galvanometer

44. A conducting circular loop of radius *r* carries a constant current *i*. It is placed in a uniform magnetic field *B* such that *B* is perpendicular to the plane of loop. What is the magnetic force acting on the loop ?

Ans. Zero.

45. Two wires of equal lengths are bent in the form of two loops. One of the loops is square shaped whereas the other loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque ? Give reasons.

Ans. For a wire of given length, the circular loop encloses greater area than the square loop. As, torque $\tau = IAB$, *i.e.*, $\tau \propto A$, so the circular current loop will experience greater torque in the magnetic field.

46. On what interaction is the principle of galvanometer based ?

Ans. The principle of galvanometer is based on the interaction of current and magnetic field.

47. What is the value of net force acting on a current carrying (*i*) rectangular coil and (*ii*) circular coil of same area placed in a uniform magnetic field ? What will be the torque acting in each case ?

Ans. The net force acting on each current carrying coil placed in magnetic field is zero. Torque on each coil is same provided the plane of each coil is equally inclined with the direction of magnetic field.

48. What is a dead beat galvanometer ?

Ans. Dead beat galvanometer is one in which the coil comes to rest at once after the passage of current through it. The deflection can be noted in no time.

49. Why is the coil wrapped on a conducting frame in a galvanometer ?

Ans. When a coil is rotated in the magnetic field, a variable magnetic flux is linked with the conducting frame of the coil, resulting the eddy currents in it, which opposes the cause producing it. These eddy currents in conducting frame help in stopping the coil soon *i.e.* in making the galvanometer dead beat.

50. What is the function of soft iron cylinder between the poles of a galvanometer ?

(CBSE 2017)

Ans. It concentrates the magnetic field and helps in making the magnetic field radial and strong.

51. Why are pole pieces of galvanometer made concave ?

Ans. To have a uniform, strong and radial magnetic field.

52. What is the function of the radial magnetic field in the moving coil galvanometer ?

(CBSE 2019)

Ans. It helps the arm of the couple and hence the torque on coil always the same in all positions. This arrangement provides linear scale to the galvanometer.

53. Define the current sensitivity of a moving coil galvanometer and state its SI unit ?
[CBSE 2013, 2013 (C)]

Ans. The current sensitivity of a moving coil galvanometer is defined as the deflection produced in the galvanometer when unit current flows through it.

Current sensitivity, $I_s = \frac{\theta}{I} = \frac{nBA}{k}$

The SI unit of current sensitivity is rad A^{-1}.

54. Write two factors by which the current sensitivity of a moving coil galvanometer can be increased ?

Ans. A current sensitivity of moving coil galvanometer is $I_s = \frac{\theta}{I} = \frac{nBA}{k}$.

Thus, current sensitivity can be increased (*i*) by increasing the strength of magnetic field B or increasing the number of turns in the coil. (*ii*) by decreasing the torsional constant k of its suspension fibre.

VII. Conversion of galvanometer into ammeter and voltmeter

55. Can a galvanometer as such be used for measuring the current ? Explain.
(CBSE 2017)

Ans. No, a galvanometer as such can not be used for measuring current as it is a very sensitive instrument as it shows full scale deflection even for a very small current.

56. What is meant by figure of merit of a galvanometer ?

Ans. Figure of merit of galvanometer is defined as the amount of current which produces one scale deflection in the galvanometer.

57. Which has greater resistance
(*a*) milliammeter or ammeter ?
(*b*) milli voltmeter or voltmeter?

Ans. The resistance of milliammeter is greater than that of ammeter. The resistance of voltmeter is greater than that of millivoltmeter.

58. Why should a voltmeter have a low current carrying capacity ?

Ans. A voltmeter should have a low current carrying capacity so that its resistance is high. It draws a very small current from the circuit. Due to it, the potential difference ($V = IR$) to be measured will not be much different from the actual value.

59. A galvanometer of resistance 50 Ω is shunted by a resistance of 5 Ω. What fraction of the main current passes through the galvanometer? Through the shunt ?
(Pb. Board 2011)

Ans. Fraction of current passing through galvanometer $= \frac{i_g}{i} = \frac{S}{G+S} = \frac{5}{50+5} = \frac{1}{11}$

Fraction of current passing through shunt

$$= \frac{i_s}{i} = \frac{i - i_g}{i} = 1 - \frac{i_g}{i} = 1 - \frac{1}{11} = \frac{10}{11}$$

60. A current of 10^{-7} A produces 50 division deflection in a galvanometer. Find its figure of merit.

Ans. Figure of merit,

$$k = \frac{I}{\theta} = \frac{10^{-7}}{50} = 2 \times 10^{-9} \text{ A/div}$$

61. Can moving coil galvanometer be used to detect an a.c. in a circuit ? Give reason.

Ans. A moving coil galvanometer cannot be used to detect a.c. in a circuit, since it measures the average value of current and the average value of a.c. over a complete cycle is zero.

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Biot Savart's law and magnetic field due to a linear current carrying wire

1. Write the relation for the force $\vec{F}$ acting on a charge carrier q moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ in vector notation. Using this relation, deduce the conditions under which this force will be
(*i*) maximum
(*ii*) minimum.
(CBSE 2007)

Ans. $\vec{F} = q(\vec{v} \times \vec{B})$

or $|\vec{F}| = q|\vec{v} \times \vec{B}| = qvB\sin\theta$

(*i*) **F will be maximum**, when $\sin\theta = 1$ or $\theta = 90°$, *i.e.*, the charged particle is moving perpendicular to the direction of magnetic field.

(*ii*) **F will be minimum,** When $\sin\theta = 0$ or $\theta = 0°$ or $180°$ *i.e.*, the charged particle is moving parallel to the direction of magnetic field.

2. What is magnetic flux density ? Define its units and give its dimensions.

Ans. Magnetic flux density at a point in a magnetic field means magnetic field induction at that point. It is defined as the force experienced by a unit charge while moving with a unit velocity, perpendicular to the direction of magnetic field at that point. Force experienced by the charged particle having charge q moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is given by

$$|\vec{F}| = q|\vec{v}\times\vec{B}| = qvB\sin\theta$$

or $$B = \frac{F}{qv\sin\theta}$$

The SI unit of B is tesla, where 1 tesla is the magnetic flux density at a point if 1 coulomb charge while moving with a velocity of 1 ms^{-1}, perpendicular to a magnetic field experiences a force of 1 N at that point.

The dimensional formula of B

$$= \frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [ML^0T^{-2}A^{-1}]$$

3. In what respect does a wire carrying a current differ from a wire, which carries no current ?

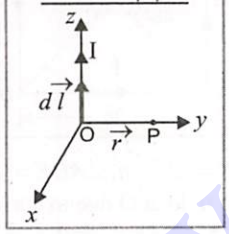
Ans. A current carrying wire produces a magnetic field. When current is flowing through a wire, the electrons move inside it along a definite direction. On the other hand, if no current is flowing through a wire, the electrons inside this wire are in random motion, their average thermal velocity is zero. Such a wire does not produce any magnetic field.

4. A current of one ampere is passed through a straight wire of length 2·0 metre. Find the magnetic field at a point in air at a distance 3 metre from one end of wire but lying on the axis of the wire.

Ans. When a point P lies on the axis of wire conductor in air, then $I\,d\vec{l}$ and $\vec{r}$ for each element of the straight wire conductor are parallel. Therefore, $I\,d\vec{l}\times\vec{r} = 0$, so the magnetic field induction at given point is **zero.**

5. State Biot Savart's law. A current *I* flows in a conductor placed perpendicular to the plane of paper. Indicate the direction of the magnetic field due to a small element $d\vec{l}$ at a point P at a distance $\vec{r}$ from the element as shown in Fig. 4(Q).33. (CBSE 2009)

FIGURE 4(Q).33



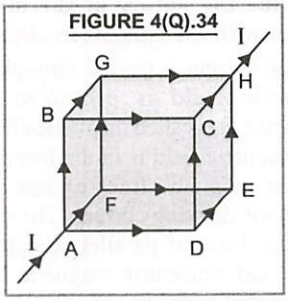
Ans. According to Biot-Savart's law, the magnitude of the magnetic field induction at a point P, due to current element of length dl, carrying current I, at a distance r from the element is given by

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I|d\vec{l}\times\vec{r}|}{r^3}$$

The direction of $d\vec{B}$ is perpendicular to the plane containing $d\vec{l}$ and $\vec{r}$ and is directed along x-axis, which is given by Right Hand Rule.

6. Fig. 4(Q).34 shows a cube made from twelve uniform wires. Find the magnetic field at the centre of the cube, if a battery is connected between the points *A* and *H*.

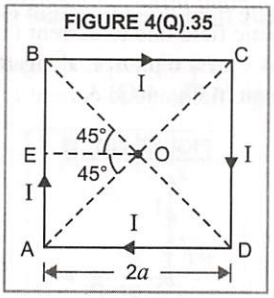
FIGURE 4(Q).34



Ans. We have the following current carrying conducting pairs *AB* and *EH*, *AF* and *CH*, *AD* and *GH*, *BG* and *DE*, *FG* and *DC*, *FE* and *BC*, for which each pair produce equal and opposite magnetic fields at the centre. As a result of it, the resultant magnetic field at the centre will **be zero.**

7. Find the expression for the magnetic field at the centre O of a coil bent in the form of a square of side 2*a*, carrying current *I*, Fig. 4(Q).35.

FIGURE 4(Q).35



Ans. Here, $OE = AE = a$, $\angle AOE = \angle BOE = 45°$

Magnetic field at O due to current in arm AB of wire

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 45° + \sin 45°] = \frac{\sqrt{2}\mu_0 I}{4\pi a}$$

It is acting vertically downwards.

The magnetic field at O due to currents in arms BC, CD and DA will also be of the same magnitude and direction as $\vec{B}_1$. Therefore, resultant magnetic field at O is

$$B = 4B_1 = \frac{4\times\sqrt{2}\mu_0 I}{4\pi a} = \frac{\sqrt{2}\mu_0 I}{\pi a}$$

It is acting **vertically downwards.**

II. Magnetic field due to a circular coil carrying current

8. What is the magnetic effect of current ? Describe the nature of the magnetic field related with the current in circular coil.

Ans. When a current is passed through a conductor, magnetic field is produced around the conductor. It is called magnetic effect of current. The magnetic field is in the form of concentric circular magnetic lines of force for a linear conductor carrying current. The magnetic field is in the form of parallel straight lines at the centre and concentric magnetic lines near the circular coil carrying current.

9. Plot a magnetic field due to a circular coil carrying current and explain the law used to find the direction of magnetic field at the centre of circular coil carrying current.

Ans. The magnetic lines of force due to circular coil carrying current are perpendicular to the wire loop and are circular near the wire and practically straight near the centre of the wire loop. Fig. 4.13. The direction of magnetic field at the centre of circular current loop is given by Right hand rule. For it refer to Art. 4.7.

10. Deduce the expression for the magnetic field induction at the centre of a circular electron orbit of radius r, and angular velocity of orbiting electron ω.

Ans. The electron moving on a circular orbit acts like a current loop. Magnetic field induction in magnitude, at the centre of the current loop is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}\frac{2\pi}{r}\left(\frac{e}{T}\right) = \frac{\mu_0}{2r}\left(\frac{e}{2\pi/\omega}\right) = \frac{\mu_0 e\omega}{4\pi r}$$

11. What is magnetic dipole moment of a current loop? Give its direction if any.

Ans. Magnetic dipole moment of a current loop = niA where n = no. of turns in a current loop; i = current through the loop and A = area of each turn of the loop. Magnetic dipole moment is a vector quantity. Its direction is perpendicular to the plane of loop directed outwards for anticlockwise current in loop and is directed inwards for clockwise current in loop.

12. A wire of length L metre carrying a current of I ampere is bent in the form of a circle. Find its magnetic moment. **(Uttarakhand Board 2012)**

Ans. $L = 2\pi R$ or $R = L/2\pi$;

Magnetic moment $= IA = I\pi R^2$

$$= I\pi\left(\frac{L}{2\pi}\right)^2 = IL^2/4\pi$$

13. A circular coil of N Turns and diameter d carries a current I. It is unwound and rewound to make another coil of diameter $2d$, current I remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil. **(CBSE 2012)**

Ans. Magnetic moment of original coil,

$M_1 = NI(\pi d^2/4)$. Let N_1 be the number of turns of new wound coil, then

$N\,2\pi(d/2) = N_1\,2\pi(2d/2)$ or $N_1 = N/2$

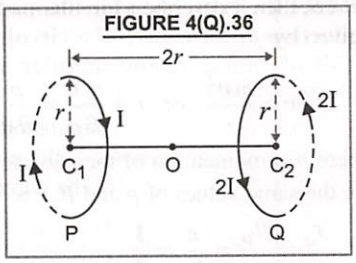
Magnetic moment of new coil,

$$M_2 = N_1 I\pi[(2d)^2/4] = \frac{N}{2}I\pi d^2$$

$$\frac{M_2}{M_1} = \frac{NI\pi d^2/2}{NI\pi d^2/4} = 2$$

14. Two identical circular loops, P and Q each of radius r and carrying currents I and $2I$ respectively are lying in parallel planes such that they have a common axis. Their centres are distance $2r$ apart. The direction of current in both the loops is clockwise as seen from O which is equidistant from both the loops. Find the magnitude of the net magnetic field at point O.

FIGURE 4(Q).36



Ans. Magnetic field induction at O due to current in coil P is

$$B_1 = \frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}\, r}$$

pointings towards OC_1

Magnetic field induction at O due to current in coil Q is

$$B_2 = \frac{\mu_0 (2I) r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 I}{2\sqrt{2}\, r}$$

pointing towards OC_2

Net magnetic field inducion at O

$$B = B_2 - B_1 = \frac{\mu_0 I}{2\sqrt{2}\, r} - \frac{\mu_0 I}{4\sqrt{2}\, r} = \frac{\mu_0 I}{4\sqrt{2}\, r}$$

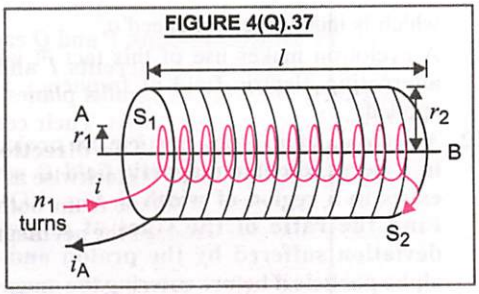
(pointing towards OC_2)

III. Ampere's circuital law, solenoid and magnetic field due to current in conducting rod

15. Two long coaxial insulated solenoids S_1 and S_2 of equal lengths are wound one over the other as shown in Fig. 4(Q).37. A steady current 'I' flows through the inner solenoid S_1 to the other end B, which is connected to the outer solenoid S_2 through which the same current flows in opposite direction so as to come out at end A. If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of net magnetic field at a point (*i*) inside on the axis and (*ii*) outside the combined system.

(CBSE 2014)

FIGURE 4(Q).37



Ans. (*i*) *At a point inside on the axis.*

Magnetic field due to current i through S_1

$B_1 = \mu_0 n_1 i$, along BA, and

Magnetic field due to current i through S_2

$B_2 = \mu_0 n_2 i$, along AB

$\therefore$ Net magnetic field $= B_1 - B_2$

$= \mu_0 i (n_1 - n_2)$, **along BA**

(*ii*) At a point outside the combined system, net magnetic field = **Zero.**

IV. Force on charged particle due to electric and magnetic fields and cyclotron

16. Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. Show that no work is done by this force on the charged particle. (CBSE 2011)

Ans. Lorentz magnetic force on a moving charged particle in the magnetic field is

$$\vec{F} = q(\vec{v} \times \vec{B})$$

This force is acting perpendicular to the plane containing $\vec{v}$ and $\vec{B}$, and is directed as given by Right Hand rule. Since work done,

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta,$$

where $\vec{s}$ is displacement of charged particle in magnetic field. Here, angle $\theta = 90°$, so

$$W = Fs \cos 90° = 0.$$

17. A charged particle is moving on a circular path of radius R in a uniform magnetic field under the Lorentz force F. How much work is done by the force in one round ? Is the momentum of the particle changing?

(CBSE 2011)

Ans. When a charged particle moving on a circular path of radius R in a uniform magnetic field, the Lorentz magnetic force F ($= q\, v\, B$) acting on the particle, provides the required centripetal force for its circular motion. It means the Lerentz force acts along the radius towards the centre of circular path. While moving on a circular path, the small displacement $\vec{dr}$ of the charged particle is always perpendicular to Lerentz force, *i.e.*, $\theta = 90°$, therefore work done

$$dW = \vec{F} \cdot \vec{dr} = F\, dr \cos 90° = 0.$$

Since the velocity of the charged particle, moving on a circular path is acting tangentially to the path whose direction is changing

continuously in circular motion of the particle, therefore the momentum of the particle is changing.

18. Two identical charged particles moving with same speed enter a region of uniform magnetic field. If one of these enters normal to the field direction and the other enters along a direction at 30° with the field, what would be the ratio of their angular frequencies ? (CBSE Sample Paper 2008)

Ans. When a charged particle of charge q, mass m, moving with velocity v enters a uniform magnetic field B, acting perpendicular to the direction of motion of charged particle, then the magnetic force on the particle provides centripetal force. So

$$q\,v\,B = mv^2/r$$

or $$v/r = qB/m$$

The particle will describe a circular path of radius r. Angular frequency of particle,

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

When the charged particle is moving, making an angle 30° with the direction of magnetic field, then due to component velocity $v \cos 30°$ (along the direction of field) no force acts on the particle. Due to component velocity $v \sin 30°$, (acting normally to the direction of field) a force is acting on the particle which provides the required centripetal force for circular motion of particle. Thus,

$$q\,(v \sin 30°)\,B = \frac{m(v \sin 30°)^2}{r}$$

or $$\frac{v \sin 30°}{r} = \frac{qB}{r}$$

Angular frequency $$\omega = \frac{v \sin 30°}{r} = \frac{qB}{r}$$

Thus, the angular frequency of both the particles is same. So ratio of their angular frequency

$$= 1 : 1$$

19. An α-particle and a proton are moving in the plane of the paper in a region where there is a uniform magnetic field $\vec{B}$ directed normal to the plane of the paper. If the two particles have equal linear momenta, what will be the ratio of their trajectories in the field ? (CBSE Sample Paper 2008)

Ans. When a charged particle of charge q moving with velocity v is subjected to normal magnetic field B, then it describe a circular path of radius r given by

$$Bqv = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{Bq} = \frac{p}{Bq}$$

where p = momentum of the particle

For the same values of p and B, $r \propto 1/q$

$$\therefore \quad \frac{r_\alpha}{r_p} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

20. A deutron and a proton are accelerated by the cyclotron. Can both be accelerated with the same oscillator frequency ? Give reason to justify your answer. (CBSE 2017)

Ans. Period of revolution of a charged particle in cyclotron is, $$T = \frac{2\pi m}{Bq}$$

Frequency of revolution of a charged particle in cyclotron is, $$\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

For deutron and a proton, charge q is same but mass m is different, so frequency of revolution of deutron and a proton in cyclotron will be different. Hence they can not be accelerated with the same oscillator frequency.

21. Show that the frequency of revolution, of a charged particle (in the X-Y plane), in a uniform magnetic field $\vec{B}\,(= B\hat{k})$ is independent of its speed.
Which practical machine makes use of this fact ?
What is the frequency of the alternating electric field used in this machine ?

Ans. The radius of the circular path traced by charged particle in a perpendicular uniform magnetic field is $$r = \frac{mv}{qB}$$

Frequency of revolution,

$$\nu_c = \frac{1}{T} = \frac{v}{2\pi r} = \frac{v \times qB}{2\pi \times mv} = \frac{qB}{2\pi m}$$

which is independent of speed v.

A cyclotron makes use of this fact in which alternating electric field of frequency ν_c is applied.

22. A proton and an alpha particle are projected in a perpendicular magnetic field B which exists in a region of width d, Fig. 4(Q).38. Find the ratio of the sines of angles of deviation suffered by the proton and the alpha particle if before entering the magnetic

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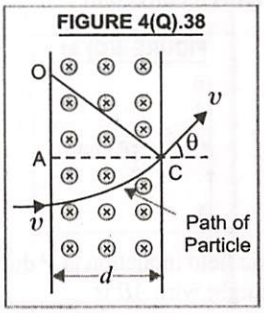
field both the particles (*i*) have the same momentum (*ii*) have the same kinetic energy.

FIGURE 4(Q).38



(CBSE Sample Paper 2003)

Ans. If m_p, m_α are the masses of proton and α-particle then $m_\alpha = 4\, m_p$.

If q_p, q_α are the charges of proton and α-particle, then $q_\alpha = 2\, q_p$.

When a charged particle describes a circular path in the magnetic field, then radius of the path is

$$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

where p is momentum and K is the kinetic energy of the particle. From figure, it is clear that θ is the angle of deviation (*i.e.*, angle between direction of final velocity and initial velocity) of the charged particle on leaving the magnetic field.

In ΔOAC,

$$\sin\theta = \frac{d}{r} = \frac{Bqd}{mv} = \frac{Bqd}{p} = \frac{Bqd}{\sqrt{2mK}}$$

Case (*i*). When $p_\alpha = p_p$, then $\sin\theta \propto q$

$$\therefore \quad \frac{\sin\theta_p}{\sin\theta_\alpha} = \frac{q_p}{q_\alpha} = \frac{1}{2}$$

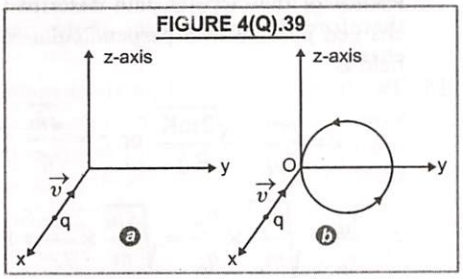
Case (*ii*). When $K_\alpha = K_p$, then $\sin\theta \propto \frac{q}{\sqrt{m}}$

$$\frac{\sin\theta_p}{\sin\theta_\alpha} = \frac{q_p}{q_\alpha} \times \sqrt{\frac{m_\alpha}{m_p}} = \frac{1}{2} \times \sqrt{\frac{4}{1}} = 1$$

23. A charge q moving along the X-axis with a velocity $\vec{v}$ is subjected to a uniform magnetic field B acting along the Z-axis as it crosses the origin O. Fig. 4(Q).39(*a*).

(*i*) Trace its trajectory. (*ii*) Does the charge gain kinetic energy as it enters the magnetic field ? Justify your answer. (CBSE 2009)

FIGURE 4(Q).39



Ans. (*i*) The trajectory of charge q will be a circle in Y-Z plane as shown in Fig. 4(Q).39(*b*).

(*ii*) The speed and kinetic energy of the charged particle remain constant, but velocity of the charged particle changes in direction.

24. A proton and a deutron having equal momenta enter in a region of uniform magnetic field at right angles to the direction of the field. Depict their trajectories in the field. (CBSE 2013)

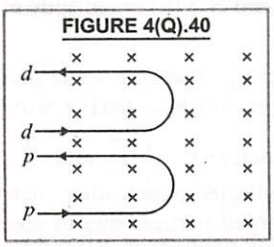
Ans. As $p = mv$ or $v = p/m$. The radius of circular path traced by a charged particle in a perpendicular magnetic field is :

$$r = \frac{mv}{Bq} = \frac{p}{Bq} \quad \text{or} \quad r \propto \frac{1}{q}$$

$$\therefore \quad \frac{r_p}{r_d} = \frac{q_d}{q_p} = \frac{e}{e} = 1$$

Therefore, both the charged particles will describe a trajectory of same radii as shown in Fig. 4(Q).40.

FIGURE 4(Q).40



25. An α-particle and proton of the same kinetic energy are in turn allowed to pass through a magnetic field B, acting normal to the direction of motion of the particles. Calculate the ratio of radii of the circular paths described by them. (CBSE 2019)

Ans. Kinetic energy of a particle mass m moving with velocity v is

$$K = \frac{1}{2} mv^2$$

or $\quad mv = \sqrt{2mK}$

Radius of the circular path described by the charged particle in a perpendicular magnetic field is

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq} \text{ or } r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \quad \frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p}} \times \frac{q_p}{q_\alpha} = \sqrt{\frac{4m}{m}} \times \frac{e}{2e} = 1$$

26. An α-particle and proton are accelerated through the same potential difference. Find the ratio (v_α/v_p) of velocities acquired by two particles. (CBSE Sample Paper 2020)

Ans. Workdone on the charged particle when accelerated under a potential difference V is equal to gain in its kinetic energy. So

$$qV = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2qV}{m}} \text{ or } v \propto \sqrt{\frac{q}{m}}$$

$$\therefore \quad \frac{v_\alpha}{v_p} = \sqrt{\frac{q_\alpha}{q_p} \times \frac{m_p}{m_\alpha}} = \sqrt{\frac{2e}{e} \times \frac{m}{4m}} = \frac{1}{\sqrt{2}}$$

27. A charged particle moving with a uniform velocity $\vec{v}$ enters a region where uniform electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present. It passes through the region without any change in its velocity. What can we conclude about the (*i*) relative directions of $\vec{E}, \vec{v}$ and $\vec{B}$? (*ii*) magnitude of $\vec{E}$ and $\vec{B}$? (CBSE 2017)

Ans. (*i*) A charged particle while passing through a region goes undeflected (*i.e.* without any change in velocity) if $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular to each other, such that the forces on charged particle due to electric field and magnetic field are equal and opposite. Due to it, they cancel out each others effect.

(*ii*) The force on charged particle q due to perpendicular magnetic field,

$$F_m = q\,v\,B \sin 90° = q\,v\,B.$$

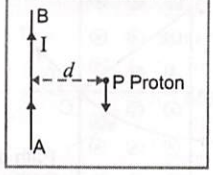
Force on charged particle due to perpendicular electric field, $F_e = q\,E$.

As the charged particle goes undeflected, so

$$q\,v\,B = q\,E \quad \text{or} \quad v = E/B.$$

28. A long straight wire *AB* carries a current *I*. A proton *P* travels with a speed *v*, parallel to the wire, at a distance *d* from it in a direction opposite to the current as shown in Fig. 4(Q).41. What is the force experienced by the proton and what is its direction ? (CBSE 2010)

FIGURE 4(Q).41



Ans. Magnetic field induction at P due to current I in long straight wire AB is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$

It acts perpendicular to the plane of paper inwards, represented by $\hat{n}$.

Lorentz force acting on proton at P can be given by

$$\vec{F} = q(\vec{v} \times \vec{B}) = e\left(\vec{v} \times \frac{\mu_0}{4\pi d}2I\,\hat{n}\right)$$

or $$\vec{F} = \frac{\mu_0 I e v}{2\pi d}(\hat{v} \times \hat{n})$$

$$\therefore \quad F = \frac{\mu_0 I e v}{2\pi d}$$

acting in the plane of paper away from the wire.

29. Explain the main functions of electric and magnetic fields in a cyclotron.

Ans. The oscillating electric field acting between the two dees of the cyclotron will accelerate the charged particle when the particle passes through the gap between the two dees. The magnetic field makes the charged particle to describe a circular path inside the dee.

30. What is the basic principle of working of cyclotron ? Write two uses of this machine.

Ans. The working of the cyclotron is based on the fact that a heavy positively charged particle can be accelerated to a sufficiently high energy with the help of smaller values of oscillating electric field, by making it to cross the same electric field time and again with the use of strong magnetic field.

A cyclotron is used (*i*) to bombard nuclei with high energy particles and to study the resulting nuclear reaction (*ii*) to produce radioactive substances which may be used in hospitals for diagnosing the diseases in the body.

Read Your Flow

V. Force on current carrying wire in magnetic field and force between two parallel conductors carrying currents

31. A body is suspended from the lower end of a vertical spring. What shall be the effect on the position of the body when a current is sent through the spring ? Does it depend upon the direction of current in the spring ?

Ans. When the current is passed through a vertically suspended spring, the direction of currents in two adjoining parallel turns of wire is the same. Hence a force of attraction will be acting there. Due to it, the spring is compressed and the body attached at the end of spring is lifted upwards. The similar observations are made if the direction of current is reversed.

32. Two long parallel wires are hanging freely. If they are connected to a battery (*i*) in series, (*ii*) in parallel, what would be the effect on their positions ?

Ans. (*i*) When a battery is connected inseries to two long parallel wires, the currents in the two wires will be in opposite directions. Due to which a force of repulsion will be acting between them and they are moving further apart. (*ii*) When a battery is connected in parallel to two long parallel wires, the currents in the two wires will be in same direction. Due to it, a force of attraction will be acting between them and they are coming closer to each other.

33. A straight wire, of length L, carrying a current I, stays suspended horizontally in mid air in a region where there is a uniform magnetic field $\vec{B}$. The linear mass density of the wire is λ. Obtain the magnitude of this magnetic field. [CBSE 2004 (C)]

Ans. The wire remains suspended if weight of wire = force exerted by magnetic field, *i.e.*,

$mg = ILB \sin 90°$ or $\lambda Lg = ILB$ or $B = \lambda g/I$

The direction of $\vec{B}$ must be horizontal and perpendicular to the length of wire.

34. A stream of protons is moving parallel to a stream of electrons. Will the two streams tend to come closer or move apart ?

Ans. The behaviour of the two streams of protons and electrons coming closer or moving apart depends on their speed. If they have large speeds, they will move apart because the magnetic interaction is more repulsive than electrostatic attraction. If they have small speeds, they will come closer to each other as electrostatic attraction between them is stronger than magnetic repulsion between them.

35. A wire of length l metre carries a current I ampere along the Y-axis. A magnetic field, $\vec{B} = B_0 (\hat{i} + \hat{j} + \hat{k})$ tesla exists in space. Find the magnitude of the force on the wire.

Ans. As the wire carries current I along the y-axis, so $\vec{l} = l\hat{j}$. Magnetic force on wire is

$$\vec{F} = I(\vec{l} \times \vec{B}) = I[l\hat{j} \times [B_0(\hat{i} + \hat{j} + \hat{k})]]$$

$$= IlB_0[\hat{j} \times \hat{i} + \hat{j} \times \hat{j} + \hat{j} \times \hat{k}]$$

$$= IlB_0[-\hat{k} + 0 + \hat{i}] = IlB_0[\hat{i} - \hat{k}]$$

Magnitude of the magnetic force is

$$F = IlB_0[(1)^2 + (-1)^2]^{1/2} = I\,l\,B_0\sqrt{2}\ N$$

36. Two long straight parallel conductors carrying steady currents I_a and I_b along the same direction are separated by a distance d. How does one explain the force of attraction between them ? If a third conductor carrying a current I_c in the opposite direction is placed just in the middle of these conductors, find the resultant force acting on the third conductor. [CBSE 2018 (C)]

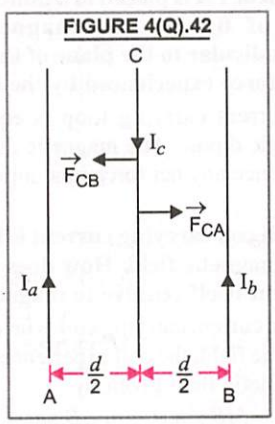
Ans. Two straight parallel conductors carrying currents (*i*) in the same direction attract each other and (*ii*) in the opposite directions repel each other. Refer to Fig. 4(Q).42, force on conductor C due to conductor A is

$$|\vec{F}_{CA}| = \frac{\mu_0}{4\pi}\frac{2I_aI_c}{d/2} \quad \text{(acting towards right)}$$

Force on conductor C due to conductor B is

$$|\vec{F}_{CB}| = \frac{\mu_0}{4\pi}\frac{2I_bI_c}{d/2} \quad \text{(acting towards left)}$$

FIGURE 4(Q).42



Net force on conductor C will be

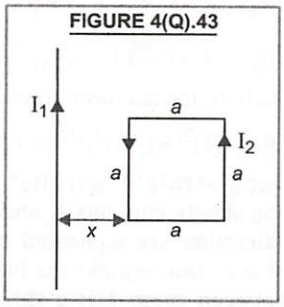
$$|\vec{F}_{net}| = \frac{\mu_0}{4\pi}\frac{4I_aI_c}{d} - \frac{\mu_0}{4\pi}\frac{4I_bI_c}{d}$$

$$= \frac{\mu_0 I_c}{\pi d}(I_a - I_b) \text{ (towards right)}$$

37. A square loop of side a carrying a current I_2 is kept at a distance x from an infinitely long straight wire carrying a current I_1 as shown in Fig. 4(Q).43. Obtain the expression for the resultant force acting on the loop.

(CBSE 2019)

FIGURE 4(Q).43



Ans. The force on the near side of square loop is repulsive and on far side is attractive. So Resultant force on square loop is

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{x} - \frac{\mu_0}{4\pi}\frac{2I_1I_2}{(x+a)}$$ acting in the plane of loop away from the long straight wire

$$= \frac{\mu_0}{2\pi} I_1 I_2 \left[\frac{1}{x} - \frac{1}{x+a}\right] = \frac{\mu_0 I_1 I_2}{2\pi}\left[\frac{a}{x(x+a)}\right]$$

VI. Torque on coil in magnetic field and galvanometer

38. A circular loop of radius 0·2 m carrying a current of 1 A is placed in a uniform magnetic field of 0·5 T. The magnetic field is perpendicular to the plane of the loop. What is the force experienced by the loop ?

Ans. The current carrying loop is equivalent to a magnetic dipole. The magnetic dipole does not experience any net force in a uniform magnetic field.

39. A small coil carrying current is held in a uniform magnetic field. How does the coil tend to orient itself relative to magnetic field ?

Ans. When a current carrying coil is held in a uniform magnetic field, the coil experiences a torque due to magnetic field given by

$\tau = MB \sin\alpha = ni\,AB \sin\alpha$.

The system will be in stable equilibrium if torque is zero ; which is so if $\alpha = 0°$. This is possible if magnetic field $\vec{B}$ is perpendicular to the plane of the coil.

40. Does the torque on planar current loop in a magnetic field change when its shape is changed without changing its geometrical area.

Ans. Torque on a planar current loop is given by $\tau = niAB \sin\alpha$, which is independent of shape if area A is unchanged. Hence torque on a planar current loop in a magnetic field does not change when its shape is changed without changing its area.

41. State the underlying principle of working of a moving coil galvanometer. Write two reasons why a galvanometer can not be used as such to measure current in a given circuit.

(CBSE 2010, 2011)

Ans. The principle of working of a moving coil galvanometer is based on the fact that if a current carrying coil is placed in a uniform radial field, it experiences a torque.

A galvanometer cannot be used as such to measure current in a circuit due to following two reasons :

(*i*) Galvanometer is very sensitive. It gives full scale deflection with a quite small current (nearly few micro ampere).

(*ii*) In order to measure the current, the galvanometer has to be connected in series of circuit. As its resistance is large, its presence in the circuit will decrease the effective current in the circuit

42. A rectangular coil of sides l and b carrying a current I is subjected to a uniform magnetic field $\vec{B}$ acting at an angle θ to its plane. Write the expression for the torque acting on it. In which orientation of the coil in the magnetic field, the torque is (*i*) minimum and (*ii*) maximum.

Ans. Torque on the coil, $\tau = IlB\,b\cos\theta = IlbB\cos\theta$.

(*i*) Torque is minimum if $\cos\theta$ is minimum, *i.e.*, $\cos\theta = 0$ or $\theta = 90°$

i.e., the plane of coil is set perpendicular to the direction of magnetic field.

(*ii*) Torque is maximum if $\cos\theta = 1$ or $\theta = 0°$. It means the plane of coil is parallel to the direction of magnetic field.

43. State properties of the material of the wire used for suspension of the coil in a moving coil galvanometer.

Ans. The properties of the material of the wire used for suspension of the coil in a moving coil galvanometer are as follows :

1. It should have low torsional constant, *i.e.*, restoring torque per unit twist should be small.
2. It should have high tensile strength.
3. It should be a non-magnetic substance.
4. It should have a low temperature coefficient of resistance.
5. It should be a good conductor of electricity.

44. What is a radial magnetic field ? How has it been achieved in moving coil galvanometer ?

(CBSE 2010)

Ans. Radial magnetic field is that field, in which the plane of the coil in all positions is parallel to the direction of the magnetic field. A radial magnetic field has been achieved by (*i*) properly cutting the magnetic pole pieces in the shape of concave faces. (*ii*) using a soft iron core within the coil.

45. Why is phosphor bronze alloy preferred for the suspension wire of a moving coil galvanometer ?

Ans. The suspension wire of phosphor bronze alloy is preferred in moving coil galvanometer because it has several advantages :

(*i*) Its restoring torque per unit twist is small. Due to it, the galvanometer is very sensitive.

(*ii*) It has great tensile strength so that even if it is thin, it will not break under the weight of the coil suspended from its end.

(*iii*) It is rust resisting. Hence it remains unaffected by the weather conditions of air in which it is suspended.

46. What is the main function of a soft iron core used in a moving coil galvanometer ?

Ans. (*i*) **This makes the magnetic field radial.** In such a magnetic field the plane of the coil is always parallel to the direction of magnetic field. Due to it, the galvanometer scale becomes linear.

(*ii*) **This increases the strength of magnetic field** due to the crowding of the magnetic lines of force through the soft iron core, which in turn increases the sensitiveness of the galvanometer.

47. The current sensitivity of a moving coil galvanometer is 10 div/mA and voltage sensitivity is 20 div/V. Find the resistance of the galvanometer.

Ans. Here, $I_s = 10 \text{ div/mA} = 10 \text{ div}/10^{-3} \text{ A}$

$= 10^4 \text{ div/A}$

$V_s = 20 \text{ div/V}$

As $V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{I_s}{R}$ or $R = \frac{I_s}{V_s} = \frac{10 \text{ div/A}}{20 \text{ div/V}}$

$= 500\ \Omega$

48. The coils, in certain galvanometers, have a fixed core made of a non-magnetic metallic material. Why does the oscillating coil come to rest so quickly in such a case ?

[CBSE 2008 (C)]

Ans. In a galvanometer the coil having a fixed core made of non-magnetic metallic material, when oscillates, the eddy currents are set up in the metallic material which opposes the motion of the coil in the magnetic field. Due to it, the coil comes to rest at once.

49. Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Justify this statement.

(CBSE 2011)

Ans. Current sensitivity of a galvanometer is given by $I_s = \frac{nBA}{k}$ where n is the no. of turns in the coil, A is the area of each turn of the coil, B is the strength of uniform magnetic field and k is the restoring torque per unit twist of suspension fibre in galvanometer.

Voltage sensitivity $V_s = \frac{I_s}{R} = \frac{nBA}{kR}$

I_s can be increased by increasing n and A. But increase of n increases the resistance R of galvanometer. Due to it, V_s will decrease.

A can not be increased beyond a limit because in that case the entire coil will not be in a uniform magnetic field. Moreover it will make the galvanometer bulky and unmanageable.

VII. Conversion of galvanometer into ammeter and voltmeter

50. What information would you wish to have about the galvanometer before you convert the galvanometer into an ammeter or voltmeter ?

Ans. Before converting the galvanometer into an ammeter or voltmeter we require the following two informations.

(*i*) The resistance of galvanometer and (*ii*) the current for full scale deflection of galvanometer.

51. Why do we not use galvanometer as an ammeter ?

Ans. A galvanometer shows a full scale deflection with a very small current. Hence a galvanometer can measure limited current. Therefore as such, a galvanometer can not be used as an ammeter, which can measure the given large current.

52. Of the two identical galvanometers one is to be converted into an ammeter and another into a milliammeter. Which of the shunts will be of larger resistance ?

Ans. The shunt of milliammeter should have larger resistance. It is so because the current measuring range of milliammeter is less than that of ammeter. Therefore the larger portion of the main current must pass through galvanometer coil and less portion of current through shunt for a converted galvanometer into ammeter/ milliammeter. It is only possible if the resistance of shunt used is of larger value in milliammeter than in ammeter.

53. Why should an ammeter have a low resistance and a high current carrying capacity ?

Ans. An ammeter is used to measure the current. It can measure the current of the circuit if connected in series of the circuit. The ammeter connected in series of the circuit can measure the current and will not disturb the current of the circuit if its resistance is low and current carrying capacity is high.

54. Why should a voltmeter have a high resistance and a low current carrying capacity ?

Ans. Voltmeter is a high resistance galvanometer. It is used to measure potential difference between two points of the circuit. To measure a potential difference between two points of a circuit, the voltmeter is connected in parallel to the circuit across those two points. The potential difference between those two points will not be affected if there is no change in the current flowing through the circuit between those two points, which will be so if practically no current flows through voltmeter. The same is possible if voltmeter has a high resistance and a low current carrying capacity. That is why a voltmeter should have high resistance and low current capacity.

55. What is an ammeter ? How is it used in an electric circuit ? How does it differ from a voltmeter ?

Ans. Ammeter is a low resistance galvanometer. The resistance of ammeter is low and that of voltmeter is high. Ammeter is connected in series and voltmeter in parallel in the circuit.

56. Which one of the two, an ammeter or a milliammeter, has a higher resistance and why ?

Ans. An ammeter measures strong current and milliammeter measures small current. We know that a galvanometer can be converted into an ammeter or milliammeter using a shunt resistance S, given by

$$S = \frac{I_g \times G}{I - I_g}$$

It shows that the shunt resistance to be used to convert a galvanometer into a milliammeter is more than that needed to convert into an ammeter. As the shunt is connected in parallel with the galvanometer, so the effective resistance of converted galvanometer into ammeter or milliammeter is, $R = \frac{GS}{G+S}$. It shows that milliammeter will have a higher resistance than that of ammeter.

57. Why is it that while using a moving coil galvanometer as a voltmeter, a high resistance in series is required whereas in an ammeter a shunt is used ? [CBSE 2010, 2012, 2013 (C)]

Ans. A galvanometer can be converted into a voltmeter of given range by connecting a high resistance of suitable value in series with it, so that most of the voltage applied drops across it, enabling the galvanometer to measure much larger required voltage.

A galvanometer can be converted into an ammeter of given range by connecting a low resistance shunt in parallel with it, so that most of the current bypasses through the shunt resistance, enabling the galvanometer to measure much larger given current.

58. Compare a voltmeter and an ammeter.

Ans. (*i*) Voltmeter is a high resistance device which is used to measure potential difference.

Ammeter is a low resistance device which is used to measure the current in electric circuit.

(*ii*) Voltmeter is obtained by using a high resistance of suitable value in series of the galvanometer.

Ammeter is obtained by using a low resistance shunt in parallel with a galvanometer.

(*iii*) Voltmeter is always connected in parallel to the conductor in circuit across which potential difference is to be determined.

Ammeter is always used in series of circuit to measure the current.

(*iv*) The range of voltmeter can be increased or decreased.

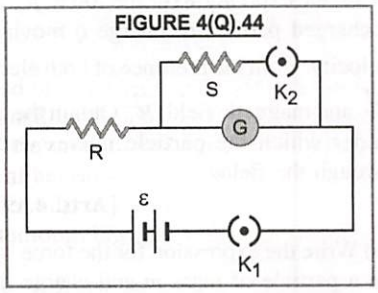
The range of ammeter can be increased but cannot be decreased.

(*v*) Ideal voltmeter has infinite resistance.

Ideal ammeter has zero resistance.

59. The current flowing in the galvanometer G when key K_2 is kept open is I. On closing the key K_2, the current in the galvanometer becomes I/n, where n is an integer. Fig.4(Q).44.

FIGURE 4(Q).44



Obtain an expression for resistance G of the galvanometer in terms of R, S and n. To what form does this expression reduce when the value of R is very large as compared to S ?

(CBSE Sample Paper 2015)

Ans. When key K_2 is open, current in the galvanometer is $I = \dfrac{\varepsilon}{R+G}$

When key K_2 is closed, the equivalent resistance of the parallel combination of G and S is

$$R' = \frac{GS}{G+S}$$

Total current in the circuit,

$$I_1 = \frac{\varepsilon}{R + GS/(G+S)}$$

This current will divide between S and G in the inverse ratio of S and G.

Hence current through the galvanometer will be

$$I_G = \frac{S}{S+G} I_1 = \frac{S}{(S+G)} \times \frac{\varepsilon\,(G+S)}{R(G+S)+GS}$$

$$= \frac{S\varepsilon}{RG+RS+GS}$$

As per question, $I_G = \dfrac{I}{n} = \dfrac{1}{n}\dfrac{\varepsilon}{(R+G)}$

$$\therefore \quad \frac{\varepsilon}{n(R+G)} = \frac{S\varepsilon}{RG+RS+GS}$$

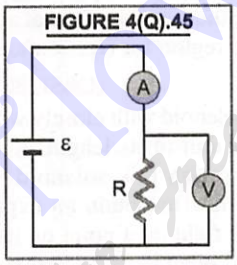
On solving, $G = \dfrac{(n-1)RS}{R-(n-1)S}$

When $R >> S$, $G = \dfrac{(n-1)RS}{R} = (n-1)S$

60. A voltmeter of resistance R_V and an ammeter of resistance R_A are connected in a circuit to measure a resistance R as shown in Fig. 4(Q).45. The ratio of the meter readings gives an apparent resistance R'. Show that R and R' are related by the relation

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_V}$$ **(CBSE Sample Paper 2003)**

FIGURE 4(Q).45



Ans. Apparent resistance,

$$R' = \frac{\text{voltmeter reading}}{\text{ammeter reading}} = \frac{V}{I}$$

Total resistance of circuit, $R_{eq} = R_A + \dfrac{R R_V}{R+R_V}$

According to Ohm's law, $\dfrac{\varepsilon}{I} = R_A + \dfrac{R R_V}{R+R_V}$

Also, $\varepsilon = IR_A + V$ or $\dfrac{\varepsilon}{I} = R_A + \dfrac{V}{I} = R_A + R'$

or $R_A + \dfrac{R R_V}{R+R_V} = R_A + R'$ or $R' = \dfrac{R R_V}{R+R_V}$

or $\dfrac{1}{R'} = \dfrac{R+R_V}{R R_V} = \dfrac{1}{R_V} + \dfrac{1}{R}$ or $\dfrac{1}{R} = \dfrac{1}{R'} - \dfrac{1}{R_V}$

SHORT ANSWER QUESTIONS — Carrying 3 marks

1. Describe Oersted's experiment and explain the law by which direction of magnetic field produced can be accounted. **[Art. 4.2]**
2. What is the force acting on a moving charge in a uniform magnetic field ? Discuss the cases when the force is maximum and minimum and define the unit of magnetic field $\vec{B}$. **[Art. 4.3]**
3. What is magnetic field ? State and explain Biot-Savart law. **(Karnataka Board 2012, Jharkhand Board 2011) [Art. 4.3 and 4.4]**
4. Derive an expression for magnetic field strength B at a point P due to current flowing through a straight conductor using Biot Savart's law. **(Raj. Board 2011, Jharkhand Board 2011) [Art. 4.6]**

5. State Biot-Savart's law. Deduce the expression for the magnetic field at a point on the axis of a current carrying circular loop of radius R, distant x from the centre. Hence write the magnetic field at the centre of a loop. Draw the magnetic field lines due to circular wire carrying current.
[CBSE 2015, 2016, 2018 (C)]
[Refer to Art. 4.4 and 4.8]

6. State Ampere's circuital law, expressing it in integral form. (CBSE 2014) [Art. 4.10]

7. A long straight wire of a circular cross-section of radius 'a' carries a steady current I. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point at distance 'r' in the region for (*i*) $r < a$ and (*ii*) $r > a$.
(CBSE 2010) [Art. 4.13]

8. A long solenoid with closely wound turns has n turns per unit of its length. A steady current I flows through this solenoid. Use Ampere's circuital law to obtain an expression, for the magnetic field, at a point on its axis and close to its mid point.
(West Bengal Board 2012, CBSE 2019, 2011)
[Art. 4.14]

9. State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius r having n-turns per unit length and carrying a steady current I. (CBSE 2015, 2018)
[Arts. 4.10 and 4.15]

10. A long solenoid with air core has n turns per unit length and carries a current I. Using Ampere's circuital law, derive an expression for the magnetic field B at an interior point on its axis. Write an expression for magnetic field H in the interior of the solenoid.
(Sample Paper CBSE 2019) [Art. 4.14]

Ans. Magnetic field B at an interior point on the axis of solenoid is

$$B = \mu_0 n I$$

But $B = \mu_0 H$

So $\mu_0 H = \mu_0 n I$ or $H = n I$

11. Discuss the motion of a charged particle when subjected to a perpendicular uniform electric field. [Art. 4.17]

12. Discuss the motion of a charged particle when subjected to a uniform magnetic field, when the direction of motion of the particle makes an angle ϕ with the direction of magnetic field.
(Tamil Nadu Board 2012) [Art. 4.18]

13. Write the expression for the force $\vec{F}$, acting on a charged particle of charge q moving with a velocity $\vec{v}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. Obtain the condition under which the particle moves undeflected through the fields. (CBSE 2012)
[Arts. 4.19 & 4.20]

14. (*a*) Write the expression for the force $\vec{F}$ acting on a particle of mass m and charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. Under what conditions will it move in (*a*) a circular path and (*ii*) a hetical path ?
(CBSE 2017) [Art 4.19]

15. Write and explain the expression of Lorextz force. What is the magnetic dipole moment of a current loop ? In what direction does it act.
(West Bengal Board 2012)
[Arts. 4.19, 4.24]

16. Explain the principle and working of a cyclotron with the help of a schematic diagram. Write the expression for cyclotron frequency.
(Kerala Board 2012, CBSE 2019, 2017, 2009)
[Art. 4.21]

17. Find the force on a current carrying conductor placed in the uniform magnetic field. Name the instrument in which this is used as a working principle. [Art. 4.22]

18. Find the force between two parallel conductors carrying currents (*i*) in the same direction (*ii*) in opposite directions and hence define one ampere.
(Karnataka Board 2012, CBSE 2011, 2009, Pb. Board 2011) [Art. 4.23]

19. Two long straight parallel conductors carrying currents I_1 and I_2 are separated by a distance d. If the currents are flowing in the same direction, show how the magnetic field produced by one exerts an attractive force on the other. Obtain the expression for the force and hence define 1 ammpere. (CBSE Sample Paper 2022-23)
[Art. 4.23]

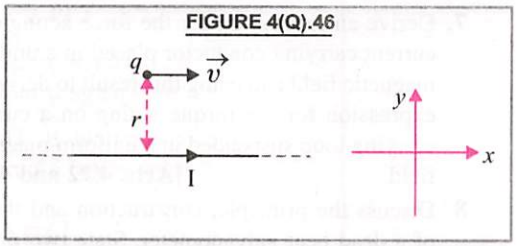
20. Derive an expression for the maximum force experienced by a straight conductor of length l, carrying a current I and kept in a uniform magnetic field B. (HP Board 2011) [Art. 4.22]

21. Derive an expression for the force between two long parallel current carrying conductors. Use this expression to define SI unit of current.
(CBSE 2012, 2010) [Art. 4.23]

22. A particle of mass m and charge q is moving at speed v parallel to a long straight conductor carrying current I as shown in Fig. 4(Q).46. Find magnitude and direction of electric field required so that the particle goes undeflected.

(Sample Paper CBSE 2019)

FIGURE 4(Q).46



Ans. The magnetic field at a point with perpendicular distance r from linear conductor carrying current is $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r}$.

It is acting perpendicular to the plane of paper upwards.

The particle will go undeflected if the force on particle due to magnetic field is equal and opposite to the force on the particle due to electric field.

i.e., $Bqv = Eq$ or $E = Bv$ or $E = \frac{\mu_0}{2}\frac{I}{r}v$

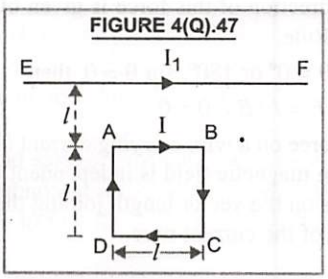
It will be so if $\vec{E}$ is acting *perpendicular to the plane of paper downwards.*

23. (*i*) Write the expression for the magnetic moment $(\vec{m})$ due to a planar square loop of side 'l' carrying a steady current I in a vector form.

(*ii*) In the Fig. 4(Q).47, this loop is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance l as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop.

(CBSE 2010)

FIGURE 4(Q).47



Sol. (*i*) The magnetic moment $(\vec{m})$ due to a planar square loop of side l carrying current I is

$$\vec{m} = I\vec{A}, \text{ where } A = l^2$$

(*ii*) The currents in AB and EF are in the same direction. So AB will be attracted towards EF with a force F_1, given by

$$F_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{l} \times (\text{length } AB) = \frac{\mu_0}{4\pi}\frac{2I_1}{l}l = \frac{\mu_0}{4\pi}2I_1.$$

The currents in CD and EF are in opposite directions. So CD will experience repulsion away from EF, with a force F_2, given by

$$F_2 = \frac{\mu_0}{4\pi}\frac{2I_1}{2l}(\text{length } CD) = \frac{\mu_0}{4\pi}\frac{2I_1}{2l}l = \frac{\mu_0}{4\pi}I_1$$

The forces on the portions BC and DA will cancel out each other's effect. Therefore, net force on loop is

$$= F_1 - F_2 = \frac{\mu_0}{4\pi}2I_1 - \frac{\mu_0}{4\pi}I_1 = \frac{\mu_0}{4\pi}I_1$$

towards EF

As the area vector is parallel to the magnetic field, $\theta = 0°$, so torque on the square loop, $\tau = IAB\sin\theta = IAB\sin 0° = 0$. Also $F_1 \neq F_2$, so no torque acts on the loop.

24. Find the torque acting on the rectangular current loop suspended in a uniform magnetic field, when it is inclined at an angle θ with the direction of magnetic field.

(CBSE 2019, 2013, Manipur Board 2011)

[Art. 4.26]

25. Define current sensitivity and voltage sensitivity of a moving coil galvanometer. How can a galvanometer be made more sensitive ?

[Art. 4.27]

26. What is shunt ? State its uses. Find the relation for the current through shunt in terms of resistance of galvanometer and shunt.

[Art. 4.29]

27. What is an ammeter ? How can a galvanometer be converted into an ammeter ? Explain.

(Pb. Board 2011, HP Board 2011) **[Art. 4.30]**

28. What is a voltmeter ? How can a galvanometer be converted into a voltmeter ? Explain.

(HP Board 2013) **[Art. 4.31]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Find the magnetic field induction (in magnitude and direction) at a point due to current flowing in a long straight conductor. Show the sketch of magnetic field produced.
(Chhatisgarh Board 2011) [Art. 4.6]
2. State the Biot-Savart's law for the magnetic field due to a current carrying element. Use this law to obtain a formula for magnetic field at the centre of a circular loop of radius R carrying a steady current I, sketch the magnetic field lines for a current loop clearly indicating the direction of the field.
(Bihar Board 2012, Uttarakhand Board 2012, Raj. Board 2011, CBSE 2007) [Arts. 4.4, 7]
3. Find the magnetic field induction at a point on the axis of a circular coil carrying current and hence find the magnetic field at the centre of circular coil carrying current. [Art. 4.8]
4. Derive an expression for the magnetic field along the axis of an air-cored solenoid, using Ampere's circuital law. Sketch the magnetic field lines for a finite solenoid. Explain why the field at the exterior mid point is weak while at the interior it is uniform and strong.
(HP Board 2011) [Art. 4.14]
5. What is a cyclotron ? Discuss its construction, working and theory. Explain cyclotron frequency.
(Bihar Board 2012, Jharkhand Board 2012, Chhattisgarh Board 2011) [Art. 4.21]
6. With the help of labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it works to accelerate the charged particles to high energies. (CBSE 2014) Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle ? Give reason. (CBSE 2011, 2009) [Art. 4.21]
7. Derive an expression for the force acting on a current carrying conductor placed in a uniform magnetic field and using this result to derive an expression for the torque acting on a current carrying loop suspended in a uniform magnetic field. [Arts. 4.22 and 4.26]
8. Discuss the principle, construction and theory of a dead beat galvanometer. State two merits of moving coil galvanometer. [Art. 4.27]
9. Draw a labelled diagram of a moving coil galvanometer. State the principle on which it works. Deduce an expression for the torque acting on a rectangular current carrying loop kept in a uniform magnetic field. Write two factors on which the current sensitivity of a moving coil galvanometer depend. (Pb. Board 2011 ; CBSE 2011, 2007) [Arts 4.27, 4.26]
10. Describe the working principle of a moving coil galvanometer. Why is it necessary to use (*i*) a radial magnetic field and (*ii*) a cylindrical soft iron core in a galvanometer ? Write the expression for current sensitivity of the galvanometer.
(CBSE Delhi 2017) [Arts 4.27]
11. Explain sensitivity of moving coil galvanometer. Show with the help of a circuit diagram that how a moving coil galvanometer can be converted into an ammeter of given range. Write necessary mathematical relation. What is the resistance of ideal ammeter ?
(Uttarakhand Board 2012) [Arts. 4.27, 4.30]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. When a linear wire of vector length $\vec{l}$, carrying current I is subjected to a uniform magnetic field $\vec{B}$, the force experienced by wire is

$$|\vec{F}| = I\,|\vec{l} \times \vec{B}| = I\,l\,B \sin\theta$$

where θ is the angle between $\vec{l}$ and $\vec{B}$.

For, $\theta = 90°$, $F = I\,l\,B \sin 90° = I\,l\,B$ (Maximum)

The direction of this force is given by Fleming's Left Hand Rule.

For, $\theta = 0°$ or $180°$, $\sin\theta = 0$, then

$F = I\,l\,B \times 0 = 0$ (Minimum)

The force on a wire carrying current of any shape in a uniform magnetic field is independent of its shape but depends on the vector length joining the beginning and ending of the current wire.

Read the above passage carefully and answer the following very short and short questions.

1. A horizontal wire placed perpendicular to a uniform magnetic field carries a current from left to right. The magnetic field is horizontal, directed towards you. What is the direction of magnetic force on the wire ?
2. When a semicircular coil of radius r, carrying current I is placed in a uniform magnetic field $\vec{B}$, find the force experienced by the coil.
3. When a closed loop of any shape, carrying current I is placed in a uniform magnetic field $\vec{B}$, find the force experienced by the coil.
4. A horizontal straight conducting rod of length l and mass m is suspended by two vertical wires at its ends. A current I is set up in the rod through the wires. How much magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ?

CASE 2. When a wire loop of n turns, each of area $\vec{A}$, carrying current I is suspended in a uniform magnetic field $\vec{B}$, then the coil experiences a torque given by

$$\vec{\tau} = nI(\vec{A} \times \vec{B})$$

or $$\tau = nI\,|\vec{A} \times \vec{B}| = nIAB\sin\theta$$

where θ is the angle between $\vec{A}$ and $\vec{B}$. This torque tends to rotate the coil about its axis of suspension till the angle θ between $\vec{A}$ and $\vec{B}$ becomes zero, *i.e.,* the coil is set with its plane perpendicular to the direction of magnetic field.

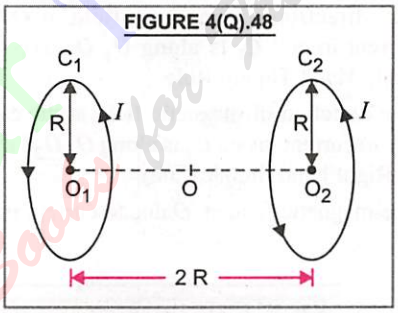
If the coil is set with its plane parallel to the direction of magnetic field B, then $\theta = 90°$, so the torque, $\tau = nIAB\sin 90° = nIAB$ (Maximum). In this situation, the magnetic field is called radial magnetic field.

Read the above passage carefully and answer the following very short and short questions.

5. What is the radial magnetic field ?
6. What are the net magnetic force and torque on a current loop in a non-uniform magnetic field ?
7. What are the conditions under which the current loop is in the equilibrium position in a uniform magnetic field ?
8. A circular coil of 20 turns and radius 10 cm carries a current 5 A. If a magnetic field of 0·1 T is applied normal to the plane of the coil, find the torque acting on the coil.

CASE 3. There are two parallel coaxial circular coils C_1 and C_2 of equal radius r and same number of turns N, carrying equal current I in the same direction separated by a distance 2 R as shown in Fig. 4(Q).48. The direction of magnetic field at a point due to circular coil can be given by Right hand thumb rule.

FIGURE 4(Q).48



Read the above passage carefully and answer the following very short and short questions.

9. What is the direction of magnetic field at the centre O_1 due to current in coil C_1 ?
10. What is the direction of the magnetic field at the centre O_1 due to current in the coil C_2 ?
11. What is the magnetic field at O due to current in coil C_1 ? $\left(\text{Use } \frac{\mu_0 NI}{R} = B_0\right)$
12. What is the total magnetic field at O due to current in coils C_1 and C_2 ?

ANSWERS

1. According to Fleming's Left Hand Rule, it is vertically downwards.
2. When a semicircular coil of radius r, carrying current I is placed in a magnetic field, then $|\vec{l}| = 2r$. Now, the total force on the semicircular coil is

$$F = I\,|\vec{l} \times \vec{B}| = I\,l\,B\sin\theta = \mathbf{I\,(2\,r)\,B\sin\theta}$$

3. For a closed loop $|\vec{l}| = 0$

   $\therefore$ Force, $F = I\,l\,B\sin\theta = I \times 0 \times B\sin\theta = \mathbf{0}$
4. Tension in the supporting wires will be zero if the weight of the rod acting vertically downwards

is balanced by the upward force on rod due to magnetic field, *i.e.*, $mg = I\,l\,B \sin 90° = I\,l\,B$

or $$B = \frac{mg}{I\,l}$$

5. It is that magnetic field in which a coil carrying current when subjected to a magnetic field experiences maximum torque. In this magnetic field, the plane of the coil is always parallel to the direction of the magnetic field.
6. The net magnetic force on a current loop in a non-uniform magnetic field is not zero but Torque may and may not be zero.
7. In equilibrium position, the net magnetic force as well as Torque on the current loop are zero.
8. Here, $n = 20$, $r = 10$ cm $= 0{\cdot}10$ m ; $I = 5$ A, $B = 0{\cdot}1$ T, $\theta = 0°$

   Torque, $\tau = n\,I\,AB \sin\theta = nI\,(\pi r^2)\,B \sin 0° = \mathbf{0}$
9. The direction of magnetic field at O_1 due to current in coil C_1 is **along $O_1\,O_2$,** according to Right Hand Thumb Rule.
10. The direction of magnetic field at the centre O_1 due to current in coil C_2 is **along $O_1\,O_2$** (according to Right hand thumb Rule).
11. The magnetic field at O due to current in coil C_1 is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi N I R^2}{(R^2+R^2)^{3/2}} \text{ acting along } O_1\,O_2$$

$$= \frac{\mu_0}{2}\frac{N I R^2}{2\sqrt{2}R^3} = \frac{\mu_0 N I}{4\sqrt{2}R} = \frac{B_0}{4\sqrt{2}} \textbf{ along } O_1O_2$$

12. Magnetic field at O due to current in coil C_1 is

$$B_1 = \frac{\mu_0}{4\sqrt{2}}\frac{NI}{R} = \frac{B_0}{4\sqrt{2}} \text{ along } O_1\,O_2$$

Magnetic field at O due to current in coil C_2 is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi N I R^2}{(R^2+R^2)^{3/2}} \text{ acting along } O_1\,O_2$$

$$= \frac{\mu_0}{2}\frac{N I R^2}{2\sqrt{2}\times R^3} = \frac{\mu_0}{4\sqrt{2}}\frac{NI}{R}$$

$$= \frac{B_0}{4\sqrt{2}} \text{ along } O_1\,O_2$$

Resultant magnetic field at O is

$$= B_1 + B_2 = \frac{B_0}{4\sqrt{2}} + \frac{B_0}{4\sqrt{2}} = \frac{B_0}{2\sqrt{2}} \textbf{ along } O_1\,O_2$$

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. A charged particle of charge q moving with velocity $\vec{v}$ in an electric field $\vec{E}$ experiences a force $q\,\vec{E}$. It is acting in the direction of $\vec{E}$. A charged particle while moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$, experiences a force $\vec{F} = q\,(\vec{v} \times \vec{B})$. This force is acting perpendicular to the plane containing $\vec{v}$ and $\vec{B}$ and is directed as given by Right Hand thumb Rule.

Based on the above paragraph, answer questions no. 1 to 4 :

1. The uniform electric and magnetic fields are produced, pointing in the East direction. An electron is projected with its velocity pointing east direction. The electron velocity will
   (*a*) increase (*b*) decrease
   (*c*) remain unchanged
   (*d*) can not be predicted.
2. When a charged particle is moving perpendicular to the direction of magnetic field, its energy
   (*a*) decreases (*b*) increases
   (*c*) remains unchanged
   (*d*) some time will decrease and sometimes will increase.

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false and R is also false.

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3. **Assertion.** If a charged particle of charge q while moving with velocity $v\ \hat{i}$ is subjected to a magnetic field $B\ \hat{j}$, then the direction of force on the particle is along + z direction.

Reason. The direction of magnetic force on the charged particle is given by Right Hand thumb rule.

4. **Assertion.** A charged particle moving perpendicular to a uniform electric field will go undeviated.

Reason. There will be no effect of electric field on the charged particle moving perpendicular to the electric field.

CASE 2. Galvanometer is a device to study the current in the circuit. A galvanometer can be converted into an ammeter of a given range by using a suitable low resistance in parallel of galvanometer. A galvanometer can be converted into a voltmeter of given range by using a suitable high resistance in series of it. The resistance of voltmeter is maximum, of ammeter is least and of galvanometer is in between voltmeter and ammeter.

Based on the above paragraph, answer questions no. 5 to 8 :

5. The resistance of a given milliammeter is

(*a*) less than that of ammeter

(*b*) more than that of micro-ammeter

(*c*) more than that of ammeter

(*d*) less than that of microammeter

6. A galvanometer of resistance 30 Ω shows full scale deflection by current 600 μA is to be converted into an ammeter of range 0 to 5 A. The shunt resistance used in ohm is

(*a*) 3·0 milli ohm (*b*) 3·6 milli ohm

(*c*) 3·2 milli ohm (*d*) 2·6 milli ohm.

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false and R is also false.

7. **Assertion.** Ammeter is used in series and voltmeter in parallel of the circuit.

Reason. Ammeter is used to measure the current and voltmeter is used to measure the potential difference.

8. **Assertion.** We can decrease the range of ammeter by using a resistance in series of ammter.

Reason. The effective resistance in series combination of resistors will become less than the individual resistance.

ANSWERS

1. (*b*) **2.** (*c*) **3.** (*a*) **4.** (*d*) **5.** (*c*, *d*) **6.** (*b*) **7.** (*b*) **8.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

1. The force on electron due to electric field $\vec{E}$ is $\vec{F}_e = -e\vec{E}$, which is acting opposite to the direction of $\vec{E}$.

The force on electron due to magnetic field is

$$\vec{F}_m = -e(\vec{v} \times \vec{B}) = -e\,|\,v\,B \sin 0^\circ\,|\,\hat{n} = 0$$

Thus total force on electron,

$$\vec{F} = \vec{F}_e + \vec{F}_m = -e\vec{E} + 0 = -e\vec{E}.$$

Therefore, velocity of the electron will **decrease**.

2. A charged particle, moving perpendicular to a magnetic field experiences a force, $F = q\,v\,B \sin 90^\circ = q\,v\,B$, which acts perpendicular to $\vec{v}$ as well as $\vec{B}$. This force will provide the centripetal force, resulting the particle to move on circular path. As $\vec{F}$ and displacement $(\vec{s})$ on circular path are perpendicular to each other, so magnitude of velocity of the particle will not change, only direction of motion changes. Hence workdone on the particle, $W = \vec{F} \cdot \vec{s} = F\,s \cos 90^\circ = 0$.

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Therefore, the speed of the charged particle remains unchanged and hence **energy of the charged particle in magnetic field remains unchanged.**

3. **Both Assertion and Reason are true and the Reason is the correct explanation of the Assertion.**

4. **Both Assertion and Reason are false.** As charged particle in the electric field will experience a force, $\vec{F} = q\vec{E}$, in the direction of $\vec{E}$ and will describe a parabolic path in the electric field.

5. The resistance of ammeter is least, more of milliammeter and maximum of microammeter.

6. $S = \dfrac{I_g G}{I - I_g} = \dfrac{(600 \times 10^{-6}) \times 30}{5 - (600 \times 10^{-6})}$

$= \dfrac{600 \times 10^{-6} \times 30}{5} = 3{\cdot}6 \times 10^{-3}\ \Omega$

= 3·6 milli ohm

7. **Both Assertion and Reason are true but the Reason is not the correct explanation of Assertion.**

8. We can not decrease the range of ammeter and the effective resistance in series combination is greater than the individual resistance. Thus **both Assertion and Reason are false.**

PROBLEMS FOR PRACTICE

TYPE I. BIOT-SAVART'S LAW AND MAGNETIC FIELD DUE TO CURRENT THROUGH THE STRAIGHT CONDUCTOR

1. A current of 10 A is flowing east to west in a long wire kept in the east-west direction. Find magnetic field in a horizontal plane at a distance of (*i*) 10 cm. north (*ii*) 20 cm south from the wire, and in a vertical plane at a distance of (*iii*) 40 cm downwards, (*iv*) 50 cm upwards.

[Ans. (*i*) 2×10^{-5} T, vertically downwards
(*ii*) 10^{-5} T, vertically upwards
(*iii*) 5×10^{-6} T, horizontally pointing south
(*iv*) 4×10^{-6} T, horizontally pointing north.]

2. Calculate the magnetic field induction at the centre of the coil in the form of a square of side 10 cm, carrying a current of 10 A.
(Pb. Board 2001) **[Ans. $1{\cdot}13 \times 10^{-4}$ T]**

3. Two long parallel wires are placed at a distance of 12 cm from each other in air. Each wire has a current of 3 A. Calculate the magnetic field at a point mid way between them when the currents in them are : (*i*) in the same direction and (*ii*) in opposite direction.

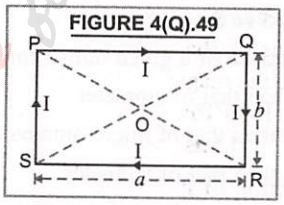
[Ans. (*i*) Zero (*ii*) 2×10^{-5} T, normally downwards]

4. A long straight wire carrying a current of 25 A is placed in an external uniform magnetic field $3{\cdot}0 \times 10^{-4}$ T parallel to the current. Find the magnitude of the resultant magnetic field at a point 1·5 cm away from the wire.

[Ans. $4{\cdot}48 \times 10^{-4}$ T]

5. A rectangular loop of conductor of length a, breadth b carrying current I as shown in Fig. 4(Q).49. Find the magnetic field induction at the centre O of the loop.

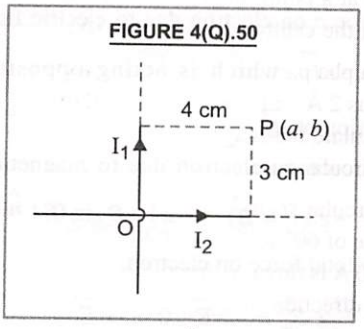
FIGURE 4(Q).49



$\left[\text{Ans. } \dfrac{2\mu_0 I\sqrt{a^2+b^2}}{\pi ab}\right]$

6. Two insulating infinite long conductors carrying currents I_1 (= 2A) and I_2 (= 4 A) lie mutually perpendicular to each other in the same plane, as shown in Fig. 4(Q).50. Find the magnetic field at the point P (a, b), where a = 4 cm ; b = 3 cm.

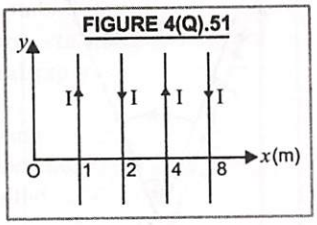
FIGURE 4(Q).50



[Ans. $1{\cdot}66 \times 10^{-5}$ T ; normally outwards]

7. Equal currents $I = 1A$ are flowing through the long wires parallel to y-axis located at, $x = +1$ m, $x = +2$ m, $x = +4$ m, $x = +8$ m and so on but in opposite directions as shown in Fig. 4(Q).51. What is the magnetic field induction at the origin O ?

FIGURE 4(Q).51



[Ans. $1{\cdot}33 \times 10^{-7}$ T $\hat{k}$]

TYPE II. MAGNETIC FIELD DUE TO CURRENT THROUGH CIRCULAR COIL

8. A straight wire of length $\pi/2$ metre, is bent into a circular shape. If the wire were to carry a current of 5 A, calculate the magnetic field, due to it, before bending, at a point distance 0·01 times the radius of the circle formed from it. Also calculate the magnetic field, at the centre of the circular loop formed, for the same value of current. **[Ans. 4×10^{-4} T ; $1{\cdot}256 \times 10^{-5}$ T]**

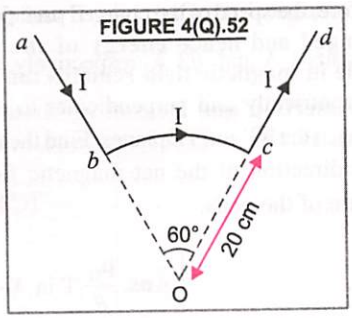
9. The electron of hydrogen atom moves along a circular path of radius $0{\cdot}5 \times 10^{-10}$ m (*i*) with a speed of $4{\cdot}0 \times 10^{6}$ ms^{-1}. (*ii*) with a frequency $6{\cdot}8 \times 10^{15}$ Hz. Calculate the magnetic field produced at the centre of the circular path. ($e = 1{\cdot}6 \times 10^{-19}$ C). **[Ans. 25·6 T, 13·4 T]**

10. The magnetic field due to a current-carrying circular loop of radius 10 cm at its centre is $0{\cdot}60 \times 10^{-4}$ T. Find the magnetic field due to this loop at a point on the axis at a distance of 4 cm from the centre. **[Ans. 4×10^{-5} T]**

11. An alpha particle moves along a circular path of radius 2 Å with a uniform speed of 2×10^{6} ms^{-1}. Calculate the magnetic field set up at the centre of circular path. **[Ans. 1·6 T]**

12. A circular segment of radius 20 cm subtends an angle of 60° at its centre. Fig. 4(Q).52. A current of 10 A is flowing through it. Find the magnitude and direction of the magnetic field produced at the centre.

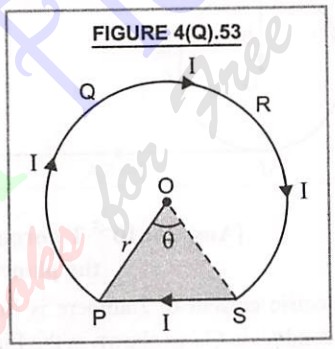
FIGURE 4(Q).52



[Ans. $5{\cdot}2 \times 10^{-6}$ T]

13. A current I (= 4A) flows along a thin wire $PQRS$ shaped as shown in Fig. 4(Q).53. The radius of the curved part of the wire is 10·0 cm. The angle $\theta = 90°$. Find the magnitude of the total magnetic field at the point O.

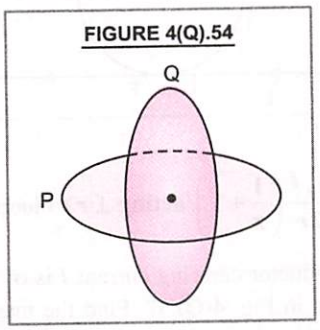
FIGURE 4(Q).53



[Ans. $2{\cdot}68 \times 10^{-5}$ T]

14. Two identical loops P and Q each of radius 5 cm are lying in perpendicular planes such that they have a common centre as shown in the Fig. 4(Q).54. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3 A and 4 A respectively. **(CBSE 2017)**

FIGURE 4(Q).54



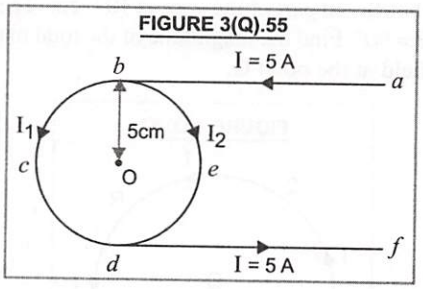
[Ans. $6{\cdot}28 \times 10^{-5}$ T ; Direction can not be determined as direction of current is not given]

15. Two identical circular coils, P and Q, carrying currents 1 A and $\sqrt{3}$ A respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils. **(CBSE 2017)**

[Ans. $\frac{\mu_0}{R}$ T in X-Z plane]

16. In the net work shown in Fig. 4(Q).55, find the magnetic field at the centre O of the coil.

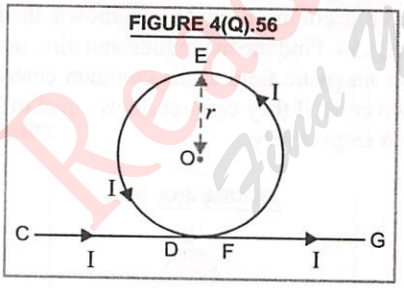
FIGURE 3(Q).55



[Ans. 2×10^{-5} T normally out of the plane of paper]

17. An electric current of I ampere is flowing in a long conductor CG as shown in the Fig. 4(Q).56. Find the magnitude and direction of magnetic induction at the centre O of circular part.

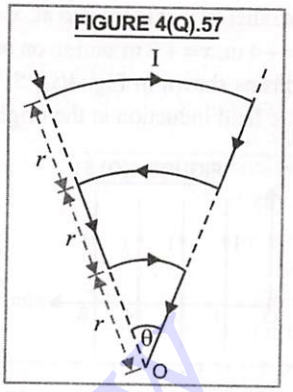
FIGURE 4(Q).56



[Ans. $\frac{\mu_0 I}{2r}\left(\frac{1}{\pi}+1\right)$ acting $\perp r$ to loop upwards]

18. A conductor carrying current I is of the type as shown in Fig. 4(Q).57. Find the magnetic field induction at the common centre O of all the three arcs.

FIGURE 4(Q).57



[Ans. $\frac{5\mu_0 I\theta}{24\pi r}$]

TYPE III. AMPERE'S CIRCUITAL LAW

19. A long straight solid metal wire of radius R carries a current i, uniformly distributed over its circular cross-section. Find the magnetic field at a distance r from axis of wire (i) inside and (ii) outside the wire.

[Ans. (i) $\frac{\mu_0 \mu_r ir}{2\pi R^2}$ (ii) $\frac{\mu_0}{4\pi}\frac{2i}{r}$]

20. A long straight solid conductor of radius 6 cm carries a current of 8 A, which uniformly distributed over its circular cross-section. Find the magnetic field (a) at a distance of 3 cm from the axis of the conductor (b) at a distance 10 cm from the axis of the conductor.

[Ans. (a) $1{\cdot}33 \times 10^{-5}$ T (b) $1{\cdot}6 \times 10^{-5}$ T]

21. If the current density in a linear conductor of radius 'a' varies with r according to relation $J = kr^2$, where k is a constant and r is the distance of a point from the axis of conductor. Find the magnetic field induction at a point distance r from the axis, when (i) $r < a$ and (ii) $r > a$.

[Ans. (i) $\frac{\mu_0 \mu_r kr^3}{4}$ (ii) $\frac{\mu_0 ka^4}{4r}$]

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TYPE IV. MAGNETIC FIELD DUE TO SOLENOID AND TOROID

22. A solenoid of length 0·20 m, having 120 turns carries a current of 2·5 A. Find the magnetic field : (*a*) in the interior of the solenoid, (*b*) at one end of the solenoid. Given $\mu_0 = 4\pi \times 10^{-7}$ S.I. units.

[Ans. (*a*) $1{\cdot}885 \times 10^{-3}$ T (*b*) $0{\cdot}9425 \times 10^{-3}$ T]

23. A long wire carries a current of 20 A along the directed axis of a long solenoid. The field due to a solenoid is 4 mT. Find the resultant field at a point 3 mm from the solenoid axis.

[Ans. 4·2 mT ; 18·4° with the directed axis]

24. A copper wire having a resistance of 0·02 Ω per metre is used to wind a 500 turns solenoid of radius 2·0 cm and length 30 cm. What should be the *emf* of the battery which when connected across the solenoid would produce a magnetic field of 10^{-2} T, near the centre of the solenoid.

[Ans. 6 V]

25. A coil wrapped around a toroid has inner radius of 10·0 cm and an outer radius of 20·0 cm. If the wire wrapped makes 600 turns and carries a current of 10·0 A, what are the maximum and minimum values of magnetic field within the toroid ? **[Ans. 12 mT, 6 mT]**

TYPE V. MOTION OF CHARGED PARTICLE IN (*i*) ELECTRIC FIELD AND (*ii*) MAGNETIC FIELD

26. An electron enters electric field of 10^4 V/m perpendicular to the field with a velocity of 10^7 m/s. Find the vertical displacement of electron after 2 milliseconds. Mass of electron = $9{\cdot}1 \times 10^{-31}$ kg ; charge on electron = $1{\cdot}6 \times 10^{-19}$ C.

[Ans. $3{\cdot}51 \times 10^9$ m]

27. A chamber is maintained at a uniform magnetic field of 5×10^{-3} T. An electron with a speed of 5×10^7 ms^{-1} enters the chamber in a direction normal to the field. Calculate (*i*) radius of the path (*ii*) frequency of revolution of the electron.

Charge of electron = $1{\cdot}6 \times 10^{-19}$ C,

Mass of electron = $9{\cdot}1 \times 10^{-31}$ kg

[Ans. (*i*) 5·7 cm (*ii*) $1{\cdot}4 \times 10^8$ Hz]

28. A proton projected in a magnetic field of 0·02 T travels along a helical path of radius 6 cm and pitch 24 cm. Find the components of velocity of the proton along and perpendicular to the magnetic field. Take the mass of the proton = $1{\cdot}6 \times 10^{-27}$ kg.

[Ans. $1{\cdot}2 \times 10^5$ ms^{-1}, $7{\cdot}6 \times 10^4$ ms^{-1}]

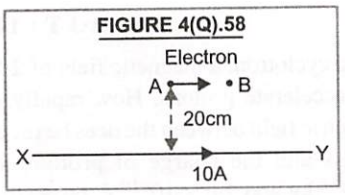
29. An electron after being accelerated through a potential difference of 100 V enters a uniform magnetic field of 0·004 T, perpendicular to its direction of motion. Calculate the radius of the path described by the electron.

[Ans. 8·4 mm]

30. A beam of protons enters a uniform magnetic field of 0·3 T with a velocity of 4×10^5 ms^{-1} at an angle of 60° to the field. Find the radius of the helical path taken by the beam. Also find the pitch of the helix (distance travelled by a proton parallel to the magnetic field during one period of rotation). Mass of proton = $1{\cdot}67 \times 10^{-27}$ kg. **[Ans. 1·2 cm ; 4·35 cm]**

31. An infinite long straight conductor *XY* is carrying a current of 10 A. An electron is moving with a speed of 10^5 ms^{-1} parallel to the conductor in air from point *A* to *B*, as shown in Fig. 4(Q).58. The perpendicular distance between the electron and the conductor *XY* is 20 cm. Calculate the magnitude of the force experienced by the electron. Write the direction of this force.

FIGURE 4(Q).58



[Ans. $1{\cdot}6 \times 10^{-19}$ N, in the plane of paper away from *XY*]

32. A proton, a deutron and an alpha particle, after being accelerated through the same potential difference enter a region of uniform magnetic field $\vec{B}$, in a direction perpendicular to $\vec{B}$. Find the ratio of their kinetic energies. If the radius of proton's circular path is 7 cm, what will be the radii of the paths of deutron and alpha particle ?

[Ans. 1 : 1 : 2, $r_d = 7\sqrt{2}$ cm, $r_\alpha = 7\sqrt{2}$ cm]

TYPE VI. (*i*) MOTION OF CHARGED PARTICLE, BOTH IN ELECTRIC AND MAGNETIC FIELDS (*ii*) CYCLOTRON

33. A beam of proton passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and perpendicular to the direction of the beam. If the magnitudes of the electric and magnetic fields are 100 kV/m, 50 mT respectively, calculate the velocity of the beam v. (CBSE 2008)

[Ans. 2×10^6 ms^{-1}]

34. If the maximum value of accelerating potential provided by a radio frequency oscillator be 10 kV, calculate the number of revolutions made by an α-particle in a cyclotron to achieve one-tenth of the speed of light. Mass of proton = $1{\cdot}67 \times 10^{-27}$ kg ; charge on proton = $1{\cdot}6 \times 10^{-19}$ C. **[Ans. 470 revolutions]**

35. A cyclotron has an oscillatory frequency of 10 M Hz and a dee radius of 60 cm. Calculate the magnetic field required to accelerate deutrons of mass $3{\cdot}3 \times 10^{-27}$ kg and charge $1{\cdot}6 \times 10^{-19}$ C. Find the energy of deutrons emerging from the cyclotron.

[Ans. 1·3 T ; 14·74 MeV]

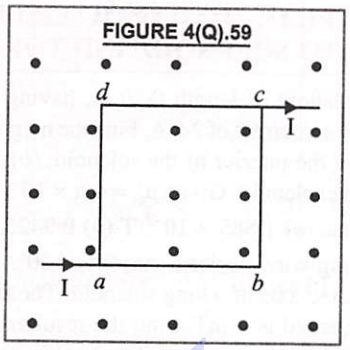
36. In a cyclotron, a magnetic field of 2·4 T is used to accelerate protons. How rapidly should the electric field between the dees be reversed ? The mass and the charge of protons are $1{\cdot}67 \times 10^{-27}$ kg and $1{\cdot}6 \times 10^{-19}$ C respectively.

[Ans. $1{\cdot}37 \times 10^{-8}$ s, $3{\cdot}64 \times 10^7$ Hz]

TYPE VII. FORCE ON CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

37. A current of 4A enters at the corner '*a*' of a square frame of side 10 cm and leaves at opposite corner '*c*'. A magnetic fild of B = 0·20 T acts in a direction perpendicular to the plane of paper directed outwards Fig. 4(Q).59. Find the magnitude and direction of the magentic forces on the four arms of the frame.

FIGURE 4(Q).59



[Ans. 0·04 N on each arm ; force on arms *ad* and *bc* is towards right and on arms *ab* and *dc* is down wards in plane of frame]

38. A horizontal wire 0·1 m long having mass 3 g carries a current of 5 A. Find the magnitude of the magnetic field which must act at 30° to the length of the wire inorder to support its weight.

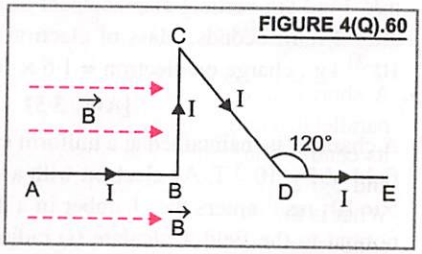
[Ans. 0·1176 T]

39. On a smooth inclined plane at 30° with the horizontal a thin current carrying metallic rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0·15 tesla in the vertical direction. For what value of current can the rod remain stationary ? The mass per unit length of rod is 0·30 kg m^{-1}.

[Ans. 11·3 A]

40. Find the magnitude of the magnetic force on the segment *CD* of length 30 cm placed in a magnetic field of 0·3 T and a current of 4 A flows through it as shown in Fig. 4(Q).60.

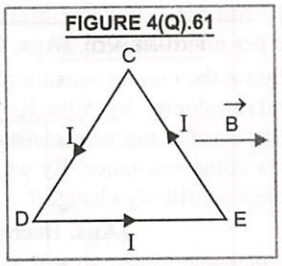
FIGURE 4(Q).60



[Ans. 0·31 N]

41. Fig. 4(Q).61, shows an equilateral triangular loop *CDE*, carrying current *I*. Length of each side of triangle is *l*. If a uniform magnetic field exists parallel to side *DE* of loop, then find the forces acting on the three wires *CD*, *DE* and *EC* separately.

FIGURE 4(Q).61



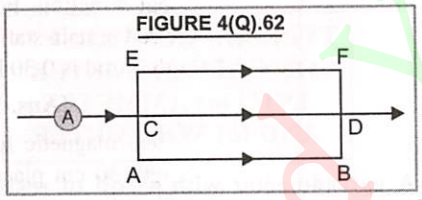
[Ans. $IlB\sqrt{3}/2$, normally outwards, zero

$IlB\sqrt{3}/2$, normally inwards]

TYPE VIII. FORCE BETWEEN TWO PARALLEL STRAIGHT CONDUCTORS CARRYING CURRENTS

42. Three long parallel wires *AB*, *CD* and *EF* of equal resistance are connected as shown in Fig. 4(Q).62. The separation between the neighbouring wires is 2·0 cm and wires *AE* and *BF* are of negligible resistance. The current is supplied to the wires and ammeter reads 12 A. Find the magnetic force per unit length

FIGURE 4(Q).62



(*a*) of wire *AB*

(*b*) of wire *CD*.

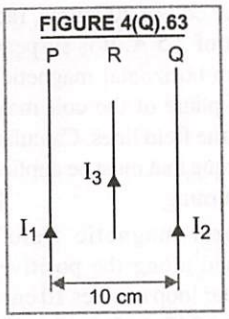
[Ans. (*a*) $2{\cdot}4 \times 10^{-4}$ Nm^{-1} directed upwards (*b*) Zero]

43. A short conductor of length 5·0 cm is placed parallel to a long conductor of length 1·5 m near its centre. The conductors carry currents 4·0 A and 3·0 A respectively in the same direction. What is the total force experienced by the long conductor, when they are 3·0 cm apart?

[Ans. 4×10^{-6} N ; attractive]

44. Two very long, straight parallel wires *P* and *Q* carry currents 5 A and 10 A respectively and are at a distance of 10 cm apart, as shown in Fig. 4(Q).63. If a third wire *R* (length 10 cm) having a current of 5 A is placed in middle between them, how much force will be acting on *R* ? The direction of current in all the wires is the same.

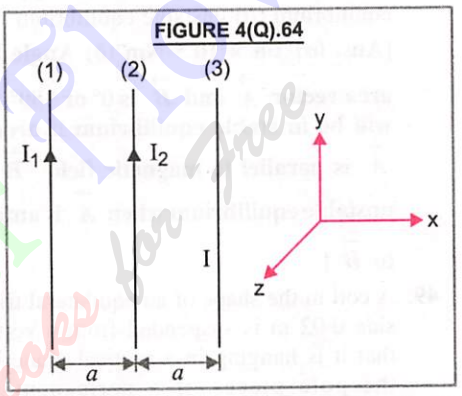
FIGURE 4(Q).63



[Ans. 10^{-5} N towards *Q*]

45. Three long straight parallel wires are kept as shown in Fig. 4(Q).64. The wire (3) carries a current *I*.

FIGURE 4(Q).64



(*i*) The direction of flow of current *I* in wire (3), is such that the net force, on wire (1), due to other two wires, is zero (*ii*) By reversing the direction of *I*, the net force, on the wire (2) due to the other two wires, becomes zero.

What will be the directions of current *I*, in the two cases ? Also obtain the relation between the magnitudes of current I_1, I_2 and *I*.

(CBSE 2016 S)

[Ans. (*i*) Downwards ; $I = 2I_2$ (*ii*) upwards; $I = I_1$]

TYPE IX. TORQUE ON A CURRENT LOOP

46. A circular coil of 200 turns and radius 10 cm is placed in a uniform magnetic field of 0·5 T, normal to the plane of the coil. If the current in the coil is 3·0 A, calculate the (*a*) total torque on the coil (*b*) total force on the coil (*c*) average force on each electron in the coil, due to the magnetic field. Assume that the area of cross-section of the wire to be 10^{-5} m^2 and the free electron density is 10^{29} m^{-3}. (CBSE 2008)

[Ans. (*a*) zero (*b*) zero (*c*) $1{\cdot}5 \times 10^{-24}$ N]

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47. A circular coil of 200 turns, radius 5 cm carries a current of 2·5 A. It is suspended vertically in a uniform horizontal magnetic field of 0·25 T, with the plane of the coil making an angle of 60° with the field lines. Calculate the magnitude of the torque that must be applied on it to prevent it from turning. **[Ans. 0·49 Nm]**

48. A uniform magnetic field of 3000 G is established along the positive Z direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. It is placed in the magnetic field. (*a*) Calculate the maximum torque the loop can experience. (*b*) Find the orientation of the loop for which the torque is zero. (*c*) In which orientation the loop would be (*i*) in stable equilibrium (*ii*) unstable equilibrium ?

[Ans. (*a*) $1{\cdot}8 \times 10^{-2}$ Nm (*b*) Angle between area vector $\vec{A}$ and $\vec{B}$ is 0° or 180° (*c*) Loop will be in stable equilibrium if area vector $\vec{A}$ is parallel to magnetic field $\vec{B}$ and in unstable equilibrium when $\vec{A}$ is antiparallel to $\vec{B}$]

49. A coil in the shape of an equilateral triangle of side 0·02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Find the couple acting on the coil. When a current of 0·1 ampere is passed through it and the magnetic field is parallel to its plane.

[Ans. $8{\cdot}66 \times 10^{-7}$ N-m]

TYPE X. MOVING COIL GALVANOMETER

50. The coil of a galvanometer is $0{\cdot}02 \times 0{\cdot}8$ m^2. It consists of 200 turns of the wire and is in a magnetic field of 0·20 T. The restoring torque constant of suspension fibre is 10^{-6} Nm/degree. Assuming magnetic field to be radial (*a*) what is the maximum current that can be measured by this galvanometer if scale can accommodate 45° deflection ? (*b*) What is the smallest current that can be detected if minimum observed deflection is 0·1 degree ? [CBSE 2013 (C)]

[Ans. (*a*) $4{\cdot}69 \times 10^{-4}$ A (*b*) $1{\cdot}56 \times 10^{-6}$ A]

51. The coil of a moving coil galvanometer has an effective area 6×10^{-2} m^2. It is suspended in a magnetic field of 3×10^{-2} Wb m^{-2}. If the torsional constant of the suspension fibre is 5×10^{-9} Nm deg^{-1}, find its current sensitivity in degree per micro-ampere. **[Ans. 0·36 deg/μA]**

52. To increase the current sensitivity of a moving coil galvanometer by 50% its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change ? (CBSE 2001)

[Ans. Decrease by 25%]

53. A galvanometer needs 30 mV for a full scale deflection of 30 divisions. Determine its voltage sensitivity. What must be its resistance if its current sensitivity is 2 div/μA ?

[Ans. 10^3 div/V ; 2000 Ω]

54. A moving coil meter has the following particulars : Number of turns, $n = 24$; Area of coil, $A = 2{\cdot}0 \times 10^{-3}$ m^2, magnetic field strength, $B = 0{\cdot}20$ T, Resisitance of the coil, $R = 14$ Ω.

(*a*) Indicate a simple way to increase the current sensitivity of the meter by 25%. (It is not easy to change A or B). (*b*) If in so doing, the resistance of the coil increases by 7 Ω, is the voltage sensitivity of the modified meter greater or lesser than the original meter ?

[Ans. (*a*) *n* should be increased from 24 to 30 (*b*) Voltage sensitivity is lesser than original]

TYPE XI. CONVERSION OF GALVANOMETER INTO (*a*) AMMETER AND (*b*) VOLTMETER

55. A galvanometer with a coil of resistance 12·0 Ω shows full scale deflection for a current 2·5 mA. How will you convert the meter into (*i*) an ammeter of range 0 to 7·5 A (*b*) a voltmeter of range 0 to 10 V. (CBSE 2005)

[Ans. (*a*) Shunt resistance $4{\cdot}0 \times 10^{-3}$ Ω (*b*) series resistance 3988 Ω.]

56. An ammeter gives full scale deflection with a current of 1 ampere. It is converted into an ammeter of range 10 ampere. Find the ratio of the resistance of ammeter to the shunt resistance used. **[Ans. 9 : 10]**

57. A galvanometer of resistance 50 Ω gives full deflection for a current of 0·05 A. Calculate the length of shunt wire required to convert the galvanometer into an ammeter of range 0 to 5 A. The diameter of the shunt wire is 2mm and its resistivity is 5×10^{-7} Ω m.

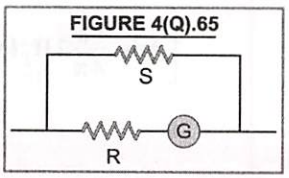
[Ans. 3·174 m in parallel]

58. A galvanometer can be converted into a voltmeter of certain range by connecting a

resistance of 880 Ω in series with it. When the resistance of 420 Ω is connected in series, the range becomes half. Find the resistance of galvanometer. **[Ans. 40 Ω]**

59. The resistance of a pivoted type galvanometer is 8 Ω and current for full scale deflection on it is 0·01 A. This galvanometer is to be converted into an ammeter of 5 A range. The only shunt available is 0·02 Ω. Find the value of R to be connected in series with the galvanometer coil, Fig. 4(Q).65.

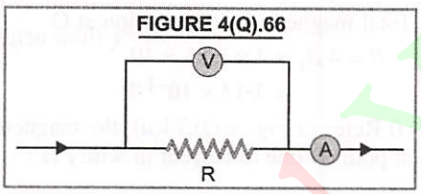
FIGURE 4(Q).65



[Ans. 1·98 Ω]

60. The voltmeter V in the Fig. 4(Q).66 reads 117 V and the ammeter A reads 0·13 A. The resistance of the voltmeter is 9000 Ω and the resistance of ammeter is 0·015 Ω. Compute (*i*) the resistance R, (*ii*) the power input to R.

FIGURE 4(Q).66



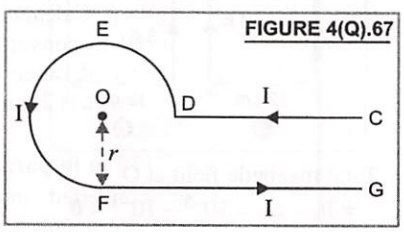
[Ans. (*i*) 1000 Ω (*ii*) 13·689 watt]

61. In a galvanometer there is a deflection of 10 divisions per mA. The internal resistance of the galvanometer is 60 Ω. If a shunt of 2·5 Ω is connected to the galvanometer and there are 50 divisions in all on the scale of galvanometer what maximum current can this galvanometer read ? [CBSE 2001 (C)] **[Ans. 125 mA]**

TYPE XII. TYPICAL PROBLEMS

62. A current I is flowing in a conductor shaped as shown in Fig. 4(Q).67. The radius of the curved part is r and length of straight portion is very large. Find the magnetic field induction at the centre O.

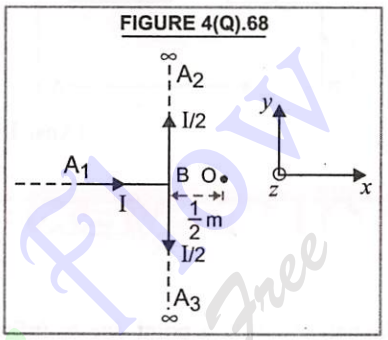
FIGURE 4(Q).67



$$\left[\textbf{Ans. } \frac{\mu_0 I}{4\pi r}\left(\frac{3\pi}{2}+1\right)\right]$$

63. A current $I = 1$ A flowing through an infinite long wire A_1 divides between two infinitely long wires A_2 and A_3 at junction B Fig. 4(Q).68. What is the magnitude of the magnetic field at point O due to currents through wires ?

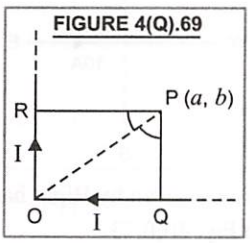
FIGURE 4(Q).68



[Ans. Zero]

64. Two infinitely long straight current carrying conductors are held at right angles to each other so that their common lies at the origin as shown in Fig. 4(Q).69. If both the conductors carry the same current I as shown, find the magnetic field at the point P (a, b).

FIGURE 4(Q).69



$$\left[\textbf{Ans. } \frac{\mu_0 I}{4\pi ab}\left[(a+b)+\sqrt{a^2+b^2}\right]\right]$$

65. A moving coil metre has the following particulars. Number of turns, $N = 20$, Area of the coil, $A = 2{\cdot}0 \times 10^{-3}$ m^2; Magnetic field strength, $B = 0{\cdot}20$ T, Resistance of the coil, $R = 12$ Ω.

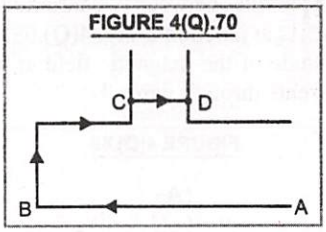
(*a*) Indicate a simple way to increase the current sensitivity of the metre by 20% (It is not easy to change A or B).

(*b*) If in so doing, the resistance of the coil inceases to 18 Ω, is the voltage sensitivity of the modified meter greater or lesser than the original meter ?

[Ans. (*a*) Increasing the turns by 20% (*b*) Lesser]

66. In the Fig. 4(Q).70, a long wire *AB* is placed on a table. A wire *CD* 10 cm long, is just above *AB*. It can slide up and down on two vertical wires. If the wires carry a current of 20 A, then how much above *AB*, will the wire *CD* settle ? The mass of the wire *CD* is 0·50 gram.

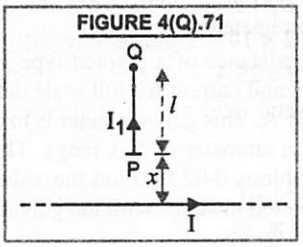
FIGURE 4(Q).70



[Ans. 1·63 mm]

67. Find the force on the conductor carrying current I_1 as shown in Fig. 4(Q).71.

FIGURE 4(Q).71



[Ans. $\frac{\mu_0}{4\pi} 2 I_1 \log_e \left(\frac{l+x}{x}\right)$**]**

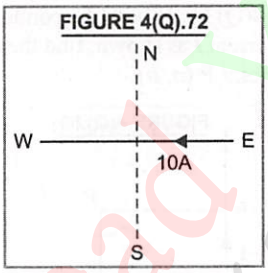
HINTS/EXPLANATIONS

For Difficult Questions

1. Magnetic field at a point due to infinite long linear conductor carrying current is

$$B = \frac{\mu_0}{4\pi} \frac{2I}{r}$$

FIGURE 4(Q).72

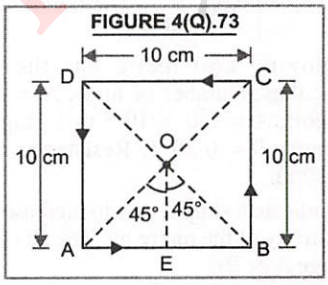


Its direction is given by Right hand thumb rule.

2. Refer to Fig. 4(Q).73,

$OE = a = 5$ cm $= 5 \times 10^{-2}$ m.

FIGURE 4(Q).73



Magnetic field induction at the centre O due to current in the arm *AB* of the wire is

$$B_1 = \frac{\mu_0}{4\pi} \frac{I}{a} [\sin\theta_1 + \sin\theta_2]$$

$$= 10^{-7} \times \frac{10}{5\times10^{-2}} [\sin 45° + \sin 45°]$$

$$= 2{\cdot}83 \times 10^{-5} \text{ T}$$

Total magnetic field induction at O

$$B = 4\, B_1 = 4 \times 2{\cdot}83 \times 10^{-5}$$

$$= \mathbf{1{\cdot}13 \times 10^{-4}\ T}$$

3. (*i*) Refer to Fig. 4(Q).74(*a*), the magnetic field at point O due to current in wire *I* is

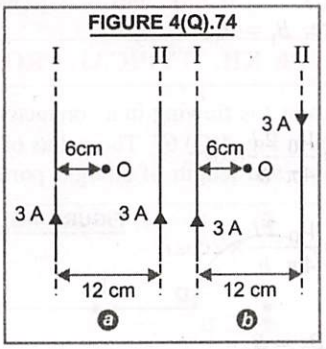
$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2I}{r} = 10^{-7} \times \frac{2\times3}{6\times10^{-2}} = 10^{-5}\, T$$

Acting normally downwards

Magnetic field at point O due to current in wire II is $B_2 = 10^{-7} \times \frac{2\times3}{6\times10^{-2}} = 10^{-5}\, T$

Acting normally upwards.

FIGURE 4(Q).74



∴ Total magnetic field at O

$= B_1 - B_2 = 10^{-5} - 10^{-5} = \mathbf{0}$

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(*ii*) Refer to Fig. 4(Q).74(*b*), total magnetic field at O is

$B = B_1 + B_2 = 10^{-5} + 10^{-5}$

$= 2 \times 10^{-5}$ **T acting normally downwards.**

4. Here, $I = 25$ A, $r = 1{\cdot}5 \times 10^{-2}$ m ; $B_2 = 3 \times 10^{-4}$ T

Magnetic field due to straight current carrying wire,

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2\times 25}{(1{\cdot}5\times 10^{-2})} = \frac{10}{3}\times 10^{-4}\ \text{T}$$

This magnetic field will act perpendicular to external magnetic field B_2 (= 3×10^{-4} T). Therefore, the magnitude of the resultant magnetic field is

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{10}{3}\times 10^{-4}\right)^2 + (3\times 10^{-4})^2}$$

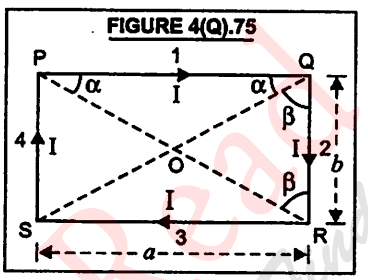
$$= 4{\cdot}48 \times 10^{-4}\ \text{T}$$

5. Let $\vec{B_1}, \vec{B_2}, \vec{B_3}$ and $\vec{B_4}$ be the magnetic fields at O due to current in arms 1, 2, 3 and 4 respectively, Fig. 4(Q).75.

The total magnetic field induction at O due to current through rectangular loop is

$$\vec{B} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \vec{B_4}$$

FIGURE 4(Q).75



where, $B_1 = B_3 = \dfrac{\mu_0}{4\pi}\dfrac{I}{(b/2)}$

$\times [\sin(90° - \alpha) + \sin(90° - \alpha)]$

$$= \frac{\mu_0}{4\pi}\frac{2I}{b}[\cos\alpha + \cos\alpha]$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{b}\times 2\cos\alpha = \frac{\mu_0}{\pi}\frac{I}{b}\times\frac{a}{\sqrt{a^2+b^2}}$$

and $B_2 = B_4 = \dfrac{\mu_0}{4\pi}\dfrac{I}{(a/2)}$

$\times [\sin(90° - \beta) + \sin(90° - \beta)]$

$$= \frac{\mu_0}{4\pi}\frac{2I}{a}[\cos\beta + \cos\beta]$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{a}2\cos\beta = \frac{\mu_0}{\pi}\frac{I}{a}\frac{b}{\sqrt{a^2+b^2}}$$

As $\vec{B_1}, \vec{B_2}, \vec{B_3}$ and $\vec{B_4}$ are acting in the same direction, *i.e.*, perpendicular to the plane of rectangular loop downwards,

$\therefore\ B = B_1 + B_2 + B_3 + B_4$

$$= \frac{\mu_0}{\pi}\frac{I}{\sqrt{a^2+b^2}}\left[\frac{a}{b} + \frac{b}{a} + \frac{a}{b} + \frac{b}{a}\right]$$

$$= \frac{\mu_0}{\pi}\frac{2I}{\sqrt{a^2+b^2}}\left[\frac{a}{b} + \frac{b}{a}\right]$$

$$= \frac{\mu_0}{\pi}\frac{2I\sqrt{a^2+b^2}}{ab}$$

6. Magnetic field at P due to current I_1 is

$B_1 = \dfrac{\mu_0}{4\pi}\dfrac{2I_1}{a}$; directed normally inwards

Magnetic field at P due to current I_2 is

$B_2 = \dfrac{\mu_0}{4\pi}\dfrac{2I_2}{b}$; directed normally outwards

Net magnetic field at point P is

$B = B_2 - B_1$; directed normally outwards.

$$= \frac{\mu_0}{4\pi}2\left[\frac{I_2}{b} - \frac{I_1}{a}\right]$$

$$= 10^{-7} \times 2\left[\frac{4}{3\times 10^{-2}} - \frac{2}{4\times 10^{-2}}\right]$$

$$= 1{\cdot}66 \times 10^{-5}\ \text{T}$$

7. Mag. field induction at O due to current through one wire is

$\vec{B_1} = \dfrac{\mu_0}{4\pi}\dfrac{2I}{r}\hat{k}$, we have

$$\vec{B} = \frac{\mu_0}{4\pi}2\times 1\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \ldots\right]\hat{k}$$

$$= \frac{\mu_0}{2\pi}\left[\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right)\hat{k} - \left(\frac{1}{2} + \frac{1}{8} + \ldots\right)\hat{k}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k} - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k}\right]$$

$$= \frac{\mu_0}{2\pi}\frac{2}{3}\hat{k} = \frac{\mu_0}{4\pi}\frac{4\hat{k}}{3}$$

$$= 10^{-7} \times \frac{4}{3}\hat{k} = 1{\cdot}33 \times 10^{-7}\ \text{T}\hat{k}$$

8. Given, $2\pi r = \pi/2$ or $r = \frac{1}{4} = 0{\cdot}25$ m ; $I = 5$ A

When wire is straight, the distance of point, $r' = 0{\cdot}01\ r = (0{\cdot}01 \times 0{\cdot}25)$ m.

This distance is very small as compared to length $(\pi/2)$ m of wire. Hence wire can be taken of infinite length.

Magnetic field induction due to straight wire is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r'} = \frac{10^{-7} \times 2 \times 5}{(0{\cdot}01 \times 0{\cdot}25)} = 4 \times 10^{-4}\ \text{T}$$

Magnetic field induction at the centre of the circular loop carrying current is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{10^{-7} \times 2\pi \times 5}{(0{\cdot}25)} = 1{\cdot}256 \times 10^{-5}\ \text{T}$$

9. (*i*) $\text{T} = 2\pi r/v$ and $i = \frac{e}{T} = \frac{ev}{2\pi r}$

(*ii*) $i = e\nu$

Now use, $\text{B} = \frac{\mu_0}{4\pi}\frac{2\pi \text{i}}{\text{r}} = \frac{\mu_0}{4\pi}\frac{2\pi}{\text{r}}\text{e}\nu$

10. Here, $r = 10$ cm $= 0{\cdot}10$; $B_1 = 0{\cdot}50 \times 10^{-4}$ T,

$x = 4$ cm $= {\cdot}04$ m

Magnetic field at the centre of a circular coil carrying current is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r}$$

Magnetic field due to circular current loop at a point on the axis of loop, distance x from the centre of loop is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi I r^2}{(r^2 + x^2)^{3/2}} = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

$$\therefore \frac{B_2}{B_1} = \frac{r^3}{(r^2 + x^2)^{3/2}}$$

$$= \frac{(0{\cdot}10)^3}{[(0{\cdot}1)^2 + (0{\cdot}04)^2]^{3/2}} = 0{\cdot}8$$

$$B_2 = B_1 \times 0{\cdot}8 = (0{\cdot}50 \times 10^{-4}) \times 0{\cdot}8$$

$$= 4 \times 10^{-5}\ \text{T}$$

11. Charge on α-particle, $q = +2e$, where e is the charge of an electron. Thus

$q = 2 \times 1{\cdot}6 \times 10^{-19}$ C.

$r = 2 \times 10^{-10}$ m ; $v = 2 \times 10^6$ ms^{-1}

Current $I = \frac{q}{T} = \frac{q}{2\pi r/v} = \frac{qv}{2\pi r}$

Magnetic field at the centre of circular path

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\left(\frac{qv}{2\pi r}\right) = \frac{\mu_0}{4\pi}\frac{qv}{r^2}$$

$$= \frac{10^{-7} \times (2 \times 1{\cdot}6 \times 10^{-19}) \times (2 \times 10^6)}{(2 \times 10^{-10})^2} = 1{\cdot}6\ \text{T}$$

12. Let $\hat{n}$ be a unit vector perpendicular to the plane of paper upwards. Then total magnetic field at O due to current through the given structure is

$$\vec{B} = \vec{B}_{ab} + \vec{B}_{bc} + \vec{B}_{cd} = 0 + \frac{\mu_0}{4\pi}\frac{I}{r}\theta(-\hat{n}) + 0$$

[As point O lies on the wire ab and cd, so $B_{ab} = 0 = B_{cd}$]

$$\therefore B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta = 10^{-7} \times \frac{10}{(20 \times 10^{-2})} \times \frac{\pi}{3}$$

$$= 5{\cdot}2 \times 10^{-6}\ \text{T}$$

13. Here, $I = 4$ A ; $r = 10{\cdot}0$ cm $= 0{\cdot}10$ m ;

As $\angle POS = 90°$ and $PO = OS = r$;

so $\angle OPS = \angle OSP = 45°$

Angle subtended at O by curved wire $PQRS$

$$= \frac{3\pi}{2}\ \text{rad}$$

Magnetic field at O due to current through portion SP of wire is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{r\cos\theta/2}\left[\sin\frac{\theta}{2} + \sin\frac{\theta}{2}\right]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r} \times 2\tan\frac{\theta}{2}$$

It is acting normally downwards.

Magnetic field at O due to current through portion $PQRS$ of wire is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{r}[2\pi - \pi/2] = \frac{\mu_0}{4\pi}\frac{I}{r} \times \frac{3\pi}{2}$$

It is acting normally downwards.

Total magnetic field at point O is

$$B = B_1 + B_2 = \frac{\mu_0}{4\pi}\frac{I}{r}\left[2\tan\frac{\theta}{2} + \frac{3\pi}{2}\right]$$

$$= 10^{-7} \times \frac{4}{0{\cdot}1}\left[2\tan\frac{90°}{2} + \frac{3\pi}{2}\right]$$

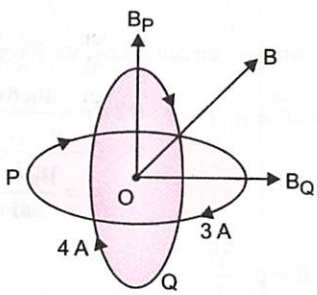
$$= 10^{-7} \times 40\left[2\tan 45° + \frac{3\pi}{2}\right]$$

$$= 10^{-7} \times 40\left[2 \times 1 + \frac{3}{2}\pi\right]$$

$= 2{\cdot}68 \times 10^{-5}$ T acting normally downwards.

14. Refer to Fig. 4(Q).76,

FIGURE 4(Q).76



$$B_P = \frac{\mu_0 I_1}{2r} = \frac{(4\pi \times 10^{-7}) \times 3}{2 \times (5 \times 10^{-2})} = 12\pi \times 10^{-6}\ \text{T}$$

$$B_Q = \frac{\mu_0 I_2}{2r} = \frac{(4\pi \times 10^{-7}) \times 4}{2 \times (5 \times 10^{-2})} = 16\pi \times 10^{-6}\ \text{T}$$

Net magnetic field

$$B = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{(12\pi \times 10^{-6})^2 + (16\pi \times 10^{-6})^2}$$

$$= 4\pi \times 10^{-6}\sqrt{9+16} = (4\pi \times 10^{-6}) \times 5$$

$$= 20 \times 3{\cdot}14 \times 10^{-6}\ \text{T} = \mathbf{6{\cdot}28 \times 10^{-5}\ T}$$

The direction of current I is not given, hence, **direction of $\vec{B}$ cannot be predicted.**

15. Refer to figure of the hint of above question, we have

$$B_P = \frac{\mu_0 I_1}{2R} = \frac{\mu_0 \times 1}{2R} = \frac{\mu_0}{2R} \text{ acting along } Z\text{-axis}$$

$$B_Q = \frac{\mu_0 I_2}{2R} = \frac{\mu_0 \times \sqrt{3}}{2R} = \frac{\sqrt{3}}{2}\frac{\mu_0}{R}$$

acting along X-axis

Net magnetic field is

$$B = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0}{2R}[(1)^2 + (\sqrt{3})^2]^{1/2}$$

$$= \frac{\mu_0}{R}\ \textbf{T (in X-Z plane)}$$

16. Here $I_1 = I_2 = \frac{I}{2} = \frac{5}{2}A$, $Ob = r = 5 \times 10^{-2}$ m

Total magnetic field induction at O due to current in the net work is

$$\vec{B} = \vec{B}_{ab} + \vec{B}_{bcd} + \vec{B}_{bed} + \vec{B}_{df}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r}\hat{n} + \frac{\mu_0}{4\pi}\frac{\pi I}{r}\hat{n} + \frac{\mu_0}{4\pi}\frac{\pi I}{r}(-\hat{n}) + \frac{\mu_0}{4\pi}\frac{I}{r}\hat{n}$$

where $\hat{n}$ is a unit vector normally out of the plane of paper.

$$\therefore\ \vec{B} = 2 \times \frac{\mu_0}{4\pi}\frac{I}{r}\hat{n} = 2 \times 10^{-7} \times \frac{5}{5 \times 10^{-2}}\hat{n}$$

$= 2 \times 10^{-5}$, normally out of the plane of paper.

17. $\vec{B}_0 = \vec{B}_{CG} + \vec{B}_{DEF}$

$$= \frac{\mu_0}{4\pi}\frac{2I}{r}\hat{n} + \frac{\mu_0}{4\pi}\frac{2\pi I}{r}\hat{n} = \frac{\mu_0 I}{2r}\left[\frac{1}{\pi} + 1\right]\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to loop upwards.

18. We know that magnetic field induction at the centre of an arc subtending an angle θ is

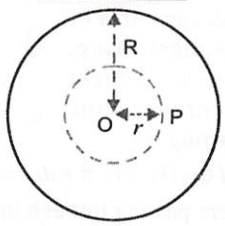
$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta$$

∴ Total magnetic field induction at O due to current through the three arcs is

$$B = \frac{\mu_0}{4\pi}I\left[\frac{1}{r} - \frac{1}{2r} + \frac{1}{3r}\right]\theta = \frac{5\mu_0 I\theta}{24\pi r}$$

19. Fig. 4(Q).77, represents the section of solid metal wire of radius R, perpendicular to its length.

FIGURE 4(Q).77



(*i*) Let the point P be lying inside the wire at a perpendicular distance r from the axis of the wire. Consider a circular path of radius r around the axis of the wire. By symmetry, the magnetic field produced due to current flowing in the wire at any point over this path is tangential to it and equal in magnitude at all points on this path.

Current enclosed by the closed path

$$= \frac{i}{\pi R^2} \times \pi r^2 = i r^2/R^2$$

Line integral of $\vec{B}$ over this closed path is

$$\oint \vec{B}.\vec{dl} = B\,2\pi r = \mu_0 \mu_r \frac{i r^2}{R^2}$$

(From Ampere's circuital law)

$$\therefore \quad B = \frac{\mu_0 \mu_r i r^2}{2\pi r R^2} = \frac{\mu_0 \mu_r i r}{2\pi R^2}$$

(*ii*) When point P is outside the wire, $r > R$ so that the current enclosed by the closed path = i

According to Ampere's circuital law

$$\oint \vec{B}.\vec{dl} = \mu_0 i \quad \text{or} \quad B\,2\pi r = \mu_0 i$$

or $$B = \frac{\mu_0 i}{2\pi r}$$

20. Here, $R = 6$ cm $= 6 \times 10^{-2}$ m ; $I = 8$ A ;

(*a*) $r = 3$ cm $= 3 \times 10^{-2}$ m. Point is inside the conductor

$$B = \frac{\mu_0}{2\pi}\frac{Ir}{R^2} = \frac{(4\pi \times 10^{-7})}{2\pi} \times \frac{8 \times (3 \times 10^{-2})}{(6 \times 10^{-2})^2}$$

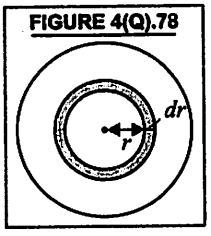
$$= \mathbf{1{\cdot}33 \times 10^{-5}\ T}$$

(*b*) $r = 10$ cm $= 0{\cdot}10$ m. Point is outside the conductor

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 8}{0{\cdot}10} = \mathbf{1{\cdot}6 \times 10^{-5}\ T}$$

21. (*i*) Consider an elementary ring of radius r thickness dr whose centre lies on the axis of the conductor. Area of the elementary ring, $dA = 2\,\pi\,r\,dr$. Current passing through the elementary ring is

FIGURE 4(Q).78



$$dI = J\,dA = (kr^2)\,(2\,\pi\,r\,dr) = k\,2\,\pi\,r^3\,dr$$

Total current passing through the closed path of radius r is,

$$I = \int_0^r k\,2\pi r^3\,dr = \frac{k\pi r^4}{2}$$

Using Ampere circuital law, we have

$$\oint \vec{B}.\vec{dl} = \mu_0 \mu_r I$$

$$B\,2\pi r = \mu_0 \mu_r \frac{\pi k r^4}{2} \quad \text{or} \quad B = \frac{\mu_0 \mu_r k r^3}{4}$$

(*ii*) If $r > a$, then net current through the closed path of radius r is,

$$I = \int_0^a k\,2\pi r^3\,dr = \frac{\pi k a^4}{2}$$

Using Ampere circuital law, we have

$$\oint \vec{B}.\vec{dl} = \mu_0 I \quad \text{or} \quad B\,2\pi r = \frac{\mu_0 \pi k a^4}{2}$$

$$B = \frac{\mu_0 k a^4}{4r}$$

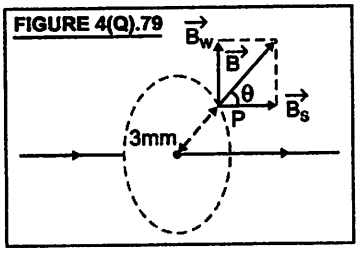
22. (*a*) $B = \mu_0 \frac{NI}{l}$;

Here, $l = 0{\cdot}20$ m ; $N = 120$; $I = 2{\cdot}5$ A

(*b*) $B = \frac{\mu_0 NI}{2l}$

23. Refer to Fig. 4(Q).79, the magnetic field due to current through solenoid, $\vec{B_S}$ is directed parallel to the axis of wire where $B_S = 4$ mT. The magnetic field due to current through straight wire is in the form of concentric circular magnetic lines of force. On one circular magnetic line of force, the magnetic field induction at all points is same in magnitude. The magnetic field induction due to current through straight wire at a perpendicular distance r (= 3 mm) is

FIGURE 4(Q).79



$$B_w = \frac{\mu_0}{4\pi}\frac{2I}{r}. \qquad \text{It is acting} \perp r \text{ to } \vec{B_S}$$

$$= \frac{10^{-7} \times 2 \times 20}{3 \times 10^{-3}} = 1{\cdot}33 \times 10^{-3}\ \text{T} = 1{\cdot}33\ \text{mT}$$

The resultant magnetic field induction at P in magnitude

$$B = \sqrt{B_S^2 + B_w^2} = \sqrt{4^2 + (1{\cdot}33)^2} = \mathbf{4{\cdot}2\,mT}$$

$$\tan\theta = \frac{1{\cdot}33}{4} = \tan 18{\cdot}4°$$

or $\theta = \mathbf{18{\cdot}4°}$ **with the directed axis**

24. Length of wire used, $L = 2\pi r \times$ no. of turns

$= 2\pi \times (2 \times 10^{-2}) \times 500$ m

Resistance per unit length $= 0{\cdot}02\ \Omega\ m^{-1}$

Total resistance of wire,

$R = 2\pi \times (2 \times 10^{-2}) \times 500 \times 0{\cdot}02 = 0{\cdot}4\pi\ \Omega$

No. of turns per unit length,

$$n = \frac{500}{30 \times 10^{-2}} = \frac{5000}{3}\ m^{-1}$$

As, $B = \mu_0 nI = \mu_0 \dfrac{n\varepsilon}{R}$

So, $\varepsilon = \dfrac{BR}{\mu_0 n} = \dfrac{10^{-2} \times (0{\cdot}4\pi)}{(4\pi \times 10^{-7}) \times (5000/3)} = \mathbf{6\ V}$

25. Here inner radius of toroid, $r_1 = 10 \times 10^{-2}$ m

Outer radius of toroid, $r_2 = 20 \times 10^{-2}$ m

$$B_{max.} = \mu_0 \frac{N}{2\pi r_1} I = \frac{(4\pi \times 10^{-7}) \times 600 \times 10}{2\pi \times (10 \times 10^{-2})}$$

$= 12 \times 10^{-3}\ T = \mathbf{12\ mT}$

$$B_{min.} = \mu_0 \frac{N}{2\pi r_2} I = \frac{(4\pi \times 10^{-7}) \times 600 \times 10}{2\pi \times (20 \times 10^{-2})}$$

$= 6 \times 10^{-3}\ T = \mathbf{6\ mT}$

26. $y = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{Ee}{m}t^2$

$$= \frac{10^4 \times (1{\cdot}6 \times 10^{-19}) \times (2 \times 10^{-3})^2}{2 \times 9{\cdot}1 \times 10^{-31}}$$

$= \mathbf{3{\cdot}51 \times 10^9\ m}$

27. Here, $B = 5 \times 10^{-3}$ T, $v = 5 \times 10^7\ ms^{-1}$,

$e = 1{\cdot}6 \times 10^{-19}$ C,

$m = 9{\cdot}1 \times 10^{-31}$ kg

(*i*) Radius of the circular path,

$$r = \frac{mv}{Be} = \frac{(9{\cdot}1 \times 10^{-31}) \times (5 \times 10^7)}{(5 \times 10^{-3}) \times (1{\cdot}6 \times 10^{-19})}$$

$= 5{\cdot}7 \times 10^{-2}\ m = \mathbf{5{\cdot}7\ cm}$

(*ii*) Frequency of revolution,

$$\nu = \frac{eB}{2\pi m} = \frac{(1{\cdot}6 \times 10^{-19}) \times (5 \times 10^{-3})}{2 \times 3{\cdot}14 \times (9{\cdot}1 \times 10^{-31})}$$

$= \mathbf{1{\cdot}4 \times 10^8\ Hz}$

28. Here, $B = 0{\cdot}02$ T, $r = 6 \times 10^{-2}$ m ;

pitch $= 24 \times 10^{-2}$ m

Let v_1 be the component of velocity of proton perpendicular to magnetic field. Then

$$r = \frac{mv_1}{qB} \quad \text{or} \quad v_1 = \frac{rqB}{m}$$

$$\therefore \quad v_1 = \frac{(6 \times 10^{-2}) \times (1{\cdot}6 \times 10^{-19}) \times 0{\cdot}02}{1{\cdot}6 \times 10^{-27}}$$

$= \mathbf{1{\cdot}2 \times 10^5\ ms^{-1}}$

Time period of revolution,

$$T = \frac{2\pi r}{v_1} = \frac{2\pi \times 6 \times 10^{-2}}{1{\cdot}2 \times 10^5} = \pi \times 10^{-6}\ s$$

Let v_2 be the component of velocity of proton along the direction of magnetic field. Then

$$v_2 = \frac{\text{pitch}}{\text{time period}} = \frac{24 \times 10^{-2}}{\pi \times 10^{-6}}$$

$= \mathbf{7{\cdot}6 \times 10^4\ ms^{-1}}$

29. Here, $V = 100$ V ; $B = 0{\cdot}004$ T ; $r = ?$

Let v be the velocity acquired by electron when accelerated through a pot. diff. V. Then

$$eV = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

As the electron describes a circular path of radius r in the perpendicular magnetic field, so

$$\frac{mv^2}{r} = evB$$

or $$r = \frac{mv}{eB} = \frac{m}{eB}\sqrt{\frac{2eV}{m}} = \frac{\sqrt{2meV}}{eB}$$

$$\therefore \quad r = \frac{\sqrt{2 \times (9{\cdot}1 \times 10^{-31}) \times (1{\cdot}6 \times 10^{-19}) \times 100}}{(1{\cdot}6 \times 10^{-19}) \times 0{\cdot}004}$$

$= 8{\cdot}4 \times 10^{-3}\ m = \mathbf{8{\cdot}4\ mm}$

30. Let v_1, v_2 be the rectangular components of proton's velocity perpendicular and parallel to the direction of magnetic field. Then

$v_1 = v \sin 60° = (4 \times 10^5) \times \sqrt{3}/2$

$= 3{\cdot}464 \times 10^5\ ms^{-1}$

$v_2 = v \cos 60° = (4 \times 10^5) \times 1/2$

$= 2 \times 10^5\ ms^{-1}$

The component velocity v_1 makes the proton to move along a circular path of radius r, given by

$$qv_1B = \frac{mv_1^2}{r}$$

or $$r = \frac{mv_1}{qB} = \frac{(1{\cdot}67\times10^{-27})\times(3{\cdot}464\times10^{5})}{(1{\cdot}6\times10^{-19})\times0{\cdot}3}$$

$$= 1{\cdot}2\times10^{-2}\text{ m} = \mathbf{1{\cdot}2\text{ cm}}$$

Time period of revolution of proton

$$T = \frac{2\pi r}{v_1} = \frac{2\times3{\cdot}142\times1{\cdot}2\times10^{-2}}{3{\cdot}464\times10^{5}}$$

$$= 21{\cdot}75\times10^{-8}\text{ s}$$

The component velocity v_2 of proton, makes the proton to move along the direction of magnetic field.

Therefore, due to velocity components v_1 and v_2, the proton moves on the helical path.

Pitch of the helix, $p = v_2 \times T$

$$= (2\times10^{5})\times(21{\cdot}75\times10^{-8})$$

$$= 4{\cdot}35\times10^{-2}\text{ m} = \mathbf{4{\cdot}35\text{ cm}}$$

31. $$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{10^{-7}\times2\times10}{0{\cdot}20} = 10^{-5}\text{ T}$$

It is acting perpendicular to the plane of paper upwards.

$$F = q\,v\,B\sin\theta$$

$$= 1{\cdot}6\times10^{-19}\times10^{5}\times10^{-5}\times\sin90^\circ$$

$$= \mathbf{1{\cdot}6\times10^{-19}\text{ N}}$$

The direction of $\vec{F}\ [= q(\vec{v}\times\vec{B})]$ is in **the plane of paper away from XY.**

32. Charge on proton = e, charge on deutron = e, Charge on α-particle = 2 e.

As $qV = \frac{1}{2}mv^2 = K$ (kinetic energy)

So, $K \propto q$

$$\therefore\quad K_p : K_d : K_\alpha = q_p : q_d : q_\alpha = e : e : 2e$$

$$= \mathbf{1 : 1 : 2}$$

Radius of circular path of a charged particle in a magnetic field is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$\left(\because K = \frac{1}{2}mv^2 \text{ or } mv = \sqrt{2mK}\right)$$

For proton, $r_p = \frac{\sqrt{2m_pK_p}}{eB} = 7\text{ cm}$

For deutron, $r_d = \frac{\sqrt{2m_dK_d}}{eB}$

$$= \sqrt{\frac{2\times2m_p\times K_p}{eB}} = \sqrt{2}\,r_p = \sqrt{2}\times7\text{ cm}$$

For α-particle, $r_\alpha = \frac{\sqrt{2m_\alpha K_\alpha}}{2eB}$

$$= \sqrt{\frac{2\times4m_p\times2K_p}{2eB}} = \sqrt{2}\,r_p = \sqrt{2}\times7\text{ cm}$$

33. Here, $E = 100\text{ kV/m} = 100\times10^{3}\text{ V/m} = 10^{5}\text{ V/m}$,

$B = 50\text{ mT} = 50\times10^{-3}\text{ T}$

$$v = \frac{E}{B} = \frac{10^{5}}{50\times10^{-3}} = \mathbf{2\times10^{6}\text{ ms}^{-1}}$$

34. Here, $V = 10\text{ kV} = 10\times10^{3}\text{ V} = 10^{4}\text{ V}$,

Mass of α-particle,

$$m_\alpha = 4\,m_p = 4\times1{\cdot}67\times10^{-27}\text{ kg};$$

Charge of α-particle,

$$q_\alpha = 2\,q_p = 2\times1{\cdot}6\times10^{-19}\text{ C}.$$

In a cyclotron, an α-particle gains energy $q_\alpha V$ when it crosses a region of potential difference V. In one revolution, the α-particle crosses the gap twice between dees, so the energy gained in each revolution = $2\,q_\alpha V$

Let α-particle makes n revolutions before coming out of the dees. The gain in its kinetic energy will be

$$\frac{1}{2}m_\alpha v_\alpha^2 = n2q_\alpha V \text{ or } n = \frac{m_\alpha v_\alpha^2}{4q_\alpha V}.$$

Given, $v_\alpha = \frac{c}{10} = \frac{3\times10^{8}}{10} = 3\times10^{7}\text{ ms}^{-1}$

$$\therefore\quad n = \frac{(4\times1{\cdot}67\times10^{-27})\times(3\times10^{7})^2}{4\times(2\times1{\cdot}6\times10^{-19})\times10^{4}}$$

$$\simeq \mathbf{470\text{ revolutions}}$$

35. Here, $\nu = 10\times10^{6}\text{ Hz} = 10^{7}\text{ Hz}$; $r = 0{\cdot}60\text{ m}$

$m = 3{\cdot}3\times10^{-27}\text{ kg}$; $q = 1{\cdot}6\times10^{-19}\text{ C}$

As $\nu = \frac{qB}{2\pi m}$; so, $B = \frac{2\pi m\nu}{q}$

$$\therefore\quad B = \frac{2\times3{\cdot}142\times(3{\cdot}3\times10^{-27})\times10^{7}}{(1{\cdot}6\times10^{-19})} = \mathbf{1{\cdot}3\text{ T}}$$

Max. KE of the emerging deutron is

$$K_{max} = \frac{q^2B^2r^2}{2m}$$

$$= \frac{(1{\cdot}6\times10^{-19})^2\times(1{\cdot}3)^2\times(0{\cdot}6)^2}{2\times(3{\cdot}3\times10^{-27})}\text{ J}$$

$$= \frac{(1{\cdot}6\times10^{-19})^2\times(1{\cdot}3)^2\times(0{\cdot}6)^2}{2\times(3{\cdot}3\times10^{-27})\times(1{\cdot}6\times10^{-13})}\text{ MeV}$$

$$= \mathbf{14{\cdot}74\text{ MeV}}$$

36. Here, $B = 2{\cdot}4$ T, $m = 1{\cdot}67\times10^{-27}$ kg ; $q = 1{\cdot}6\times10^{-19}$ C.

Time taken by proton to complete semicircular path

$$t = \frac{T}{2} = \frac{2\pi m}{2Bq} = \frac{\pi m}{Bq}$$

$$= \frac{3{\cdot}14\times1{\cdot}67\times10^{-27}}{2{\cdot}4\times1{\cdot}6\times10^{-19}}$$

$$= \mathbf{1{\cdot}37\times10^{-8}\text{ s}}$$

The direction of electric field should be reversed after every $1{\cdot}37\times10^{-8}$ s.

Time period of oscillation of electric field is

$$T = 2t = 2\times1{\cdot}37\times10^{-8}\text{ s}$$

Frequency of oscillation of applied electric field

$$\nu = \frac{1}{T} = \frac{1}{2\times1{\cdot}37\times10^{-8}}$$

$$= \mathbf{3{\cdot}64\times10^{7}\text{ Hz}}$$

37. Here, $I = 4$A. This current divides equally in arms *abc* and *adc*, *i.e.*, the current through each arm of square is, $I' = 2$A. Length of square $l = 0{\cdot}10$ m ; $B = 0{\cdot}20$ T ; $\theta = 90°$.

Magnitude of the force on each arm of square frame is

$$F = I'lB\sin\theta = 2\times(0{\cdot}10)\times(0{\cdot}20)\times\sin 90°$$

$$= \mathbf{0{\cdot}04\text{ N}}$$

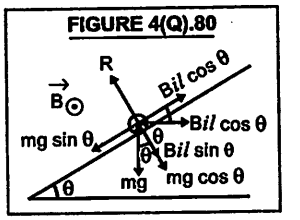
According to Fleming's left hand rule, the forces on arms *ad* and *bc* will be towards right and on arms *ab* and *dc* will be downwards in the plane of square frame.

38. $F = BIl\sin\theta = mg$

$$\text{or } B = \frac{mg}{il\sin\theta} = \frac{(3\times10^{-3})\times9{\cdot}8}{5\times0{\cdot}1\times\sin30°} = \mathbf{0{\cdot}1176\text{ T}}$$

39. In equilibrium position, Fig. 4(Q).80

FIGURE 4(Q).80



$$BIl\cos\theta = mg\sin\theta$$

$$\text{or } i = \frac{mg\sin\theta}{Bl\cos\theta} = \left(\frac{m}{l}\right)\frac{g}{B}\tan\theta$$

$$= \frac{0{\cdot}3\times9{\cdot}8\times\tan30°}{0{\cdot}15} = \mathbf{11{\cdot}3\text{ A}}$$

40. Here, $l = 30$ cm $= 0{\cdot}30$ m ; $B = 0{\cdot}3$ T, $I = 4$A,

$\theta = 180° - 120° = 60°$

Force, $F = IBl\sin\theta = 4\times0{\cdot}3\times0{\cdot}3\times\sin60°$

$$= 4\times0{\cdot}3\times0{\cdot}3\times\sqrt{3}/2 = \mathbf{0{\cdot}31\text{ N}}$$

41. For current in wire *CD*, the angle between $\vec{l}$ and $\vec{B}$ is 120°, *i.e.*, $\theta = 120°$. Therefore, force on wire *CD* is

$$\vec{F}_{CD} = I[(\overrightarrow{CD})\times\vec{B}] = IlB\sin120°\,\hat{n}$$

$$\text{or } F_{CD} = IlB\sin120° = \mathbf{IlB\times\sqrt{3}/2}$$

Its direction according to Right Hand Rule is **normally out of the plane of paper.**

For arm *DE*, the angle between $\vec{l}$ and $\vec{B}$ is zero *i.e.*, $\theta = 0°$. Therefore, force on wire *DE* is $IlB\sin0° = \mathbf{0}$

For arm *EC*, the angle between $\vec{l}$ and $\vec{B}$ is 120° *i.e.*, $\theta = 120°$. Therefore, force on wire *EC* is

$$F_{CE}\,I(CE)\,B\sin120° = \mathbf{IlB\times\sqrt{3}/2}$$

Its direction is along **normal into the plane of paper.**

42. As three parallel wires are of equal resistance and the total current supplied is 12 *A*, so current through each wire is 4 *A*.

(*a*) Force per unit length of *AB* due to current in *CD* is

$$F_1 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} = 10^{-7}\times\frac{2\times4\times4}{(2\times10^{-2})}$$

$$= 1{\cdot}6\times10^{-4}\text{ Nm}^{-1}\text{, directed upward}$$

Force per unit length of AB due to current in EF is $F_2 = \frac{\mu_0}{4\pi}\frac{2I_1I_3}{r_1} = 10^{-7} \times \frac{2\times4\times4}{(4\times10^{-2})}$

$= 0{\cdot}8 \times 10^{-4}$ Nm^{-1}, directed upward

Total force per unit length of AB is

$F = F_1 + F_2 = 1{\cdot}6 \times 10^{-4} + 0{\cdot}8 \times 10^{-4}$

$= \mathbf{2{\cdot}4 \times 10^{-4}\ Nm^{-1}}$ **directed upwards**

(b) Force per unit length of CD due to current in AB is

$F_1' = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} = 10^{-7} \times \frac{2\times4\times4}{(2\times10^{-2})}$

$= 1{\cdot}6 \times 10^{-4}$ Nm^{-1} directed downwards

Force per unit length of CD due to current in EF is

$F_2' = \frac{\mu_0}{4\pi}\frac{2I_1I_3}{r} = 10^{-7} \times \frac{2\times4\times4}{2\times10^{-2}}$

$= 1{\cdot}6 \times 10^{-4}$ Nm^{-1} directed upwards

$= -1{\cdot}6 \times 10^{-4}$ Nm^{-1} directed downwards

Total force per unit length of CD is

$F' = F_1' + F_2' = (1{\cdot}6 \times 10^{-4}) + (-1{\cdot}6 \times 10^{-4}) = \mathbf{0}$

43. Force on long conductor is equal and opposite to the force on small conductor $= \frac{\mu_0}{4\pi}\frac{2I_1I_2l}{r}$

Here, $I_1 = 4{\cdot}0$ A ; $I_2 = 3{\cdot}0$ A ;

$r = 3{\cdot}0 \times 10^{-2}$ m ; $l = 5{\cdot}0 \times 10^{-2}$ m.

44. Force on R due to wire P is

$F_1 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_1}l = \frac{10^{-7}\times2\times5\times5\times(0{\cdot}10)}{(5\times10^{-2})}$

$= 10^{-5}$ N towards P

Force on R due to wire Q is

$F_2 = \frac{\mu_0}{4\pi}\frac{2I_3I_2l}{r_2} = \frac{10^{-7}\times2\times10\times5\times(0{\cdot}10)}{(5\times10^{-2})}$

$= 2 \times 10^{-5}$ N towards Q

Net force on $R = F_2 - F_1 = 2 \times 10^{-5} - 10^{-5}$

$= \mathbf{10^{-5}\ N}$ **towards** $\mathbf{Q}$

45. (i) The net force on wire (1) will be zero if the force on wire (1) due to wire (2) is equal and opposite to the force on wire (1) due to wire (3). It is so when current direction in wire (3) is **downwards.** Then

$\frac{\mu_0}{4\pi}\frac{2I_1I_2}{a} = \frac{\mu_0}{4\pi}\frac{2I_1I}{(a+a)}$ or $I = 2I_2$

(ii) When the direction of current in wire (3) is reversed, *i.e.*, in upwards direction, Fig. 4(Q).81, then the net force on wire (2) will be zero if force on wire (2) due to wire (1) is equal and opposite to the force on wire (2) due to wire (3), *i.e.*,

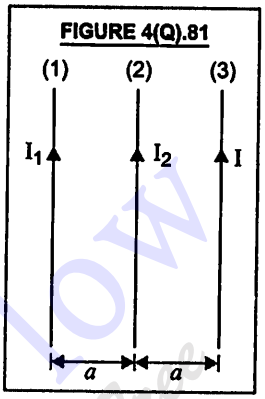


FIGURE 4(Q).81

$\frac{\mu_0}{4\pi}\frac{2I_1I_2}{a} = \frac{\mu_0}{4\pi}\frac{2II_2}{a}$ or $I = I_1$

46. Here, $n = 200$, $r = 0{\cdot}10$ m ;

$A = \pi r^2 = \pi\,(0{\cdot}1)^2$ m^2,

$B = 0{\cdot}5$ T, $\theta = 0°$; $I = 3{\cdot}0$ A

(a) Torque $= n\,I\,AB \sin\theta = 200 \times 3{\cdot}0 \times [\pi \times (0{\cdot}1)^2] \times 0{\cdot}5 \times \sin 0°$

$= \mathbf{0}$

(b) Magnetic forces on the two semi circular parts of coil are equal and opposite acting in the same plane, hence the total force on the coil is **zero.**

(c) Here $A = 10^{-5}$ m^2, $N = 10^{29}$ m^{-3}

Force on each electron

$F = evB = e\left(\frac{I}{NAe}\right)B$

$\left[\because I = NAev \text{ or } v = \frac{I}{NAe}\right]$

$\therefore \quad F = \frac{IB}{NA} = \frac{3{\cdot}0\times0{\cdot}5}{10^{29}\times10^{-5}} = \mathbf{1{\cdot}5 \times 10^{-24}\ N}$

47. $\tau = nIBA \sin\alpha$; Here, $n = 200$, $I = 2{\cdot}5$ A ;

$B = 0{\cdot}25$ T ; $\alpha = 90° - 60° = 30°$,

$A = \pi r^2 = \frac{22}{7} \times (0{\cdot}05)^2\ m^2$

48. Here, $B = 3000$ G $= 0{\cdot}3$ T ; $I = 12$ A

$A = 10 \times 5 = 50$ cm$^2 = 50 \times 10^{-4}$ m^2.

(*a*) $\tau_{max.} = IAB \sin 90° = 12 \times (50 \times 10^{-4}) \times 0{\cdot}3$
$= \mathbf{1{\cdot}8 \times 10^{-2}\ Nm}$

(*b*) Torque is zero when $\sin\theta = 0$ *i.e.* $\theta = 0°$ or $180°$

(*c*) Loop will be in stable equilibrium if $\vec{A}$ is parallel to $\vec{B}$ and in unstable equilibrium if $\vec{A}$ is antiparallel to $\vec{B}$.

49. Let the coil of triangular area A carrying current I be suspended in a uniform magnetic field B ($= 5 \times 10^{-2}$ T). Let θ be the angle between magnetic field and normal to the surface of the loop.

Here, area, $A = \frac{1}{2}$(base × height)

base = 0·02 m ;
height = 0·02 sin 60° = 0·07132 m.
Then torque acting on the closed loop is
$\tau = BIA \sin\theta$
$= (5 \times 10^{-2}) \times 0{\cdot}1 \times \frac{1}{2} \times (0{\cdot}02 \times 0{\cdot}07132) \times \sin 90°$
$= \mathbf{8{\cdot}66 \times 10^{-7}\ Nm}$

50. Here, $A = 0{\cdot}02 \times 0{\cdot}8\ m^2$; $n = 200$;
$B = 0{\cdot}20$ T ; $k = 10^{-5}$ Nm/degree
(*a*) $I_{max.} = ?$; $\theta = 45°$; (*b*) $I_{min.} = ?$; $\theta = 0{\cdot}1°$

Use the relation, $I = \frac{k}{nBA}\theta$

51. Here, $n = 1$, $A = 6 \times 10^{-2}\ m^2$,
$B = 3 \times 10^{-2}$ Wbm^{-2}, $k = 5 \times 10^{-9}$ Nm deg^{-1}

Current sensitivity $I_s = \frac{\theta}{I} = \frac{nBA}{k}$

$= \frac{1 \times (3 \times 10^{-2}) \times (6 \times 10^{-2})}{5 \times 10^{-9}}$
$= 3{\cdot}6 \times 10^5$ deg A^{-1}
$= \frac{3{\cdot}6 \times 10^5}{10^6}$ deg/μA $= \mathbf{0{\cdot}36\ deg/\mu A}$

52. Here, $I_s' = I_s + \frac{50}{100}I_s = \frac{150}{100}I_s = \frac{3}{2}I_s$; $R' = 2R$

Initial voltage sensitivity,

$$V_s = \frac{I_s}{R}$$

New voltage sensitivity,

$$V_s' = \frac{I_s'}{R'} = \frac{\frac{3}{2}I_s}{2R} = \frac{3}{4}V_s$$

% decrease in voltage sensitivity

$$= \frac{V_s - V_s'}{V_s} \times 100 = \left(1 - \frac{V_s'}{V_s}\right) \times 100$$

$$= \left(1 - \frac{3}{4}\right) \times 100 = \mathbf{25\%}$$

53. Voltage needed, $V = 30$ mV $= 30 \times 10^{-3}$ V
Deflection of division, $n = 30$

$\therefore$ Voltage sensitivity, $V_s = \frac{n}{V} = \frac{30}{30 \times 10^{-3}}$
$= \mathbf{10^3\ div/V}$

Resistance of galvanometer is

$$R_G = \frac{I_s}{V_s} = \frac{2\text{div}/\mu A}{10^3\ \text{div}/V} = \frac{2 \times 10^6\ \text{div}/A}{10^3\ \text{div}/V}$$
$$= \mathbf{2000\ \Omega}$$

54. (*a*) Current sensitivity $I_s = \frac{\theta}{I} = \frac{nBA}{k}$

Since it is easier to change n than k, A or B, therefore the current sensitivity can be increased by increasing n. In order to increase the current sensitivity by 25%, n should be increased by 25%. Therefore new value of n should be

$$n' = n + \frac{25}{100}n = \frac{125}{100}n = \frac{5}{4} \times 24 = \mathbf{30}$$

(*b*) New resistance, $R' = 14 + 7 = 21\ \Omega$. Voltage sensitivity $V_s = \frac{I_s}{R} = \frac{nBA}{kR} = \frac{24BA}{k \times 14} \approx \frac{1{\cdot}7BA}{k}$

New voltage sensitivity,

$$V_s' = \frac{n'BA}{kR'} = \frac{30BA}{k \times 21} \approx 1{\cdot}4\frac{BA}{k}$$

So, $\frac{V_s'}{V_s} = \frac{1{\cdot}4}{1{\cdot}7} < 1$ or $V_s' < V_s$

55. (*a*) For conversion into ammeter :
$G = 12{\cdot}0\ \Omega$, $I_g = 2{\cdot}5$ mA $= 2{\cdot}5 \times 10^{-3}$ A,
$I = 7{\cdot}5$ A

$$S = \frac{I_g G}{I - I_g} = \frac{(2{\cdot}5 \times 10^{-3}) \times 12}{7{\cdot}5 - (2{\cdot}5 \times 10^{-3})} = \frac{2{\cdot}5 \times 12 \times 10^{-3}}{7{\cdot}4975}$$
$$= \mathbf{4{\cdot}0 \times 10^{-3}\ \Omega\ in\ parallel}$$

(*b*) For conversion into voltmeter
$G = 12{\cdot}0\ \Omega$, $I_g = 2{\cdot}5 \times 10^{-3}$ A ; $V = 10$ V

$$R = \frac{V}{I_g} - G = \frac{10}{2{\cdot}5 \times 10^{-3}} - 12$$
$$= 4000 - 12 = \mathbf{3988\ \Omega\ in\ series}$$

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56. $S = \dfrac{I_g G}{I - I_g} = \dfrac{1 \times G}{10 - 1} = \dfrac{G}{9}\Omega$;

Resistance of ammeter formed

$$R_p = \frac{S \times G}{G + S} = \frac{(G/9) \times G}{G + G/9} = \frac{G/9}{10/9} = \frac{G}{10}$$

$$\therefore \quad \frac{R_p}{S} = \frac{G/10}{G/9} = \frac{9}{10}$$

57. $S = \dfrac{I_g G}{I - I_g} = \dfrac{0{\cdot}05 \times 50}{5 - 0{\cdot}05} = \dfrac{0{\cdot}05 \times 50}{4{\cdot}95} = \dfrac{50}{99}\Omega$

Now, $l = \dfrac{S \pi r^2}{\rho} = \dfrac{50}{99} \times \dfrac{22}{7} \times \dfrac{(10^{-3})^2}{5 \times 10^{-7}}$

$= \mathbf{3{\cdot}174\ m}$

58. Let G be the resistance of galvanometer. If a resistance R is connected in series of galvanometer it works as a voltmeter of range 0 to V volt. The current for full scale deflection of a voltmeter is $I_g = \dfrac{V}{R + G}$

In first case, $I_g = \dfrac{V}{880 + G}$

In second case, $I_g = \dfrac{V/2}{420 + G} = \dfrac{V}{2(420 + G)}$

$$\therefore \quad \frac{V}{880 + G} = \frac{V}{2(420 + G)}$$

or $\quad 880 + G = 840 + 2G$ or $G = \mathbf{40\ \Omega}$

59. $I_g (G + R) = (I - I_g) S$

or $0{\cdot}01\ (8 + R) = (5 - 0{\cdot}01)\ 0{\cdot}02 = 4{\cdot}99 \times 0{\cdot}02$

or $R = \dfrac{4{\cdot}99 \times 0{\cdot}02}{0{\cdot}01} - 8 = 9{\cdot}98 - 8 = \mathbf{1{\cdot}98\ \Omega}$

60. Current through voltmeter, $I_1 = \dfrac{117}{9000} = 0{\cdot}013\ A$

Current through R, $I_2 = I - I_1$

$= 0{\cdot}13 - 0{\cdot}013 = 0{\cdot}117\ A$

(*i*) Resistance, $R = \dfrac{117}{0{\cdot}117} = \mathbf{1000\ \Omega}$

(*ii*) Power input to $R = I_2^2 R$

$= (0{\cdot}117)^2 \times 1000$

$= \mathbf{13{\cdot}689\ watt}$

61. Since the galvanometer has 50 divisions, so current for full scale deflection is

$I_g = \dfrac{1}{10} \times 50\,mA = 5\ mA = 5 \times 10^{-3}\ A$,

$G = 60\ \Omega$, $S = 2{\cdot}5\ \Omega$.

Let I be the maximum current which a galvanometer can read when shunted with resistance S, then

$$I_g = \frac{IS}{G + S}$$

or $\quad I = \dfrac{I_g (G + S)}{S} = \dfrac{(5 \times 10^{-3})\ (60 + 2{\cdot}5)}{2{\cdot}5}$

$= 125 \times 10^{-3}\ A = \mathbf{125\ mA}$

62. $\vec{B}_0 = \vec{B}_{CD} + \vec{B}_{DEF} + \vec{B}_{FG}$; Here, $B_{CD} = 0$;

$$\vec{B}_{DEF} = \frac{\mu_0}{4\pi} \frac{\frac{3}{4} \times 2\pi I}{r} \hat{n}$$

and $\vec{B}_{FG} = \dfrac{\mu_0}{4\pi} \dfrac{I}{r} [\sin 90° + \sin 0°]\, \hat{n} = \dfrac{\mu_0}{4\pi} \dfrac{I}{r} \hat{n}$

$$\therefore \quad \vec{B}_0 = 0 + \frac{\mu_0}{4\pi} \times \frac{3\pi}{2r} I \hat{n} + \frac{\mu_0}{4\pi} \frac{I}{r} \hat{n}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r} \left(\frac{3\pi}{2} + 1 \right) \hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the plane of conductor outwards.

63. Let Mag. fields at O due to current through wires A_1, A_2, A_3 be $\vec{B}_1$, $\vec{B}_2$ and $\vec{B}_3$ respectively. Then

$\vec{B}_1 = 0$, $\because$ O lies on the axis of wire A_1

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{I}{(l/2)} [\sin 90° + \sin 0°]\, (-\hat{k})$$

$$= 10^{-7} \times 1 \times 2\ [1 + 0]\ (-\hat{k})$$

$$= -2 \times 10^{-7}\ \hat{k}$$

$$\vec{B}_3 = \frac{\mu_0}{4\pi} \frac{I}{(l/2)} [\sin 0° + \sin 90°]\, \hat{k}$$

$$= 10^{-7} \times 1 \times 2\ [0 + 1]\ \hat{k} = 2 \times 10^{-7}\ \hat{k}$$

$\therefore$ Total magnetic field at O is

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$= 0 + (-2 + 10^{-7}\ \hat{k}) + (2 \times 10^{-7}\ \hat{k}) = 0$$

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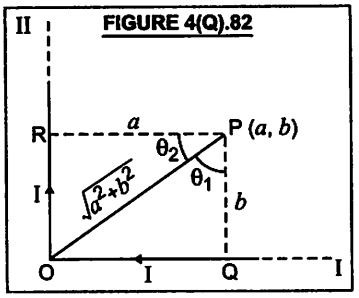
64. Refer to Fig. 4(Q).82, magnetic field at point P due to the conductor *I* is

FIGURE 4(Q).82



$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{b}\left[\sin\theta_1 + \sin\frac{\pi}{2}\right] = \frac{\mu_0 I}{4\pi b}\left[\frac{a}{\sqrt{a^2+b^2}} + 1\right]$$

Its direction is normally into the plane of paper.

Magnetic field at point *P* due to conductor II is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin\theta_2 + \sin\frac{\pi}{2}\right] = \frac{\mu_0 I}{4\pi a}\left[\frac{b}{\sqrt{a^2+b^2}} + 1\right]$$

It is also acting normally into the plane of paper.

Resultant magnetic field at *P*.

$$B = B_1 + B_2$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{1}{\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{1}{a}+\frac{1}{b}\right)\right]$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{\sqrt{a^2+b^2}}{ab} + \frac{a+b}{ab}\right]$$

$$= \frac{\mu_0 I}{4\pi ab}\left[\sqrt{a^2+b^2} + (a+b)\right]$$

65. (*i*) Current sensitivity, $I_s = \frac{\theta}{I} = \frac{nBA}{k}$

As, it is not easy to change *A* or *B*, hence sensitivity can be increased either by increasing *n* or by decreasing *k*. As it is easier to change *n*, than *k* hence current sensitivity can be increased by increasing *n*, I_s can be increased by 20% if *n* is increased by 20% *i.e.* by increasing *n* from 20 to 25.

(*ii*) Voltage sensitivity, $V_s = \frac{\theta}{V} = \frac{nBA}{kR}$

Voltage sensitivity of modified meter will be

$$V_s' = \frac{n'BA}{kR'}$$

Hence $\frac{V_s'}{V_s} = \frac{n'}{R'}\frac{R}{n} = \frac{25}{18}\times\frac{12}{20} = \frac{5}{6} = 0{\cdot}83 < 1$

or $V_s' < V_s$

Therefore, the ***voltage sensitivity of the modified meter is lesser than the original meter.***

66. Let the wire *CD* settle at distance *r* above the wire *AB*. The magnetic force on *CD* of length *l* due to current in *AB* is $F = \frac{\mu_0}{4\pi}\cdot\frac{2i_1 i_2}{r}l$

Here, $l = 10$ cm. $= 0{\cdot}1$ m, $i_1 = i_2 = 20$ A

$$\therefore \quad F = \frac{10^{-7}\times 2\times 20\times 20\times 0{\cdot}1}{r} = \frac{8\times10^{-6}}{r}\text{N}$$

Since the currents in *AB* and *CD* are in opposite directions, therefore force on *CD* will be upwards. The gravitational force on *CD* acting vertically downwards will be

$$F_1 = mg = 0{\cdot}5\times10^{-3}\times 9{\cdot}8 \text{ N}$$

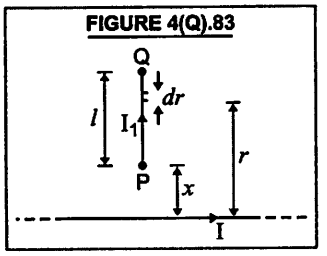
In the state of equilibrium, $F = F_1$

or $\frac{8\times10^{-6}}{r} = 0{\cdot}5\times10^{-3}\times 9{\cdot}8$

or $r = \frac{8\times10^{-6}}{0{\cdot}5\times9{\cdot}8\times10^{-3}} = \mathbf{1{\cdot}63\times10^{-3}\ m}$

67. Refer to Fig. 4(Q).83, the magnetic field induction at a point distance *r* from the long linear conductor carrying current is

FIGURE 4(Q).83



$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

Force on the current element of length *dr* carrying current I_1 is

$$dF = \frac{\mu_0}{4\pi}\frac{2I}{r}I_1\,dr = \frac{\mu_0}{4\pi}2II_1\frac{dr}{r}$$

Total force on the conductor of length *l* placed perpendicularly is

$$F = \frac{\mu_0}{4\pi}2II_1\int_x^{(l+x)}\frac{dr}{r} = \frac{\mu_0}{4\pi}2II_1\log_e\frac{(l+x)}{x}$$

NCERT EXERCISES

WITH SOLUTIONS

Q. 1. A circular coil of wire consisting of 100 turns, each of radius 8·0 cm carries a current of 0·40 A. What is the magnitude of the magnetic field $\vec{B}$ at the centre of the coil ?

Sol. Here, $n = 100$; $r = 8{\cdot}0 \times 10^{-2}$ m ; $I = 0{\cdot}40$ A ; $B = ?$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r} = 10^{-7} \times 2 \times \frac{22}{7} \times \frac{100 \times 0{\cdot}4}{8 \times 10^{-2}} = \mathbf{3{\cdot}1 \times 10^{-4}\ T}$$

Q. 2. A long straight wire carries a current of 35 A. What is the magnitude of the field $\vec{B}$ at a point 20 cm from the wire ?

Sol. Here, $I = 35$ A, $r = 20$ cm $= 20 \times 10^{-2}$ m ; $B = ?$

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 35}{20 \times 10^{-2}} = \mathbf{3{\cdot}5 \times 10^{-5}\ T}$$

Q. 3. A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of $\vec{B}$ at a point 2·5 m east of the wire.

Sol. Here, $I = 50$ A ; $r = 2{\cdot}5$ m ; $B = ?$

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 50}{2{\cdot}5} = \mathbf{4 \times 10^{-6}\ T}$$

It acts **upwards, perpendicular to the plane of wire.**

Q. 4. A horizontal overhead power lines carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1·5 m below the line ?

Sol. Here, $i = 90$ A ; $r = 1{\cdot}5$ m.

Now, $$B = \frac{\mu_0}{4\pi}\frac{2i}{r} = \frac{10^{-7} \times 2 \times 90}{1{\cdot}5} = \mathbf{1{\cdot}2 \times 10^{-5}\ T}$$

It acts towards **south.**

Q. 5. What is the magnitude of the magnetic force per unit length on a wire carrying a current of 8 A making an angle of 30° with the direction of a uniform magnetic field of 0·15 T ?

Sol. Here, $I = 8$ A ; $\theta = 30°$; $B = 0{\cdot}15$ T ; $F = ?$; $l = 1$ m.

$F = B\,I\,l \sin\theta = 0{\cdot}15 \times 8 \times 1 \times \sin 30° = 0{\cdot}15 \times 8 \times 1 \times (1/2) = \mathbf{0{\cdot}6\ Nm^{-1}}$

Q. 6. A 3·0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0·27 T. What is the magnetic force on the wire ?

Sol. Here, $l = 3{\cdot}0$ cm $= 3 \times 10^{-2}$ m ; $I = 10$ A ; $B = 0{\cdot}27$ T, $\theta = 90°$; $F = ?$

$F = B\,I\,l \sin\theta = 0{\cdot}27 \times 10 \times (3 \times 10^{-2}) \times \sin 90° = 0{\cdot}27 \times 10 \times 3 \times 10^{-2} \times 1 = \mathbf{8{\cdot}1 \times 10^{-2}\ N}$

The **direction of force is perpendicular to the direction of current as well as of magnetic field.**

Q. 7. Two long and parallel straight wires *A* and *B* carrying currents of 8·0 A and 5·0 A in the same direction are separated by a distance of 4·0 cm. Estimate the force on a 10 cm section of wire *A*.

Sol. Here, $I_1 = 8{\cdot}0$ A ; $I_2 = 5{\cdot}0$ A ; $r = 4{\cdot}0 \times 10^{-2}$ m ; $l = 10$ cm $= 0{\cdot}1$ m ; $F = ?$

$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}l = 10^{-7} \times \frac{2 \times 8 \times 5 \times 0{\cdot}1}{(4{\cdot}0 \times 10^{-2})} = \mathbf{2 \times 10^{-5}\ N}$$

This force is attractive force normal to *A* towards *B*.

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Q. 8. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1·8 cm. If the current carried is 8·0 A estimate the magnitude of $\vec{B}$ inside the solenoid near its centre.

Sol. Here, $l = 80$ cm $= 0{\cdot}80$ m ; $N = 5 \times 400 = 2000$; $i = 8{\cdot}0$ A ; $D = 1{\cdot}8$ cm.

Magnitude of magnetic field induction at a point well inside the solenoid is

$$B = \frac{\mu_0 N i}{l} = \frac{4\pi \times 10^{-7} \times 2000 \times 8{\cdot}0}{0{\cdot}80} = 8\pi \times 10^{-3}\ \text{T} \approx \mathbf{2{\cdot}5 \times 10^{-2}\ T}$$

Q. 9. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0·80 T. What is the magnitude of torque experienced by the coil ?

Sol. Here, $l = 10$ cm $= 0{\cdot}10$ m ; $n = 20$; $I = 12$ A ; $\alpha = 30°$; $B = 0{\cdot}80$ T ; $\tau = ?$

Area, $A = l \times l = 0{\cdot}10 \times 0{\cdot}10 = (0{\cdot}1)^2\ \text{m}^2$

$$\tau = n\,B\,I\,A \sin\alpha = 20 \times 0{\cdot}80 \times 12 \times (0{\cdot}1)^2 \times \sin 30° = \mathbf{0{\cdot}96\ Nm}$$

Q. 10. Two moving coil metres M_1 and M_2 have the following particulars :

$R_1 = 10\ \Omega$; $N_1 = 30$; $A_1 = 3{\cdot}6 \times 10^{-3}\ \text{m}^2$; $B_1 = 0{\cdot}25$ T ;

$R_2 = 14\ \Omega$, $N_2 = 42$, $A_2 = 1{\cdot}8 \times 10^{-3}\ \text{m}^2$; $B_2 = 0{\cdot}50$ T

(The spring constants are identical for the two metres). Determine the ratio of (*a*) current sensitivity and (*b*) voltage sensitivity of M_2 and M_1.

Sol. For meter M_1, $R_1 = 10\ \Omega$; $N_1 = 30$; $A_1 = 3{\cdot}6 \times 10^{-3}\ \text{m}^2$; $B_1 = 0{\cdot}25$ T ; $k_1 = k$

For meter M_2, $R_2 = 14\ \Omega$, $N_2 = 42$; $A_2 = 1{\cdot}8 \times 10^{-3}\ \text{m}^2$; $B_2 = 0{\cdot}50$ T ; $k_2 = k$.

As, current sensitivity $I_s = NB\,A/k$.

(*a*) So, $$\frac{I_{s_2}}{I_{s_1}} = \frac{N_2 B_2 A_2 / k_2}{N_1 B_1 A_1 / k_1} = \frac{42 \times 0{\cdot}50 \times 1{\cdot}8 \times 10^{-3}/k}{30 \times 0{\cdot}25 \times 3{\cdot}6 \times 10^{-3}/k} = \mathbf{1{\cdot}4}$$

(*b*) Voltage sensitivity $V_s = \dfrac{NBA}{k\,R}$

So, $$\frac{V_{s_2}}{V_{s_1}} = \frac{N_2 B_2 A_2 /(k_2 R_2)}{N_1 B_1 A_1 /(k_1 R_1)} = \frac{N_2 B_2 A_2 R_1 k_1}{N_1 B_1 A_1 R_2 k_2} = \frac{42 \times 0{\cdot}50 \times (1{\cdot}8 \times 10^{-3}) \times 10 \times k}{30 \times 0{\cdot}25 \times (3{\cdot}6 \times 10^{-3}) \times 14 \times k} = \mathbf{1}$$

Q. 11. In a chamber, a uniform magnetic field of 6·5 G (1G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of $4{\cdot}8 \times 10^6\ \text{ms}^{-1}$ normal to the field. (i) Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1{\cdot}6 \times 10^{-19}$ C, $m_e = 9{\cdot}1 \times 10^{-31}$ kg).

Sol. Here, $B = 6{\cdot}5 \times 10^{-4}$ T, $v = 4{\cdot}8 \times 10^6$ m/s, $e = 1{\cdot}6 \times 10^{-19}$ C ; $\theta = 90°$; $m = 9{\cdot}1 \times 10^{-31}$ kg ; $r = ?$

(*i*) Force on the moving electron due to magnetic field will be, $F = e\,v\,B \sin\theta$.

The direction of this force is perpendicular to $\vec{v}$ as well as $\vec{B}$ therefore this force will only change the direction of motion of the electron without affecting its velocity *i.e.* this force will provide the centripetal force to the moving electron and hence, the electron will move on the circular path. If r is the radius of circular path traced by electron, then

$$evB \sin 90° = mv^2/r$$

or $$r = \frac{mv}{Be} = \frac{(9{\cdot}1 \times 10^{-31}) \times (4{\cdot}8 \times 10^6)}{(6{\cdot}5 \times 10^{-4}) \times (1{\cdot}6 \times 10^{-19})} = \mathbf{4{\cdot}2 \times 10^{-2}\ m = 4{\cdot}2\ cm}$$

Q. 12. In Q.11, obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron ? Explain.

Sol. $$Bev = \frac{mv^2}{r} \text{ or } Be = \frac{mv}{r} = \frac{m}{r}(r\omega) = m\omega = m\,2\,\pi\,v$$

or $$v = \frac{Be}{2\pi m} = \frac{(6{\cdot}5\times10^{-4})\times(1{\cdot}6\times10^{-19})}{2\times(22/7)\times9{\cdot}1\times10^{-31}} = \mathbf{18{\cdot}18\times10^{6}\ Hz}$$

The relation for frequency is independent of speed of electron, hence the frequency of revolution of electron is **independent of speed of electron.**

Q. 13. (a) A circular coil of 30 turns and radius 8.0 cm. carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make on angle of 60° with the normal to the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area ? (All other particulars are also unaltered).

Sol. Here, $n = 30$, $I = 6.0$ A, $B = 1.0$ T, $\alpha = 60°$, $r = 8.0$ cm $= 8\times10^{-2}$ m.

$\therefore$ Area of the coil, $A = \pi r^2 = \frac{22}{7}\times(8\times10^{-2})^2 = 2.01\times10^{-2}$ m^2

(*a*) Now, $\tau = nIBA \sin\alpha = 30\times6.0\times1.0\times(2.01\times10^{-2})\times\sin 60°$

$= 30\times6.0\times2.01\times10^{-2}\times\sqrt{3}/2 = \mathbf{3.133\ N\text{-}m}$

(*b*) Since the torque on the planar loop does not depend upon the shape, in case the area of the loop is the same, the **torque will remain unchanged.**

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NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\vec{B} = B_0 \hat{k}$.

(*a*) They have equal z-components of momenta

(*b*) They must have equal charges

(*c*) They necessarily represent a particle-anti-particle pair

(*d*) The charge to mass ratio satisfy :

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

2. Biot-Savart law indicates that the moving electrons (velocity $\vec{v}$) produce a magnetic field $\vec{B}$ such that

(*a*) $\vec{B} \perp \vec{v}$ (*b*) $\vec{B} \parallel \vec{v}$

(*c*) it obeys inverse cube law

(*d*) it is along the line joining the electron and point of observation

3. A current carrying circular loop of radius *R* is placed in the *x*-*y* plane with centre at the origin. Half of the loop with $x > 0$ is now bent so that it now lies in the *y*-*z* plane.

(*a*) The magnitude of magnetic moment now diminishes

(*b*) The magnetic moment does not change

(*c*) The magnitude of $\vec{B}$ at (0,0,*z*), $z >> R$ increases

(*d*) The magnitude of $\vec{B}$ at (0,0,*z*), $z >> R$ is unchanged

4. An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true ?

(*a*) The electron will be accelerated along the axis

(*b*) The electron path will be circular about the axis

(*c*) The electron will experience a force at 45° to the axis and hence execute a helical path

(*d*) The electron will continue to move with uniform velocity along the axis of the solenoid

5. In a cyclotron, a charged particle

(*a*) undergoes acceleration all the time.

(*b*) speeds up between the dees because of the magnetic field

(*c*) speeds up in a dee

(*d*) slows down within a dee and speeds up between dees

6. A circular current loop of magnetic moment *M* is in an arbitrary orientation in an external magnetic field $\vec{B}$. The work doen to rotate the loop by 30° about an axis perpendicular to its plane is

(*a*) *MB* (*b*) $\sqrt{3}\frac{MB}{2}$

(*c*) $\frac{MB}{2}$ (*d*) zero

MULTIPLE CHOICE QUESTIONS-II

7. The gyro-magnetic ratio of an electron in an H-atom, according to Bohr model, is

(*a*) independent of which orbit it is in

(*b*) negative (*c*) positive

(*d*) increases with the quantum number *n*

8. Consider a wire carrying a steady current, *I* placed in a uniform magnetic field $\vec{B}$ perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do not work. This implies that

(*a*) motion of charges inside the conductor is unaffected by $\vec{B}$ since they do not absorb energy

(*b*) Some charges inside the wire move to the surface as a result of $\vec{B}$

(*c*) if the wire moves under the influence of $\vec{B}$, no work is done by the force

(*d*) if the wire moves under the influence of $\vec{B}$, no work is done by the magnetic force on the ions, assumed fixed within the wire

9. Two identical current carrying coaxial loops, carry current *I* in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as *C*,

(*a*) $\oint_C \vec{B}\cdot d\vec{l} = \mp 2\mu_0 I$

(*b*) the value of $\oint_C \vec{B}\cdot d\vec{l}$ is independent of sense of *C*

(*c*) there may be a point on *C* where $\vec{B}$ and $d\vec{l}$ are perpendicular

(*d*) $\vec{B}$ vanishes everywhere on *C*

10. A cubical region of space is filled with some uniform electric and magnetic fields. An electron enters the cube across one of its faces with velocity $\vec{v}$ and a positron enters via opposite face with velocity $-\vec{v}$. At this instant,

(*a*) the electric forces on both the particles cause identical accelerations

(*b*) the magentic forces on both the particles cause equal accelerations

(*c*) both particles gain or loose energy at the same rate

(*d*) the motion of the centre of mass (CM) is determined by $\vec{B}$ alone

11. A charged particle would continue to move with a constant velocity in a region wherein,

(*a*) $\vec{E} = 0,\ \vec{B} \neq 0$ (*b*) $\vec{E} \neq 0,\ \vec{B} \neq 0$

(*c*) $\vec{E} \neq 0,\ \vec{B} = 0$ (*d*) $\vec{E} = 0,\ \vec{B} = 0$

ANSWERS

1. (*d*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*a*)	**6.** (*d*)	**7.** (*a,b*)	**8.** (*b,d*)
9. (*b,c*)	**10.** (*b,c,d*)	**11.** (*a,b,d*)					

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. In a uniform magnetic field, the two charged particles will traverse identical helical paths in a completely opposite sense if the charge/mass ratio of these two particles is same and charges on them are of opposite character. In this situation $(e/m)_1 + (e/m)_2 = 0$, holds good.

2. According to Biot Savart's law, the magnetic field $\vec{B}$ at a point distance *r* from a charge *q* moving with a velocity $\vec{v}$ is given by, $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$ or $B = \frac{\mu_0}{4\pi}\frac{v\sin\theta}{r^2}$

The direction of $\vec{B}$ is along $(\vec{v}\times\vec{r})$, *i.e.*, perpendicular to the plane containing $\vec{v}$ and $\vec{r}$. $\vec{B}$ at a point obeys inverse square law and not inverse cube law.

3. For a circular loop of radius *R*, carrying current *I* in *x*-*y* plane, the magnetic moment $M = I \times \pi R^2$. It acts perpendicular to the loop along *z*-direction. When half of the current loop is bent in *y*-*z* plane, then magnetic moment due to half current loop in *x*-*y* plane, $M_1 = I(\pi R^2/2)$ acting along *z*-direction. Magnetic moment due to half current loop in *y*-*z* plane, $M_2 = I(\pi R^2/2)$ along *x*-direction.

Effective magnetic moment due to entire bent current loop,

$$M' = \sqrt{M_1^2 + M_2^2} = \sqrt{(I\pi R^2/2)^2 + (I\pi R^2/2)^2} = \frac{I\pi R^2}{2}\sqrt{2} < M$$

i.e., **magnetic moment diminishes.**

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The magnitude of $\vec{B}$ at a point on the axis of loop, distance z from the centre of current loop in x-y plane is $$B = \frac{\mu_0}{4\pi}\frac{2\pi I R^2}{(R^2+z^2)^{3/2}}$$

The magnitude of $\vec{B}$ at a point distance z from the centre of bent current loop, whose half part is in x-y plane and half part is in y-z plane, is

$$B' = \sqrt{\left[\frac{\mu_0}{4\pi}\frac{\pi I R^2}{(R^2+z^2)^{3/2}}\right]^2 + \left[\frac{\mu_0}{4\pi}\frac{\pi I R^2}{(R^2+z^2)^{3/2}}\right]^2} = \frac{\mu_0}{4\pi}\frac{\pi I R^2}{(R^2+z^2)^{3/2}}\sqrt{2} < B.$$

4. There is a uniform magnetic field $\vec{B}$ inside the current carrying long solenoid acting along the axis of solenoid. The magnitude of force on the electron of charge ($-e$) moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is $$|\vec{F}| = -e|\vec{v}\times\vec{B}| = -evB\sin\theta$$

Here angle θ between $\vec{v}$ and $\vec{B}$ is zero, *i.e.*, $\theta = 0°$ and $\sin\theta = 0$. Therefore $F = 0$.
It means the electron will continue to move with a uniform velocity along the axis of the solenoid.

5. In a cyclotron, a charged particle describes the circular paths inside the dees and is accelerated while going from one dee to another due to electric field. During circular path, the charged particle has centripetal acceleration which is provided by the magnetic force due to magnetic field. Therefore, a charged particle undergoes acceleration all the time, inside the cyclotron.

6. Let magnetic moment $\vec{M}$ of current loop be making an angle α with the direction of $\vec{B}$. When a current circular loop is rotated in a magnetic field by 30° about an axis perpendicular to its plane, there is no change in the angle α between magnetic moment $\vec{M}$ and magnetic field $\vec{B}$. Therefore, $\theta_1 = \alpha$, $\theta_2 = \alpha$.
Workdone, $$W = MB(\cos\theta_1 - \cos\theta_2) = MB(\cos\alpha - \cos\alpha) = 0$$

Multiple Choice Questions – II

7. Consider the electron of mass m, charge e be revolving in nth orbit of hydrogen atom. According to Bohr's theory, angular momentum of electron (L) = $nh/2\pi$.
Magnetic moment of electron, $M = n \times$ Bohr magneton = $n\,[eh/(4\pi m)]$

$$\text{Gyromagnetic ratio} = \frac{\text{magnetic moment}}{\text{angular momentum}} = \frac{neh/(4\pi m)}{nh/(2\pi)} = \frac{e}{2m} = \textbf{a constant}$$

As value of e is negative, so **gyromagnetic ratio is negative.**

8. (*a*) Motion of charges inside the conductor is affected by magnetic field $\vec{B}$, due to magnetic force $\vec{F}$, given by $$\vec{F} = q(\vec{v}\times\vec{B})$$

(*b*) Due to magnetic force, some charges inside the wire **move to the surface of wire.**

(*c*) The force on wire of length l, carrying current I when subjected to magnetic field $\vec{B}$ is, $\vec{F} = I(\vec{l}\times\vec{B})$. It acts perpendicular to the plane containing $\vec{l}$ and $\vec{B}$ and is directed as given by Right Hand rule. If the wire moves under the influence of $\vec{B}$ at an angle θ, where $\theta \neq 90°$, then work done, $W = Fs\cos\theta$, can not be zero.

(*d*) When wire moves under the influence of $\vec{B}$, then displacement of the ions is perpendicular to the magnetic force $\vec{F}$. Therefore **work done is zero.**

9. Consider a simple amperian loop passing once through both the identical current carrying coaxial loops.

(*i*) According to Ampere circuital law, $\oint_C \vec{B}.d\vec{l} = \mu_0 (I - I) = 0$. Hence, option (*a*) is wrong.

(*ii*) As $\oint_C \vec{B}.d\vec{l} = 0$, therefore $\oint_C \vec{B}.d\vec{l}$ is independent of sense of *C*. Thus **option (*b*) is correct.**

(*iii*) There will be a point on loop *C*, lying at the axis of two loops *A* and *B*, where $\vec{B}$ and $d\vec{l}$ are perpendicular to each other. **Thus option (*c*) is correct.**

(*iv*) The value of $\vec{B}$ does not vanish on various points of *C*. Thus option (*d*) is wrong.

10. Acceleration of electron due to electric field is

$$\vec{a}_E = -e\vec{E}/m.$$

Its direction is opposite to the direction of $\vec{E}$.

Acceleration of positron due to electric field is $\vec{a}'_E = e\vec{E}/m$; Its direction is along the direction of $\vec{E}$.

Thus $\vec{a}'_E \neq \vec{a}_E$. Hence, option (*a*) is wrong.

Magnetic force on electron, $\vec{F} = -e(\vec{v} \times \vec{B})$

Acceleration of electron, $\vec{a}_m = \dfrac{\vec{F}}{m} = -\dfrac{e(\vec{v} \times \vec{B})}{m}$

Magnetic force on positron, $\vec{F}' = e(-\vec{v} \times \vec{B}) = -e(\vec{v} \times \vec{B})$

Acceleration of positron, $\vec{a}'_m = \dfrac{\vec{F}'}{m} = -\dfrac{-e(\vec{v} \times \vec{B})}{m}$ $\therefore$ $\vec{a}_m = \vec{a}'_m$. Hence, **option (*b*) is true.**

As both the particles (electron and positron) are of same mass and same charge in magnitude having same acceleration due to magnetic field, hence they gain or loose the energy at the same rate. **Thus option (*c*) is true.**

Due to electric field, net electric force on electron-positron pair, $= -e\vec{E} + e\vec{E} = 0$.

Net magnetic force on electron-positron pair $= -e(\vec{v} \times \vec{B}) + [-e(\vec{v} \times \vec{B})] = -2e(\vec{v} \times \vec{B})$

Therefore, the motion of the centre of mass (*CM*) is determined by magnetic field alone. Thus **option (*d*) is true.**

11. A charged particle will move with a constant velocity in a region if the force on it due to electron field as well as magnetic field is zero or force on charged particle due to electric field is equal and opposite to the force on it due to magnetic field.

Force on charged particle due to electric field, $\vec{F}_E = q\vec{E}$

Force on charged particle due to magnetic field, $\vec{F}_m = q(\vec{v} \times \vec{B})$ or $F_m = qvB\sin\theta$.

Now, $F_E = 0$ if $E = 0$ and $F_m = 0$ if $\sin\theta = 0$ or $\theta = 0°$ or $180°$

A charged particle would continue to move with a constant velocity in a region if the force due to electric field and magnetic field is zero, *i.e.*,

(*i*) $E = 0$ and particle is moving along the direction of magnetic field (*i.e.*, $\theta = 0°$ or $180°$). Hence, $B \neq 0$. **Option (*a*) is correct.**

(*ii*) $E = 0$ and $B = 0$. **Option (*d*) is correct.**

(*iii*) The resultant force, $q\vec{E} + q(\vec{v} \times \vec{B}) = 0$. In this case $E \neq 0$ and $B \neq 0$, *i.e.*, **Option (*b*) is correct.**

(*iv*) If $B = 0$ but $E \neq 0$, then the charged particle will be accelerated by electric field. Option (*c*) is wrong.

VERY SHORT ANSWER QUESTIONS

12. Verify that the cyclotron frequency $\omega = eB/m$ has the correct dimensions of $[T]^{-1}$.

Ans. If a charged particle is moving perpendicular to the magnetic field, then $qvB = \frac{mv^2}{r}$ or $\frac{qB}{m} = \frac{v}{r} = \omega$

$\therefore$ Dimensions of $\omega = \frac{qB}{m} = \frac{v}{r} = \frac{LT^{-1}}{L} = [T^{-1}]$

13. Show that a force that does no work must be a velocity dependent force.

Ans. Let a force $\vec{F}$ does no work for the displacement $d\vec{l}$ while moving with velocity $\vec{v}$ for time dt.

Then work done, $dW = \vec{F}.d\vec{l} = 0$ or $\vec{F}.\vec{v}\,dt = 0$ or $\vec{F}.\vec{v} = 0$ or $Fv\cos\theta = 0$ or $\theta = 90°$.

Therefore, for no work done, $\vec{F}$ must be velocity dependent if angle between $\vec{F}$ and $\vec{v}$ is 90°. If $\vec{v}$ changes its direction, then $\vec{F}$ should also change its direction so that angle between $\vec{F}$ and $\vec{v}$ is 90°. It is so when a charged particle is moving in a perpendicular uniform magnetic field.

14. The magnetic force depends on $\vec{v}$ which depends on the inertial frame of reference. Does then the magnetic force differ from inertial frame to frame ? Is it reasonable that the net acceleration has a different value in different frames of reference ?

Ans. Magnetic force on a charged particle, $\vec{F} = q(\vec{v} \times \vec{B})$ or $F = qvB\sin\theta$.

Thus the magnetic force is a velocity dependent. It differs from one inertial frame to another. The net acceleration arising due to magnetic force is independent from frame to frame for inertial frames of reference.

15. Describe the motion of a charged particle in a cyclotron if the frequency of the radio frequency (rf) field were doubled.

Ans. Time period of revolution of charged particle in cyclotron, $T = \frac{2\pi m}{Bq}$

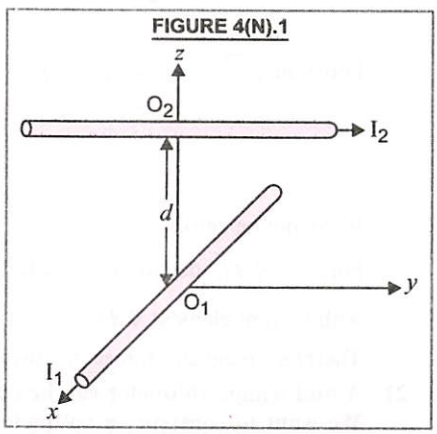
Frequency of revolution, $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

If $\nu' = 2\nu$, then $T' = \frac{1}{\nu'} = \frac{1}{2\nu} = \frac{1}{2} \times \frac{2\pi m}{Bq} = \frac{\pi m}{Bq} = \frac{\pi m}{Bq}$ = a constant.

It means, the charged particle will accelerate and decelerate alternately between the dees of cyclotron. So the radius of path in the dees will remain unchanged.

16. Two long wires carrying current I_1 and I_2 are arranged as shown in Fig. 4(N).1. The one carrying current I_1 is along the x-axis. The other carrying current I_2 is along a line parallel to the y-axis given by $x = 0$ and $z = d$. Find the force exerted at O_2 because of the wire along the x-axis.

FIGURE 4(N).1



Ans. Magnetic field at point O_2 due to current I_1 in long wire along x-axis is

$B_{O_2} = \frac{\mu_0}{4\pi}\frac{2I}{d}$; It will be acting along the y-axis. As second wire is along y-axis, so the angle between the directions of $I_2\vec{l}$ and $\vec{B}_{O_2}$ is zero, *i.e.*, $\theta = 0°$.

Force on wire lying along y-axis is,

$f = I_2 |\vec{l} \times \vec{B}_{O_2}| = I_2 l B_{O_2} \sin 0° = \mathbf{0}$.

SHORT ANSWER QUESTIONS

17. A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-z and z-x planes with their centres at the origin, joined together. Find the direction and magnitude to $\vec{B}$ at the origin.

Ans. $\vec{B} = \vec{B}_{xy} + \vec{B}_{yz} + \vec{B}_{zx} = \left(\frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{2}\hat{k}\right) + \left(\frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{2}\hat{i}\right) + \left(\frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{2}\hat{j}\right) = \frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{2}(\hat{i}+\hat{j}+\hat{k})$

$$= \frac{\mu_0 I}{8R}(\hat{i}+\hat{j}+\hat{k})$$

18. A charged particle of charge e and mass m is moving in an electric field $\vec{E}$ and magnetic field $\vec{B}$. Construct dimensionless quantities and quantities of dimension $[T]^{-1}$.

Ans. A dimensionless quantity cannot be constructed with e, m, $\vec{E}$ and $\vec{B}$. A quantity of dimensions $[T^{-1}] = \omega = \left[\frac{eB}{m}\right]$

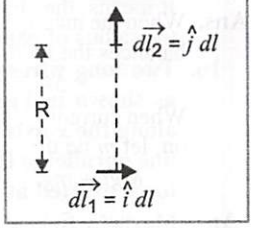
19. An electron enters with a velocity $\vec{v} = v_0\hat{i}$ into a cubical region (faces parallel to coordinate planes) in which there are uniform electric and magnetic fields. The orbit of the electron is found to spiral down inside the cube in plane parallel to the x-y plane. Suggest a configuration of fields $\vec{E}$ and $\vec{B}$ that can lead to it.

Ans. As per question, the orbit of electron is spiral down inside the cube in a plane parallel to the x-y plane, therefore, $\vec{B}$ must be along + z direction, *i.e.*, $\vec{B} = B_0\hat{k}$. Here the electric field should be along x-axis, *i.e.*, $\vec{E} = E_0\hat{i}$, where $E_0 > 0$.

20. Do magnetic forces obey Newton's third law. Verify for two current elements $d\vec{l}_1 = dl\,\hat{i}$ located at the origin and $d\vec{l}_2 = dl\,\hat{j}$ located at (0, R, 0). Both carry current I.

Ans. Refer Fig. 4(N).2, magnetic field due to current element $I\,d\vec{l}_1$ at a point on element $d\vec{l}_2$, $\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl}{R^2}\hat{k}$

FIGURE 4(N).2



Force on $d\vec{l}_2$ is, $\vec{F} = I\,dl\,\hat{j} \times \vec{B} = I\,dl\,\hat{j} \times \frac{\mu_0}{4\pi}\frac{I\,dl}{R^2}\hat{k}$

$$= \frac{\mu_0}{4\pi}\frac{I^2\,dl^2}{R^2}(\hat{j}\times\hat{k}) = \frac{\mu_0}{4\pi}\frac{I^2\,dl^2}{R^2}\hat{i}$$

It can not be zero.

Force on $I\,d\vec{l}_1$ due to magnetic field by current $I\,d\vec{l}_2$ is zero as magnetic field of $I\,d\vec{l}_2$ has no interaction with current element $I\,d\vec{l}_1$.

Therefore magnetic forces on current elements in this case **do not obey Newton's third law.**

21. A multirange voltmeter can be constructed by using a galvanometer circuit as shown in Fig. 4(N).3. We want to construct a voltmeter that can measure 2 V, 20 V and 200 V using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.

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Ans. Here, $G = 10\ \Omega$, $I_g = 1\ \text{mA} = 10^{-3}\ \text{A}$

Case (*i*), $V = 2\ \text{V}$, $R_1 = ?$

$$R_1 = \frac{V}{I_g} - G = \frac{2}{10^{-3}} - 10 = 1990\ \Omega \approx 2\ \text{k}\Omega$$

Case (*ii*), $V = 20\ \text{V}$, $(R_1 + R_2) = ?$

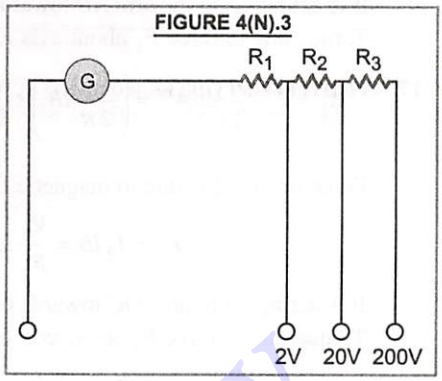
$$(R_1 + R_2) = \frac{20}{10^{-3}} - 10 = 20{,}000 - 10 \approx 20\ \text{k}\Omega$$

$$\therefore \quad R_2 = 20\ \text{k}\Omega - 2\ \text{k}\Omega = \mathbf{18\ k\Omega}$$

Case (*iii*), $V = 200\ \text{V}$, $(R_1 + R_2 + R_3) = ?$

$$\therefore \quad R_1 + R_2 + R_3 = \frac{200}{10^{-3}} - 10 \approx 200\ \text{k}\Omega \quad \therefore \quad R_3 = 200\ \text{k}\Omega - 20\ \text{k}\Omega \approx \mathbf{180\ k\Omega}.$$

FIGURE 4(N).3



22. A long straight wire carrying current of 25 A rests on a table as shown in Fig. 4(N).4. Another wire PQ of length 1 m, mass 2·5 g carries the same current but in the opposite direction. The wire *PQ* is free to slide up and down. To what height will *PQ* rise ?

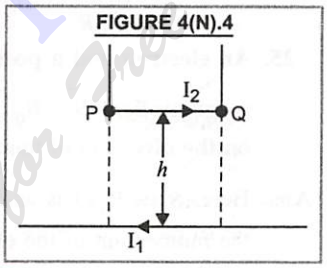
Ans. Here, $I_1 = 25\ \text{A}$, $I_2 = 25\ \text{A}$, $l = 1\ \text{m}$, $m = 2{\cdot}5\ \text{g} = 2{\cdot}5 \times 10^{-3}\ \text{kg}$

In equilibrium position

$$mg = \frac{\mu_0}{4\pi}\frac{2I_1 I_2 l}{h}$$

$$\text{or} \quad h = \frac{\mu_0}{4\pi}\frac{2I_1 I_2 l}{mg} = \frac{10^{-7} \times 2 \times 25 \times 25 \times 1}{(2{\cdot}5 \times 10^{-3}) \times 9{\cdot}8} = 51 \times 10^{-4}\ \text{m} = \mathbf{0{\cdot}51\ cm}$$

FIGURE 4(N).4



LONG ANSWER QUESTIONS

23. A 100 turn rectangular coil ABCD (in *XY* plane) is hung from one arm of a balance (Fig. 4(N).5). A mass 500 g is added to the other arm to balance the weight of the coil. A current 4·9 A passes through the coil and a constant magnetic field of 0·2 T acting inward (in *xz* plane) is switched on such that only arm *CD* of length 1 cm lies in the field. How much additional mass '*m*' must be added to regain the balance ?

Ans. When the magnetic field is off and the weight added in one pan of a balance, balances the rectangular coil in the other pan of balance. Then

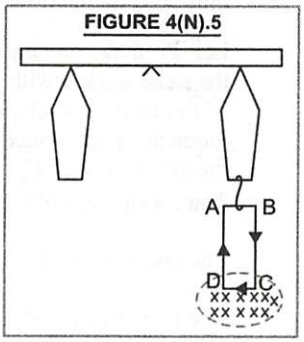
$$Mgl = W_{\text{coil}}\, l \text{ or } W_{\text{coil}} = Mg = 500 \times 9{\cdot}8\ \text{N}$$

When current I is passed through the coil and the magnetic field is switched on, let m be the mass added in a pan to balance the beam. Then

$$Mgl + mgl = W_{\text{coil}}\, l + IBL \sin 90°\, l$$

$$\text{or} \quad mgl = IBL\, l \quad \text{or} \quad m = \frac{IBL}{g} \quad \therefore \quad m = \frac{4{\cdot}9 \times 0{\cdot}2 \times (1 \times 10^{-2})}{9{\cdot}8} = 10^{-3}\ \text{kg} = \mathbf{1\ g}$$

FIGURE 4(N).5



24. A rectangular conducting loop consists of two wires on two opposite sides of length *l* joined together by rods of length *d*. The wires are each of the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance *R* and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in a uniform magnetic field *B* at 45° to its plane. Find τ, the torque exerted by the magnetic field on the loop about an axis through the centres of rods.

Ans. Refer to Fig. 4(N).6, force on arm *AB* due to magnetic field ,

$$F_1 = I_1 lB = \frac{V_0}{R_1} lB = \frac{V_0}{2R} lB$$

It is acting $\perp r$ to the arm AB towards observer.

Torque due to force F_1 about axis EF is,

$$\tau_1 = F_1 \times \frac{d}{2}\sin 45° = \left(\frac{V_0}{2R}\, lB\right) \times \frac{d}{2} \times \frac{1}{\sqrt{2}} = \frac{V_0\, l\, Bd}{4\sqrt{2}\, R}$$

(anticlockwise)

Force on arm DC due to magnetic field,

$$F_2 = I_2\, lB = \frac{V_0}{R_2}\, lB = \frac{V_0}{R}\, lB.$$

It is acting $\perp r$ to arm DC towards observer.

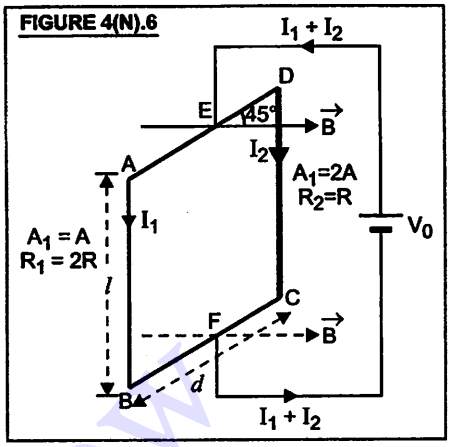
Torque due to force F_2 about axis EF is

$$\tau_2 = F_2 \times \frac{d}{2}\sin 45° = \left(\frac{V_0}{R}\, lB\right)\frac{d}{2} \times \frac{1}{\sqrt{2}} = \frac{V_0\, l\, Bd}{2\sqrt{2}\, R} \quad \text{(clockwise)}$$

Net torque, $\tau = \tau_2 - \tau_1$ (clockwise)

$$= \frac{V_0\, l\, Bd}{2\sqrt{2}\, R}\left[1 - \frac{1}{2}\right] = \frac{V_0\, l\, Bd}{4\sqrt{2}\, R} = \frac{V_0\, AB}{4\sqrt{2}\, R} \qquad (\because\ l \times d = A)$$

FIGURE 4(N).6



25. An electron and a positron are released from (0, 0, 0) and (0, 0, 1·5 R) respectively, in a uniform magnetic field $\vec{B} = B_0\,\hat{i}$, each with an equal momentum of magnitude $p = e\,BR$. Under what conditions on the direction of momentum will the orbits be non-intersecting circles ?

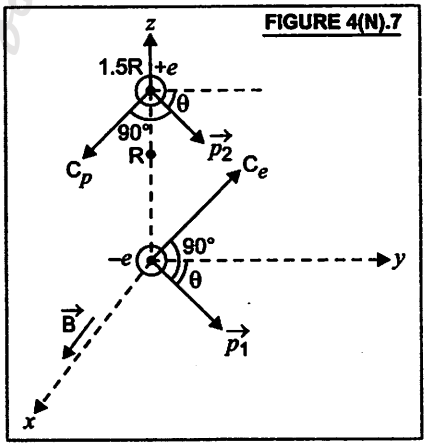
Ans. Here, $\vec{B}$ $(= B_0\,\hat{i})$ is acting along the x-axis. For a circular orbit, the momentum of the electron and positron are in y-z plane. Let $\vec{p_1}$ and $\vec{p_2}$ be the momentum of the electron and positron respectively. Both of them due to same momentum $(= e\ BR)$ move on circular orbits, each of radius R, but in opposite sense.

Let $\vec{p_1}$ make an angle θ with the y-axis and $\vec{p_2}$ must make the same angle θ with y-axis. Refer to Fig. 4(N).7. The centres of the respective circular orbits must be perpendicular to the momenta at a distance R. Let the centre of the circular orbit of the electron be at C_e and of the positron be at C_p.

The coordinates of C_e is $(0, R\sin\theta, R\cos\theta)$

The coordinates of C_p is $\left[0, -R\sin\theta, \left(\frac{3}{2}R - R\cos\theta\right)\right]$

FIGURE 4(N).7



The two circular orbits will not intersect if the distance between their two centres is greater than $2R$.

Let d be the distance between C_p and C_e. Then

$$d^2 = [-R\sin\theta - (R\sin\theta)]^2 + \left[\left(\frac{3}{2}R - R\cos\theta\right) - (R\cos\theta)\right]^2$$

$$= 4R^2\sin^2\theta + \frac{9}{4}R^2 - 6R^2\cos\theta + 4R^2\cos^2\theta = 4R^2(\sin^2\theta + \cos^2\theta) + \frac{9}{4}R^2 - 6R^2\cos\theta$$

$$\therefore \quad = \frac{25}{4}R^2 - 6R^2\cos\theta$$

The two circular orbits will not intersect each other if $d > 2R$ or $d^2 > 4R^2$

$$\therefore \quad \frac{25}{4}R^2 - 6R^2\cos\theta > 4R^2 \quad \text{or} \quad \frac{25}{4} - 6\cos\theta > 4 \quad \text{or} \quad \frac{9}{4} > 6\cos\theta \quad \text{or} \quad \cos\theta < 3/8$$

26. A uniform conducting wire of length 12 a and resistance R is wound up as a current carrying coil in the shape of (i) an equilateral triangle of side a ; (ii) a square of sides a and, (iii) a regular hexagon of sides a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

Ans. (*i*) **Refer to Fig. 4(N).8(*a*) ;** No. of turns, $n = 4$

Area of triangle $A = \frac{1}{2} a \times a \sin 60° = a^2\sqrt{3}/4$

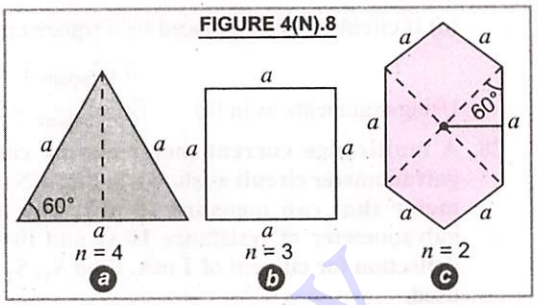
(*ii*) **Refer to Fig. 4(N).8(*b*) ;**

$n = 3$, area $A = a \times a = a^2$

(*iii*) **Refer to Fig. 4(N).8(*c*) ;**

$n = 2$, area $A = 6\left[\frac{a}{2} \times \frac{a}{2} \tan 60°\right] = \frac{3\sqrt{3}}{2} a^2$

FIGURE 4(N).8



Magnetic moment, $M = nIA$

For (*i*), $M_1 = 4 \times I \times a^2\sqrt{3}/4 = \sqrt{3}a^2 I$

For (*ii*), $M_2 = 3 \times I \times a^2 = 3a^2 I$

For (*iii*), $M_3 = 2 \times I \times \frac{3\sqrt{3}}{2} a^2 = 3\sqrt{3}a^2 I$

27. Consider a circular current-carrying loop of radius R in the x-y plane with centre at origin. Consider the line integral $\Im(L) = \left| \int_{-L}^{L} \vec{B}.d\vec{l} \right|$ taken along z-axis.

(*a*) Show that $\Im(L)$ monotonically increases with L.

(*b*) Use an appropriate Amperian loop to show that $\Im(\infty) = \mu_0 I$, where I is the current in the wire.

(*c*) Verify directly the above result.

(*d*) Suppose we replace the circular coil by a square coil of sides R carrying the same current I. What can you say about $\Im(L)$ and $\Im(\infty)$?

Ans. (*a*) As circular loop carrying current is lying in x-y plane, the magnetic field $\vec{B}$ is along the z-axis, in the direction of line integral. Hence, line integral, $\Im(L) = \left| \int_{-L}^{+L} \vec{B}.d\vec{l} \right| = \int_{-L}^{+L} B\,dl\cos 0° = \int_{-L}^{+L} B\,dl = 2BL.$

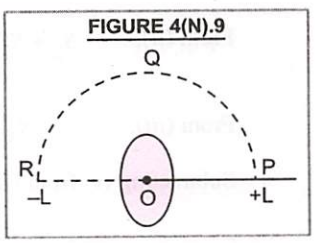
Therefore $\Im(L)$ is monotonically increasing with L.

(*b*) Consider a closed amperian path $PQRP$ as shown in Fig. 4(N).9.

According to Ampere circuital law, line integral of $\vec{B}$ over the closed path $PQRP$ is

$$\oint_{PQRP} \vec{B}.d\vec{l} = \int_{PQR} \vec{B}.d\vec{l} + \int_{RP} \vec{B}.d\vec{l} = \int_{PQR} \vec{B}.d\vec{l} + \int_{-L}^{+L} \vec{B}.d\vec{l} = \mu_0 I$$

FIGURE 4(N).9



or $\int_{PQR} \vec{B}.d\vec{l} + \Im(L) = \mu_0 I$ ...(*i*)

When $L \to \infty$, then the value of $B \to 0$ (as $B \propto 1/r^3$)

$\therefore \int_{PQR} \vec{B}.d\vec{l} = 0$. From (*i*), $0 + \Im(\infty) = \mu_0 I$ or $\Im(\infty) = \mu_0 I$

(*c*) Magnetic field at a point on the axis of circular coil at a distance z from the centre of a circular coil of radius R carrying current I is $B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$ $\therefore \int_{-\infty}^{+\infty} B_z\,dz = \int_{-\infty}^{+\infty} \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}\,dz$

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Put $z = R\tan\theta$, $dz = R\sec^2\theta\, d\theta$ $\therefore$ $\int_{-\infty}^{+\infty} B_z\, dz = \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \cos\theta\, d\theta = \mu_0 I.$

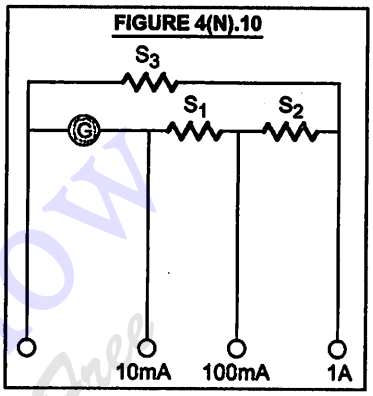
(*d*) If circular coil is replaced by a square coil of side R, carrying the same current I, then

$$B(z)_{\text{square}} < B(z)_{\text{circular coil}} \quad \therefore \quad \Im(L)_{\text{square}} < \Im(L)_{\text{circular coil}}$$

Using arguments as in (*b*) $\Im(\infty)_{\text{square}} = \Im(\infty)_{\text{circular}} = \mu_0 I.$

28. A multirange current meter can be constructed by using a galvanometer circuit as shown in Fig. 4(N).10. We want a current meter that can measure 10 mA, 100 mA and 1 A using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find S_1, S_2 and S_3 that have to be used.

FIGURE 4(N).10



Ans. Here, $G = 10\ \Omega$, $I_g = 1\text{ mA} = 10^{-3}$ A.

To convert a galvanometer into an ammeter of given range of current 0 to I, the shunt resistance required is $S = \frac{I_g G}{I - I_g}$.

Case (*i*), $I = 10\text{ mA} = 10 \times 10^{-3}$ A, $S = S_1 + S_2 + S_3$.

$$\therefore \quad S_1 + S_2 + S_3 = \frac{1\text{ mA} \times 10\ \Omega}{(10-1)\text{ mA}} = \frac{10}{9}\ \Omega \qquad \ldots(i)$$

Case (*ii*), When $I = 100$ mA, $S = S_2 + S_3$, galvanometer resistance $= G + S_1$

$$\therefore \quad S_2 + S_3 = \frac{I_g(G + S_1)}{I - I_g} = \frac{1\text{ mA} \times (10 + S_1)}{(100-1)\text{ mA}} = \frac{10 + S_1}{99} \qquad \ldots(ii)$$

Case (*iii*), When $I = 1$ A, $S = S_3$, galvanometer resistance $= (G + S_1 + S_2)$

$$\therefore \quad S_3 = \frac{I_g(G + S_1 + S_2)}{I - I_g} = \frac{1\text{mA}\,[10 + S_1 + S_2]}{(1000-1)\text{ mA}} = \frac{10 + S_1 + S_2}{999} \qquad \ldots(iii)$$

Putting the value of (*ii*) in (*i*), we get

$$S_1 + \frac{10 + S_1}{99} = \frac{10}{9} \quad \text{or} \quad S_1\left(1 + \frac{1}{99}\right) = \frac{10}{9} - \frac{10}{99} = \frac{100}{99}$$

$$\therefore \quad S_1 \times \frac{100}{99} = \frac{100}{99} \quad \text{or} \quad S_1 = 1\ \Omega$$

From (*ii*), $S_2 + S_3 = \frac{1}{99}(10 + 1) = \frac{1}{9}$ ...(*iv*)

From (*iii*), $S_3 = \frac{10 + 1 + S_2}{999} = \frac{11 + S_2}{999}$ or $S_3 - \frac{S_2}{999} = \frac{11}{999}$ ....(*v*)

Subtracting (*v*) from (*iv*), we get,

$$S_2 + \frac{S_2}{999} = \frac{1}{9} - \frac{11}{999} = \frac{100}{999} \quad \text{or} \quad \frac{S_2 \times 1000}{999} = \frac{100}{999} \quad \text{or } S_2 = \frac{1}{10} = 0{\cdot}1\ \Omega$$

From (*iv*), $\frac{1}{10} + S_3 = \frac{1}{9}$ or $S_3 = \frac{1}{9} - \frac{1}{10} = \frac{1}{90}$ or $S_3 = 0{\cdot}011\ \Omega$

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29. Five long wires *A, B, C, D* and *E*, each carrying current *I* are arranged to form edges of a pentagonal prism as shown in Fig. 4(N).11. Each carries current out of the plane of paper.

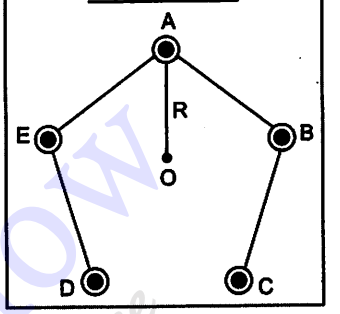
(a) What will be magnetic induction at a point on the axis *O* ? Axis is at a distance *R* from each wire.

(b) What will be the field if current in one of the wires (say *A*) is switched off ?

(*c*) What if current in one of the wire (say) *A* is reversed ?

Ans. (*a*) Let the five wires *A, B, C, D* and *E* be perpendicular to the plane of paper at locations as shown in Fig. 4(N).11.

FIGURE 4(N).11



Resultant magnetic field induction at *O* due to current through all the five wires is = **0** (as magnetic field induction due to five wires will be represented by various sides of a closed pentagon in one order, lying in the plane of paper).

(*b*) Total Magnetic field induction at *O* due to currents through the wires *B, C, D* and *E* is equal and opposite to the magnetic field induction at *O* due to current through wire *A*.

∴ Magnetic field induction at *O* due to current through wire *A* is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R} \text{ acting } \perp r \text{ to } AO \text{ towards right.}$$

∴ Magnetic field induction at *O* due to currents through the wires *B, C, D* and *E* is $= \frac{\mu_0}{4\pi}\frac{2I}{R}$, **acting $\perp r$ to *AO* towards left.**

(*c*) If current in wire *A* is reversed, then total magnetic field induction at *O*

= mag. field induction due to wire *A* + mag. field induction due to wires *B, C, D* and *E*

$$= \frac{\mu_0}{4\pi}\frac{2I}{R} \text{ (acting } \perp r \text{ } AO \text{ towards left)} + \frac{\mu_0}{4\pi}\frac{2I}{R} \text{ (acting } \perp r \text{ } AO \text{ towards left)}$$

$$= \frac{\mu_0 I}{\pi R} \textbf{ acting } \perp r \textbf{ } AO \textbf{ towards left.}$$

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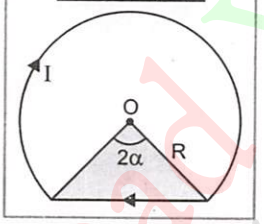
I. Multiple Choice Questions (with One Correct Answer)

I. Biot Savart's law and magnetic field due to a linear current carrying wire

1. The magnetic field intensity due to a thin wire carrying current I shown in Fig. 4(CF).1., is

FIGURE 4(CF).1



(a) $\frac{\mu_0 I}{2\pi R}[\pi - \alpha + \tan\alpha]$

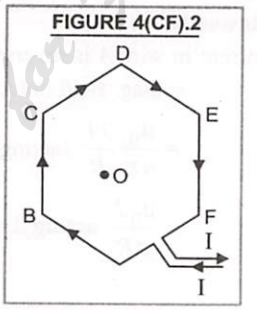
(b) $\frac{\mu_0 I}{2\pi R}[\pi - \alpha]$

(c) $\frac{\mu_0 I}{2\pi R}[\pi + \alpha]$

(d) $\frac{\mu_0 I}{2\pi R}[\pi + \alpha - \tan\alpha]$ **(DPMT 2006)**

2. A current I is flowing in a hexagonal coil of side a Fig. 4(CF).2. The magnetic field induction at the centre O of the coil will be

FIGURE 4(CF).2



(a) $\frac{\mu_0 I}{3\sqrt{3}\pi a}$ (b) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

(c) $\frac{3\sqrt{3}\mu_0 I}{\pi a}$ (d) $\frac{\mu_0 I}{\sqrt{3}\pi a}$.

3. The current I ampere is flowing in an equilateral triangle of side a the magnetic field induction at the centroid will be

(a) $\frac{9\mu_0 I}{2\pi a}$ (b) $\frac{5\sqrt{2}\mu_0 I}{3\pi a}$

(c) $\frac{3\mu_0 I}{2\pi a}$ (d) $\frac{\mu_0 I}{3\sqrt{3}\pi a}$.

4. Current I flows through a long conducting wire bent at right angles as shown in the Fig. 4(CF).3. The magnetic field at a point P and the right bi-sector of the angle COD at a distance r from O is

ANSWERS

1. (a) 2. (b) 3. (a)

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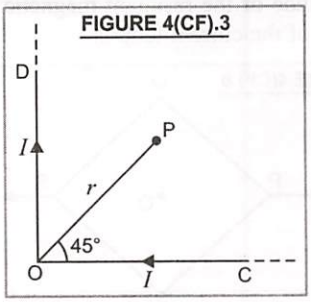
FIGURE 4(CF).3



(a) $\frac{\mu_0 I}{\pi r}$ (b) $\frac{2\mu_0 I}{\pi r}$

(c) $\frac{\mu_0 I}{4\pi r}(\sqrt{2}+1)$ (d) $\frac{\mu_0}{4\pi}\frac{2I}{r}(\sqrt{2}+1)$

5. Two identical long conducting wires *AOB* and *COD* are placed at right angle to each other, with one above other such that *O* is their common point for the two. The wires carry I_1 and I_2 currents, respectively. Point *P* is lying at distance *d* from *O* along a direction perpendicular to the plane containing the wires. The magnetic field at the point *P* will be

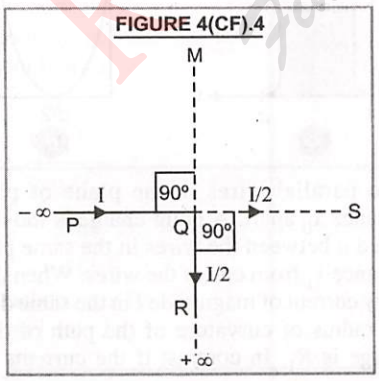
(a) $\frac{\mu_0}{2\pi d}\left(\frac{I_1}{I_2}\right)$ (b) $\frac{\mu_0}{2\pi d}(I_1+I_2)$

(c) $\frac{\mu_0}{2\pi d}(I_1^2-I_2^2)$ (d) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{1/2}$

(AIPMT 2014)

6. An infinitely long conductor *PQR* is bent to form a right angle as shown in Fig. 4(CF).4. A current *I* flows through *PQR*. The magnetic field due to this current at the point *M* is H_1. Now another infinitely long straight conductor *QS* is connected at *Q* so that the current is *I*/2 in *QR* as well as in *QS*, the current in *PQ* remaining unchanged. The magnetic field at *M* is now H_2. The ratio H_1/H_2 is given by

FIGURE 4(CF).4

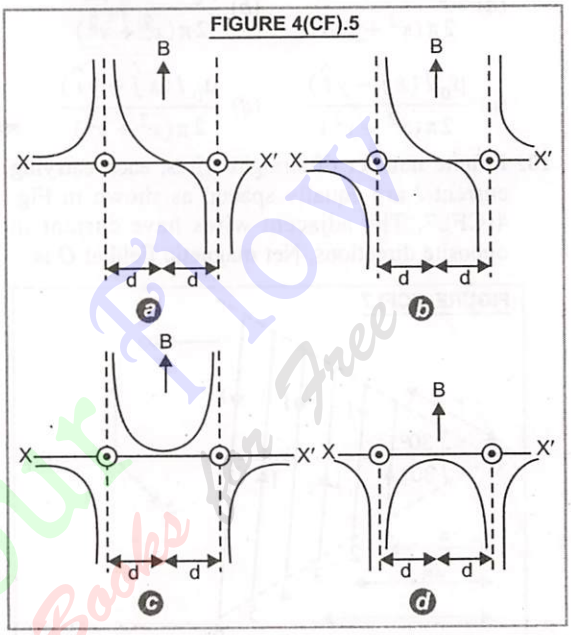


(a) 1/2 (b) 1

(c) 2/3 (d) 2.

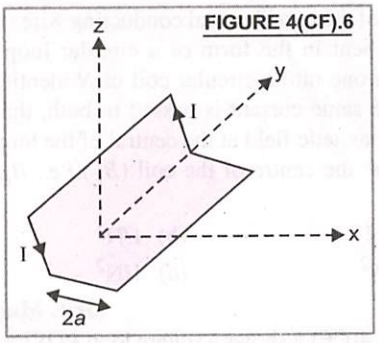
7. Two long parallel wires are at a distance 2 *d* apart. They carry steady equal currents flowing out of the plane of paper as shown in Fig. 4(CF).5. The variation of the magnetic field *B* along the line *XX′* is given by

FIGURE 4(CF).5



8. A non-planar loop of conducting wire carrying a current *I* is placed as shown in the Fig. 4(CF).6. Each of the straight sections of the loop is of length 2 *a*. The magnetic field due to this loop at the point *P* (*a*, 0, *a*) points in the direction

FIGURE 4(CF).6



(a) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$

ANSWERS

4. (d) 5. (d) 6. (c) 7. (b)

(c) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

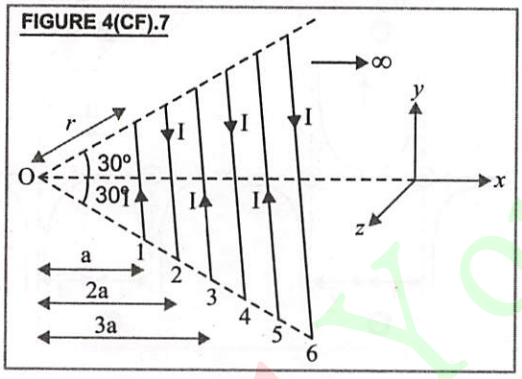
9. A long straight wire along z-axis carries a current I in the negative z-direction. The magnetic field vector $\vec{B}$ at a point having coordinates (x, y) in the $z = 0$ plane is

(a) $\frac{\mu_0 I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)}$ (b) $\frac{\mu_0 I(x\hat{i}+y\hat{j})}{2\pi(x^2+y^2)}$

(c) $\frac{\mu_0 I(x\hat{j}-y\hat{i})}{2\pi(x^2+y^2)}$ (d) $\frac{\mu_0 I(x\hat{j}+y\hat{i})}{2\pi(x^2+y^2)}$

10. Infinite number of straight wires, each carrying current I are equally spaced as shown in Fig. 4(CF).7. The adjacent wires have current in opposite directions. Net magnetic field at O is

FIGURE 4(CF).7



(a) Zero (b) $\frac{\mu_0 I}{8\pi}\frac{\log_e 2}{r\sqrt{3}}$

(c) $\frac{\mu_0 I}{4\pi}\frac{\log_e 2}{r\sqrt{3}}$ (d) $\frac{\mu_0 I}{2\pi}\frac{\log_e 2}{r\sqrt{3}}$

11. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C) *i.e.*, B_L/B_C will be :

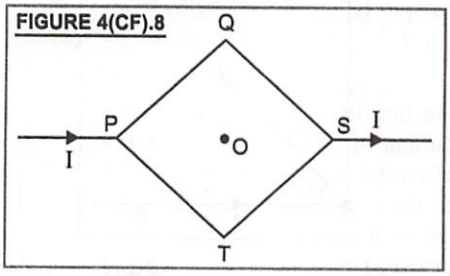
(a) N (b) 1/N

(c) N^2 (d) $1/N^2$

(JEE Main 2019)

12. Fig. 4(CF).8 shows a square loop $PQST$ with edge length r. The resistance of the wire PQS is R and that of PTS is 2 R. If the wire is uniform, the magnitude of the resultant magnetic field at the centre of the current loop is

FIGURE 4(CF).8

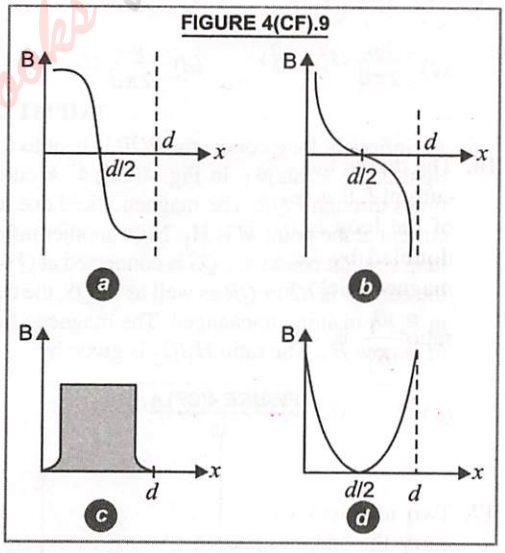


(a) $\frac{2\mu_0 I}{\pi r}$ (b) $\frac{\sqrt{2}\mu_0 I}{\pi r}$

(c) $\frac{\sqrt{2}\mu_0 I}{3\pi r}$ (d) $\frac{2\mu_0 I}{3\pi r}$

13. Two parallel beams of protons and electrons, carrying equal currents are fixed at a separation d. The protons and electrons move in opposite directions. There is a point P on the straight perpendicular line joining the two beams at a distance x from one beam. The magnetic field at this point is B. If B is plotted against x, it can be represented by the curve.

FIGURE 4(CF).9



14. Two parallel wires in the plane of paper are distance x_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance x_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast if the currents I in the

ANSWERS

8. (d) **9.** (a) **10.** (d) **11.** (d) **12.** (c) **13.** (b)

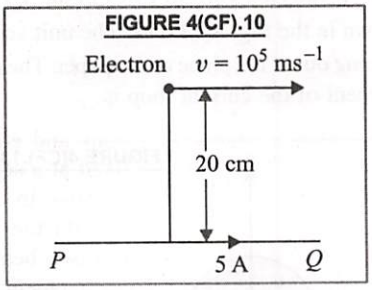
two wires have directions opposite to each other. The value of curvature of the path is R_2. If $x_0/x_1 = 3$, the value of R_1/R_2 is :

(a) 1 (b) 2 (c) 3 (d) 4

(JEE Advanced Paper 1, 2014)

15. An infinitely long straight conductor carries a current of 5 A as shown in Fig. 4(CF).10. An electron is moving with a speed of 10^5 m/s parallel to the conductor. The perpendicular distance between the electron and the conductor is 20 cm at an instant. Calculate the magnitude of the force experienced by the electron at that instant.

FIGURE 4(CF).10



(a) 8×10^{-20} N (b) 4×10^{-20} N

(c) $8\pi \times 10^{-20}$ N (d) $4\pi \times 10^{-20}$ N

(NEET 2021)

II. Magnetic field due to a circular coil carrying current

16. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

(JEE Main 2018)

17. Two identical wires A and B, each of length l carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then B_A/B_B is :

(a) $\pi^2/8$ (b) $\pi^2/16\sqrt{2}$

(c) $\pi^2/16$ (d) $\pi^2/8\sqrt{2}$

(JEE Main 2016)

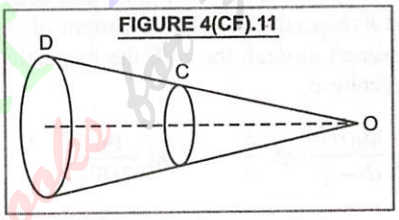
18. Charge q is uniformly distributed on a disc of radius r. If the disc is rotated with a frequency ν, then magnetic field induction at the centre will be

(a) $\frac{\mu_0 q}{\nu r}$ (b) $\frac{\mu_0 q \nu}{r}$

(c) $\frac{\mu_0 \nu}{q r}$ (d) $\frac{\mu_0}{q \nu r}$.

19. Two circular coils C and D have equal number of turns and carry equal currents in the same direction in the same sense and subtend same solid angle at point O. Fig. 4(CF).11. The smaller coil C is midway between O and D. If we represent mag- netic field induction due to bigger coil and smaller coil C at O as B_D and B_C respectively, then B_D/B_C is

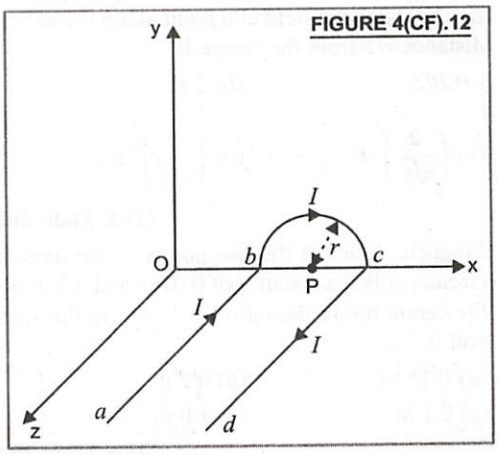
FIGURE 4(CF).11



(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

20. A long wire bent as shown in Fig. 4(CF).12, carries a current I. If the radius of the semi-circular portion is r, the magnetic field induction at the centre P is

FIGURE 4(CF).12



ANSWERS

14. (c) **15.** (a) **16.** (c) **17.** (d) **18.** (b) **19.** (c)

(a) $\frac{\mu_0 I}{4r}$ (b) $\frac{\mu_0 I}{4\pi r}\sqrt{\pi^2+4}$

(c) $\frac{\mu_0 I}{4r}+\frac{\mu_0 I}{2\pi r}$ (d) $\frac{\mu_0 I}{4\pi r}\sqrt{\pi^2-4}$

(AIPMT 2015)

21. Two similar coils of radius R and number of turns N are lying concentrically with their planes at an angle 60° to each other. The currents flowing in them are I and 2 I respectively. The resultant magnetic induction at the centre will be

(a) $\frac{\mu_0 NI}{R}$ (b) $\frac{\sqrt{3}\mu_0 NI}{2R}$

(c) $\frac{\sqrt{5}\mu_0 NI}{2R}$ (d) $\frac{\sqrt{7}\mu_0 NI}{2R}$

22. A coil of n number of turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current of strength I is passed through the coil, the magnetic field at its centre is

(a) $\frac{\mu_0 nI}{(b-a)}\log_e\frac{a}{b}$ (b) $\frac{\mu_0 nI}{2(b-a)}$

(c) $\frac{2\mu_0 nI}{b}$ (d) $\frac{\mu_0 nI}{2(b-a)}\log_e\frac{b}{a}$

(IIT 2011, Karnataka CET 2009)

23. The magnetic field at the centre of a circular coil of radius r, due to current I flowing through it, is B. The magnetic field at a point along the axis at a distance $r/2$ from the centre is

(a) $B/2$ (b) $2B$

(c) $\left(\frac{2}{\sqrt{5}}\right)^3 B$ (d) $\left(\frac{2}{\sqrt{3}}\right)^3 B$

(JEE Main 2022)

24. Magnetic fields at the two points on the axis of a circular coil at a distance of 0·05 m and 0·2 m from the centre are in the ratio 8 : 1. The radius of the coil is ..... .

(a) 0·15 m (b) 0·2 m

(c) 0·1 m (d) 1·0 m

(JEE Main 2021)

25. A current I flows in an infinitely long wire with cross section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is

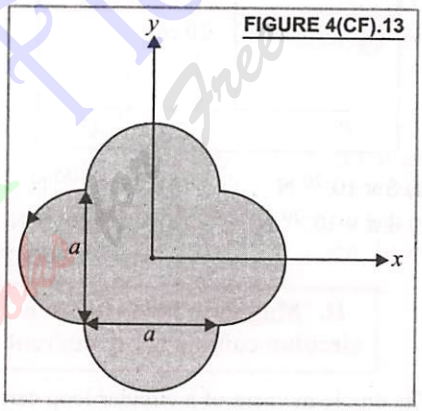
(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$

(c) $\frac{\mu_0 I}{4\pi^2 R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

(AIEEE 2011)

26. A loop carrying current I lies in the x-y plane as shown in the Fig. 4(CF).13. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

FIGURE 4(CF).13



(a) $a^2 I\hat{k}$ (b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (d) $(2\pi+1)\,a^2 I\hat{k}$

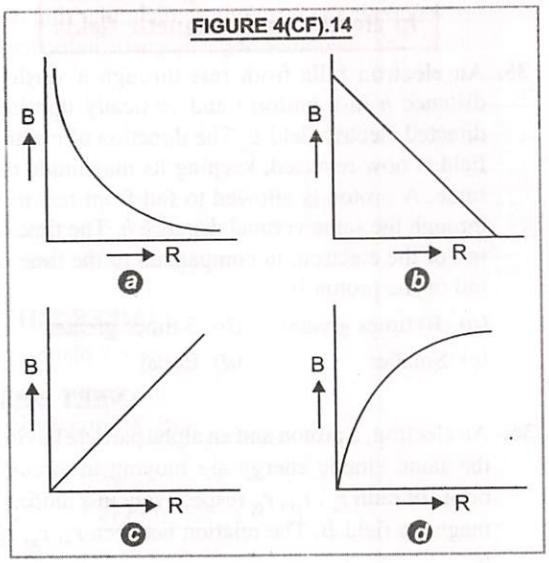
(IIT 2012)

27. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure

ANSWERS

20. (b) 21. (d) 22. (d) 23. (c) 24. (c) 25. (d) 26. (b)

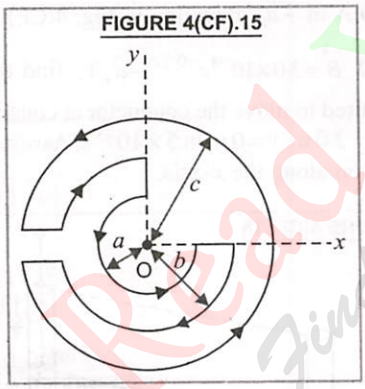
FIGURE 4(CF).14



(AIEEE 2012)

28. Refer to Fig. 4(CF).15, where $c = 3\,a$ and $a < b < c$, the magnetic field induction at point O will be zero due to current I in the various portion of the network when

FIGURE 4(CF).15



(a) $b = \frac{5}{7}a$ (b) $b = \frac{7a}{9}$

(c) $b = \frac{9a}{13}$ (d) $b = \frac{13a}{15}$

29. One of the two identical wires of length L is bent inform of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), *i.e.*, $\frac{B_L}{B_C}$ will be

(a) $\frac{1}{N}$ (b) N^2

(c) N (d) $\frac{1}{N^2}$

(JEE Main 2020)

III. Ampere circuital law, Solenoid and Toroid

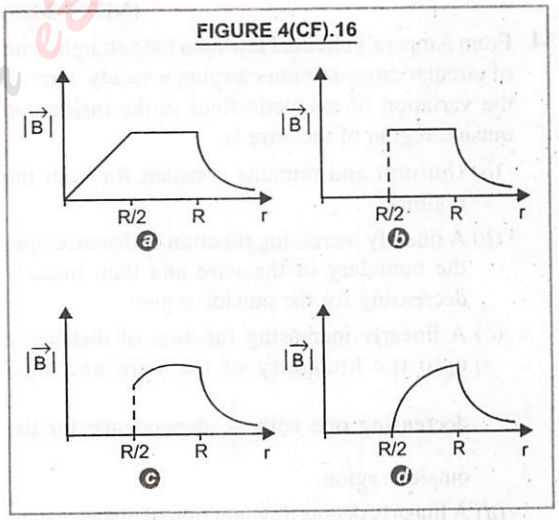
30. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $a/2$ and $2\,a$ is

(a) 1/2 (b) 1/4 (c) 4 (d) 1

(NEET 2016, AIEEE 2007)

31. An infinitely long hollow conducting cylinder with inner radius $\frac{R}{2}$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

FIGURE 4(CF).16



(IIT 2012)

32. A long straight metal rod has a very long hole of radius r_1 made parallel to the rod axis as shown

ANSWERS

27. (a) 28. (c) 29. (d) 30. (d) 31. (d)

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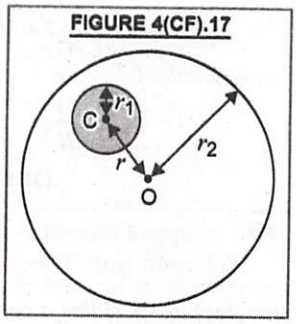
in Fig. 4(CF).17. Here, $OC = r$. If I is the current flowing through the rod, then the value of magnetic field induction on the axis of the hole is

FIGURE 4(CF).17



(a) $\frac{\mu_0 I r}{4\pi(r_2^2 - r_1^2)}$ (b) $\frac{\mu_0 I r}{2\pi(r_2^2 - r_1^2)}$

(c) $\frac{\mu_0 I r}{\pi(r_2^2 - r_1^2)}$ (d) $\frac{\mu_0 I r}{6\pi(r_2^2 - r_1^2)}$

33. A long solenoid of 50 cm length having 100 turns carries a current of 2·5 A. The magnetic field at the centre of the solenoid is :

$[\mu_0 = 4\pi \times 10^{-7}\ \text{Tm A}^{-1}]$

(a) $3{\cdot}14 \times 10^{-4}$ T (b) $6{\cdot}28 \times 10^{-5}$ T

(c) $3{\cdot}14 \times 10^{-5}$ T (d) $6{\cdot}28 \times 10^{-4}$ T

(NEET 2020)

34. From Ampere's circuital law for a long straight wire of circular cross-section carrying a steady current, the variation of magnetic field in the inside and outside region of the wire is

(a) Uniform and remains constant for both the regions.

(b) A linearly increasing function of distance upto the boundary of the wire and then linearly decreasing for the outside region.

(c) A linearly increasing function of distance r upto the boundary of the wire and then decreasing one with $\frac{1}{r}$ dependence for the outside region.

(d) A linearly decreasing function of distance upto the boundary of the wire and then a linearly increasing one for the outside region.

(NEET 2022)

IV. Force on charged particle in electric and magnetic fields

35. An electron falls from rest through a vertical distance h in a uniform and vertically upwards directed electric field E. The direction of electric field is now reversed, keeping its magnitude the same. A proton is allowed to fall from rest in it through the same vertical distance h. The time of fall of the electron, in comparison to the time of fall of the proton is

(a) 10 times greater (b) 5 times greater

(c) Smaller (d) Equal

(NEET 2018)

36. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

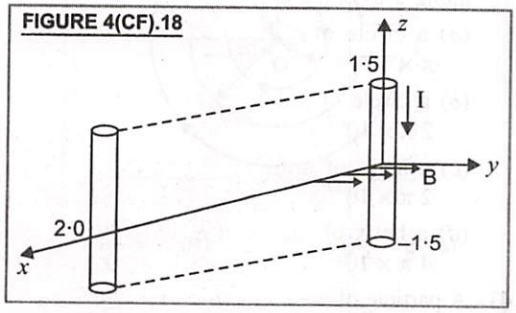
(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$

(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

(JEE Main 2018)

37. A conductor lies along the z-axis at $-1{\cdot}5 \le z < 1{\cdot}5$ m and carries a fixed current of 10·0 A in $-\hat{a}_z$ direction in Fig. 4(CF).18. For a field $\vec{B} = 3{\cdot}0 \times 10^{-4} e^{-0{\cdot}2x} - \hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2{\cdot}0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis.

FIGURE 4(CF).18



(a) 14·85 W (b) 29·7 W

(c) 1·57 W (d) 2·97 W

(JEE Main 2014)

38. A particle having the same charge as that of electron moves in a circular path of radius 0·5 cm

ANSWERS

32. (b) 33. (d) 34. (c) 35. (c) 36. (b) 37. (d)

under the influence of a magnetic field of 0·5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron = $1{\cdot}6 \times 10^{-19}$ C)

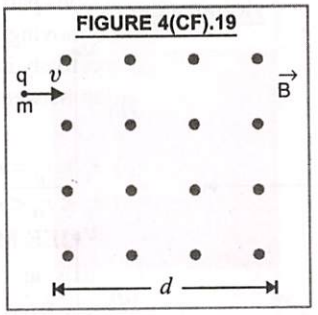
(*a*) $9{\cdot}1 \times 10^{-31}$ kg (*b*) $1{\cdot}6 \times 10^{-27}$ kg

(*c*) $1{\cdot}6 \times 10^{-19}$ kg (*d*) $2{\cdot}0 \times 10^{-24}$ kg

(JEE Main 2019)

39. A charged particle of mass m and charge q is projected into a uniform magnetic field of induction $\vec{B}$ with speed v which is perpendicular to $\vec{B}$. The width of the magnetic field is d. The impulse imparted to the particle by the field is ($d << mv/qB$)

FIGURE 4(CF).19



(*a*) qBd (*b*) qBv/d

(*c*) $mv/(qBd)$ (*d*) $mv^2/(qBd)$

40. A proton of mass $1{\cdot}67 \times 10^{-27}$ kg and charge $1{\cdot}6 \times 10^{-19}$ C is projected with a speed of 2×10^6 m/s at an angle of 60° to the X-axis. If a uniform magnetic field of 0·104 tesla is applied along Y-axis, the path of proton is

(*a*) a circle of radius 0·2 m and time period $\pi \times 10^{-7}$ s

(*b*) a circle of radius = 0·1 m and time period $2\pi \times 10^{-7}$ s

(*c*) a helix of radius = 0·1 m and time period $2\pi \times 10^{-7}$ s

(*d*) a helix of radius = 0·2 m and time period $4\pi \times 10^{-7}$ s.

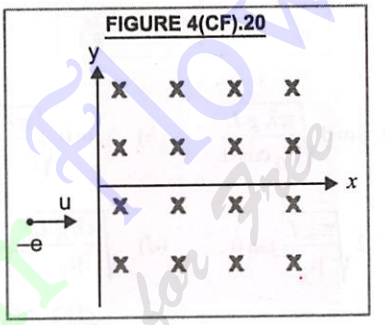
41. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The mininmum value of v required so that the particle can just enter the region $x > b$ is

(*a*) $\frac{qbB}{m}$ (*b*) $\frac{q(b-a)B}{m}$

(*c*) $\frac{qaB}{m}$ (*d*) $\frac{q(b+a)B}{2m}$

42. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field $\vec{B} = -B_0\hat{k}$ which exists to the right of y-axis, Fig. 4(CF).20. The electron exits from the region after some time with speed v at coordinate y, then

FIGURE 4(CF).20



(*a*) $v > u, y < 0$ (*b*) $v = u, y > 0$

(*c*) $v > u, y > 0$ (*d*) $v = u, y < 0$

43. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(*a*) $\vec{v} = \vec{B} \times \vec{E}/E^2$ (*b*) $\vec{v} = \vec{E} \times \vec{B}/B^2$

(*c*) $\vec{v} = \vec{B} \times \vec{E}/B^2$ (*d*) $\vec{v} = \vec{E} \times \vec{B}/E^2$

(AIEEE 2007)

44. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly [use $g = 10$ ms^{-2}]

(*a*) 1×10^{-5} V (*b*) 1×10^{-7} V

(*c*) 1×10^{-9} V (*d*) 1×10^{-10} V

(IIT 2012)

ANSWERS

38. (*d*) **39.** (*a*) **40.** (*c*) **41.** (*b*) **42.** (*d*) **43.** (*b*) **44.** (*c*)

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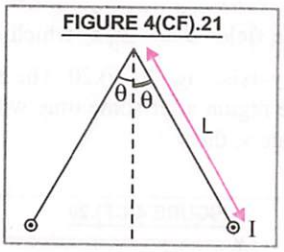
45. Two long current carrying thin wires, both with current I are held by insulating threads of length L and are in equilibrium as shown in Fig. 4(CF).21, with threads making an angle θ with the vertical. If wires have mass λ per unit length, then the value of I is : (g = gravitational acceleration)

FIGURE 4(CF).21



(a) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$

(c) $2\sqrt{\frac{\pi g L}{\mu_0}}\tan\theta$ (d) $\sqrt{\frac{\pi\lambda g L}{\mu_0}}\tan\theta$

(JEE Main 2015)

46. When a proton is released from rest in a room, it starts with an initial acceleration a_0 towards west. When it is projected towards north with a speed v_0 it moves with an initial acceleration $3\,a_0$ towards west. The electric and magnetic fields in the room are

(a) $\frac{ma_0}{e}$ east, $\frac{3ma_0}{ev_0}$ down

(b) $\frac{ma_0}{e}$ west, $\frac{2ma_0}{ev_0}$ up

(c) $\frac{ma_0}{e}$ west, $\frac{2ma_0}{ev_0}$ down

(d) $\frac{ma_0}{e}$ east, $\frac{3ma_0}{ev_0}$ up **(AIPMT 2013)**

47. In the product,

$$\vec{F} = q(\vec{v}\times\vec{B}) = q\,\vec{v}\times(B\,\hat{i} + B\,\hat{j} + B_0\,\hat{k})$$

For $q = 1$ and $\vec{v} = 2\,\hat{i} + 4\,\hat{j} + 6\,\hat{k}$

and $\vec{F} = 4\,\hat{i} - 20\,\hat{j} + 12\,\hat{k}$

What will be the complete expression for $\vec{B}$?

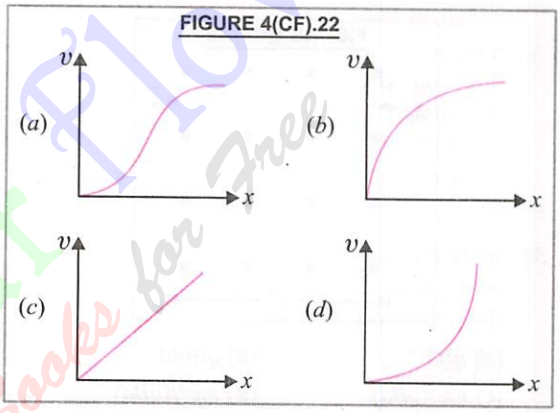
(a) $6\,\hat{i} + 6\,\hat{j} - 8\,\hat{k}$ (b) $-8\,\hat{i} - 8\,\hat{j} - 6\,\hat{k}$

(c) $-6\,\hat{i} - 6\,\hat{j} - 8\,\hat{k}$ (d) $8\,\hat{i} + 8\,\hat{j} - 6\,\hat{k}$

(NEET 2021)

48. A particle of mass m and charge q is released from rest in uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

FIGURE 4(CF).22



(JEE Main 2020)

49. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0·3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to :

(Mass of the proton = $1{\cdot}67 \times 10^{-27}$ kg, charge of the proton = $1{\cdot}67 \times 10^{-19}$ C)

(a) 4 cm (b) 2 cm

(c) 12 cm (d) 5 cm

(JEE Main 2020)

50. A proton, a deutron and an α-particle are moving with the same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is ..... and their speeds are in the ratio ..... ,

(a) 2 : 1 : 1 and 4 : 2 : 1

(b) 1 : 2 : 4 and 2 : 1 : 1

(c) 1 : 2 : 4 and 1 : 1 : 2

(d) 4 : 2 : 1 and 2 : 1 : 1 **(JEE Main 2021)**

ANSWERS

45. (b) 46. (c) 47. (c) 48. (b) 49. (a) 50. (a)

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51. Ionised hydrogen atoms and α-particles with same momenta enter perpendicular to a constant magnetic field B. The ratio of their radii of their paths $r_\mu : r_\alpha$ will be :

(*a*) 2 : 1 (*b*) 1 : 2

(*c*) 4 : 1 (*d*) 1 : 4 **(NEET 2019)**

V. Force on current carrying conductor in magnetic field

52. A metallic rod of mass per unit length 0.5 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 30° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of induction 0.25 T is acting on it in the vertical direction. The current flowing in the rod to keep it stationary is

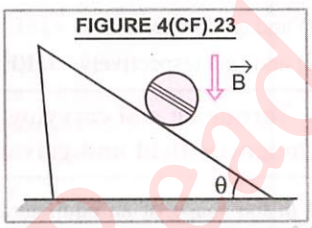
(*a*) 14.76 *A* (*b*) 5.98 *A*

(*c*) 7.14 *A* (*d*) 11.32 *A*

(NEET 2018)

53. In the Fig. 4(CF).23, a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the plane and lies in the equatorial plane of the sphere. Current in the coil is I. The value of B if the sphere is in equilibrium is

FIGURE 4(CF).23



(*a*) $mg \cos \theta/(\pi IR)$ (*b*) $mg/\pi IR$

(*c*) $mg \tan \theta/\pi IR$ (*d*) $mg \sin \theta/\pi IR$

54. An electric current runs counter clockwise in a rectangular loop around the outside edge of this page, which lies flat on your table. A uniform magnetic field is then turned on, directed parallel to the page from top to bottom, the magnetic force on the page will cause

(*a*) the bottom edge to lift

(*b*) the top edge to lift

(*c*) the left edge to lift

(*d*) the right edge to lift

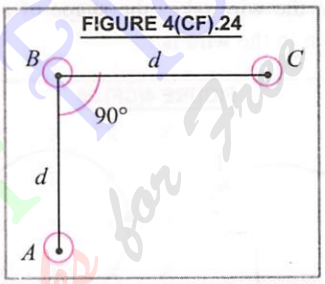
55. A horizontal rod of mass 10 g and length 10 cm is placed on a smooth plane inclined at an angle of 60° with the horizontal with the length of the rod parallel to the edge of the inclined plane. A uniform magnetic field induction B is applied vertically downwards. If the current through the rod is 1·73 ampere, the value of B for which the rod remains stationary on the inclined plane is [g = 10 m/s^2]

(*a*) 0·5 T (*b*) 1·0 T

(*c*) 1·5 T (*d*) 1·7 T

56. An arrangement of three parallel straight wires placed perpendicular to plane of paper carry same current 'I' along the same direction as shown in Fig. 4(CF).24. Magnitude of force per unit length on middle wire 'B' is given by

FIGURE 4(CF).24



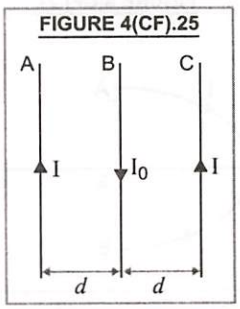
(*a*) $\frac{\mu_0 i^2}{2\pi d}$ (*b*) $\frac{2\mu_0 i^2}{\pi d}$

(*c*) $\frac{\sqrt{2}\mu_0 i^2}{\pi d}$ (*d*) $\frac{\mu_0 i^2}{\sqrt{2}\pi d}$

(NEET 2017)

57. Three very long straight current carrying conductors are placed parallel to each other as shown in 4(CF).25. The conductors A and C are fixed whereas conductor B is free to move. If the conductor B is pulled towards right through a very small distance r, then what is the net force acting on it and the angular frequency of the resulting oscillation?

FIGURE 4(CF).25



ANSWERS

51. (*a*) 52. (*d*) 53. (*b*) 54. (*b*) 55. (*b*) 56. (*d*)

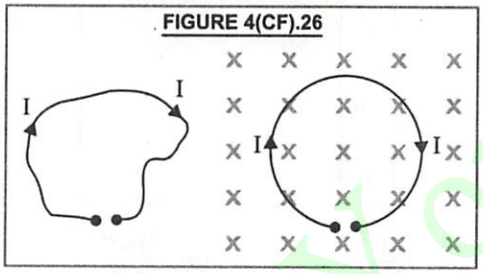
(a) $\frac{\mu_0 2II_0}{\pi d^2}, \sqrt{\frac{\mu_0 II_0}{\pi md^2}}$ (b) $\frac{\mu_0 2II_0}{4\pi r^2}, \sqrt{\frac{\mu_0 II_0}{2\pi md^2}}$

(c) $\frac{\mu_0 II_0}{2\pi r^2}, \sqrt{\frac{\mu_0 II_0}{4\pi md}}$

(d) $\frac{\mu_0 II_0}{2\pi}\left[\frac{(d+r)-(d-r)}{d^2-r^2}\right], \sqrt{\frac{\mu_0 II_0}{\pi md^2}}$

58. A thin flexible wire of length L is connected to adjacent fixed points and carries a current I in the clockwise direction, as shown in Fig. 4(CF).26. When the system is put in a uniform magnetic field of strength $\vec{B}$ going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is :

FIGURE 4(CF).26

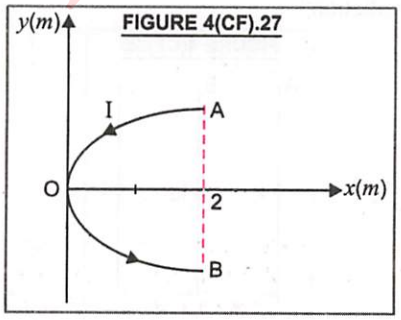


(a) IBL (b) $\frac{IBL}{\pi}$

(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$. **(IIT 2010)**

59. A conducting wire bent in the form of a parabola $y^2 = 2x$, carries a current $I = 2$ A as shown in Fig. 4(CF).27. The wire is subjected to a uniform magnetic field $\vec{B} = -2\hat{k}$ tesla. The magnetic force on the wire in (newton) is

FIGURE 4(CF).27

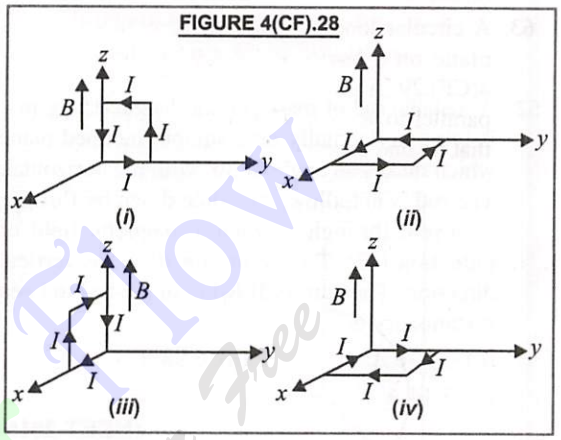


(a) $-8\hat{i}$ (b) $-16\hat{i}$

(c) $8\hat{i}$ (d) $16\hat{i}$

60. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below :

FIGURE 4(CF).28



If there is a uniform magnetic field of 0·3 T in the positive z-direction, in which orientations the loop would be in (*i*) stable equilibrium and (*ii*) unstable equilibrium.

(a) (*i*) and (*ii*) respectively

(b) (*i*) and (*iii*) respectively

(c) (*ii*) and (*iv*) respectively

(d) (*ii*) and (*iii*) respectively **(JEE Main 2015)**

VI. Torque on coil carrying current in magnetic field and galvanometer

61. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of permanent magnet. Such that $\vec{B}$ is in plane of the coil. If due to a current I in the triangle, a torque τ acts on it, the side l of the triangle is :

(a) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{BI}\right)$ (b) $2\left(\frac{\tau}{\sqrt{3}BI}\right)^{1/2}$

(c) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{BI}\right)^{1/2}$ (d) $\frac{1}{\sqrt{3}}\left(\frac{\tau}{BI}\right)$

(AIPMT 2005)

ANSWERS

57. (*d*) 58. (*c*) 59. (*d*) 60. (*c*) 61. (*b*)

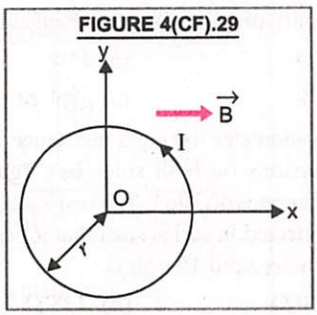
62. A conducting wire of length l is turned in the form of a circular coil and a current I is passed through it. For the torque, due to magnetic field produced at its centre, to be maximum, the number of turns in the coil will be

(*a*) one (*b*) two

(*c*) three (*d*) more than three.

63. A circular loop of mass m and radius r in X-Y plane on a horizontal table as shown in Fig. 4(CF).29. A uniform magnetic field B is applied parallel to X-axis. The current I in the loop, so that its one edge just lifts from the table is

FIGURE 4(CF).29

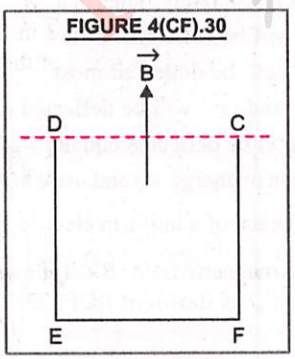


(*a*) $\frac{mg}{\pi r B}$ (*b*) $\frac{mg}{2\pi r B}$

(*c*) $\frac{mg}{\pi r^2 B}$ (*d*) $\frac{2\pi r B}{mg}$

64. A wire of cross-section A, density ρ forms three sides of a square of each length a. It is free to rotate about axis CD. It is held vertically in a uniform magnetic field $\vec{B}$ acting vertically upwards. When current I is passed through it, the structure is deflected by an angle θ from the vertical, then the value of θ is given by

FIGURE 4(CF).30

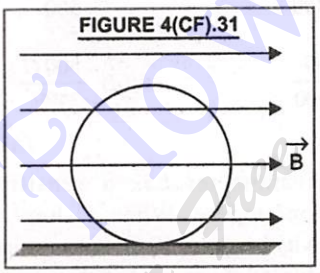


(*a*) $\sin\theta = \frac{A\rho g}{IB}$ (*b*) $\cos\theta = \frac{A\rho B}{2IB}$

(*c*) $\tan\theta = \frac{2A\rho g}{IB}$ (*d*) $\cot\theta = \frac{2A\rho g}{IB}$

65. A conducting ring of mass 100 gram and radius 0·5 m is placed on a smooth horizontal plane. The ring carries a current of I = 4 A. A horizontal magnetic field B = 10 T is switched on at time t = 0 as shown in Fig. 4(CF).31. The initial angular acceleration of the ring is

FIGURE 4(CF).31



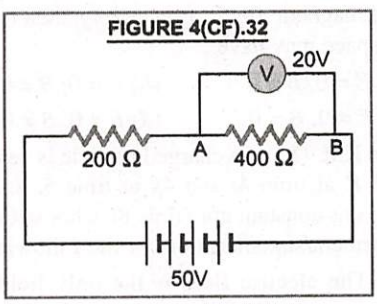
(*a*) 200 π rad/s^2 (*b*) 400 π rad/s^2

(*c*) 800 π rad/s^2 (*d*) 1200 π rad/s^2

VII. Conversion of galvanometer into ammeter and voltmeter

66. In the circuit given below, a voltmeter reads 20 V when it is connected across 400 Ω resistance. Calculate what the same voltmeter will read when connected across the 200 Ω resistance.

FIGURE 4(CF).32



(*a*) 100 V (*b*) 50 V

(*c*) 30 V (*d*) 10 V

67. A galvanometer of resistance 25 Ω is connected to a battery of 2 volt along with a resistance in series. When the value of this resistance is 3000 Ω,

ANSWERS

62. (*a*) 63. (*b*) 64. (*d*) 65. (*c*) 66. (*d*)

a full scale deflection of 30 units is obtained in the galvanometer. In order to reduce this deflection to 20 units, the resistance in series will be

(*a*) 4514 Ω (*b*) 5413 Ω

(*c*) 2000 Ω (*d*) 6000 Ω.

68. In an ammeter 0·2% of main current passes through the galvanometer. If resistance of galvanometer is G, the resistance of ammeter will be

(*a*) $\frac{G}{499}$ (*b*) $\frac{499}{500}G$

(*c*) $\frac{G}{500}$ (*d*) $\frac{500\,G}{499}$

(AIPMT 2014)

69. A galvanometer has a sensitivity of 60 division/ampere. When a shunt is used its sensitivity becomes 10 divisions/ampere. What is the value of shunt used if the resistance of the galvanometer is 20 Ω ?

(*a*) 2 Ω (*b*) 3 Ω

(*c*) 4 Ω (*d*) 6 Ω

70. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is

(*a*) $2{\cdot}535 \times 10^3$ Ω (*b*) $4{\cdot}005 \times 10^3$ Ω

(*c*) $1{\cdot}985 \times 10^3$ Ω (*d*) $2{\cdot}045 \times 10^3$ Ω

(JEE Main 2017)

71. The current sensitivity of a moving coil galvanometer increases by 35%, when its resistance is increased by a factor 3. The voltage sensitivity of galvanometer changes by a factor

(*a*) 35% (*b*) 45%

(*c*) 55% (*d*) none of the above

72. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0·005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt is

(*a*) 100 Ω (*b*) 125 Ω

(*c*) 80 Ω (*d*) 120 Ω

(JEE Main 2019)

II. Multiple Choice Questions (with One or More than One Correct Answers)

73. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represents the electric fields and magnetic fields respectively, then this region of space may have :

(*a*) $E = 0, B = 0$ (*b*) $E = 0, B \neq 0$

(*c*) $E \neq 0, B = 0$ (*d*) $E \neq 0, B \neq 0$

74. The K.E. (K) of a charged particle is zero at time t, 2 K at time $4t$ and $4k$ at time $5t$ and then it remains constant upto time $8t$. Choose the correct statement/statements out of the following :

(*a*) The electric field is the only field present between t and $4t$

(*b*) Between $5t$ and $8t$ only magnetic field is present

(*c*) Both, electric and magnetic fields may be present

(*d*) Both, electric and magnetic fields may be present between t and $4t$, but electric field is definitely present.

75. H^+, He^+ and O^{2+}, all having same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 amu, 4 amu and 16 amu respectively. Then

(*a*) H^+ will be deflected most

(*b*) O^{2+} will be deflected most

(*c*) He^+ and O^{2+} will be deflected equally

(*d*) all will be deflected equally

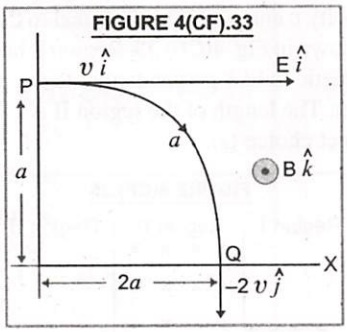
76. A particle of charge + q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and uniform magnetic field $B\hat{k}$ follows a trajectory from P to Q as shown in 4(CF).33. The velocities

ANSWERS

67. (*a*) 68. (*c*) 69. (*c*) 70. (*c*) 71. (*c*) 72. (*c*) 73. (*a,b,d*)
74. (*c,d*) 75. (*a,c*)

at P and Q are $v\hat{i}$ and $-2v\hat{j}$ respectively which of following statement(s) is/are correct ?

FIGURE 4(CF).33



(*a*) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(*b*) Rate of work done by the electric field at P is $\frac{3}{4}\left[\frac{mv^3}{a}\right]$

(*c*) Rate of work done by the electric field at $P = 0$

(*d*) Rate of work done by both fields at Q is zero

77. For a charged particle moving in a magnetic field, the velocity vector and acceleration vector at some instant are $v = 3\hat{i} + 4\hat{j}$ and $\vec{a} = 2\hat{i} + x\hat{j}$ respectively. Select the correct alternative (s) :

(*a*) $x = -1{\cdot}5$

(*b*) $x = 3$

(*c*) magnetic field is along z-direction

(*d*) K.E. of particle is constant

78. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true ?

(*a*) They will never come out of the magnetic field region

(*b*) They will come out travelling along parallel paths

(*c*) They will come out at the same time

(*d*) They will come out at different times.

(IIT 2011)

79. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is (are) correct for time $t > 0$?

(*a*) If $\theta = 0°$, the charge moves in a circular path in the x-z plane

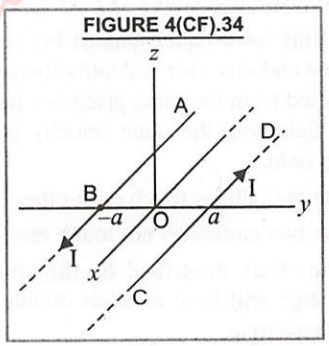
(*b*) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis

(*c*) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis

(*d*) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis **(IIT 2012)**

80. Two long parallel wires, AB and CD, carry equal currents in opposite directions. They lie in the x-y plane, parallel to the x-axis, and pass through the points $(0, -a, 0)$ and $(0, a, 0)$ respectively, Fig. 4(CF).34. The resultant magnetic field is :

FIGURE 4(CF).34



(*a*) zero on the x-axis

(*b*) maximum on the x-axis

(*c*) directed along the z-axis at the origin, but not at other points on the z-axis

(*d*) directed along the z-axis at all points on the z-axis.

81. A beam of electrons starts to accelerate from rest due to a uniform electric field in vacuum, the moving electrons :

ANSWERS

76. (*a,b,d*) **77.** (*a,c,d*) **78.** (*b,d*) **79.** (*c,d*) **80.** (*b,d*)

(*a*) initially experience a force of mtual repulsion

(*b*) experience a force of mutual attraction after travelling a certain distance

(*c*) will continue to diverge due to electrostatic repulsion

(*d*) will follow parallel lines because there is no force of attraction between them.

82. Current flows through a straight cylindrical conductor of radius r. The current is distributed uniformly over its cross-section. The magnetic field at a distance x from the axis of the conductor has magnitude B :

(*a*) $B = 0$ at the axis

(*b*) $B \propto x$ for $0 \le x \le r$

(*c*) $B \propto 1/x$ for $x > r$

(*d*) B is maximum for $x = r$

83. A microammeter has a resistance of 100 Ω and a full scale range of 50 μ A. It can be used as a voltmeter or a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s).

(*a*) 50 V range with 10 k Ω resistance in series

(*b*) 10 V range with 200 k Ω resistance in series

(*c*) 5 mA range with 1 Ω resistance in parallel

(*d*) 10 mA range with 1 Ω resistance in parallel

84. Two ions have equal masses but one is singly ionized and the other is doubly ionized. They are projected from the same place in a uniform magnetic field with the same velocity perpendicular to the field,

(*a*) the two circles touch each other

(*b*) the two circles do not touch each other

(*c*) the circle described by the singly-ionized charge will have a radius double that of the other circle

(*d*) both the ions will go along circles of equal radii.

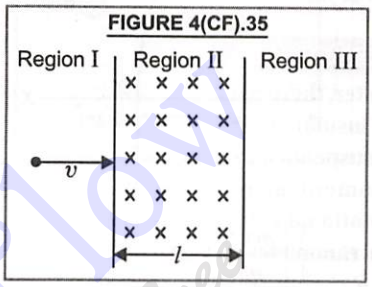
85. Let $\vec{E}$ and $\vec{B}$ denote electric and magnetic fields in a frame S and $\vec{E'}$ and $\vec{B'}$ in another frame S' moving with respect to S at a velocity $\vec{v}$. Two of the followinge quations are wrong. Identify them

(*a*) $E_y' = E_y - \dfrac{v B_z}{c^2}$ (*b*) $E_y' = E_y + v B_z$

(*c*) $B_y' = B_y + v E_z$ (*d*) $B_y' = B_y + \dfrac{v E_z}{c^2}$.

86. A particle of mass m and charge q moving with velocity v enters region II normal to the boundary as shown in Fig. 4(CF).35. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the region II is l. Choose the correct choice (*s*).

FIGURE 4(CF).35



(*a*) The particle enters region III only if its velocity $v > qlB/m$

(*b*) The particle enters region III only if its velocity $v < qlB/m$

(*c*) Path length of the particle in region II is maximum when velocity $v = \dfrac{qlB}{m}$

(*d*) Time spent in region II is same for any velocity v as long as the particle returns to region I

(IIT 2008)

87. In cyclotron, an ion is made to travel successively along semicircles of increasing radius under the action of a magnetic field. The angular velocity of the ion is independent of

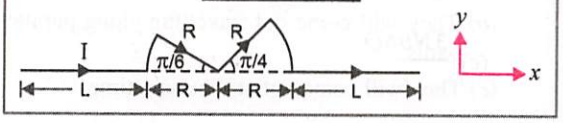
(*a*) speed of ion (*b*) radius of the circle

(*c*) mass of the ion (*d*) charge of the ion

(National Standard Exam in Physics 2012)

88. A conductor shown in Fig. 4(CF).36 carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are)

FIGURE 4(CF).36



ANSWERS

81. (*a*,*b*) 82. (*a*,*b*,*c*,*d*) 83. (*b*,*c*) 84. (*a*,*c*) 85. (*a*,*c*) 86. (*a*,*c*,*d*) 87. (*a*,*b*)

(a) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$

(b) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(c) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$

(d) If $\vec{B}$ is along $\hat{z}$, $F = 0$

(JEE Advanced 2015)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1 **In a moving coil galvanometer, there is a coil of copper having number of insulated turns N, each of area A. The coil is suspended in a radial magnetic field B. The moment of inertia of the coil about its rotational axis is I. The scale divisions in the galvanometer are n and resistance of the coil is R.**

Answer the following questons

89. If a current i_0 in the coil produces a deflection of $\pi/3$ radian to the pointer of galvanometer, the value of torsional constant of the spring is

(a) $\frac{NBAi_0}{3\pi}$ (b) $\frac{3NBAi_0}{2\pi}$

(c) $\frac{NBAi_0\pi}{3}$ (d) $\frac{3NBAi_0}{\pi}$

90. The voltage sensitivity of the galvanometer (in rad/volt) is

(a) $\frac{\pi}{3R}$ (b) $\frac{3i_0R}{\pi}$

(c) $\frac{\pi}{3i_0R}$ (d) $\frac{\pi}{i_0R}$

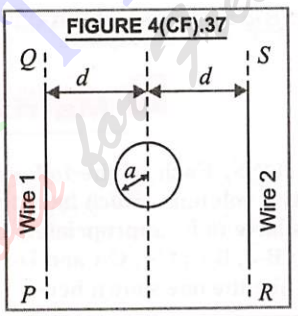
91. If the charge Q is passed almost instantaneously through the coil, the angular speed of the coil immediately after this is

(a) $\frac{NBAQ}{I}$ (b) $\frac{2NBAQ}{I}$

(c) $\frac{3NBAQ}{I}$ (d) $\frac{NBAQ^2}{I}$

Comprehension 2 **Fig. 4(CF).37 shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in plane of paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I.**
The current in the loop is in the counter clockwise direction if seen from the above.

FIGURE 4(CF).37



Answer the following questions :

92. When $d = a$, but wires are not touching the loop. It is found that the net magnetic field on the axis of the loop is zero at height h above the loop. In that case

(a) current in wire 1 and wire 2 is in the direction PQ and RS, respectively and $h = a$

(b) current in wire 1 and wire 2 is in the direction PQ and SR, respectively and $h = a$

(c) current in wire 1 and wire 2 is in the direction PQ and SR, respectively and $h = 1{\cdot}2\ a$

(d) current in wire 1 and wire 2 is in the direction PQ and RS, respectively and $h = 1{\cdot}2\ a$

(JEE Advanced 2014)

93. Consider $d >> a$ and the loop is rotated about its diameter parallel to the wires by 30° from the position in the Fig. 4(CF).45. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (consider

ANSWERS

88. (a,b,c) 89. (d) 90. (c) 91. (a) 92. (c)

that the net field due to the wires is constant over the loop)

(a) $\frac{\mu_0 I^2 a^2}{d}$ (b) $\frac{\mu_0 I^2 a^2}{2d}$ (c) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (d) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

(JEE Advanced 2014)

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices.

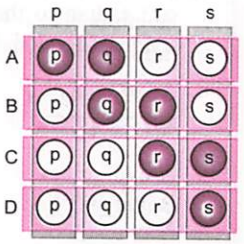
94.

	Column I		Column II
(A)	Ampere Swimming rule	(*p*)	Direction of induced current in a conductor
(B)	Fleming's Left hand rule	(*q*)	Direction of magnetic field lines due to current through circular coil
(C)	Fleming's Right hand rule	(*r*)	Direction of deflection of magnetic needle due to current in a straight conductor
(D)	Right hand thumb rule	(*s*)	Direction of force on the current carrying conductor due to magnetic field

(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*r* ; C-*s* ; D-*p* (*c*) A-*r*, B-*s* ; C-*p* ; D-*q* (*d*) A-*s* ; B-*p* ; C-*q* ; D-*r*

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*p*, A-*q* ; B-*q*, B-*r* ; C-*r*, C-*s* and D-*s*, then the correctly bubbled matrix will look like the one shown here.



95. There are some regions having electric or/and magnetic fields in column I and the possible trajectories of the positive test charge in rest/motion in column II are given :

	Column I		Column II
(A)	A positively charged particle is projected with some velocity in a region with uniform electric field	(*p*)	Straight line
(B)	A positively charged particle is projected with some velocity in a region with a uniform magnetic field.	(*q*)	Parabola
(C)	A positively charged particle is projected with a velocity lying in a plane normal to a region containing, parallel uniform electric and magnetic fields.	(*r*)	Circle
(D)	A positively charged particle is released from rest in a region containing perpendicular uniform electric and magnetic fields.	(*s*)	Helix

ANSWERS

93. (*b*) **94.** (*c*) **95.** A-*p*, *q* ; B-*p*, *q*, *r* ; C-*s* ; D-*p*, *s*

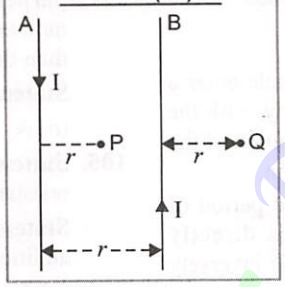
VI. Integer Type Questions

A	B	C	D
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

96. There are two infinite long parallel straight current carrying wires, A and B separated by a distance r. Fig. 4(CF).38. The current in each wire is I. The ratio of magnitude of magnetic field at points P and Q when points P and Q lie in the plane of wires is :

FIGURE 4(CF).38

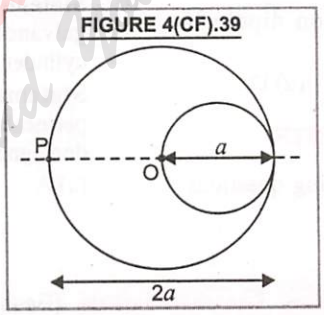


97. A length of wire carries a steady current I. It is bent first to form a circular plane coil of one turn. The same length is now bent more sharply to give double loop of smaller radius. If the same current I is passed, the ratio of the magnitude of magnetic field at the centre with its first value is :

98. A current 1 amp is flowing in the sides of an equilateral triangle of side $4{\cdot}5 \times 10^{-2}$ m. The magnetic field at the centroid of the triangle in the units of (10^{-5} T) is :

99. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2 a as shown in the Fig. 4(CF).39. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 a J$, then the value of N is **(IIT 2012)**

FIGURE 4(CF).39



100. A galvanometer gives full scale deflection with 0·006 A current. By connecting it to a 4990 W resistance, it can be converted into a voltmeter of range 0·30 V. If connected to a $\frac{2n}{249}\ \Omega$ resistance, it becomes an ammeter of range 0 – 1·5 A. The value of n is : **(JEE Advanced 2014)**

ANSWERS

96. (8) **97.** (4) **98.** (4) **99.** (5) **100.** (5)

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if :

(A) both **Assertion** and **Reason** are true and the **Reason** is the correct explanation of the **Assertion.**

(B) both **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

(C) **Assertion** is true but the **Reason** is false.

(D) both **Assertion** and **Reason** are false.

101. Assertion. If a proton and an α-particle enter a uniform magnetic field perpendicularly, with the same speed, then time period of revolution of the α-particle is double than that of proton.

Reason. In a magnetic field, the time period of revolution of a charged particle is directly proportional to mass of particle and is inversely proportional to charge of particle.

(*a*) A (*b*) B (*c*) C (*d*) D

102. Assertion. If two long parallel wires, hanging freely are connected to a battery in series, they come closer to each other.

Reason. Force of attraction acts between the two parallel wires in series carrying current.

(*a*) A (*b*) B (*c*) C (*d*) D

103. Assertion. When a magnetic dipole is placed in a non uniform magnetic field, only a torque acts on the dipole.

Reason. Force would also act on dipole if magnetic field were uniform.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if,

(A) Statement-1 is true ; Statement-2 is true ; Statement-2 is a correct explanation of Statement-1.

(B) Statement-1 is true ; Statement-2 is true ; Statement-2 is not a correct explanation of Statement-1.

(C) tatement-1 is true ; Statement-2 is false.

(D) Statement-1 is false ; Statement-2 is true.

104. Statement-1. If an electron and proton enter a perpendicular magnetic field with equal momentum, then path of electron is more curved than that of proton.

Statement-2. Electron is lighter particle.

(*a*) A (*b*) B (*c*) C (*d*) D

105. Statement-1. Higher the range, greater is the resistance of ammeter.

Statement-2. To increase the range of ammeter, additional shunt needs to be used across it.

(*a*) A (*b*) B (*c*) C (*d*) D

(JEE Main 2013)

106. Statement-1. A proton and an α-particle are projected with the same kinetic energy at right angles to a uniform magnetic field, then the α-particle will move along a circular path of smaller radius than the proton.

Statement-2. $Bqv = \frac{mv^2}{r}$

(*a*) A (*b*) B (*c*) C (*d*) D

107. Statement-1. The sensitivity of a moving coil galvanometer is increased by using a soft iron cylinder as a core inside the coil.

Statement-2. Soft iron has a high magnetic permeability and cannot be easily magnetised or demagnetised.

(*a*) A (*b*) B (*c*) C (*d*) D

(IIT 2008)

VIII. Multiple Choice Questions (Based on Experimental Skills)

108. A galvanometer with resistance 1000 Ω gives full scale deflection at 0·1 mA. What value of resistance should be added to 1000 Ω to increase its current range 10 *A*.

(*a*) 0·01 Ω in series (*b*) 0·01 Ω in parallel

(*c*) 0·10 Ω in series (*d*) 0·10 Ω in parallel

109. A galvanometer of resistance 3663 Ω gives full scale deflection for a certain current I_g. The value of the resistance of the shunt which when joined to the galvanometer coil will result in 1/34 of the

ANSWERS

101. (*a*) **102.** (*d*) **103.** (*d*) **104.** (*d*) **105.** (*d*) **106.** (*d*) **107.** (*c*) **108.** (*b*)

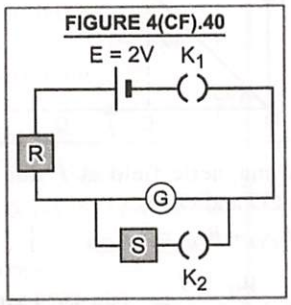
total current passing through the galvanometer is

(*a*) 100 Ω (*b*) 200 Ω

(*c*) 300 Ω (*d*) none of the above

110. In the circuit, shown in Fig. 4(CF).40, $R = 5000\ \Omega$. If key K_1 is closed, galvanometer shows a deflection of 30 scale division. On closing key K_2 and making $S = 20\ \Omega$, the deflection of galvanometer reduces to 15 division. The resistance of galvanometer is

FIGURE 4(CF).40



(*a*) 50 Ω (*b*) 30 Ω

(*c*) 20 Ω (*d*) 15 Ω

111. A galvanometer having 20 division scale and 50 Ω resistance is connected in series to a cell of e.m.f. 1·5 *V* through a resistance of 100 Ω, shows full scale deflection. The figure of merit of the galvanometer in microampere/division is

(*a*) 330 (*b*) 660

(*c*) 500 (*d*) 600

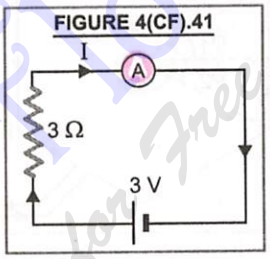
112. A resistance of 980 Ω is connected in series with a voltmeter, after which the scale division becomes 50 times larger. The resistance of voltmeter is

(*a*) 10 Ω (*b*) 20 Ω

(*c*) 100 Ω (*d*) 200 Ω

113. In the circuit shown in Fig. 4(CF).41 ammeter *A* is a galvanometer of resistance 60 Ω shunted with resistance 0·02 Ω. What is the current recorded by ammeter ?

FIGURE 4(CF).41



(*a*) 0·5 *A* (*b*) 1·0 *A*

(*c*) 2·0 *A* (*d*) 10 *A*

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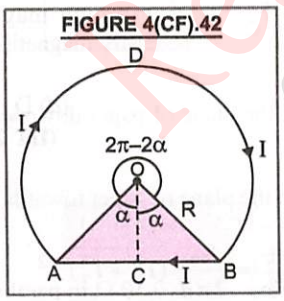
I. Multiple Choice Questions (with One Correct Answer)

1.

FIGURE 4(CF).42



Magnetic field due to current in wire *BA*,

$$B_{BA} = \frac{\mu_0 I}{4\pi\,(OC)}\,[\sin\alpha + \sin\alpha]$$

$$= \frac{\mu_0 I}{4\pi R\cos\alpha}\,(2\sin\alpha) \quad (\text{as } OC = R\cos\alpha)$$

$$\text{or } B_{BA} = \frac{\mu_0 I}{2\pi R}\tan\alpha$$

It is acting vertically downwards.

$$B_{ADB} = \left[\frac{\mu_0 I}{4\pi R}\,(2\pi - 2\alpha)\right] = \frac{\mu_0 I}{2\pi R}\,(\pi - \alpha)$$

It is acting vertically downwards.

Total magnetic field, $B = B_{BA} + B_{ADB}$

$$= \frac{\mu_0 I}{2\pi R}\,(\pi - \alpha + \tan\alpha)$$

ANSWERS

109. (*d*) **110.** (*c*) **111.** (*c*) **112.** (*b*) **113.** (*b*)

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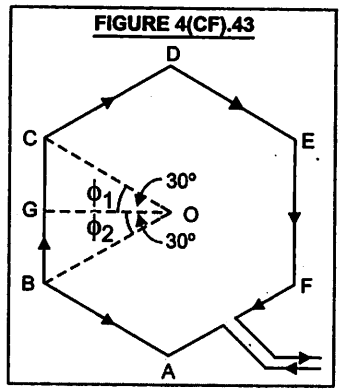
2. The edge of every side of hexagonal will make an angle 60° with the line joining the edge to the centre O. So $\phi_1 = 30°$ and $\phi_2 = 30°$.
Total magnetic field induction at O due to current through all the six sides will be

$$B = 6 \times \frac{\mu_0}{4\pi} \frac{I}{OG} (\sin\phi_1 + \sin\phi_2)$$

FIGURE 4(CF).43



Here, $OG = a\cos 30° = \sqrt{3}\,a/2$

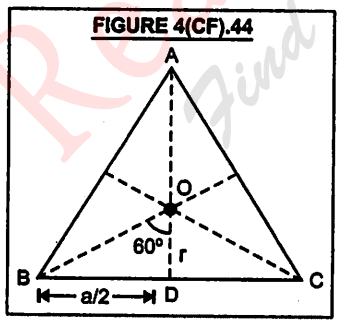
$$\therefore \quad B = \frac{6\mu_0}{4\pi} \frac{I}{(\sqrt{3}a/2)} \left(\frac{1}{2} + \frac{1}{2}\right) = \frac{\sqrt{3}\mu_0 I}{\pi a}$$

3. Refer to Fig. 4(CF).44,

In ΔOBD, $r = \frac{a}{2}\cot 60° = \frac{a}{2} \times \frac{1}{\sqrt{3}}$

Magnetic field induction at O due to current through all the three sides of a triangle will be

FIGURE 4(CF).44



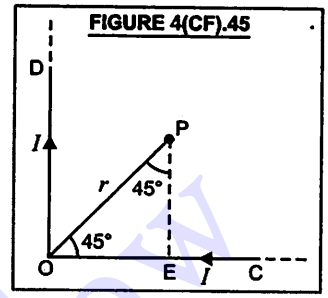
$$B = 3 \times \frac{\mu_0}{4\pi} \frac{I}{r} (\sin 60° + \sin 60°)$$

$$= 3 \times \frac{\mu_0}{4\pi} \frac{I}{\left(\frac{a}{2\sqrt{3}}\right)} \left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right]$$

$$= 3 \times \frac{\mu_0}{4\pi} \frac{I}{a} \times 2 \times 3 = 9\frac{\mu_0 I}{2\pi a}$$

4. Refer Fig. 4(CF).45. Magnetic field at P due to current through CO is same as that due to current through OD.

FIGURE 4(CF).45



$\therefore$ Total magnetic field at P due to currents through CO and OD is

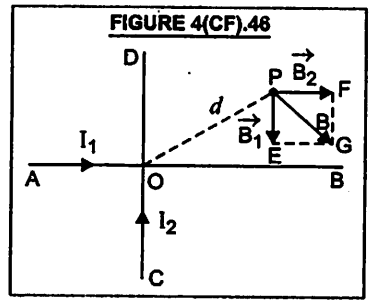
$$B = B_{CO} + B_{OD} = 2B_{CO} \qquad [\because B_{CO} = B_{OD}]$$

$$= 2 \times \frac{\mu_0}{4\pi} \frac{I}{(r/\sqrt{2})} [\sin 45° + \sin 90°]$$

$$= \frac{\mu_0}{4\pi} \frac{2I}{r} \sqrt{2} \left[\frac{1}{\sqrt{2}} + 1\right] = \frac{\mu_0}{4\pi} \frac{2I}{r} [1 + \sqrt{2}]$$

5. Refer to Fig. 4(CF).46

FIGURE 4(CF).46



$B_1 = \frac{\mu_0}{2\pi} \frac{I_1}{d}$ acting in the plane of paper downwards

$B_2 = \frac{\mu_0}{2\pi} \frac{I_2}{d}$ acting in the plane of paper towards right.

$$\therefore \quad B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$$

6. As per question, it is clear that current through QS should be $I/2$ in the direction Q to S.
Magnetic field induction at a distance r from a long straight conductor carrying current I is

$$B = \frac{\mu_0}{4\pi} \frac{I}{r} (\sin\phi_1 + \sin\phi_2)$$

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As the point M is at the end side of a long straight conductor, so $\phi_1 = 90°$ and $\phi_2 = 0°$. Therefore magnetic field induction at M due to current through PQ is

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin 90° + \sin 0°)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r},$$ acting perpendicular to the plane of paper outwards.

The magnetic field induction at M due to current in QR will be zero. As per question

$$H_1 = \frac{\mu_0}{4\pi}\frac{I}{r}$$

When straight conductor QS is connected at Q,

then $$H_2 = \frac{\mu_0}{4\pi}\frac{I}{r} + \frac{\mu_0}{4\pi}\frac{I/2}{r} = \frac{\mu_0}{4\pi}\left(\frac{3}{2}\frac{I}{r}\right)$$

So $$\frac{H_1}{H_2} = \frac{\mu_0 I/(4\pi r)}{\frac{\mu_0}{4\pi}\left(\frac{3}{2}\frac{I}{r}\right)} = \frac{2}{3}$$

7. Magnetic field induction due to a straight long conductor at a distance r away from it is given by $B = \frac{\mu_0}{4\pi}\frac{2I}{r}$. Its direction is given by Right hand thumb rule. The magnetic field induction due to current in the two wires in the space between them is in opposite direction, therefore, the net magnetic field is $$B = \frac{\mu_0}{4\pi}\frac{2I}{x}\hat{j} - \frac{\mu_0}{4\pi}\frac{2I}{(d-x)}\hat{j}$$

At mid point, $x = d/2$, the magnetic field B is zero. At $x < d/2$, the magnetic field B is along $\hat{j}$. At $x > d/2$ the magnetic field is along $-\hat{j}$. When x is negative, then the value of B is negative. When $x > d$, then the value of B is positive. Therefore, the correct option is as **shown in Fig. 4(CF).5(*b*).**

8. Refer to Fig. 4(CF).47, the direction of magnetic field induction at a point $(a, 0, a)$ for the current due to portion ABCD is pointing along z-axis *i.e.* represented by $\hat{k}$. The direction of magnetic field induction at the given point due to current in portion DEFA is pointing $+x$ axis *i.e.* represented by $\hat{i}$. Therefore, the effective direction of magnetic field induction is along the resultant of $\hat{i}$ and $\hat{k}$ *i.e.*, $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

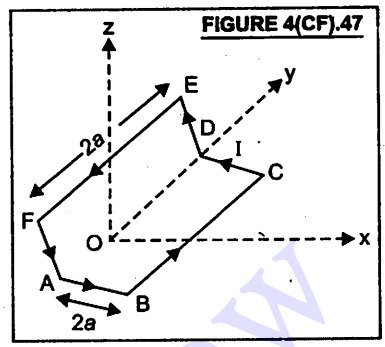

FIGURE 4(CF).47

9. Refer to Fig. 4(CF).48,

$$\vec{r} = x\hat{i} + y\hat{j}; \text{ and } \hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}}$$

Magnetic field induction at P,

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0}{2\pi}\frac{I}{\sqrt{x^2+y^2}}$$

It acts perpendicular to position vector.

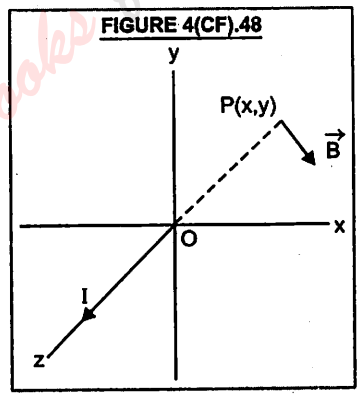

FIGURE 4(CF).48

Unit vector perpendicular to the position vector

$$\hat{n} = \hat{r}\times\hat{k} = \frac{(x\hat{i}+y\hat{j})\times\hat{k}}{\sqrt{x^2+y^2}} = \frac{y\hat{i}-x\hat{j}}{\sqrt{x^2+y^2}}$$

$$\therefore \vec{B} = |\vec{B}|\hat{n} = \frac{\mu_0 I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)}$$

10. Refer to Fig. 4(CF).49, the magnetic field at O due to straight wire carrying current is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 30° + \sin 30°]\hat{k}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{1}{2}+\frac{1}{2}\right]\hat{k} = \frac{\mu_0}{4\pi}\frac{I}{a}\hat{k}$$

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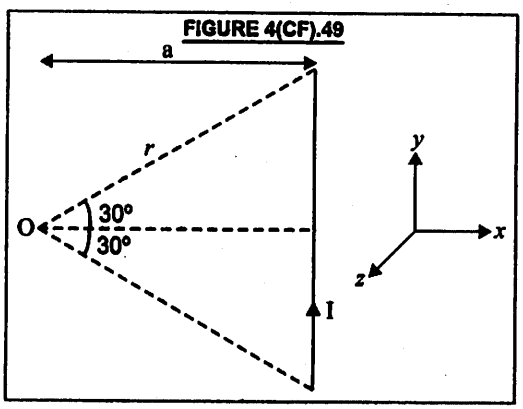
FIGURE 4(CF).49



Total magnetic field at O due to current through all the straight parallel wires carrying currents

$$\vec{B} = B_1\hat{k} + B_2(-\hat{k}) + B_3(\hat{k}) + B_4(-\hat{k}) + ...$$

$$B = B_1 - B_2 + B_3 - B_4 + ...$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\log_e(1+1) = \frac{\mu_0}{4\pi}\frac{I}{a}\log_e 2$$

Here, $\cos 30° = \frac{a}{r}$ or $a = r\cos 30° = r\sqrt{3}/2$

$$\therefore\ B = \frac{\mu_0}{4\pi}\frac{I}{(r\sqrt{3}/2)}\log_e 2 = \frac{\mu_0}{2\pi}\frac{I}{r\sqrt{3}}\log_e 2$$

11. $L = 2\pi r = 2\pi r_1$ N or $r = L/2\pi$ and $r_1 = L/2\pi$ N

$$B_L = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2(L/2\pi)};$$

$$B_C = \frac{\mu_0 N I}{2r_1} = \frac{\mu_0 N I}{2(L/2\pi N)} \quad \therefore \quad \frac{B_L}{B_C} = \frac{1}{N^2}$$

12. Let I_1 and I_2 be the currents in arms PQS and PTS respectively. Then $I = I_1 + I_2$.

As resistance of arm $PQS = R$

and resistance of arm $PTS = 2R$

Therefore, $I_1 = \frac{2I}{3}$ and $I_2 = \frac{I}{3}$

Magnetic field induction at O due to current through PQ or QS is

$$B_{PQ} = B_{QS} = \frac{\mu_0}{4\pi}\frac{I_1}{r/2}\left[\sin\frac{\pi}{4} + \sin\frac{\pi}{4}\right]$$

$$= \frac{\mu_0}{4\pi}\frac{2I/3}{r/2} \times \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\mu_0 I}{3\pi r}$$

acting perpendicular to the plane of loop downwards.

Magnetic field induction at O due to current through PT or TS is

$$B_{PT} = B_{TS} = \frac{\mu_0}{4\pi}\frac{I/3}{r/2}\left(\sin\frac{\pi}{4} + \sin\frac{\pi}{4}\right) = \frac{\sqrt{2}\mu_0 I}{6\pi r}$$

acting perpendicular to the plane of loop upwards.

The resultant magnetic field induction at O is

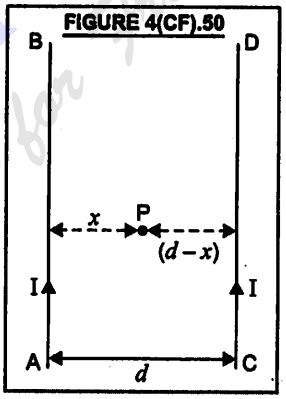
$$B = B_{PQ} + B_{QS} - B_{PT} - B_{TS}$$

$$= \frac{\sqrt{2}\mu_0 I}{3\pi r} + \frac{\sqrt{2}\mu_0 I}{3\pi r} - \frac{\sqrt{2}\mu_0 I}{6\pi r} - \frac{\sqrt{2}\mu_0 I}{6\pi r}$$

$$= \frac{\sqrt{2}\mu_0 I}{3\pi r}$$

13. The two parallel beams of protons and electrons will be equivalent to two linear parallel line wires carrying currents in the same direction. Refer Fig. 4(CF).50, the resultant magnetic field at P is

FIGURE 4(CF).50

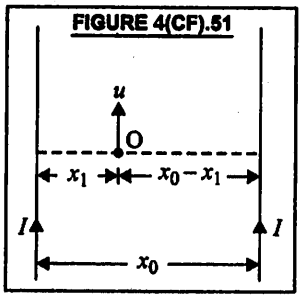


$$B = \frac{\mu_0}{4\pi}\frac{2I}{x} - \frac{\mu_0}{4\pi}\frac{2I}{(d-x)} = \frac{\mu_0}{4\pi}2I\left[\frac{1}{x} - \frac{1}{d-x}\right]$$

If $x = d/2$, $B = 0$. If $x < d/2$, B is positive and if $x > d/2$ then B is negative. Thus variation between B and x will be represented by **option (b)**.

14. Refer to Fig. 4(CF).51, when currents in the two wires are in the same direction, then magnetic field induction at O is

FIGURE 4(CF).51



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$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x_1} - \frac{1}{(x_0 - x_1)}\right] = \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - 2x_1}{x_1(x_0 - x_1)}\right]$$

When the currents in the two wires are in opposite directions, then

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x_1} + \frac{1}{(x_0 - x_1)}\right] = \frac{\mu_0 I}{2\pi}\left[\frac{x_0}{x_1(x_0 - x_1)}\right]$$

In magnetic field, the radius of curvature of curved path of the moving charged particle is related to B as

$$R \propto \frac{1}{B}$$

or $$\frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{x_0}{(x_0 - 2x_1)} = \frac{x_0/x_1}{(x_0/x_1 - 2)} = \frac{3}{3-2} = 3$$

15. Refer to Fig. 4(CF).52, the magnetic field at point A due to current in long wire PQ is

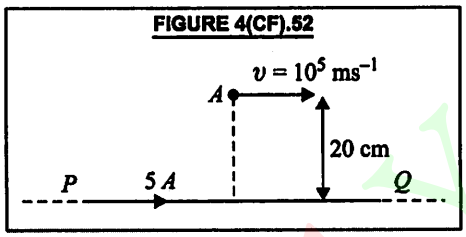


FIGURE 4.13

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 5}{(20 \times 10^{-2})} = \frac{1}{2} \times 10^{-5}\,\text{T}$$

Its direction is upwards, perpendicular to the plane of paper.

Now magnitude of force acting on the electron due to this field is

$$|\vec{F}| = q\,|\vec{v} \times \vec{B}| = qvB\sin\theta$$

Here, $v = 10^5$ ms^{-1}, $B = \frac{1}{2} \times 10^{-5}$ T,

$q = 1{\cdot}6 \times 10^{-19}$ C, $\theta = 90°$

$$\therefore\ |\vec{F}| = (1{\cdot}6 \times 10^{-19}) \times 10^5 \times \left(\frac{1}{2} \times 10^{-5}\right) \times \sin 90°$$

$$= 0{\cdot}8 \times 10^{-19}\ \text{N}$$

$$= 8 \times 10^{-20}\ \text{N}$$

16. Let R be the radius of the circular loop

Magnetic dipole moment, $m = I \times \pi R^2$

When the magnetic dipole moment is doubled (*i.e.* $m' = 2m$),

Let R' be the radius of circular loop, then

$$m' = 2m = I\pi R'^2 \text{ or } I\pi R'^2 = 2 \times I \times \pi R^2$$

or $$R' = \sqrt{2}R$$

Magnetic field $$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} = \frac{\mu_0 I}{2R}$$

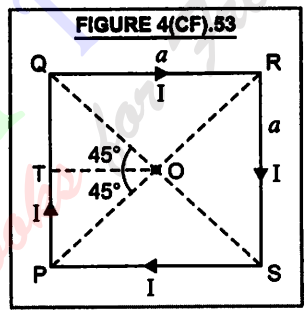
and $$B_2 = \frac{\mu_0 I}{2 \times (\sqrt{2}R)}$$

$$\therefore\ \frac{B_1}{B_2} = \sqrt{2}$$

17. For A, $l = 2\pi R$ or $R = l/2\pi$

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2 \times l/2\pi} = \frac{\mu_0 \pi I}{l}$$

For B, Refer to Fig. 4(CF).53



$l = 4a$ or $a = l/4$

$OT = a/2 = l/8$

Magnetic field at O due to current through a square is

$$B_B = 4 \times \frac{\mu_0}{4\pi}\frac{I}{(OT)}\,[\sin 45° + \sin 45°]$$

$$= 4 \times \frac{\mu_0}{4\pi}\frac{I}{l/8}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] = \frac{16\mu_0 I}{\pi\sqrt{2}\,l}$$

$$\therefore\ \frac{B_A}{B_B} = \frac{(\mu_0 \pi I/l)}{(16\mu_0 I/\sqrt{2}\,\pi l)} = \frac{\sqrt{2}\,\pi^2}{16} = \frac{\pi^2}{8\sqrt{2}}$$

18. Consider a small element of the disc of radius x width dx, Fig. 4(CF).54. Face area of the small element of disc $= 2\pi x\,dx$

Charge on the elementary part of disc,

$$dq = \frac{q}{\pi r^2} \times 2\pi x\,dx$$

or $$dq = \frac{2qx\,dx}{r^2}$$

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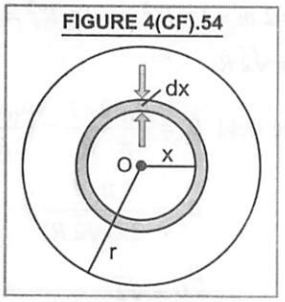
FIGURE 4(CF).54



Magnetic field induction at O due to rotation of elementary ring with frequency v is

$$dB = \frac{\mu_0}{4\pi}\frac{2\pi dq\, v}{x} \qquad (\because dI = dq \times v)$$

$$= \frac{\mu_0}{2x} \times \frac{2q\,x\,dx}{r^2} \times v = \frac{\mu_0\, q\, v}{r^2}\, dx$$

$\therefore$ Total magnetic field conduction at O due to rotation of whole disc with frequency v is

$$B = \int_0^r \frac{\mu_0 qv}{r^2}\, dx = \frac{\mu_0 qv}{r^2}(r-0) = \frac{\mu_0\, q\, v}{r}$$

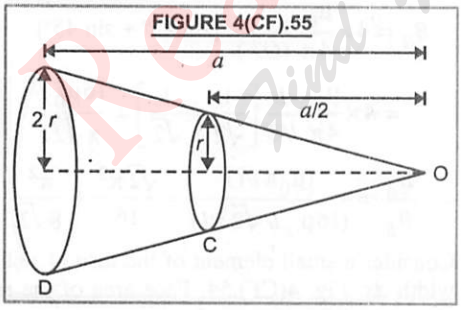
19. Since the two coils subtend the same solid angle at O, hence area of coil D = 4 × area of coil C,

[because solid angle $= \dfrac{\text{area}}{(\perp r \text{ distance})^2}$].

Therefore, radius of coil D = 2 × radius of coil C

$$\therefore \quad B_D = \frac{\mu_0}{4\pi}\frac{2\pi I (2r)^2}{[(2r)^2 + a^2]^{3/2}}$$

FIGURE 4(CF).55



$$B_C = \frac{\mu_0}{4\pi}\frac{2\pi I r^2}{[r^2 + (a/2)^2]^{3/2}}$$

$$\therefore \quad \frac{B_D}{B_C} = \frac{4}{(4r^2+a^2)^{3/2}} \times \left[\frac{4r^2+a^2}{4}\right]^{3/2} = \frac{4}{8} = \frac{1}{2}$$

20. Total magnetic field induction at P is

$$\vec{B} = \vec{B}_{ab} + \vec{B}_{bc} + \vec{B}_{cd}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{r}(-\hat{j}) + \frac{\mu_0}{4\pi}\frac{\pi I}{r}(-\hat{k}) + \frac{\mu_0}{4\pi}\frac{I}{r}(-\hat{j})$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{r}(-\hat{j}) + \frac{\mu_0}{4\pi}\frac{\pi I}{r}(-\hat{k})$$

$$\therefore \quad |\vec{B}| = \left[\left(\frac{\mu_0}{4\pi}\frac{2I}{r}\right)^2 + \left(\frac{\mu_0}{4\pi}\frac{\pi I}{r}\right)^2\right]^{1/2}$$

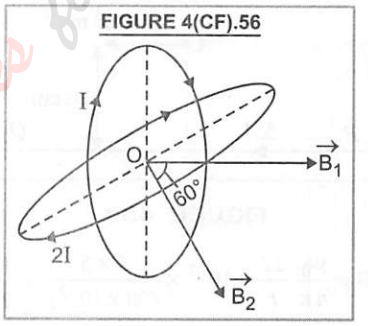
$$= \frac{\mu_0}{4\pi}\frac{I}{r}\sqrt{4+\pi^2}$$

21. Refer to Fig. 4(CF).56.

Magnetic field induction at centre O due to coil carrying current I is $B_1 = \dfrac{\mu_0 NI}{2R}$

Magnetic field induction due to other coil at O, carrying current 2 I is

FIGURE 4(CF).56



$$B_2 = \frac{\mu_0 N 2I}{2R} = \frac{\mu_0 NI}{R} = 2B_1$$

Angle between $\vec{B}_1$ and $\vec{B}_2$ is $\theta = 60°$.

Resultant magnetic field induction at O is

$$B = \sqrt{B_1^2 + B_2^2 + 2B_1 B_2 \cos 60°}$$

$$= \sqrt{B_1^2 + (2B_1)^2 + 2B_1 \times 2B_1 XY_2}$$

$$= \sqrt{7}\, B_1 = \frac{\sqrt{7}\,\mu_0 NI}{2R}$$

22. Take an element of thickness dr at a distance r from the centre of spiral coil as shown in Fig. 4(CF).57.

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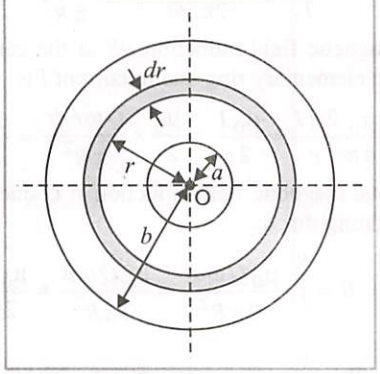
FIGURE 4(CF).57



No. of turns per unit thickness of spiral coil

$$= \frac{N}{(b-a)}$$

No. of turns in the element of thickness dr is

$$n = \frac{N}{(b-a)}\,dr$$

Magnetic field induction at the centre of the spiral coil due to current in element of thickness dr is

$$dB = \frac{\mu_0}{4\pi}\,\frac{2\pi n I}{r} = \frac{\mu_0}{2}\left(\frac{N\,dr}{b-a}\right)\frac{I}{r} = \frac{\mu_0 NI}{2(b-a)}\,\frac{dr}{r}$$

Total magnetic field induction at the centre of spiral wire due to current through the wire is

$$B = \int_a^b \frac{\mu_0 NI}{2(b-a)}\,\frac{dr}{r} = \frac{\mu_0 NI}{2(b-a)}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$$

23. Magnetic field at the centre of a circular coil of radius r, due to current I is $B = \dfrac{\mu_0}{4\pi}\,\dfrac{2\pi I}{r} = \dfrac{\mu_0 I}{2r}$

Magnetic field at a point on the axis of a circular coil at a distance x ($= r/2$) from the centre of the coil is

$$B' = \frac{\mu_0}{4\pi}\,\frac{2\pi I r^2}{(r^2+x^2)^{3/2}} = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$$

$$= \frac{\mu_0 I r^2}{2[r^2+(r/2)^2]^{3/2}} = \frac{\mu_0 I}{2r(5/4)^{3/2}} = B\left(\frac{4}{5}\right)^{3/2}$$

$$= B\left(\frac{2}{\sqrt{5}}\right)^3$$

24. Magnetic field at a point on the axis of circular coil is

$$B = \frac{\mu_0}{4\pi}\,\frac{2\pi N I R^2}{(R^2+x^2)^{3/2}} = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}$$

At $x_1 = 0{\cdot}05$ m, $B_1 = \dfrac{\mu_0 N I R^2}{2[R^2+(0{\cdot}05)^2]^{3/2}}$

At $x_2 = 0{\cdot}2$ m, $B_2 = \dfrac{\mu_0 N I R^2}{2[R^2+(0{\cdot}2)^2]^{3/2}}$

$$\frac{B_1}{B_2} = \frac{(R^2+0{\cdot}04)^{3/2}}{(R^2+0{\cdot}0025)^{3/2}}$$

As per question, $\dfrac{B_1}{B_2} = \dfrac{8}{1}$

$$\therefore \quad \frac{8}{1} = \frac{(R^2+0{\cdot}04)^{3/2}}{(R^2+0{\cdot}0025)^{3/2}}$$

or $$\left(\frac{8}{1}\right)^{2/3} = \frac{(R^2+0{\cdot}04)}{R^2+0{\cdot}0025}$$

or $$\frac{4}{1} = \frac{R^2+0{\cdot}04}{R^2+0{\cdot}0025}$$

or $4(R^2+0{\cdot}0025) = R^2+0{\cdot}04$

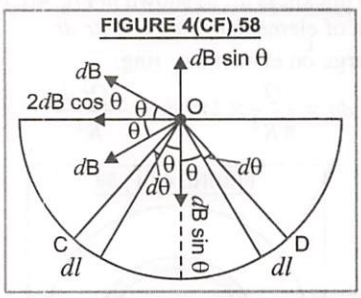
or $4R^2+0{\cdot}0100 = R^2+0{\cdot}04$

or $3R^2 = 0{\cdot}04-0{\cdot}01 = 0{\cdot}03$

or $R^2 = 0{\cdot}01$ or $R = \mathbf{0{\cdot}1\ m}$

25. Consider two elements on semicircular cross-section of long wire at C and D, each of length dl symmetrically situated with respect to point O lying on the axis of wire, having semicircular cross-section as shown in Fig. 4(CF).58.

FIGURE 4(CF).58



Current through each element,

$$dI = \frac{I}{\pi r}\,dl \quad \text{and} \quad dl = r\,d\theta\,;$$

$$\therefore \quad dI = \frac{I}{\pi r}\,r\,d\theta = \frac{I}{\pi}\,d\theta$$

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Magnitude of magnetic field induction at O due to current dI in element at C or D is

$$dB = \frac{\mu_0}{4\pi}\frac{2dI}{r} = \frac{\mu_0 dI}{2\pi r}$$

Net magnetic field induction at O due to both the elements carrying current dI is

$$= 2\,(dB)\cos\theta = 2\times\frac{\mu_0}{2\pi}\frac{dI}{r}\cos\theta$$

$$= \frac{\mu_0}{\pi r}\times\left(\frac{I}{\pi}d\theta\right)\cos\theta = \frac{\mu_0 I}{\pi^2 r}\cos\theta\, d\theta$$

Total magnetic field induction at O due to current through the entire wire is

$$B = \frac{\mu_0 I}{\pi^2 r}\int_0^{\pi/2}\cos\theta\, d\theta = \frac{\mu_0 I}{\pi^2 r}\left[\sin\frac{\pi}{2} - \sin 0\right] = \frac{\mu_0 I}{\pi^2 r}$$

26. Area enclosed by the given current loop is

A = Area of 4 semisphere each of radius $(a/2)$ plus area of a square of each side a

$$= \frac{1}{2}[4\pi(a/2)^2] + a^2 = \frac{\pi a^2}{2} + a^2 = \left(\frac{\pi}{2}+1\right)a^2$$

Area vector, $\vec{A} = \left(\frac{\pi}{2}+1\right)a^2\hat{k}$

Magnetic moment of the current loop,

$$\vec{M} = I\vec{A} = I\left(\frac{\pi}{2}+1\right)a^2\hat{k} = \left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$$

27. Face area of disc $= \pi R^2$

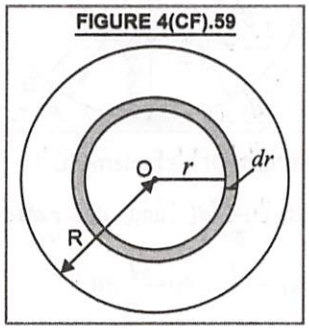
Charge per unit area of disc $= \dfrac{Q}{\pi R^2}$

Consider an elementary ring of disc of radius r and thickness dr, as shown in Fig. 4(CF).59. Face area of elementary ring $= 2\pi r\, dr$

Charge on elementary ring,

$$dq = \frac{Q}{\pi R^2}\times 2\pi r\, dr = \frac{2Qr\, dr}{R^2}$$

FIGURE 4(CF).59



Time period of rotation of disc, $T = 2\pi/\omega$

Current I associated with rotating charge dq is

$$I = \frac{dq}{T} = \frac{2Q\,r\,dr/R^2}{2\pi/\omega} = \frac{Q\omega r\, dr}{\pi R^2}$$

Magnetic field induction dB at the centre O of the elementary ring due to current I is

$$dB = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\times\frac{Q\omega r\, dr}{\pi R^2} = \frac{\mu_0 Q\omega dr}{2\pi R^2}$$

Total magnetic field induction at O due to entire rotating disc is

$$B = \int_0^R \frac{\mu_0 Q\omega dr}{2\pi R^2} = \frac{\mu_0 Q\omega R}{2\pi R^2} = \frac{\mu_0 Q\omega}{2\pi R}$$

$$\therefore\ B \propto \frac{1}{R} \qquad (\because\ Q \text{ and } \omega \text{ are unchanged})$$

Hence, variation of B and R should be a **rectangular hyperbola as represented in option (a).**

28. Magnetic field induction at O will be

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a}\times\frac{3\pi}{2}\hat{n} + \frac{\mu_0}{4\pi}\frac{I}{b}\frac{3\pi}{2}(-\hat{n}) + \frac{\mu_0}{4\pi}\frac{I}{c}2\pi(\hat{n})$$

As per question, $B = 0$.

$$\therefore\ 0 = \frac{\mu_0}{4\pi}\frac{I}{a}\frac{3\pi}{2} - \frac{\mu_0}{4\pi}\frac{I}{b}\frac{3\pi}{2} + \frac{\mu_0}{4\pi}\frac{I}{c}2\pi$$

$$\text{or}\quad 0 = \frac{\mu_0}{4\pi}I\pi\left[\frac{3}{2a} - \frac{3}{2b} + \frac{2}{c}\right]$$

$$\text{or}\quad \frac{3}{2a} - \frac{3}{2b} + \frac{2}{3a} = 0 \qquad (\text{as } c = 3a)$$

$$\text{or}\quad \frac{3}{2a} + \frac{2}{3a} = \frac{3}{2b}$$

On solving, we get,

$$b = \frac{9a}{13}$$

29. $L = 2\pi r_1 = N\,2\pi r_2$

$$\therefore\ r_1 = \frac{L}{2\pi} \quad \text{and} \quad r_2 = \frac{L}{2\pi N}$$

$$B_L = \frac{\mu_0 I}{2r_1} = \frac{\mu_0 I}{2(L/2\pi)} = \frac{\mu_0 I\pi}{L}$$

$$B_C = \frac{N\mu_0 I}{2r_2} = \frac{N\mu_0 I}{2(L/2\pi N)} = \frac{N^2\mu_0\pi I}{L}$$

$$\therefore\quad \frac{B_L}{B_C} = \frac{1}{N^2}$$

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30. Since uniform current is flowing through a straight wire, the current enclosed by the first amperian path is $i = \frac{I\pi r_1^2}{\pi R^2} = \frac{I r_1^2}{R^2}$

Using, Ampere circuital law,

$$\oint B \cdot \vec{dl} = \mu_0 i$$

$$\therefore B\, 2\pi r_1 = \mu_0 i$$

or $$B = \frac{\mu_0 i}{2\pi r_1} = \frac{\mu_0}{2\pi r_1} \times \frac{I r_1^2}{R^2} = \frac{\mu_0 I r_1}{2\pi R^2}$$

Magnetic field induction at a distance r_2 is

$$B' = \frac{\mu_0}{4\pi}\frac{2I}{r_2} \quad \therefore \quad \frac{B}{B'} = \frac{r_1 r_2}{R^2} = \frac{(a/2)\,2a}{a^2} = \mathbf{1}$$

31. Let I be the current through the long hollow conducting cylinder, as shown in Fig. 4(CF).60.

When point P is inside the hollow cylinder (i.e., $r < R/2$), then

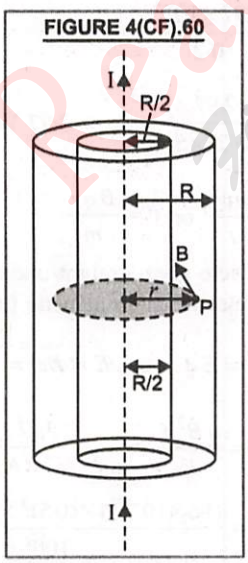
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \times \text{current threading the closed path}$$

$$B\, 2\pi r = \mu_0 \times 0 = 0 \quad \text{or} \quad B = 0$$

When point P is inbetween inner and outer part of conducting cylinders. (i.e., $R/2 \le r < R$)

$$B\, 2\pi r = \mu_0 \frac{I\pi[r^2 - (R/2)^2]}{\pi[R^2 - (R/2)^2]}$$

FIGURE 4(CF).60



$$B = \frac{\mu_0}{2\pi r}\frac{I[r^2 - (R/2)^2]}{3R^2/4} = \frac{2\mu_0 I}{3\pi R^2}\left[r - \frac{R^2}{4r}\right]$$

$$= \frac{2\mu_0}{3\pi R^2} J\pi\,[(R^2 - (R/2)^2]\left[r - \frac{R^2}{4r}\right]$$

$$[\because I = J\pi(R^2 - R^2/4)]$$

$$= \frac{\mu_0 J}{2}\left[r - \frac{R^2}{4r}\right], \text{ i.e., } B \propto (r - R^2/4r)$$

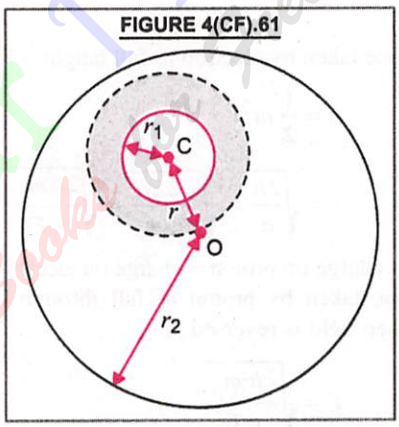
When point P is outside the cylinder, i.e., $r > R$, then

$$B\, 2\pi r = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r}, \text{ i.e., } B \propto \frac{1}{r}$$

Therefore, **option (*d*) is true.**

32. Draw a cylindrical gaussian surface whose axis lies at the axis of hole as shown in Fig. 4(CF).61.

FIGURE 4(CF).61



Let r be the radius of this gaussian surface.

Area of crosssection of this gaussian surface $= \pi r^2$

Current threading the gaussian surface

$$I' = \frac{I \times \pi r^2}{\pi(r_2^2 - r_1^2)} = \frac{I r^2}{(r_2^2 - r_1^2)}$$

Magnetic field induction at point C is

$$B = \frac{\mu_0}{4\pi}\frac{2I'}{r} = \frac{\mu_0}{4\pi}\frac{2}{r}\frac{I r^2}{(r_2^2 - r_1^2)} = \frac{\mu_0 I r}{2\pi(r_2^2 - r_1^2)}$$

33. Here, $l = 50$ cm $= 0{\cdot}50$ m, $N = 100$, $I = 2{\cdot}5$ A

Magnetic field at the centre of the solenoid is

$$B = \frac{\mu_0 N I}{l} = \frac{(4\pi \times 10^{-7}) \times 100 \times 2{\cdot}5}{0{\cdot}50}$$

$$= \mathbf{6{\cdot}28 \times 10^{-4}\ T}$$

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34. For a long straight wire of radius R, carrying current I, the magnetic field at a distance r ($< R$), from the axis of the wire for a point inside the wire, according to Ampere's circuital law is

$$B = \frac{\mu_0}{2\pi}\frac{Ir}{R^2} \quad i.e., \; B \propto r$$

For a point outside the wire, at distance r from the axis of the wire,

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \quad i.e., \; B \propto \frac{1}{r}$$

Thus, **option (c) is true.**

35. Acceleration of electron while falling in the electric field, $a = \dfrac{eE}{m_e}$

Time taken by electron to fall height h is

$$h = \frac{1}{2}at^2$$

or $$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h}{eE/m_e}} = \sqrt{\frac{2hm_e}{eE}}$$

As charge on proton = charge on electron = e, so time taken by proton to fall through height h when field is reversed is

$$t' = \sqrt{\frac{2hm_p}{eE}}$$

$$\therefore \quad \frac{t'}{t} = \sqrt{\frac{m_p}{m_e}} > 1 \quad \text{or} \quad t' > t$$

It means electron will take **smaller time to fall.**

36. $F = qvB = \dfrac{mv^2}{r}$ or $r = \dfrac{mv}{qB}$

Kinetic energy, $K = \dfrac{1}{2}mv^2$ or $mv = \sqrt{2mK}$

$$\therefore \quad r = \frac{\sqrt{2mK}}{qB} \quad \text{or} \quad r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \quad r_e : r_p : r_\alpha = \frac{\sqrt{m_e}}{q_e} : \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} : \frac{\sqrt{4m_p}}{2e} = \frac{\sqrt{m_e}}{e} ; \frac{\sqrt{m_p}}{e} : \frac{\sqrt{m_p}}{e}$$

As mass of electron (m_e) is least so r_e is least.

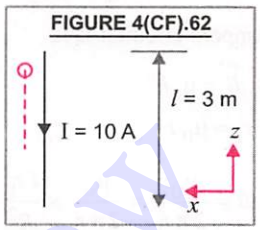
but $r_p = r_\alpha$ So $r_e < r_p = r_\alpha$

37. Refer to Fig. 4(CF).62,

Here, $l = 1{\cdot}5 + 1{\cdot}5 = 3{\cdot}0$ m ; $I = 10{\cdot}0$ A,

Average Power $= \dfrac{\text{work}}{\text{time}}$

FIGURE 4(CF).62



$$W = \int_0^2 F\,dx = \int_0^2 BIl\,dx$$

$$= \int_0^2 3{\cdot}0\times10^{-4}e^{-0{\cdot}2x}\times10\times3\,dx$$

$$= 9\times10^{-3}\times\int_0^2 e^{-0{\cdot}2x}\,dx$$

$$= 9\times10^{-3}\left[\frac{e^{-0{\cdot}2x}}{-0{\cdot}2}\right]_0^2 = 9\times10^{-3}\left[\frac{e^{-0{\cdot}2\times2}-1}{-0{\cdot}2}\right]$$

$$= \frac{9\times10^{-3}}{0{\cdot}2}\times[1-e^{-0{\cdot}4}] = \frac{9\times10^{-3}}{0{\cdot}2}\times0{\cdot}33$$

$$= \frac{2{\cdot}97\times10^{-3}}{0{\cdot}2}\ \text{J}$$

$$P = \frac{2{\cdot}97\times10^{-3}}{(0{\cdot}2)\times5\times10^{-3}} = \mathbf{2{\cdot}97\ W}$$

38. $Bqv = \dfrac{mv^2}{r}$ or $v = \dfrac{Bqr}{m}$

when particle goes straight under the combined effect of electric and magnetic field, then

$$Bqv = Eq \quad \text{or} \quad E = Bv = B\left(\frac{Bqr}{m}\right)$$

or $$m = \frac{qB^2r}{E}$$

$$= \frac{(1{\cdot}6\times10^{-19})\times(0{\cdot}5)^2\times(0{\cdot}5\times10^{-2})}{100}$$

$$= \mathbf{2\times10^{-24}\ kg}$$

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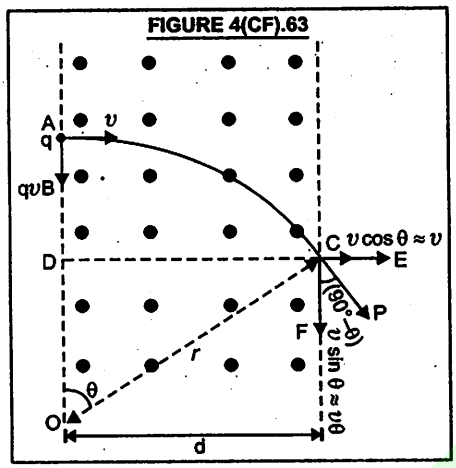
39. Force on the charged particle in magnetic field is $F = q\,v\,B$, acting in the direction as shown in Fig. 4(CF).63. It provides the required centripetal force. Due to it, the charged particle describes a circular path AC of radius r with centre at O. Then

$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB} \quad \text{or} \quad \frac{v}{r} = \frac{qB}{m}$$

FIGURE 4(CF).63



In Fig. 4(CF).63, $d = r \sin\theta$

When $d << r$, $\sin\theta \approx \theta = d/r$

Change in velocity of charged particle from A to C is, $\Delta v = v \sin\theta = v\,\theta$ (for small θ), acting along CF.

Thus impulse imparted = change in momentum

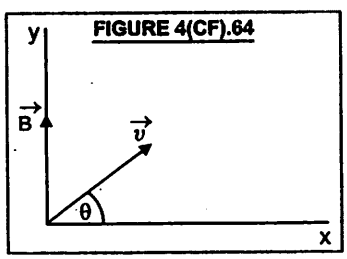
$$= \Delta p = m\Delta v = mv\theta = mv\,(d/r) = md\,(qB/m)$$
$$= qBd$$

40. Since the proton is entering the magnetic field at some angle other than 90°, its path is **helix.**

Component velocity of proton along X-axis, Fig. 4(CF).64

$v_x = v \cos 60° = 2 \times 10^6 \times 1/2 = 10^6\ \text{ms}^{-1}$.

FIGURE 4(CF).64



Due to component velocity v_x, the radius of the helix is described and is given by the relation

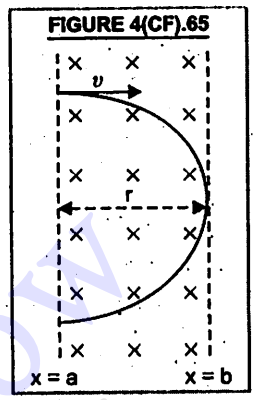
$$r = \frac{m\,v_x}{q\,B} = \frac{1{\cdot}67 \times 10^{-27} \times 10^6}{1{\cdot}6 \times 10^{-19} \times 0{\cdot}104} = 0{\cdot}1\,\text{m}$$

Now $T = \dfrac{2\pi r}{v_x} = \dfrac{2\pi \times 0{\cdot}1}{10^6}$

$$= 2\pi \times 10^{-7}\ \text{s}$$

41. Refer to Fig. 4(CF).65,

FIGURE 4(CF).65



$$\frac{mv^2}{r} = qvB$$

or $\dfrac{mv}{b-a} = qB$

or $v = \dfrac{qB(b-a)}{m}$

42. Given $\vec{u}$ and $\vec{B}$ are $\perp r$ to each other. Force on electron due to magnetic field is, $\vec{F} = -e(\vec{u} \times \vec{B})$.

Since this force acts perpendicular to $\vec{u}$, therefore, it does not do any work. Due to it, the magnitude of velocity of electron remains constant. Due to the perpendicular magnetic field acting on the moving electron, the electron will describe a circular path in the magnetic field in clockwise direction. As a result of it, the **condition (d) i.e., $v = u$ and $y < 0$ is satisfied.**

43. When charged particle goes undefected, then

$qE = q\,v\,B$ or $v = E/B$

The two forces acting on moving charged particle due to electric and magnetic fields oppose each other if $\vec{v}$ is along $\vec{E} \times \vec{B}$

Now, $v = \dfrac{EB}{B^2} = \dfrac{EB\sin 90°}{B^2} = \dfrac{|\vec{E} \times \vec{B}|}{B^2}$

$\therefore \quad \vec{v} = \dfrac{(\vec{E} \times \vec{B})}{B^2}$

44. Refer to Fig. 4(CF).66,

X = potential difference applied between two large vertical plates held distance d = 1 cm = 10^{-2} m apart.

Electric field intensity in a region between the two plates, $E = \dfrac{\text{pot.diff.}}{\text{distance}} = \dfrac{X}{10^{-2}}\ \text{V/m}$

When proton is released at O, the force on it due to electric field = $q\,E$ acting along the direction of field, *i.e.*, along OA.

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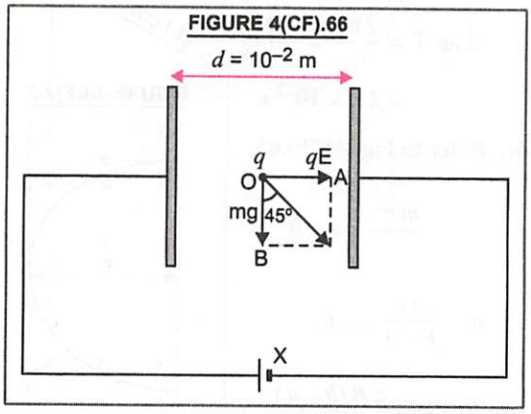
FIGURE 4(CF).66



Weight of proton = mg acting vertically downwards, *i.e.*, along OB.

Since proton moves at 45° to the vertical just after the release, so $\tan 45° = qE/mg$

or $qE = mg$ or $q\dfrac{X}{d} = mg$

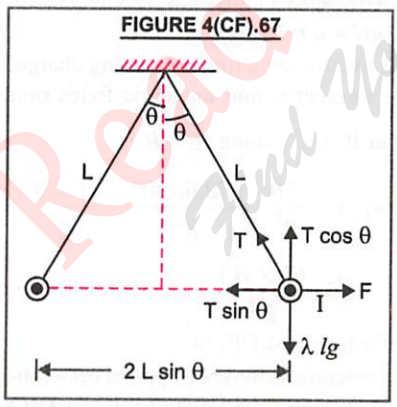
or $X = \dfrac{mg\,d}{q} = \dfrac{(1{\cdot}6\times10^{-27})\times10\times10^{-2}}{1{\cdot}6\times10^{-19}}$

$= \mathbf{1\times10^{-9}\ V}$

45. Refer to Fig. 4(CF).67, let l be the length of a wire under study. In equilibrium state

$$T\cos\theta = mg = \lambda l g \quad \text{or} \quad l = \frac{T\cos\theta}{\lambda g} \qquad ...(i)$$

FIGURE 4(CF).67



$$T\sin\theta = F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}l$$

$$= \frac{\mu_0}{2\pi}\frac{I\times I}{(2L\sin\theta)}\times\frac{T\cos\theta}{\lambda g}$$

or $$I = 2\sin\theta\sqrt{\frac{\pi L\lambda g}{\mu_0\cos\theta}}$$

46. In this question, the proton moves from rest towards west. It is due to a force on proton by virtue of electric field along west.

Acceleration of proton due to electric field

$$= \frac{eE}{m} = a_0 \quad \text{or} \quad E = \frac{ma_0}{e} \text{ west}$$

When proton is projected towards north with a speed v_0, it moves with an acceleration 3 a_0 towards west, shows that the proton is experiencing forces due to electric field along west and magnetic field acting vertically downwards. Therefore, the acceleration of proton due to magnetic field = 3 $a_0 - a_0 = 2\ a_0$.

Force on proton due to magnetic field

$$m\,(2\ a_0) = e\,v_0\,B$$

or $$B = \frac{2ma_0}{ev_0} \textbf{ downwards}$$

Choice (*c*) is correct.

47. Given, $q = 1$, $\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k}$,

$$\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}, \quad \vec{B} = B\hat{i} + B\hat{j} + B_0\hat{k}$$

$$(\vec{v}\times\vec{B}) = (2\hat{i} + 4\hat{j} + 6\hat{k})\times(B\hat{i} + B\hat{j} + B_0\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B & B & B_0 \end{vmatrix}$$

$$= \hat{i}\,(4B_0 - 6B) + \hat{j}\,(6B - 2B_0) + \hat{k}\,(2B - 4B)$$

$$= (4B_0 - 6B)\,\hat{i} + (6B - 2B_0)\,\hat{j} - 2B\hat{k}$$

Given, $\vec{F} = q\,(\vec{v}\times\vec{B})$

$\therefore\ (4\hat{i} - 20\hat{j} + 12\hat{k})$

$$= 1[(4B_0 - 6B)\,\hat{i} + (6B - 2B_0)\,\hat{j} - 2B\hat{k}]$$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ on both the sides, we have

$4 = 4B_0 - 6B$ or $2B_0 - 3B = 2$ ...(*i*)

$-20 = 6B - 2B_0$ or $B_0 - 3B = 10$ ...(*ii*)

$12 = -2B$ or $B = -6$ ...(*iii*)

From (*ii*), $B_0 = 10 + 3B = 10 + 3\times(-6) = -8$

Hence, $\vec{B} = -6\hat{i} - 6\hat{j} - 8\hat{k}$

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48. Force on the particle in the electric field, $F = qE$

Work done on the particle by electric field when particle moves through distance $x = qEx$

Gain in KE of particle $= \frac{1}{2}mv^2$

Using work-energy theorem, we have

$$qEx = \frac{1}{2}mv^2 \quad \text{or} \quad v^2 = \frac{2qE}{m}x$$

which is an equation of parabola.

Thus **option (*b*) is true.**

49. Pitch $= v\cos 60° \times T = v \times \frac{1}{2} \times \frac{2\pi m}{qB}$

$$= (4\times10^5)\times\frac{1}{2}\times\frac{(2\times3{\cdot}14)\times(1{\cdot}67\times10^{-27})}{(1{\cdot}67\times10^{-19})\times0{\cdot}3}$$

$$= \frac{4\times3{\cdot}14}{0{\cdot}3}\times10^{-3} \approx 4\times10^{-2}\ \text{m} \simeq \mathbf{4\ cm}$$

50. Momentum, $p = mv$ or $v = p/m$

Force on moving charged particle in magnetic field, $F = qvB = q(p/m)B = qpB/m$

$\therefore$ $F_p = q_p pB/m_p$, $v_p = p/m_p$

$F_d = q_d pB/m_d$, $v_d = p/m_d$

$F_\alpha = q_\alpha pB/m_\alpha$, $v_\alpha = p/m_\alpha$

$$\therefore F_p : F_d : F_\alpha = \frac{q_p}{m_p} : \frac{q_d}{m_d} : \frac{q_\alpha}{m_\alpha} = \frac{q}{m} : \frac{q}{2\text{m}} : \frac{2q}{4\text{m}}$$

$$= 1 : \frac{1}{2} : \frac{1}{2} = \mathbf{2:1:1}$$

$$v_p : v_d : v_\alpha = \frac{1}{m_p} : \frac{1}{m_d} : \frac{1}{m_\alpha} = \frac{1}{m} : \frac{1}{2\text{m}} : \frac{1}{4\text{m}}$$

$$= 1 : \frac{1}{2} : \frac{1}{4} = \mathbf{4:2:1}$$

51. $Bqv = \frac{mv^2}{r}$ or $r = \frac{mv}{Bq} = \frac{p}{Bq}$ $\therefore$ $r \propto \frac{1}{q}$

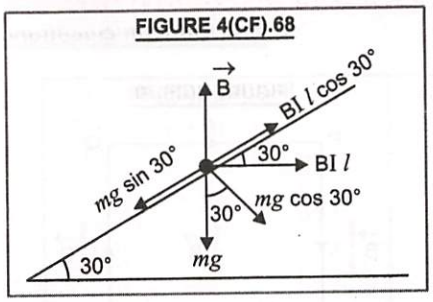
Hence, $\frac{r_\mu}{r_\alpha} = \frac{q_\alpha}{q_\mu} = \frac{2e}{l} = \frac{2}{1}$

52. Refer to Fig. 4(CF).68, as the rod is just in equilibrium state, so

$mg \sin 30° = BIl \cos 30°$

or $I = \frac{mg}{Bl}\tan 30° = \frac{0{\cdot}5\times9{\cdot}8}{0{\cdot}25\times1}\times\frac{1}{\sqrt{3}} = \mathbf{11{\cdot}32\ A}$

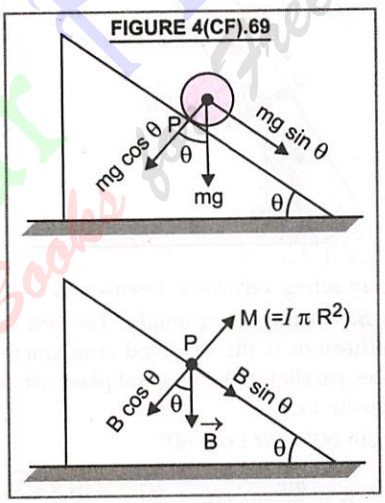
FIGURE 4(CF).68



53. Refer to Fig. 4(CF).69, Magnetic dipole moment of the coil carrying current, $M = I\pi R^2$. Its direction is perpendicular to the plane upwards.

Resolving $\vec{B}$ into two rectangular components we have, $B\cos\theta$ acts perpendicular to the plane downwards and $B\sin\theta$ acts along the plane downwards.

FIGURE 4(CF).69



For rotational equilibrium of sphere about point P, $(mg\sin\theta)R = M(B\sin\theta)$

or $mgR\sin\theta = I\pi R^2 B\sin\theta$ or $B = \dfrac{mg}{\pi IR}$

54. The force on a linear conductor carrying current is $F = IlB\sin\theta$

where θ is the smaller angle between $I\vec{l}$ and $\vec{B}$. In Fig. 4(CF).70, for arm PS and RQ, $\theta = 0°$ and $180°$, and $\sin\theta = 0$, so no force acts on them.

The force on arm $QP = IlB\sin 90°$ acting perpendicular to the plane of paper upwards.

The force on arm $SR = IlB\sin 90°$ acting perpendicular to the plane of paper downwards.

Therefore, the **top edge QP of loop will be lifted up.**

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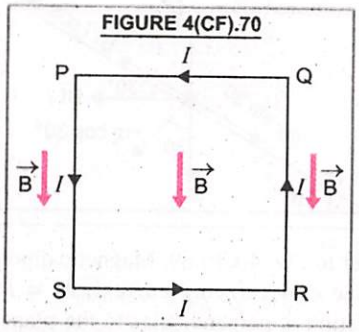
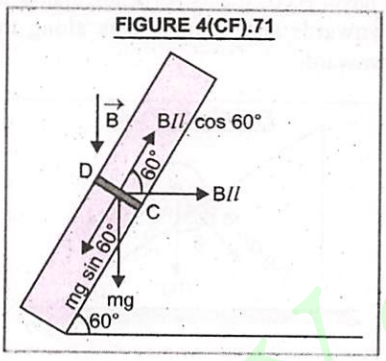
FIGURE 4(CF).70



55. Refer Fig.4(CF).71, the forces acting on the rod are

FIGURE 4(CF).71



(*i*) mg acting vertically downwards

(*ii*) $BI\,l$ acting horizontally. The rod will be in equilibrium if the resolved components of the forces parallel to the inclined plane are equal and opposite *i.e.*,

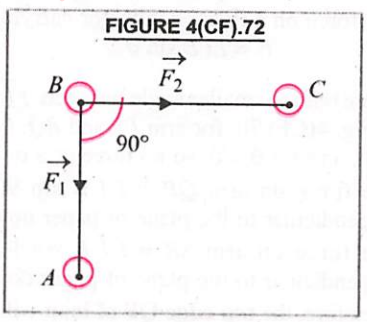
$mg \sin 60° = BI\,l \cos 60°$

or $B = \dfrac{mg}{Il} \tan 60° = \dfrac{10^{-2} \times 10 \times \sqrt{3}}{1{\cdot}73 \times 0{\cdot}1} = 1\,\text{T}$

56. Refer to Fig. 4(CF).72, $AB = BC = d$

Magnetic force acting per unit length of wire B due to currents in wires A and B is

FIGURE 4(CF).72



$$F_1 = \frac{\mu_0}{4\pi} \frac{2I \times I}{d} = \frac{\mu_0 I^2}{2\pi d} \text{ along } BA$$

Magnetic force acting per unit length on wire B due to currents in wires B and C is

$$F_2 = \frac{\mu_0 I^2}{2\pi d} \text{ along } BC$$

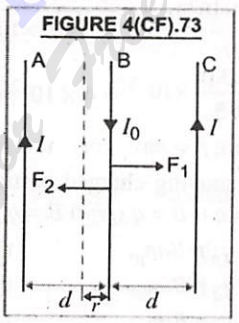
$\therefore$ Net magnetic force per unit length on the wire B is

$$F = \sqrt{F_1^2 + F_2^2} = F_1\sqrt{2} \qquad (\because F_1 = F_2)$$

$$F_2 = \frac{\mu_0 I^2}{2\pi d}\sqrt{2} = \frac{\mu_0 I^2}{\sqrt{2}\pi d}$$

57. The net force acting on the conductor after it gets displaced through a very small distance r is given by

FIGURE 4(CF).73



$$F = |F_1 - F_2| = \left| \frac{\mu_0 I I_0}{2\pi(d-r)} - \frac{\mu_0 I I_0}{2\pi(d+r)} \right|$$

$$= \frac{\mu_0 I I_0}{2\pi}\left[\frac{(d+r)-(d-r)}{d^2 - r^2}\right]$$

As it is given that $r << d$, so $F = \dfrac{\mu_0 I I_0 r}{\pi d^2}$

Now $mr\omega^2 = \dfrac{\mu_0 I I_0 r}{\pi d^2} \Rightarrow \omega^2 = \dfrac{\mu_0 I I_0}{\pi m d^2}$

$$\omega = \sqrt{\frac{\mu_0 I I_0}{\pi m d^2}}$$

58. Consider a small element of length dL of the wire of radius R, as shown in Fig. 4(CF).74

Force on the element : $B\,I\,(dL) = 2\,T \sin\theta$

$B\,I\,(R\,2\,\theta) = 2\,T\,\theta \qquad (\because \theta \text{ is small})$

or $T = BIR = BI\dfrac{L}{2\pi} \qquad (\because L = 2\pi R)$

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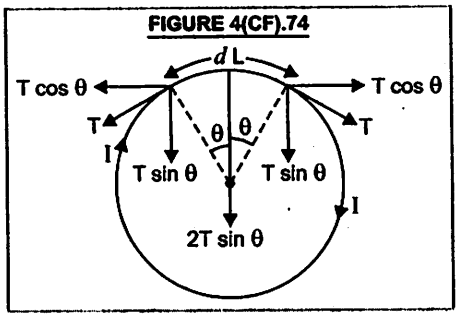
FIGURE 4(CF).74



59. When $x = 2$ m ;

$$y^2 = 2 \times 2 = 4 \text{ or } y = 2 \text{ m}$$

$\therefore$ length $AB = 2 + 2 = 4$ m

This bent conducting wire will be of vector length

$$\vec{l} = \overrightarrow{AB} = 4(-\hat{j}) = -4\hat{j}$$

Force on wire, $\vec{F} = I\,\vec{l} \times \vec{B}$

$$= 2[(-4\hat{j}) \times (-2\hat{k})] = 16\hat{i}$$

60. Torque on the coil, $\tau = IAB \sin\theta = MB \sin\theta$

where θ is the angle between $\vec{M}$ and $\vec{B}$. For equilibrium, torque is zero.

In case of stable equilibrium, $\vec{M}$ is parallel to $\vec{B}$ and in unstable equilibrium $\vec{M}$ is antiparallel to $\vec{B}$. The same is true for fig. (*ii*) and fig. (*iv*) respectively.

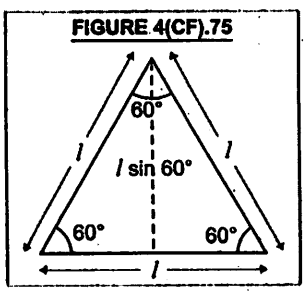
61. $\tau = IAB \sin 90° = IAB$

[Angle between A and B is 90°]

Area, $A = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times l \times l \sin 60° = \frac{\sqrt{3}}{4} l^2$$

FIGURE 4(CF).75



$$\tau = I\left(\frac{\sqrt{3}}{4} l^2\right) B \quad \text{so} \quad l^2 = \frac{4\tau}{\sqrt{3}\,BI}$$

So $$l = 2\left(\frac{\tau}{\sqrt{3}\,BI}\right)^{1/2}$$

62. Let n be the no. of turns formed of circular coil of radius r, in length l of the wire. Then

$$l = (2\pi r)\, n \quad \text{or} \quad r = \frac{l}{2\pi n}$$

Maximum torque, $\tau_{max.} = n\, I\, AB = n\, I\, \pi r^2 B$

$$= n\, I\, \pi \times (l/2\pi n)^2 B = \frac{l^2 I B}{4\pi n}$$

τ will be maximum if n is minimum. If will be so if $n = 1$.

63. Here, area of the coil $A = \pi r^2$. The direction of $\vec{A}$ is normal to the plane of coil, making an angle 90° with the field direction. Therefore, the torque on the coil is $\tau = BIA = BI\pi r^2$

Since only one edge of circular coil lifts from the table, the torque required is $= \frac{mg}{2} r$

$$\therefore \quad BI\pi r^2 = \frac{mgr}{2} \quad \text{or} \quad I = \frac{mg}{2\pi r B}$$

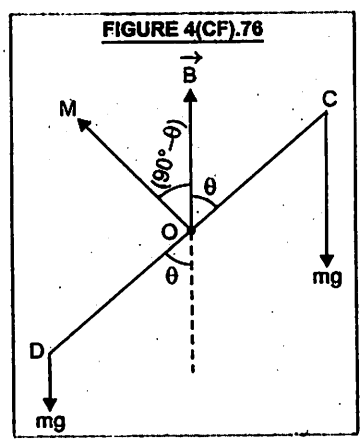
64. Mass of each side of square, $m = A\, a\, \rho$

Magnetic moment of current loop, $M = I\, a^2$;

acting perpendicular to the plane of current loop.

In the deflected position of current loop, Fig. 4(CF).76, the perpendicular distance of C or D from axis of rotation $= \frac{a}{2} \sin\theta$.

FIGURE 4(CF).76



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Perpendicular distance of weight mg of wire EF from axis of rotation $CD = a \sin\theta$

Torque on current square loop in magnetic field

$= MB \sin(90° - \theta) = MB \cos\theta$

$= I a^2 B \cos\theta$

Torque on loop due to gravity

$= 2\, mg\, (a/2) \sin\theta + mg\, a \sin\theta$

$= 2\, mg\, a \sin\theta = 2\, A\, a\, \rho\, g \times a \sin\theta$

$= 2\, A\, a^2\, \rho\, g \sin\theta$

In deflected position, the above two torques are equal and opposite, so

$I a^2 B \cos\theta = 2\, A\, a^2\, \rho\, g \sin\theta$

or $\cot\theta = \dfrac{2A\rho g}{IB}$

65. Here, mass of ring, $m = 100$ g $= 0{\cdot}1$ kg ; radius of ring, $R = 0{\cdot}5$ m ; $I = 4$ A, $B = 10$ T

Magnetic moment of ring, $M = I(\pi R^2)$ acting perpendicular to the plane of ring.

Torque on the ring due to current

$$\tau = MB \sin 90° = I \pi R^2 B$$

Moment of inertia of ring, about an axis of rotation tangent to the ring

$$I' = \frac{1}{2} m R^2$$

If α is the initial angular acceleration of the ring, then initial torque on ring

$$\tau' = I'\alpha = \frac{1}{2} m R^2 \times \alpha$$

$$\therefore \quad \frac{1}{2} m R^2 \alpha = I \pi R^2 B$$

or $\alpha = \dfrac{2 I \pi B}{m} = \dfrac{2 \times 4 \times \pi \times 10}{0{\cdot}1} = \mathbf{800\,\pi\ rad/s^2}$

66. Let G be the resistance of voltmeter connected in parallel to 400 Ω resistance. Net resistance

$$R_P = \frac{400G}{400+G}$$

Now, Resistance 200 Ω and R_P are in series. Total effective resistance

$$R_S = 200 + \left(\frac{400G}{400+G}\right)$$

Now, current in the circuit $I = \dfrac{V}{R_S}$

$$I = \frac{50}{200 + \left(\dfrac{400G}{400+G}\right)} = \frac{50(400+G)}{200(400+G) + 400G}$$

Pot. difference between A and $B = IR_p = 20$

$$\therefore \quad \frac{50(400+G)}{200(400+G)+400G} \times \frac{400G}{400+G} = 20$$

or $\dfrac{50}{200[400+3G]} \times 400G = 20$

$100\,G = 8000 + 60\,G$

or $G = 200\ \Omega$

When the voltmeter is connected in parallel to the resistance 200 Ω, Net resistance becomes

$$R_p' = \frac{200 \times 200}{200+200} = 100\,\Omega$$

Total resistance of the circuit = 100 + 400 = 500 Ω

Current $I' = \dfrac{50}{500} = \dfrac{1}{10}$ A

Reading of voltmeter = Potential difference across R_p'

$$= I' R_p' = \frac{1}{10} \times 100 = \mathbf{10\ V}$$

67. Current through galvanometer,

$$i_g = \frac{V}{R+G} = \frac{2}{3000+25} = k \times 30 \qquad \text{...(i)}$$

Where k is figure of merit of galvanometer. The current coresponding to 20 unit deflection of galvanometer,

$$i = \frac{i_g}{30} \times 20 = \frac{2}{3} i_g = \frac{2}{3} \times \frac{2}{3025}\ \text{A}$$

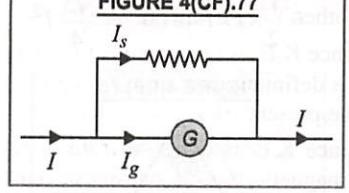
If R' is the resistance to be used in series of galvanometer, then

$$i = \frac{V}{R'+G} \quad \text{or} \quad \frac{2}{3} \times \frac{2}{3025} = \frac{2}{R'+25}.$$

On solving, we get, $R' = \mathbf{4514\ \Omega}$

68. Refer to Fig. 4(CF).77, Fractional current passing through galvanometer,

FIGURE 4(CF).77



$$\frac{I_g}{I} = \frac{S}{G+S} = \frac{0{\cdot}2}{100} = \frac{1}{500} \quad \text{or} \quad G + S = 500\,S \qquad \text{...(i)}$$

Resistance of ammeter $= \dfrac{GS}{G+S} = \dfrac{GS}{500\,S} = \dfrac{G}{500}$

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69. Here, $I \propto 60$ and $I_g \propto 10$

$$\therefore \quad \frac{I_g}{I} = \frac{10}{60} = \frac{1}{6} \; ; \; G = 20\ \Omega$$

As, $\frac{I_g}{I} = \frac{S}{G+S}$

$$\therefore \quad \frac{1}{6} = \frac{S}{20+S}$$

On solving, we get $S = 4\ \Omega$

70. Here, $I_g = 5 \times 10^{-3}$ A ; $G = 15\ \Omega$, $V = 10$ V,

$$R = \frac{V}{I_g} - G = \frac{10}{5\times10^{-3}} - 15$$

$$= 2000 - 15 = \mathbf{1{\cdot}985 \times 10^3\ \Omega}$$

71. Given $I_s' = I_s + \frac{35}{100} I_s = \frac{135}{100} I_s$

Initial voltage sensitivity, $V_s = \frac{I_s}{R}$

New voltage sensitivity, $V_s' = \frac{I_s'}{R'}$

$$= \left(\frac{135}{100} I_s\right) \times \frac{1}{3R}$$

$$= \frac{9}{20} V_s$$

% decrease in voltage sensitivity

$$= \left(\frac{V_s - V_s'}{V_s}\right) \times 100 = \frac{V_s - \frac{9}{20} V_s}{V_s} \times 100$$

$$= \mathbf{55\%}$$

72. Current for full scale deflection,

$I_g = 30 \times 0{\cdot}005 = 0{\cdot}150$ A

A galvanometer can be converted into a voltmeter of given range by connecting a resistance R in series, given by

$$R = \frac{V}{I_g} - G = \frac{15}{0{\cdot}15} - 20 = \mathbf{80\ \Omega}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

73. When $E \neq 0$, the velocity of proton changes as $F_e = qE$ when $B \neq 0$, velocity of proton may be parallel or antiparallel to that of field as such $F_m = 0$ and velocity of proton does not change. Thus, : (*a*) When $E = 0$, $B = 0$, $F_e = F_m = 0$, velocity doesn't change

(*b*) When $E = 0$, $B \neq 0$, proton moves along $\vec{B}$ and velocity doesn't change.

(*d*) When $E \neq 0$, $B \neq 0$, F_e and F_m may balance each other.

74. (*a*) Since K.E. is increasing from t to $4t$, electric field is definitely present but magnetic field may also be present. Hence *incorrect* statement.

(*b*) Since K.E. is constant from $5t$ to $8t_1$ either only magnetic field is present or both fields are present but cancel out each other hence *incorrect* statement.

Thus, **'c' and 'd' are correct statements.**

75. (*c*) As $r = \frac{mv}{qB}$ and $K = \frac{1}{2} mv^2$, $r = \frac{\sqrt{2mK}}{qB}$

$$i.e., \; r \propto \frac{\sqrt{m}}{q} \qquad \text{[for same } K \text{ and } B\text{]}$$

$$r_{H^+} \propto \frac{\sqrt{1}}{1}, \; r_{He^+} \propto \frac{\sqrt{4}}{1} \text{ and } r_{O^{2+}} \propto \frac{\sqrt{16}}{2} \propto \frac{\sqrt{4}}{1}$$

r_{He^+} and $r_{O^{2+}}$ are equal, So **He^+ and O^{2+} will be deflected equally.**

(*a*) Since $\frac{r_{H^+}}{r_{He^+}} = \frac{\sqrt{1}/1}{\sqrt{4}/1} = \frac{1}{2}$,

$$r_{H^+} = \frac{1}{2} \, He^+ = \frac{1}{2} \, r_{O^{2+}}$$

H^+ will be deflected most.

76. (*a*) Increase in K.E. = work done by electric field

$$\frac{1}{2} m (2v)^2 - \frac{1}{2} mv^2 = qE \, (2a)$$

or $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

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(*b*) Rate of work done by electric field at P
= Electric force × Velocity at P

$$= qE \times v = q\left[\frac{3}{4}\frac{mv^2}{qa}\right]v = \frac{3}{4}\frac{mv^3}{a}$$

(*d*) At Q, force acting on particle due to electric field is zero (as $2v\hat{j}$ is perpendicular to both $E\hat{i}$ and $B\hat{k}$) and hence **rate of work done by both fields is zero.**

77. (*a*) Since magnetic force (F_m) is always perpendicular to the velocity vector $\vec{v}$, *i.e.*,

$$\vec{F}_m \perp \vec{v} \text{ or } \vec{a} \perp \vec{v} \text{ or } \vec{a}\cdot\vec{v} = 0$$

or $(2\hat{i} + x\hat{j})\cdot(3\hat{i} + 4\hat{j}) = 0,\ x = -1{\cdot}5.$

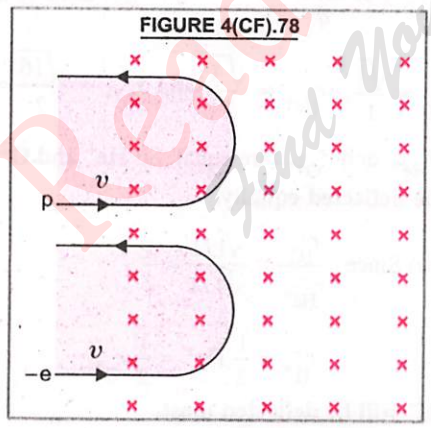
(*c*) Since $\vec{B}$ is ⊥ to the plane containing the velocity vector, in this case (XY-plane), $\vec{B}$ **is along Z-direction.**

(*d*) Since work done by magnetic force is zero, So **K.E. of particle remains constant.**

78. Both will travel in semicircular path as shown in Fig. 4(CF).78, and they will **come out of magnetic field travelling along parallel paths.**

Time spent by the charged particle to describe semicircular path in the magnetic field is

FIGURE 4(CF).78



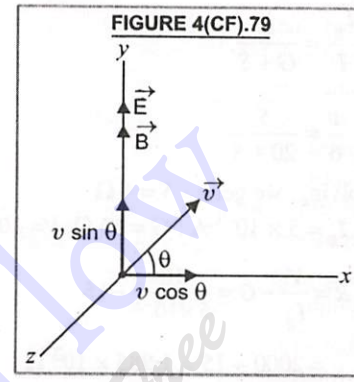
$$t = \frac{\pi m}{Bq} \quad \text{or} \quad t \propto m$$

As mass (m) of electron is different from that of proton, hence **time for their semicircular paths will be different.**

79. Here, $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$.

Let m, q be the mass and charge of the positive point charge which is moving with velocity $\vec{v}$ in the electric and magnetic fields acting along Y-axis.

FIGURE 4(CF).79



Resolving $\vec{v}$ into two rectangular components we have, $v\cos\theta$ acts along X-axis and $v\sin\theta$ acts along Y-axis.

Due to component velocity $v\sin\theta$, the charged particle is accelerated due to electric field with acceleration, $a_y = E_0\, q/m$ along Y-axis.

The particle does not experience any force due to magnetic field.

Due to component velocity $v\cos\theta$, the particle is accelerated due to electric field with acceleration, $a_y = E_0\, q/m$ along Y-axis.

The particle experiences maximum force due to magnetic field and describes a circular path with time period, $T = 2\pi\, m/Bq$ which is independent of r and v.

Due to combined electric and magnetic fields along Y-axis, **when $\theta = 10°$,** the particle will describe a helical path whose pitch increases with time along the Y-axis (***i.e.*, option (*c*) is true).** **When $\theta = 90°$,** the particle does not experience any force due to magnetic field. Hence it is accelerated along y-axis due to electric field alone. Thus **option (*d*) is true.**

80. The magnetic field at a point on x-axis due to currents through AB and CD will be maximum, given by

$$B = B_1 + B_2 = \frac{\mu_0}{4\pi}\frac{2I}{a} + \frac{\mu_0}{4\pi}\frac{2I}{a} = \frac{\mu_0 I}{a}$$

acting along z-axis

The resultant magnetic field will be directed along the **z-axis at all points on the z-axis.**

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81. The beam of electrons consists of number of electrons. In moving beam of electrons two types of forces play their part. (*i*) Electrostatic respulsion due to like charges on electrons and (*ii*) Magnetic attraction, as the two moving electrons in the same direction behave as two parallel linear conductors carrying current in the same direction. Initially electrostatic respulsion is more effective than the magnetic attraction as speed of electrons is small. When speed of electrons becomes high, the magnetic attraction becomes more prominent than electrostatic repulsion.

82. Magnetic field induction at a point inside the cylindrical conductor is $B \propto x$

Magnetic field induction at a point outside the cylindrical conductor is $B \propto 1/x$

Magnetic field induction at a point on the axis of cylindrical conductor is zero.

For $x = r$, B is maximum.

83. Here, $G = 100\ \Omega$; $I_g = 50 \times 10^{-6}\ A = 5 \times 10^{-5}\ A$

For a voltmeter of range 50 V,

$$R = \frac{50}{5\times10^{-5}} - 100 = 10^6 - 10^2 = 9{\cdot}999 \times 10^5\ \Omega$$

For a voltmeter of range 10 *V*,

$$R = \frac{10}{5\times10^{-5}} - 100 = 199{,}900\ \Omega$$

$$\approx \mathbf{200\ k\Omega\ in\ series}$$

For an ammeter of range 5 mA,

$$S = \frac{5\times10^{-5}\times100}{5\times10^{-3} - 5\times10^{-5}} \approx \mathbf{1\ \Omega\ in\ parallel}$$

For an ammeter of range 10 mA

$$S = \frac{5\times10^{-5}\times100}{10\times10^{-3} - 5\times10^{-5}} = 0{\cdot}5\ \Omega \text{ in parallel}$$

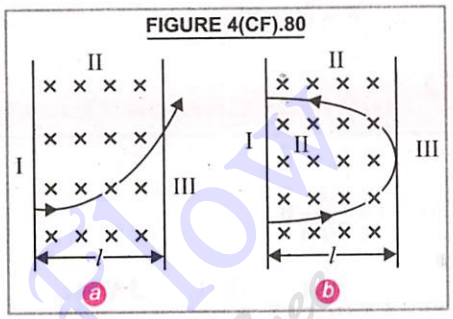
84. When a charged particle is subjected to a perpendicular magnetic field, it will describe a circular path of radius, $r = mv/B\,q$ *i.e.* $r \propto 1/(q/m)$. Thus **option (*c*) is true.** As time period of revolution, $T = \dfrac{2\pi m}{B\,q}$ is independent of v and r, so the particle will return to its initial position after one revolution, hence the two particles of different charges will return to their initial positions after one revolution and **their paths will touch each other there.**

85. Check the accuracy of each relation by the method of dimensions.

86. In region II, the magnetic field is uniform and velocity of the particle in region II is normal to the magnetic field, so the particle will describe a circular path of radius r given by, $r = \dfrac{mv}{Bq}$

For particle to enter region III, Fig. 4(CF).80(*a*)

FIGURE 4(CF).80



$$r > l \quad \text{or} \quad \frac{mv}{Bq} > l \quad \text{or} \quad v > \frac{\mathbf{Bql}}{\mathbf{m}}$$

For path length of particle in region II to be maximum, $l = r$ or $v = \dfrac{\mathbf{Bql}}{\mathbf{m}}$

The time of revolution of charged particle in region II is $t = \dfrac{\pi r}{v} = \dfrac{\pi l}{v} = \dfrac{\pi m}{Bq}$ which is independent of v. Therefore, all those particles moving with any velocity, which **return to region I, spend the same time in region II.**

87. Here, as we know in cyclotron centripetal force on charge is equal to magnetic force $F_c = F_m$

So $\dfrac{mv^2}{r} = qvB \Rightarrow \dfrac{v}{r} = \dfrac{qB}{m}$

$$\left[\text{As, } v = r\omega, \text{ So } \frac{v}{r} = \omega\right]$$

$$\therefore \quad \omega = \frac{qB}{m} \qquad [\textit{i.e.}, \textbf{Independent of } v, r]$$

88. While finding the magnetic force on a current carrying wire in magnetic field, the vector length of the wire is taken. Here, the vector length of the wire is

$$\vec{l} = (L + R + R + L)\,\hat{i} = 2(L + R)\,\hat{i}$$

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Force on conductor of length l, carrying current I when placed in a magnetic field $\vec{B} = (B\hat{k})$ is

$$\vec{F} = I[\vec{l} \times \vec{B}] = I\,[2(L+R)\,\hat{i} \times B\hat{k}]$$

$$= 2IB(L+R)\,(-\hat{j})$$

$$\therefore \quad F \propto (L+R)$$

When $\vec{B}$ is along x-axis *i.e.*, $\vec{B} = B\hat{i}$, then

$$\vec{F} = I\,[2(L+R)\,\hat{i} \times B\,\hat{i}] = 0 \quad (\because \hat{i} \times \hat{i} = 0)$$

When $\vec{B}$ is along y-axis *i.e.*, $\vec{B} = B\,\hat{j}$, then

$$\vec{F} = I\,[2(L+R)\,\hat{i} \times B\,\hat{j}] = 2I(L+R)B\hat{k}$$

So, $F \propto (L+R)$

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

89. As $i = \dfrac{k}{NBA}\theta$;

so $k = \dfrac{NBAi}{\theta} = \dfrac{NBAi_0}{\pi/3} = \dfrac{3NBAi_0}{\pi}$

90. Voltage sensitivity, $V_s = \dfrac{\theta}{iR} = \dfrac{\pi/3}{i_0 R} = \dfrac{\pi}{3i_0 R}$

91. Corresponding to charge Q, if the current i flows for time dt, then

Torque, $\tau = NBA\,i$ or $\tau\,dt = NBA\,idt = NBAQ$

It ω is the angular speed of the coil produced when charge Q passes through the coil, then, change in angular momentum

$$= I\,\omega = \tau\,dt = NBAQ$$

or $\omega = \dfrac{NBAQ}{I}$

92. Let O be a point at a height h above the loop. Refer to Fig. 4(CF).81, the magnetic field induction at O due to current loop

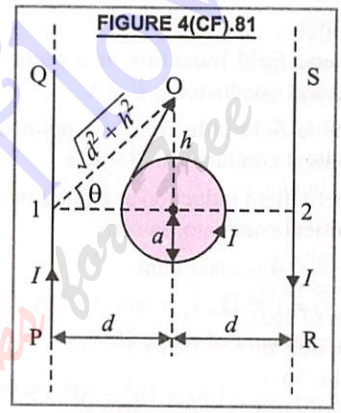
$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi I a^2}{(a^2+h^2)^{3/2}} = \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}}$$

Magnetic field induction at O due to currents in the wires 1 and 2.

$$B_2 = 2\times\frac{\mu_0}{4\pi}\frac{2I}{\sqrt{d^2+h^2}}\times\frac{d}{\sqrt{d^2+h^2}}$$

$$= \frac{\mu_0 I d}{\pi(d^2+h^2)} = \frac{\mu_0 I a}{\pi(a^2+h^2)} \quad (\because d = a)$$

FIGURE 4(CF).81



If net magnetic field at O is zero, then

$$B_1 = B_2 \text{ or } \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}} = \frac{\mu_0 I a}{\pi(a^2+h^2)}$$

or $\dfrac{a}{2\sqrt{a^2+h^2}} = \dfrac{1}{\pi}$

or $\dfrac{a^2}{4(a^2+h^2)} = \dfrac{1}{\pi^2} = \dfrac{1}{10}$ $[\because \pi^2 \approx 10]$

or $10\,a^2 = 4\,a^2 + 4\,h^2$ or $h^2 = 3\,a^2/2$

or $h = \mathbf{1{\cdot}2\,a}$

93. When $d >> a$, then magnetic field at O due to currents in wires 1 and 2 will be

$$B = 2\times\frac{\mu_0}{4\pi}\frac{2I}{d} = \frac{\mu_0 I}{\pi d}$$

Torque on the loop = $B\,I\,A \sin 30°$

$$= \frac{\mu_0 I}{\pi d}\times I\times(\pi a^2)\times\frac{1}{2} = \frac{\mu_0 I^2 a^2}{2d}$$

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IV. Matching Type Questions

94. Ampere swimming rule tells the direction of deflection of the magnetic needle due to current in a straight conductor, when magnetic needle is held below the conductor.

Fleming's left hand rule tells the direction of force on the current carrying conductor placed perpendicular to a uniform magnetic field.

Fleming's Right hand rule tells the direction of induced current.

Right hand thumb rule tells the direction of magnetic field lines due to current through circular coil.

V. Matrix-Match Type Questions

95. A positively charged particle is projected with some velocity in a region with a uniform electric field, it will describe a straight line if field is acting parallel to the motion of charged particle and parabolic path if field is acting at some angle or perpendicular to the motion of charged particle.

A positively charged particle is projected with some velocity in a uniform magnetic field, it will describe straight line if field is parallel to the motion of charged particle since no force will be acting due to magnetic field. It will describe circular path if magnetic field is acting perpendicular to the direction of motion of charged particle. It will describe helix if the magnetic field is acting at some angle with the direction of motion of charged particle.

A positively charged particle is projected with a velocity lying in a plane normal to a region containing parallel uniform electric and magnetic fields, the particle will describe helical path of increasing radius.

A positively charged particle is projected in a region containing perpendicular uniform electric and magnetic fields will describe straight line if force due to electric field is equal and opposite to that due to magnetic field. If the force on charged particle due to two fields is not same, then the particle will describe a helical path.

VI. Integer Type Questions

96. Magnetic field at P due to currents in two wires will be acting perpendicular to the plane of wires, upwards and is given by

$$B_P = \frac{\mu_0}{4\pi}\frac{2I}{(r/2)} + \frac{\mu_0}{4\pi}\frac{2I}{(r/2)} = \frac{2\mu_0 I}{\pi r}$$

Magnetic field at Q due to current in A is perpendicular to the plane of wire upwards and due to current in B is perpendicular to the plane of wire downwards and is given by

$$B_Q = -\frac{\mu_0 2I}{4\pi 2r} + \frac{\mu_0 2I}{4\pi r} = \frac{\mu_0 I}{4\pi r}$$

$$\therefore \quad \frac{B_P}{B_Q} = \frac{(2\mu_0 I/\pi r)}{(\mu_0 I/4\pi r)} = 8$$

97. When wire is taken in the form of one turn circular coil, then length, $l = 2\pi r$ or $r = \frac{l}{2\pi}$, $n = 1$

Magnetic field induction at the centre of circular coil due to current I is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r} = \frac{\mu_0}{4\pi}\frac{2\pi\times 1\times I}{(l/2\pi)} = \frac{\mu_0 \pi I}{l}$$

When wire is taken in the form of double loop,

then $l = 2\times 2\pi r_1$ or $r_1 = \frac{l}{4\pi}$ and $n = 2$

$\therefore$ Magnetic field induction at the centre of the circular coil,

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi\times n\times I}{r_1} = \frac{\mu_0}{4\pi}\frac{2\pi\times 2\times I}{(l/4\pi)}$$

$$= 4\times\mu_0\frac{\pi I}{l}$$

$$\therefore \quad \frac{B_1}{B} = 4$$

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98. Refer Fig. 4(CF).82, the magnetic field induction at the centroid O due to current I through one side BC of the triangle will be

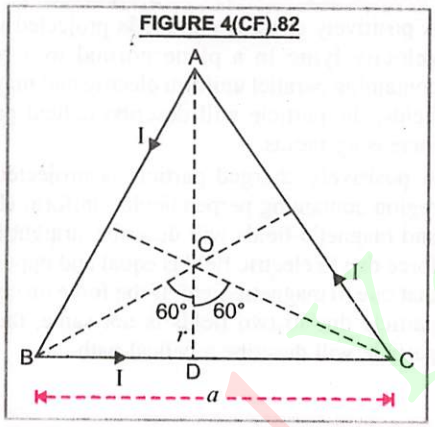
$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\theta_1 + \sin\theta_2)$$

It will be acting perpendicular to the plane of triangle upwards.

Total magnetic field induction at O due to current through all the three sides of the triangle will be

$$B = 3B_1 = \frac{3\mu_0}{4\pi}\frac{I}{r}[\sin\theta_1 + \sin\theta_2]$$

FIGURE 4(CF).82



Here, $I = 1$ A, $\theta_1 = 60° = \theta_2$

and $r = OD = \dfrac{BD}{\tan 60°} = \dfrac{a/2}{\sqrt{3}}$

$$= \frac{a}{2\sqrt{3}} = \frac{4{\cdot}5\times10^{-2}}{2\sqrt{3}}\text{ m}.$$

$$\therefore\quad B = 3\times10^{-7}\times\frac{1}{(4{\cdot}5\times10^{-2}/2\sqrt{3})} \times [\sin 60° + \sin 60°]$$

On solving, $B = \mathbf{4\times10^{-5}\ T}$

99. Due to current in complete cylinder, the magnetic field induction at P is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2I}{a} = \frac{\mu_0}{4\pi}\frac{2}{a}(\pi a^2 J)$$

$$= \frac{\mu_0 aJ}{2}$$

current through the cylindrical cavity of radius $a/2$ is

$$I' = \frac{I}{\pi a^2}\times\pi(a/2)^2 = \frac{I}{4}$$

Due to current in cylindrical cavity, the magnetic field induction at P is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2}{}\frac{(I/4)}{(a+a/2)}$$

$$= \frac{\mu_0}{4\pi}\frac{2I}{4\times(3a/2)}$$

$$= \frac{\mu_0}{4\pi}\frac{2}{}\frac{(\pi a^2 J)}{6a} = \frac{\mu_0 aJ}{12}$$

Net magnetic field induction at P is

$$B = B_1 - B_2 = \frac{\mu_0 aJ}{2} - \frac{\mu_0 aJ}{12}$$

$$= \frac{\mu_0 aJ}{2}\left[1-\frac{1}{6}\right] = \frac{5\mu_0 aJ}{12}$$

As per question, $= \dfrac{5\mu_0 aJ}{12} = \dfrac{N\mu_0 aJ}{12}$

$$\therefore\qquad N = 5$$

100. Here, $I_g = 0{\cdot}006$ A, $R = 4990\ \Omega$, $V = 30$ V. Let G be the resistance of galvanometer, then

$$V = I_g(G+R)$$

or $$G = \frac{V}{I_g} - R = \frac{30}{0{\cdot}006} - 4990 = 10\,\Omega$$

When galvanometer works as an ammeter, then shunt resistance required is

$$S = \frac{I_g G}{I - I_g} = \frac{0{\cdot}006\times10}{(1{\cdot}5 - {\cdot}006)} = \frac{6}{149{\cdot}4}\,\Omega$$

As per question,

$$\frac{2n}{249} = \frac{6}{149{\cdot}4}$$

or $$n = \frac{6\times249}{2\times149{\cdot}4} = 5$$

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VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

101. Time period, $T = \dfrac{2\pi m}{Bq}$ *i.e.* $T \propto \dfrac{m}{q}$.

Both the Assertion and Reason are correct and **Reason is the correct explanation of Assertion.**

102. When two long parallel wires, are connected to a battery in series. They carry currents in opposite directions, hence they repel each other.

Both Assertion and Reason are false.

103. In a non uniform magnetic field, both a torque and a net force act on the dipole.

If magnetic field were uniform, net force on dipole would be zero.

Both Assertion and Reason are false.

FOR ENGINEERING STUDENTS

104. Momentum, $p = mv$ or $v = p/m$. In a magnetic field, $q\,v\;B = mv^2/r$ or $r = mv/q\,B = p/q\,B$

i.e. $r \propto 1/q$

$\therefore\quad r_1/r_2 = q_2/q_1 = e/e = 1.$

Thus statement-1 is false and statement-2 is true.

105. In order to increase the range of ammeter, its resistance should be decreased, *i.e.*, additional shunt is connected in parallel. **Thus statement-1 is false. Statement-2 is true.**

106. $E = \dfrac{1}{2}mv^2$ or $v = \sqrt{2E/m}$

$Bq\,v = mv^2/r$

or $r = \dfrac{mv}{Bq} = \dfrac{m}{Bq}\sqrt{\dfrac{2E}{m}} = \dfrac{\sqrt{2mE}}{Bq}$

i.e., $r \propto \dfrac{\sqrt{m}}{q}$

$\therefore\quad \dfrac{r_p}{r_\alpha} = \sqrt{\dfrac{m_p}{m_\alpha}} \times \dfrac{q_\alpha}{q_p} = \sqrt{\dfrac{1}{4}} \times \dfrac{2}{1} = 1$

i.e., radius of both the particles should be the same. Hence **statement-1 is wrong** and **statement-2 is correct.**

107. In a moving coil galvanometer,

$I = \dfrac{k}{n\,BA}\theta$ or $\theta = \dfrac{n\,BA}{k}I$

For the given value of current I, θ increases if B increases. The use of iron core increases the magnetic field, so the deflection θ increases for the same value of current I making the galvanometer more sensitive. Hence, **statement-1 is correct.**

We know that soft iron can be easily magnetised or demagnetised, hence **statement-2 is wrong.**

VIII. Multiple Choice Questions (Based on Experimental Skills)

108. Here, $G = 1000\ \Omega$, $I_g = 0{\cdot}1\ \text{mA} = 10^{-4}\ \text{A}$;
$I = 10\ \text{A}$

$$S = \frac{I_g G}{I - I_g} = \frac{10^{-4} \times 1000}{10 - 10^{-4}}$$

$= \mathbf{0{\cdot}01\ A\ in\ parallel.}$

109. If I is the total current, then

$I_g = I/34$

or $I = 34\,I_g$

Now, $S = \dfrac{I_g G}{I - I_g} = \dfrac{I_g \times 3663}{34 I_g - I_g} = \mathbf{111\Omega}$

110. $G = \dfrac{RS}{R - S} = \dfrac{5000 \times 20}{5000 - 20} = \mathbf{20\Omega}$

111. $I = nk = \dfrac{E}{R + G}$

or $k = \dfrac{E}{n(R + G)}$

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$$k = \frac{1{\cdot}5}{20\,(100+50)} = 0{\cdot}5 \times 10^{-3}\ \text{A/div}$$

$$= 500 \times 10^{-6}\ \text{A/div} = \mathbf{500\ \mu\ A/div}$$

112. Let R be the resistance of voltmeter and n be the number of divisions in the voltmeter. When current I_g flows through voltmeter, the voltage recorded by each division of voltmeter is

$$I_g \frac{R}{n} = V \qquad ...(i)$$

When resistance 980 Ω is connected in series, then

$$I_g \frac{(R+980)}{n} = 50\,\text{V} \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we have

$$R + 980 = 50\,R$$

or $$R = 980/49 = \mathbf{20\ \Omega}$$

113. Resistance of ammeter,

$$G' = \frac{GS}{G+S} = \frac{60\times 0{\cdot}02}{60+0{\cdot}02}$$

$$\therefore \quad \text{Current } I = \frac{E}{R+G'}$$

$$= \frac{3{\cdot}0}{3{\cdot}0 + \left(\dfrac{60\times 0{\cdot}02}{60+0{\cdot}02}\right)} = \mathbf{1{\cdot}0A}$$

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5

MAGNETISM AND MATTER

5.1. INTRODUCTION

In the last chapter, we studied magnetic effects of current. We learnt that electricity and magnetism are not independent phenomena. Infact, moving electric charges or electric currents produce magnetic fields.

In this chapter, we trace the origin of the science of magnetism, and its subsequent growth. A Greek Philosopher, Thales of Miletus had observed as long back as 600 B.C, that a naturally occuring ore of iron attracted small pieces of iron towards it. This ore was found in the district of Magnesia in Asia Minor in Greece. Hence the ore was named **magnetite.** *The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc. towards the ore was called* **magnetism.** *The iron ore showing this effect was called a* ***natural magnet.*** *The structural formula of this natural ore was* Fe_3O_4.

The Chinese discovered that a piece of magnetite, when suspended freely, always points out roughly in the north south direction. ***Thus a natural magnet has attractive and directive properties.*** A magnetic compass based on directive property of magnets was used by navigators to find their way in steering the ships.

That is why magnetite was also called **'lode stone'** in the sense of leading stone.

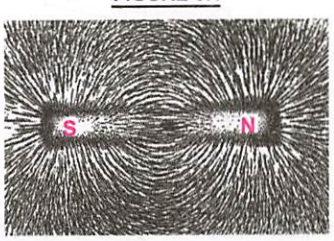
The natural magnets have often irregular shape and they are weak. It is found that a piece of iron or steel can acquire magnetic properties, on rubbing with a magnet. Such magnets made out of iron and steel are called **artificial magnets.** Artificial magnets can have desired shape and desired strength. A bar magnet, a horse shoe magnet, magnetic needle, compass needle etc. all are artificial magnets.

5.2. THE BAR MAGNET

It is the most commonly used form of an artificial magnet.

When we hold a sheet of glass over a short bar magent and sprinkle some iron filings on the sheet, the iron filings rearrange themselves as shown in Fig. 5.1. The pattern suggests that attraction is maximum at the two ends of the bar magnet. These ends are called **poles** of the magnet.

FIGURE 5.1



5.3. SOME BASICS OF MAGNETISM

Some of the commonly known ideas of magnetism are :

1. ***The earth behaves as a magnet.*** This is as if a huge bar magnet is held at the centre of earth with its north pole pointing approximately towards geographic south. The south pole of the imaginary bar magnet would point approximately towards geographic south.

2. ***Every magnet attracts small pieces of magnetic substances like iron, cobalt, nickel and steel towards it.*** The attraction is maximum at the two ends of the magnet, as shown in Fig. 5.1. These ends are called **poles** of the magnet.

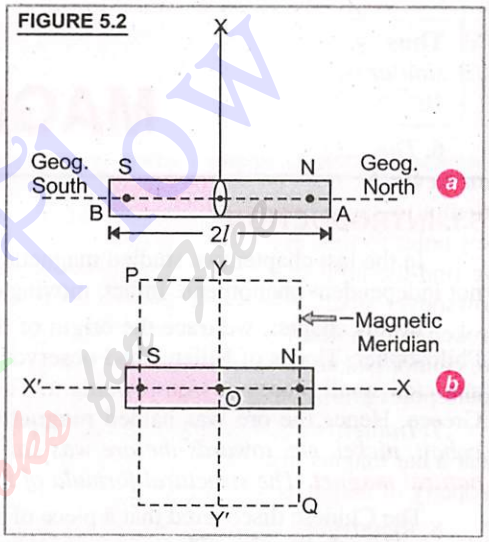
3. ***When a magnet is suspended freely with the help of an unspun silken thread, it comes to rest along the north south direction.*** If it is turned slightly from this direction and left, it again returns to this direction. The pole of the magnet which points towards geographic north is called north pole of the magnet (N) and the pole which points towards geographic south is called south pole (S), Fig. 5.2(a).

Note that ***poles exist always in pairs ; two poles of a magnet are always unlike but of equal strength.*** Further, poles N and S are situated a *little inwards* from the geometrical ends A and B of the magnet. The **magnetic length** (NS) of magnet is roughly 6/7 of its **geometric length** (AB). We represent NS by $2l$,* (*and not* l).

The straight line XX' passing through north and south poles of a magnet is called **axial line** of the magnet. A line YY' passing through centre of the magnet in a direction perpendicular to the length of the magnet is called **equatorial line** of the magnet.

The straight line joining north and south poles of a freely suspended magnet represents magnetic N–S direction. *A vertical plane PQ passing through N-S line of a freely suspended magnet is called* **magnetic meridian.** These are shown in Fig. 5.2(b).

FIGURE 5.2



4. ***Like poles repel each other and unlike poles attract each other.***

To show this, we suspend a bar magnet with the help of a thread. When we bring N pole of another magnet near the N pole of suspended magnet, we observe repulsion, Fig. 5.3. Similarly, south pole of one magnet repels south pole of the other, Fig. 5.4. However, when S pole of one is brought near N pole of suspended magnet, there is attraction, Fig. 5.5.

FIGURE 5.3

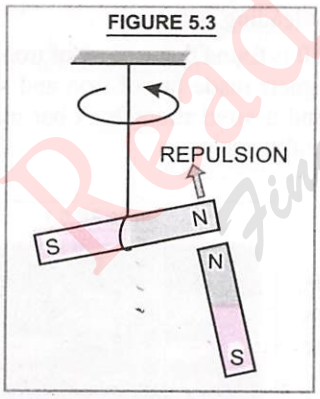


FIGURE 5.4

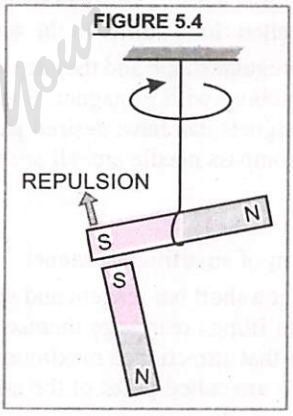
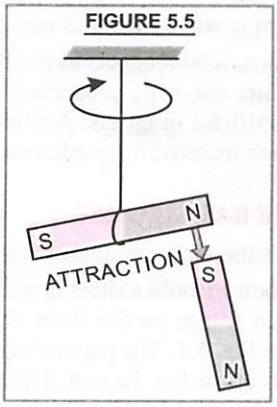


FIGURE 5.5



5. ***The force of attraction or repulsion F between two magnetic poles of strengths m_1 and m_2 separated by a distance r in space is directly proportional to the product of pole strengths and inversely proportional to the square of the distance between their centres, i.e.,***

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = K\frac{m_1 m_2}{r^2}, \quad \text{where } K \text{ is magnetic force constant.}$$

*Conventionally, this is done for simplification of calculations, as distance of each pole from the centre would be l.

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In SI units, $K = \frac{\mu_0}{4\pi} = 10^{-7}\ Wb\ A^{-1}\ m^{-1}$

where μ_0 is **absolute magnetic permeability of free space (air/vacuum).**

$$\therefore \quad F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} \qquad ...(1)$$

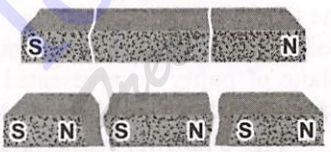
This is called **Coulomb's law of magnetic force*.** However, in cgs system, the value of $K = 1$.

SI Unit of magnetic pole strength

Suppose $m_1 = m_2 = 1$ Am, $r = 1$ *m*. Then from (1), $F = 10^{-7} \times \frac{1\times1}{1^2} = 10^{-7}$ N.

Thus, ***strength of a magnetic pole is said to be one ampere-metre, if it repels an equal and similar pole, when placed in vacuum (or air) at a distance of one metre from it, with a force of*** 10^{-7} **N.**

FIGURE 5.6



6. ***The magnetic poles always exist in pairs. The poles of a magnet can never be separated i.e. magnetic monopoles do not exist.*** In an attempt to separate the magnetic poles, if we break a magnet, we find new poles formed at the broken ends. If the two pieces are broken again, we find the broken ends contain new poles, Fig. 5.6. Thus each piece, howsoever small, is a complete magnet in itself. Even if a magnet is broken into molecules, each molecule shall be a complete magnet.Thus, we cannot isolate the north or south pole of a magnet. Unlike electric charges, isolated magnetic north and south poles known as magnetic monopoles do not exist.

7. ***Inductive Property.*** When a piece of a magnetic material like soft iron, cobalt, nickel etc. is placed near a bar magnet, it acquires magnetism. The magnetism so acquired is called induced magnetism and this property of magnetism is called inductive property.

8. ***Repulsion is the sure test of magnetism.*** Suppose we have two identical pieces, one is iron and other is a magnet. To distinguish which is which, we take another bar magnet. This bar magnet would attract the iron piece and also one end of this bar magnet would attract the opposite pole of the given magnet. However, one end of this bar magnet would repel only like pole of the given magnet.

Hence when repulsion occurs between the testing bar magnet and one of the two given pieces, that piece must be a magnet. That is why we say that repulsion is sure test of magnetism.

9. We can make artificial magnets out of iron and its alloys.

Sample Problem **Two unlike magnetic poles of strength 10 A – m each are held in air at a distance of 0·5 m from eachother. What is the magnetic force of attraction between them ?**

Sol. Here, $m_1 = m_2 = 10$ A-m, $r = 0{\cdot}5$ m, $F = ?$

According to Coulomb's Law of magnetic force,

$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = 10^{-7} \times \frac{10\times10}{(0{\cdot}5)^2} = 4\times10^{-5}\ \text{N}$$

5.4. THE MAGNETIC FIELD LINES

We know that **Magnetic Field is** the space around a magnet or the space around a conductor carrying current, in which its magnetic effect can be felt.

In order to visualise a magnetic field graphically, Faraday introduced the concept of the magnetic field lines. These lines give us a visual and intuitive realisation of the magnetic field.

* This corresponds to Coulomb's law in electrostatics.

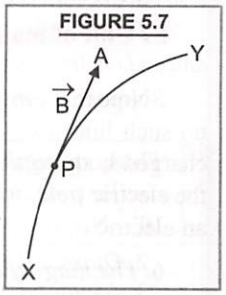
According to Faraday,

***Magnetic field line** is an imaginary curve, the tangent to which at any point gives us the direction of magnetic field $\vec{B}$ at that point.*

If we imagine a number of small compass needles around a magnet, each compass needle experiences a torque due to the field of the magnet. The torque acting on a compass needle aligns it in the direction of the magnetic field.

The path along which the compass needles are aligned is known as magnetic field line.

FIGURE 5.7



In Fig.5.7, *XY* represents a magnetic field line. At any point *P* on the line, we have drawn a tangent *PA* to the curve *XY*. *This tangent PA represents the direction of magnetic field* $\vec{B}$ *at P.*

In a region, magnetic field may be uniform or non-uniform.

Magnetic field in a region is said to be **uniform** if the field has same strength and same direction at all the points in the region. **Uniform magnetic field in the plane of paper** is represented by a set of equidistant parallel field lines as shown in Fig. 5. 8(*a*).

FIGURE 5.8

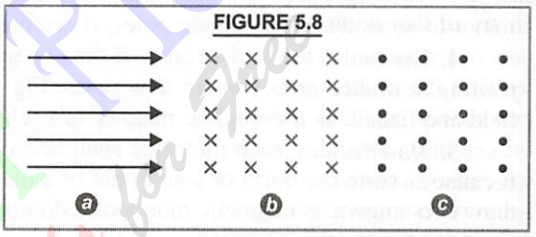
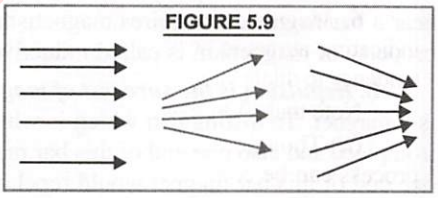


Fig. 5.8(*b*) represents a **uniform magnetic field perpendicular to the plane of the paper and *inwards*.** Fig. 5.8(*c*) represents a **uniform magnetic field perpendicular to the plane of paper and outwards.**

Magnetic field in a region is said to be **non-uniform** if it has different strengths at different points in the region. A non- uniform magnetic field is represented by *unequally spaced* field lines, which may be parallel or converging or diverging as shown in Fig. 5.9

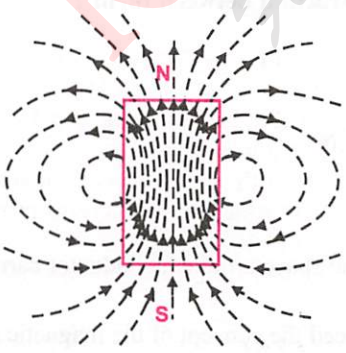
FIGURE 5.9



A strong magnetic field is represented by closely spaced field lines, and a weak magnetic field is represented by widely spaced field lines.

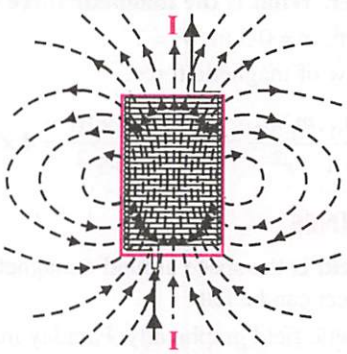
In Fig. 5.10(*a*), we have shown magnetic field lines due to a bar magnet *NS*. Fig. 5.10(*b*) shows magnetic field lines due to a finite solenoid carrying current. For the sake of comparison, we have shown *electric field lines* due to an electric dipole in Fig. 5.10(*c*).

FIGURE 5.10



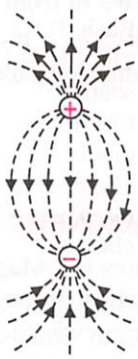
Magnetic field lines due to a bar magnet

a



Magnetic field lines due to a solenoid carrying current

b



Electric filed lines due to an electric dipole

c

We find that magnetic field lines of a bar magnet and a current carrying solenoid resemble very closely. However, the two are different from electric field lines due to an electric dipole shown in Fig. 5.10(*c*).

5.5. PROPERTIES OF MAGNETIC FIELD LINES

1. *The magnetic field lines of a magnet (or of a solenoid carrying current) form closed continuous loops.* We imagine magnetic field lines to be extending through the body of the magnet/solenoid from south pole to north pole, as shown in Figs. 5.10(*a*) and (*b*).

This is unlike electric field lines, which begin from a positive charge and end on the negative charge or escape to infinity, as shown in Fig. 5.10(c).

Note that electric field lines are discontinuous only in the sense that no such lines exist inside a charged body. However, from a positively charged body to a negatively charged body, there is no discontinuity in the electric field lines. That is why at very far off points, field lines due to an electric dipole and a magnetic dipole appear identical.

DO YOU KNOW ?

Magnetic field inside a long solenoid carrying current is a uniform magnetic field. On the contrary, magnetic field of a bar magnet is a non-uniform magnetic field.

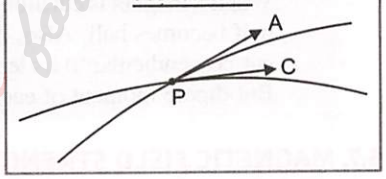
2. *Outside the body of the magnet, the direction of magnetic field lines is from north pole to south pole.*

3. At any given point, *tangent to the magnetic field line represents the direction of net magnetic field* $(\vec{B})$ *at that point.*

4. *The magnitude of magnetic field at any point is represented by the number of magnetic field lines passing normally through unit area around that point.* Therefore, crowded lines represent a strong magnetic field and lines which are not so crowded represent a weak magnetic field.

5. *No two magnetic field lines can intersect eachother.* This is because as shown in Fig. 5.11, at the point of intersection, say *P*, we can draw two tangents *PA* and *PC* to the two magnetic field lines. This would mean two directions of magnetic field at the same point *P*. This is not possible.

FIGURE 5.11



6. *Magnetic field lines have a tendency to contract longitudinally* indicating attracton between unlike magnetic poles. The lines also have a tendency to dilate laterally, indicating repulsion between like magnetic poles.

Note that (*i*) In a particular region, where there is no magnetic field, there would be no magnetic field lines.

(*ii*) Though magnetic field lines are imaginary, they can be mapped using a small compass needle. This process can be compared to photographing a virtual object.

(*iii*) Magnetic field lines exist inside every magnetised material.

Note. Magnetic field lines due to a bar magnet can be plotted on a sheet of paper using a small magnetic compass. This will be only a two dimensional plot. In actual practice, field lines exist all around the magnet in three dimensions.

Retain in Memory

Basic Difference between electricity and magnetism

1. The simplest source of electricity is a point charge. It can be positive or negative. The simplest source of magnetism is a magnetic dipole as magnetic monopoles are not known to exist.
2. Electric charge is scalar. It experiences a force in an electric field. The magnetic dipole has a dipole moment, which is a vector. It experiences a torque in a magnetic field.
3. The direction of magnetic dipole moment is from south pole to north pole. This is analogous to the direction of electric dipole moment of an electric dipole from negative charge to positive charge.

5.6. MAGNETIC DIPOLE

A magnetic dipole consists of two unlike poles of equal strength and separated by a small distance.

For example, a bar magnet, a compass needle etc. are magnetic dipoles. We shall show that a current loop behaves as a magnetic dipole. An atom of a magnetic material behaves as a dipole due to electrons revolving around the nucleus.

The two poles of a magnetic dipole (or a magnet), called north pole and south pole are always of equal strength, and of opposite nature. Further such two magnetic poles exist always in pairs and cannot be separated from each other.

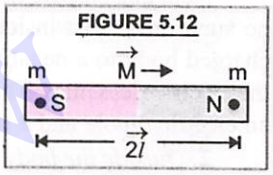
The distance between the two poles of a bar magnet is called the **magnetic length** of the magnet. It is a vector directed from *S*-pole of magnet to its *N*-pole, and is represented by $2\vec{l}$.

Magnetic dipole moment ***is the product of strength of either pole (m) and the magnetic length*** $(2\vec{l})$ ***of the magnet.***

It is represented by $\vec{M}$.

Magnetic dipole moment = strength of either pole × magnetic length

$$\vec{M} = m(2\vec{l}) \quad \text{...(2)}$$



Magnetic dipole moment is a vector quantity directed from South to North pole of the magnet, as shown in Fig. 5.12.

The SI unit of *M* is **joule/tesla** or **ampere metre²**. ∴ SI unit of pole strength is **Am.**

The dimensional formula of magnetic moment is $[M^0 L^2 T^0 I]$, and the dimensional formula of pole strength is $[M^0 L T^0 I]$.

Retain in Memory

When a magnet is cut along its length (longitudinally) into two equal parts, the pole strength of each half becomes half. Also, dipole moment of each half becomes half. However, when the magnet is cut perpendicular to its length (*i*.e., transversally), the pole strength of each half remains the same. But dipole moment of each half becomes half.

5.7. MAGNETIC FIELD STRENGTH AT A POINT DUE TO BAR MAGNET

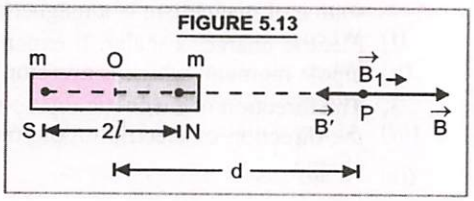
The strength of magnetic field at any point ***is defined as the force experienced by a hypothetical unit north pole placed at that point. It is a vector quantity. The direction of magnetic field*** $(\vec{B})$ ***is the direction along which hypothetical unit north pole would tend to move if free to do so.***

Note that we have used the word hypothetical unit north pole in the above definition. This is because an isolated magnetic pole does not exist. The simplest magnetic structure that exists is a magnetic dipole.

(*a*) When point lies on axial line of bar magnet

Let $2l$ be the magnetic length of a bar magnet with pole strength m and centre O. M is the magnetic dipole moment of the magnet, where, $M = m \times 2l$; $OP = d$, is the distance of the point P on the axial line from the centre of the magnet, Fig. 5.13. Magnetic field strength at P due to N pole of magnet is

$$B = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{NP^2} = \frac{\mu_0}{4\pi} \frac{m}{(d-l)^2}, \text{ along } NP \text{ produced.}$$



Magnetic field strength at P due to S pole of magnet,

$$B' = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{SP^2} = \frac{\mu_0}{4\pi} \frac{m}{(d+l)^2}, \text{ along } PS$$

∴ Magnetic field strength at P due to the bar magnet

$$B_1 = B - B' = \frac{\mu_0}{4\pi} \frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi} \frac{m}{(d+l)^2} = \frac{\mu_0 m}{4\pi} \left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2} \right]$$

$$B_1 = \frac{\mu_0 m}{4\pi} \left[\frac{(d+l)^2 - (d-l)^2}{(d^2-l^2)^2} \right] = \frac{\mu_0 m.4ld}{4\pi (d^2-l^2)^2} = \frac{\mu_0 (m \times 2l)\, 2d}{4\pi (d^2-l^2)^2}$$

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2} \quad ...(3)$$

When the magnet is **short,** $l^2 << d^2$ $\therefore$ $$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{d^4} = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \quad ...(4)$$

The direction of $\vec{B}_1$ is along NP produced.

(b) When point lies on equatorial line of bar magnet

In Fig. 5.14, the point P is shown on equatorial line of the same bar magnet, where $OP = d$. Magnetic field strength at P due to N pole of magnet,

$$B = \frac{\mu_0}{4\pi}\frac{m \times 1}{NP^2} = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}, \text{ along } NP \text{ produced.}$$

It is represented by PA.

Magnetic field strength at P due to S pole of magnet, $B' = \frac{\mu_0}{4\pi}\frac{m \times 1}{SP^2} = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}$, along PS.

It is represented by PC.

As $B = B'$ in magnitude, their components $B \sin\theta$ along OP produced and $B' \sin\theta$ along PO will cancel out. However, components along PX parallel to NS will add up. Therefore,

Magnetic field strength at P due to the bar magnet,

$$B_2 = B\cos\theta + B'\cos\theta = 2B'\cos\theta, \text{ along } PX$$

$$B_2 = 2\frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)} \times \frac{l}{\sqrt{d^2 + l^2}} = \frac{\mu_0}{4\pi}\frac{m \times 2l}{(d^2 + l^2)^{3/2}}$$

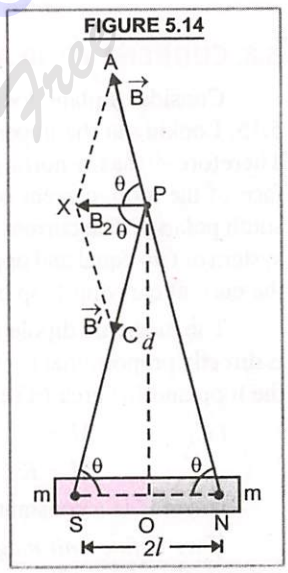
$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}} \quad ...(5)$$

If the magnet is short, $l^2 << d^2$ $\therefore$ $$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M}{d^3} \quad ...(6)$$

The direction of $\vec{B}_2$ is along PX, a line parallel to NS, as shown in Fig. 5.14.

Dividing (4) by (6), we get $$\frac{B_1}{B_2} = 2 \quad \text{or} \quad B_1 = 2B_2$$

FIGURE 5.14



Hence, ***magnetic field due to a short bar magnet at any point on axial line of magnet is twice the magnetic field at a point at the same distance on the equatorial line of the magnet.***

Retain in Memory

(*i*) When point lies on *axial line* of magnetic dipole, it is known as ***end-on*** **position** or ***tangent A*** **position.**

(*ii*) When point lies on *equatorial line* of magnetic dipole, it is known as ***broadside on position*** **or** ***tangent B*** **position.**

(*iii*) At any point on **axial line** of a dipole, magnetic field strength $\vec{B}_1$ **is along** $\vec{M}$.

(*iv*) At any point on **equatorial line** of a dipole, magnetic field strength $\vec{B}_2$ **is in a direction opposite to** $\vec{M}$.

Sample Problem 1 **A magnet 2 cm long has a pole strength of 60 Am. Find the magnitude of magnetic field strength B at a point on its axis at a distance of 20 cm from it. What would be the value of B, if the point were to lie at the same distance on equatorial line of magnet ?**

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Sol. Here, $2l = 2\text{ cm} = 0{\cdot}02\text{ m}$; $m = 60\text{ Am}$, $B = ?$, $d = 20\text{ cm} = 0{\cdot}2\text{ m}$

On axial line, as $l < d$,

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{\mu_0}{4\pi}\frac{2\times m\times 2l}{d^3} = \frac{10^{-7}\times 2\times 60\times 0{\cdot}02}{(0{\cdot}2)^3} = \mathbf{3{\cdot}0\times 10^{-5}\ T}$$

At the same distance, on equatorial line, $B_2 = \frac{1}{2}B_1$, $\therefore \quad B_2 = \frac{1}{2}\times 3{\cdot}0\times 10^{-5}\text{ T} = \mathbf{1{\cdot}5\times 10^{-5}\ T}$

Sample Problem 2 **Calculate the magnetic field due to a bar magnet 2 cm long and having pole strength of 100 A-m at a point 10 cm from each pole.**

Sol. Here, $B = ?\ 2l = 2\text{ cm} = 2\times 10^{-2}\text{ m}$

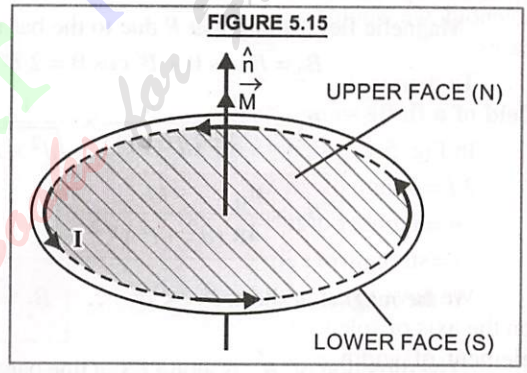
$$m = 100\text{ Am}, (d^2 + l^2)^{1/2} = 10\text{ cm} = \frac{1}{10}\text{ m}$$

On the equatorial line of bar magnet,

$$B = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{(m\times 2l)}{(d^2+l^2)^{3/2}} = \frac{10^{-7}\times 100\times 2\times 10^{-2}}{(1/10)^3} = \mathbf{2\times 10^{-4}\ T}$$

5.8. CURRENT LOOP AS A MAGNETIC DIPOLE

Consider a plane loop of wire carrying current, Fig. 5.15. Looking at the upper face, current is anticlockwise. Therefore, it has a north polarity. Looking at the lower face of the loop, current is clockwise. Therefore, it has a south polarity. The current carrying loop thus behaves as a system of two equal and opposite magnetic poles and hence the current carrying loop behaves as a **magnetic dipole.**



The magnetic dipole moment of the current loop (M) is directly proportional to (i) strength of current (I) through the loop and (ii) area (A) enclosed by the loop.

i.e., $\quad M \propto I \quad$ and $\quad M \propto A$

$$\therefore \quad M = K\,I\,A \qquad \text{...(7)}$$

where K is a constant of proportionality.

If we define unit magnetic dipole moment as that of a small one turn loop of unit area carrying unit current, then from (7), $\quad 1 = K\times 1\times 1 \quad$ or $\quad K = 1$

$\therefore$ from (7), $\quad M = IA$

For N such turns, $\quad \boxed{M = NIA} \qquad \text{...(8)}$

The SI unit of M is **ampere metre^2.**

One ampere metre^2 is the magnetic moment of one turn loop of area one square metre carrying a current of one ampere.

In vector form, we can rewrite Eqn. (8) as $\quad \boxed{\vec{M} = NIA\,\hat{n}} \qquad \text{...(9)}$

where $\hat{n}$ is unit vector perpendicular to the plane of the loop in a direction given by right handed screw rule.

The factor NI is called **ampere turns** of the circular current loop. Thus magnetic dipole moment of current loop = ampere turns × area of the loop.

Note that eqn. (9) is valid for planar current loop of any shape and size.

Sample Problem **A wire of length L metre carrying current I ampere is bent in the form of a circle. Find its magnetic moment.** (Uttarakhand Board 2012)

Sol. If r is radius of the circle, then $2\pi r = L$ or $r = \dfrac{L}{2\pi}$

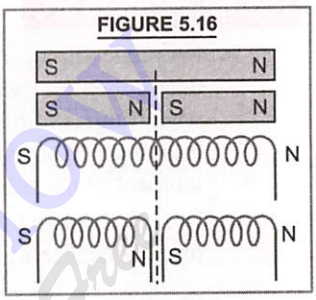
$$M = IA = I\pi r^2 = I\pi\left(\frac{L}{2\pi}\right)^2 = \frac{I L^2}{4\pi}$$

5.9. BAR MAGNET AS AN EQUIVALENT SOLENOID

We know that a current loop acts as a magnetic dipole. According to Ampere's hypothesis, all magnetic phenomena can be explained in terms of circulating currents.

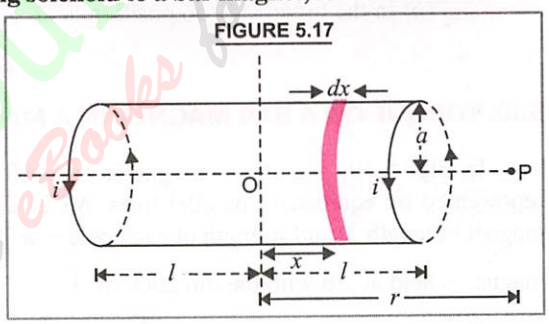
In Figs. 5.10(a) and (b), magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. ***Therefore, a bar magnet can be thought of as a large number of circulating currents in analogy with a solenoid.*** Cutting a bar magnet is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The magnetic field lines remain continuous, emerging from one face of one solenoid and entering into other face of other solenoid, Fig. 5.16. If we were to move a small compass needle in the neighbourhood of a bar magnet and a current carrying solenoid, we would find that the deflections of the needle are similar in both cases.

FIGURE 5.16



To demonstrate the similarity of a current carrying solenoid to a bar magnet, let us calculate axial field of a finite solenoid carrying current.

FIGURE 5.17



In Fig. 5.17, suppose a = radius of solenoid,

$2l$ = length of solenoid with centre O

n = number of turns per unit length of solenoid,

i = strength of current passed through the solenoid

We have to calculate magnetic field at any point P on the axis of solenoid, where $OP = r$. Consider a small element of width dx of the solenoid, at a distance x from O.

Number of turns in the element = $n\,dx$.

Using eqn. (16) on page 4/12, magnitude of magnetic field at P due to this current element is

$$dB = \frac{\mu_0\, i\, a^2\, (n\, dx)}{2\,[(r-x)^2 + a^2]^{3/2}} \qquad \text{...(10)}$$

If P lies at a very large distance from O, *i.e.*, $r >> a$ and $r >> x$, then $[(r-x)^2 + a^2]^{3/2} \approx r^3$

$$dB = \frac{\mu_0\, i\, a^2\, n\, dx}{2\, r^3} \qquad \text{...(11)}$$

As range of variation of x is from $x = -l$ to $x = +l$, therefore the magnitude of total magnetic field at P due to current carrying solenoid

$$B = \frac{\mu_0\, nia^2}{2\, r^3} \int_{x=-l}^{x=+l} dx = \frac{\mu_0\, nia^2}{2\, r^3}\,[x]_{x=-l}^{x=+l}$$

$$B = \frac{\mu_0\, ni}{2}\,\frac{a^2}{r^3}\,(2l) = \frac{\mu_0}{4\pi}\,\frac{2n\,(2l)\, i\,\pi\, a^2}{r^3} \qquad \text{...(12)}$$

If M is magnetic moment of the solenoid, then

$$M = \text{total no. of turns} \times \text{current} \times \text{area of cross section}$$

$$M = n\,(2\,l) \times i \times (\pi\, a^2)$$

$$\therefore \quad \boxed{B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}} \qquad \text{...(13)}$$

This is the expression for magnetic field on the axial line of a short bar magnet. For details, see Art. 5.7.

Thus, the axial field of a finite solenoid carrying current is same as that of a bar magnet. Hence, ***for all practical purposes, a finite solenoid carrying current is equivalent to a bar magnet.***

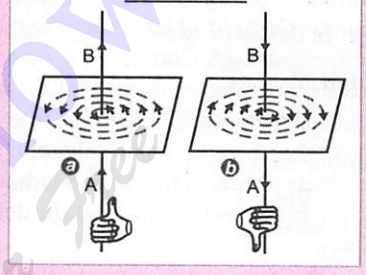
Retain in Memory

1. Magnetic dipole moment of a current carrying solenoid is determined by *right hand thumb rule*. According to this rule, when fingers of right hand curl in the direction of current, the thumb of right hand gives the direction of magnetic dipole moment of the solenoid Fig. 5.18
2. When two small magnets of magnetic moments M_1 and M_2 are placed distance r apart, then the force acting between them

   (*i*) in the end on position is, $F = \dfrac{\mu_0}{4\pi}\dfrac{6M_1M_2}{r^4}$

   (*ii*) in the broad side on position is, $F = \dfrac{\mu_0}{4\pi}\dfrac{3M_1M_2}{r^4}$

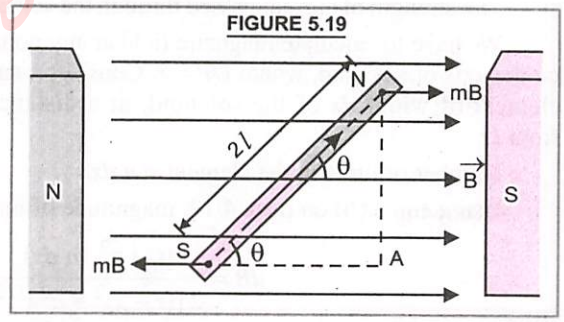
FIGURE 5.18



5.10. TORQUE ON A BAR MAGNET IN A MAGNETIC FIELD

In Fig. 5.19, a uniform magnetic field $\vec{B}$ is represented by equidistant parallel lines. NS is a bar magnet of length $2l$ and strength of each pole $= m$. The magnet is held at $\angle\theta$ with the direction of $\vec{B}$.

FIGURE 5.19



Force on N-pole $= mB$, along $\vec{B}$

Force on S-pole $= mB$, opposite to $\vec{B}$

These forces being equal, unlike and parallel will form a couple, which tends to rotate the magnet clockwise so as to align it along $\vec{B}$.

Draw NA perpendicular to $\vec{B}$ and $SA \parallel \vec{B}$

$\therefore$ Torque acting on the bar magnet

$$\tau = \text{moment of the couple} = \text{force} \times \text{perp. distance} = mB \times NA \qquad \text{...(14)}$$

In ΔNAS, $\quad \sin\theta = \dfrac{NA}{NS} = \dfrac{NA}{2l} \quad \therefore \quad NA = 2l\sin\theta$

Put in (14), $\quad \tau = mB \times 2l\sin\theta$. As $\quad M = m \times 2l \quad \therefore \quad \boxed{\tau = MB\sin\theta} \qquad \text{...(15)}$

In vector form, we can rewrite this equation as $\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$

The direction of $\vec{\tau}$ is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$, and is given by right handed screw rule.

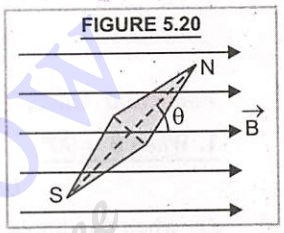
When $B = 1$ and $\theta = 90°$, then from (15), $\tau = M \times 1 \sin 90° = M$ or $M = \tau$

Hence we may define

> ***Magnetic dipole moment** is the torque acting on a magnetic dipole held perpendicular to a uniform magnetic field of unit strength.*

Unit of M is unit of τ divided by unit of B. Therefore, the SI unit of $\vec{M}$ is joule per tesla (JT^{-1}).

5.11. DETERMINATION OF MAGNITUDE OF MAGNETIC FIELD USING A COMPASS NEEDLE



In Fig. 5.20 equidistant parallel lines represent a uniform magnetic field $\vec{B}$ in the plane of the paper. *NS* is a small compass needle of magnetic moment *M*, held at an angle θ with $\vec{B}$.

Torque acting on the compass needle, $\tau = MB \sin\theta$. This is *restoring torque* as it tries to align the compass along the direction of $\vec{B}$..

But $\tau = I\alpha$, where I is moment of inertia of the compass needle and α is its angular acceleration. When θ is small, $\sin\theta \simeq \theta$.

$$\therefore \qquad \tau = -MB\theta = I\alpha \quad \text{or} \quad \alpha = \frac{-MB}{I}\theta \text{ or } \alpha \propto \theta \qquad \text{(Minus sign for restoring torque)}$$

The motion of compass needle is simple harmonic. Time period of oscillation of compass needle is

$$T = 2\pi\sqrt{\frac{\theta}{\alpha}}$$

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Squaring both sides, we get

$$T^2 = 4\pi^2\left(\frac{I}{MB}\right) \quad \text{or} \quad \boxed{B = \frac{4\pi^2 I}{MT^2}}$$

Knowing *M*, *I* and *T*, we can calculate ***B***.

5.12. POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A MAGNETIC FIELD

> ***Potential energy of a magnetic dipole** in a magnetic field is the energy possessed by the dipole due to its particular position in the field.*

When a magnetic dipole of moment $\vec{M}$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$, the magnitude of the torque acting on the dipole is

$$\tau = MB \sin\theta \qquad \text{...(16)}$$

This torque tends to align the dipole in the direction of the field. Work has to be done in rotating the dipole against the action of the torque. This work done is stored in the magnetic dipole as potential energy of the dipole.

Now, small amount of work done in rotating the dipole through a small angle $d\theta$ against the restoring torque is

$$dW = \tau\, d\theta = MB \sin\theta\, d\theta$$

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Total work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is

$$W = \int_{\theta_1}^{\theta_2} MB \sin\theta \, d\theta = MB\,[-\cos\theta]_{\theta_1}^{\theta_2} = -MB\,[\cos\theta_2 - \cos\theta_1]$$

$\therefore$ Potential energy of the dipole is $\boxed{U = W = -MB\,(\cos\theta_2 - \cos\theta_1)}$ ...(17)

When $\theta_1 = 90°$, and $\theta_2 = \theta$, then $U = W = -MB\,(\cos\theta - \cos 90°)$

$$U = W = -MB\cos\theta \quad \text{...(18)}$$

In vector notation, we may rewrite (18) as

$$\boxed{U = -\vec{M}\,.\,\vec{B}} \quad \text{...(19)}$$

Particular Cases

1. *When* $\theta = 90°$

$$U = -MB\cos\theta = -MB\cos 90° = 0$$

i.e, when the dipole is perpendicular to magnetic field its potential energy is zero.

Hence to calculate potential energy of dipole at any position making angle θ *with* $\vec{B}$., *we use*

$$U = -MB\,(\cos\theta_2 - \cos\theta_1)$$

and take $\theta_1 = 90°$ and $\theta_2 = \theta$. Therefore,

$$U = -MB\,(\cos\theta - \cos 90°) = -MB\cos\theta$$

2. *When* $\theta = 0°$

$U = -MB\cos 0° = -MB$, which is *minimum.*

This is the position of *stable equilibrium, i.e.*, ***when the magnetic dipole is aligned along the magnetic field, it is in stable equilibrium having minimum P.E.***

3. *When* $\theta = 180°$

$U = -MB\cos 180° = MB$, which is *maximum.* ***This is the position of unstable equilibrium.***

Sample Problem 1 **A magnet having a magnetic moment of $4{\cdot}0 \times 10^4$ J/T is free to rotate in a horizontal plane where a magnetic field 4×10^{-5} T exists. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.**

Sol. Here, $M = 4{\cdot}0 \times 10^4$ J/T, $B = 4 \times 10^{-5}$ T, $W = ?$, $\theta_1 = 0°$, $\theta_2 = 60°$

$$W = -MB\,(\cos\theta_2 - \cos\theta_1) = -4{\cdot}0 \times 10^4 \times 4 \times 10^{-5}\,(\cos 60° - \cos 0°)$$

$$= -1{\cdot}6\left(\frac{1}{2} - 1\right) = \mathbf{0{\cdot}8\ J}$$

Sample Problem 2 **Calculate the magnitude of torque required to hold a bar magnet of magnetic moment 200 Am^2 along a direction making an angle of 30° with the direction of uniform magnetic field of 0·36 G.**

Sol. Here, $\tau = ?$, $M = 200\ Am^2$, $\theta = 30°$, $B = 0{\cdot}36\ G = 0{\cdot}36 \times 10^{-4}$ T

$$\tau = MB\sin\theta = 200 \times 0{\cdot}36 \times 10^{-4} \sin 30° = \mathbf{3{\cdot}6 \times 10^{-3}\ N\text{-}m}$$

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5.13. THE ELECTROSTATIC ANALOGUE OF MAGNETISM

We have studied that two unlike magnetic poles of equal strength (m) separated by a distance $(2\vec{l})$ form a magnetic dipole. It has a magnetic moment $\vec{M} = m(2\vec{l})$, which is directed from south to north through the magnet.

The magnetic dipole is analogous to an electric dipole, which consists of two equal charges of opposite sign ($\pm q$) separated by a certain distance $(2\vec{a})$. It has an electric dipole moment $\vec{p} = q(2\vec{a})$, which is directed from $-q$ to $+q$ through the dipole.

The equations for magnetic field $\vec{B}$ due to a magnetic dipole (or bar magnet) can be obtained from the equations of electric field $\vec{E}$ due to an electric dipole by making the **following changes :**

$$\vec{E} \longrightarrow \vec{B} \; ; \; \vec{p} \longrightarrow \vec{M} \; ; \; \frac{1}{4\pi\epsilon_0} \longrightarrow \frac{\mu_0}{4\pi}$$

Therefore, for any point on axial line of short bar magnet at a distance (d) from the centre of magnet, we write $$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

And for any point on equatorial line of short bar magnet at a distance (d) from the centre of magnet, we write $$B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

Further, torque on magnetic dipole in an external magnetic field $\vec{B}$ is, $\vec{\tau} = \vec{M} \times \vec{B}$

This corresponds to torque on electric dipole in an external electric field $\vec{E}$ as, $\vec{\tau} = \vec{p} \times \vec{E}$

Similarly, the potential energy of magnetic dipole in an external magnetic field $\vec{B}$ is, $U = -\vec{M} \cdot \vec{B}$

This corresponds to potential energy of an electric dipole in an external electric field $\vec{E}$ as, $U = -\vec{p} \cdot \vec{E}$

The analogy between magnetism and electrostatics is summarised in the following table :

S. No.	Physical Quantity	Magnetism	Electrostatics
1.	Dipole moment	$\vec{M} = m \times 2\vec{l}$	$\vec{p} = q \times 2\vec{a}$
2.	Torque in external field	$\vec{\tau} = \vec{M} \times \vec{B}$	$\vec{\tau} = \vec{p} \times \vec{E}$
3.	Potential energy of dipole	$u = -\vec{M} \cdot \vec{B}$	$u = -\vec{p} \cdot \vec{E}$
4.	Field intensity on axial line	$B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M}{d^3}\right)$	$E_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{2p}{r^3}\right)$
5.	Field intensity on equatorial line	$B_2 = \frac{\mu_0}{4\pi}\left(\frac{M}{d^3}\right)$	$E_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^3}\right)$

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There is **one fundamental difference between electricity and magnetism.** Whereas in electricity, an isolated charge can exist ; in magnetism, an isolated magnetic pole does not exist. The simplest magnetic structure that can exist is only a magnetic dipole or current loop characterized by magnetic dipole moment $\vec{M}$. This is unlike a scalar charge q which characterizes electricity.

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In an external magnetic field, the magnetic dipole experiences a torque (unlike the force experienced by a charge q in an external electric field). The effect of torque is to align the dipole along the external magnetic field. The directive property of a magnet is attributed to the torque acting on the magnetic dipole in the magnetic field of earth.

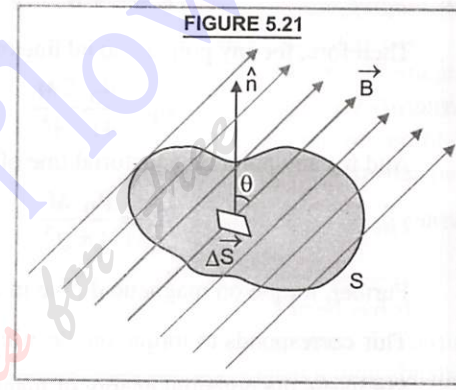
5.14. GAUSS'S LAW FOR MAGNETISM

According to Gauss's law for magnetism, the net magnetic flux (ϕ_B) through any closed surface is always zero.

The law implies that the number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it.

Suppose a closed surface S is held in a uniform magnetic field $\vec{B}$. Consider a small vector area element $\Delta \vec{s}$ of this surface, Fig. 5.21. Magnetic flux through this area element is defined as $\Delta \phi_B = \vec{B} . \Delta \vec{s}$. Summing over all small area elements of the surface, we obtain according to Gauss's law for magnetism,

FIGURE 5.21



$$\phi_B = \sum_{\text{all}} \Delta \phi_B = \sum_{\text{all}} \vec{B} . \Delta \vec{s} = 0 \quad \text{...(20)}$$

If the area elements are really small, we can rewrite this equation as

$$\phi_B = \oint_S \vec{B} . \vec{ds} = 0 \quad \text{...(21)}$$

Compare this equation with Gauss's law in electrostatics, *i.e.*, electric flux through a closed surface S is given by

$$\phi_E = \oint_S \vec{E} . \vec{ds} = \frac{q}{\epsilon_0} \quad \text{...(22)}$$

where q is electric charge enclosed by the closed surface. If an electric dipole were enclosed by the surface equal and opposite charges in the dipole add upto zero. Therefore, ϕ_E would be zero. The fact that $\phi_B = 0$ indicates that ***the simplest magnetic element is a dipole or current loop. The isolated magnetic poles, called magnetic monopoles are not known to exist.*** All magnetic phenomena can be explained in terms of an arrangement of magnetic dipoles and/or current loops.

Gauss's theorem in magnetism establishes that :

(*i*) *isolated magnetic poles called magnetic monopoles do not exist*, or

(*ii*) *in magnetism, there is no counterpart of isolated free charge in electricity*, or

(*iii*) *magnetic poles always exist in unlike pairs of equal strength.*

Thus corresponding to eqn. (22) of Gauss's theorem in electrostatics, we can visualize eqn. (21) as

$$\phi_B = \oint_S \vec{B} . \vec{ds} = \mu_0 (m) + \mu_0 (-m) = 0$$

where m is strength of N-pole and $-m$ is strength of S-pole of same magnet ; μ_0 is absolute magnetic permeability of free space.

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5.15. MAGNETIC FIELD OF EARTH

Sir William Gilbert was the first to suggest in the year 1600, that ***earth itself is a huge magnet. His statement was based on the following evidence :***

(*i*) A magnet suspended from a thread and free to rotate in a horizontal plane comes to rest along the north-south direction. On disturbing, the magnet returns quickly to its north-south direction again. This is as if a huge bar magnet is placed approximately along the axis of rotation of earth and deep in its interior. The north pole of this fictitious magnet must be towards geographic south so as to attract south pole of the suspended magnet and *vice-versa.* ***We must add here that fictitious magnet is just a conceptual device adopted to explain the earth's magnetic field and its peculiarities. In reality, there is no such bar magnet in the interior of the earth.***

DO YOU KNOW ?

- Though there are large deposits of iron inside the earth, it is not at all likely that a large solid block of iron stretches from north pole to south pole.

(*ii*) When a soft iron piece is buried under the surface of earth in the north south direction, it is found to acquire the properties of a magnet after some time.

(*iii*) When we draw field lines of a magnet, we come across *neutral points.* At these points, magnetic field due to the magnet is neutralized or cancelled exactly by the magnetic field of earth. If earth had no magnetism of its own, we would never observe neutral points.

The branch of Physics which deals with the study of magnetism of earth is called terrestrial magnetism or geomagnetism.

It has been established that earth's magnetic field is fairly uniform. The strength of this field at the surface of earth is approximately 0·1 gauss or 10^{-5} tesla. The field is not confined only to earth's surface. It extends upto a height nearly 5 times the radius of earth ($\approx 5 \times 6400$ km = 32000 km).

5.16. CAUSE OF EARTH'S MAGNETISM

The exact cause of earth's magnetism is not yet known fully. However, some important postulates in this respect are as follows :

(*a*) As stated earlier, William Gilbert suggested in the year 1600 that the earth itself is a huge magnet.

(*b*) According to Prof. Brackett, *earth's magnetism may be due to rotation of earth about its axis.* This is because every substance is made up of charged particles (protons and electrons). Therefore, a substance rotating about an axis is equivalent to circulating currents, which are responsible for the magnetisation.

(*c*) In the outer layers of earth's atmosphere, gases are in the ionised state, primarily on account of cosmic rays. *As earth rotates, strong electric currents are set up due to movement of (charged) ions.* These currents might be magnetising the earth.

(*d*) The earth's core is extremely hot and molten. *Circulating ions of iron and nickel in the highly conducting liquid region of the earth's core might be forming current loops and producing earth's magnetic field.* This hypothesis is supported by the following facts :

(*i*) Moon has no molten core and hence no magnetic field. (*ii*) Venus has a slower rate of rotation and hence a weaker magnetic field. (*iii*) Amongst the planets, Jupiter has the fastest rotation rate. Therefore, it has a fairly strong magnetic field.

5.17. GENERAL FEATURES OF EARTH'S MAGNETIC FIELD

The earth's magnetic field is approximately like that of a giant magnetic dipole, whose axis is inclined roughly 11·3° west of the axis of rotation of earth, which is along geographic north N_g and geographic south S_g, Fig. 5.22. The south pole of this giant magnetic dipole is towards geographic north and north pole of this dipole is towards geographic south. The strength of earth's magnetic field at its surface is of the order of 10^{-5} tesla. For example, at Delhi, earth's field is 0·35 gauss = $3{\cdot}5 \times 10^{-5}$ T.

As the north pole of a freely suspended magnet points approximately towards N_g, therefore, earth's magnetic pole near N_g is called north magnetic pole N_m. Similarly, the magnetic pole near S_g is called south magnetic pole S_m.

Note that N_m and S_m are points on earth's surface and not in the interior of earth. Therefore, we can specify their location exactly in terms of longitude and latitude. At present, N_m is located at a place in North Canada, longitude 71·8° W and latitude 79·74° N. Obviously, S_m will be at a place diametrically opposite to N_m, *i.e.*, longitude 108·2° *E* and latitude 79·74° S. The magnetic poles are approximately 2000 km away from geographic poles.

As is clear from Fig. 5.22, magnetic equator divides the earth's surface into two (magnetic) hemi spheres. The magnetic field lines enter into the hemisphere containing N_g and come out of the hemisphere containing S_g.

Note that the straight line passing through the geographic north and south poles of earth is called the **geographic axis.** It is the axis of rotation of earth.

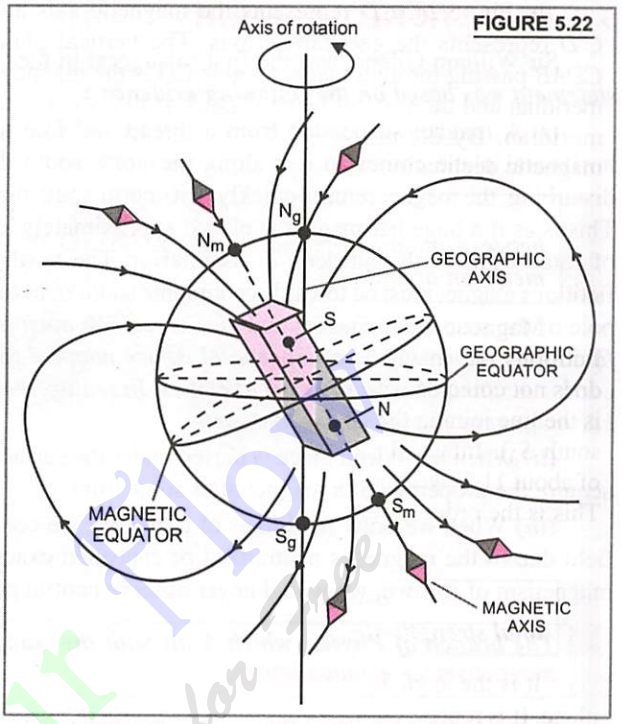


FIGURE 5.22

The straight line passing through magnetic north and south poles of earth is called **magnetic axis.** The great circle on the earth perpendicular to the magnetic axis is called **magnetic equator.**

The vertical plane passing through geographic north and south poles is called **geographic meridian.** And the vertical plane passing through the magnetic axis of a freely suspended magnet is called **magnetic meridian.**

5.18. MAGNETIC ELEMENTS

***Magnetic elements of earth** at a place are the quantities which describe completely in magnitude as well as direction, the magnetic field of earth at that place.*

Following are the three magnetic elements of earth :

1. Magnetic declination (θ),
2. Magnetic inclination or Magnetic dip (δ),
3. Horizontal component (H) of earth's magnetic field.

1. Magnetic declination.

When a magnet is suspended freely by an unspun silken thread, the ends of the magnet do not point out exactly along geographic north south direction, as shown in Fig. 5.23. As stated already, the line joining the north and south poles of a freely suspended magnet is called **magnetic axis.** The axis about which earth rotates is the **geographic axis.**

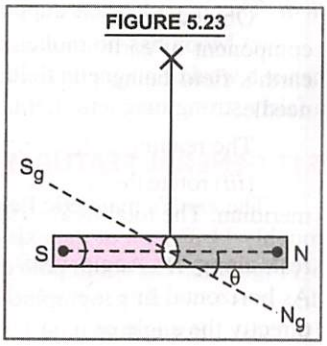


FIGURE 5.23

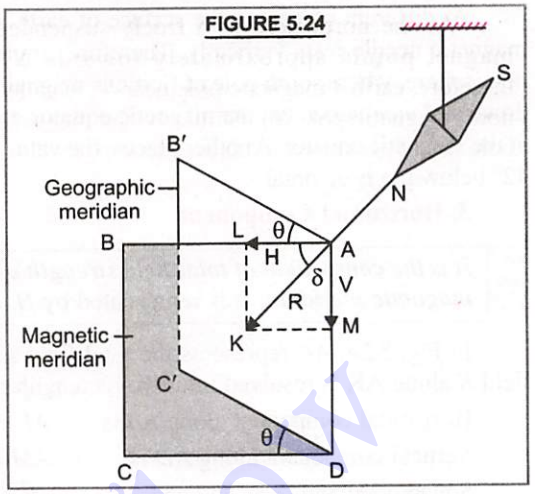
The small angle between magnetic axis and geographic axis at a place is defined as the magnetic declination at that place. It is represented by θ, Fig. 5.23.

In Fig. 5.24, CD represents the magnetic axis and $C'D$ represents the geographic axis. The vertical plane $CDAB$ passing through magnetic axis CD is the magnetic meridian and the vertical plane $C'DAB'$ is the geographic meridian. By definition, $\angle CDC' = \angle BAB' = \theta$ is the magnetic declination. Hence, we may also define

***Magnetic declination** at a place is the angle between magnetic meridian and geographic meridian at that place.*

Magnetic declination arises because the axis of the fictitious bar magnet responsible for earth's magnetism does not coincide with the axis of rotation of earth (which is the line joining the geographic north N_g and geographic south S_g). Infact, the imaginary magnet lies at an angle of about 11·3° west of the polar axis as shown in Fig. 5.22. This is the order of magnetic declination at the place.

FIGURE 5.24



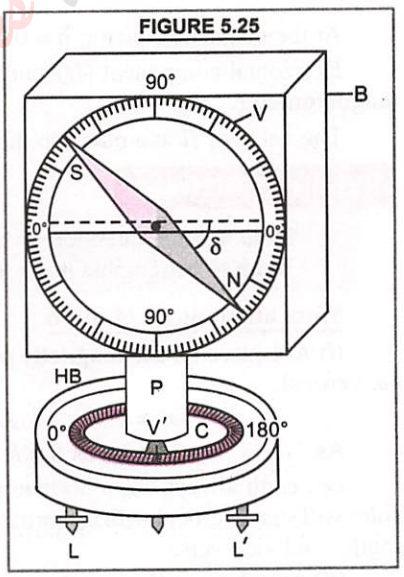
2. Magnetic Dip or Magnetic Inclination

***Magnetic dip** or **Magnetic inclination** at a place is defined as the angle which the direction of total strength of earth's magnetic field makes with a horizontal line in magnetic meridian.*

It is the angle by which total intensity of earth's magnetic field dips or comes up out of the horizontal plane. It is represented by δ. The value of magnetic dip at a place can be measured using an instrument called **'Dip circle'.**

The essential parts of a dip circle are shown in Fig. 5.25. A magnetic needle NS is pivoted on a horizontal axle at the centre of a circular vertical scale V. The ends of magnetic needle are free to move over this scale graduated in degrees marked 0° – 0° in the horizontal direction and 90° – 90° in the vertical direction. The needle and the scale are enclosed in a rectangular box B with glass windows. The box is mounted on a vertical pillar P on horizontal base HB provided with levelling screws L & L'. A circular scale C (0° to 360°) is engraved on the HB. The box can be rotated over the horizontal base and its position can be read on the circular scale C with the help of a vernier scale V' which can slide over circular scale.

FIGURE 5.25



To use the dip circle, (*i*) adjust the levelling screws till the base is horizontal and scale V inside the box is vertical.

(*ii*) rotate the box B till the ends of magnetic needle NS read 90° – 90° on the vertical scale.

Obviously, the needle in this position is aligned with the vertical component of earth's magnetic field. The horizontal component of earth's field being perpendicular to this plane does not affect the needle.

The reading of the vernier V' is noted on horizontal circular scale C.

(*iii*) rotate the box exactly through 90° from this position. The plane of vertical scale V is now in magnetic meridian. The total field strength R of earth's magnetic field is in the same plane and hence magnetic needle rests along $\vec{R}$. Angle which $\vec{R}$ makes with the horizontal direction in magnetic meridian is the angle of dip. As horizontal line is marked 0 – 0, the readings of the ends of magnetic needle on the vertical scale give us directly the angle of dip (δ), Fig. 5.25.

At different places on the surface of earth, δ is different. For example, at the two magnetic poles, the magnetic needle rests vertically. Therefore, angle of dip at the two poles is 90°. Obviously in the northern hemisphere, where south pole of ficitious magnet lies, the north pole of the mounted magnetic needle will dip down and *vice-versa*. On the magnetic equator, the angle of dip is zero as the needle would rest horizontally at the magnetic equator. At other places, the value of δ lies between 0° and 90°. The dip angle at Delhi is about 42° below the horizontal.

3. Horizontal Component

It is the component of total field strength of earth's magnetic field in the horizontal direction in magnetic meridian. It is represented by *H*.

In Fig. 5.24, *AK* represents the total field strength of earth's magnetic field, $\angle BAK = \delta$. The resultant field *R* along AK is resolved into two rectangular components :

Horizontal component along *AB* is $AL = H = R\cos\delta$ ...(23)

Vertical component along *AD* is $AM = V = R\sin\delta$ ...(24)

Square (23) and (24), and add $H^2 + V^2 = R^2(\cos^2\delta + \sin^2\delta) = R^2$

$\therefore$ $$R = \sqrt{H^2 + V^2}$$ ...(25)

Dividing (24) by (23), we get

$$\frac{R\sin\delta}{R\cos\delta} = \frac{V}{H} \quad \text{or} \quad \tan\delta = \frac{V}{H}$$ ...(26)

The value of horizontal component $H = R\cos\delta$ is different at different places. At the magnetic poles, $\delta = 90°$ $\therefore$ $H = R\cos 90° = \text{zero}$

At the magnetic equator, $\delta = 0°$ $\therefore$ $H = R\cos 0° = R = \text{maximum}$.

Horizontal component (*H*) can be measured using both, a **vibration magnetometer** and **a deflection magnetometer.**

The value of *H* at a place on the surface of earth is of the order of $3{\cdot}2 \times 10^{-5}$ tesla.

Retain in Memory

Note that the direction of horizontal component *H* of earth's magnetic field is from geographic south to geographic north above the surface of earth. (if we ignore declination).

More about angle of dip δ

(*i*) At a place **on the magnetic poles**, total earth's magnetic field is perpendicular to the surface of earth *i.e.* vertical.

$\therefore$ $R = V$ As $V = R\sin\delta$ $\therefore$ $\sin\delta = 1$; $\delta = 90°$

As $H = R\cos\delta = R\cos 90° = 0$ therefore, *at magnetic poles*, $\delta = 90°$ and $H = 0$

i.e., earth always has a horizontal component except at magnetic poles. A freely suspended magnet at poles will stand vertical with its north pole pointing towards earth's geographical north pole (which is magnetic south), and vice-versa.

(*ii*) At a place **on the equator,** total earth's magnetic field is parallel to the surface of earth *i.e.* horizontal.

$\therefore$ $R = H$

As $H = R\cos\delta$ $\therefore$ $\cos\delta = 1$; $\delta = 0°$

As $V = R\sin\delta = R\sin 0° = 0$. Therefore, at the equator, $\delta = 0°$ and $V = 0$

i.e., earth always has a vertical component except at equator. A freely suspended magnet at equator will stand horizontal.

(*iii*) In a vertical plane at an angle θ with the magnetic meridian.

$H' = H\cos\theta$ and $V' = V$

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Therefore, observed angle of dip δ' in a vertical plane making $\angle\theta$ with magnetic meridian is given by

$$\tan\delta' = \frac{V'}{H'} = \frac{V}{H\cos\theta} = \frac{\tan\delta}{\cos\theta}, \quad i.e., \quad \boxed{\tan\delta' = \frac{\tan\delta}{\cos\theta}} \qquad ...(27)$$

Here, δ is true value of dip and δ' is apparent value of dip.

For a vertical plane other than magnetic meridian, $\theta > 0°$, $\cos\theta < 1$

$\therefore$ $\delta' > \delta$ *i.e.*, angle of dip increases.

when $\theta = 90°$, $\quad \tan\delta' = \dfrac{\tan\delta}{\cos 90°} = \infty$

$\therefore$ $\delta' = 90°$ *i.e.*, in a plane perpendicular to magnetic meridian, dip needle will stand vertical.

(*iv*) If δ_1 and δ_2 are observed angles of dip in two arbitrary vertical planes–which are perpendicular to eachother, then the true angle of dip δ is given by the relation :

$$\boxed{*\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2} \qquad ...(28)$$

(*v*) If λ is magnetic latitude at a place, then it can be shown that the angle of dip δ at that place is given by

$$\boxed{\tan\delta = 2\tan\lambda} \qquad ...(29)$$

Retain in Memory

A magnet suspended freely at equator will set itself parallel to the surface of earth while one suspended at pole will stand vertical.

Sample Problem 1 **At a place, the horizontal component of earth's magnetic field is B and angle of dip is 60°. What is the value of horizontal component of earth's magnetic field at equator ?** (CBSE 2017)

Sol. Here, $H = B$ and $\delta = 60°$.

From $H = R\cos\delta$

$B = R\cos 60° = R/2 \quad \therefore \quad R = 2B$

At equator, $\delta = 0 \quad \therefore \quad H = R\cos 0° = 2B\cos 0° = \mathbf{2B}$

Sample Problem 2 **The horizontal component of earth's magnetic field at a place is $0{\cdot}4 \times 10^{-4}$ T. If angle of dip is 45°, what are the values of vertical component and total field strength of earth's field ?**

Sol. Here, $H = 0{\cdot}4 \times 10^{-4}$ T, $\delta = 45°$, $H = ?$ $R = ?$

As $\dfrac{V}{H} = \tan\delta \quad \therefore \quad V = H\tan\delta = 0{\cdot}4 \times 10^{-4}\tan 45° = \mathbf{0{\cdot}4 \times 10^{-4}\ T}$

From $H = R\cos\delta$, $\quad R = \dfrac{H}{\cos\delta} = \dfrac{0{\cdot}4\times10^{-4}}{\cos 45°} = 0{\cdot}4\sqrt{2}\times10^{-4}\ \text{T} = \mathbf{0{\cdot}56 \times 10^{-4}\ T}$

5.19. VARIATION OF EARTH'S MAGNETIC FIELD

(*a*) Global Variation (*i.e.* from place to place).

Strength of terrestrial magnetism as we know, is small ($\approx 4 \times 10^{-5}$ T). Smaller still is the back ground field of our galaxy, the Milky way ($\approx 2 \times 10^{-12}$ T).

*For proof, see Q. 25 of Problems for Practice, Page 5/73.

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At about 30000 km above the surface of earth, the earth's field falls below 10^{-6} T. Beyond this height, solar wind disturbs the dipole pattern. The solar wind consists of streams of charged particles that emerge continuously from the sun. These particles get trapped near the magnetic poles of earth. They ionise the atmosphere above these poles causing a spectacular display of light.Magnetic maps showing the variation of magnetic elements from place to place have been drawn.

On these magnetic maps :

(*i*) The lines joining places of equal declination are called **isogonic lines.**

(*ii*) The lines joining places of zero declination are called **agonic lines.**

(*iii*) The lines joining places of equal dip/inclination are called **isoclinic lines.**

(*iv*) The lines joining places of zero dip/inclination are called **aclinic lines.** Similarly,

(*v*) The lines joining places of equal values of horizontal component (H) of earth's magnetic field are called **Isodynamic lines.**

(*b*) Temporal Variation (*i.e.* from time to time)

The magnetic field of earth is found to change with time. These changes may be characterised as short term and long term changes.

In *short term changes,* the magnetic poles of earth S_m and N_m keep on shifting their position. For example, magnetic south pole (in northern arctic region of Canada) has been found to move in north-westerly direction at a rate of 10 km per year. Detailed maps of earth's magnetic field are maintained and revised from time to time.

The *long term changes* are on geological time scale $\approx 10^5$ to 10^6 years. It appears that the direction of earth's magnetic field has reversed itself every million year or so. It is believed that once in a million years or so, the currents in the earth's core slow down, come to halt and then pick up in the reverse direction.

5.20. NEUTRAL POINTS

When we trace magnetic lines of force around a magnet using a compass needle, what we obtain is the resultant of the magnetic field of magnet and that of the earth. As earth's magnetic field is fixed, the resultant magnetic field would depend on the direction in which the magnet is placed. In the plot of the resultant magnetic field, we come across points at which field (B) due to the magnet becomes equal and opposite to the horizontal component (H) of earth's field *i.e.* $B = H$. Therefore, the net magnetic field at these points will be zero. ***These points where net magnetic field due to the magnet and magnetic field due to the earth is zero are called*** **Neutral points.** A small compass needle placed at a neutral point shall experience no force/torque. Therefore, it can set itself in any direction, which may be different from the usual N–S direction.

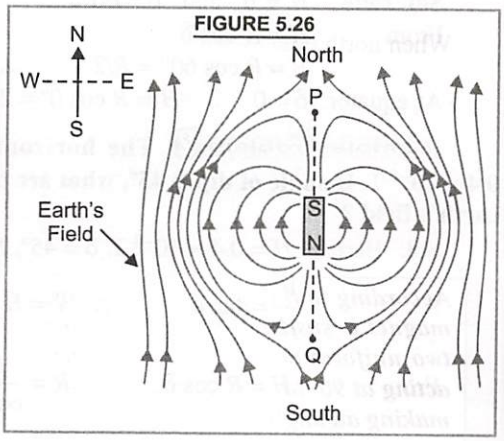

FIGURE 5.26

We can show that (*i*) ***When a magnet is placed with its north pole towards geographic south,*** neutral points P and Q lie on axial line of the magnet as shown in Fig. 5.26.

At each neutral point,

$$B_1 = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = H$$

If the magnet is short $l << d$,

$$B_1 = \frac{\mu_0}{4\pi} \frac{2M}{d^3} = H$$

(*ii*) ***When a magnet is placed with its north pole towards geographic north,*** neutral points P and Q lie on equatorial line of the magnet as shown in Fig. 5.27.

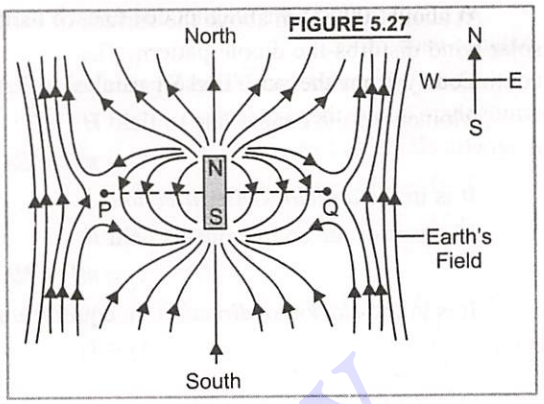

FIGURE 5.27

At each neutral point,

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = H$$

If the magnet is short, $l << d$

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3} = H$$

In both the formulae, symbols have their usual meaning.

Note. Comparison of Figs. 5.26 and 5.27 shows that when north pole of bar magnet is turned through 180°, neutral points turn in the same direction through 180°/2 = 90°.

Sample Problem 1 **A short bar magnet is placed with its north pole pointing south. The neutral point is 10 cm away from the centre of the magnet. If H = 0·4 gauss, calculate magnetic moment of the magnet.**

Sol. Here, d = 10 cm = 0·1 m, H = 0·4 gauss = $0{\cdot}4 \times 10^{-4}$ T, M = ?

Neutral points in this case, lie on axial line of magnet, such that $\frac{\mu_0}{4\pi}\frac{2M}{d^3} = H$

$$\therefore \quad 10^{-7} \times \frac{2M}{(0{\cdot}1)^3} = 0{\cdot}4 \times 10^{-4} \quad \text{or} \quad M = 0{\cdot}2 \text{ A m}^2$$

Sample Problem 2 **A short bar magnet is placed with its north pole pointing north. If magnetic moment of magnet is 0·4 Am^2 and horizontal component of earth's magnetic field is 0·4 gauss, what is the distance of neutral point from the centre of the magnet ?**

Sol. Here, M = 0·4 Am^2, H = 0·4 gauss = $0{\cdot}4 \times 10^{-4}$ T , d = ?

When north pole is pointing north, $B = \frac{\mu_0}{4\pi}\frac{M}{d^3} = H$

$$10^{-7} \times \frac{0{\cdot}4}{d^3} = 0{\cdot}4 \times 10^{-4}$$

$$d^3 = 10^{-3},\ d = 10^{-1} \text{ m} = \mathbf{10 \text{ cm}}$$

5.21. THE TANGENT LAW IN MAGNETISM

According to the tangent law in magnetism, when a magnet is suspended under the combined action of two uniform magnetic fields of strengths F and H, acting at 90° to each other, the magnet comes to rest making an angle θ with the direction of H, such that

$$F = H \tan \theta \qquad ...(30)$$

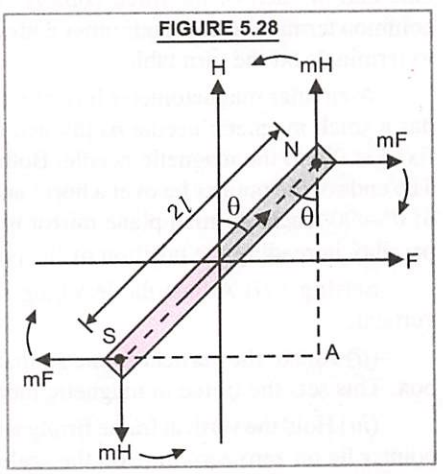

FIGURE 5.28

In Fig. 5.28, NS is a bar magnet of length $2l$, strength of each pole = m and dipole moment $M = m \times 2l$. Under the combined action of two mutually perpendicular fields F and H, the magnet comes to rest at an angle θ with H.

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A couple acts on the magnet due to each magnetic field. In equilibrium, the moments of the two couples balance each other.

Draw NA parallel to H and SA parallel to F. Let them meet at A.

Moment of the couple due to field H

$$\tau_1 = mH \times SA$$

It is in the *anticlockwise direction.*

Moment of the couple due to field F

$$\tau_2 = mF \times NA$$

It is in the *clockwise direction*. In equilibrium, the moments of two couples must be equal and opposite *i.e.*,

$$\tau_2 = \tau_1$$

$$\therefore \quad mF \times NA = mH \times SA \quad \text{or} \quad F = H \times \frac{SA}{NA} = H \tan\theta, \quad \textit{i.e.} \quad \boxed{F = H \tan\theta} \qquad \text{...(31)}$$

which proves the tangent law.

Usually, H is horizontal component of earth's field and F is field due to a magnetic dipole or current loop.

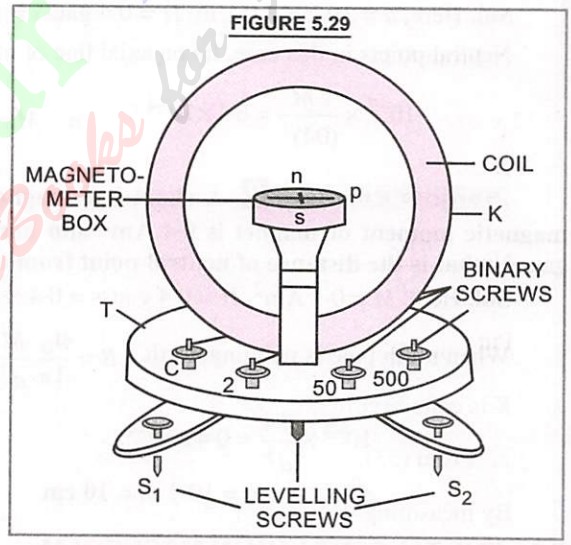
5.22. TANGENT GALVANOMETER

It is an instrument which is used for detection and measurement of low electric currents.

Principle. It is based on *tangent law* in magnetism.

Construction. A tangent galvanometer consists of circular frame K of a non magnetic material like wood, ebonite or brass. It is mounted vertically on a horizontal base called turn table (T), provided with levelling screws S_1 and S_2 at the base. The frame can be rotated about a vertical axis passing through its centre, Fig. 5.29.

FIGURE 5.29



Three coils of fine insulated copper wire having 2, 50 and 500 number of turns are wound on the circular frame. The coils are well insulated from one another. One end of each of the three coils is connected to a common terminal C and their other ends are connected to terminals on the turn table.

A circular magnetometer box of non-magnetic material is held at the centre of circular frame. This box has a small magnetic needle ns pivoted at its centre on a vertical axis with a long thin aluminium pointer p fixed at 90° to the magnetic needle. Both, the magnetic needle and the pointer are free to move as one body. The ends of the pointer lie over a horizontal circular scale graduated in degrees and divided into four quadrants of 0° – 90° each. A small plane mirror is fixed at the base under the pointer. This would remove error due to parallax in reading the position of the pointer on the scale.

Setting : (*i*) Adjust the levelling screws so that the turn table is horizontal and the frame is exactly vertical.

(*ii*) Rotate the vertical frame so that the plane of the frame lies exactly over the magnetic needle in the box. This sets the frame in magnetic meridian.

(*iii*) Hold the vertical frame firmly with hand and turn the magnetometer box, so that ends of the aluminium pointer lie on zero zero line, on the scale.

Theory and Working. In Fig. 5.30, *XY* is cross section of the circular frame perpendicular to the plane of the paper.

As the frame is set already in magnetic meridian, therefore, earth's horizontal field *H* is along *YX*, (*i.e.*, along south north direction). When a current *I* is passed through any one coil* of tangent galvanometer, a magnetic field *F* develops along the axis of the circular coil and perpendicular to the plane of the coil *i.e.* along east west direction. The magnetic needle mounted in the magnetometer box is *under the combined action of two mutually perpendicular fields F* and *H*. The needle gets deflected and comes to rest making an angle θ with the direction of *H*.

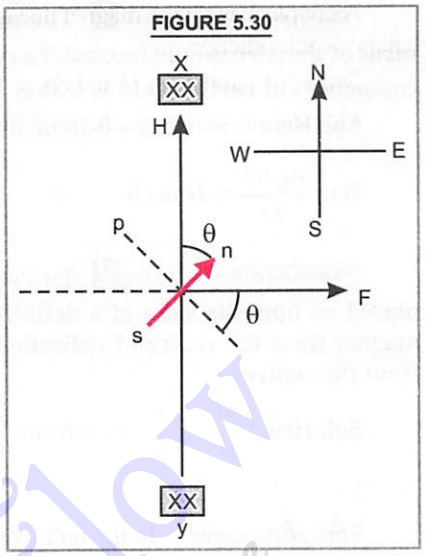


FIGURE 5.30

The pointer also turns through the same angle θ. As pointer was already set on 0–0 line, therefore angle θ is read from the scale under the ends of the pointer.

According to tangent law in magnetism,

$$F = H \tan \theta \qquad \text{...(32)}$$

But $$F = \frac{\mu_0}{4\pi} \frac{2\pi n I^{**}}{r}$$

where *n* is number of turns in the coil, *r* is radius of the coil and μ_0 is magnetic permeability of free space.

$\therefore$ From (32), $$\frac{\mu_0}{4\pi} \frac{2\pi n I}{r} = H \tan \theta \quad \text{or} \quad I = \frac{4\pi r\, H \tan \theta}{\mu_0 . 2\pi n}$$

or $$I = \frac{2rH}{n\mu_0} \tan \theta = \frac{H}{n\mu_0/2r} \tan \theta \qquad \text{...(33)}$$

Now $\frac{n\mu_0}{2r} = G$, the galvanometer constant. $\therefore$ $I = \frac{H}{G} \tan \theta$

Put $$\frac{H}{G} = \frac{H}{n\mu_0/2r} = \frac{2rH}{n\mu_0} = K,$$ another constant.

K is called **reduction factor** of tangent galvanometer.

$\therefore$ From (33), $$I = K \tan \theta \qquad \text{...(34)}$$

By measuring θ, and knowing *K*, current *I* can be calculated.

Reduction factor of Tangent Galvanometer

When θ = 45°, from (34) ; $I = K \tan 45° = K \times 1$ or $K = I$

Hence reduction factor of a tangent galvanometer is equal to current, required to be passed through tangent galvanometer to produce a deflection of 45°. K is measured in ampere.

The value of *K* depends on number of turns (*n*), radius of coil (*r*) and horizontal component of earth's magnetic field (*H*) at the place. Obviously, reduction factor of a tangent galvanometer increases with increase in radius of the coil, and decreases with increase in number of turns of the coil.

Note that a tangent galvanometer is also called **moving magnet type galvanometer,** because, here the coil is stationary and the magnetic needle moves. This is contrary to moving coil galvanometer, Art. 4.27.

*The coil is chosen according to the strength of current *I* to be measured. For low currents, coil with larger number of turns is used. This is because from (33), $I \times n \propto \tan \theta$

** Refer to Art. 4.7, *B* is replaced by *F* here.

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Sample Problem 1 **The radius of the coil of a tangent galvanometer is 16 cm. How many turns of the wire should be used if a current of 40 mA is to produce a deflection of 45°. Given, horizontal component of earth's field is $0{\cdot}36 \times 10^{-4}$ T.**

Sol. Here, $r = 16$ cm $= 0{\cdot}16$ m, $n = ?$, $I = 40$ mA $= 40 \times 10^{-3}$ A, $\theta = 45°$, $H = 0{\cdot}36 \times 10^{-4}$ T

As $\dfrac{\mu_0 n I}{2r} = H \tan\theta$ $\quad\therefore\quad$ $n = \dfrac{2rH\tan\theta}{\mu_0 I} = \dfrac{2\times 0{\cdot}16\times 0{\cdot}36\times 10^{-4}\tan 45°}{4\pi\times 10^{-7}\times(40\times 10^{-3})} = \mathbf{229}$

Sample Problem 2 **Two short magnets having magnetic moments in the ratio 8 : 1, when placed on opposite sides of a deflection magnetometer produce no deflection. If distance of stronger magnet from the centre of deflection magnetometer is 6 cm, what is the distance of weaker magnet from the centre ?**

Sol. Here, $\dfrac{M_1}{M_2} = \dfrac{8}{1}$, $d_1 = 6$ cm $\quad d_2 = ?$

For short magnets, In the null method $\dfrac{M_1}{M_2} = \left(\dfrac{d_1}{d_2}\right)^3 = \dfrac{8}{1} = (2)^3$

$\therefore$ $\dfrac{d_1}{d_2} = 2$; $d_2 = \dfrac{d_1}{2} = \dfrac{6}{2} = \mathbf{3\ cm}$

5.23. BOHR MAGNETON OR MAGNETIC DIPOLE MOMENT OF AN ATOM DUE TO REVOLVING ELECTRON

In every atom, electrons revolve around the nucleus. A revolving electron is like a loop of current, which has a definite magnetic dipole moment. When electron revolves in anticlockwise direction, the equivalent current is clockwise, Fig. 5.31(a). Therefore, upper face of the electron loop acts as south pole and lower face acts as north pole, Fig. 5.31(b). Hence an atom behaves as a magnetic dipole.

If e is the charge on an electron revolving in an orbit of radius r with a uniform angular velocity ω, then equivalent current

$$i = \frac{\text{charge}}{\text{time}} = \frac{e}{T}$$

where, T = the period of revolution of electron $= 2\pi/\omega$

$$\therefore \quad i = \frac{e}{2\pi/\omega} = \frac{\omega e}{2\pi} \qquad \text{...(35)}$$

Area of the orbit $A = \pi r^2$

Magnetic moment of the atom is given by

$$M = iA = \frac{\omega e}{2\pi}\pi r^2 \quad \text{or} \quad \boxed{M = \frac{1}{2} e\omega r^2} \qquad \text{...(36)}$$

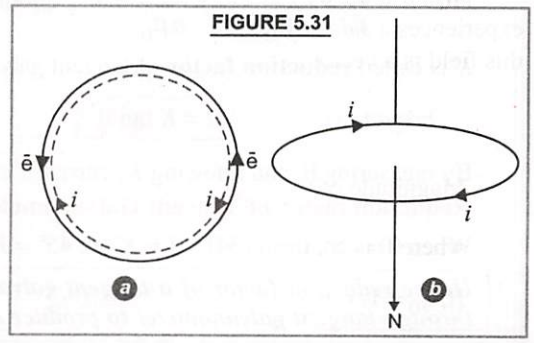

FIGURE 5.31

According to **Bohr's theory,** an electron in an atom can revolve only in certain ***stationary orbits***, in which angular momentum of electron ($m\,v\,r$) is an integral multiple of $h/2\,\pi$, where h is Planck's constant.

Thus, $mvr = \dfrac{nh}{2\pi}$, where $n = 1, 2, 3.....$ denotes the number of the orbit. Using $v = r\omega$, we get

$$m(r\omega)r = \frac{nh}{2\pi} \quad \text{or} \quad \omega r^2 = \frac{nh}{2\pi m} \qquad \text{...(37)}$$

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Put in (36), $M = \frac{1}{2} e . \frac{nh}{2\pi m} = n \frac{eh}{4\pi m} = n(\mu_B)$ ...(38)

where $\mu_B = eh/4\pi m$ ...(39)

From (38), it is clear that the magnetic moment of an atom is quantised having values 1 μ_B, 2 μ_B, 3 μ_B and so on, where μ_B is the least value of atomic dipole moment, $\mu_B = \frac{eh}{4\pi m}$. This is called **Bohr Magneton.** It serves as the unit of atomic magnetic dipole moment. We can calculate its value.

$$\mu_B = \frac{eh}{4\pi m} = \frac{1.6\times10^{-19}\times6.6\times10^{-34}}{4\times\frac{22}{7}\times9\times10^{-31}} \quad ...(40)$$

$\mu_B = 9.27 \times 10^{-24}$ ampere metre^2. We may define

***Bohr magneton** as the minimum magnetic dipole moment associated with an atom due to orbital motion of an electron in the first stationary orbit of the atom.*

Again, we define, ***Gyromagnetic ratio of electron*** $= \frac{\text{magnetic moment}}{\text{angular momentum}} = \frac{\frac{1}{2} e \omega r^2}{m \omega r^2}$

$$= \frac{eh/4\pi m}{h/2\pi} = \frac{e}{2m} = \frac{1{\cdot}6\times10^{-19}}{2\times9\times10^{-31}} = 8{\cdot}8\times10^{10} \text{ C/kg}$$

5.24. SOME IMPORTANT TERMS USED IN MAGNETISM

To describe the magnetic properties of materials, we define the following few terms, which should be clearly understood:

1. Magnetic field strength ; Magnetic field Induction or Magnetic flux density (*B*)

To understand this term, we make use of Lorentz force equation.

When a positive test charge q_0 is fired with a velocity v through a point P and the moving charge experiences a *sideways* force F, we assert that a magnetic field is present at P. The magnetic induction B of this field is a vector satisfying Lorentz force equation

$$\vec{F} = q_0 (\vec{v} \times \vec{B}) \quad ...(41)$$

Magnitude of this force is, $F = q_0 v B \sin\theta$ ...(42)

where θ is smaller angle between $\vec{v}$ and $\vec{B}$.

If, $q_0 = 1$, $v = 1$ and $\theta = 90°$, then from (42), $B = F$. We may define

***Magnetic field strength** or **magnetic field induction** or **flux density of magnetic field** being equal to the force experienced by a unit positive charge moving with unit velocity in a direction perpendicular to the magnetic field.*

We may also define Magnetic field strength or Magnetic field induction or Magnetic flux density (B) as the number of magnetic field lines passing normally through unit area of the substance.

If ϕ is the magnetic flux passing normally through an area A, then

$\phi = \vec{B} . \vec{A} = BA\cos\theta$, where θ is the angle between magnetic field strength $\vec{B}$ and area vector $\vec{A}$.

When magnetic field lines pass normally through the area,

$$\theta = 0°, \phi = BA \cos 0° = BA \text{ or } B = \phi/A$$

SI unit of magnetic flux density is **tesla** (T) or Weber/metre^2

The CGS unit of B is **gauss,** where $\boxed{1 \text{ gauss} = 10^{-4} \text{ T}}$

2. Magnetising force or Magnetising Intensity ($\vec{H}$)

The degree to which a magnetic field can magnetise a material is represented in terms of magnetising force or magnetising intensity ($\vec{H}$).

Let us consider a toroidal solenoid with n turns per unit length carrying a current I wound round a ring of a magnetic material. The magnetic induction of the field produced in the material of the toroidal solenoid is

$$\boxed{B = \mu n I} \quad \text{...(43)}$$

The product (nI) is called the *magnetising force* or *magnetising intensity H. i.e.*

$$\boxed{H = n I} \quad \text{...(44)}$$

so that from (43), $\boxed{B = \mu H}$ Hence, ...(45)

The magnitude of magnetising force may be defined as the number of ampere turns flowing round unit length of toroidal solenoid to produce the magnetic induction B, in the solenoid.

If inside the toroidal solenoid, there is free space, then magnetic induction

$$\boxed{B_0 = \mu_0 H} \quad \text{...(46)}$$

where μ_0 is magnetic permeability of free space. H is a vector quantity.

The SI units of H are ampere turn/metre *i.e.* Am^{-1}.

The dimensional formula of H is $[M^0 L^{-1} T^0 A^1]$.

3. Magnetic Permeability (μ)

It is the ability of a material to permit the passage of magnetic lines of force through it i.e. the degree or extent to which magnetic field can penetrate or permeate a material is called magnetic permeability of the material. It is represented by μ.

Magnetic permeability of a material is equal to the ratio of the magnitude of magnetic induction (B) to the magnetising force or magnetising intensity (H). It is represented by μ.

i.e. $$\mu = \frac{B}{H} \quad \text{or} \quad B = \mu H.$$

SI unit of magnetic permeability $= \dfrac{\text{Unit of } B}{\text{Unit of } H}$

$$\mu = \frac{T}{Am^{-1}} = Tm\,A^{-1}$$

$$\mu = \frac{\text{Weber}}{m^2} \cdot mA^{-1} = Wb\,m^{-1}\,A^{-1}$$

As $Wb\ A^{-1}$ = henry (H), therefore, SI unit of μ is also $H\ m^{-1}$.

The dimensional formula of magnetic permeability is $[MLT^{-2}\ A^{-2}]$.

***Relative magnetic permeability of a material** is defined as the ratio of the number of magnetic field lines per unit area in that material (i.e. flux density B in the material) to the number of magnetic field lines per unit area that would be present, if the medium were replaced by vacuum. (i.e. flux density B_0 in vacuum).*

i.e., $$\mu_r = \frac{B}{B_0} \qquad \text{...(47)}$$

μ_r has no dimensions. Its value for vacuum is one.

***Relative magnetic permeability of a material** may also be defined as the ratio of magnetic permeability of the material (μ) and magnetic permeability of free space (μ_0)*

$$\therefore \quad \mu_r = \frac{\mu}{\mu_0} \quad \text{or} \quad \mu = \mu_r \mu_0 \qquad \text{...(48)}$$

We know that $\mu_0 = 4\pi \times 10^{-7}$ weber/amp-metre ($Wb\ A^{-1}\ m^{-1}$) or henry/metre (Hm^{-1}).

4. Intensity of Magnetisation ($\vec{I}$)

It represents the extent to which a specimen is magnetised, when placed in a magnetising field. Quantitatively,

***The intensity of magnetisation of a magnetic material** is defined as the magnetic moment per unit volume of the material placed in the magnetising field.*

$$I = \frac{\text{magnetic moment}}{\text{volume}} = \frac{M}{V} \qquad \text{...(49)}$$

If a = uniform area of cross-section of the magnetised specimen (a rectangular bar)

$2l$ = magnetic length of the specimen, m = strength of each pole of the specimen,

From (49), $$I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a}$$ Hence, ...(50)

***Intensity of magnetistation of a magnetic material** is also defined as the pole strength per unit area of cross-section of the material, placed in the magnetising field.*

As $$I = \frac{\text{magnetic moment}}{\text{volume}}$$. Its SI unit is $$I = \frac{\text{Amp. metre}^2}{\text{metre}^3} = \text{A m}^{-1}.$$

The SI unit of I are the same as SI units of H.

Intensity of magnetisation is a vector quantity. Its dimensional formula is $[M^0 L^{-1} T^0 A]$.

5. Magnetic susceptibility (χ_m)

It is a property which determines how easily a specimen can be magnetised when placed in the magnetising field. Quantitatively,

***Susceptibility of a magnetic material** is defined as the ratio of the intensity of magnetisation (I) induced in the material to the magnetising force (H) applied on it. Magnetic susceptibility is represented by χ_m.*

Thus $$\chi_m = \frac{I}{H} \qquad \text{...(51)}$$

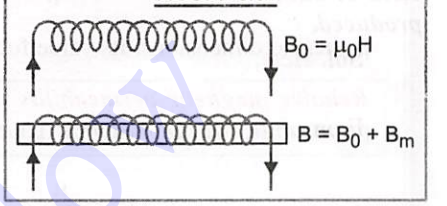
Since by definition, I is magnetic moment per unit volume, χ_m is usually called ***volume susceptibility*** of the material.

As units of H and I are same (A m^{-1}), therefore (χ_m) has no units and no dimension. It is only a number.

5.25. RELATION BETWEEN MAGNETIC PERMEABILITY AND MAGNETIC SUSCEPTIBILITY

When a magnetic material is placed in a magnetising field of magnetising intensity H, the material gets magnetised. The total magnetic induction B in the material is the sum of the magnetic induction B_0 in vacuum produced by the magnetising intensity and magnetic induction B_m, due to magnetisation of the material, Fig. 5.32.

FIGURE 5.32



Therefore, $B = B_0 + B_m$ ...(52)

But, $B_0 = \mu_0 H$ and $B_m = \mu_0 I$,

where I is the intensity of magnetisation induced in the magnetic material. Therefore, from (52)

$B = \mu_0 H + \mu_0 I = \mu_0 (H + I)$, *i.e.*, $\boxed{B = \mu_0 (H + I)}$ ...(53)

Now as $\chi_m = \dfrac{I}{H}$ $\therefore$ $I = \chi_m H$

From (53), $B = \mu_0 (H + \chi_m H) = \mu_0 H (1 + \chi_m)$

But $B = \mu H$ $\therefore$ $\mu H = \mu_0 H (1 + \chi_m)$ or $\dfrac{\mu}{\mu_0} = 1 + \chi_m$ or $\boxed{\mu_r = 1 + \chi_m}$ ...(54)

This is the relation between relative magnetic permeability and magnetic susceptibility of the material.

Note. We know that the cgs unit of magnetic induction B is **gauss** which is 10^{-4} tesla.

The cgs unit of magnetising intensity H is the **oersted**.

From (46), $H = B_0/\mu_0$ $\therefore$ 1 oersted $= \dfrac{1 \text{ gauss}}{\mu_0} = \dfrac{10^{-4}\ \text{T}}{4\pi \times 10^{-7}\ \text{TA}^{-1}\ \text{m}}$

$$\boxed{1 \text{ oersted} = 80\ \text{Am}^{-1}}$$

Table 5.1 gives a summary of important terms used in magnetism.

TABLE 5.1. Important Terms used in Magnetism

Physical Quantity	Symbol	Nature	Dimensions	Units	Remarks
1. Magnetic field induction, Magnetic flux density or Magnetic field strength	$\vec{B}$	vector	$[MT^{-2}A^{-1}]$	T	$1\ T = 10^4$ *gauss*
2. Magnetising intensity or Magnetising force	$\vec{H}$	vector	$[L^{-1}A]$	Am^{-1}	
3. Intensity of magnetisation	$\vec{I}$	vector	$[L^{-1}A]$	Am^{-1}	$\vec{I} = \dfrac{\vec{M}}{V}$
4. Magnetic moment	$\vec{M}$	vector	$[L^2A]$	Am^2	
5. Magnetic Permeability	μ	scalar	$[MLT^{-2}A^{-2}]$	$Tm\,A^{-1}$	
6. Magnetic Permeability of free space	μ_0	scalar	$[MLT^{-2}A^{-2}]$	$Tm\,A^{-1}$	$\dfrac{\mu_0}{4\pi} = 10^{-7}$
7. Relative magnetic permeability	μ_r	scalar	No	No	$\mu_r = \dfrac{\mu}{\mu_0}$
8. Magnetic Susceptibility	χ_m	scalar	No	No	$\mu_r = 1 + \chi_m$

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Sample Problem 1 **A bar magnet made of steel has a magnetic moment of 2·5 Am^2 and a mass of 6·6 g. If density of steel is $7·9 \times 10^3$ kg/m^3, what is the intensity of magnetisation ?**

Sol. Here, $M = 2·5\ Am^2$, $m = 6·6\ g = 6·6 \times 10^{-3}\ kg$

$\rho = 7·9 \times 10^3\ kg/m^3$, $I = ?$

$$I = \frac{M}{V} = \frac{M}{m/\rho} = \frac{M\rho}{m} = \frac{2·5 \times 7·9 \times 10^3}{6·6 \times 10^{-3}} = \mathbf{2·99 \times 10^6\ Am^{-1}}$$

Sample Problem 2 **An iron rod of 0·2 cm^2 cross-sectional area is subjected to a magnetising field of 1200 Am^{-1}. The susceptibility of iron *is 599. Find the permeability and the magnetic flux produced.***

Sol. Here, $A = 0·2\ cm^2 = 0·2 \times 10^{-4}\ m^2$

$H = 1200\ Am^{-1}$, $\chi_m = 599$, $\mu = ?$ $\phi = ?$

From $\mu = \mu_0 (1 + \chi_m) = 4\pi \times 10^{-7} (1 + 599) = \mathbf{7·536 \times 10^{-4}\ Tm\ A^{-1}}$

$\phi = BA = (\mu H)\ A = 7·536 \times 10^{-4} \times 1200 \times 0·2 \times 10^{-4} = \mathbf{1·81 \times 10^{-5}\ Wb}$

5.26. CLASSIFICATION OF MAGNETIC MATERIALS

There is a large variety of elements and compounds on earth. Some new elements, alloys and compounds have been synthesized in the laboratory. **Faraday** classified these substances on the basis of their magnetic properties, into the following three categories :

(*i*) Diamagnetic substances,

(*ii*) Paramagnetic substances, and

(*iii*) Ferromagnetic substances.

Their main characteristics are discussed below :

(a) Diamagnetic Substances

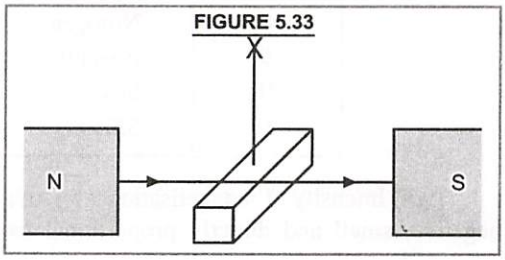
The diamagnetic substances are those in which the individual atoms/molecules/ions do not possess any net magnetic moment on their own. When such substances are placed in an external magnetising field, they get feebly magnetised in a direction opposite to the magnetising field.

Examples of some diamagnetic materials are : Antimony, Bismuth, Copper, Lead, Gold, Silver, Diamond, Zinc, Quartz ; Water, Alcohol, Mercury ; Air, Hydrogen, Nitrogen and all inert gases like Helium, Neon, Argon, etc.

Some of the other important properties of diamagnetic substances are :

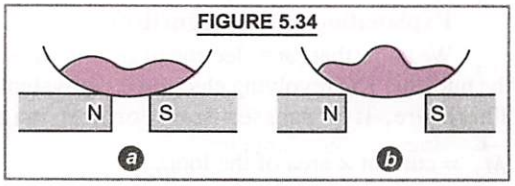
(*i*) **When suspended in a uniform magnetic field, they set their *longest axis at right angles to the direction of the field,* i.e.,** the shortest axis is along the direction of the field. This is shown in Fig. 5.33.

FIGURE 5.33



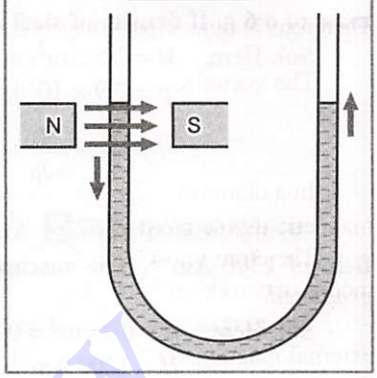
(*ii*) **When placed in a non-uniform magnetic field, these substances have a tendency *to move from stronger parts of the field to the weaker parts.*** For example, consider a diamagnetic liquid put in a watch glass placed on two pole pieces of an electromagnet held *close* to each other (≈ 2 mm or so). When the current is switched on, the liquid level acquires the shape shown in Fig. 5.34 (*a*). The liquid accumulates on the sides where field is weaker. The depression in the middle is due to stronger field at the centre. If the distance between the poles is increased, the effect is reversed, as in this case, the field is weaker at the centre. This is shown in Fig. 5.34 (*b*).

FIGURE 5.34



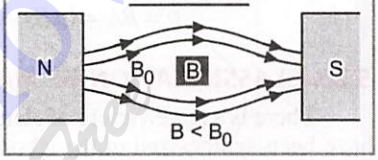
(*iii*) **The level of a diamagnetic liquid in U tube is depressed, in the limb which is between the poles of a strong magnet** as shown in Fig. 5.35. It confirms movement of diamagnetics from stronger parts of the field to weaker parts. This property implies that whereas a magnet attracts metals like iron, *it would repel a diamagnetic substance like copper.*

FIGURE 5.35



(*iv*) **When a specimen of a diamagnetic material is placed in a magnetising field, *the magnetic field lines prefer not to pass through the specimen,*** Fig. 5.36. It implies that the magnetic field lines are repelled or expelled and the field inside the material is reduced. This reduction is slight, being one part in 10^5.

FIGURE 5.36



(*v*) **Relative magnetic permeability of diamagnetic substances is always less than unity.**

$$\mu_r = \frac{B}{B_0} \quad \text{and} \quad B < B_0 \quad \therefore \quad \mu_r < 1$$

(*vi*) From the relation $\mu_r = (1 + \chi_m)$, as $\mu_r < 1$, χ_m is negative. *Hence **susceptibility of diamagnetic substances has a small negative value.***

(*vii*) ***Susceptibility of diamagnetics does not change with temperature.*** Bismuth at low temperature is an exception to this general property.

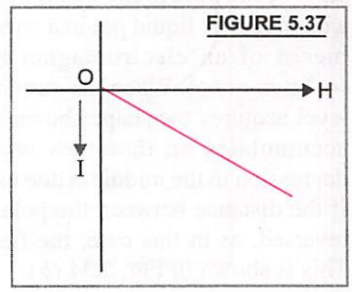
Values of Magnetic susceptibility of some diamagnetic substances at room temperature (≈ 300 K) are given in Table 5.2.

TABLE 5.2. Magnetic susceptibility of some diamagnetic substances at 300 K

S. No.	Diamagnetic substance	χ_m
1.	Copper	$-9{\cdot}6 \times 10^{-6}$
2.	Lead	$-1{\cdot}6 \times 10^{-5}$
3.	Diamond	$-2{\cdot}2 \times 10^{-5}$
4.	Gold	$-3{\cdot}6 \times 10^{-5}$
5.	Water	$-9{\cdot}1 \times 10^{-6}$
6.	Mercury	$-3{\cdot}2 \times 10^{-5}$
7.	Hydrogen	$-2{\cdot}1 \times 10^{-9}$
8.	Nitrogen	$-5{\cdot}0 \times 10^{-9}$
9.	Bismuth	$-1{\cdot}6 \times 10^{-6}$
10.	Silver	$-2{\cdot}6 \times 10^{-5}$
11.	Silicon	$-4{\cdot}2 \times 10^{-5}$

(*viii*) Intensity of magnetisation ($\vec{I}$) of a diamagnetic substance is negative, small and directly proportional to the applied magnetising force ($\vec{H}$) as shown in Fig. 5.37.

FIGURE 5.37



Explanation of Diamagnetism

We know that each electron in an atom is revolving in an orbit around the nucleus. The revolving electron is equivalent to a tiny loop of current. Therefore, it possesses some orbital magnetic dipole moment $\vec{M}_l$ = current × area of the loop.

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In addition to the orbital motion, every electron is assumed to have a spin motion around its axis. Therefore, another dipole magnetic moment called spin magnetic moment $\vec{M}_s$ is also associated with electron.

The vector sum of $\vec{M}_l$ and $\vec{M}_s$ provides the net magnetic dipole moment $\vec{M}$ to the atom, *i.e.*,

$$\vec{M} = \vec{M}_l + \vec{M}_s$$

In a diamagnetic substance, $\vec{M}_l$ and $\vec{M}_s$ cancel each other for every atom so that the atom has no net magnetic dipole moment.

Therefore, motion of all the electrons in an atom of a diamagnetic material is assumed to be reduced to motion of two electrons revolving with same angular velocity in a circular orbit of same radius, but in opposite sense.

The magnetic moment of the two being equal and opposite, cancel each other (in the absence of any external magnetic field) so that net magnetic moment of each atom is zero, Figs. 5.38 and 5.39.

FIGURE 5.38

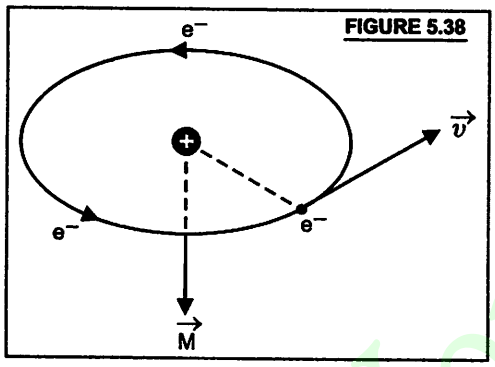
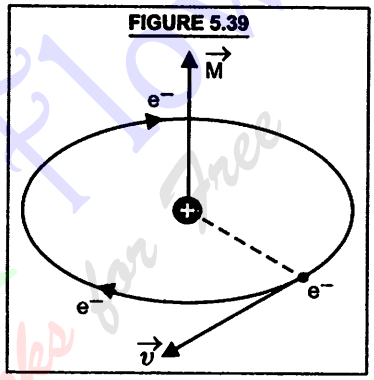


FIGURE 5.39



Let a uniform external magnetic field of induction $\vec{B}$ be applied perpendicular to the plane of rotation of electron and directed away from the reader. Each electron experiences a magnetic Lorentz force $F = Bev$. According to Fleming's left hand rule, the Lorentz force F acts radially outwards on the electron revolving anti-clockwise. It tends to decrease the centripetal force and hence decrease the velocity of electron to $(\vec{v} - \Delta\vec{v})$. As a result, the magnetic moment of electron decreases to $(\vec{M} - \Delta\vec{M})$, Fig. 5.40. On the electron revolving clockwise, the Lorentz force acts radially inwards, tending to increase the centripetal force and hence the velocity of electron to $(\vec{v} + \Delta\vec{v})$. Therefore, the magnetic moment of electron increases to $(\vec{M} + \Delta\vec{M})$, Fig. 5.41. The vector addition of these two magnetic moments gives rise to a net dipole magnetic moment $2\Delta\vec{M}$, directed towards the reader *i.e.* opposite to external field $\vec{B}$.

FIGURE 5.40

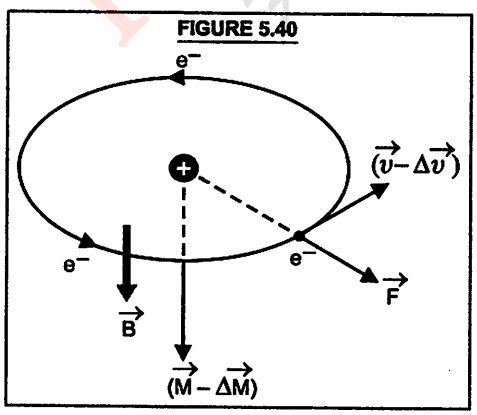
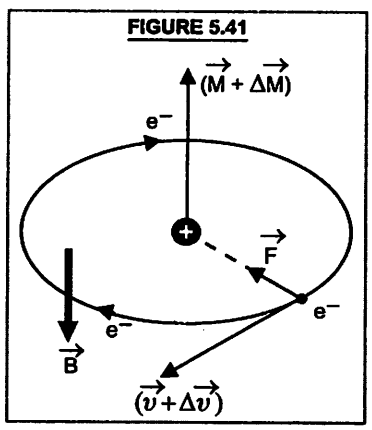


FIGURE 5.41



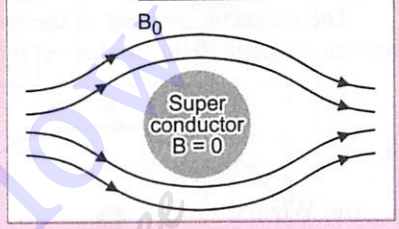
The net magnetic moment $2\Delta\vec{M}$ so developed is called *induced magnetic moment*. The magnetic moments induced in different atoms add vectorially to give a net magnetisation of the material in a direction opposite to $\vec{B}$. This accounts for the diamagnetic behaviour of materials.

Further, as the appearance of induced magnetic moment in atoms is not affected by the thermal motion of the atoms, therefore, magnetic susceptibility of such materials does not depend on temperature of the materials.

Retain in Memory

1. **Superconductors** are the most exotic diamagnetic materials. These are metals cooled to very low temperatures, which have zero resistance, *i.e.*, perfect conductivity is associated with perfect diamagnetism. The magnetic field lines are completely expelled from super- conductors, as shown in Fig. 5.42.

FIGURE 5.42



$$\therefore \quad \mu_r = \frac{B}{B_0} = 0$$

i.e., relative magnetic permeability of super-conductors is zero

As $\mu_r = 1 + \chi_m = 0$

$\therefore \quad \chi_m = -1$

i.e., magnetic susceptibility of superconductors is minus one.

2. *A superconductor repels a magnet and in turn, is repelled by the magnet.*
3. The phenomenon of perfect diamagnetism in superconductors is called **Meissner effect.** Superconducting magnets have been used for running magnetically leviated superfast trains.

(b) Paramagnetic substances

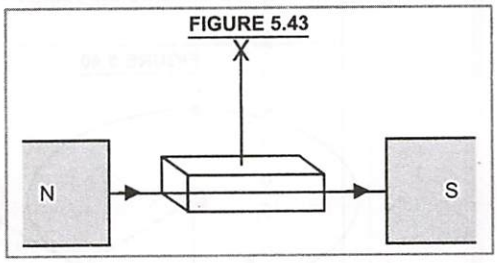
***Paramagnetic substances** are those in which each individual atom/molecule/ion has some net non zero magnetic moment of its own. When such substances are placed in an external magnetising field, they get feebly magnetised in the direction of the magnetising field.*

Examples of some of the paramagnetic substances are Aluminium,Chromium, Manganese, Lithium, Magnesium, Platinum, Tungsten, Sodium, Potasium, Copper chloride, liquid Oxygen, etc.

Some of the other important properties of paramagnetic substances are :

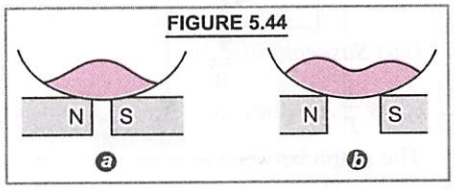
(*i*) **When suspended in a uniform magnetic field, they rotate so as to bring their *longest axis along the direction of the field.*** This is shown in Fig. 5.43.

FIGURE 5.43



(*ii*) **When placed in a non-uniform magnetic field, they tend to *move from weaker parts of the field to the stronger parts.*** It implies that paramagnetic substances get weakly attracted to a magnet. For example, consider a paramagnetic powder or liquid put in a watch glass placed on two pole pieces of an electromagnet held *close* to each other (≈ 2 mm or so). As the current is switched on, the liquid level acquires a shape shown in Fig. 5.44(*a*), accumulating at the centre, where the field is the strongest. If the distance between the poles is increased, the effect is reversed, as in that case, the field is weaker at the centre. This is shown in Fig. 5.44(*b*).

FIGURE 5.44



(*iii*) **When a magnetic field is applied to the level of a paramagnetic liquid in one limb of a U-tube, the liquid level rises as** shown in Fig. 5.45. It confirms that paramagnetic substances move from weaker to stronger parts of the magnetic field.

FIGURE 5.45

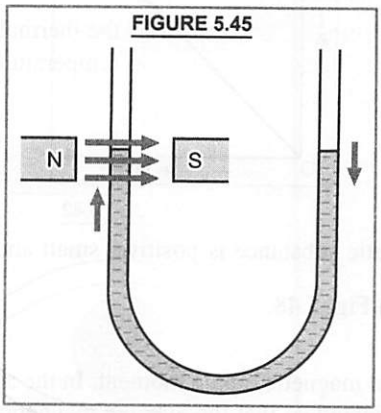
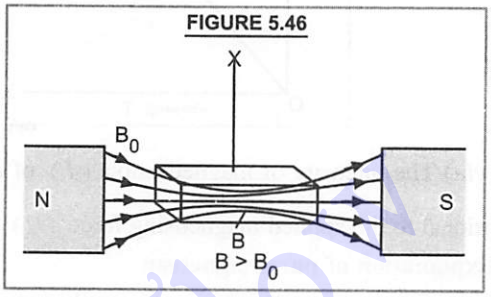


FIGURE 5.46



(*iv*) **When a specimen of a paramagnetic substance is placed in a magnetising field, the magnetic field lines prefer to pass through the specimen rather than through air,** as shown in Fig. 5.46. Thus magnetic induction B inside the sample is more than the magnetic induction B_0 outside the sample, *i.e.*, $B > B_0$ This enhancement is slight, being one part in 10^5.

(*v*) **Relative magnetic permeability of paramagnetic substances is always more than unity.**

As $\mu_r = \dfrac{B}{B_0}$ and $B > B_0$ $\therefore$ $\mu_r > 1$.

It means paramagnetic substances have a tendency to pull in magnetic field lines when placed in a magnetising field.

(*vi*) From the SI relation, $\mu_r = 1 + \chi_m$, as $\mu_r > 1$, therefore, χ_m must be positive. *Hence,* ***susceptibility of paramagnetic substances is positive, though small.***

Values of magnetic susceptibility of some paramagnetic substances at room temperature ($\approx$ 300 K) are given in Table 5.3.

TABLE 5.3. Magnetic susceptibility of some paramagnetic substances at 300 K

S. No.	Paramagnetic substances	χ_m
1.	Aluminium	$2{\cdot}3 \times 10^{-5}$
2.	Calcium	$1{\cdot}9 \times 10^{-5}$
3.	Chromium	$2{\cdot}7 \times 10^{-4}$
4.	Lithium	$2{\cdot}1 \times 10^{-5}$
5.	Magnesium	$1{\cdot}2 \times 10^{-5}$
6.	Niobium	$2{\cdot}6 \times 10^{-5}$
7.	Oxygen (STP)	$2{\cdot}1 \times 10^{-6}$
8.	Platinum	$2{\cdot}9 \times 10^{-4}$
9.	Tungsten	$6{\cdot}8 \times 10^{-5}$

(*vii*) ***Susceptibility of paramagnetic substances varies inversely as the temperature of the substance,*** *i.e.*, $\chi_m \propto \dfrac{1}{T}$, *i.e.*, they lose their magnetic character with rise in temperature.

The graph between temperature T and $(1/\chi_m)$ is a straight line as shown in Fig. 5.47.

FIGURE 5.47

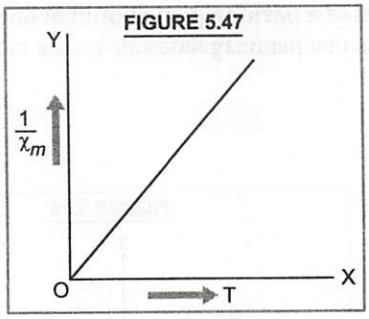
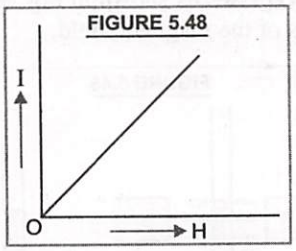


FIGURE 5.48



(*viii*) The intensity of magnetisation ($\vec{I}$) of a paramagnetic substance is positive, small and directly proportional to the applied magnetising force ($\vec{H}$) as shown in Fig. 5.48.

Explanation of paramagnetism

In paramagnetic materials, every atom has some permanent magnetic dipole moment. In the absence of an external magnetic field, the atomic dipoles are randomly oriented so that the average magnetic moment per unit volume of the material is zero. Therefore, a paramagnetic material does not behave as a magnet in the absence of an external magnetic field, Fig. 5.49.

FIGURE 5.49

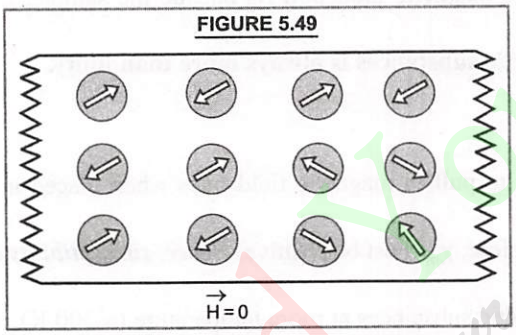
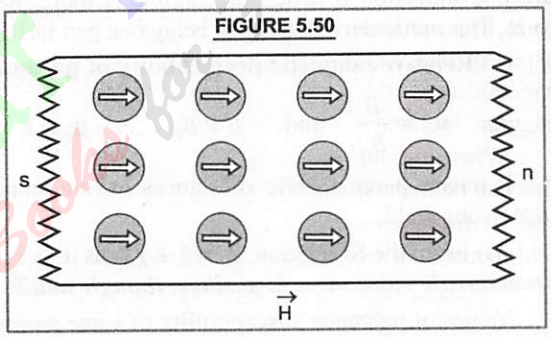


FIGURE 5.50



When an external magnetic field is applied, it tries to align the atomic magnetic dipoles in the direction of the field, Fig. 5.50. That is why the specimen gets magnetised weakly in the direction of the field. This is paramagnetism.

When we raise the temperature of the material, the atomic magnetic dipoles acquire some kinetic energy. This tends to disorient the dipoles. That is why magnetic susceptibility of the paramagnetic materials decreases with rise in temperature.

(c) Ferromagnetic substances

***Ferromagnetic substances** are those in which each individual atom/molecule/ion has a non zero magnetic moment, as in a paramagnetic substance.*

When such substances are placed in an external magnetising field, they get strongly magnetised in the direction of the field.

Examples of ferromagnetic materials are Iron, Cobalt, Nickel, Gadolinium and a number of their alloys.

The ferromagnetic materials show all the properties of paramagnetic substances, but to a much greater degree. For example,

(*i*) **They are strongly magnetised in the direction of external magnetising field in which they are placed.**

(*ii*) They have a strong tendency to move from a region of weak magnetic field to the region of strong magnetic field, *i.e.*, **they get strongly attracted to a magnet.**

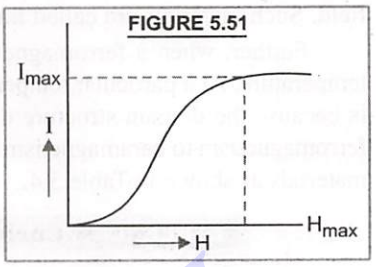
(*iii*) **Relative magnetic permeability of ferromagnetic materials is very large** ($\approx 10^3$ to 10^5)

(*iv*) **The susceptibility of ferromagnetic materials is also very large.** ($\because \chi_m = \mu_r - 1$)

That is why they can be magnetised easily and strongly.

(*v*) **With rise in temperature, susceptibility of ferromagnetics decreases.** At a certain temperature, ferromagnetics change over to paramagnetics. This **transition temperature** is called **Curie temperature.** For example, curie temperature of iron is about 1000 K.

FIGURE 5.51



The intensity of magnetisation ($\vec{I}$) of a ferromagnetic substance is positive, large and varies non linearly with the applied magnetising force ($\vec{H}$) as shown in Fig. 5.51.

Explanation of Ferromagnetism

Ferromagnetism has been explained by Weiss on the basis of domain theory in addition to the usual electron theory.

Like paramagnetic substances, each atom of a ferromagnetic substance is a tiny magnetic dipole having permanant dipole moment. However, in ferromagnetic materials, atoms form a very large number of small effective regions called *domains.* Each domain has a linear dimension $\approx$ 1000 A° and contains about 10^{10} atoms. Within each domain, a special interaction called *exchange coupling* renders dipole moments of all the atoms in a particular direction. Thus each domain is a strong magnet without any external magnetic field. Inspite of this, a ferromagnetic substance does not behave as a magnet, because in the absence of external magnetic field, the magnetic moments of different domains are randomly oriented so that their resultant magnetic moment in any direction is zero, Fig. 5.52.

When an external magnetising field is applied on the ferromagnetic substance, it gets strongly magnetised. This can be understood in terms of

(*i*) *displacement of boundaries of the domains i.e.* domains which are oriented in the direction of applied field increase in size and the domains which are oriented opposite to the field decrease in size, Fig. 5.53.

FIGURE 5.52

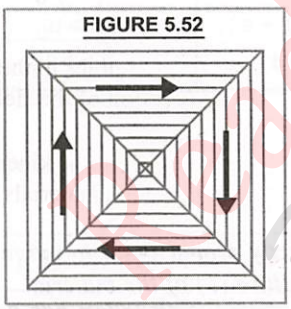


FIGURE 5.53

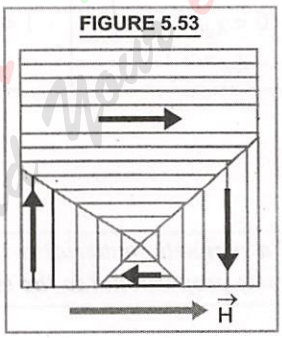
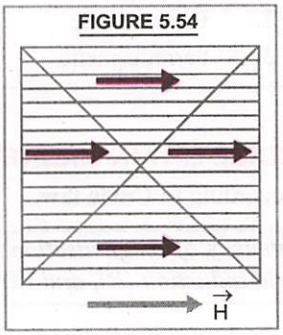


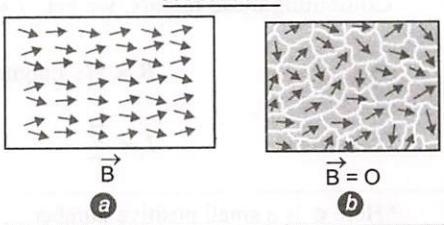
FIGURE 5.54



(*ii*) *rotation of the domains i.e.* the domains rotate till their magnetic moments are aligned in the direction of the applied magnetic field, Fig. 5.54. This would happen only when the magnetic field applied is very strong.

Fig. 5.55(*a*) shows the state when the domains have partially aligned along $\vec{B}$ and amalgamated to form a single giant domain. This is how ferromagnetic material gets strongly magnetised in the direction of the applied field. In contrast, Fig. 5.55(*b*) shows the state when $\vec{B} = 0$.

FIGURE 5.55



When external magnetic field is removed, magnetization disappears in some of the materials like soft iron. These materials are called **soft ferromagnets.** However, in materials like Alnico (an alloy of iron, aluminium, nickel, cobalt and copper), the magnetization persists even after removal of external magnetic field. Such materials are called **hard ferromagnets.**

Further, when a ferromagnetic material is heated, its magnetization decreases gradually with rise in temperature. At a particular temperature, ferromagnetic material changes over to paramagnetic material. This is because the domain structure disintegrates with rise in temperature. The temperature of transition from ferromagnetism to paramagnetism is called the **Curie temperature.** It is different for different ferromagnetic materials as shown in Table 5.4.

TABLE 5.4. Curie temperature of some ferromagnetic materials

Material	Curie Temperature
Cobalt	1394 K
Iron	1043 K
Fe_2O_3	893 K
Nickel	631 K
Gadolinium	317 K

Table 5.5 displays the values of susceptibility (χ_m), relative magnetic permeability (μ_r) and absolute magnetic permeability (μ) of the three types of magnetic substances.

TABLE 5.5. χ_m, μ_r and μ of magnetic substances

	Substance	χ_m	μ_r	μ
1.	Diamagnetic	$-1 \le \chi_m < 0$	$0 \le \mu_r < 1$	$\mu < \mu_0$
2.	Paramagnetic	$0 < \chi_m < \in$*	$1 < \mu_r < (1 + \in)$	$\mu > \mu_0$
3.	Ferromagnetic	$\chi_m >> 1$	$\mu_r >> 1$	$\mu >> \mu_0$

5.27. CURIE LAW IN MAGNETISM

According to **Curie law,**

Intensity of magnetisation (I) of a magnetic material is (i) directly proportional to magnetic field induction (B), and (ii) inversely proportional to the temperature (T) of the material.

i.e., $I \propto B$, and $I \propto \frac{1}{T}$

Combining these factors, we get $I \propto \frac{B}{T}$

As $B \propto H$, magnetising intensity

$\therefore$ $I \propto \frac{H}{T}$ or $\frac{I}{H} \propto \frac{1}{T}$

*Here $\in$ is a small positive number

But $\frac{I}{H} = \chi_m \quad \therefore \quad \chi_m \propto \frac{1}{T}$ or $\boxed{\chi_m = \frac{C}{T}}$ ...(56)

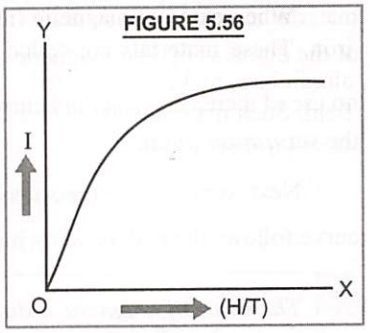
where C is a constant of proportionality and is called ***Curie constant.***

Thus for a paramagnetic material, both χ_m and μ_r depend not only on the material, but also on the sample temperature.

The law is physically reasonable as increasing H tends to align the elementary dipoles in the specimen increasing I, whereas increasing T tends to interfere with this alignment, decreasing I.

The variation of I with H/T is shown in Fig. 5.56.

FIGURE 5.56



At very high fields or at very low temperatures, the magnetisation approaches its maximum value, when all atomic dipole moments are aligned. This is called the **saturation magnetisation** value, as is indicated by the portion of graph parallel to X-axis.

Note that magnetic thermometers are based on Curie law. These thermometers are used to measure very low temperatures (< 1 K) by measuring the susceptibility of a paramagnetic substance.

Curie's law is well verified experimentally provided the ratio H/T is not too large.

The variation of $1/\chi_m$ with temperature (T) for a paramagnetic material is a straight line as shown already in Fig. 5.47.

DO YOU KNOW ?

The susceptibility of ferromagnetics, above the Curie temperature *i.e.* in the paramagnetic phase is given by

$$\chi_m = \frac{C}{T - T_c}$$

where T_c is Curie temperature and sample temperature $T > T_c$. This is the form Curie law assumes for ferromagnetic materials above the Curie temperature.

Sample Problem **Effective dipole moment of a paramagnetic salt is 4·5 joule/tesla at 4·2 K under magnetic field of 0·8 T. Find magnetic moment at 2·8 K, when magnetic field is 1 T. Assume Curie's Law.**

Sol. Here, $M_1 = 4{\cdot}5$ J/T, $T_1 = 4{\cdot}2$ K, $B_1 = 0{\cdot}8$ T

$M_2 = ?\ T_2 = 2{\cdot}8$ K, $B_2 = 1{\cdot}0$ T

According to Curie's Law

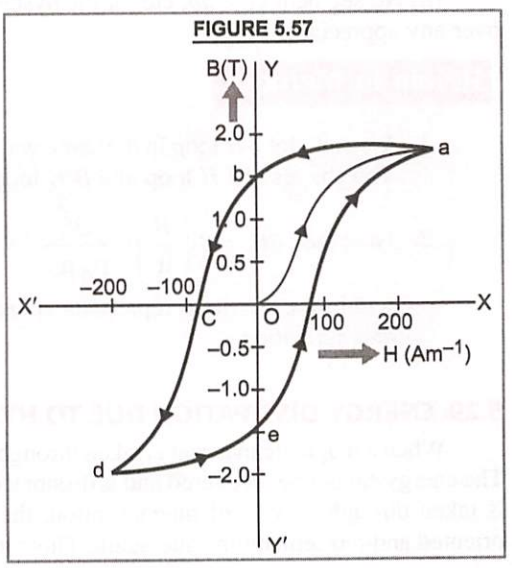
$$\frac{M_2}{B_2} \times T_2 = \frac{M_1}{B_1} \times T_1$$

$$M_2 = \frac{M_1}{B_1} \times \frac{T_1}{T_2} \times B_2 = \frac{4{\cdot}5 \times 4{\cdot}2 \times 1{\cdot}0}{0{\cdot}8 \times 2{\cdot}8} = \mathbf{8{\cdot}437\ JT^{-1}}$$

5.28. HYSTERISIS CURVE

The hysterisis curve represents the relation between magnetic field induction $\vec{B}$ (or intensity of magnetization $\vec{I}$) of a ferromagnetic material with magnetizing force or magnetic intensity $\vec{H}$. The shape of the hysterisis curve is shown in Fig. 5.57. It represents the behaviour of the material as it is taken through a cycle of magnetization.

FIGURE 5.57



Suppose the material is unmagnetised initially, *i.e.*, $\vec{B} = 0$ and $\vec{H} = 0$. This state is represented by the origin O. We place the material in a solenoid and increase the current through the solenoid gradually. The magnetising force $\vec{H}$ increases. The magnetic field induction $\vec{B}$ in the

material increases and saturates as depicted in the curve *oa*. This behaviour represents alignment and merger of the domains of ferromagnetic material until no further enhancement in $\vec{B}$ is possible. Therefore, there is no use of increasing solenoid current and hence magnetising intensity $\vec{H}$ beyond this. The point *a* represents the ***saturation point.***

Next, we decrease the solenoid current and hence magnetising intensity $\vec{H}$ till it reduces to zero. The curve follows the path *ab* showing that when $\vec{H} = 0$, $\vec{B} \neq 0$. Thus, some magnetism is left in the specimen.

The value of magnetic induction $\vec{B}$ left in the specimen when the magnetising force is reduced to zero is called Retentity or Remanence or Residual magnetism of the material.

It shows that the *domains are not completely randomised* even when the magnetising force is removed.

Next, the current in the solenoid is reversed and increased slowly. Certain domains are flipped until the net magnetic field induction $\vec{B}$ inside is reduced to zero. This is represented by the curve *bc*. ***It means to reduce the residual magnetism or retentivity to zero, we have to apply a magnetising force = OC in opposite direction. This value of magnetising force is called coercivity of the material.***

As the reverse current in solenoid is increased in magnitude, we once again obtain saturation in the reverse direction at *d*. The variation is represented by the curve *cd*. Next, the solenoid current is reduced (curve *de*), reversed and increased (curve *ea*). The cycle repeats itself. From Fig. 5.57, we find that saturated magnetic induction B_s is of the order of 1·5 T and coercivity is of the order of $-90\ \text{Am}^{-1}$.

From the above discussion, it is clear that when a specimen of a magnetic material is taken through a cycle of magnetisation, *the intensity of magnetisation (I) and magnetic field induction (B) lag behind the magnetising force (H).* Thus, even if the magnetising force *H* is made zero, the values of *I* and *B* do not reduce to zero *i.e.,* the specimen tends to retain the magnetic properties.

This phenomenon of lagging of I or B behind H when a specimen of a magnetic material is subjected to a cycle of magnetisation is called hysteresis. The curve abcdea is called hysteresis loop.

From Fig. 5.57, we make the following observations :

(*i*) For a given value of H ; value of B or I is not unique, but depends on previous history of the sample.

(*ii*) No segment O *a*, *ab*, etc. of the hysterisis curve is linear. Therefore, B or I is not proportional to H over any appreciable range.

Retain in Memory

1. We can plot *I-H* loop in the same way as we have plotted *B-H* loop of the ferromagnetic material. The shapes of *I-H* loop and *B-H* loop for a given material are identical.
2. Note that $BH = B\left(\frac{B}{\mu}\right) = \frac{B^2}{\mu_0 \mu_r}$ has the dimensions of energy per unit volume. That is why area within the *BH* loop represents *energy dissipated per unit volume in the material per cycle of magnetisation.*

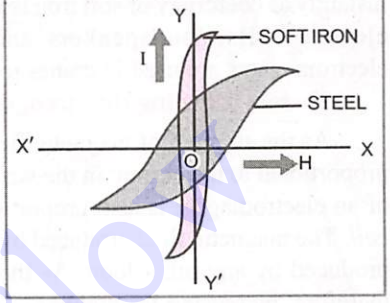
5.29. ENERGY DISSIPATION DUE TO HYSTERESIS

When a magnetic material is taken through a cycle of magnetisation, some energy is spent in the process. The energy cannot be recovered and is dissipated in the form of heat in the specimen. Infact, when a specimen is taken through a cycle of magnetisation, the molecular magnets (magnetic dipoles) in the specimen are oriented and reoriented time and again. This molecular motion within the specimen results in the production

of heat. It can be shown that the loss of energy per unit volume of specimen per cycle of magnetisation in cgs system is equal to the area of the *I-H* loop of the specimen and in SI, it is equal to the area of the *B-H* loop. The exact shape and size of the *I-H* and *B-H* loops depend upon the nature of the material and history of the specimen. By studying the hysteresis loops of various magnetic materials, we can study the difference in their properties *e.g.*, retentivity, coercivity, permeability, susceptibility and energy loss, etc. Such studies enable us to select suitable materials for different purposes *e.g.* for electromagnets, for transformer cores, for permanent magnets etc.

For example, ***hysteresis loop for soft iron is narrow and large, whereas the hysteresis loop for steel is wide and short,*** [Fig. 5.58].

FIGURE 5.58



The hysterisis loops of soft iron and steel reveal that

(*i*) *The retentivity of soft iron is greater than the retentivity of steel,*

(*ii*) *Soft iron is more strongly magnetised than steel,*

(*iii*) *Coercivity of soft iron is less than coercivity of steel. It means soft iron loses its magnetism more rapidly than steel does.*

(*iv*) *As area of I-H loop for soft iron is smaller than the area of I-H loop for steel, therefore, hysterisis loss in case of soft iron is smaller than the hysterisis loss in case of steel.*

Sample Problem **The hysteresis loss for a sample of 12 kg is 300 J m^{-3} cycle^{-1}. If density of iron is 7500 kg/m^3, calculate energy loss per hour at 50 cycles/sec.**

Sol. Here, energy loss/volume, per cycle = 300 Jm^{-3} cycle^{-1}.

$$\text{Volume of sample} = \frac{\text{mass}}{\text{density}} = \frac{12}{7500}\ \text{m}^3\ ; \quad \text{Total energy loss/cycle} = 300 \times \frac{12}{7500} = \frac{12}{25}$$

$$\text{Energy loss/sec} = \frac{12}{25} \times 50 = 24\ \text{J/sec}\ ; \quad \text{Energy loss/hour} = 24 \times 60 \times 60\ \text{J} = \mathbf{8{\cdot}64 \times 10^4\ J}$$

5.30. USES OF FERROMAGNETICS

Important properties of ferromagnetic materials are studied from their hysteresis curves. These properties help us in the proper selection of materials for various practical applications as detailed below :

(a) Permanent Magnets

Permanent magnets are the materials which retain at room temperature, their ferromagnetic properties for a long time. The material chosen should have

(*i*) *high retentivity* so that the magnet is strong,

(*ii*) *high coercivity* so that the magnetisation is not erased by stray magnetic fields, temperature changes or mechanical damage due to rough handling etc.

(*iii*) *high permeability* so that it can be magnetised easily.

Steel is preferred for making permanent magnets. Though retentivity of steel is slightly smaller than that of soft iron, yet its coercivity is much larger than that of soft iron. Other suitable materials for permanent magnets are (*i*) **cobalt steel**, (52% iron, 36% cobalt, 7% tungsten, 4·0% chromium, 0·5% manganese and 0·5% carbon) (*ii*) **carbon steel** (98% iron ; 0·86% carbon and 0·9% manganese), (*iii*) **the alloy alnico** (55% iron, 10% aluminium, 17% nickel, 12% cobalt and 6% copper), (*iv*) **Ticonal** containing tin, cobalt, nickel and aluminium.

An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass current through the solenoid. The magnetic field of the solenoid magnetises the rod.

DO YOU KNOW ?

That ferromagnetic materials have been divided into two types :

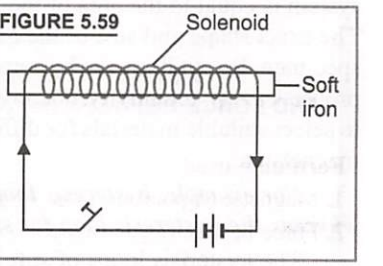
1. Soft magnetic materials, which have low retentivity, low coercivity and low hysterisis loss. Soft iron, mμ metal and stalloy are some of the examples of these materials, used primarily for electromagnets, cores of transformers, motors and generators.

2. Hard magnetic materials, which have high retentivity, high coercivity and large hysterisis loss. Steel, alnico, alcomax and ticonal are some of the examples of these materials, which have been used for making permanent magnets of electric metres and loud speakers, etc.

(b) Electromagnets

The core of electromagnets is made of ferromagnetic materials, which have **high permeability, low coercivity** and **low retentivity. Soft iron** is a suitable material for this purpose. When a soft iron rod is placed in a solenoid and current is passed through the solenoid as shown in Fig. 5.59, magnetism of the solenoid is increased by a thousand fold. When the solenoid current is switched off, the magnetism is removed instantly as coercivity of soft iron is very low. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery, etc.

FIGURE 5.59



Factors affecting the strength of an electromagnet

As the strength of magnetic field around a current carrying wire is proportional to the current in the wire, therefore, the strength of the field of an electromagnet is also proportional to *current flowing through the coil.* The magnetic field produced by each loop of the coil is same as that produced by any other loop. As these fields are in the same direction, therefore, *increasing the number of loops* in an electromagnet increases the strength of the magnetic field.

Further, the strength of an electromagnet can be increased by *placing an iron rod or core inside the coil.* The field inside the coil magnetizes the core by induction. The magnetic strength of the core adds to that of the coil to produce a much stronger electromagnet.

DO YOU KNOW ?

An exciting application of superconducting electromagnets is in making a magnet transportation train which can move with high speed. While moving, this train floats several inches above the rail lines and its speed is limited only by air friction and passanger comfort.

Hence, we conclude that the factors affecting the strength of electromagnets are : (*i*) Number of loops in the coil, (*ii*) Strength of current passed, (*iii*) Nature of the core of electromagnet. (*iv*) Temperature. As magnetism is lost at high temperature, therefore, to retain magnetism is a given material, it must be maintained at low temperature.

(c) Transformer cores

The materials used for making transformer cores, chokes, diaphragms of telephones etc. are subjected to *ac* cycle of magnetisation for a long period. Therefore, low hysterisis loss is the first essential condition for such materials. The hysterisis curve of such materials must be narrow. The energy dissipated and therefore, heating of the material will be small. The material chosen must also have high resistivity to decrease energy losses due to eddy currents. Soft iron is more suitable for transformer cores. More effective alloys have also been developed for this purpose. Some of them are *perm alloys, mu metal* (76% nickel, 17% iron, 5% copper and 2% chromium) and *radio metal*, etc.

Curiosity Question

Q. What are some of the important technological applications of magnetism ?

Ans. The actual list of technological applications of magnetism is really long. Some of these are :

(*i*) Magnets are used in meters, motors, electric generators, loud speakers etc.

(*ii*) Large electromagnets are used to pick up heavy loads.

(*iii*) Large electromagnets are being used to leviate modern trains. The so-called *maglev* trains are faster and provide a much smoother ride than the ordinary track system, due to absence of friction between the train and the track.

(*iv*) Magnetic tapes are used in sound and video recording equipment.

(*v*) Magnetic recording material is used on computer discs.

(*vi*) Superconducting magnets are used to contain the plasmas used in controlled nuclear fusion research.

SOLVED EXAMPLES

TYPE I. MAGNETIC DIPOLE MOMENT AND FORCE BETWEEN MAGNETIC POLES

Formulae used.

1. Magnetic (dipole) moment $M = m\,(2\,l)$
2. Force between two magnetic poles in air,

$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$$

Units used. M is in JT $^{-1}$ or A m^2, distance (r), magnetic length ($2\,l$) in metre ; m_1, m_2 are pole strengths in Am.

Standard Value. $\frac{\mu_0}{4\pi} = 10^{-7}\,Wb\,A^{-1}\,m^{-1}$

Example 1 **A magnetised steel wire 31.4 cm long has a pole strength of 0·2 Am. It is bent in the form of a semicircle. Calculate its magnetic moment.**

Solution. Here, $l = 31{\cdot}4$ cm, $m = 0{\cdot}2$ Am,

$M = ?$

Let r be radius of semicircle.

$$\therefore\ l = \pi r,\ r = \frac{l}{\pi} = \frac{31{\cdot}4}{3{\cdot}14} = 10 \text{ cm} = 0{\cdot}1 \text{ m}$$

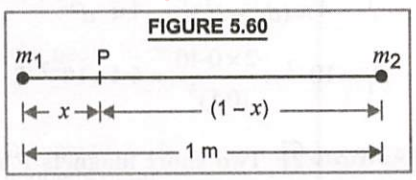
$$M = m \times 2\,r = 0{\cdot}2 \times 2 \times 0{\cdot}1 = \mathbf{0{\cdot}04\ Am^2}$$

Example 2 **Two similar magnetic poles, having pole strengths in the ratio 1 : 2 are placed 1 m apart. Find the point where a unit pole experiences no net force due to the two poles.**

Solution. Let $m_1 = m$ and $m_2 = 2$ m, $r = 1$ m.

Let unit pole placed at P, at a distance x from m_1 experience no net force, Fig. 5.60.

FIGURE 5.60



$$\therefore\ \left(\frac{\mu_0}{4\pi}\right)\frac{m_1 \times 1}{x^2} = \left(\frac{\mu_0}{4\pi}\right)\frac{m_2 \times 1}{(1-x)^2}$$

$$\frac{m_1}{m_2} = \frac{x^2}{(1-x)^2} = \frac{1}{2}$$

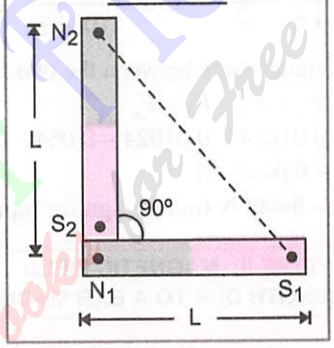
$$2\,x^2 = (1-x)^2 \quad \text{or} \quad \sqrt{2}\,x = 1 - x$$

$$\therefore\ x = \frac{1}{1+\sqrt{2}} = \mathbf{0{\cdot}414\ m}$$

Example 3 **Two identical thin bar magnets, each of length L and pole strength m are placed at right angles to each other, with the N pole of one touching the S-pole of the other. Find the magnetic moment of the system.**

Solution. As is clear from Fig. 5.61, the system behaves as a magnet whose pole strength is m and length is N_2S_1

FIGURE 5.61



$$\text{Now,}\quad N_2S_1 = \sqrt{N_1S_1^2 + N_2S_2^2}$$

$$= \sqrt{L^2 + L^2} = L\sqrt{2}$$

$\therefore$ Magnetic moment of the system

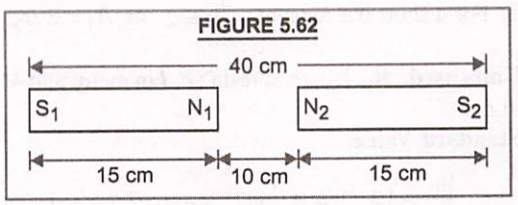
M = pole strength $\times\ N_2S_1$

$$M = m.L\sqrt{2} = \sqrt{2}\,mL$$

Example 4 **Calculate the force acting between two magnets of length 15 cm each and pole strength 80 Am each when the separation between their north poles is 10 cm and that between south poles is 40 cm.**

Solution. The given situation is shown in Fig. 5.62.

FIGURE 5.62



Here, $m_1 = m_2 = 80\ Am$

Force of attraction between N_1 and S_2

$$F_1 = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = 10^{-7} \times \frac{80 \times 80}{(0{\cdot}25)^2}$$

$$= 0{\cdot}01024 \text{ N}$$

Force of attraction between N_2 and S_1

$$F_2 = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = F_1 = 0{\cdot}01024 \text{ N}$$

Force of repulsion between N_1 and N_2

$$F_3 = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = 10^{-7} \times \frac{80 \times 80}{(0{\cdot}1)^2} = 0{\cdot}064 \text{ N}$$

Force of repulsion between S_1 and S_2

$$F_4 = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = 10^{-7} \times \frac{80 \times 80}{(0{\cdot}40)^2} = 0{\cdot}004 \text{ N}$$

$\therefore$ Resultant force between the two magnets

$$F = F_1 + F_2 - F_3 - F_4$$
$$= 0{\cdot}01024 + 0{\cdot}01024 - 0{\cdot}064 - 0{\cdot}004$$
$$= -0{\cdot}04752 \text{ N}$$

$F = -$ **0·048 N** (minus sign for repulsion).

TYPE II. MAGNETIC FIELD STRENGTH DUE TO A BAR MAGNET

Formulae used. **1.** *Magnetic field strength at a point on axial line of a bar magnet.*

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2}$$

When the magnet is short, $l^2 << d^2$, $B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$

2. *Magnetic field strength at a point on equatorial line of a bar magnet.*

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}$$

If the magnet is short, $l^2 << d^2$ $\therefore$ $B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

3. For a short bar magnet, $\frac{B_1}{B_2} = 2$ or $B_1 = 2\,B_2$.

Units used. B_1, B_2 are in tesla ; d, l in metre and M in Am^2.

Standard Value.

$$\frac{\mu_0}{4\pi} = 10^{-7} Wb\ A^{-1} m^{-1} \text{ or } 10^{-7}\ TmA^{-1}.$$

Example 5 **The earth's magnetic field at the equator is approximately 0·4 G. Estimate the earth's dipole moment.**

(NCERT Solved Example)

Solution. If we assume that earth's magnetic field is due to a bar magnet at the centre of earth, held along the polar axis of earth, then the equatorial magnetic field is $B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

where $d = R$ = radius of earth = $6{\cdot}4 \times 10^6$ m

Using $B = 0{\cdot}4$ G $= 0{\cdot}4 \times 10^{-4}$ T,

and $\frac{\mu_0}{4\pi} = 10^{-7}$ Wb A^{-1} m^{-1}, we get

$$0{\cdot}4 \times 10^{-4} = 10^{-7}\frac{M}{(6{\cdot}4 \times 10^6)^3}$$

$$M = \frac{0{\cdot}4 \times 10^{-4}(6{\cdot}4 \times 10^6)^3}{10^{-7}}$$

$$= \mathbf{1{\cdot}05 \times 10^{23}\ Am^2}$$

Example 6 **What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5·0 cm at a distance of 50 cm from its mid-point ? The magnetic moment of the bar magnet is 0·40 Am².** **(NCERT Solved Example)**

Solution. Here, $B_2 = ?$, $B_1 = ?$, $2\,l = 5{\cdot}0$ cm,

$l = 2{\cdot}5$ cm $= 2{\cdot}5 \times 10^{-2}$ m,

$d = 50$ cm $= 0{\cdot}5$ m, $M = 0{\cdot}40$ Am^2

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{M}{(d^2 + l^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M}{d^3} \quad (\because l << d)$$

$$= 10^{-7} \times \frac{0{\cdot}40}{(0{\cdot}5)^3} = \mathbf{3{\cdot}2 \times 10^{-7}\ T}$$

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2} = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \quad (\because l << d)$$

$$B_1 = 10^{-7} \times \frac{2 \times 0{\cdot}40}{(0{\cdot}5)^3} = \mathbf{6{\cdot}4 \times 10^{-7}\ T}$$

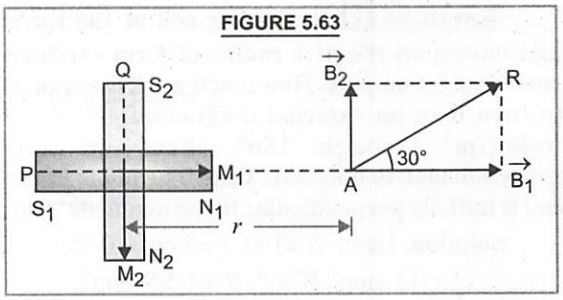
Example 7 **Two short magnets P and Q are placed one over another with their magnetic axes mutually perpendicular to eachother. It is found that resultant magnetic field at a point on the prolongation of magnetic axis of P is inclined at 30° with this axis. Compare the magnetic moments of the two magnets.**

Solution. In Fig. 5.63, at any point A, distant r from the centre of two magnets.

magnetic field due to P, $B_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{r^3}$

magnetic field due to Q, $B_2 = \frac{\mu_0}{4\pi}\frac{M_2}{r^3}$.

FIGURE 5.63



As resultant field is inclined at 30° with B_1, therefore,

$$\tan\beta = \frac{B_2}{B_1} = \frac{M_2}{2M_1} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{\sqrt{3}}{2}$$

TYPE III. MAGNETIC MOMENT OF A CURRENT LOOP

Formulae used.

Magnetic moment of a current loop, $M = NIA$

Units used. N is a number with no units, I in ampere, A in m^2, M in Am^2.

Example 8 **An electron in an atom revolves around the nucleus in an orbit of radius 0·5 Å. Calculate the equivalent magnetic moment if the frequency of revolution of electron is 10^{10} MHz.** [CBSE 2000 (C)]

Solution. Revolving electron is equivalent to a current loop. Its magnetic moment is

$$M = NIA = 1 \cdot (ev)\,\pi r^2$$
$$= (1{\cdot}6 \times 10^{-19} \times 10^{16})\, 3{\cdot}14\, (0{\cdot}5 \times 10^{-10})^2$$
$$= 1{\cdot}256 \times 10^{-23}\ Am^2$$

Example 9 **A loop of wire having 50 turns carries a current of 10 A in anticlockwise direction. If diameter of loop is 10 cm, what is the magnitude and direction of magnetic moment of current loop ?**

Solution. Here, $N = 50$, $I = 10$ A,

$$r = \frac{10}{2}\text{ cm} = 5\times10^{-2}\ m$$

M = ? From $M = NIA = NI(\pi r^2)$

$$M = 50 \times 10 \times 3{\cdot}14\, (5 \times 10^{-2})^2$$

$$M = 3{\cdot}925\ Am^2$$

According to right hand thumb rule, direction of $\vec{M}$ is perpendicular to the plane of loop and away from observer.

TYPE IV. TORQUE ACTING ON A MAGNETIC DIPOLE AND POTENTIAL ENERGY OF DIPOLE IN A MAGNETIC FIELD

Formulae used. **1.** Torque, $\tau = MB\sin\theta$

2. Potential energy, $U = -MB(\cos\theta_2 - \cos\theta_1)$

Units used. B in tesla, M in JT^{-1} or Am^2, τ in N–m, U in joule ; θ_1, θ_2 in degrees.

Example 10 **A short bar magnet placed with its axis at 30° to a uniform magnetic field of 0·2 T experiences a torque of 0·06 N-m. Calculate magnetic moment of the magnet. What orientation of magnet corresponds to its stable equilibrium in the magnetic field ?** (CBSE 2002)

Solution. Here, $\theta = 30°$, $B = 0{\cdot}2$ T,

$\tau = 0{\cdot}06$ N-m, M = ?

From, $\tau = MB\sin\theta$,

$$M = \frac{\tau}{B\sin\theta} = \frac{0{\cdot}06}{0{\cdot}2\sin 30°} = 0{\cdot}6\ Am^2$$

Potential energy of magnetic dipole

$$U = -MB\cos\theta$$

In stable equilibrium, P.E. is minimum

$\therefore \quad \cos\theta = 1$ or $\theta = 0°$

i.e., bar magnet will be in stable equilibrium.

when its magnetic moment $\vec{M}$ is parallel to magnetic field $\vec{B}$.

Example 11 **An electron moves around the nucleus in a hydrogen atom of radius 0·51 Å with a velocity of 2×10^6 m/s. Calculate (*i*) the equivalent current due to orbital motion. (*ii*) magnetic field produced at the centre of nucleus. (*iii*) magnetic moment associated with electron.**

(CBSE 2008)

Solution. Here, $r = 0{\cdot}51$ Å $= 0{\cdot}51 \times 10^{-10}$ m,

$v = 2 \times 10^6$ m/s

(*i*) $I = \dfrac{e}{T} = \dfrac{e}{2\pi r/v} = \dfrac{ev}{2\pi r}$

$= \dfrac{1{\cdot}6\times10^{-19}\times2\times10^{6}}{2\times3{\cdot}14\times0{\cdot}51\times10^{-10}} = \mathbf{10^{-3}\ A}$

(*ii*) $B = \dfrac{\mu_0 I}{2r} = \dfrac{4\pi\times10^{-7}\times10^{-3}}{2\times0{\cdot}51\times10^{-10}} = \mathbf{12{\cdot}3\ T}$

(*iii*) $M = IA = \dfrac{ev}{2\pi r}\times\pi r^2$

$= 10^{-3} \times 3{\cdot}14\ (0{\cdot}51 \times 10^{-10})^2$

$= \mathbf{8{\cdot}16 \times 10^{-24}\ Am^2}$

Example 12 **A short bar magnet of magnetic moment 0·9 JT^{-1} is placed with its axis at 30° to a uniform magnetic field. It experiences a torque of 0·063 J. Calculate the magnitude of magnetic field. In which orientation will the bar magnet be in stable equilibrium in the magnetic field ?** (CBSE 2012)

Solution. Here, $M = 0{\cdot}9\ JT^{-1}$, $\theta = 30°$,

$\tau = 0{\cdot}063$ J, $B = ?$

$\tau = MB \sin\theta$, $B = \dfrac{\tau}{M\sin\theta} = \dfrac{0{\cdot}063}{0{\cdot}9\times\sin 30°}$

$= \mathbf{0{\cdot}14\ T}$

For stable equilibrium, the magnetic moment $\vec{M}$ of magnet must be parallel to magnetic field $\vec{B}$.

Example 13 **A bar magnet of magnetic moment 6 J/T is aligned at 60° with a uniform external magnetic field of 0·44 T. Calculate (*a*) the work done in turning the magnet to align its magnetic moment (*i*) normal to the magnetic field (*ii*) opposite to the magnetic field ; and (*b*) the torque on the magnet in the final orientation in case (*ii*).** (CBSE 2018)

Solution. Here, $M = 6$ J/T, $\theta_1 = 60°$,

$B = 0{\cdot}44$ T

(*a*) (*i*) $W = ?\ \theta_2 = 90°$

(normal to magnetic field)

As $W = -MB(\cos\theta_2 - \cos\theta_1)$

$\therefore\ W = -6 \times 0{\cdot}44\ (\cos 90° - \cos 60°)$

$= -2{\cdot}64\ (0 - 0{\cdot}5) = \mathbf{1{\cdot}32\ J}$

(*ii*) $W = ?\ \theta_2 = 180°$

(opposite to magnetic field)

$W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -6 \times 0{\cdot}44\ (\cos 180° - \cos 60°)$

$= -2{\cdot}64\ (-1 - 0{\cdot}5) = \mathbf{3{\cdot}96\ J}$

(*b*) $\tau = ?\ \theta = \theta_2 = 180°$

$\tau = MB\sin\theta = 6 \times 0{\cdot}44 \sin 180° = \mathbf{Zero}$

Example 14 **A circular coil of 100 turns and having an effective radius of 5 cm carries a current of 0·1 ampere. How much work is required to turn it in an external magnetic field of 1·5 weber/m^2 through 180° about an axis perpendicular to magnetic field. The plane of the coil is initially perpendicular to the magnetic field.**

Solution. Here, $N = 100$, $r = 5$ cm $= 0{\cdot}05$ m,

$I = 0{\cdot}1$ amp., $W = ?$, $B = 1{\cdot}5$ Wb/m^2,

$\theta_1 = 0°$, $\theta_2 = 180°$

Now, area $A = \pi r^2 = 3{\cdot}14\ (0{\cdot}05)^2$ *sq m*

$W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -(NIA)\,B(\cos\theta_2 - \cos\theta_1)$

$= -100 \times 0{\cdot}1 \times 3{\cdot}14\ (0{\cdot}05)^2$

$\times\ 1{\cdot}5\ [\cos 180° - \cos 0°]$

$= -31{\cdot}4\ (0{\cdot}05)^2 \times 1{\cdot}5\ (-1 - 1)$

$= \mathbf{0{\cdot}24\ J}$

Example 15 **A current of 5 A is flowing through a 10 turns circular coil of radius 7 cm. The coil lies in *x*-*y* plane. What is the magnitude and direction of dipole moment associated with it ?**

If this coil were placed in a uniform external magnetic field directed along the *x*-axis, in which plane would the coil lie in equilibrium ? $\pi = 22/7$. (CBSE Sample Paper 2003)

Solution. Here, $I = 5$A, $N = 10$, $r = 7 \times 10^{-2}$ m

$M = NIA = NI(\pi r^2)$

$= 10\times5\times\dfrac{22}{7}\ (7\times10^{-2})^2 = \mathbf{0{\cdot}77\ Am^2}$

The direction of $\vec{M}$ is perpendicular to the plane of the coil, **along Z-axis.**

Torque on the current loop, $\tau = MB\sin\theta$

where θ is angle between $\vec{M}$ and $\vec{B}$.

For stable equilibrium, $\tau = 0\ \therefore\ \theta = 0°$.

Hence $\vec{B}$ should be perpendicular to plane of the coil. Hence in the condition of stable equilibrium, coil will be in *Y*-*Z* plane.

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Example 16 **A current of 7·0 A is flowing in a plane circular coil of radius 1·0 cm having 100 turns. The coil is placed in a uniform magnetic field of 0·2 Wb/m^2. If the coil is free to rotate, what orientation would correspond to its (*i*) stable equilibrium and (*ii*) unstable equilibrium ? Calculate potential energy of the coil in the two cases.** (CBSE 2000)

Solution. Here, $I = 7{\cdot}0$ A, $r = 1{\cdot}0$ cm $= 10^{-2}$ m,
$N = 100$, $B = 0{\cdot}2$ Wb/m^2.

$$M = NIA = NI\pi r^2$$

$$= 100 \times 7{\cdot}0 \times \frac{22}{7} (10^{-2})^2 = 0{\cdot}22 \text{ Am}^2$$

(*i*) Stable equilibrium corresponds to $\theta = 0°$

i.e. $\vec{M}$ parallel to $\vec{B}$.

$$u_{min} = -MB \cos 0° = 0{\cdot}22 \times 0{\cdot}2 \times 1$$

$$= \mathbf{-0{\cdot}044 \text{ J}}$$

(*ii*) unstable equilibrium corresponds to $\vec{M}$ antiparallel to $\vec{B}$. P.E. is maximum

$$u_{max.} = -MB \cos 180° = -0{\cdot}22 \times 0{\cdot}2 \times (-1)$$

$$= \mathbf{0 \cdot 044 \text{ J}}$$

TYPE V. MAGNETIC ELEMENTS OF EARTH

Formulae used. 1. Declination, θ = angle between magnetic meridian and geographic meridian

2. Horizontal component, $H = R \cos \delta$,
vertical component, $V = R \sin \delta$
where δ is the angle of dip at the place.

3. $R = \sqrt{H^2 + V^2}$ **4.** $\tan \delta = V/H$

5. When vertical plane carrying magnetic needle in magnetic meridian is turned through θ degree, apparent value of dip (δ') is given by :

$$\tan \delta' = \frac{V}{H'} = \frac{V}{H \cos \theta} = \frac{\tan \delta}{\cos \theta}$$

Units used. All angles θ, δ in degrees, resultant field of earth, R and its components H, V in tesla.

Example 17 **A compass needle whose magnetic moment is 60 Am^2 pointing geographic north at a certain place where horizontal component of earth's magnetic field is 40 μWb/m^2 experiences a torque of $1{\cdot}2 \times 10^{-3}$ Nm. What is the declination of the place ?**

Solution. Here, $M = 60$ Am^2,
$H = 40\ \mu$ Wb/m^2 $= 40 \times 10^{-6}$ Wb/m^2
$\tau = 1{\cdot}2 \times 10^{-3}$ Nm. Declination, θ = ?

As is known, in stable equilibrium, a compass needle points along magnetic N–S and experiences no torque. But when compass needle is turned through angle of declination (θ), it points towards geographic N–S and experiences torque,

$$\tau = MB \sin \theta$$

$$\sin \theta = \frac{\tau}{MB} = \frac{1{\cdot}2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = \frac{1}{2}$$

$$\therefore \quad \theta = 30°$$

Example 18 **The value of dip at a place is 45°. The plane of the dip circle is turned through 60° from the magnetic meridian. Find the apparent value of dip.**

Solution. Here, $\delta = 45°$, $\theta = 60°$, δ' = ?

$$\tan \delta' = \frac{\tan \delta}{\cos \theta} = \frac{\tan 45°}{\cos 60°} = \frac{1}{1/2} = 2$$

$$\therefore \quad \delta' = \tan^{-1}(2) = \mathbf{63{\cdot}4°}$$

Example 19 **The magnetic field at a point on magnetic equator is $3{\cdot}1 \times 10^{-5}$ T. If radius of earth is taken as 6400 km, what is magnetic moment of the assumed dipole at the centre of earth ?**

Solution. As is clear from Fig. 5.64, any point on the magnetic equator lies on equatorial line of the assumed magnetic dipole at the centre of earth.

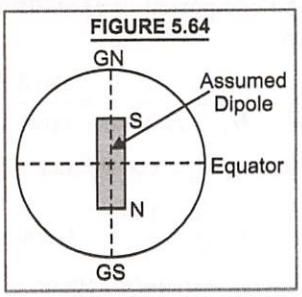


FIGURE 5.64

$$\therefore \quad B_{equa} = \frac{\mu_0}{4\pi} \frac{M}{R^3}$$

$$M = \frac{4\pi}{\mu_0} B_{equa} \times R^3$$

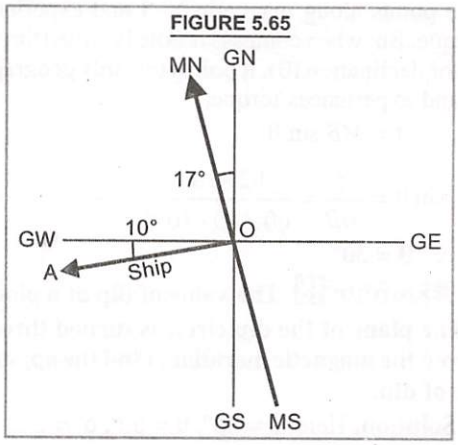
$$= 10^7 (3{\cdot}1 \times 10^{-5}) (6400 \times 10^3)^3$$

$$M = \mathbf{8{\cdot}1 \times 10^{22} \text{ Am}^2}$$

Example 20 **A ship is to reach a place 10° south of west. In what direction should it be steered if declination at the place is 17° west ?**

Solution. As the ship is to reach a place 10° south of west *i.e.* along *OA*, Fig. 5.65, therefore, it should be steered west of (magnetic) north at an angle of (90–17+10) = **83°**

FIGURE 5.65



Example 21 **A magnet suspended at 30° with magnetic meridian makes an angle of 45° with the horizontal. What shall be the actual value of the angle of dip ?** (Uttarakhand Board 2012)

Solution. Here, $\theta = 30°$; Apparent value of dip, $\delta_1 = 45°$; Actual value of dip, $\delta = ?$

If H is horizontal component of earth's magnetic field in magnetic meridian, then $\tan\delta = \frac{V}{H}$

Let H_1 be component of H at 30° to magnetic meridian, then

$$\tan\delta_1 = \frac{V}{H_1} = \frac{V}{H\cos\theta} = \frac{\tan\delta}{\cos\theta}$$

or $\tan\delta = \tan\delta_1 \times \cos\theta = \tan 45° \times \cos 30°$

$$= 1 \times \frac{\sqrt{3}}{2} = \frac{1{\cdot}732}{2} = 0{\cdot}866$$

$$\therefore \quad \delta = \tan^{-1}(0{\cdot}866) = \mathbf{40{\cdot}9°}$$

Example 22 **In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0·26 G and dip angle is 60°. What is the magnetic field of earth at this location ?** (NCERT Solved Example)

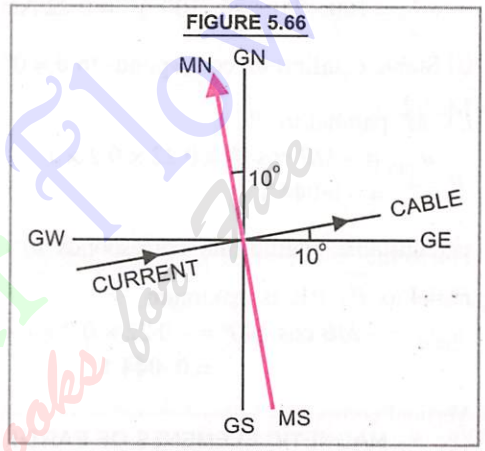
Solution. Here, $H = 0{\cdot}26$ G, $\delta = 60°$, $R = ?$

As $H = R\cos\delta$

$$\therefore \quad R = \frac{H}{\cos\delta} = \frac{0{\cdot}26}{\cos 60°} = \frac{0{\cdot}26}{(1/2)} = \mathbf{0{\cdot}52\ G}$$

Example 23 **A long straight horizontal cable carries a current of 2.5 amp. in the direction 10° south of west to 10° north of east, Fig. 5.66. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0.33 G and the angle of dip is zero. Locate the line of neutral points (Ignore the thickness of the cable). [At neutral points, magnetic field due to a current cable is equal and opposite to the horizontal component of earth's magnetic field.]**

FIGURE 5.66



Solution. Here, $i = 2.5$ amp. ;

$R = 0.33\ G = 0.33 \times 10^{-4}$ T ; $\delta = 0°$

Horizontal component of earth's field

$$H = R\cos\delta = 0.33 \times 10^{-4} \cos 0°$$

$$= 0.33 \times 10^{-4} \text{ tesla}$$

Let the neutral points lie at a distance r from the cable.

Strength of magnetic field on this line due to current in the cable $= \frac{\mu_0 i}{2\pi r}$

At neutral point, $\frac{\mu_0 i}{2\pi r} = H$

$$r = \frac{\mu_0 i}{2\pi H} = \frac{4\pi\times10^{-7}\times2.5}{2\pi\times0.33\times10^{-4}} = 1.5 \times 10^{-2} \text{ m}$$

$$= \mathbf{1.5\ cm}$$

Hence neutral points lie on a straight line parallel to the cable at a perpendicular distance of 1.5 cm above the plane of the paper.

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Example 24 **A telephonic cable at a place has four long straight horizontal wires carrying a current of 1.0 amp. in the same direction east to west. The earth's magnetic field at the place is 0.39 G and the angle of dip is 35°. The magnetic declination is almost zero. What are the resultant magnetic fields at points 4·0 cm below and above the cable ?**

Solution. Here, No. of wires, $n = 4$, $i = 1.0$ amp. ;

earth's field, $R = 0.39\ G = 0.39 \times 10^{-4}$ T

dip, $\delta = 35°$; declination $\theta = 0°$

$R_1 = ?$, $R_2 = ?$;

$r = 4$ cm each $= 4 \times 10^{-2}$ m

Magnetic field at 4 cm due to currents in 4 wires

$$B = 4 \times \frac{\mu_0 i}{2\pi r} = 4 \times \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 4 \times 10^{-2}} = 2 \times 10^{-5} \text{ tesla}$$

Horizontal component of earth's field

$$H = R \cos \delta = 0.39 \times 10^{-4} \cos 35°$$
$$= 0.39 \times 10^{-4} \times 0.8192$$
$$= 3.19 \times 10^{-5} \text{ tesla}$$

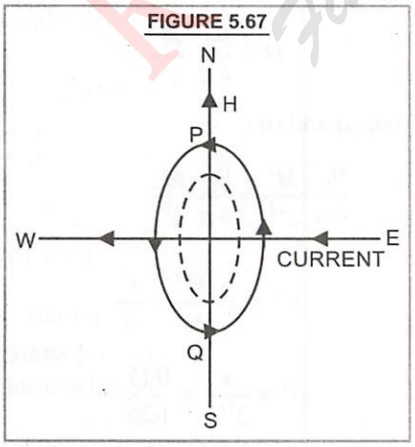
Vertical component of earth's field

$$V = R \sin \delta = 0.39 \times 10^{-4} \sin 35°$$
$$= 0.39 \times 10^{-4} \times 0.5736$$
$$= 2.2 \times 10^{-5} \text{ tesla}$$

At point Q, 4 cm below the wire, horizontal component due to earth's field and field due to current are in opposite directions, Fig. 5.67, so that

$$H_1 = H - B$$

FIGURE 5.67



$$\therefore \quad H_1 = 3.19 \times 10^{-5} - 2 \times 10^{-5}$$
$$= 1.19 \times 10^{-5} \text{ tesla}$$

Hence, $R_1 = \sqrt{H_1^2 + V^2}$

$$= \sqrt{(1.19 \times 10^{-5})^2 + (2.2 \times 10^{-5})^2}$$
$$= \mathbf{2.5 \times 10^{-5}\ tesla}$$

At point P, 4 cm above the wire, horizontal component of earth's field and field due to current are in the same direction, Fig. 5.67,

$$\therefore H_2 = H + B = 3.19 \times 10^{-5} + 2 \times 10^{-5}$$
$$= 5.19 \times 10^{-5} \text{ T}$$

$$R_2 = \sqrt{H_2^2 + V^2}$$
$$= \sqrt{(5{\cdot}19 \times 10^{-5})^2 + (2{\cdot}2 \times 10^{-5})^2}$$
$$= \mathbf{5{\cdot}54 \times 10^{-5}\ tesla}$$

TYPE VI. NEUTRAL POINTS

Formulae used. 1. When magnet is placed with its N pole towards S of earth, neutral points lie on axial line of magnet, such that

$$B_1 = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = H$$

2. When magnet is placed with its N pole towards N of earth, neutral points lie on equatorial line of magnet, such that

$$B_2 = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}} = H$$

Units used. M is in Am^2 or JT^{-1} ; d and l are in metre, B_1, B_2 and H are in tesla.

Standard Value. $\frac{\mu_0}{4\pi} = 10^{-7}$ Wb A^{-1} m^{-1}

Example 25 **A small bar magnet has a magnetic moment 5 Am^2. The neutral point is obtained on axial line when it is placed in magnetic meridian with its N pole pointing S of earth and neutral point is obtained on equatorial line, when it is placed with its N pole pointing towards north of earth. If horizontal component of earth's field is 0·38 G, find the position of neutral points in the two cases.**

Solution. Here, $M = 5\ Am^2$,

$$H = 0{\cdot}38\ G = 0{\cdot}38 \times 10^{-4}\ T.$$

If d_1 is distance of neutral point when N pole of magnet is pointing S of earth, then

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d_1^3} = H$$

$$d_1^3 = \frac{\mu_0}{4\pi}\frac{2M}{H} = 10^{-7} \times \frac{2\times 5}{0{\cdot}38\times 10^{-4}}$$

$$= 26{\cdot}3 \times 10^{-3}\ m$$

$$d_1 = (26{\cdot}3 \times 10^{-3})^{1/3}\ m = 2{\cdot}97 \times 10^{-1}\ m$$

$$= \mathbf{29{\cdot}7\ cm}$$

In the second case, $B_2 = \frac{\mu_0}{4\pi}\frac{M}{d_2^3} = H$

$$d_2^3 = \frac{\mu_0}{4\pi}\frac{M}{H} = \frac{10^{-7}\times 5}{0{\cdot}38\times 10^{-4}} = 13{\cdot}15 \times 10^{-3}$$

$$d_2 = (13{\cdot}15 \times 10^{-3})^{1/3}\ m = 2{\cdot}36 \times 10^{-1}\ m$$

$$= \mathbf{23{\cdot}6\ cm}$$

Example 26 **The magnetic moment of a short bar magnet is 1·6 A m^2. It is placed in the magnetic meridian with north pole pointing south. The neutral point is obtained at 20 cm from the centre of the magnet. Calculate the horizontal component H of earth's field. If magnet be reversed i.e. north pole pointing north, find the position of the neutral point.**

Solution. Here, $M = 1{\cdot}6\ A\ m^2$,

$$d_1 = 20\ cm = \frac{1}{5}\ m,\ H = ?$$

When north pole of magnet is pointing south, neutral points lie on axial line of magnet. At neutral point,

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d_1^3} = H$$

or $$H = \frac{10^{-7}\times 2\times 1{\cdot}6}{(1/5)^3} = \mathbf{4 \times 10^{-5}\ tesla}$$

When magnet is reversed *i.e.* north pole is pointing north, neutral points lie on equatorial line of the magnet. We have to calculate d_2.

As $$B_2 = \frac{\mu_0}{4\pi}\frac{M}{d_2^3} = H$$

$$\therefore\quad d_2^3 = \frac{\mu_0}{4\pi}\frac{M}{H}$$

$$d_2^3 = \frac{10^{-7}\times 1{\cdot}6}{4\times 10^{-5}} = 0{\cdot}4\times 10^{-2}\ m^3$$

$$= 0{\cdot}4 \times 10^{-2} \times 10^6\ cm^3$$

$$= 4000\ cm^3$$

$$d_2 = (4000)^{1/3} = \mathbf{15{\cdot}87\ cm}$$

Example 27 **A magnet placed in the magnetic meridian with its north pole pointing north of the earth produces a neutral point at a distance of 0·15 m from either pole. It is then broken into two equal parts and one such piece is placed in a similar position. Find the position of the neutral point.**

Solution. As N-pole of magnet is pointing north, neutral points lie on equatorial line of magnet at a distance x from each pole.

$$\therefore\quad H = B_{eqn.} = \frac{\mu_0}{4\pi}\frac{M}{x^3} \qquad ...(i)$$

When the magnet is broken into two equal parts, pole strength remains unchanged, but magnetic moment is reduced to half

i.e., $M' = M/2$

If x' is distance of neutral point from each pole of the part, then

$$H = \frac{\mu_0}{4\pi}\frac{M'}{x'^3} \qquad ...(ii)$$

From (*i*) and (*ii*),

$$\frac{\mu_0}{4\pi}\frac{M'}{x'^3} = \frac{\mu_0}{4\pi}\frac{M}{x^3}$$

$$x'^3 = x^3\frac{M'}{M} = \frac{x^3}{2}$$

$$x' = \frac{x}{2^{1/3}} = \frac{0{\cdot}15}{1{\cdot}26}$$

$$= \mathbf{0{\cdot}119\ m\ from\ each\ pole}$$

TYPE VII. TANGENT LAW, TANGENT GALVANOMETER

Formulae used.

1. $F = H \tan\theta$, where θ is angle with H

2. $I = K \tan\theta$, where $K = \dfrac{H}{G} = \dfrac{H}{\mu_0 n/2r} = \dfrac{2rH}{\mu_0 n}$

Units used. The field intensities F and H are in tesla. Current I is in ampere, radius r is in metre. Magnetic moment M is in Am^2. Time period T in sec. Frequency ν is in hertz.

Example 28 **Two tangent galvanometers A and B have coils of radii 8 cm and 16 cm respectively and resistance 8 ohm each. They are connected in parallel to a cell of emf 4 V and negligible internal resistance. The deflections produced are 30° and 60° respectively. A has 2 turns. What is the number of turns in B ?** [CBSE 2007 (C)]

Solution. Here, $r_1 = 8$ cm, $r_2 = 16$ cm,

$R_1 = R_2 = 8$ ohm

$E = 4$ V, $\theta_1 = 30°$, $\theta_2 = 60°$, $n_1 = 2$, $n_2 = ?$

$$I = \frac{E}{R} = \frac{2r_1 H}{\mu_0 n_1}\tan\theta_1 = \frac{2r_2 H}{\mu_0 n_2}\tan\theta_2$$

$$\therefore\ n_2 = \frac{n_1 r_2 \tan\theta_2}{r_1 \tan\theta_1}$$

$$= \frac{2\times 16\tan 60°}{8\tan 30°} = \frac{4.\sqrt{3}}{1/\sqrt{3}} = 12$$

Example 29 **The coil in a tangent galvanometer is 16 cm in radius. How many turns of the wire should be wound on it if a current of 20 mA is to produce a deflection of 45° ? Take horizontal component of earth's field = 0.36×10^{-4} T.**

Solution. Here, $r = 16$ cm $= 16 \times 10^{-2}$ m,

$n = ?$; $I = 20$ m A $= 20 \times 10^{-3}$ amp.,

$\theta = 45°$; $H = 0.36 \times 10^{-4}$ tesla

From $$I = \frac{2rH}{\mu_0 n}\tan\theta$$

$$n = \frac{2rH}{\mu_0 I}\tan\theta = \frac{2\times 16\times 10^{-2}\times 0.36\times 10^{-4}}{4\pi\times 10^{-7}\times 20\times 10^{-3}}$$

$$= \mathbf{458\ turns}$$

TYPE VIII. ATOM AS A MAGNETIC DIPOLE

Formula used. 1. Magnetic moment of an atom due to revolving electron

$$M = n\left(\frac{eh}{4\pi m}\right) = n\mu_B,$$

where $\mu_B = \dfrac{eh}{4\pi m}$ = Bohr magneton

$= 9{\cdot}27 \times 10^{-24}\ Am^2$.

Units used. M and μ_B in Am^2 ; e in coulomb ; m in kg and h in Js.

Standard Values. $e = 1{\cdot}6 \times 10^{-19}$ C, $m = 9{\cdot}1 \times 10^{-31}$ kg, $h = 6{\cdot}6 \times 10^{-34}$ J.s

Example 30 **In a hydrogen atom, the electron moves in an orbit of radius 0·5 Å, making 10^{16} rps. Calculate the magnetic moment associated with the orbital motion of electron.**

Solution. Here, $r = 0{\cdot}5$ Å $= 0{\cdot}5 \times 10^{-10}$ m ;

$\nu = 10^{16}$ rps, M = ?

Charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C

Equivalent current, $I = \dfrac{e}{T} = e\nu$

Area of the orbit, $A = \pi r^2$

As $M = IA$

$\therefore\ M = e\nu(\pi r^2) = 1{\cdot}6 \times 10^{-19} \times 10^{16} \times 3{\cdot}14 \times (0{\cdot}5 \times 10^{-10})^2$

$= \mathbf{1{\cdot}256 \times 10^{-23}\ Am^2}$

Example 31 **An electron moves around the nucleus in a hydrogen atom of radius 0·51 Å with a velocity of 2×10^6 m/s. Calculate the following :**

(*i*) the equivalent current due to orbital motion of electron, (*ii*) magnetic field produced at the centre of nucleus, (*iii*) the magnetic moment associated with the electron. (CBSE 2008)

Solution. Here, $r = 0{\cdot}51$ Å $= 0{\cdot}51 \times 10^{-10}$ m

$v = 2 \times 10^6$ m/s

(*i*) $$I = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

$$= \frac{1{\cdot}6\times 10^{-19}\times 2\times 10^6}{2\times 3{\cdot}14\times 0{\cdot}51\times 10^{-10}} = \mathbf{10^{-4}\ A}$$

(*ii*) $B = \dfrac{\mu_0 I}{2r} = \dfrac{4\pi\times10^{-7}\times10^{-4}}{2\times0{\cdot}51\times10^{-10}} = \mathbf{1{\cdot}23\ T}$

(*iii*) $M = IA = \dfrac{ev}{2\pi r}\times\pi r^2 = \dfrac{1}{2}evr$

$= \dfrac{1}{2}\times1{\cdot}6\times10^{-19}\times2\times10^{6}\times0{\cdot}51\times10^{-10}$

$= \mathbf{8{\cdot}16\times10^{-25}\ Am^2}$

TYPE IX. MAGNETIC PROPERTIES OF MATERIALS

Formulae used. 1. $\mu = \dfrac{B}{H}$

2. $\chi_m = \dfrac{I}{H}$, where I = magnetic moment/volume

3. $\mu_r = \dfrac{\mu}{\mu_0}$

4. $B = \mu_0 (H + I)$

5. $\mu_r = 1 + \chi_m$.

Units used. I and H are in Am^{-1}, B in tesla, μ in $TA^{-1}m$, M in Am^2, Volume V in m^3, μ_r and χ_m have no units.

Standard value. $\mu_0 = 4\pi\times10^{-7}\ Wb\ A^{-1}m^{-1}$

Example 32 **The core of a toroid having 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. The magnetic field in the core for a current of 0·70 A is 2·5 T. Calculate relative permeability of the core ?**

Solution. Here, $N = 3000$, $r_1 = 11$ cm,

$r_2 = 12$ cm, $I = 0{\cdot}70$ A, $B = 2{\cdot}5$ T, $\mu_r = ?$

Mean radius $= \dfrac{r_1 + r_2}{2} = \dfrac{11+12}{2} = 11{\cdot}5$ cm

$= 11{\cdot}5\times10^{-2}$ m

Number of turns per unit length,

$$n = \frac{N}{2\pi r} = \frac{3000}{2\times3{\cdot}14\times11{\cdot}5\times10^{-2}}$$

As $B = \mu n I$

$$\therefore\quad \mu = \frac{B}{nI} = \frac{2{\cdot}5\times2\times3{\cdot}14\times11{\cdot}5\times10^{-2}}{3000\times0{\cdot}7}$$

$$= 8{\cdot}6\times10^{-4}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{8{\cdot}6\times10^{-4}}{4\pi\times10^{-7}} = \mathbf{684{\cdot}4}$$

Example 33 **The coercivity of a certain permanent magnet is $4{\cdot}0\times10^4\ Am^{-1}$. This magnet is placed inside a solenoid 15 cm long and having 600 turns. Calculate the current required to demagnetise the magnet completely.**

Solution. Here, coercivity $= 4{\cdot}0\times10^4\ Am^{-1}$ of magnet implies that magnetising intensity $H = 4{\cdot}0\times10^4\ Am^{-1}$ is required to be applied in opposite direction to demagnetise the magnet.

n = number of turns per unit length

$$= \frac{600}{15\times10^{-2}} = 4000\ \text{turns/m}$$

As $H = nI$

$$\therefore\quad I = \frac{H}{n} = \frac{4{\cdot}0\times10^4}{4000} = \mathbf{10\ A}$$

Example 34 **A toroidal solenoid has 3000 turns and a mean radius of 10 cm. It has a soft iron core of relative magnetic permeability 2000. Find the magnitude of the magnetic field in the core when a current of 1.0 amp. is passed through the solenoid.**

Solution. Here, $n = \dfrac{3000}{2\pi r}$, $r = 10$ cm $= 0.1$ m.

$\mu_r = 2000$, $i = 1$ amp., $B = ?$

$$B = \mu_0\mu_r n i = 4\pi\times10^{-7}\times2000\times\frac{3000}{2\pi\times0{\cdot}1}\times1$$

$$= \mathbf{12\ tesla}$$

Example 35 **A magnetising field of 1600 Am^{-1} produces a magnetic flux of $2{\cdot}4\times10^{-5}$ weber in a bar of iron of cross section 0·2 cm^2. Calculate permeability and susceptibility of the bar.**

Solution. Here, $H = 1600\ Am^{-1}$,

$\phi = 2{\cdot}4\times10^{-5}$ Wb.

$a = 0{\cdot}2\ cm^2 = 0{\cdot}2\times10^{-4}\ m^2$, $\mu = ?$, $\chi_m = ?$

$$B = \frac{\phi}{a} = \frac{2{\cdot}4\times10^{-5}}{0{\cdot}2\times10^{-4}} = 1{\cdot}2\ \text{weber/m}^2$$

$$\mu = \frac{B}{H} = \frac{1{\cdot}2}{1600} = \mathbf{7{\cdot}5\times10^{-4}\ TA^{-1}m}$$

As $\mu = \mu_0 (1 + \chi_m)$

$$\therefore\quad \chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7{\cdot}5\times10^{-4}}{4\pi\times10^{-7}} - 1$$

$$\chi_m = \frac{7{\cdot}5\times10^3}{4\pi} - 1 = 597{\cdot}1 - 1 = \mathbf{596{\cdot}1}$$

Example 36 **A solenoid of 500 turns per metre is carrying a current of 3 A. Its core is made of iron, which has a relative permeability of 5000. Determine the magnitudes of magnetic intensity, intensity of magnetisation and magnetic field inside the core.**

Solution. Here, $n = 500$ turns/m ;

$i = 3$ A ; $\mu_r = 5000$.

Magnetic intensity,

$H = n\,i = 500\ \text{m}^{-1} \times 3\ \text{A} = \mathbf{1500\ Am^{-1}}$

As $\mu_r = 1 + \chi_m$

$\therefore \quad \chi_m = \mu_r - 1 = 5000 - 1 = 4999 \approx 5000$

Also, $\mu_r = \dfrac{\mu}{\mu_0} = 5000 \quad \therefore \ \mu = 5000\,\mu_0$

Intensity of magnetisation,

$I = \chi_m H = 5000 \times 1500 = \mathbf{7{\cdot}5 \times 10^6\ Am^{-1}}$

Magnetic field inside the core,

$B = \mu H = 5000\,\mu_0 H = 5000\,(4\pi \times 10^{-7}) \times 1500$

$B = 3\pi = 3 \times \dfrac{22}{7} = \mathbf{9{\cdot}4\ T}$

Example 37 **A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2 A. If the number of turns is 1000 per metre, calculate (*i*) H (*ii*) B (*iii*) Intensity of magnetisation I, and the magnetising current.** (NCERT Solved Example)

Solution. Here, $\mu_r = 400$,

$I' = 2$ A, $n = 1000$ per metre

(*i*) $H = n\,I' = 1000 \times 2 = \mathbf{2 \times 10^3\ Am^{-1}}$

(*ii*) $B = \mu H = \mu_0\,\mu_r H$

$= 4\pi \times 10^{-7} \times 400\ (2 \times 10^3) = \mathbf{1{\cdot}0\ T}$

(*iii*) From $B = \mu_0 (H + I)$, where I is intensity of magnetisation,

$$I = \frac{B}{\mu_0} - H = \frac{1{\cdot}0}{4\pi \times 10^{-7}} - 2 \times 10^3$$

$$= 7{\cdot}95 \times 10^5 - 0{\cdot}02 \times 10^5$$

$$= \mathbf{7{\cdot}93 \times 10^5\ Am^{-1}}$$

(*iv*) The magnetising current I_m is the additional current that needs to be passed through the windings of the solenoid in the absence of the core, which would produce a B value as in the presence of the core. Thus, $B = \mu_0\,n\,(I' + I_m)$

$1{\cdot}0 = 4\pi \times 10^{-7} \times 1000\ (2 + I_m)$

$$I_m = \frac{1{\cdot}0}{4\pi \times 10^{-4}} - 2 = 796 - 2 = \mathbf{794\ A}$$

Example 38 **An ideal solenoid having 2000 turns per meter has an iron core of permeability 500 and carries a current of 1·0 A. Calculate (*i*) magnetising intensity (H) at the centre of solenoid (*ii*) magnetic permeability of iron (*iii*) magnetic susceptibility of iron (*iv*) intensity of magnetisation.**

Solution. Here, $n = 2000$, $I = 1{\cdot}0$ A

(*i*) Magnetising intensity, $H = nI$

$= 2000 \times 1 = \mathbf{2000\ Am^{-1}}$

(*ii*) Magnetic permeability

$\mu = \mu_0\,\mu_r = 4\pi \times 10^{-7} \times 500$

$= \mathbf{6{\cdot}28 \times 10^{-4}\ Tm\ A^{-1}}$.

(*iii*) $\chi_m = \mu_r - 1 = 500 - 1 = \mathbf{499}$.

(*iv*) $I = \chi_m H = 499 \times 2000 = \mathbf{9{\cdot}98 \times 10^5\ Am^{-1}}$

Example 39 **An iron atom has a dipole moment of 9×10^{-24} Am^2. Density of iron is 8 g/cm^3 and molecular mass is 55 g mole^{-1}. If domain size of iron is a cube of side 10^{-6} m find (*i*) number of atoms in the domain (*ii*) maximum dipole moment (*iii*) intensity of magnetisation.**

Solution. Dipole moment of each atom

$= 9 \times 10^{-24}$ Am^2

Density of iron = 8 g/cc $= 8 \times 10^3$ kg/m^3

Molecular mass = 55 g $= 55 \times 10^{-3}$ kg

Volume of domain, $= (1 \times 10^{-6}\ \text{m})^3 = 10^{-18}\ \text{m}^3$

Mass of domain = Vol. × density

$= 10^{-18} \times 8 \times 10^3 = 8 \times 10^{-15}$ kg

Number of atoms in the domain

$$= \frac{\text{mass of domain} \times \text{Avogadro number}}{\text{molecular mass}}$$

$$= \frac{8 \times 10^{-15} \times 6{\cdot}023 \times 10^{23}}{55 \times 10^{-3}}$$

$= 8{\cdot}76 \times 10^{10}$

Maximum dipole moment

$= 9 \times 10^{-24} \times 8{\cdot}76 \times 10^{10}$

$M = 7{\cdot}88 \times 10^{-13}$ Am^2

Intensity of magnetisation,

$$I = \frac{M}{V} = \frac{7{\cdot}88 \times 10^{-13}}{10^{-18}}$$

$$= \mathbf{7{\cdot}88 \times 10^5\ Am^{-1}}$$

Example 40 **Find the percentage increase in the magnetic field B when the space within a current carrying toroid is filled with aluminium of susceptibility $2{\cdot}1 \times 10^{-5}$.**

Solution. In the absence of aluminium,

$$B_0 = \mu_0 H.$$

In the presence of aluminium,

$$B = \mu H = \mu_0 (1 + \chi_m) H$$

Increase in magnetic field induction

$$= B - B_0 = \mu_0 \chi_m H$$

$$\text{Percentage increase} = \frac{B - B_0}{B_0} \times 100$$

$$= \frac{\mu_0 \chi_m H}{\mu_0 H} \times 100$$

$$= 2{\cdot}1 \times 10^{-5} \times 100 = \mathbf{2{\cdot}1 \times 10^{-3}\%}$$

Example 41 **A bar of 5×10^{-3} m^2 area is subjected to a field of 600 Am. If susceptibility of material is 499, calculate (*i*) permeability (*ii*) flux density and (*iii*) magnetic flux.**

Solution. Here, $A = 5 \times 10^{-3}$ m^2, $H = 600$ Am

$\chi_m = 499$, $\mu = ?$ $B = ?$ $\phi = ?$

$$\mu = \mu_0 (1 + \chi_m) = 4\pi \times 10^{-7} (1 + 499)$$

$$= \mathbf{6{\cdot}28 \times 10^{-4}\ T\ A^{-1}\ m}$$

$$B = \mu H = 6{\cdot}28 \times 10^{-4} \times 600 = \mathbf{0{\cdot}377\ T}$$

$$\phi = BA = 0{\cdot}377 \times 5 \times 10^{-3}$$

$$= \mathbf{1{\cdot}885 \times 10^{-3}\ Wb}$$

TYPE X. CURIE LAW AND HYSTERISIS LOSS

Formulae used. 1. $\chi_m = \frac{C}{T}$, where C is a constant.

2. Energy loss/volume/cycle = area of IH loop

$$= \frac{1}{4\pi} \times \text{area of BH loop}$$

Units used. T is in K. χ_m has no units. Energy is in joule.

Example 42 **A specimen of iron weighing 6 kg loses $8{\cdot}64 \times 10^4$ J of energy per hour when subjected to 50 c/s a.c. If density of iron is 6000 km/m^3, what is the hysterisis loss per unit volume per cycle ?**

Solution. Here, $m = 6$ kg

Energy lost/hour = $8{\cdot}64 \times 10^4$ J/hr.

$\nu = 50$ c/s, $\rho = 6000$ kg/m^3 ;

hysterisis loss = ?

$$\text{volume, } V = \frac{m}{\rho} = \frac{6}{6000}\ m^3 = 10^{-3}\ m^3$$

If E is hysterisis loss (energy/vol./cycle), then

energy lost/hour = $E \times V \times 50 \times 60 \times 60$

$= 8{\cdot}64 \times 10^4$

$$E \times 10^{-3} \times 18 \times 10^4 = 8{\cdot}64 \times 10^4$$

$$E = \frac{8{\cdot}64 \times 10^4}{18 \times 10} = \mathbf{4{\cdot}8 \times 10^2\ J/m^3/cycle}$$

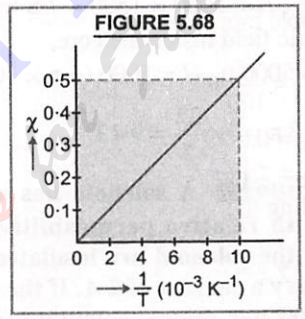
Example 43 **Find curie constant for a sample from χ versus $1/T$ graph shown in Fig. 5.68.**

Solution. From Fig. 5.68,

$$\text{for } \chi = 0{\cdot}5,\ \frac{1}{T} = 10 \times 10^{-3} K^{-1} = 10^{-2}\ K^{-1}$$

$$\text{As } \chi = \frac{C}{T} \quad \therefore\ C = \chi \cdot T = 0{\cdot}5 \times \frac{1}{10^{-2}} = \mathbf{50\ K}$$

FIGURE 5.68



Example 44 **Hysterisis loss for a sample of 120 kg is 300 J m^{-3}/cycle. Frequency of supply is 50 cycle/s. If density of iron is 7500 kg/m^3, find energy loss in 60 minutes.**

Solution. Here, $m = 120$ kg

Hysterisis loss = 300 J m^{-3}/cycle.

Frequency = 50 c/s

Density of iron = 7500 kg/m^3

Total time = 60 mins = 60×60 s

$$\text{Volume of specimen} = \frac{\text{mass}}{\text{density}} = \frac{120}{7500}\ m^3$$

$$\text{Loss of energy/cycle} = 300 \times \frac{120}{7500} = \frac{360}{75}$$

$$\text{Loss of energy/sec} = \frac{360}{75} \times 50$$

$$\text{Total loss of energy} = \frac{360}{75} \times 50 \times 60 \times 60$$

$$= 240 \times 3600$$

$$= \mathbf{86{\cdot}4 \times 10^4\ J}$$

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Example 45 **A sample of paramagnetic salt contains 2×10^{24} atomic dipoles, each of moment 1.5×10^{-23} JT^{-1}. The sample is placed under a homogeneous magnetic field of 0.64 T and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K. (Assume Curie's law).**

Solution. Here, no. of dipoles, $n = 2 \times 10^{24}$

Magnetic moment of each dipole,

$$M' = 1.5 \times 10^{-23} \text{ JT}^{-1}$$

Total dipole moment of sample $= n \times M'$

$$= 2 \times 10^{24} \times 1.5 \times 10^{-23}$$
$$= 30 \text{ JT}^{-1}$$

As saturation achieved is 15%, therefore, effective dipole moment

$$M_1 = \frac{15}{100} \times 30 = 4.5 \text{ JT}^{-1};$$

$B_1 = 0.64$ T, $T_1 = 4.2$ K

$M_2 = ?$ $B_2 = 0.98$ T, $T_2 = 2.8$ K

According to Curie's law,

$$\chi_m = \frac{C}{T} = \frac{I}{H}$$

or $$I = \frac{CH}{T}$$

As $I \propto M$ and $H \propto B$

$$\therefore \quad M \propto \frac{CB}{T}, \quad \frac{M_2}{M_1} = \frac{B_2}{B_1} \cdot \frac{T_1}{T_2}$$

or $$M_2 = \frac{B_2 T_1 M_1}{T_2 B_1} = \frac{0.98 \times 4.2 \times 4.5}{2.8 \times 0.64},$$

$$= \mathbf{10.34 \text{ JT}^{-1}}$$

TYPE XI. TYPICAL EXAMPLES

Example 46 **A short bar magnet placed with its axis at 30° experiences a torque of 0·016 N-m in an external field of 800 G.**

(*a*) What is the magnetic moment of the magnet ? (*b*) What is the work done by an external force in moving it from its most stable to most unstable position ? (*c*) What is the work done by force due to external magnetic field in the process mentioned in (*b*) ? (*d*) The bar magnet is replaced by a solenoid of cross sectional area 2×10^{-4} m^2 and 1000 turns, but the same magnetic moment. Determine the current flowing through the solenoid. (NCERT Solved Example)

Solution. (*a*) Here, $\theta = 30°$, $B = 800 \times 10^{-4}$ T,

$\tau = 0{\cdot}016$ Nm, $M = ?$

As $\tau = MB \sin\theta$

$$\therefore \quad M = \frac{\tau}{B \sin\theta} = \frac{0{\cdot}016}{800 \times 10^{-4} \times \sin 30°}$$
$$= \mathbf{0{\cdot}40 \text{ Am}^2}$$

(*b*) For most stable position, $\theta = 0°$ and for most unstable position, $\theta = 180°$.

$\therefore$ work done by external force

$$= -MB(\cos 180° - \cos 0°) = 2 \text{ MB}$$
$$= 2 \times 0{\cdot}40 \times 800 \times 10^{-4} = \mathbf{0{\cdot}064 \text{ J}}$$

(*c*) Work done by external mag. field

$$= \mathbf{-0{\cdot}064 \text{ J}}$$

(*d*) As $M = NIA$

$$\therefore \quad 0{\cdot}40 = 1000 \times I \times 2 \times 10^{-4};$$

$$I = \frac{0{\cdot}40}{0{\cdot}2} = \mathbf{2 \text{ A}}$$

Example 47 **A magnetic needle has magnetic moment of $6{\cdot}7 \times 10^{-2}$ Am^2 and moment of inertia of $7{\cdot}5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6·70 s. What is the magnitude of the magnetic field ?** (NCERT Solved Example)

Solution. Here, $M = 6{\cdot}7 \times 10^{-2}$ Am^2,

$I = 7{\cdot}5 \times 10^{-6}$ kg m^2

Time for one oscillation,

$$T = \frac{6{\cdot}70}{10} = 0{\cdot}67 \text{ s}; \; B = ?$$

From $$T^* = 2\pi\sqrt{\frac{I}{MB}};$$

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4 \times (22/7)^2 \times 7{\cdot}5 \times 10^{-6}}{6{\cdot}7 \times 10^{-2} (0{\cdot}67)^2} = \mathbf{0{\cdot}01 \text{ T}}$$

Example 48 **A domain in ferromagnetic iron is in the form of a cube of side length 10^{-4} m. Estimate the number of iron atoms in the domain and the maximum possible dipole moment and magnetisation of the domain. The molecular mass**

*For derivation of this formula, refer to Art. 5.11.

of iron is 55 g/mole, and its density is 7·9 g/cm^3. Assume that each iron atom has a dipole moment of 9·27 × 10^{-24} Am^2.

(NCERT Solved Example)

Solution. Here, length of cubic domain

$= l = 10^{-4}$ m

Volume of domain = $V = l^3 = (10^{-4})^3$

$= 10^{-12}$ m^3 = 10^{-6} cm^3

Mass of domain = Volume × density

$= 10^{-6} \times 7{\cdot}9$ gram.

It is given that 55 gram of iron contain 6·023 × 10^{23} iron atoms (Avogadro's number)

$\therefore$ Number of atoms in the domain

$$= \frac{6{\cdot}023 \times 10^{23} \times 7{\cdot}9 \times 10^{-6}}{55}$$

$$= \mathbf{8{\cdot}65 \times 10^{16}\ atoms.}$$

Maximum possible dipole moment is achieved when all the atomic dipole moments are perfectly aligned (which of course is unrealistic).

$\therefore$ $M_{max} = (8{\cdot}65 \times 10^{16}) \times 9{\cdot}27 \times 10^{-24}$

$= \mathbf{8{\cdot}0 \times 10^{-7}\ Am^2}$

Max. Intensity of magnetisation

$$I_{max} = \frac{M_{max}}{\text{Volume of domain}} = \frac{8{\cdot}0 \times 10^{-7}}{10^{-12}}$$

$$= \mathbf{8 \times 10^5\ Am^{-1}}$$

REVISION AT A GLANCE

1. According to **molecular theory,** every molecule of a magnetic substance (whether magnetised or not), is a complete magnet in itself.

   In an unmagnetised substance, the molecular magnets are randomly oriented such that they form closed chains. That is why resultant magnetism of the unmagnetised specimen is zero. On magnetising the substance, the molecular magnets are realigned in the direction of the field.

2. **Magnetic field lines** are hypothetical lines which enable us to understand certain phenomena in magnetism.

   Tangent to field line at a point gives us the direction of magnetic field $\vec{B}$ at that point. No two magnetic field lines can intersect each other.

   Magnetic field lines are continuous curves from north to south, outside the body of the magnet and from south to north inside the body of the magnet. A current carrying solenoid from outside resembles a bar magnet. Inside the solenoid, there is a strong magnetic field, which can magnetise a specimen. The solenoid is hollow from inside, whereas the bar magnet is solid.

3. **A magnetic dipole** consists of two unlike poles of equal strength (m) and separated by a small distance ($2\vec{l}$). It has a magnetic dipole moment, $\vec{M} = m(2\vec{l})$, directed from south to north. The SI units of M are joule/tesla or ampere metre^2

   When a bar magnet of magnetic moment M is held at $\angle\theta$ with the direction of a uniform magnetic field $\vec{B}$, a **torque** $\tau = MB \sin\theta$ acts on the magnet. It tries to align the dipole in the direction of the field. Work is required to be done in rotating the dipole against the action of the torque. This work is stored as **potential energy of the dipole.** $P.E. = W = -MB(\cos\theta_2 - \cos\theta_1)$

   Here, θ_1 = initial angle & θ_2 is final angle of bar magnet with $\vec{B}$.
   Usually, θ_1 is taken as 90°, which is the position of zero P.E.

4. **A loop of current** acts as a dipole of magnetic moment $\vec{M} = IA\hat{n}$ where I is strength of current through the loop and A is area enclosed by the loop, $\hat{n}$ is unit vector along $\vec{M}$.

5. According to **Gauss's theorem in magnetism**, surface integral of magnetic field induction over a surface (closed or open) is always zero, *i.e.* $\oint_S \vec{B}.\vec{ds} = 0$

   This theorem establishes that magnetic poles always exist in equal and unlike pairs.

6. **Terrestrial magnetism** deals with the magnetic field of earth. It is described in terms of three quantities, which are called **magnetic elements of earth.** These are :
(*i*) Magnetic declination (θ) (*ii*) Magnetic inclination or Dip (δ) (*iii*) Horizontal component (*H*) of earth's magnetic field.
Magnetic declination at a place is the angle between magnetic axis and the geographic axis. It is also equal to the angle between magnetic meridian and geographic meridian at that place.
Magnetic Inclination or dip at a place is defined as the angle, which the direction of total earth's magnetic field makes with a horizontal line in magnetic meridian.
Horizontal component of earth's magnetic field is the component of total earth's magnetic field (*R*) in the horizontal direction in magnetic meridian. Obviously.

$$H = R\cos\delta\ ;\ V = R\sin\delta$$

$$\therefore \qquad \sqrt{H^2+V^2} = R \quad \text{and} \quad \frac{V}{H} = \tan\delta$$

At magnetic poles, $\delta = 90°$ and at magnetic equator, $\delta = 0°$

7. **Magnetic Field strength** B_1 due to a bar magnet at any point on the *axial line* of the magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2}$$

where *d* is distance of the point from the centre of the magnet. The direction of $\vec{B}_1$ ***is along SN produced.***
Magnetic Field strength B_2 due to a bar magnet at any pt. on *equatorial* line of the bar magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}$$

The direction of $\vec{B}_2$ ***is along a line parallel to NS.***

8. **Neutral points** are the points where the resultant magnetic field due to the field of the bar magnet and due to field of earth is zero. When a magnet is placed with its *N* pole towards geographic north, neutral points lie on equatorial line of the magnet. At each neutral point, $B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = H$

where *H* is horizontal component of earth's magnetic field.
When a bar magnet is placed with its *N* pole towards geographic south, the neutral points lie on axial line of the magnet. At each neutral point, $B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2} = H$

9. **Tangent Law.** When a magnet is suspended freely under the combined action of two uniform magnetic fields of strengths *F* and *H* acting at 90° to each other and the magnet comes to rest making an angle θ with the direction of *H*, then $F = H\tan\theta$
This is tangent law. A **tangent galvanometer** is based on tangent law.
In a tangent galvanometer, $I = \frac{2rH}{\mu_0 n}\tan\theta = \frac{H}{G}\tan\theta = K\tan\theta$

where $K = \frac{H}{G}$ = ***reduction factor*** **of tangent galvanometer** and $G = \mu_0 n/2r$ is ***galvanometer constant.***

10. Every atom behaves as a magnetic dipole, because electrons in the atom revolve around the nucleus. The magnetic moment *M* associated with an atomic dipole is $M = \frac{neh}{4\pi m} = n(\mu_B)$

where $n = 1, 2, 3,$....denotes the number of orbit

and $\mu_B = \frac{eh}{4\pi m}$ = least value of dipole moment of atom = 9.27×10^{-24} Am^2. μ_B is called ***Bohr magneton.***

11. Some important terms used in magnetism are :

(*i*) **Magnetic field strength, Magnetic field induction or Magnetic flux density** (B) is defined from Lorentz force eqn. $\vec{F} = q_0 (\vec{v} \times \vec{B})$

SI unit of B is *tesla* and cgs unit of B is gauss, where 1 tesla = 10^4 gauss.

(*ii*) **Magnetising force or magnetising intensity (H)** = number of ampere turns flowing round unit length of toroidal solenoid to produce magnetic field induction B in the solenoid. $H = n I$. SI unit of H is Am^{-1}.

(*iii*) **Relative Magnetic permeability** $\mu_r = \frac{B}{B_0} = \frac{\mu}{\mu_0}$. SI unit of μ, μ_0 is Wb A^{-1} m^{-1}.

Relative magnetic permeability has no units.

(*iv*) **Intensity of magnetisation** $I = \frac{M}{V} = \frac{m}{a}$. SI unit of I is Am^{-1} ; same as that of H.

(*v*) **Magnetic susceptibility** $(\chi_m) = \frac{I}{H}$. It has no units.

Important relations governing these terms are :

$$B = \mu_0 (H + I) \; ; \; \mu_r = 1 + \chi_m$$

12. Classification of magnetic materials.

On the basis of magnetic properties, different materials have been classified into three categories : *Diamagnetic, Paramagnetic* and *Ferromagnetic.*

Diamagnetic substances *are those in which the individual atoms/molecules/ions do not possess any net magnetic moment on their own. These are feebly magnetised in a direction opposite to that of the magnetising field in which they are placed.* For example, bismuth, antimony, copper, gold, quartz, mercury, water, alcohol, air, hydrogen etc. The permeability (μ) of diamagnetic substances is less than one and their susceptibility is negative, which does not change with temperature.

Paramagnetic substances *are those in which each individual atom/molecule/ion has a net non zero magnetic moment of its own. These are weakly magnetised in the direction of the magnetising field, in which they are placed.* For example, aluminium, platinum, chromium, manganese, crown glass, oxygen etc. The permeability μ of paramagnetic substances is greater than one, and their susceptibility is positive. The susceptibility (χ_m) of such materials decreases with rise in temperature.

Ferromagnetic substances *are those in which each individual atom/molecule/ion has a non zero megnetic moment on its own. They are magnetised strongly in the direction of the magnetising field in which they are placed.* For example, iron, cobalt, nickel and a number of their alloys. Ferromagnetic materials show all the properties of paramagnetic substances to a much greater degree.

At a certain temperature, ferromagnetics pass over to paramagnetics. This transition temperature is called **Curie temperature.** For soft iron, Curie temp. is 1000 K.

The three kinds of behaviour (dia, para, ferro) has been explained on the basis of *electron theory of magnetism.*

13. Hysteresis is the phenomenon of lagging of magnetic induction (B) and intensity of magnetisation (I) behind the magnetising field (H), when a specimen is taken through a cycle of magnetisation.

From the hysteresis loop of a material, we can study about **retentivity, coercivity** etc. of the material. The loss of energy per unit volume of the specimen per cycle of magnetisation is equal to area of the I—H loop (in cgs system) or the area of B–H loop (in SI units) of the specimen. The study of these characteristics enables us to select suitable materials for different purposes.

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OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. If the magnetizing field on a ferromagnetic material is increased, its permeability

(*a*) decreases (*b*) increases

(*c*) remains unchanged

(*d*) first decreases and then increases

(CBSE Sample Paper 2022-23)

2. The horizontal component of earth's magnetic field at a place is 0·2 G, whereas its total magnetic field is 0·4 G. The angle of dip at the place is

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 90° **(CBSE 2022)**

3. A bar magnet has magnetic dipole moment $\vec{M}$. Its initial position is parallel to the direction of uniform magnetic field $\vec{B}$. In this position, the magnitudes of **torque** and **force** acting on it are :

(*a*) 0 and *MB* (*b*) zero and zero

(*c*) *MB* and *MB* (*d*) $|\vec{M} \times \vec{B}|$ and zero

(CBSE 2022)

4. Which of the following statements is correct ?

(*a*) Magnetic field lines do not form closed loops

(*b*) Magnetic field lines start from *N*-pole and end at *S*-pole of a magnet

(*c*) Two magnetic field lines may intersect each than

(*d*) Tangent at a point on every field line gives the direction of magnetic field at that point

(CBSE 2022)

5. Which of the following is weakly repelled by a magnetic field :

(*a*) Iron (*b*) Cobalt

(*c*) Steel (*d*) Copper

6. If a diamagnetic material is placed in a magnetic field, the magnetic field inside the material compared to that outside will be

(*a*) Slightly less (*b*) Slightly more

(*c*) Very high (*d*) Same

7. The permanent magnetic material is characterised by :

(*a*) Narrow hysteresis loop

(*b*) Broad hysteresis loop

(*c*) High mechanically hardness, all over

(*d*) Mechanically hard surface

8. The area of *B-H* loop for soft iron, as compared to that for steel is :

(*a*) More (*b*) Less

(*c*) Equal (*d*) Zero

9. A stationary magnet does not interact with :

(*a*) iron rod (*b*) moving charge

(*c*) magnet (*d*) stationary charge

10. The value of the magnetic susceptibility for a superconductor is :

(*a*) zero (*b*) Infinity

(*c*) highly positive (*d*) highly negative

11. A bar magnet *AB* with magnetic moment *M* is cut into two equal parts perpendicular to its axis. One part is kept over the other so that end *B* is exactly over *A*. What will be the magnetic moment of the combination so formed ?

(*a*) Zero (*b*) *M*/4

(*c*) *M* (*d*) 3 M/4

12. Two unlike magnetic poles of strength 10 A-m each are held in air at a distance of 0·10 m from each other. What is the magnetic force of attraction between them ?

(*a*) 3×10^{-3} N (*b*) $2{\cdot}5 \times 10^{-3}$ N

(*c*) $1{\cdot}5 \times 10^{-3}$ N (*d*) 1×10^{-3} N

13. S.I. unit of magnetic pole strength is

(*a*) Ampere/meter (*b*) Ampere-meter

(*c*) volt/meter (*d*) Ampere/meter^2

14. Which of the following is an example for diamagnetic substances ?

(*a*) copper (*b*) nickel

(*c*) aluminum (*d*) iron

15. At a certain place, the horizontal component of earth's magnetic field is B_0 and angle of dip is 45°. The total intensity of the field at that place is

(*a*) B_0 (*b*) $\sqrt{2}\, B_0$

(*c*) $2\, B_0$ (*d*) B_0^2

16. The susceptibility of a ferromagnetic material is χ at 27°C. At what temperature will its susceptibility be 0·5 χ.

(*a*) 54°C (*b*) 327°C

(*c*) 600°C (*d*) 237°C

17. Horizontal component of earth's magnetic field at a place is $3{\cdot}2 \times 10^{-5}$ T and angle of dip is 60°. The resultant intensity of earth's magnetic field at the place is

(*a*) $3{\cdot}2 \times 10^{-5}$ T (*b*) $6{\cdot}4 \times 10^{-5}$ T

(*c*) $1{\cdot}6 \times 10^{-5}$ T (*d*) $12{\cdot}8 \times 10^{-5}$ T

18. If the distance between two similar poles of equal strength is doubled, to get the same repulsive force, the pole strength of each pole should be

(*a*) increased by $\sqrt{2}$ times

(*b*) increased by 4 times

(*c*) halved

(*d*) doubled

19. The SI unit of magnetic field intensity is

(*a*) Am N^{-1} (*b*) NA^{-1} m^{-1}

(*c*) NA^{-2} m^{-2} (*d*) NA^{-1} m^{-2}

20. Two wires of the same length are shaped into a square of side '*a*' and a circle with radius *r*. If they carry same current, the ratio of their magnetic moment is

(*a*) 2 : π (*b*) π : 2

(*c*) π : 4 (*d*) 4 : π

21. A magnet of magnetic moment $50\hat{i}$ Am^2 is placed along the *x*-axis in a magnetic field, $\vec{B} = (0{\cdot}5\hat{i} + 3{\cdot}0\hat{j})$ *T*. The torque acting on the magnet is

(*a*) $175\hat{k}$ Nm (*b*) $75\hat{i}$ Nm

(*c*) $150\hat{k}$ Nm (*d*) $25\sqrt{37}\hat{k}$ Nm

22. A current *i* flows in a circular coil of radius *r*. If the coil is placed in a uniform magnetic field *B* with its plane parallel to the field, magnitude of the torque that acts on the coil is

(*a*) zero (*b*) $2\pi r i B$

(*c*) $\pi r^2 i B$ (*d*) $2\pi r^2 i B$

23. The vertical component of earth's magnetic field at a place is $\frac{1}{\sqrt{3}}$ times the horizontal component. The angle of dip at the place is

(*a*) 0° (*b*) 30°

(*c*) 45° (*d*) 60° **(CBSE 2022)**

24. Which of the following is not affected by the presence of a magnetic field ?

(*a*) Current carrying conductor

(*b*) A moving charge

(*c*) A stationary charge

(*d*) A rectangular current loop with its plane parallel to the field **(CBSE 2022)**

25. What is the angle of dip at a place, where horizontal and vertical components of earth's magnetic field are equal ?

(*a*) 30° (*b*) 60°

(*c*) 45° (*d*) 90° **(CBSE 2021)**

26. When a magnetic needle is kept in a non-uniform magnetic field, if experiences

(*a*) a force and a torque

(*b*) only a force

(*c*) only a torque

(*d*) neither a force nor a torque

27. Which of the following statements is incorrect about hysteresis ?

(*a*) This effect is common to all ferromagnetic substances

(*b*) The hysteresis loop area is proportional to the thermal energy developed per unit volume of the material.

(*c*) The hysteresis loop area is independent of the thermal energy developed per unit volume of the substance.

(*d*) The shape of the hysteresis loop is characteristic of the material.

28. Relative permeability of iron is 5500. Its magnetic susceptibility is :

(*a*) 5500×10^{-7} (*b*) 5500×10^{7}

(*c*) 5499 (*d*) 5501

29. Retentivity is maximum for

(*a*) steel (*b*) soft iron

(*c*) copper (*d*) silver

II. Assertion-Reason Type Questions

Direction. For question numbers 30 to 38, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

(*d*) A is false, and R is also false.

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30. Assertion. A circular coil carrying current behaves as a magnetic dipole.

Reason. A magnetic dipole possesses magnetic north and south poles of equal strength a very large distance apart.

31. Assertion. At the magnetic poles of the earth, a compass needle will be vertical.

Reason. At the magnetic poles of the earth, the magnetic field lines enter or leave the surface of earth normally.

32. Assertion. Soft iron is most suitable as a core of the transformer.

Reason. The retentivity and coercivity of soft iron are very small.

33. Assertion. A bar magnet of magnetic moment m is divided into two equal parts by cutting it perpendicular to its length, the magnetic moment of either piece is also $m/4$.

Reason. Magnetic moment of bar magnet is the product of pole strength and semi-length of the magnet.

34. Assertion. The magnetic field strength due to a short bar magnet on its axial line at distance r is double than at the equitorial line at the same distance.

Reason. Magnetic field strength at a point on axial line,

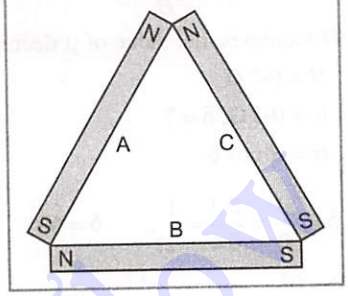
$$B_a = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

and on equitorial line

$$B_{eq} = \frac{\mu_0}{4\pi}\frac{M}{r^3}.$$

35. Assertion. Three similar bar magnets, each of magnetic moment M are placed to form an equilateral triangle as shown in Fig. 5(Q).1. The magnetic moment of the system is zero.

FIGURE 5(Q).1



Reason. The given equilateral structure of three magnets do not have two free poles as required for magnets.

36. Assertion. Magnetic lines of force exert longitudinal tension.

Reason. There is a force of attraction between N-pole and S-pole of magnets.

37. Assertion. The magnetic field lines of a magnet (or of a solenoid carrying current) are not closed continuous loop.

Reason. The magnetic field lines start from north pole and end on south pole.

38. Assertion. The magnetic lines of force exert lateral pressure.

Reason. The magnetic line of force is a straight or curved path in the magnetic field, tangent to it at a point gives the direction of magnetic field at that point.

ANSWERS

I. Multiple Choice Questions

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*d*)	**5.** (*d*)	**6.** (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*d*)	**10.** (*d*)
11. (*a*)	**12.** (*d*)	**13.** (*b*)	**14.** (*a*)	**15.** (*b*)	**16.** (*b*)	**17.** (*b*)	**18.** (*d*)	**19.** (*b*)	**20.** (*c*)
21. (*c*)	**22.** (*c*)	**23.** (*b*)	**24.** (*c*)	**25.** (*c*)	**26.** (*a*)	**27.** (*c*)	**28.** (*c*)	**29.** (*b*)	

II. Assertion-Reason Type Questions

30. (*c*)	**31.** (*a*)	**32.** (*c*)	**33.** (*d*)	**34.** (*a*)	**35.** (*d*)	**36.** (*a*)	**37.** (*d*)	**38.** (*b*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. When magnetising field on a ferromagnetic material is increased, its permeability decreases.

As, $\mu = 1 + \lambda_m = 1 + \frac{I}{H}$;

So as H increases, the value of μ decreases.

2. Here, $H = 0{\cdot}2$ G

and $R = 0{\cdot}4$ G, $\delta = ?$

From $H = R\cos\delta$

$$\cos\delta = \frac{H}{R} = \frac{0{\cdot}2}{0{\cdot}4} = \frac{1}{2}, \qquad \delta = 60°$$

3. Here, $\theta = 0°$

$\tau = MB\sin\theta = MB\sin 0° =$ **Zero**

$F = MB - MB =$ **Zero**

4. Tangent at a point on a magnetic field line gives us the direction of magnetic field at that point. Magnetic lines of force form closed loops. They go from N-pole to S-pole out side of a magnet and S-pole to N-pole inside the magnet. The two magnetic lines of force do not intersect each other as at the point of intersection two tangents can be drawn which gives two directions of magnetic field at that point, which is impossible.

5. Copper is a diamagnetic which is weakly repelled by a magnetic field.

6. The diamagnetic materials when placed in a magnetic field, the magnetic lines of force prefer not to pass through the specimen.

7. Knowledge based question.

8. The area of B–H loop measures energy dissipated per unit volume in the material per cycle of magnetisation, which is less for soft iron than steel.

9. A magnetic can interact with a magnet but can not interact with a stationary charge. A moving charge also has a magnetic field around it which can be interacted by a magnet.

10. Superconductor behaves as a good diamagnetic material. Its magnetic susceptibility is highly negatives.

11. Magnetic moment of each part of the magnet $= \vec{M}/2$

Total magnetic moment of the combination of two parts $\vec{M} = (\vec{M}/2) + (-\vec{M}/2) = 0$

12. $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = 10^{-7} \times \frac{10\times 10}{(0{\cdot}10)^2}$

$= 1 \times 10^{-3}$ N

13. Pole strength, $m = \frac{M}{2l}$

SI unit of $m = \frac{Am^2}{m} = Am$

15. Here, $H = B_0$; $\delta = 45°$

Total intensity of earth's magnetic field is

$R = H/\cos\delta = B_0/\cos 45° = B_0\sqrt{2}$

16. As magnetic susceptibility, $\chi \propto \frac{1}{T}$

$\therefore \quad \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$

or $\frac{0{\cdot}5\,\chi}{\chi} = \frac{27+273}{t+273}$ or $t = 327°C$

17. $R = \frac{H}{\cos\theta} = \frac{3{\cdot}2\times10^{-5}}{\cos 60°} = \frac{3{\cdot}2\times10^{-5}}{1/2}$

$= 6{\cdot}4 \times 10^{-5}$ T

18. Let m be the pole strength of each pole and r be the distance between them. The force of repulsion between them is

$$F = \frac{\mu_0}{4\pi}\frac{m\times m}{r^2} \qquad ...(1)$$

When $r' = 2r$ and $m = m'$,

then as per question

$$F = \frac{\mu_0}{4\pi}\frac{m'\times m'}{(2r)^2} = \frac{\mu_0}{4\pi}\frac{m\times m}{r^2}$$

$\therefore \quad (m')^2 = 4m^2$

or $\quad m' = 2m$

19. From $F = BIl\sin\theta$

$$B = \frac{F}{Il\sin\theta} = \frac{N}{Am\times 1} = \text{NA}^{-1}\,\text{m}^{-1}$$

20. Here, $4a = 2\pi r$ or $4a/2\pi$

$$\frac{M_1}{M_2} = \frac{IA_1}{IA_2} = \frac{a^2}{\pi r^2} = \frac{a^2}{\pi\left(\frac{4a}{2\pi}\right)^2} = \frac{\pi}{4}$$

21. Torque, $\vec{\tau} = \vec{M} \times \vec{B}$

$$= (50\hat{i}) \times (0{\cdot}5\hat{i} + 3{\cdot}0\hat{j})$$

$$= 50 \times 0{\cdot}5\,(\hat{i} \times \hat{i}) + (50 \times 3{\cdot}0)\,(\hat{i} \times \hat{j})$$

$$= 25\,(0) + 150\hat{k} = \mathbf{150\hat{k}\ Nm}$$

22. Area of the circular coil, $A = \pi r^2$. The direction of $\vec{A}$ is perpendicular to the plane of area. Thus angle between $\vec{A}$ and magnetic field $\vec{B}$ is 90°, *i.e.*, $\theta = 90°$. Hence, magnitude of torque on the coil is

$$\tau = i\,AB \sin\theta$$
$$= i \times \pi r^2 \times B \times \sin 90°$$
$$= \pi r^2 i B$$

23. $V = \frac{1}{\sqrt{3}} H$

$$\tan\delta = \frac{V}{H} = \frac{V}{V\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$$

$$\therefore \quad \delta = \pi/6 = 30°$$

24. A current carrying conductor experiences a force in the magnetic field, $F = B\,I\,l$. A moving charge experiences a force in the magnetic field, $F = q\,v\,B \sin\theta$. For a stationary charge $v = 0$, then force, $F = 0$. A rectangular current loop with its plane parallel to the field, will have a magnetic moment M, perpendicular to the plane of the loop and thus experiences a couple $= M\,B \sin 90° = M\,B$. Due to it, the current loop will be affected.

25. $\tan\delta = \frac{V}{H} = \frac{H}{H} = 1 \ \therefore \delta = 45°$

26. Knowledge based question.

27. **Option (c) is incorrect** because hysteresis loop area depends on the thermal energy developed per unit volume of the substance.

28. Here, $\mu_r = 5500$; we know that

$$\mu_r = 1 + \chi_m$$

or $\chi_m = \mu_r - 1 = 5500 - 1 = \mathbf{5499}$

29. **Retentivity is maximum for soft iron** because during hysteresis loop, when magnetising field is reduced to zero, the intensity of magnetisation left in the material is maximum (related to retentivity) for soft iron as compared to other materials.

II. Assertion-Reason Type Questions

30. Here, **Assertion is true but Reason is false** as a magnetic dipole has north and south pole of equal strength a little distance apart.

31. Both **Assertion and Reason are true** and **Reason is the correct explanation of Assertion** because a freely suspended magnetic needle comes to rest with its axis along the direction of magnetic field and tangent at a point on the magnetic line of force tells the direction of magnetic field at that point.

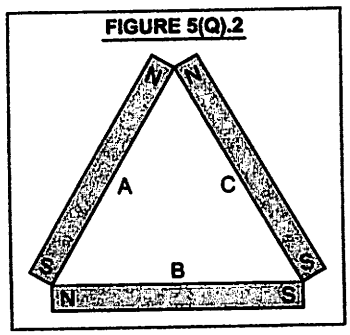
32. Here, **Assertion is true but Reason is false** as the soft iron has high retentivity but small coercivity.

33. Here, **Both Assertion and Reason are false,** as the magnetic moment of a bar magnet is equal to the product of pole strength and magnetic length of the magnet. On dividing a bar magnet into two equal parts, each part will act as a magnet of same pole strength but with a magnetic length half of the previous value. Thus the magnetic moment of each piece of bar magnet will be $M/2$.

34. **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

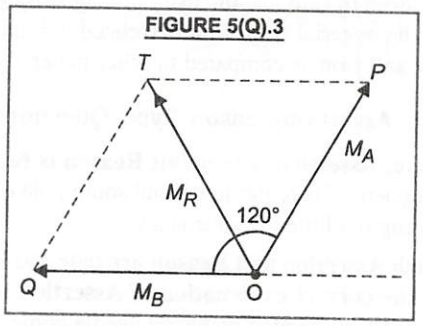
35. **Both Assertion and Reason are false** as the given structure behaves as two magnets [one as a combination of magnets A and B and other magnet C] with magnetic moment M each placed on each other.

FIGURE 5(Q).2



Refer to Fig. 5(Q).3, the magnetic moment of magnet A, $M_A = M$, represented by $\overrightarrow{OP}$. The magnetic moment of magnet B, $M_B = M$, represented by $\overrightarrow{OQ}$.

FIGURE 5(Q).3



$\angle POQ = 120°$. The resultant magnetic moment of magnets A and B is

$$M_R = \sqrt{M_A^2 + M_B^2 + 2M_A M_B \cos 120°}$$

$$= \sqrt{M^2 + M^2 + 2M \times M \times (-1/2)} = M$$

As $M_A = M_B$,

so $\vec{M}_R$ will bisect the angle POQ

i.e., $\angle POT = 60°$ which is parallel to the magnetic moment of magnet C with like poles in the same direction.

Hence total magnetic moment of the system

$$= M_R + M_C$$

$$= M + M$$

$$= 2M$$

36. Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

37. Both Assertion and Reason are false because magnetic field lines are continuous closed loops. They emerge from N-pole and enter into magnet at S-pole and inside the magnet, they go from S-pole to N-pole of magnet.

38. Both Assertion and Reason are true but Reason is not the true explanation of Assertion.

CONCEPTUAL PROBLEMS

I. Magnetic moment and force between magnetic poles

1. Why is a current loop considered a magnetic dipole ?

Sol. Like a bar magnet/magnetic dipole, a current loop possesses magnetic moment $M = NIA$. In an external magnetic field, the current loop experiences a torque and aligns its axis parallel to magnetic field.

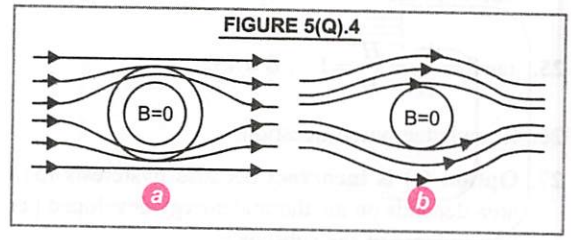
2. Does the length of an iron bar change when it is magnetised ?

Sol. Yes, on magnetisation, the molecular magnets are aligned parallel to the field. Therefore, the length of the bar in the direction of magnetisation increases. This effect is called *magnetostriction effect*. This effect is used for producing ultrasonic waves.

3. What is meant by magnetic screening or shielding ?

Sol. Magnetic screening/shielding is the phenomenon of protection of a region against any external magnetic effects. For example, when a soft iron ring is placed in a magnetic field, most of the lines are found to pass through the ring and *no lines pass through the space inside the ring*. Thus space inside the ring is shielded, Fig. 5(Q).4(a).

FIGURE 5(Q).4



Superconductors also provide perfect magnetic screening as no magnetic lines of force pass through the superconductor, Fig. 5(Q).4(b).

Magnetic screening has been used to protect costly wrist watches from external magnetic fields by enclosing them in a soft iron core.

4. Many of the Figs. 5(Q).5 show magnetic field lines wrongly (thick lines in the figs). Point out what is wrong with them. Some of them may describe electrostatic field lines correctly. Point out which ones. Remember we are talking of only static electric or magnetic field. (NCERT Solved Example)

Sol. (*a*) Wrong, because magnetic lines of force never emanate from a single point as shown. The field lines shown represent electric field of a single positive charge.

(*b*) Wrong, because magnetic field lines do not cross one another.

(*c*) Correct, magnetic lines of force are confined within a toroid. Field lines form closed loops as each loop encloses a region across which a current passes.

(*d*) Wrong, field lines cannot be completely straight. The lines should curve out at the edges and form closed loops.

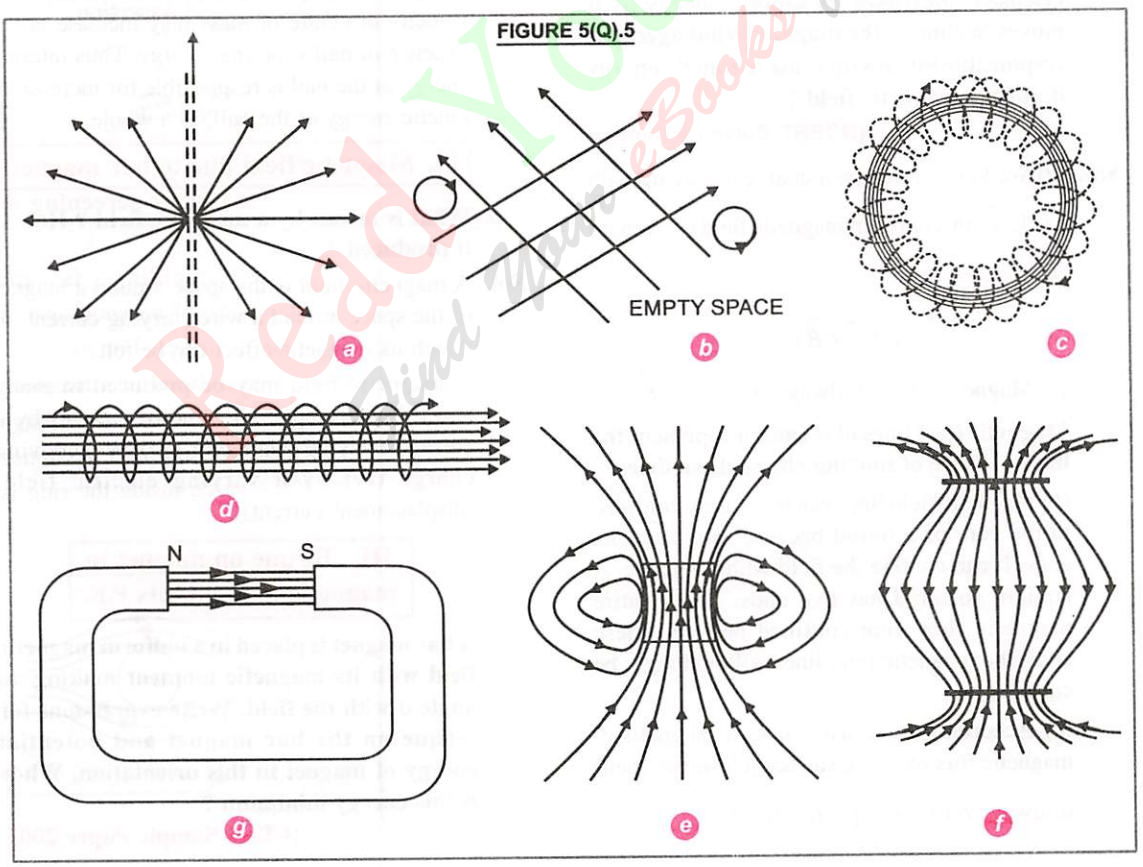
(*e*) Correct, around both, the N-pole and S-pole, the net flux of the field is zero.

(*f*) Wrong, these lines cannot represent a magnetic field because all the field lines cannot emanate from the upper shaded (plate).

The given field lines infact show the electrostatic field lines between positively charged upper plate and negatively charged lower plate.

(*g*) Wrong, magnetic field lines between two poles cannot be precisely straight at the ends. Some deviation of lines is inevitable, otherwise, Ampere's law is violated. This is true also for electric field lines.

FIGURE 5(Q).5



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5. (a) Magnetic field lines show the directions (at every point) which a small magnetised needle takes up (at that point). Do the magnetic field lines also represent the lines of force of a moving charged particle at every point ?

(b) Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why ?

(c) If magnetic monopoles existed, how would Gauss's law of magnetism be modified ?

(d) Does a bar magnet exert a torque on itself due to its own field ? Does one element of a current carrying wire exert a force on another element of the same wire ?

(e) Magnetic field arises due to charges in motion. Can a system have magnetic moment even though its net charge is zero ?

(f) Magnetic force is always normal to the velocity of a charge and therefore does no work. An iron nail held near a magnet, when released, increases its kinetic energy as it moves to cling to the magnet. What agency is responsible for this increase in kinetic energy if not the magnetic field ?

(NCERT Solved Example)

Sol. (*a*) We know, force on a charge q moving with a vel, $\vec{v}$ in a uniform magnetic field of strength $\vec{B}$ is

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$\therefore$ Magnetic force is always normal to $\vec{B}$

Magnetic field lines of B cannot represent the lines of force of moving charged particle.

(*b*) Magnetic field lines can be entirely confined to the core of a toroid because toroid has no ends. It can confine the field within its core. A straight solenoid has two ends. If the entire magnetic flux were confined between these ends, the magnetic field lines will no longer be continuous.

(*c*) According to Gauss's law in magnetism, magnetic flux over any surface (closed or open) is always zero, *i.e.*, $\oint \vec{B} \cdot \vec{ds} = 0$

If monopoles existed, the magnetic flux would no longer be zero, but equal to μ_0 times the pole strength enclosed by the surface, *i.e.*,

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 m$$

(*d*) **No,** there is no force or torque on an element due to the field produced by that element itself. But there is a force (or torque) on an element of the same wire. However, for the special case of a straight wire, this force is zero.

(*e*) **Yes,** a system can have magnetic moment even if its net charge is zero. For example, every atom of para and ferromagnetic materials has a magnetic moment, though every atom is electrically neutral. Again, a neutron has no charge, but it does have some magnetic moment.

(*f*) An iron nail is made up of a large number of atoms, in which so many electronic charges are in motion. All these charges in motion experience a magnetic force when held near a magnet. The magnetic forces do not change speed of the charges, but they do change their velocity. The velocity of centre of mass may increase at the expense of nail's internal energy. Thus internal energy of the nail is responsible for increase in kinetic energy of the nail, as a whole.

II. Magnetic field due to bar magnet

6. What is meant by a magnetic field ? How is it produced ?

Sol. A magnetic field is the space around a magnet or the space around a wire carrying current, in which its magnetic effect can be felt.

A magnetic field may be produced in many ways. For example, (*i*) by a magnet (*ii*) by a current carrying conductor (*iii*) by a moving charge (*iv*) by a varying electric field. (displacement current)

III. Torque on magnet in magnetic field and its P.E.

7. A bar magnet is placed in a uniform magnetic field with its magnetic moment making an angle θ with the field. Write expressions for torque on the bar magnet and potential energy of magnet in this orientation. When is this energy minimum ?

(CBSE Sample Paper 2003)

Sol. $\tau = MB \sin\theta$

P.E. $= -MB\cos\theta$ when $\theta = 0$,

P.E. $= -MB\cos 0° = -MB$ = minimum

Thus, P.E of magnet is minimum when its magnetic moment $\vec{M}$ is aligned parallel to the field $\vec{B}$.

8. A bar magnet of magnetic moment M is aligned parallel to the direction of a uniform magnetic field B. Calculate work done to align the magnetic moment (*i*) opposite to the field (*ii*) normal to field direction ? (CBSE 2001)

Sol. Work done in turning a dipole from orientation θ_1 to θ_2 is given by

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

(*i*) In aligning $\vec{M}$ opposite to magnetic field,

$\theta_1 = 0, \theta_2 = 180°$

$\therefore\ W = -MB(\cos 180° - \cos 0°)$

$= -MB(-1-1) = \mathbf{2\ MB}$

(*ii*) In aligning $\vec{M}$ perpendicular to magnetic field, $\theta_1 = 0°, \theta_2 = 90°$

$W = -MB(\cos 90° - \cos 0°) = +\mathbf{MB}$

9. (a) What happens if a bar magnet is cut into two pieces (i) transverse to its length (ii) along its length ? (Pb. Board 2011)

(b) What happens if an iron bar magnet is melted ? Does it retain its magnetism ?

(c) A magnetised needle in a uniform magnetic field experiences a torque but no net force. However, an iron nail near a bar magnet experiences a force of attraction in addition to a torque, explain.

(d) Must every magnetic field configuration have a north pole and a south pole ? What about the field due to a toroid ?

(e) Can you think of magnetic field configuration with three poles ?

(f) Two identical looking iron bars A and B are given, one of which is definitely known to be magnetised. How would one ascertain whether or not both are magnetised ? If only one is magnetised how does one ascertain which one? Use nothing else but the bars A and B. (NCERT Solved Example)

Sol. (*a*) In both the cases, we get two magnets each having north and south poles.

When we cut transverse to the length, pole strength of each new magnet is same as that of original magnet but magnetic moment is halved because length is halved.

When we cut along the length, pole strength of each new magnet is half the pole strength of original magnet ; magnetic moment is also halved, as length remains the same.

(*b*) On melting, iron bar magnet loses its magnetism to some extent. This is because its temperature exceeds Curie temperature ($\approx 750°C$) for iron.

(*c*) When a magnetised needle is put in a uniform magnetic field, forces on north and south poles of the needle are equal and unlike. Therefore, net force is zero. But these forces form a torque which aligns the magnetic needle in the direction of the field.

An iron nail is unmagnetised. It experiences force of attraction; gets magnetised, then experiences a torque and gets aligned along the field.

(*d*) **No,** it is not necessery that every magnetic field configuration must have a north pole and a south pole. The poles exist only when the source has some net magnetic moment. For example, in case of a toroid and infinite straight conductor carrying current, there are no poles, *as net magnetic moment in both the cases is zero.*

(*e*) Magnetic poles always exist in pairs. However, one can imagine magnetic field configuration with three poles-when north poles of two magnets are glued together or south poles of two magnets are glued together to provide a three pole field configuration.

(*f*) Place bar B horizontally on a table. Take bar A. Touch any one end of A at the middle of B. If A experiences no force then B is magnetised and A is unmagnetised.

However, if A experiences the same force at the middle of B as at the ends of B, then A is magnetised and B is not magnetised.

IV. Magnetism of Earth, Magnetic elements and Neutral Points

10. The angle of dip at a location in southern India is about 18°. Would you expect a greater or lesser dip angle in Britain ?

Sol. Britain is located close to geographic north pole of earth. Therefore, dip angle would be much greater in Britain than the angle of dip in South India.

11. How many neutral points on a horizontal board are there when a magnet is held vertically on the board ?

Sol. There will be only one neutral point on the horizontal board. This is because field of earth is from south to north ; and the field of magnetic pole on the board is radially outwards. At any point towards south of magnetic pole, field of earth and field of magnetic pole will cancel out to give a neutral point.

V. Tangent law and Tangent Galvanometer

12. A compass needle, pivoted about the horizontal axis and forced to move in the magnetic meridian is observed to point along (*i*) vertical direction at place A, (*ii*) horizontal direction at place *B*. What are the angles of dip at the two places ? (CBSE Sample Paper 2004)

Sol. At place *A*, $\delta = 90°$ and at place *B*, $\delta = 0°$

13. A compass box and a dip circle were taken to magnetic north pole of earth. What would one observe with regard to the direction of their needles and why ?

Sol. At north pole of earth, horizontal component of earth's magnetic field is zero. Therefore, compass needle may point out in any random direction. The axis of dip needle would become vertical as earth's magnetic field is along the vertical.

VI. Magnetic properties of materials

14. Mention two characteristic properties of the material suitable for making core of transformer. (CBSE 2012)

Sol. Two core material of transformer must have (*i*) high permeability (*ii*) low hysterisis loss.

15. Give two essential characteristics of a material used for preparing an electro-magnet. (CBSE 2004)

Sol. The material used for preparing electromagnets must have (*i*) high permeability (*ii*) low retentivity.

16. Why do we prefer to use the alloy alnico for making permanent magnets ? (CBSE 2004)

Sol. This is because this alloy has high retentivity and high coercivity. The only disadvantage is that alnico is brittle.

VII. Classification of magnetic materials

17. The susceptibility of a magnetic material is $1{\cdot}9 \times 10^{-5}$. What type of material does it represent ? (CBSE 2011)

Sol. As susceptibility is positive, the material must be paramagnetic.

18. The susceptibility of a magnetic material is $-4{\cdot}2 \times 10^{-6}$. What type of material does it represent ? (CBSE 2010)

Sol. As susceptibility is negative, the material must be diamagnetic.

19. Two substances A and B have relative permeabilities slightly greater and less than unity respectively. What is their magnetic nature ? (ICSE 2001)

Sol. $\mu_r = 1 + \chi$ or $\chi = \mu_r - 1$

For A, $\mu_r > 1$

$\therefore$ χ is small and positive.

It must be paramagnetic.

For B, $\mu_r < 1$

$\therefore$ χ is small and negative.

Hence, *B* must be diamagnetic.

20. How does the intensity of magnetisation of a paramagnetic material vary with increasing applied magnetic field ? (CBSE 2006)

Sol. At low field values, intensity of magnetisation increases almost linearly with field. At sufficiently high field values, intensity of magnetisation gets saturated becoming independent of field.

21. Why do magnetic lines of force prefer to pass through ferromagnetic materials ? (CBSE 2011)

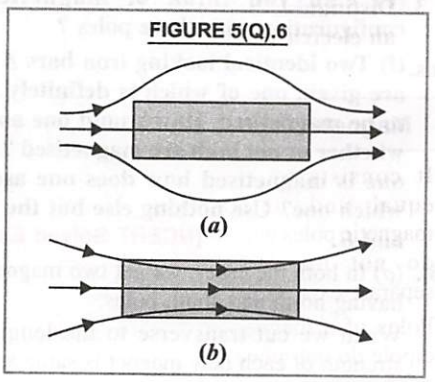
Sol. This is because permeability (μ) and susceptibility (χ_m) of such materials are very high.

22. What is the basic use of hysteresis curve ?

Sol. The basic use of hysteresis curve lies in the selection of suitable materials for different purposes like core of transformer/generator, electromagnets, permanent magnets etc. The choice is made on the basis of properties like retentivity, coercivity, energy loss etc. which are revealed by the hysteresis loop.

23. A uniform magnetic field gets modified as shown in Fig. 5(Q).6, when two specimens are

FIGURE 5(Q).6



placed in it. (*i*) Identify the specimen *A* and *B* (*ii*) How is the magnetic susceptibility of specimen *A* different from that of specimen *B*. (CBSE Sample Paper 2022-23)

Sol. (*i*) In Fig. 5(Q).6(*a*), the specimen is **diamagnetic** as magnetic lines of force prefer not to pass through it.

In Fig. 5(Q).6(*b*), the specimen is **paramagnetic** as magnetic lines of force prefer its pass through it.

(*ii*) Magnetic susceptibility of (*a*) is small negative and that of (*b*) is small positive.

24. An iron bar magnet is heated to 1000°C and then cooled in a magnetic field free space. Will it retain magnetism ? (Raj. Board 2012)

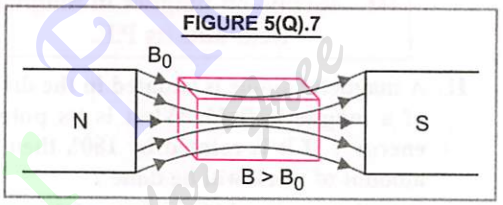
Sol. No, it will not retain magnetism. Curie temperature for iron is about 750°*C*. At this temperature, it will become paramagnetic and lose its magnetism on heating further. Hence it will not retain magnetism on cooling in a magnetic field free space.

25. The susceptibility of a magnetic material is 0·9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field. (CBSE 2018)

Sol. Here, $\chi_m = 0{\cdot}9853$. As magnetic susceptibility of the material is positive, though small, the magnetic material must be *paramagnetic*.

When a piece of this material is placed in a uniform magnetic field, the magnetic field lines prefer to pass through the specimen (as shown in Fig. 5(Q).7 rather than through air. The magnetic induction *B* inside the sample is more than the magnetic induction B_0 outside the sample *i.e.*, $B > B_0$. However, this enhancement is small, being one part in 10^5.

FIGURE 5(Q).7



QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Magnetic moment and force between magnetic poles

1. What are the SI units of pole strength and magnetic moment ? (CBSE 2003)

Ans. SI unit of pole strength is ampere-metre and SI unit of magnetic moment is ampere metre^2.

2. Write the formula for magnetic moment of a current loop. (CBSE 2002)

Ans. $M = NIA$, when N is number of turns I, the current and A = area of cross-section of loop.

3. Distinguish between a magnetic dipole and an electric dipole.

Ans.

Magnetic Dipole	Electric Dipole
1. It consists of two equal and unlike magnetic poles which do not have any separate existence.	1. It consists of two equal and opposite charges which have a separate existence.
2. Poles of a magnetic dipole do not move.	2. Charges of an electric dipole can move.

4. Magnetic lines of force are endless. Comment.

Ans. True. This is because magnetic lines of force are closed continuous loops. These lines exist even inside the magnet.

5. What is the significance of Gauss's Law in magnetism ?

Ans. Magnetic poles exist in unlike pairs of equal strengths.

6. Give two examples of magnetic dipole.

Ans. Every atom of para and ferro magnetic substances; a loop of current are magnetic dipoles.

7. On what factors does the pole strength of a magnet depend ?

Ans. The pole strength of a magnet may depend on its cross section ; nature and state of magnetisation.

8. Give one important point of distinction between magnetism and electricity.

Ans. Whereas in electricity, an isolated charge exists ; in magnetism, an isolated magnetic pole does not exist.

II. Magnetic field due to bar magnet

9. Write an expression for magnetic field intensity at any point an axial line of a bar magnet.

Ans. $B_1 = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2}$; along SN

where symbols have usual meaning.

10. On equatorial line of bar magnet, what is the formula for magnetic field strength ?

Ans. $B_2 = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}}$, parallel to NS.

III. Torque on magnet in magnetic field and its P.E.

11. A magnetic dipole is situated in the direcion of a magnetic field. What is its potential energy ? If it is rotated by 180°, then what amount of work will be done ?

Ans. P.E. of dipole $= -MB \cos 0° = -MB$

Work done $= MB (\cos 0° - \cos 180°)$

$= MB (1 + 1)$

$= 2MB$.

12. What is the strength of earth's magnetic field at the surface of earth ?

Ans. It is of the order of 10^{-5} tesla.

13. What is the torque acting on a magnet of magnetic moment M held at an angle θ with the direction of magnetic field B ?

Ans. $\tau = MB \sin \theta$

14. What does the torque do ?

Ans. The torque tries to align the magnet along the direction of magnetic field ($\theta \rightarrow$ Zero).

IV. Magnetism of Earth, Magnetic elements and Neutral Points

15. Where is the vertical component of earth's magnetic field zero ?

Ans. At earth's magnetic equator.

16. A magnetic needle placed on a piece of cork is floating on the calm surface of a lake in the northern hemisphere. Would the magnetic needle alongwith the cork move towards north ?

Ans. No, the magnetic needle will come to rest along N–S direction.

17. What is the approximate distance upto which earth's magnetic field extends ?

Ans. The magnetic field of earth extends to nearly five times the radius of the earth *i.e.*

$5 \times (6{\cdot}4 \times 10^3)$ km $= 3{\cdot}2 \times 10^4$ km.

18. What are isogonic, isoclinic and isodynamic lines ?

Ans. Isogonic lines are the lines joining places of equal declination.

Isoclinic lines are the lines joining places of equal dip/inclination.

Isodynamic lines are the lines joining places of equal horizontal component of earth's field.

19. What is the maximum value of angle of dip ? At what place does it occur ?

Ans. 90°, at earth's magnetic poles.

20. What is the angle of dip at a place where horizontal and vertical components of earth's field are equal ?

Ans. 45°. This is because $\tan \delta = \frac{V}{H} = 1$; $\delta = 45°$.

21. The angles of dip at two places are respectively 0° and 90°. Where are these values on earth ? (CBSE 2011)

Ans. At magnetic equator, angle of dip = 0° and at earth's magnetic poles, angle of dip = 90°.

22. In the northern hemisphere, do magnetic lines of force due to earth's field point towards or away from earth ? (Hr. Board 2002)

Ans. Towards the earth because the artifical magnet inside the earth has south pole on that side.

23. Name the elements or parameters of earth's magnetic field. (Hr. Board 2002)

Ans. The three parameters of earth's magnetic field are : Declination, Dip and Horizontal component of earth's magnetic field.

V. Tangent law and Tangent Galvanometer

24. What is a reduction factor of tangent galvanometer and its unit of measurement ?

Ans. Reduction factor of a tangent galvanometer is equal to current, required to be passed through galvanometer to produce a deflection of 45°. Reduction factor is measured in ampere.

25. Before using tangent galvanometer for the measurement of current, why is the plane of coil of tangent galvanometer set in the magnetic meridian ?

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Ans. When plane of the coil of tangent galvanometer is set in the magnetic meridian and current is passed through the coil, then the magnetic field produced due to current in coil is perpendicular to the plane of coil in East-West direction. The horizontal component of earth is in South-North direction. Then the magnetic needle of tangent galvanometer will obey tangent law, and current (I) can be measured by noting the deflection θ.

VI. Magnetic properties of materials

26. What is the order of magnetic moment of an atom ?

Ans. Magnetic moment of an atom is of the order of 10^{-23} Am^2.

27. What are the SI units of magnetising force or magnetising intensity ?

Ans. SI units of magnetising force or magnetising intensity are Ampere/metre.

28. Name the cgs unit of magnetising intensity. How is it related to SI unit of intensity.

Ans. The cgs unit of magnetising intensity H is *oersted.*

1 oersted = 80 Am^{-1}

29. What is relative magnetic permeability of superconductors ?

Ans. For superconductors, $\mu_r = 0$.

30. What is magnetic susceptibility of super conductors.

Ans. As $\mu_r = 1 + \chi_m = 0$, for superconductors,

$\therefore \chi_m = -1$

31. What is Meissner effect ?

Ans. It is the phenomenon of perfect diamagnetism of superconductors.

32. What are units of magnetic permeability ?

Ans. Tesla metre/ampere. (Tm A^{-1})

33. How does the magnetic susceptibility of a paramagnetic material change with temperature ?

Ans. The magnetic suscetibility of a paramagnetic material varies inversely with temperature, *i.e.*, $\chi_m \propto 1/T$.

34. What are SI units of magnetic susceptibility ?

Ans. $\chi_m = \dfrac{I}{H} = \dfrac{A\,m^{-1}}{A\,m^{-1}} = 1$ $\therefore$ χ_m has **no units.**

VII. Classification of magnetic materials

35. Which of the following substances are diamagnetic ? (CBSE 2013)

Copper, Aluminium, Sodium, Bismuth

Ans. Copper and Bismuth are diamagnetic.

36. Which of the following substances are paramagnetic ? [CBSE 2013 (C)]

Al, Bi, Cu, Ca, Pb, Ni.

Ans. Aluminium and calcium are paramagnetic.

37. The permeability of Bismuth is 0·9983. To which class of magnetic material Bi belongs. (CBSE 2011)

Ans. Bismuth is diamagnetic as its permeability is less than one.

38. What is the basic difference between the atom/molecule of a diamagnetic and paramagnetic material ? (CBSE Sample Paper 2008)

Ans. The atoms/molecules of a diamagnetic substance do not possess any net magnetic moment on their own. In paramagnetic substances, each individual atom/molecule has non zero magnetic moment of its own.

39. Why are electromagnets made of soft iron ? (Kerala Board 2012)

Ans. This is because coercivity of soft iron is small.

40. Magnetisation and demagnetisation of soft iron is easier/more difficult as compared to steel. Why ?

Ans. Magnetisation and demagnetisation of soft iron is easier compared to steel, because coercivity, of soft iron is smaller than the coercivity of steel.

41. Classify the following into dia and para magnetic substances : aluminium, copper, water, mercury, oxygen, hydrogen.

Ans. *Dia* : copper, water, mercury, hydrogen.

Para : aluminium, oxygen.

42. Write the names of three ferromagnetic substances. (Kerala Board 2012)

Ans. Iron, cobalt and nickel.

43. Magnetic moment of atoms of certain materials is zero. Name such materials.

Ans. Diamagnetic materials, like copper, bismuth etc.

44. Which materials have relative magnetic permeability > 1 ?

Ans. Para and ferromagnetic materials.

45. Can there be a material, which is non magnetic ?

Ans. No, every substance is atleast diamagnetic.

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SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Magnetic moment and force between magnetic poles

1. How is magnetic force between two poles affected when strength of each pole is doubled and distance between them is halved ?

Ans. Force becomes 16 times. This is because

$$F' = \frac{\mu_0}{4\pi}\frac{m_1' m_2'}{r'^2} = \frac{\mu_0}{4\pi}\frac{(2m_1)(2m_2)}{(r/2)^2}$$

$$= 16\ F$$

2. No two magnetic lines of force can intersect. Why ?

Ans. This is because at the point of intersection, there will be two tangents to the two lines of force, which would mean two directions of magnetic field intensity at that point. This is not possible.

3. Can ever there be a magnet (*a*) with no pole (*b*) with two similar poles (*c*) with three poles ?

Ans. (*a*) Yes, there can be a magnet with no pole *e.g.* in case of a toroid carrying current.

(*b*) Yes, there can be a magnet with two similar poles when like poles of two magnets are glued together.

(*c*) Yes, there can be a magnet with three poles, again when like poles of two magnets are glued together.

4. Define unit pole from Coulomb's law of magnetism.

Ans. From Coulomb's law of magnetism,

$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$$

If $m_1 = m_2 = m$; $r = 1$ m and $F = 10^{-7}$ N.

Then $10^{-7} = 10^{-7}\dfrac{m \times m}{1^2}$

or $m^2 = 1$ or $m = \pm 1$

Hence, unit magnetic pole is that pole which when placed in free space at a distance of 1 m from a like pole of same strength would repel it with a force of 10^{-7} N.

II. Magnetic field due to bar magnet

5. For a short magnetic dipole, intensity at any point an axial line is same as intensity at same distance on equatorial line. Is it true ?

Ans. No, the statement is not true. Infact, at a given distance from the centre of magnet, intensity on axial line is twice the intensity on equatorial line.

III. Torque on magnet in magnetic field and its P.E.

6. Under what situation a magnet suspended in a uniform magnetic field will be (*a*) in stable equilibrium and (*b*) in unstable equilibrium ?

Ans. When axis of the magnet of magnetic moment M makes an angle θ with the direction of magnetic field B, then torque on magnet is

$$\tau = MB \sin\theta$$

and its PE, $U = -MB\cos\theta$.

(*a*) The magnet will be in stable equilibrium if $\tau = 0$ and U is minimum. It will be so if the magnetic moment of magnet acts in the direction of magnetic field, *i.e.*, $\theta = 0°$.

(*b*) The magnet will be in unstable equilibrium if $\tau = 0$ and U is maximum. It will be so if the magnetic moment of magnet acts in opposite direction to that of magnetic field.

7. When will a magnet in an external magnetic field be in unstable equilibrium ?

Ans. The magnet is in unstable equilibrium, when $\vec{M}$ and $\vec{B}$ are directed opposite to each other *i.e.*, when $\theta = 180°$. Torque $= MB \sin 180° =$ zero and P.E. $= -MB\cos 180° = MB =$ maximum.

IV. Magnetism of Earth, Magnetic elements and Neutral Points

8. What is the proof of earth's magnetism ?

Ans. It is based on the following evidences :

(*i*) A magnet suspended from a thread and free to rotate in a horizontal plane comes to rest along the north-south direction.

(*ii*) When a soft iron piece is buried under the earth in the north-south direction, it is found to acquire the properties of a magnet after some time.

(*iii*) When we plot magnetic field lines of a magnet, neutral points are obtained.

9. Is there a strong' magnet inside the earth responsible for earth's magnetism ? If there is a magnet, what is its inclination w.r.t. to north-south direction ?

Ans. Actually there is no permanent strong magnet inside the earth responsible for earth's magnetism. However to explain the earth's magnetism, Dr. Gilbert considered a fictitious strong magnet inside the earth, whose magnetic axis is inclined roughly 11·3° west of axis of

rotation of earth, which is along geographic north and geographic south.

10. At what positions, the neutral points will lie for a bar magnet when magnetic axis of magnet is lying in the magnetic meridian (*i*) with N-pole of magnet pointing North (*ii*) with S-pole of magnet pointing North ?

Ans. (*i*) The neutral points will lie on equatorial line of bar magnet of magnetic moment M and magnetic length $2\,l$, where

$$H = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}$$

where d is the perpendicular distance of neutral point from the centre of magnet.

(*ii*) The neutral points will be on axial line of bar magnet, where

$$H = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2}$$

11. How does a magnetic compass behave at a neutral point ?

Ans. At a neutral point, net magnetic field due to magnet and earth is zero. Therefore, a compass needle may point out in any random direction.

V. Tangent law and Tangent Galvanometer

12. Write mathematical form of tangent law in magnetism.

Ans. $F = H \tan\theta$, where F and H are strengths of two magnetic fields acting perpendicular to each other and θ is the angle which freely suspended magnet makes with the direction of H.

13. To work with a tangent galvanometer, how do you set its coil ?

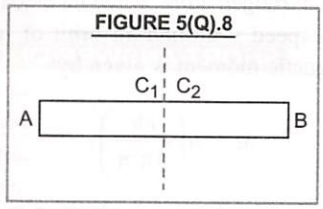
Ans. The coil of tangent galvanometer is set in magnetic meridian by making it parallel to the magnetic needle in the magnetometer box.

VI. Magnetic properties of materials

14. A bar magnet AB is cut into two equal parts one part is kept over the other so that pole C_2 is above C_1. If M is magnetic moment of original magnet, what will be the magnetic moment of the combination so formed ?
(CBSE Sample Paper 2008)

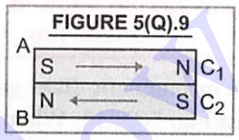
Ans. As shown in Fig. 5(Q).8, each part has magnetic moment $M/2$. These are aligned in opposite directions, Fig. 5(Q).9.

FIGURE 5(Q).8



$\therefore$ Net magnetic moment of combination
$= M/2 - M/2$
$= 0.$

FIGURE 5(Q).9



15. Two circular loops of radii r and $2r$ have currents I and $I/2$ flowing through them in clockwise and anticlockwise sense respectively. If their equivalent magnetic moments are M_1 and M_2, what is the relation between M_1 and M_2 ? (CBSE Sample Paper 2011)

Ans. $M_1 = I(\pi r^2)$

$$M_2 = \frac{I}{2}(\pi 4r^2) = I(2\pi r^2)$$

$$\therefore \frac{M_1}{M_2} = \frac{1}{2}$$

The direction of $\vec{M}_1$ is opposite to the direction of $\vec{M}_2$.

16. State two methods to destroy the magnetism of a magnet.

Ans. (*i*) By heating the magnet.

(*ii*) By applying magnetic field in the reverse direction.

17. Define the term : magnetic dipole moment of a current loop. Write the expression for the magnetic moment when an electron revolves at a speed v around an orbit of radius r in hydrogen atom. (CBSE 2008)

Ans. A current carrying loop behaves as a system of two equal and opposite magnetic poles separated by a distance. Hence it behaves as a magnetic dipole. Magnetic dipole moment of current loop is the product of current I and area A enclosed by the loop of current, *i.e.*

$$M = IA = \frac{e}{T}\times\pi r^2 = \frac{e}{2\pi r/v}\times\pi r^2 = \frac{evr}{2}.$$

In a hyrdogen atom, when an electron revolves at a speed v around an orbit of radius r, the magnetic moment is given by

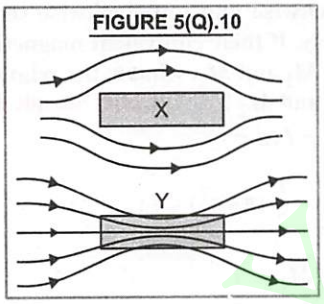
$$M = n\left(\frac{eh}{4\pi m}\right)$$

where e is charge on electron, m is mass of electron ; n denotes the number of orbit and h is Plack's constant.

VII. Classification of magnetic materials

18. A uniform magnetic field gets modified as shown in Fig. 5(Q).10. When two specimens X and Y are placed in it. Identify the specimens X and Y. (CBSE 2013)

FIGURE 5(Q).10



Ans. X is diamagnetic as it expels magnetic lines

Y is paramagnetic or ferromagnetic as it pulls the magnetic lines into it.

19. How are wrist watches protected from powerful magnets ?

Ans. The machine parts of a wrist watch are surrounded by an iron ring, which acts as a screen against external magnetic fields.

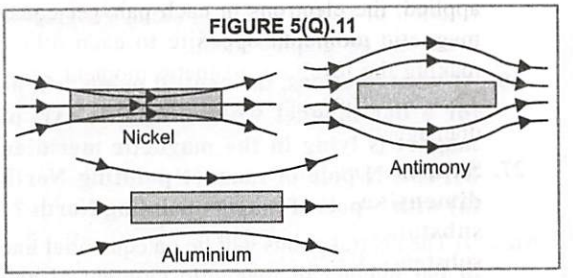
20. Why does the magnetisation of a paramagnetic salt increase on cooling ?

Ans. At lower temperatures, tendency to disrupt the alignment of dipoles with applied magnetic field arising from random thermal motion is reduced.

21. Three identical specimens of magnetic materials nickel, antimony and aluminium are kept in a non-uniform magnetic field. Draw the modifications in the field lines in each case. Justify your answer. (CBSE 2010)

Ans. The modifications in field lines are as shown in Fig. 5(Q).11. This is because, Nickel is ferromagnetic, Antimony is diamagnetic and Aluminium is paramagnetic.

FIGURE 5(Q).11



22. What is the susceptbility and permeability of a perfectly diamagnetic substance ? (Kerala Board 2012)

Ans. For a perfectly diamagnetic substance,

$$B = \mu_0 (H + I) = 0 \quad \therefore \quad I = -H.$$

Therefore, $\chi_m = \dfrac{I}{H} = -1$.

Also, $\mu_r = 1 + \chi_m = 1 - 1 = 0$

$\therefore \quad \mu = \mu_0 \mu_r =$ Zero.

23. Comment on the state of magnetisation of a substance whose atoms contain odd number of electrons.

Ans. An atom which contains odd number of electrons will have atleast one electron left unpaired. So it has some permanent magnetic dipole moment. Hence the atom must have a paramagnetic character.

24. Which material is used to make electro-magnets and why ? (CBSE 2017, 2010)

Ans. Soft iron is used to make electromagnets, because hysteresis loop for soft iron is narrow. Therefore, energy loss/volume/cycle is small.

25. Which material is used in making permanent magnets and why ? (CBSE 2017, 2010)

Ans. For making permanent magnets, a material having high coercivity is used. Steel is a better choice compared to soft iron.

26. What is the basic difference between the atom/molecule of a diamagnetic and a paramagnetic material ? Why are elements with even atomic number more likely to be diamagnetic ?

Ans. Atoms/molecules of diamagnetic substances have electrons that can be grouped in pairs with spins in opposite sense. In paramagnetic materials, atoms/molecules have excess electrons which spin in the same sense.

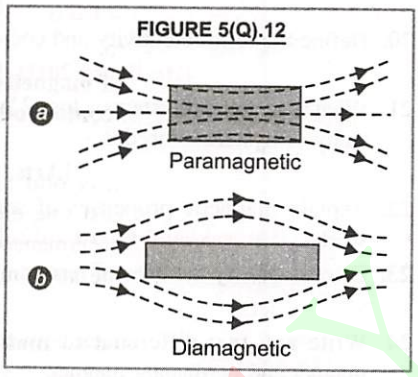
Elements with even atomic number have electrons which can be grouped into pairs with spins in opposite sense. When external field is

applied, the electrons in each pair get equal magnetic moments opposite to each other, making the resultant magnetic moment zero. Therefore, such elements are more likely to be diamagnetic.

27. Suppose you have two bars of identical dimensions, one made of paramagnetic substance and the other of diamagnetic substance. If you place these bars along a uniform magnetic field, show diagramatically what modification in the field pattern would take place in each case. [CBSE 2017 (C)]

Ans. Inside a paramagnetic bar, field concentrates slightly in the bar, Fig. 5(Q).12(*a*). Inside a diamagnetic bar, magnetic field lines avoid to enter. This is shown in Fig. 5(Q).12(*b*).

FIGURE 5(Q).12



28. Out of the two magnetic materials. 'A' has relative permeability slightly greater than unity while 'B' has less than unity. Identify the nature of materials 'A' and 'B'. Will their susceptibilities be positive or negative ? (CBSE 2014)

Ans. For material *A*, $\mu_r > 1$. Therefoe, '*A*' must be paramagnetic.

For material *B*, $\mu_r < 1$. Therefore, '*B*' must be diamagnetic.

As $\chi_m = \mu_r - 1$

$\therefore$ Susceptibility of '*A*' is positive and susceptibility of '*B*' is negative.

29. (*a*) Define the term magnetic susceptibility and write its relation in terms of relative magnetic permeability.

(*b*) Two magnetic materials *A* and *B* have relative magnetic permeabilities of 0·96 and 500. Identify the magnetic materials *A* and *B*.

[CBSE 2018 (C)]

Ans. (*a*) Magnetic susceptibility is a property which determines how easily a specimen can be magnetised, when placed in the magnetising field.

Magnetic susceptibility of a material is defined as the ratio of intensity of magnetisation (*I*) induced in the material to the magnetising force (*H*) applied on it. It is represented by χ_m.

i.e., $\chi_m = \dfrac{I}{H}$

We can show that relative magnetic permeability (μ_r) is related to (χ_m) by the relation

$$\mu_r = 1 + \chi_m$$

(*b*) For magnetic material *A*, $\mu_r = 0{\cdot}96 < 1$

$\therefore$ A must be diamagnetic in nature.

For magnetic material *B*, $\mu_r = 500$ which is high. Therefore, *B* must be a ferromagnetic material.

SHORT ANSWER QUESTIONS

Carrying 3 marks

1. Discuss the basic properties of a magnet. (Pb. Board 2011) [Art. 5.3]
2. What are magnetic lines of force ? Give their important properties. [Arts. 5.4 & 5]
3. What is the magnetic dipole moment of a current loop ? In what direction does it act ? (West Bengal Board 2012, Karnataka Board 2012) [Art. 5.6]
4. What is magnetic dipole moment ? Calculate the magnetic dipole moment of a revolving electron. (Jharkhand Board 2011) [Arts. 5.6 & 5.8]
5. Draw the magnetic field lines due to a circular loop of area $\vec{A}$ carrying current *I*. Show that it acts as a bar magnet of magnetic moment $\vec{M} = I\vec{A}$ (CBSE 2015) [Art. 5.8]
6. Derive an expression for the magnetic field due to a solenoid of length 2 *l*, radius *a*, having *n* number of turns per unit length and carrying a steady current *I*, at a point on the axial line, distant *r* from the centre of the solenoid. How does this expression compare with the axial magnetic field due to a bar magnet of magnetic moment *M* ? (CBSE 2015) [Art. 5. 9]

7. In a current carrying solenoid, current is flowing clockwise from left end. Draw the magnetic field produced in it. (Raj. Board 2011) [Art. 5.9]
8. Define strength of magnetic field. Obtain an expression for strength of magnetic field on the axis of a bar magnet. (Chhatisgarh Board 2011) [Art. 5.7]
9. Derive an expression for torque acting on a bar magnet held at an angle with the direction of a uniform magnetic field. (AP Board 2012, Jharkhand Board 2012) [Art. 5.10]
10. Calculate potential energy of a magnetic dipole in a magnetic field. (Jharkhand Board 2012) [Art. 5.12]
11. Find expression for work done in rotating a bar magnet in a uniform magnetic field. (Chhatisgarh Board 2012) [Arts. 5.12]
12. What are magnetic elements at a place ? Define them. (Uttarakhand Board 2012) [Art. 5.18]
13. (*a*) State Gauss's law for magnetism. Explain its significance.

    (*b*) Write the four important properties of the magnetic field lines due to a bar magnet. (CBSE 2019) [Art. 5.14, 5.5]
14. State tangent law in magnetism. How will the reduction factor of a tangent galvanometer change when the number of turns of its coil is doubled ? Explain. (AP Board 2012, Chhatisgarh Board 2012) [Arts. 5.21 & 5.22]
15. Deduce an expression for the magnetic dipole moment of an electron orbiting around the central nucleus. (Jharkhand Board 2011, Raj. Board 2011, CBSE 2010, 2009) [Art. 5.23]
16. Explain magnetic permeability, magnetising force and magnetic field induction. State the relation between them. [Refer to Art. 5.24]
17. Write three points of differences between para-, dia- and ferro-magnetic materials, giving one example of each. (CBSE 2019) [Art. 5.26]
18. Explain paramagnetism on the basis of electron theory. (AP Board 2012) [Art. 5.26]
19. State and explain Curie law in magnetism. [Art. 5.27]
20. Define the term retentivity and coercivity. (Hr. Board 2011) [Art. 5.28]
21. What is meant by hysteresis loss ? Where is the study of hysteresis used ? [Arts. 5.28, 5.30]
22. Explain magnetic properties of soft iron and steel. (Uttarakhand Board 2012) [Art. 5.29]
23. Discuss briefly the uses of ferromagnetics. [Art. 5.30]
24. Write any two differences between electro-magnet and permanant magnet. (Bihar Board 2012) [Art. 5.30]
25. Mention two characteristic properties of the material suitable for making core of a transformer. (CBSE 2012) [Art. 5.30]

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Depict the field line pattern due to a current carrying solenoid of finite length,

   (*i*) In what way do these lines differ from those due to an electric dipole.

   (*ii*) Why can't two magnetic field lines intersect each other ? (CBSE 2011) [Art. 5.4]
2. What are the magnetic elements of earth ? Explain them briefly. (Raj. Board 2012) [Art. 5.18]
3. Explain what is meant by neutral points ? How will you calculate magnetic moment of a bar magnet by locating the neutral points ? [Art. 5.20]
4. State and prove tangent law in magnetism. Give two factors by which tangent galvanometer can be made more sensitive. [Arts. 5.21 & 22]
5. Describe the principle, construction, theory and working of a tangent galvanometer. [Art. 5.22]
6. Define magnetising field, magnetic induction, permeability, intensity of magnetisation and susceptibility. Establish relation between permeability and susceptibility. [Arts. 5.24 & 25]
7. Distinguish between the magnetic properties of dia, para and ferromagnetic substances in terms of

(*i*) susceptibility

(*ii*) magnetic permeability and

(*iii*) Coercivity. Give one example of each of these materials.

Draw the field lines due to an external magnetic field near a

(*i*) diamagnetic

(*ii*) paramagnetic substance.

(CBSE 2007) **[Art. 5.26]**

8. What are dia, para and ferro magnetic materials ? Discuss their important properties.

(HP Board 2011) **[Art. 5.26]**

9. Discuss briefly electron theory of magnetism. How does it account for the three kinds of behaviour of magnetic substances ?

[Art. 5.26]

10. What is meant by hysteresis ? Discuss briefly the dissipation of energy due to hysteresis. Draw hysteresis curves of soft iron and steel.

(Raj. Board 2011) **[Arts. 5.28 & 29]**

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. According to **molecular Theory,** every molecule of a magnetic substance (whether magnetised or not) is a complete magnet in itself. In an unmagnetised substance, molecular magnets are randomly oriented such that they form closed chains. That is why the resultant magnetism of the unmagnetised specimen is zero. On magnetising the specimen, the molecular magnets are realigned in the direction of the field.

Magnetic field lines are hypothetical lines, which enable us to understand certain phenomena in magnetism. Tangent to field line at a point gives us the direction of magnetic field $\vec{B}$ at that point. No two magnetic field lines intersect eachother.

A **magnetic dipole** consists of two unlike poles of equal strength(m) separated by a small distance ($2\vec{l}$). It has a magnetic dipole moment $\vec{M} = m(2\vec{l})$ directed from **south pole to north pole** of dipole through dipole.

S.I. units of magnetic moment $\vec{M}$ are ampere meter^2 or joule/tesla.

When a bar magnet of magnetic moment M is held at $\angle\theta$ with the direction of uniform magnetic field $\vec{B}$, a torque $\tau = MB \sin\theta$ acts on the magnet. It tries to align the dipole along the field. Work (W) is required to be done in rotating the dipole against the action of the torque. This work is stored as **potential energy** of the depole.

$$\text{P.E.} = W = -MB(\cos\theta_2 - \cos\theta_1)$$

Usually, $\theta_1 = 90°$, which is the position of zero potential energy.

A loop of current acts as a dipole of magnetic moment $\vec{M} = IA\hat{n}$, where I is the strength of current through the loop, A is area enclosed by the loop, $\hat{n}$ is unit vector along $\vec{M}$.

Based on the above paragraph, answer questions no. 1 to 4 :

1. What is the magnitude and direction of magnetic dipole moment of current loop ?

2. Which face of current loop behaves as a S-pole of magnetic dipole ?

3. When a bar magnet of magnetic moment M is placed along the direction of a uniform magnetic field B, then what are the total force and torque on the bar magnet ?

4. What is the work done in rotating the magnetic dipole from equilibrium position to 180° in a uniform magnetic field ?

CASE 2. According to **Gauss's theorem** in magnetism, surface integral of magnetic field over a surface, closed or open is always zero, *i.e.*,

$$\oint_S \vec{B}\cdot d\vec{s} = 0$$

This theorem establishes that magnetic poles always exist in equal and unlike pairs.

Magnetism of earth is called **terrestrial magnetism.** It is described in terms of three quantities, called **magnetic elements** of earth.

(*i*) Magnetic declination (θ) at a place is the angle between magnetic axis (or magnetic meridian) and geographic axis (or geographic meridian).

(*ii*) Magnetic inclination or **dip (δ)** at a place is the angle, which direction of total earth's magnetic field makes with a horizontal line in magnetic meridian.

(*iii*) Horizontal Component of earth's magnetic field (*H*). It is the component of earth's total magnetic

field (R) in horizontal direction in magnetic meridian. clearly,

$$H = R\cos\delta,\ V = R\sin\delta$$

$$\sqrt{H^2+V^2} = R \quad \text{and} \quad \frac{V}{H} = \tan\delta$$

At magnetic equator, $\delta = 0°$

and at magnetic poles, $\delta = 90°$.

Magnetic field strength (B_1) due to a bar magnet at any point on axial line of magnet.

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2};\ \text{along SN produced.}$$

Magnetic field strength (B_2) due to a bar magnet at any point on equatorial line of bar magnet.

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}$$

The direction of $\vec{B_2}$ is along a line parallel to NS.

Based on the above paragraph, answer questions no. 5 to 8 :

5. With usual notation, $\oint \vec{B}\cdot d\vec{s} = \text{zero}$. What does it represent ?
6. What are magnetic elements of earth?
7. What are the values of magnetic inclination at poles and at equator ?
8. What is the ratio of magnetic field strength at any point on axial line and on equatorial line of a short bar magnet ?

CASE 3. Neutral Points are the points where the resultant magnetic field due to field of the bar magnet and the field of earth is zero.

When a magnet is placed with its north pole towards geographic south, the neutral points lie on axial line of the magnet. At each neutral point,

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2} = H$$

Here, H is horizontal component of earth's magnetic field.

When a bar magnet is placed with its north pole towards geographic north, neutral points lie on equatorial line of the magnet. At each neutral point

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = H$$

Based on the above paragraph, answer questions no. 9 to 12 :

9. What are neutral points ?
10. To obtain neutral points on axial line of a bar magnet, how will you place the magnet ?
11. On axial line or equatorial line of a bar magnet, how does the field due to magnet vary ?
12. What are the directions of magnetic fields due to bar magnet and Earth at neutral points when
    (*i*) point is on the axial line of bar magnet
    (*ii*) point is on the equatorial line of bar magnet.

ANSWERS

1. Magnitude of magnetic dipole moment of current loop, $M = nIA$, where n is the number of turns in the current loop, I is the current in loop and A is the area of each turn of the loop.

   Direction of $\vec{M}$ is from S-pole to N-pole through the dipole.
2. That face of current loop will show south polarity where the direction of current in the loop is **clockwise**.
3. Let m be the pole strength of each pole and M be the magnetic moment of the bar magnet.

   Total force on the bar magnet in magnetic field.

   $$= mB - mB = 0$$

   Torque on the bar magnet in the magnetic field

   $$= MB\sin 0° = 0.$$
4. Workdone, $W = -MB(\cos\theta_2 - \cos\theta_1)$

   Here, $\theta_1 = 0°$

   and $\theta_2 = 180°$, so

   $$W = MB[\cos 180° - \cos 0°]$$
   $$= -MB[(-1) - 1]$$
   $$= 2MB$$
5. It represents Gauss's theorem in magnetism.
6. Three magnetic elements of earth are
   (*i*) Magnetic declination (θ).
   (*ii*) Magnetic inclination or magnetic dip (δ).
   (*iii*) Horizontal component of earth's magnetic field (H).
7. At magnetic poles, $\delta = 90°$ and at magnetic equator, $\delta = 0°$.

8. For a short bar magnet $d >> l$, so magnetic field strength due to short bar magnet at any point on the axial line of short bar magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2} = \frac{\mu_0}{4\pi}\frac{2Md}{d^4} = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

Magnetic field strength at any point on equatorial line of short bar magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M}{(d^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

$$\therefore \quad \frac{B_1}{B_2} = 2$$

9. Neutral points are the positions where resultant of magnetic field due to the bar magnet and due to earth cancel out so that net magnetic field is zero.

10. To obtain neutral points on axial line of a bar magnet, the magnet must be placed with its north pole towards geographic south.

11. On axial line or equatorial line of a short bar magnet, magnetic field due to the magnet varies as $1/d^3$ where d is distance of the point from the centre of the magnet.

12. (*i*) When neutral points are on the axial line of a bar magnet, then the north pole of bar magnet is towards geographic south, then at neutral points, the direction of magnetic field due to bar magnet is north to south direction and the direction of earth's magnetic field is from south to north direction.

(*ii*) When neutral points are on the equatorial line of bar magnet, then the north pole of magnet is towards geographic north, then at neutral points, the direction of magnetic field due to bar magnet is north to south direction and the direction of earth's magnetic field is from south to north direction.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. As electrons in an atom revolve around the neucleus, therefore, every atom behaves as **magnetic diple.** Atomic dipole moment is $M = \frac{n(eh)}{4\pi m} = n(\mu_B)$

where $n = 1, 2, 3.....$ denotes the number of orbit

and $\mu_B = \frac{eh}{4\pi m}$ = least value of dipole moment of atom = $9{\cdot}27 \times 10^{-24}$ Am^2 = **Bohr magneton.**

Some important terms used in magnetism are :

(*i*) **Magnetic field strength** or magnetic field induction or magnetic flux density (B). It is measured in **tesla** or **gauss**, where 1 tesla = 10^4 gauss.

(*ii*) **Magnetising force** or **Magnetising intensity** (H) = $n\,I$. It is measured in Am^{-1}.

(*iii*) **Relative Magnetic permeability** $\mu_r = \frac{B}{B_0} = \frac{\mu}{\mu_0}$. It has **no units.**

(*iv*) **Intensity of magnetisation,** $I = \frac{M}{V} = \frac{m}{a}$.

It is measured in Am^{-1}, same as the unit of magnetising force.

(*v*) **Magnetic susceptibility** $(\chi_m) = \frac{I}{H}$. It has no unit.

Important relations are, $B = \mu_0 (H + I)$; $\mu_r = 1 + \chi_m$.

Based on the above paragraph, answer questions no. 1 to 4 :

1. Least value of dipole moment of an atom is called **Bohr magneton,** whose value is

(*a*) $9{\cdot}27 \times 10^{24}$ Am^2 (*b*) $9{\cdot}27 \times 10^{-24}$ Am^2

(*c*) $9{\cdot}27 \times 10^{-13}$ Am^2 (*d*) $9{\cdot}27 \times 10^{23}$ Am^2

2. Tesla is SI unit of

(*a*) magnetic permeability (*b*) magnetic field strength

(*c*) intensity of megnetisation (*d*) magnetic susceptibility

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, A and R are true and R is correct explanation of A

(*b*) Both, A and R are true, but R is not a correct explanation of A

(*c*) A is true but R is false

(*d*) Both, A and R are false.

3. **Assertion (A).** Relative magnetic permeability has no units.

Reason (R). This is beacuse $\mu_r = \frac{\mu}{\mu_0}$, where symbols have standard meaning.

4. **Assertion (A).** Magnetising intensity (H) and intensity of magnetisation (I) have same units.

Reason (R). The two represent absolutely same quantities.

CASE 2. On the basis of magnetic properties, different materials have been classified into three categories : **Diamagnetic ; Paramagnetic** and **Ferromagnetic.**

1. Diamagnetic Substances are those in which individual atoms/molecules/ions donot possess any net magnetic moment on their own. These are feebly magnetised in a direction opposite to the magnetising field in which they are placed. For example : Bi, Sb, Au, Cu, quartz, water, alcohol, air, hydrogen etc.

Permeability (μ) of diamagnetic substances is less than one and their susceptibility is negative, which does not change with temperature.

2. Paramagnetic substances are those in which each individual atom/molecule/ion has some net magnetic moment of its own. These substances are weakly magnetised in the direction of the field in which they are placed. For example, aluminium, platinum, chromium, manganese, crown glass, oxygen etc. The susceptibility of paramagnetic substances is positive and it decreases with rise in temperature.

3. Ferromagnetic substances are those in which each individual atom/molecule/ion has some net magnetic moment of its own. They are magnetised strongly in the direction of the magnetising field in which they are placed. For example, iron, cobalt, nickel and a number of their alloys. Ferromagnetic materials exhibit all the properties of paramagnetic materials to a much greater degree. At a certain temperature, ferromagnetics pass over to paramagnetics. This temperature is known as **Curie Temperature.** For soft iron, curie temperature is 1000 K.

The three kinds of behaviour has been explained on the basis of **electron theory of magnetism.**

Based on the above paragraph, answer questions no. 5 to 8 :

5. Which of the following is **not** diamagnetic ?

(*a*) Bismuth (*b*) Antimony

(*c*) Quartz (*d*) Iron

6. Which of the following is **not** magnetised strongly in the direction of magnetising field ?

(*a*) cobalt (*b*) Nickel

(*c*) Platinum (*d*) Gold

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) Both A and R are false.

7. **Assertion.** Different materials have been classified into three categories on the basis of their magnetic properties.

Reason. Magnetic properties exhibited by them are distinct.

8. **Assertion.** At a certain temperature, ferromagnetics pass over to paramagnetics. This temperature is called **curie temperature.**

Reason. For all ferromagnetic materials, curie temperature is the same.

ANSWERS

1. (*b*) **2.** (*b*) **3.** (*a*) **4.** (*c*) **5.** (*d*) **6.** (*d*) **7.** (*a*) **8.** (*c*)

HINTS/EXPLANATIONS For Difficult Questions

1. One Bohr magneton = $9{\cdot}27 \times 10^{-24}$ Am^2.

2. Tesla is SI unit of magnetic field strength.

3. **Both, A and R are true, and R is correct explanation of A.**

4. Magnetising intensity $H = n\,i$. Its unit is Ampere turns per metre *i.e.*, Am^{-1}, and intensity of magnetisation

$$I = \frac{\text{magnetic moment}}{\text{volume}}.$$

Its unit is $\frac{Am^2}{m^3} = Am^{-1}$.

Thus **Assertion is true but Reason is false.**

5. Iron is not diamagnetic.
6. Gold is not magnetised strongly in the direction of magnetising field.
7. **Both, A and R are true and R is correct explanation of A.**
8. **Assertion is true, but the Reason is false.**

PROBLEMS FOR PRACTICE

TYPE I. MAGNETIC DIPOLE MOMENT AND FORCE BETWEEN MAGNETIC POLES

1. A steel wire of length l has a magnetic moment M. It is bent into a semicircular arc. What is the new magnetic moment ? **[Ans. $2M/\pi$]**
2. Two thin bar magnets of pole strengths 25 Am and 48 Am respectively and lengths 0·20 m and 0·25 m respectively are placed at right angles to each other with the N-pole of first touching the S-pole of the second. Find the magnetic moment of the system. **[Ans. 13 Am^2]**
3. Two identical magnets with a length 10 cm and weight 50 gf each are arranged freely with their like poles facing in a vertical glass tube. The upper magnet hangs in air above the lower one so that the distance between the nearest poles of the magnets is 3 mm. Determine the pole strength of the poles of these magnets. **[Ans. 6·64 Am]**

TYPE II. MAGNETIC FIELD STRENGTH DUE TO A BAR MAGNET

4. A bar magnet of length 10 cm has a pole strength of 10 Am. Calculate the magnetic field at a distance of 0·2 m from its centre at a point on its (*i*) axial line (*ii*) equatorial line.
[Ans. $2{\cdot}84 \times 10^{-5}$ T ; $1{\cdot}14 \times 10^{-5}$ T]
5. Two small magnets are placed horizontally perpendicular to magnetic meridian. Their north poles are at 30 cm east and 20 cm west from a compass needle. Compare the magnetic moments of the magnets, if compass needle remains undeflected. **[Ans. 8 : 27]**
6. The magnetic field at a point on the magnetic equator is found to be $3{\cdot}1 \times 10^{-5}$ T. Taking the earth's radius to be 6400 km, calculate the magnetic moment of the assumed dipole at the earth's centre. **[Ans. $8{\cdot}1 \times 10^{22}$ Am^2]**

TYPE III. MAGNETIC MOMENT OF A CURRENT LOOP

7. A current of 5 A is flowing through a 10 turn circular coil of radius 7 cm. The coil lies in XY plane. What is the magnitude and direction of magnetic dipole moment associated with it ?
(CBSE Sample Paper 2003)
[Ans. 0·77 Am^2, along Z-axis]
8. The magnetic dipole moment of earth is $6{\cdot}4 \times 10^{21}$ A m^2. If we consider it to be due to a current loop wound around the magnetic equator of the earth, then what should be the magnitude of the current ? Take earth to be a sphere of radius 6400 km. **[Ans. 5×10^7 A]**

TYPE IV. TORQUE ACTING ON A MAGNETIC DIPOLE AND POTENTIAL ENERGY OF DIPOLE IN A MAGNETIC FIELD

9. A magnetised needle of magnetic moment $4{\cdot}8 \times 10^{-2}$ JT^{-1} is placed at 30° with the direction of uniform magnetic field of magnitude 3×10^{-2} T. What is the torque acting on the needle ?
(CBSE 2001) **[Ans. $7{\cdot}2 \times 10^{-4}$ N-m]**
10. A bar magnet placed in a uniform magnetic field of strength 0·3 T with its axis at 30° to the field experiences a torque of 0·06 N-m. What is the magnetic moment of the bar magnet ?
(ICSE 2006) **[Ans. 0·4 Am^2]**

11. A circular coil of 100 turns and having a radius of 0·05 m carries a current of 0·1 A. Calculate the work required to turn the coil in an external magnetic field of 1·5 T through 180° about an axis perpendicular to the magnetic field. The plane of the coil is initially at right angles to the magnetic field. **[Ans. 0·24 J]**

12. A magnetic needle lying parallel to a magnetic field needs 2 J work to turn it through 60°. Calculate the torque required to maintain the needle in this position. **[Ans. 3·46 N-m]**

13. A bar magnet with poles $\sqrt{3}$ cm apart and pole strength 14·4 Am rests with its centre on a frictionless pivot. The magnet is held in equilibrium at an angle of 60° to a uniform magnetic field of intensity 0·20 T by applying a force F at right angles to its axis, 0·1 m from its pivot. What is the value of force F ?

[Ans. 0·432 N]

TYPE V. MAGNETIC OF EARTH

14. Vertical component of earth's magnetic field at a place is $\sqrt{3}$ times its horizontal component. If total intensity of earth's magnetic field at the place is 0·4 Gauss, find (*i*) angle of dip (*ii*) horizontal component of earth's magnetic field. **(CBSE 2010) [Ans. 60° ; 0·2 G]**

15. The declination at a place is 15° west of north. In which direction should a ship be steered so that it reaches a place due east ? **[Ans. 105° east]**

16. If the horizontal component of earth's magnetic field at a place is $0{\cdot}4 \times 10^{-4}$ T where dip is 60°, what are the values of vertical component and resultant magnetic field ?

[Ans. $0{\cdot}69 \times 10^{-4}$ T ; $0{\cdot}8 \times 10^{-4}$ T]

17. A dip circle shows an apparent dip of 60° at a place where true dip is 45°. If dip circle is rotated through 90°, what apparent value of dip will it show ? **[Ans. 51°]**

18. A compass needle whose magnetic moment is 60 Am^2 points geographic north at a certain place where H = 40 μT. The compass needle experiences a torque of $1{\cdot}2 \times 10^{-3}$ N-m. What is the declination at the place ? **[Ans. 30°]**

19. A magnetic needle free to rotate in a vertical plane parallel to magnetic meridian has its north tip down at 60° with the horizontal. The horizontal component of earth's magnetic field at that place is 0·4 G. Determine the magnitude of earth's magnetic field at the place.

(CBSE 2011) [Ans. 0·8 G]

20. The declination at a place is 20° west. In what direction should a ship be steered so that it reaches geographic north east ?

[Ans. 65° east of magnetic north]

21. The true dip at a place is 30°. In what plane is the dip apparently 60° ? **[Ans. 70·5°]**

22. A ship sails due east according to compass. If declination at that place is 19° east of north. What is the true direction of ship ?

[Ans. 19° south of east]

TYPE VI. NEUTRAL POINTS

23. A bar magnet of length 10 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north. A neutral point is obtained at a distance of 12 cm from the centre of the magnet. Find the magnetic moment of the magnet, when H = 0·34 gauss.

[Ans. 0·747 JT^{-1}]

24. A very short bar magnet has magnetic moment 1·4175 J T^{-1}. It is placed (*i*) with its north pole pointing towards geographic north (*ii*) with its north pole pointing towards geographic south. If horizontal component of earth's field at the place is 0·42 gauss, calculate the distance of the neutral points from the magnet.

[Ans. 15 cm ; 18·9 cm]

25. A neutral point is found on the axis of a bar magnet at a distance of 10 cm from its one end. If the length of the magnet be 10 cm, and H = 0·3 G. Find the magnetic moment of the magnet. **[Ans. 0·506 Am^2]**

TYPE VII. TANGENT LAW AND TANGENT GALVANOMETER

26. Two tangent galvanometers, A and B have their number of turns in the ratio 3 : 1 and diameters of coils in the ratio 4 : 1. Explain with reason (*i*) Which galvanometer has greater reduction factor ? (*ii*) Which galvanometer shows greater deflection, when both are connected in series to a d.c. source ? **[Ans. (*i*) A, (*ii*) B]**

27. Two tangent galvanometers differ only in the matter of number of turns in the coil. On passing current through the two joined in series, the first

shows a deflection of 35° and the other shows 45° deflection. Compute the ratio of their number of turns. Take tan 35° = 0·7. **[Ans. 7 : 10]**

28. A 2 turn coil of radius 10 cm is placed with its plane in magnetic meridian. A small magnetic needle is suspended at the centre of the coil by a torsion free silk thread. On passing a current through the coil, the needle is deflected through 45°. Calculate the strength of the current if horizontal component of earth's field is $1{\cdot}6 \times 10^{-5}$ T **[Ans. 1·27 A]**

TYPE VIII. ATOM AS A MAGNETIC DIPOLE

29. An electron is revolving in a hydrogen atom in a circular orbit of radius 4 Å making 10^{15} rps. What is the magnetic moment associated with this electron ? **[Ans. $8{\cdot}038 \times 10^{-23}$ Am^2]**

30. The dipole moment of a coil of 200 turns carrying a current of 3 A is 9·24 A m^2. What is the diameter of the coil ? **[Ans. 14 cm]**

TYPE IX. MAGNETIC PROPERTIES OF MATERIALS

31. A magnet of magnetic moment 2·5 Am^2 weighs 66 g. If density of material of the magnet is 7500 kg/m^3, find the intensity of magnetisation. **[Ans. $2{\cdot}84 \times 10^5$ Am^{-1}]**

32. Assume that each iron atom has a permanent magnetic moment = $1{\cdot}85 \times 10^{-23}$ Am^2. The number density of atoms in iron is $8{\cdot}52 \times 10^{28}$ m^{-3}. Find maximum magnetisation and maximum magnetic induction in the iron bar. **[Ans. $1{\cdot}58 \times 10^6$ Am^{-1} ; 1·985 T]**

33. A magnetising field of 1500 A/m produces a flux of $2{\cdot}4 \times 10^{-5}$ weber in a bar of iron of cross-sectional area 0·5 cm^2. Calculate the permeability and susceptibility of the iron bar used. **(CBSE 2008) [Ans. 255 ; 254]**

34. An iron rod of 0·2 cm^2 area of cross-section is subjected to a magnetising field of 1200 A m^{-1}. If susceptibility of iron is 599, calculate

(*i*) permeability, (*ii*) magnetic flux produced.

[Ans. $7{\cdot}54 \times 10^{-4}$ TA^{-1} m ; $1{\cdot}81 \times 10^{-5}$ Wb]

35. Calculate the magnetic field intensity at a distance of 20 cm. from a pole of strength 40 Am in air. Find the magnetic induction at the same point. **[Ans. 79·54 Am^{-1} ; 10^{-4} Wb/m^2]**

36. The magnetic field B and the magnetic intensity H in a material are found to be 1·6 T and 1000 A/m respectively. Calculate the relative permeability μ_r and the susceptibility χ_m of material. **[Ans. $1{\cdot}3 \times 10^3$ each]**

37. The maximum value of permeability of μ-metal (*77% Ni, 16% Fe, 5% Cu, 2% Cr*) is 0·126 T-m/A. Find the maximum relative permeability and susceptbility. **[Ans. $1{\cdot}00 \times 10^5$ each]**

38. Calculate permeability and susceptibility of a magnetic bar of cross-section 0·1 cm^2 having magnetic flux of $2{\cdot}41 \times 10^{-5}$ Wb due to magnetic intensity of 3200 Am^{-1}. **[Ans. $7{\cdot}53 \times 10^{-4}$ Tm A^{-1} ; 596·1]**

TYPE X. CURIE LAW AND HYSTERISIS LOSS

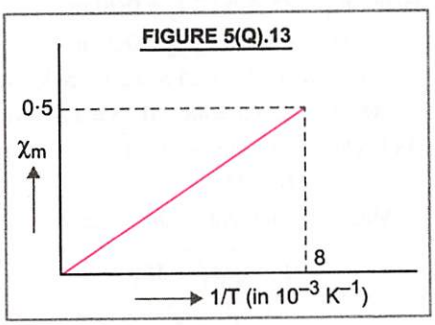
39. The magnetic susceptibility of magnesium at 300 K is $1{\cdot}2 \times 10^5$. At what temperature will its magnetic susceptibility become $1{\cdot}44 \times 10^5$? **(CBSE 2019) [Ans. 250 K]**

40. An iron sample having mass 8·4 kg is repeatedly taken over cycles of magnetisation and demagnetisation at a frequency of 50 c/s. It is found that $3{\cdot}2 \times 10^4$ J of energy is dissipated as heat in the sample in 30 min. If density of iron is 7200 km/m^3, what is the hysterisis loss ? **[Ans. 304·8 Jm^{-3} cycle^{-1}]**

41. For a magnetising field of intensity 2×10^3 Am^{-1}, aluminium at 280 K acquires intensity of magnetisation of $4{\cdot}8 \times 10^{-2}$ Am^{-1}. Find the susceptibility of aluminium at 280 K. If the temperature of the metal is raised to 320 K, what will be its susceptibility and intensity of magnetisation ?

[Ans. $2{\cdot}4 \times 10^{-5}$; $2{\cdot}1 \times 10^{-5}$, $4{\cdot}2 \times 10^{-2}$ Am^{-1}]

42. Fig. 5(Q).13 shows the variation of susceptibility of a material with 1/T, where T is the temperature. Calculate curie constant of the material.



[Ans. 62·5 K]

TYPE XI. TYPICAL PROBLEMS

43. A long straight horizontal cable carries a current of 3·3 A in the direction 10° south of west to 10° north of east. The magnetic meridian of the place happens to be 10° west of the geographic meridian. The earth's magnetic field at the location is 0·33 G and the angle of dip is zero degree. Locate the positions of neutral points ?

[Ans. A set of neutral points parallel to and above the cable are located at distance of 2 cm from cable]

44. A magnet 15 cm long with poles of strength 250 Am lies on a table. Find the magnitude of the magnetic intensity B at a point P, 20 cm directly above the north pole of the magnet.

[Ans. 388·1 μ T]

45. Two identical short bar magnets each of magnetic moment 12·5 Am^2 are placed at a separation of 10 cm between their centres, such that their axes are perpendicular to eachother. Find the magnetic field at a point midway between the two magnets.

[Ans. $2{\cdot}24 \times 10^{-2}$ T, $\theta = 26{\cdot}6°$]

46. A closely wound solenoid of 2000 turns and area of cross section $1{\cdot}6 \times 10^{-4}$ m^2, carrying a current of 4·0 A is suspended through its centre allowing it to turn in a horizontal plane. What is the magnetic moment associated with the solenoid ? What are the force and torque on the solenoid if a uniform horizontal field of $7{\cdot}5 \times 10^{-2}$ T is set up at an angle of 30° with the axis of the solenoid ? (CBSE 2015)

[Ans. 1·28 Am^2 ; $F = 0$, $\tau = 0{\cdot}048$ Nm]

47. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0·5 A is passed through the solenoid, find the magnetic moment of the rod. **[Ans. 25 Am^2]**

48. A bar magnet has pole strength 4·5 Am, magnetic length 12 cm and cross sectional area 0·9 cm^2. Find (*a*) intensity of magnetisation (I) (*b*) magnetising intensity (H) at the centre and (*c*) magnetic induction (B) at the centre of the magnet. **[Ans. (*a*) 5×10^4 Am^{-1}, (*b*) 199 Am^{-1}, (*c*) $6{\cdot}26 \times 10^{-2}$ T]**

HINTS/EXPLANATIONS For Difficult Questions

1. Here, pole strength, $m = \dfrac{M}{l}$

If r is radius of semicircle, then

$$\pi r = l, \quad r = \frac{l}{\pi}$$

New magnetic moment $M' = m \times 2r$

$$= \frac{M}{l} \cdot \frac{2l}{\pi} = \frac{2M}{\pi}$$

2. Here, $m_1 = 25$ Am ; $2l_1 = 0{\cdot}20$ m

$m_2 = 48$ Am ; $2l_2 = 0{\cdot}25$ m

$\therefore \quad M_1 = m_1 (2l_1) = 25 \times 0{\cdot}20 = 5{\cdot}0$ Am^2

$M_2 = m_2 (2l_2) = 48 \times 0{\cdot}25 = 12{\cdot}0$ Am^2

As is clear from Fig. 5(Q).14,

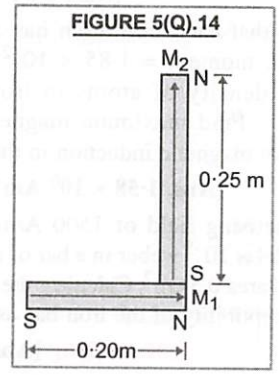
$M_1 \perp M_2$

$\therefore$ Magnetic moment of the system

$$M = \sqrt{M_1^2 + M_2^2}$$

$$= \sqrt{(5{\cdot}0)^2 + (12{\cdot}0)^2} = \mathbf{13\ Am^2}$$

FIGURE 5(Q).14



3. Let $m_1 = m_2 = m$, $r = 3$ mm $= 3 \times 10^{-3}$ m

For hanging in air in balanced position,

$F = 50\ gf = 50 \times 10^{-3} \times 9{\cdot}8$ N

As, $F = \dfrac{\mu_0}{4\pi} \dfrac{m_1 m_2}{r^2}$

$$\therefore \quad 50 \times 10^{-3} \times 9{\cdot}8 = 10^{-7} \frac{m.m}{(3 \times 10^{-3})^2}$$

$m^2 = 9 \times 5 \times 9{\cdot}8 \times 10^{-1}$, $m = \mathbf{6{\cdot}64\ Am.}$

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4. Here, $2l = 10$ cm, $l = 5$ cm $= 0{\cdot}05$ m,
$m = 10$ Am
$B_1 = ?$, $B_2 = ?$, $d = 0{\cdot}2$ m ;
$M = m \times 2l = 10 \times 0{\cdot}1 = 1$ Am^2

$$B_1 = B_{axial} = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2 - l^2)^2} = \frac{10^{-7} \times 2 \times 1 \times 0{\cdot}2}{(0{\cdot}2^2 - 0{\cdot}05^2)^2}$$

$$= \frac{0{\cdot}4 \times 10^{-7}}{(0{\cdot}0375)^2} = \mathbf{2{\cdot}84 \times 10^{-5}\ T}$$

$$B_2 = B_{equatorial}$$

$$= \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}} = \frac{10^{-7} \times 1}{(0{\cdot}2^2 + 0{\cdot}05^2)^{3/2}}$$

$$= \frac{10^{-7}}{(0{\cdot}0425)^{3/2}} = \frac{10^{-7}}{8{\cdot}76 \times 10^{-3}} = \mathbf{1{\cdot}14 \times 10^{-5}\ T}$$

5. The compass needle at C lies on axial line of two magnets, Fig. 5(Q).15. As it remains undeflected, therefore, fields due to two magnets must be equal and opposite.

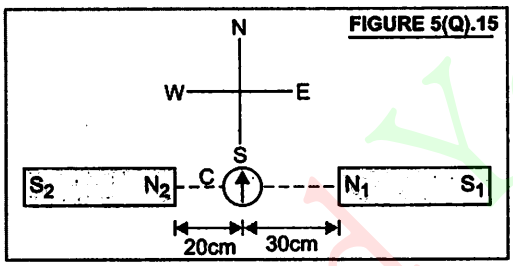

FIGURE 5(Q).15

$$B_1 = B_2$$

$$\frac{\mu_0}{4\pi} \times \frac{2M_1}{d_1^3} = \frac{\mu_0}{4\pi}\frac{2M_2}{d_2^3}$$

$$\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 = \left(\frac{20}{30}\right)^3 = \mathbf{8 : 27}$$

6. A point on magnetic equator is in broadside-on position of the earth's assumed dipole. Therefore,

$$B = \frac{\mu_0}{4\pi}\frac{M}{d^3} \quad \text{or} \quad M = \frac{Bd^3}{\mu_0/4\pi}$$

$$\therefore\ M = \frac{3{\cdot}1 \times 10^{-5}(6{\cdot}4 \times 10^6)^3}{10^{-7}}$$

$$= \mathbf{8{\cdot}1 \times 10^{22}\ Am^2}$$

7. Here, $l = 5$ A, $N = 10$, $r = 7$ cm $= 7 \times 10^{-2}$ m,
$M = NIA = NI(\pi r^2) = 10 \times 5 \times 3{\cdot}14\ (7 \times 10^{-2})^2$
$= \mathbf{0{\cdot}77\ Am^2}$

The direction of magnetic dipole moment is perpendicular to the plane of the coil, **along Z-axis.**

8. $$I = \frac{M}{\pi r^2} = \frac{6{\cdot}4 \times 10^{21}}{(22/7) \times (6{\cdot}4 \times 10^6)^2} = \mathbf{5 \times 10^7\ A}$$

9. Here, $M = 4{\cdot}8 \times 10^{-2}$ JT^{-1}, $\theta = 30°$,
$B = 3 \times 10^{-2}$ T, $\tau = ?$
$\tau = MB \sin\theta = 4{\cdot}8 \times 10^{-2} \times 3 \times 10^{-2} \times \sin 30°$
$= \mathbf{7{\cdot}2 \times 10^{-4}\ N\text{-}m}$

10. Here, $B = 0{\cdot}3$ T, $\theta = 30°$,
$\tau = 0{\cdot}06$ N-m, $M = ?$

As $\tau = MB \sin\theta$, $M = \dfrac{\tau}{B\sin\theta} = \dfrac{0{\cdot}06}{0{\cdot}3 \sin 30°}$
$= \mathbf{0{\cdot}4\ Am^2}$

11. Here, $n = 100$, $r = 0{\cdot}05$ m, $I = 0{\cdot}1$ A
$W = ?$, $B = 1{\cdot}5$ T,
$\theta_1 = 0°$, (When plane of coil is at 90° to B)
$\theta_2 = 180°$
$M = nIA = nI(\pi r^2)$
$= 100 \times 0{\cdot}1\ (3{\cdot}14)\ (0{\cdot}05)^2$
$= 7{\cdot}85 \times 10^{-2}$ Am^2
$W = -MB(\cos\theta_2 - \cos\theta_1)$
$= -7{\cdot}85 \times 10^{-2} \times 1{\cdot}5\ (\cos 180° - \cos 0°)$
$W = \mathbf{0{\cdot}24\ J}$

12. Here, $\theta_1 = 0°$, $W = 2$ J, $\theta_2 = 60°$, $\tau = ?$
$W = -MB(\cos\theta_2 - \cos\theta_1)$
$2 = -MB(\cos 60° - \cos 0°)$
$$= -MB\left(\frac{1}{2} - 1\right) = \frac{MB}{2}$$
$MB = 2 \times 2 = 4$
Torque required, $\tau = MB \sin\theta = 4 \sin 60°$
$$= 4 \times \frac{\sqrt{3}}{2} = \mathbf{3{\cdot}46\ Nm}$$

13. Here, $2l = \sqrt{3}$ cm $= \sqrt{3} \times 10^{-2}$ m
$m = 14{\cdot}4$ Am, $\theta = 60°$, $B = 0{\cdot}20$ T
$F = ?$ $r = 0{\cdot}1$ m
Now, $\tau = rF \sin 90° = MB \sin\theta$

$$F = \frac{MB\sin\theta}{r} = \frac{m(2l)B\sin\theta}{r}$$

$$F = \frac{14{\cdot}4 \times \sqrt{3} \times 10^{-2} \times 0{\cdot}2 \sin 60°}{0{\cdot}1} = \mathbf{0{\cdot}432\ N}$$

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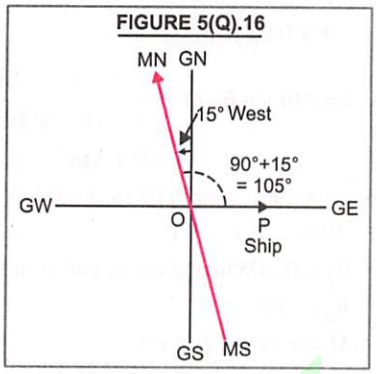
14. $V = \sqrt{3}\,H$, $R = 0{\cdot}4$ G, $\delta = ?$ $H = ?$

$$\tan\delta = \frac{V}{H} = \frac{\sqrt{3}H}{H} = \sqrt{3} \quad \therefore \quad \delta = \mathbf{60^\circ}$$

$$H = R\cos\delta = 0{\cdot}4\cos 60^\circ = 0{\cdot}4\times\frac{1}{2} = \mathbf{0{\cdot}2\ G}$$

15. In Fig. 5(Q).16, Magnetic *NS* is at 15° west of Geographic *N-S*. The ship is to go along *OP* towards geographic east. Therefore, the ship must be steered 90° + 15° = **105° east of the compass needle (Mag. N-S).**

FIGURE 5(Q).16



16. Here, $H = 0{\cdot}4 \times 10^{-4}$ T, $\delta = 60^\circ$, $V = ?$, $R = ?$

$$V = H\tan\delta = 0{\cdot}4\times10^{-4}\tan 60^\circ$$

$$= 0{\cdot}4\sqrt{3}\times10^{-4}\ \text{T} = \mathbf{0{\cdot}69\times10^{-4}\ T}$$

$$R = \frac{H}{\cos\delta} = \frac{0{\cdot}4\times10^{-4}}{\cos 60^\circ} = \mathbf{0{\cdot}8\times10^{-4}\ T}$$

17. Here, $\delta_1 = 60^\circ$, $\delta = 45^\circ$, $\delta_2 = ?$

As $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$

$\therefore\ \cot^2\delta_2 = \cot^2\delta - \cot^2\delta_1$

$$= (\cot 45^\circ)^2 - (\cot 60^\circ)^2$$

$$= 1 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{2}{3} = 0{\cdot}67$$

$$\cot\delta_2 = \sqrt{0{\cdot}67} = 0{\cdot}816$$

$\therefore\quad \delta_2 = \cot^{-1}(0{\cdot}816) = \mathbf{51^\circ}$

18. When compass needle points along magnetic N-S, no torque acts on it. However, when it points towards geographic N-S, torque on it is

$$\tau = MH\sin\theta$$

$$\sin\theta = \frac{\tau}{MH} = \frac{1{\cdot}2\times10^{-3}}{60\times40\times10^{-6}} = \frac{1}{2}$$

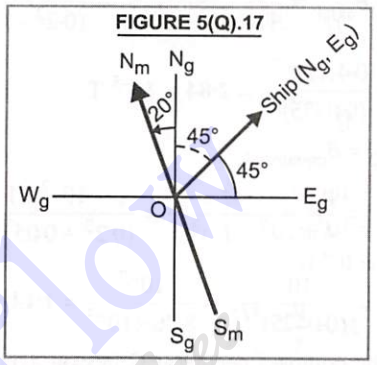
$$\boldsymbol{\theta = 30^\circ}$$

19. Here, $\delta = 60^\circ$; $H = 0{\cdot}4$ G ; $R = ?$

$$R = \frac{H}{\cos\delta} = \frac{0{\cdot}4}{\cos 60^\circ} = \frac{0{\cdot}4}{1/2} = \mathbf{0{\cdot}8\ G}$$

20. As is clear from Fig. 5(Q).17, $N_m\,S_m$ is 20° west of $N_g\,S_g$. To reach geographic *NE*, the ship must

FIGURE 5(Q).17



be steered at (20° + 45°) = **65° east of magnetic north.**

21. Here, $\delta = 30^\circ$ and $\delta' = 60^\circ$, $\theta = ?$

$$\tan\delta = \frac{V}{H} \quad \text{and} \quad \tan\delta' = \frac{V}{H'} = \frac{V}{H\cos\theta}$$

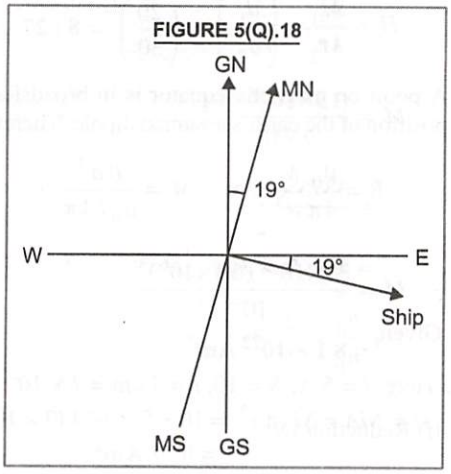
$$\tan\delta' = \frac{\tan\delta'}{\cos\theta}$$

$$\therefore\ \cos\theta = \frac{\tan\delta}{\tan\delta'} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1}{\sqrt{3}.\sqrt{3}} = \frac{1}{3}$$

$$\cos\theta = 0{\cdot}3333 \quad \therefore \quad \theta = \mathbf{70{\cdot}5^\circ}$$

22. In Fig. 5(Q).18, compass needle is represented by *MN* – *MS* at 19° east of north. As the ship is sailing due east according to the campass, therefore the ship is actually sailing at **19° south of east** as shown.

FIGURE 5(Q).18



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23. Here, $2l = 10$ cm, $l = 5$ cm $= 5 \times 10^{-2}$ m,
$d = 12$ cm, $H = 0{\cdot}34$ $G = 0{\cdot}34 \times 10^{-4}$ T
As N-pole of magnet is towards geographic north, therefore neutral points lie on equatorial line of the magnet.

$$\therefore \quad H = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}}$$

$$M = \frac{4\pi}{\mu_0} (d^2 + l^2)^{3/2} H$$

$$= 10^7 [12^2 + 5^2]^{3/2} (10^{-4})^{3/2} \times 0{\cdot}34 \times 10^{-4}$$
$$= 10^7 \times 13 \times 13 \times 13 \times 10^{-6} \times 0{\cdot}34 \times 10^{-4}$$
$$M = \mathbf{0{\cdot}747\ JT^{-1}}$$

24. (*i*) $H = \dfrac{\mu_0}{4\pi} \dfrac{M}{d^3}$

$$\text{or} \quad d = \left[\frac{\mu_0}{4\pi} \frac{M}{H}\right]^{1/3} = \left[\frac{10^{-7} \times 1{\cdot}4175}{0{\cdot}42 \times 10^{-4}}\right]^{1/3}$$
$$= 0{\cdot}15 \text{ m} = \mathbf{15 \text{ cm}}$$

(*ii*) $H = \dfrac{\mu_0}{4\pi} \dfrac{2M}{d^3}$

$$\text{or} \quad d = \left[\frac{\mu_0}{4\pi} \frac{2M}{H}\right]^{1/3} = \left[\frac{10^{-7} \times 2 \times 1{\cdot}4175}{0{\cdot}42 \times 10^{-4}}\right]^{1/3}$$
$$= 0{\cdot}189 \text{ m}$$
$$= \mathbf{18{\cdot}9 \text{ cm}}$$

25. Here, $2l = 10$ cm, $H = 0{\cdot}3$ G, $M = ?$
$r = 10 + l = 10 + 5 = 15$ cm $= 15 \times 10^{-2}$ m
As neutral point is on the axis of bar magnet, therefore,

$$H = \frac{\mu_0}{4\pi} = \frac{2M}{r^3} = 10^{-7} \frac{2 \times M}{(15 \times 10^{-2})^3}$$

$$M = \frac{(15 \times 10^{-2})^3 H}{2 \times 10^{-7}}$$

$$= \frac{3375 \times 10^{-6} \times 0{\cdot}3 \times 10^{-4}}{2 \times 10^{-7}}$$
$$= \mathbf{506{\cdot}25 \times 10^{-3} = 0{\cdot}506\ Am^2}$$

26. Given, $\dfrac{n_A}{n_B} = \dfrac{3}{1}$ and $\dfrac{2r_A}{2r_B} = \dfrac{4}{1}$ or $\dfrac{r_A}{r_B} = \dfrac{4}{1}$

(*i*) Reduction factor, $K = \dfrac{2rH}{n\mu_0}$ *i.e.*, $K \propto \dfrac{r}{n}$

$$\therefore \quad \frac{K_A}{K_B} = \frac{r_A}{r_B} \times \frac{n_B}{n_A} = \frac{4}{1} \times \frac{1}{3} = \frac{4}{3} > 1$$

i.e. $K_A > K_B$

(*ii*) When both the tangent galvanometer are connected in series, there will be same current in them. Then

$$I = K_A \tan\theta_A = K_B \tan\theta_B$$

$$\text{or} \quad \frac{\tan\theta_A}{\tan\theta_B} = \frac{K_B}{K_A} = \frac{3}{4} < 1$$

i.e. then $\theta_A < \tan\theta_B$ or $\theta_A < \theta_B$

27. As the two tangent galvanometers are connected in series, they carry the same current.

$$\therefore \quad I_1 = \frac{2rH}{\mu_0 n_1} \tan\theta_1 = I_2 = \frac{2rH}{\mu_0 n_2} \tan\theta_2$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{\tan 35°}{\tan 45°} = \frac{0{\cdot}7}{1} = \frac{7}{10}$$

28. Here, $n = 2$, $r = 10$ cm $= 0{\cdot}1$ m, $\theta = 45°$, $I = ?$
$H = 1{\cdot}6 \times 10^{-5}$ T

As $I = \dfrac{2rH}{\mu_0 n} \tan\theta$

$$\therefore \quad I = \frac{2 \times 0{\cdot}1 \times 1{\cdot}6 \times 10^{-5} \tan 45°}{4\pi \times 10^{-7} \times 2} = \mathbf{1{\cdot}27\ A}$$

29. Here, $r = 4$ Å $= 4 \times 10^{-10}$ m, $v = 10^{15}$ rps, $M = ?$

$$M = IA = (ev)\,\pi r^2$$
$$= (1{\cdot}6 \times 10^{-19} \times 10^{15}) \times 3{\cdot}14\ (4 \times 10^{-10})^2$$
$$= \mathbf{8{\cdot}038 \times 10^{-23}\ Am^2}$$

30. Here, $N = 200$, $I = 3$ A, $M = 9{\cdot}24$ A m^2 $d = ?$
If r is radius of coil, then

$$M = NIA = NI\pi \frac{d^2}{4}$$

$$d = \sqrt{\frac{4M}{\pi NI}} = \sqrt{\frac{4 \times 9{\cdot}24}{3{\cdot}14 \times 200 \times 3}}$$
$$= 0{\cdot}14 \text{ m} = \mathbf{14 \text{ cm}}$$

31. Here, $M = 2{\cdot}5\ Am^2$,
mass, $m = 66$ g $= 66 \times 10^{-3}$ kg,
$\rho = 7500$ kg/m^3, $I = ?$

$$I = \frac{M}{V} = \frac{M}{m/\rho} = \frac{M\rho}{m} = \frac{2{\cdot}5 \times 7500}{66 \times 10^{-3}}$$
$$= \mathbf{2{\cdot}84 \times 10^5\ Am^{-1}}$$

32. Here, magnetic moment of each atom
$= 1{\cdot}85 \times 10^{-23}$ Am^2
Number of atoms/volume $= 8{\cdot}52 \times 10^{28}$ m^{-3}
$\therefore$ Max. magnetisation,

$$I = \frac{M}{V} = 1{\cdot}85 \times 10^{-23} \times 8{\cdot}52 \times 10^{28}$$
$$= \mathbf{1{\cdot}58 \times 10^6\ Am^{-1}}$$

Max. mag. induction $B = \mu_0 (H + I)$
$= 4\pi \times 10^{-7} (0 + 1{\cdot}58 \times 10^6) = \mathbf{1{\cdot}985\ T}$

33. Here, $H = 1500$ A/m, $\phi = 2{\cdot}4 \times 10^{-5}$ weber
$a = 0{\cdot}5\ cm^2 = 0{\cdot}5 \times 10^{-4}\ m^2$
$\mu_r = ?,\ \chi_m = ?$

Now $B = \frac{\phi}{a} = \frac{2{\cdot}4 \times 10^{-5}}{0{\cdot}5 \times 10^{-4}} = 4{\cdot}8 \times 10^{-1}$ T

$$\mu = \frac{B}{H} = \frac{4{\cdot}8 \times 10^{-1}}{1500} = 3{\cdot}2 \times 10^{-4}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{3{\cdot}2 \times 10^{-4}}{4\pi \times 10^{-7}} = 0{\cdot}255 \times 10^3 = \mathbf{255}$$

As $\mu_r = 1 + \chi_m$ $\therefore$ $\chi_m = \mu_r - 1 = 255 - 1 = \mathbf{254}$

34. Here, $A = 0{\cdot}2\ cm^2 = 0{\cdot}2 \times 10^{-4}\ m^2$;
$H = 1200\ Am^{-1}$; $\chi_m = 599$
$\mu_r = 1 + \chi_m = 1 + 599 = 600$
$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 600$
$= \mathbf{7{\cdot}54 \times 10^{-4}\ Tm\ A^{-1}}$
Magnetic flux $\phi = \mu H A$
$= (7{\cdot}54 \times 10^{-4}) \times 1200 \times (0{\cdot}2 \times 10^{-4})$
$= \mathbf{1{\cdot}81 \times 10^{-5}\ Wb}$

35. Here, $d = 20$ cm $= 1/5$ m, $m = 40$ Am,
$H = ?, B = ?$

$$H = \frac{1}{4\pi}\frac{m \times 1}{d^2} = \frac{1 \times 7}{4 \times 22} \times \frac{40}{(1/5)^2} = \mathbf{79{\cdot}54\ Am^{-1}}$$

$B = \mu_0 H = 4\pi \times 10^{-7} \times 79{\cdot}54 = \mathbf{10^{-4}\ Wb\ m^{-2}}$

36. $\mu = \frac{B}{H} = \frac{1{\cdot}6}{1000} = 1{\cdot}6 \times 10^{-3}$ Tm A^{-1}

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1{\cdot}6 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{1{\cdot}3 \times 10^3}$$

$\chi_m = \mu_r - 1 = 1{\cdot}3 \times 10^3 - 1 = \mathbf{1{\cdot}3 \times 10^3}$

37. $\mu_r = \frac{\mu}{\mu_0} = \frac{0{\cdot}126}{4\pi \times 10^{-7}} = \mathbf{1{\cdot}00 \times 10^5}$

Susceptbility, $\chi_m = \mu_r - 1 = 1{\cdot}00 \times 10^5 - 1$
$= \mathbf{1{\cdot}00 \times 10^5}$

38. Here, $A = 0{\cdot}1\ cm^2 = 0{\cdot}1 \times 10^{-4}\ m^2$, $\mu = ?, \chi_m = ?$
$\phi = 2{\cdot}41 \times 10^{-5}$ Wb,
$H = 3200\ Am^{-1}$

$$\mu = \frac{B}{H} = \frac{\phi/A}{H} = \frac{\phi}{AH} = \frac{2{\cdot}41 \times 10^{-5}}{0{\cdot}1 \times 10^{-4} \times 3200}$$
$$= \mathbf{7{\cdot}53 \times 10^{-4}\ TmA^{-1}}$$

$$\chi_m = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \frac{7{\cdot}53 \times 10^{-4}}{4\pi \times 10^{-7}} - 1$$

$\chi_m = 597{\cdot}1 = \mathbf{596{\cdot}1}$

39. Here, $T_1 = 300$ K, $\chi_1 = 1{\cdot}2 \times 10^5$,
$T_2 = ?, \lambda_2 = 1{\cdot}44 \times 10^5$

As $\frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$

or $T_2 = \frac{\chi_1}{\chi_2} T_1 = \frac{1{\cdot}2 \times 10^5}{1{\cdot}44 \times 10^5} \times 300 = \mathbf{250\ K}$

40. Here, $m = 8{\cdot}4$ kg, $\nu = 50$ c/s,
$t = 30$ min $= 30 \times 60$ s
Energy dissipated, $E = 3{\cdot}2 \times 10^4$ J,
$\rho = 7200\ kg/m^3$

Volume, $V = \frac{m}{\rho} = \frac{8{\cdot}4}{7200}\ m^3$

Energy dissipated/volume $= \frac{3{\cdot}2 \times 10^4 \times 7200}{8{\cdot}4}$

Hysterisis loss = Energy dissipated/volume/cycle

$$= \frac{3{\cdot}2 \times 10^4 \times 7200}{8{\cdot}4 \times 30 \times 60 \times 50} = \mathbf{304{\cdot}8\ Jm^{-3}\ cycle^{-1}}$$

41. Here, $H = 2 \times 10^3\ Am^{-1}$, $T = 280$ K.
$I = 4{\cdot}8 \times 10^{-2}\ Am^{-1}$

$$\chi_m = \frac{I}{H} = \frac{4{\cdot}8 \times 10^{-2}}{2 \times 10^3} = \mathbf{2{\cdot}4 \times 10^{-5}}$$

$T' = 320$ K, $\chi'_m = ?$

According to Curie's law, $\frac{\chi'_m}{\chi_m} = \frac{T}{T'}$

$\therefore$ $\chi'_m = \frac{T}{T'} \times \chi_m = \frac{280}{320} \times 2{\cdot}4 \times 10^{-5}$
$= \mathbf{2{\cdot}1 \times 10^{-5}}$

As $\chi'_m = \frac{I'}{H}$

$\therefore$ $I' = \chi'_m H = 2{\cdot}1 \times 10^{-5} \times 2 \times 10^3$
$= \mathbf{4{\cdot}2 \times 10^{-2}\ Am^{-1}}$

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42. From the graph in Fig. 5(Q).13,

when $\chi_m = 0{\cdot}5$, $\frac{1}{T} = 8{\cdot}0\times10^{-3}$ K^{-1}

According to Curie's Law

$$\chi_m = \frac{C}{T}$$

or $C = \chi_m \times T = 0{\cdot}5\times\frac{1}{8{\cdot}0\times10^{-3}} = 0{\cdot}5\times\frac{1000}{8{\cdot}0}$

$$= \mathbf{62{\cdot}5\ K}$$

43. Here, $I = 3{\cdot}3$ A ; $B = 0{\cdot}33$ G $= 0{\cdot}33 \times 10^{-4}$ T, $\delta = 0°$. Horizontal component of earth's magnetic field $H = B\cos\delta = 0{\cdot}33 \times 10^{-4}\cos 0° = 0{\cdot}33 \times 10^{-4}$ T. The neutral points will be parallel and above the cable. Let r be the distance of neutral point from the cable. Then

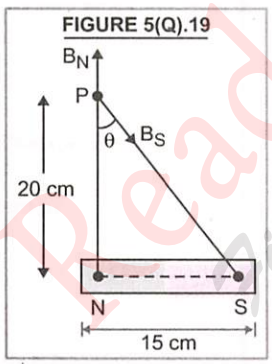
$$H = \frac{\mu_0}{4\pi}\frac{2I}{r} \quad \text{or} \quad r = \frac{\mu_0}{4\pi}\times\frac{2I}{H}$$

$$r = 10^{-7}\times\frac{2\times3{\cdot}3}{0{\cdot}33\times10^{-4}} = 2\times10^{-2}\text{ m} = 2\text{ cm}$$

Thus **the set of neutral points will lie parallel to cable, and above the cable at distance of 2 cm from cable.**

44. As is clear from Fig. 5(Q).19,

FIGURE 5(Q).19



$$B_N = \frac{\mu_0}{4\pi}\frac{m_1\times m_2}{(NP)^2} = \frac{10^{-7}\times250\times1}{(0{\cdot}2)^2}$$

$$= 625\times10^{-6}\text{ T} = 625\ \mu\text{T}$$

Now $SP = \sqrt{(0{\cdot}2)^2 + (0{\cdot}15)^2} = 0{\cdot}25$ m

$$B_S = \frac{10^{-7}\times250\times1}{(0{\cdot}25)^2} = 400\ \mu\text{T};$$

Vertical component of B_S is

$$B_S\cos\theta = 400\times\frac{20}{25} = 320\ \mu\text{T, opposite to } B_N$$

Horizontal components of B_S is

$$B_S\sin\theta = 400\times\frac{15}{25} = 240\ \mu\text{T}$$

Resultant vertical component = 625 – 320

$$= 305\ \mu\text{T}$$

Resultant field at $P = \sqrt{(305)^2 + (240)^2}$

$$= \mathbf{388{\cdot}1\ \mu\ T}$$

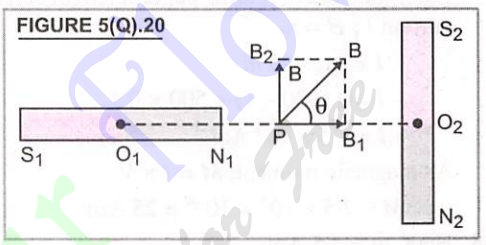
45. Here, $M_1 = M_2 = 12{\cdot}5$ Am^2.

$O_1\,O_2 = 10$ cm, Fig. 5(Q).20,

P is midway between O_1 and O_2

$\therefore \quad d = O_1P = O_2P = 5$ cm $= 5\times10^{-2}$ m

FIGURE 5(Q).20



As P lies an axial line of $N_1\,S_1$,

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{(d)^3} = \frac{10^{-7}\times2\times12{\cdot}5}{(5\times10^{-2})^3}$$

$$= 0{\cdot}02\text{ T, along } O_1PO_2$$

Again, P lies on equatorial line of $N_2\,S_2$

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi}\frac{M_2}{d^3} = \frac{10^{-7}\times12{\cdot}5}{(5\times10^{-2})^3} = 0{\cdot}01, \perp PO_2$$

Resultant magnetic field strength at P

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(0{\cdot}02)^2 + (0{\cdot}01)^2}$$

$$= \mathbf{2{\cdot}24\times10^{-2}\ T}$$

If θ is the angle which $\vec{B}$ makes with $\vec{B_1}$, then

$$\tan\theta = \frac{B_2}{B_1} = \frac{0{\cdot}01}{0{\cdot}02} = 0{\cdot}5\ ;\ \theta = \mathbf{26{\cdot}6°}$$

46. Here, $N = 2000$, $A = 1{\cdot}6\times10^{-4}$ m^2, $I = 4{\cdot}0$ A

$$M = NIA = 2000\times4{\cdot}0\times1{\cdot}6\times10^{-4}$$

$$= \mathbf{1{\cdot}28\ Am^2}$$

Net force experienced by the magnetic dipole in uniform horizontal field, $F = 0$

The magnitude of torque exerted by the magnetic field $\vec{B}$ on the solenoid is

$$\tau = MB\sin\theta = 1{\cdot}28\times7{\cdot}5\times10^{-2}\times\sin30°$$

$$= \mathbf{0{\cdot}048\ Nm}$$

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The torque tends to align the axis of the solenoid, *i.e.*, its magnetic moment $\vec{M}$ along the field $\vec{B}$.

47. Here, $n = 5$ turns/cm $= 500$ turns/m

$V = 10^{-4}\ m^3$, $\mu_r = 1000$, $i = 0{\cdot}5$ A

We know that $B = \mu_0 (H + I)$

$$\therefore \quad I = \frac{B}{\mu_0} - H = \frac{\mu H}{\mu_0} - H$$

$$= \mu_r H - H = (\mu_r - 1) H$$

For a solenoid of n turns per unit length carrying current i ; $H = ni$.

$$\therefore \quad I = (\mu_r - 1)\, ni.$$

$$I = (1000 - 1) \times 500 \times 0{\cdot}5$$

$$I = 2{\cdot}5 \times 10^5\ Am^{-1}$$

As magnetic moment, $M = I \times V$

$$\therefore \quad M = 2{\cdot}5 \times 10^5 \times 10^{-4} = \mathbf{25\ Am^2}$$

48. Here, m $= 4{\cdot}5$ Am.,

$2l = 12$ cm $= 0{\cdot}12$ m

$a = 0{\cdot}9\ cm^2 = 0{\cdot}9 \times 10^{-4}\ m^2$

(*a*) $$I = \frac{m}{a} = \frac{4{\cdot}5}{0{\cdot}9 \times 10^{-4}}$$

$$= \mathbf{5 \times 10^4\ Am^{-1}\ from\ S\ to\ N}$$

(*b*) Magnetic intensity at the centre, due to its *N*-pole

$$H_1 = \frac{m}{4\pi r^2} = \frac{4{\cdot}5}{4\pi(0{\cdot}06)^2} \text{ where } r = 2\ l/2$$

$$= 99{\cdot}5\ Am^{-1}, \text{ towards } S\text{-pole}$$

Magnetic intensity at the centre due to its *S* pole

$$H_2 = \frac{m}{4\pi r^2} = \frac{4{\cdot}5}{4\pi(0{\cdot}06)^2}$$

$$= 99{\cdot}5\ Am^{-1}, \text{ towards } S\text{-pole.}$$

Resultant magnetic intensity at the centre

$$H = H_1 + H_2$$

$$= 99{\cdot}5 + 99{\cdot}5$$

$$= \mathbf{199\ Am^{-1}, towards\ S\text{-}pole}$$

(*c*) Now, $B = \mu_0 (H + I)$

$$= 4\pi \times 10^{-7} (-199 + 5 \times 10^4)$$

$$= \mathbf{6{\cdot}26 \times 10^{-2}\ T,\ towards\ N\ pole}$$

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NCERT EXERCISES WITH SOLUTIONS

Q. 1. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0·25 T experiences a torque of magnitude equal to $4·5 \times 10^{-2}$ J. What is the magnitude of magnetic moment of the magnet ?

Sol. Here, $\theta = 30°$, $B = 0·25$ T, $\tau = 4·5 \times 10^{-2}$ J, $M = ?$

As, $\tau = M B \sin\theta$ $\quad\therefore\quad M = \dfrac{\tau}{B\sin\theta} = \dfrac{4·5\times10^{-2}}{0·25\sin 30°} = \mathbf{0·36\ JT^{-1}}$

Q. 2. A short bar magnet of moment 0·32 JT^{-1} is placed in a uniform external magnetic field of 0·15 T. If the bar is free to rotate in the plane of the field, which orientations would correspond to its, (i) stable and (ii) unstable equilibrium ? What is the potential energy of the magnet in each case ?

Sol. Here, $M = 0·32\ JT^{-1}$, $B = 0·15$ T

(*i*) In stable equilibrium, the bar magnet is aligned along the magnetic field, *i.e.* $\theta = 0°$

Potential Energy $= -$ MB cos $0° = -0·32 \times 0·15 \times 0 = \mathbf{-4·8 \times 10^{-2}\ J}$

(*ii*) In unstable equilibrium, the magnet is so oriented that magnetic moment is at 180° to the magnetic field *i.e.* $\theta = 180°$

Potential energy $= -MB \cos 180° = -0·32 \times 0·15\ (-1) = \mathbf{4·8 \times 10^{-2}\ J.}$

Q. 3. A closely wound solenoid of 800 turns and area of cross section $2·5 \times 10^{-4}$ m^2 carries a current of 3·0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment ?

Sol. Here, $n = 800$, $A = 2·5 \times 10^{-4}$ m^2. $I = 3·0$ A

A magnetic field develops along the axis of the solenoid. Therefore, the current carrying solenoid behaves like a bar magnet.

$M = n\,I\,A = 800 \times 3·0 \times 2·5 \times 10^{-4} = \mathbf{0·6\ JT^{-1}}$**, along the axis of solenoid.**

Q. 4. If the solenoid in the above question is free to turn about the vertical direction, and a uniform horizontal magnetic field of 0·25 T is applied, what is the magnitude of the torque on the solenoid when its axis makes on angle of 30° with the direction of the applied field ?

Sol. Here, $M = 0·6\ JT^{-1}$ (from Ques. 5) $\quad B = 0·25$ T ; $\quad \tau = ? \quad \theta = 30°$

As, $\tau = M B \sin\theta$ $\quad\therefore\quad$ $\tau = 0·6 \times 0·25 \sin 30° = \mathbf{0·075\ N.m.}$

Q. 5. A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work done to turn the magnet so as to align its magnetic moment

(i) normal to the field direction, (ii) opposite to the field direction ?

(b) What is the torque on the magnet in cases (i) and (ii) ?

Sol. Here, $M = 1.5\ JT^{-1}$, $B = 0.22$ T , $W = ?$

(*a*) (*i*) Here, $\theta_1 = 0°$ (along the field), $\quad \theta_2 = 90°$ ($\perp$ to the field)

As $W = -MB\,(\cos\theta_2 - \cos\theta_1)$

$\therefore$ $W = -1.5 \times 0.22\ (\cos 90° - \cos 0°) = -0.33\ (0 - 1) = \mathbf{0.33\ J}$

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(*ii*) Here, $\theta_1 = 0°, \theta_2 = 180°$

$W = -1.5 \times 0.22 (\cos 180° - \cos 0°) = -0.33 (-1 - 1) = \mathbf{0.66\ J}$

(*b*) Torque $\tau = M B \sin\theta$

(*i*) Here, $\theta = 90°$, $\tau = 1.5 \times 0.22 \sin 90° = \mathbf{0.33\ Nm}$

(*ii*) Here, $\theta = 180°$, $\tau = 1.5 \times 0.22 \sin 180° = \mathbf{0}$

Q. 6. A closely wound solenoid of 2000 turns and area of cross section $1.6 \times 10^{-4}\ m^2$, carrying a current of 4 amp. is suspended through its centre allowing it to turn in a horizontal plane :

(a) What is the magnetic moment associated with the solenoid ?

(b) What are the force and torque on the solenoid if a uniform horizontal magnetic field of $7{\cdot}5 \times 10^{-2}$ T is set up at an angle of 30° with the axis of the solenoid ?

Sol. (*a*) $N = 2000, A = 1.6 \times 10^{-4}\ m^2, I = 4$ amp., $M = ?, B = 7{\cdot}5 \times 10^{-2}$ T.

As, $M = NIA$ $\therefore$ $M = 2000 \times 4 \times 1.6 \times 10^{-4} = \mathbf{1.28\ JT^{-1}}$

(*b*) Net force on the solenoid = 0

Torque, $\tau = MB \sin\theta = 1.28 \times 7.5 \times 10^{-2} \sin 30° = 1.28 \times 7.5 \times 10^{-2} \times \frac{1}{2} = \mathbf{4.8 \times 10^{-2}\ Nm.}$

Q. 7. A short bar magnet has a magnetic moment of $0{\cdot}48\ JT^{-1}$. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (i) the axis (ii) the equatorial line (normal bisector) of the magnet.

Sol. Here, $M = 0{\cdot}48\ JT^{-1}, B = ?$; $d = 10\ cm = 0{\cdot}1\ m$

(*i*) On the axis of the magnet, $B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = 10^{-7} \times \frac{2 \times 0{\cdot}48}{(0{\cdot}1)^3} = \mathbf{0{\cdot}96 \times 10^{-4}\ T}$ **along *S*-*N* direction.**

(*ii*) On the equatorial line of the magnet

$$B = \frac{\mu_0}{4\pi} \times \frac{M}{d^3} = 10^{-7} \times \frac{0{\cdot}48}{(0{\cdot}1)^3} = \mathbf{0{\cdot}48 \times 10^{-4}\ T,}$$ **along *N*–*S* direction.**

Q. 8. A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0·36 G and the angle of dip is zero. What is the total magnetc field on the normal bisector of the magnet at the same distance as the null points (*i.e.* 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field).

Sol. As null points are on the axis of the magnet, therefore

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = H$$

On the equatorial line of magnet at same distance (d), field due to the magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3} = \frac{B_1}{2} = \frac{H}{2}$$

$\therefore$ Total magnetic field at this point on equatorial line is

$$B = H + B_2 = H + \frac{H}{2} = \frac{3}{2}H = \frac{3}{2} \times 0{\cdot}36 = \mathbf{0{\cdot}54G}$$

Q. 9. If the bar magnet in the above problem is turned around by 180°, where will the new null points be located ?

Sol. When the bar magnet is turned through 180°, neutral points would lie on equatorial line, so that

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{d_2^3} = H \qquad ...(i)$$

In the previous question, $B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d_1^3} = H$ ...(ii)

From (*i*) and (*ii*), $\frac{\mu_0}{4\pi}\frac{M}{d_2^3} = \frac{\mu_0}{4\pi}\frac{2M}{d_1^3}$ $\therefore$ $d_2^3 = \frac{d_1^3}{2} = \frac{(14)^3}{2}$

$$d_2 = \frac{14}{2^{1/3}} = \mathbf{11{\cdot}1\,cm}$$

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NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. A toroid of n turns, mean radius R and cross-sectional radius a carries current I. It is placed on a horizontal table taken as x-y plane. Its magnetic moment $\vec{M}$

(a) is non-zero and points in the z-direction by symmetry

(b) points along the axis of the toroid ($\vec{M} = M\hat{\phi}$)

(c) is zero, otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the torioid

(d) is pointing radially outwards

2. The magnetic field of Earth can be modelled by that of a point dipole placed at the centre of the Earth. The dipole axis makes an angle of 11·3° with the axis of Earth. At Mumbai, declination is nearly zero. Then,

(a) the declination varies between 11·3° W to 11·3° E

(b) the least declination is 0°

(c) the plane defined by dipole axis and Earth axis passes through Greenwich

(d) declination averaged over Earth must be always negative

3. In a permanent magnet at room temperature

(a) magnetic moment of each molecule is zero

(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned

(c) domains are partially aligned

(d) domains are all perfectly aligned

4. Consider the two idealized systems : (i) a parallel plate capacitor with large plates and small separation and (ii) a long solenoid of length $L >> R$, radius of cross-section. In (i) $\vec{E}$ is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental laws as below :

(a) case (i) contradicts Gauss's law for electrostatic fields

(b) case (ii) contradicts Gauss's law for magnetic fields.

(c) case (i) agrees with $\oint \vec{E} \cdot d\vec{l} = 0$

(d) case (ii) contradicts $\oint \vec{H} \cdot d\vec{l} = I_{en}$

5. A paramagnetic sample shows a net magnetisation of 8 Am^{-1} when placed in an external magnetic field of 0·6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0·2 T at a temperature of 16 K, the magnetisation will be

(a) $\frac{32}{3}$ Am^{-1} (b) $\frac{2}{3}$ Am^{-1}

(c) 6 Am^{-1} (d) 2·4 Am^{-1}.

MULTIPLE CHOICE QUESTIONS-II

6. S is the surface of a lump of magnetic material.

(a) Lines of $\vec{B}$ are necessarily continuous across S

(b) Some lines of $\vec{B}$ must be discontinuous across S

(c) Lines of $\vec{H}$ are necessarily continuous across S

(d) Lines of $\vec{H}$ can not all be continuous across S

7. The primary origin(s) of magnetism lies in

(a) atomic currents

(*b*) Pauli exclusion principle
(*c*) polar nature of molecules
(*d*) intrinsic spin of electron

8. A long solenoid has 1000 turns per metre and carries a current of 1 A. It has a soft iron core of μ_r = 1000. The core is heated beyond the Curie temperature, T_c.
(*a*) The $\vec{H}$ field in the solenoid is (nearly) unchanged but the $\vec{B}$ field decreases drastically
(*b*) The $\vec{H}$ and $\vec{B}$ fields in the solenoid are nearly unchanged
(*c*) The magnetisation in the core reverses direction
(*d*) The magnetisation in the core diminishes by a factor of about 10^8

9. Essential difference between electrosatic shielding by a conducting shell and magnetostatic shielding is due to
(*a*) electrostatic field lines can end on charges and conductors have free charges
(*b*) lines of $\vec{B}$ can also end but conductors can not end them
(*c*) lines of $\vec{B}$ can not end on any material and perfect shielding is not possible
(*d*) shells of high permeability materials can be used to divert lines of $\vec{B}$ from the interior region

10. Let the magnetic field on earth be modelled by that of a point magnetic dipole at the centre of earth. The angle of dip at a point on the geographical equator
(*a*) is always zero
(*b*) can be zero at specific points
(*c*) can be positive or negative
(*d*) is bounded

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*c*) **4.** (*b*) **5.** (*b*) **6.** (*a*, *d*) **7.** (*a*, *d*) **8.** (*a*, *d*)
9. (*a*, *c*, *d*) **10.** (*b*, *c*, *d*)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. In case of a toroid, the magnetic field is only confined inside the body of toroid in the form of concentric magnetic lines of force and there is no magnetic field outside the body of toroid. Thus the magnetic moment of toroid is zero. Hence, **option (*c*) is correct.**

2. Since the axis of the magnetic dipole placed at the centre of earth makes an angle 11·3° with the axis of earth, two possibilities arises as shown in Fig. 5(N).1 and 5(N).2. Hence the declination varies between 11·3° W to 11·3° E.

FIGURE 5(N).1

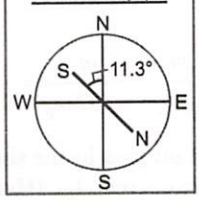
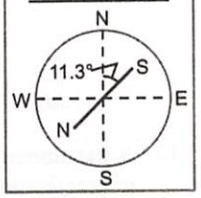


FIGURE 5(N).2



3. In a permanent magnet at room temperature domains of a magnet are partially alingned due to thermal agitation.

4. According to Gauss's law for electrostatic field, $\oint_S \vec{E}.\vec{ds} = Q/\epsilon_0$

It does not contradict for electrostatic fields as the electric field lines do not form a continuous closed path.

According to Gauss's law in magnetic field, $\oint_S \vec{E}.\vec{ds} = 0$

It contradicts for magnetic field because there is a magnetic field inside the solenoid and no field outside the solenoid carrying current but the magnetic field lines form the closed paths.

5. Here, $I_1 = 8\ \text{Am}^{-1}$; $B_1 = 0{\cdot}6$ T, $t_1 = 4$ K ; $I_2 = ?$, $B_2 = 0{\cdot}2$ T, $t_2 = 16$ K

According to Curie law, $I \propto \dfrac{B\,(\text{magnetic field induction})}{t\,(\text{temp in kelvin})}$

$$\therefore \quad \frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{t_1}{t_2} = \frac{0{\cdot}2}{0{\cdot}6} \times \frac{4}{16} = \frac{1}{12} \quad \text{or} \quad I_2 = I_1 \times \frac{1}{12} = \frac{8}{12} = \frac{2}{3}\,\text{Am}^{-1}$$

Multiple Choice Questions – II

6. The lines of magnetic field induction $\vec{B}$ are necessarily continuous across the surface S of a lump of magnetic material. Outside the lump of magnetic material, $H = B/\mu_0$ and inside the lump of magnetic material, $H = B/\mu_0\,\mu_r$ where μ_r is the relative permeability of material. Thus the lines of H cannot all be continuous across surface S.

7. The primary origin of magnetism lies in the fact that the electrons are revolving and spinning about nucleus of an atom, which gives rise to currents and hence to magnetism. Thus, **options (*a*) and (*c*) are true.**

8. In the solenoid, $H = n\,I$ = a constant and $B = \mu_0\,\mu_r\,n\,I$ changes due to variation in μ_r. When temperature of the iron core of solenoid (which is ferromagnetic material) is raised beyond curie temperature, then soft iron core behaves as para magnetic material. We know that $(\chi_m)_{\text{para}} \approx 10^{-5}$. $(\chi_m)_{\text{Ferro}} \approx 10^3$

$$\therefore \quad \frac{(\chi_m)_{\text{Ferro}}}{(\chi_m)_{\text{Para}}} = \frac{10^3}{10^{-5}} = 10^8$$

Thus magnetisation of the core diminishes by a factor 10^8. Hence, **options (*a*) and (*d*) are correct.**

9. Knowledge based question.

10. The angle of dip at a point on the geographical equator satisfies the **options (*b*), (*c*) and (*d*).**

VERY SHORT ANSWER QUESTIONS

11. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials ?

Ans. Magnetic dipole moment due to spinning proton or electron, $M = \dfrac{eh}{4\pi m}$ or $M \propto \dfrac{1}{m}$.

$$\therefore \quad \frac{M_p}{M_e} = \frac{m_e}{m_p} = \frac{m_e}{1837\,m_e} = \frac{1}{1837} << 1 \qquad \therefore \; M_p << M_e$$

12. A permanent magnet in the shape of a thin cylinder of length 10 cm has $M = 10^6$ A/m. Calculate the magnetisation current I_M. (Here M is the intensity of magnetisation).

Ans. Here, $l = 10$ cm $= 0{\cdot}1$ m ; $M = 10^6$ A/m

As $M = \dfrac{I_M\ (\text{magnetisation current})}{l\ (\text{length})}$ $\qquad \therefore \; I_M = M \times l = 10^6 \times 0{\cdot}1 = \mathbf{10^5}$ A

13. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2 ($\sim 5 \times 10^{-9}$) (at STP) and Cu ($\sim 10^{-5}$).

Ans. Density of nitrogen, $\rho_{N_2} = \dfrac{28\,\text{g}}{22{\cdot}4\,\text{litre}} = \dfrac{28\,\text{g}}{22400\,\text{cc}}$

We know, density of copper, $\rho_{\text{cu}} = 8$ g/cc

$$\therefore \quad \frac{\rho_{N_2}}{\rho_{Cu}} = \frac{28}{22400} \times \frac{1}{8} = 1{\cdot}6 \times 10^{-4}.$$

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Given, $$\frac{\chi_{N_2}}{\chi_{Cu}} = \frac{5\times10^{-9}}{10^{-5}} = 5\times10^{-4} \quad ...(i)$$

We know that $\chi = \dfrac{\text{intensity of magnetisation } (I)}{\text{magnetising force } (H)} = \dfrac{I}{H} = \dfrac{\text{magnetic moment } (M)/\text{volume } (V)}{H}$

$$= \frac{M}{HV} = \frac{M}{H\,[\text{mass } (m)/\text{density } (\rho)]} = \frac{M\rho}{Hm}$$

$\therefore \quad \chi \propto \rho$ for the given value of M/Hm.

Hence, $$\frac{\chi_{N_2}}{\chi_{Cu}} = \frac{\rho_{N_2}}{\rho_{Cu}} = \mathbf{1{\cdot}6\times10^{-4}} \quad ...(ii)$$

From (i) and (ii), we note that the major difference in susceptibility of N_2 and Cu is due to their density.

14. From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.

Ans. Susceptibility of magnetic material, $\chi = I/H$ where I is the intensity of magnetisation induced in the material and H is the magnetising force.

Diamagnetism is due to orbital motion of electrons in an atom developing magnetic moments opposite to applied field. Due to it, the resultant magnetic moment of the diamagnetic material is zero. Hence, susceptibility χ of diamagnetic material is not much affected by temperature.

Paramagnetism and ferromagnetism is due to alignments of atomic magnetic moments in the direction of the applied field. As temperature is raised, this alignment is disturbed, resulting decrease in susceptibility of both, with increase in temperature.

15. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet. (i) In which direction will it move ? (ii) What will be the direction of it's magnetic moment ?

Ans. A superconducting material and nitrogen are diamagnetic in nature. When a ball of superconducting material is dipped in liquid nitrogen, it behaves as a diamagnetic material. When it is placed near a bar magnet, it will be feebly magnetised opposite to the direction of magnetising field. Due to it, (i) it will be repelled, *i.e.*, moving away from magnet (ii) its direction of magnetic moment will be opposite to the direction of magnetic field of magnet.

SHORT ANSWER QUESTIONS

16. Verify the Gauss's law for magnetic field of a point dipole of dipole moment $\vec{M}$ at the origin for the surface which is a sphere of radius R.

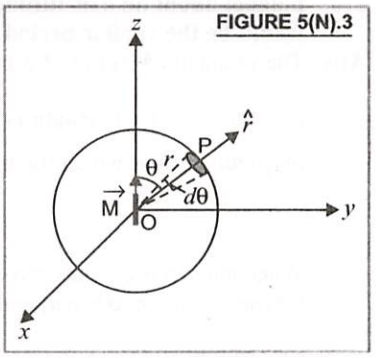


FIGURE 5(N).3

Ans. According to Gauss's law in magnetism, $\oint \vec{B}.\vec{ds} = 0$. To explain it as per question, refer Fig. 5(N).3, magnetic moment of dipole at origin O, $\vec{M} = M\hat{k}$. Let P be a point at distance r from O and OP makes an angle θ with z-axis. Component of $\vec{M}$ along $OP = M\cos\theta$.

The point P is on the axial line of dipole of magnetic moment $M\cos\theta$. The magnetic field induction at P due to dipole of moment $M\cos\theta$ is $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{2M\cos\theta}{r^3}\hat{r}$

Consider a spherical surface of radius r with centre at O lying in y–z plane. Take an elementary area ds of the surface at P. Then $\vec{ds} = r(r\sin\theta\, d\theta)\hat{r} = r^2\sin\theta\, d\theta\,\hat{r}$

$$\therefore \oint \vec{B}.\vec{ds} = \oint \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}\hat{r}.(r^2\sin\theta\, d\theta\,\hat{r}) = \frac{\mu_0}{4\pi}\frac{M}{r}\int_0^{2\pi} 2\sin\theta\cos\theta\, d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{M}{r}\int_0^{2\pi}\sin 2\theta\, d\theta = \frac{\mu_0}{4\pi}\frac{M}{r}\left(\frac{-\cos 2\theta}{2}\right)_0^{2\pi} = -\frac{\mu_0}{4\pi}\frac{M}{2r}[\cos 4\pi - \cos 0] = -\frac{\mu_0}{4\pi}\frac{M}{2r}[1-1] = \mathbf{0}.$$

17. Three identical bar magnets are rivetted together at centre in the same plane as shown in Fig. 5(N).4. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet are shown in the Fig. 5(N).4. Determine the poles of the remaining two.

FIGURE 5(N).4

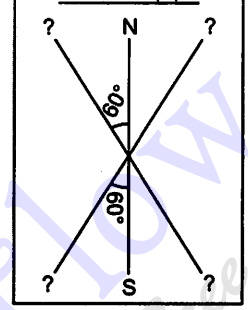
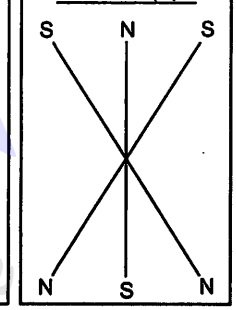


FIGURE 5(N).5



Ans. The system will be in stable equilibrium if the net force on the system is zero and net torque on the system is also zero. It will be so if the poles of the remaining two magnets are as shown in Fig. 5(N).5.

18. Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of (i) electric dipole $\vec{p}$ in an electrostatic field $\vec{E}$ and (ii) magnetic dipole $\vec{M}$ in a magnetic field $\vec{B}$. Write down a set of conditions on $\vec{E}, \vec{B}, \vec{p}, \vec{M}$ so that the two motions are verified to be identical. (Assume identical initial conditions).

Ans. Let θ be the angle between $\vec{p}$ and $\vec{E}$. Torque on electric dipole of moment $\vec{p}$ in electric field $\vec{E}$, $\tau = pE\sin\theta$. Let θ be the angle between $\vec{M}$ and $\vec{B}$. Torque on magnetic dipole moment $\vec{M}$ in magnetic field $\vec{B}$, $\tau' = MB\sin\theta$.

Two motions will be identical if $pE\sin\theta = MB\sin\theta$ or $pE = MB$

But $E = cB$ $\therefore$ $pcB = MB$ or $p = \frac{M}{c}$

19. A bar magnet of magnetic moment M and moment of inertia I (about centre, perpendicular to length) is cut into two equal pieces, perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid point, perpendicular to length, in a magnetic field $\vec{B}$. What would be the similar period T' for each piece ?

Ans. The moment of inertia of a bar magnet of mass m, length l about an axis passing through its centre and perpendicular to its length is $I = \frac{ml^2}{12}$. Let M be the magnetic moment of the magnet, B is the uniform magnetic field in which the magnet is oscillating, then time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad ...(i)$$

When magnet is cut into two equal pieces, perpendicular to length, then moment of inertia of each piece of magnet about an axis perpendicular to length passing through its centre is

$$I' = \frac{m}{2}\frac{(l/2)^2}{12} = \frac{ml^2}{12}\times\frac{1}{8} = \frac{I}{8}$$

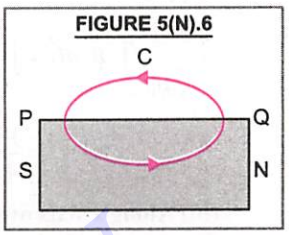
Magnetic dipole moment, $M' = M/2$

Its time period of oscillation is $T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{(M/2)B}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}} = \frac{T}{2}$

20. Use (i) the Ampere's law for $\vec{H}$ and (ii) continuity of lines of $\vec{B}$, to conclude that inside a bar magnet, (a) lines of $\vec{H}$ run from the *N* pole to *S* pole, while (b) lines of $\vec{B}$ must run from the *S* pole to *N* pole.

Ans. Consider a magnetic field line of $\vec{B}$ through the bar magnet as shown in Fig. 5(N).6. It must be a closed loop. Let *C* be the amperian loop. Then

FIGURE 5(N).6



$$\int_Q^P \vec{H}.d\vec{l} = \int_Q^P \frac{\vec{B}}{\mu_0}.d\vec{l} > 0 \qquad (i.e., \text{ positive})$$

It is so because the angle between $\vec{B}$ and $d\vec{l}$ is less than 90° inside the bar magnet. So $\vec{B}.d\vec{l}$ is positive.

Hence, *the lines of* $\vec{B}$ *must run from S pole to N pole inside the bar magnet.*

According to Ampere's law, $\oint_{PQP} \vec{H}.d\vec{l} = 0 \quad \therefore \quad \oint_{PQP} \vec{H}.d\vec{l} = \int_P^Q \vec{H}.d\vec{l} + \int_Q^P \vec{H}.d\vec{l} = 0$

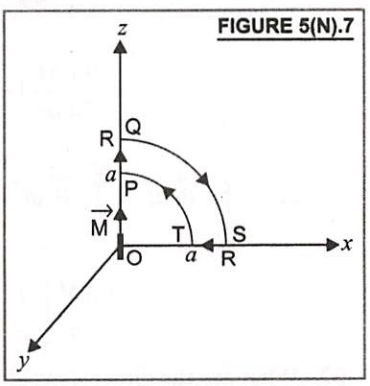
As $\int_Q^P \vec{H}.d\vec{l} > 0$, so $\int_P^Q \vec{H}.d\vec{l} < 0$ (*i.e.*, negative)

It will be so if angle between $\vec{H}$ and $d\vec{l}$ is more than 90°, so that cos θ is negative. It means the line of $\vec{H}$ must run from *N* pole to *S* pole inside the bar magnet.

LONG ANSWER QUESTIONS

21. Verify the Ampere's law for magnetic field of a point dipole of dipole moment $\vec{M} = M\hat{k}$. Take *C* as the closed curve running clockwise along (i) the z-axis from $z = a > 0$ to $z = R$; (ii) along the quarter circle of radius *R* and centre at the origin, in the first quadrant of *x*-*z* plane; (iii) along the *x*-axis from $x = R$ to $x = a$, and (iv) along the quarter circle of radius *a* and centre at the origin in the first quadrant of *x*-*z* plane.

FIGURE 5(N).7



Ans. Refer Fig. 5(N).7, From *P* to *Q*, every point on the *z*-axis lies at the axial line of magnetic dipole of moment $\vec{M}$. Magnetic field induction at a point distance *z* from the magnetic dipole of moment $\vec{M}$ $(= M\hat{k})$ is

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2|\vec{M}|}{z^3} = \frac{\mu_0 M}{2\pi z^3}$$

(*i*) **Along *z*-axis from *P* to *Q*.**

$$\int_P^Q \vec{B}.d\vec{l} = \int_P^Q B\,dl\cos 0° = \int_a^R B\,dz$$

$$= \int_a^R \frac{\mu_0}{2\pi}\frac{M}{z^3}dz = \frac{\mu_0 M}{2\pi}\left(-\frac{1}{2}\right)\left(\frac{1}{R^2} - \frac{1}{a^2}\right) = \frac{\mu_0 M}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right)$$

(*ii*) **Along the quarter circle *QS* of radius *R*.** Refer Fig. 5(N).8, point *A* lies on the equatorial line of the magnetic dipole of moment *M* sin θ.

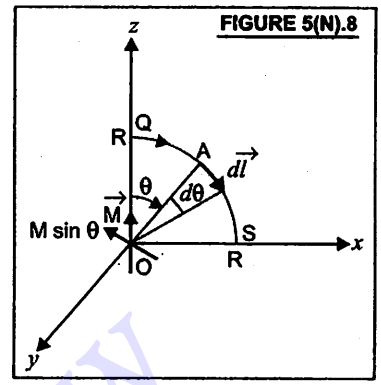
Magnetic field at point A on the circular arc is

$$B = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{R^3} \; ; \quad dl = R\,d\theta$$

$$\therefore \int_{\text{circular arc QS}} \vec{B}.\vec{dl} = \int B\,dl\cos 0 = \int_0^{\pi/2} \frac{\mu_0}{4\pi}\frac{M\sin\theta}{R^3} R\,d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{M}{R^2}(-\cos\theta)_0^{\pi/2} = \frac{\mu_0}{4\pi}\frac{M}{R^2}$$

FIGURE 5(N).8



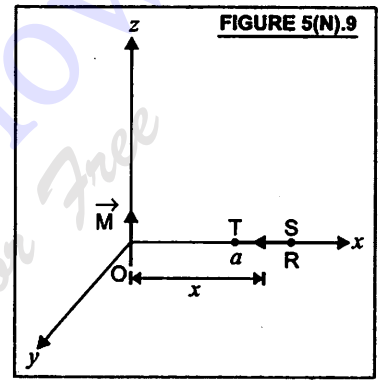
(*iii*) **Along *x*-axis over the path *ST*.** Fig. 5(N).9, every point lies on the equatorial line of magnetic dipole. Magnetic field induction at a point distance x from the dipole is

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{\vec{M}}{x^3}$$

$$\therefore \int_S^T \vec{B}.\vec{dl} = \int_R^a -\frac{\mu_0}{4\pi x^3}\vec{M}.\vec{dl} = 0$$

[$\because$ angle between $(-\vec{M})$ and $\vec{dl}$ is 90°]

FIGURE 5(N).9



(*iv*) **Along the quarter circle *TP* of radius *a*.** Refer Fig. 5(N).10.

Refer case (*ii*) ; Line integral of $\vec{B}$ along the quarter circle *TP* of radius a is

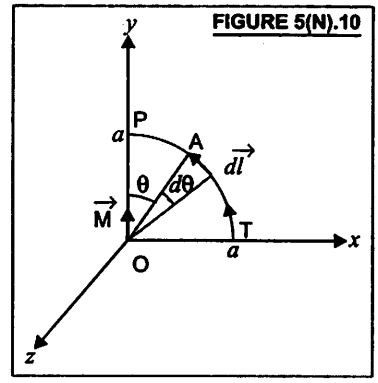
$$\int_{\text{circular arc TP}} \vec{B}.\vec{dl} = \int_{\pi/2}^{0} \frac{\mu_0}{4\pi}\frac{M\sin\theta}{a^3} a\,d\theta = \frac{\mu_0}{4\pi}\frac{M}{a^2}\int_{\pi/2}^{0} \sin\theta\,d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{M}{a^2}[-\cos\theta]_{\pi/2}^0 = \frac{\mu_0}{4\pi}\frac{M}{a^2}$$

$$\therefore \oint_{PQST} \vec{B}.\vec{dl} = \int_P^Q \vec{B}.\vec{dl} + \int_Q^S \vec{B}.\vec{dl} + \int_S^T \vec{B}.\vec{dl} + \int_T^P \vec{B}.\vec{dl}$$

$$= \frac{\mu_0 M}{4\pi}\left[\frac{1}{a^2} - \frac{1}{R^2}\right] + \frac{\mu_0}{4\pi}\frac{M}{R^2} + 0 + \left(-\frac{\mu_0}{4\pi}\frac{M}{a^2}\right) = 0$$

FIGURE 5(N).10



22. What are the dimensions of χ, the magnetic susceptibility ? Consider an H-atom. Guess an expression for χ upto a constant by constructing a quantity of dimensions of χ, out of parameters of the atom : e, m, v, R and μ_0. Here, m is the electronic mass, v is electronic velocity, R is Bohr radius. Estimate the number so obtained and compare with the value of $|\chi| \sim 10^{-5}$ for many solid materials.

Ans. Magnetic susceptibility, $\chi_m = \dfrac{I}{H}\dfrac{\text{(intensity of magnetisation)}}{\text{(magnetising force)}}$

As I and H have same units and dimensions, hence χ has no dimensions.

In this question, χ is to be related with e, m, v, R and μ_0. We know that dimensions of $\mu_0 = [MLQ^{-2}]$

[From Biot Savart's law, $dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\sin\theta}{r^2}$

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or $$\mu_0 = \frac{4\pi r^2\, dB}{I\, dl \sin\theta} = \frac{4\pi r^2}{I\, dl \sin\theta} \times \frac{F}{q v \sin\theta'} \qquad \left[\because dB = \frac{F}{q v \sin\theta'}\right]$$

$\therefore$ Dimensions of $\mu_0 = \dfrac{L^2 \times (MLT^{-2})}{(QT^{-1})(L) \times 1 \times (Q)(LT^{-1}) \times (1)} = [MLQ^{-2}]$ where Q is for charge]

As χ is dimensionless, it should have no involvement of charge Q in its dimensional formula. It will be so if μ_0 and e together should have the value $\mu_0 e^2$, as e has the dimensions of charge.

Let $$\chi = \mu_0 e^2 m^a v^b R^c \qquad ...(i)$$

where a, b, c are the powers of m, v and R such that relation (i) is satisfied.

Dimensional equation of (i) is $M^0 L^0 T^0 Q^0 = (M^1 L^1 Q^{-2}) \times (Q^2)\,(M^a) \times (LT^{-1})^b \times (L)^c$

$$= M^{1+a} L^{1+b+c} T^{-b} Q^0$$

Equating the powers of M, L and T, we get

$0 = 1 + a$ or $a = -1$; $\qquad 0 = 1 + b + c$ ...(ii)

$0 = -b$ or $b = 0$; From (ii), $0 = 1 + 0 + c$ or $c = -1$

Putting values in (i), we get

$$\chi = \mu_0 e^2 m^{-1} v^0 R^{-1} = \frac{\mu_0 e^2}{mR} \qquad ...(iii)$$

Here, $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1} ; $e = 1{\cdot}6 \times 10^{-19}$ C,
$m = 9{\cdot}1 \times 10^{-31}$ kg ; $R = 10^{-10}$ m

$\therefore$ $$\chi = \frac{(4\pi \times 10^{-7}) \times (1{\cdot}6 \times 10^{-19})^2}{(9{\cdot}1 \times 10^{-31}) \times 10^{-10}} \approx 10^{-4}$$

$\therefore$ $$\frac{\chi}{\chi_{(\text{given solid})}} = \frac{10^{-4}}{10^{-5}} = \mathbf{10}$$

23. Assume the dipole model for earth's magnetic field B which is given by B_V = vertical component of magnetic field $= \dfrac{\mu_0}{4\pi}\dfrac{2M\cos\theta}{r^3}$; B_H = Horizontal component of mangetic field $= \dfrac{\mu_0}{4\pi}\dfrac{\sin\theta\, M}{r^3}$; $\theta = 90° -$ latitude as measured from magnetic equator.

Find loci of points for which (i) $|\vec{B}|$ is minimum ; (ii) dip angle is zero ; and (iii) dip angle is $\pm 45°$.

Ans. (i) Given, vertical component of earth's magnetic field, $B_V = \dfrac{\mu_0}{4\pi}\dfrac{2M\cos\theta}{r^3}$

Horizontal component of earth's magnetic field $B_H = \dfrac{\mu_0}{4\pi}\dfrac{M\sin\theta}{r^3}$

$\therefore$ $$B = \sqrt{B_V^2 + B_H^2} = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{3\cos^2\theta + 1} \qquad ...(i)$$

From (i), we note that B is minimum if $\cos\theta = 0$ or $\theta = \dfrac{\pi}{2}$ which is so, at magnetic equator. Thus, B is **minimum at magnetic equator which is the loci of points.**

(ii) If δ is the angle of dip, then $\tan\delta = \dfrac{B_V}{B_H} = \dfrac{2\cos\theta}{\sin\theta} = 2\cot\theta$.

Angle of dip δ is zero if $\cot\theta = 0$. It will be so if $\theta = \pi/2$. So, **the angle of dip is zero on magnetic equator which is the loci of points.**

(iii) If δ is 45°, then $\tan 45° = 2\cot\theta = 2/\tan\theta$ or $\tan\theta = 2$ or $\theta = \tan^{-1}(2)$ **is the loci of points.**

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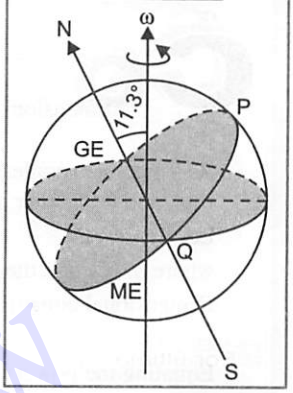
24. Consider the plane S formed by the dipole axis and the axis of earth. Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators Obtain the declination and dip angles at P and Q.

FIGURE 5(N).11



Ans. Refer Fig. 5(N).11 ;

(*i*) The point P is on the magnetic equator and in S, so at P,

declination = **0** and angle of dip = **0**

(*ii*) The point Q is at the intersection of geographical equator (GE) and magnetic equator (ME), so dip = **0** and declination = **11·3°**

25. There are two current carrying planar coils made each from identical wires of length L. C_1 is circular (radius R) and C_2 is square (side a). They are so constructed that they have same frequency of oscillation when they are placed in the same uniform $\vec{B}$ and carry the same current i. Find a in terms of R.

Ans. **For circular coil C_1,** $n_1 = \dfrac{L}{2\pi R}$; Magnetic moment, $M_1 = n_1 i A_1 = \dfrac{L}{2\pi R} \times i \times \pi R^2 = \dfrac{LiR}{2}$

For square coil C_2, $n_2 = \dfrac{L}{4a}$; Magnetic moment, $M_2 = n_2 i A_2 = \dfrac{L}{4a} \times i \times a^2 = \dfrac{Lia}{4}$

Moment of inertia of circular coil about the diameter as axis, $I_1 = \dfrac{\text{mass} \times (\text{radius})^2}{2} = \dfrac{mR^2}{2}$

Moment of inertia of square coil about an axis passing through its centre parallel to breadth

$$I_2 = \frac{ma^2}{12}.$$

Time period of oscillation of the magnet in magnetic field is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \text{and} \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{MB}{I}}$$

$$\therefore \quad \omega_1^2 = \frac{M_1 B}{I_1} \quad \text{and} \quad \omega_2^2 = \frac{M_2 B}{I_2}$$

Given, $\omega_1^2 = \omega_2^2$, so $\dfrac{M_1}{I_1} = \dfrac{M_2}{I_2}$ or $\dfrac{LIR/2}{mR^2/2} = \dfrac{LIa/4}{ma^2/12}$

On solving, a = **3 R**

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I. Multiple Choice Questions (with One Correct Answer)

I. Force, torque and potential energy of a magnet in magnetic field

1. A magnet with moment M is given. If it is bent into a semicircular form, its new magnetic moment will be :

(a) M/π (b) $M/2$

(c) M (d) $2M/\pi$

(DPMT 2002)

2. An insulating thin rod of length l has a linear charge density $\rho_x = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is :

(a) $\pi n \rho l^3$ (b) $\frac{\pi}{3} n \rho l^3$

(c) $\frac{\pi}{4} n \rho l^3$ (d) $n \rho l^3$

(JEE Main 2019)

3. A magnetic needle suspended parallel to a magnetic field requires $\sqrt{3}$ J of work to turn it through 60°. The torque needed to maintain the needle in this position will be :

(a) $2\sqrt{3}$ J (b) 3 J

(c) $\sqrt{3}$ J (d) $\frac{3}{2}$ J

[AIPMT (Main) 2012]

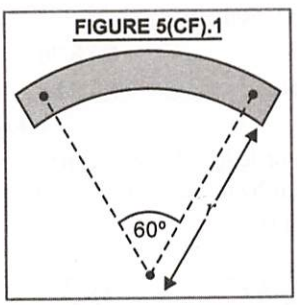
4. An insulating rod of length L carries a charge Q distributed uniformly on it. The rod is pivoted at its mid-point and is rotated at a frequency v about a fixed axis perpendicular to the length of rod passing through the pivot. The magnetic moment of the rod system is

(a) $\frac{\pi Q v L^2}{2}$ (b) $\frac{\pi Q v L^2}{3}$

(c) $\frac{\pi Q v L^2}{6}$ (d) $\frac{\pi Q v L^2}{12}$

5. A bar magnet of length 'l' and magnetic dipole moment 'M' is bent in the form of an arc as shown in Fig 5(CF).1. The new magnetic dipole moment will be

FIGURE 5(CF).1



ANSWERS

1. (d) 2. (b) 3. (b) 4. (d)

(a) $\frac{M}{2}$ (b) M

(c) $\frac{3}{\pi}M$ (d) $\frac{2}{\pi}M$

(AIPMT 2013)

6. A 250 turn rectangular coil of length 2·1 cm and width 1·25 cm carries a current of 85 μA and subjected to a magnetic field of strength 0·85 T. Work done for rotating the coil by 180° against the torque is

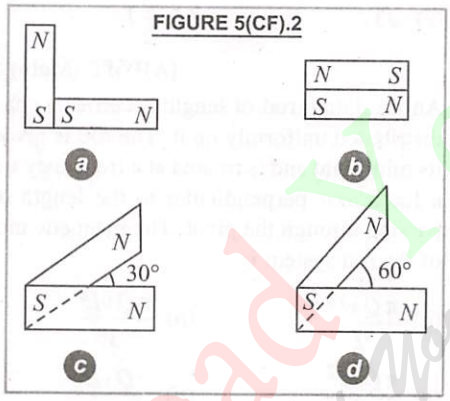
(a) 9·1 μJ (b) 4·55 μJ

(c) 2·3 μJ (d) 1·15 μJ

(NEET 2017)

7. Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment (m). Which configuration has highest value of magnetic dipole moment ?

FIGURE 5(CF).2



(AIPMT 2014)

8. A magnetic needle of magnetic moment 6·7 × 10^{-2} Am^2 and moment of inertia 7·5 × 10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0·01 T. Time taken for 10 complete oscillations is

(a) 6·98 s (b) 8·76 s

(c) 6·65 s (d) 8·89 s

(JEE Main 2017)

9. A wire of length L metre carrying a current of I ampere is bent in the form of a circle. Its magnetic moment is :

(a) $\frac{IL^2}{4\pi}$ Am^2 (b) $\frac{IL^2}{4}$ Am^2

(c) $\frac{I\pi L^2}{4A}$ Am^2 (d) $\frac{2IL^2}{\pi}$ Am^2

(NEET 2020)

10. A uniform conducting wire of length 12 a and resistance 'R' is wound up as a current carrying coil in the shape of,

(i) an equilateral triangle of side 'a'

(ii) a square of side 'a'

The magnetic dipole moments of the coil in each case respectively are

(a) 4 Ia^2 and 3 Ia^2 (b) $\sqrt{3}$ Ia^2 and 3 Ia^2

(c) 3 Ia^2 and Ia^2 (d) 3 Ia^2 and 4 Ia^2

(NEET 2021)

11. A wire of length L metre carrying a current of I ampere is bent in the form of a circle of two turns. Its magnetic moment is

(a) $IL^2/4\pi$ Am^2 (b) $IL^2/8\pi$ Am^2

(c) $I\pi L^2/8$ Am^2 (d) $4IL^2/\pi$ Am^2

II. Magnetic field due to magnet ; neutral points and vibration magnetometer

12. Two identical magnetic dipoles of magnetic moment 1·0 Am^2 each, placed at a separation of 2 m with their axes perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

(a) 5 × 10^{-7} T (b) $\sqrt{5}\times10^{-7}$ T

(c) 10^{-7} T (d) 2 × 10^{-7} T

13. Two identical magnetic dipoles of magnetic moments 2 Am^2 are placed at a separation of 2 m with their axes perpendicular to each other in air. The resultant magnetic field at a mid point between the dipoles is

(a) $4\sqrt{5}\times10^{-5}$ T (b) $2\sqrt{5}\times10^{-5}$ T

(c) $4\sqrt{5}\times10^{-7}$ T (d) $2\sqrt{5}\times10^{-7}$ T

(e) $4\sqrt{2}\times10^{-6}$ T **(Kerala PET 2011)**

14. A vibration magnetometer placed in magnetic meridian has a small bar magnet. The magnet executes oscillations with a time period of 2 sec in earth's horizontal magnetic field of 24 micro-tesla. When a horizontal field of 18 microtesla is

ANSWERS

5. (c) 6. (a) 7. (c) 8. (c) 9. (a) 10. (b) 11. (b) 12. (b) 13. (d)

produced opposite to the earth's field by placing a current carrying wire, the new time period of magnet will be

(*a*) 1 s (*b*) 2 s

(*c*) 3 s (*d*) 4 s **(AIPMT 2010)**

15. A magnet is cut in three equal parts by cutting it perpendicular to its length. The time period of original magnet is T_0 in a uniform magnetic field B. Then, the time period of each part in the same magnetic field is

(*a*) $\frac{T_0}{2}$ (*b*) $\frac{T_0}{3}$

(*c*) $\frac{T_0}{4}$ (*d*) none of these

(AIIMS 2014)

16. A small bar magnet placed with its axis at 30° with an external field of 0·6 T experiences a torque of 0·018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is

(*a*) $9{\cdot}2 \times 10^{-3}$ J (*b*) $6{\cdot}4 \times 10^{-2}$ J

(*c*) $11{\cdot}7 \times 10^{-3}$ J (*d*) $7{\cdot}2 \times 10^{-2}$ J

(JEE Main 2020)

III. Magnetic elements at a place

17. If θ_1 and θ_2 be the apparent angles of dip observed in two vertical planes at right angles to each other, then the true angle of dip θ is given by

(*a*) $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

(*b*) $\tan^2\theta = \tan^2\theta_1 + \tan^2\theta_2$

(*c*) $\cot^2\theta = \cot^2\theta_1 - \cot^2\theta_2$

(*d*) $\tan^2\theta = \tan^2\theta_1 - \tan^2\theta_2$ **(NEET 2017)**

18. At a point A on the earth's surface the angle of dip, $\delta = +25°$. At a point B on the earth's surface the angle of dip, $\delta = -25°$. We can interpret that :

(*a*) A and B are both located in the northern hemisphere

(*b*) A is located in the southern hemisphere and B is located in the northern hemisphere

(*c*) A is located in the northern hemisphere and B is located in the southern hemisphere

(*d*) A and B are both located in the southern hemisphere **(NEET 2019)**

19. An iron rod of susceptibility 599 is subjected to a magnetising field of 1200 A m^{-1}. The permeability of the material of the rod is :

($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(*a*) $8{\cdot}0 \times 10^{-5}$ T m A^{-1}

(*b*) $2{\cdot}4\pi \times 10^{-5}$ T m A^{-1}

(*c*) $2{\cdot}4\pi \times 10^{-7}$ T m A^{-1}

(*d*) $2{\cdot}4\pi \times 10^{-4}$ T m A^{-1} **(NEET 2020)**

20. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties. Of the following, which property best matches for the type of magnet required ?

(*a*) T : Large retentivity ; Small coercivity

(*b*) P : Small retentivity ; Large coercivity

(*c*) T : Large retentivity ; Large coercivity

(*d*) P : Large retentivity ; Large coercivity

(JEE Main 2020)

21. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0·5 A is passed through the solenoid, then the magnetic moment of the rod will be

(*a*) $0{\cdot}5 \times 10^2$ Am^2 (*b*) 50×10^2 Am^2

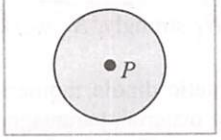
(*c*) 5×10^2 Am^2 (*d*) 500×10^2 Am^2

(JEE Main 2020)

IV. Dia, Para and Ferromagnetic materials

22. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is

FIGURE 5(CF).3



(*a*) much larger than $|\vec{B}|$ and parallel to $\vec{B}$

(*b*) zero (*c*) $\vec{B}$

(*d*) much larger than $|\vec{B}|$ but opposite to $\vec{B}$

(JEE Main 2020)

ANSWERS

14. (*d*) **15.** (*b*) **16.** (*d*) **17.** (*a*) **18.** (*c*) **19.** (*d*) **20.** (*d*) **21.** (*c*) **22.** (*b*)

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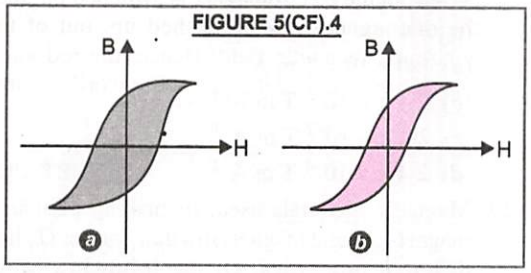
23. Hysteresis loop for two magnetic materials *A* and *B* are given below :

FIGURE 5(CF).4



These materials are used to make magnets for electric generators, transformer core and electromagnetic core. Then it is proper to use :

(*a*) *A* for electric generators and transformers

(*b*) *A* for electromagnets and *B* for electric generators

(*c*) *A* for transformers and *B* for electric generators

(*d*) *B* for electromagnets and transformers.

(JEE Main 2016)

24. For which of the following magnetic materials, $B = \mu H$ is valid, where *B* is the magnetic induction and *H* is the magnetising field and μ is nearly a constant for a wide range of *H* ?

(*a*) Diamagnetic and ferromagnetic materials only

(*b*) Diamagnetic and paramagnetic materials only

(*c*) Paramagnetic and ferromagnetic materials only

(*d*) Diamagnetic, paramagnetic and ferromagnetic materials **(IAS 2010)**

25. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when close to them, will

(*a*) attract N_1 strongly, but repel N_2 and N_3 weekly

(*b*) attract all three of them

(*c*) attract N_1 and N_2 strongly but repel N_3

(*d*) attract N_1 strongly, N_2 weakly and repel N_3 weakly **(AIEEE 2006)**

26. If the magnetic dipole moment of an atom of diamagnetic material, paramagnetic material and ferromagnetic material are denoted by μ_d, μ_p and μ_f respectively, then :

(*a*) $\mu_p = 0$ and $\mu_f \neq 0$

(*b*) $\mu_d \neq 0$ and $\mu_p = 0$

(*c*) $\mu_d \neq 0$ and $\mu_f \neq 0$

(*d*) $\mu_d = 0$ and $\mu_p \neq 0$

27. Ferro magnetic materials used in transformer must have

(*a*) low permeability and high hysteresis loss

(*b*) high permeability and low hysteresis loss

(*c*) high permeability and high hysteresis loss

(*d*) low permeability and low hysteresis loss

(Karntataka CET 2009)

28. If the magnetising field on a ferromagnetic material is increased, its permeability is

(*a*) decreased (*b*) increased

(*c*) is unaffected

(*d*) may be increased or decreased

(AIIMS 2015)

29. A 250 turn rectangular coil of length 2·1 cm and width 1·25 cm carries a current of 85 μA and subjected to a magnetic field of strength 0·85 T. Work done for rotating the coil by 180° against the torque is

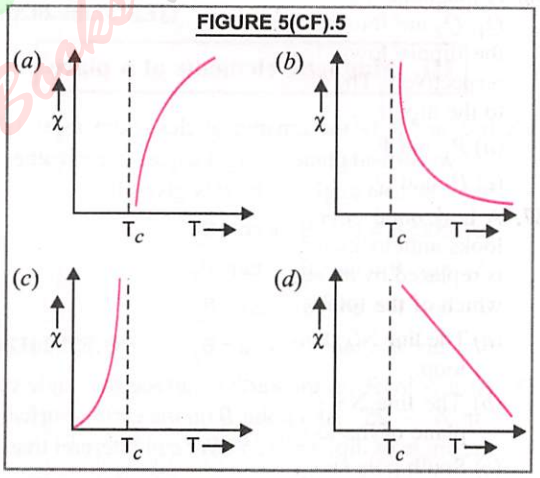
(*a*) 9·1 μJ (*b*) 4·55 μJ

(*c*) 2·3 μJ (*d*) 1·15 μJ

(NEET 2017)

30. The variation of magnetic susceptibility χ with the temperature *T* of a ferromagnetic material can be plotted as

FIGURE 5(CF).5



(AIIMS 2015)

31. A bar magnet is demagnetised by inserting it inside a solenoid of length 0·2 m, 100 turns and carrying a current of 5·2 A. The coercivity of the bar magnet is

(*a*) 285 A/m (*b*) 2600 A/m

(*c*) 520 A/m (*d*) 1200 A/m

(JEE Main 2019)

ANSWERS

23. (*d*) **24.** (*d*) **25.** (*d*) **26.** (*d*) **27.** (*b*) **28.** (*a*) **29.** (*a*) **30.** (*b*) **31.** (*b*)

32. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid is :

(*a*) 3 A (*b*) 6 A

(*c*) 30 mA (*d*) 60 mA

(JEE Main 2014)

33. A superconductor has T_c (0) = 100 K. When a magnetic field of 7·5 tesla is applied, its T_c decreases to 75 K. For this material, one can definitely say that when

(*a*) B = 5 tesla, T_c (B) = 80 K

(*b*) B = 5 tesla, 75 K < T_c (B) < 100 K

(*c*) B = 10 tesla, 75 K < T_c (B) < 100 K

(*d*) B = 10 tesla, T_c (B) = 70 K

(AIPMT 2010)

34. A thin diamagnetic rod is placed vertically between the poles of an electromagnet. When the current in the electromagnet is switched on, then the diamagnetic rod is pushed up, out of the horizontal magnetic field. Hence, the rod gains gravitational potential energy. The work required to do this comes from

(*a*) The lattice structure of the material of the rod

(*b*) The magnetic field

(*c*) The current source

(*d*) The induced electric field due to the changing magnetic field **(NEET 2018)**

35. A bar magnet is osciallating in the earth's magnetic field with a period T. What happens to its period of motion if its mass is quadrupled without changing its dimensions ?

(*a*) Motion remains *SHM* with period = T/2

(*b*) Motion remains *SHM* with period = 2 T

(*c*) Motion remains *SHM* with period = 4 T

(*d*) Motion is not *SHM*, but period = T

II. Multiple Choice Questions (with One or More than One Correct Answers)

36. A magnetic dipole is placed north south. P_1, P_2, Q_1, Q_2 are four points at the same distance from the dipole towards north, south, east and west respectively. The directions of magnetic fields due to the dipole are same at

(*a*) P_1 and P_2 (*b*) Q_1 and Q_2

(*c*) P_1 and Q_1 (*d*) P_2 and Q_2.

37. A horizontal circular loop carries a current that looks anticlockwise when viewed from above. It is replaced by an equivalent magnetic dipole NS, which of the following is true ?

(*a*) The line NS should be along a diameter of the loop.

(*b*) The line NS should be perpendicular to the plane of the loop.

(*c*) South pole should be below the loop.

(*d*) North pole should be below the loop.

38. In the given diagram, a line of force of a particular field is shown in Fig. 5(CF).6. Out of the following options, it can never represent

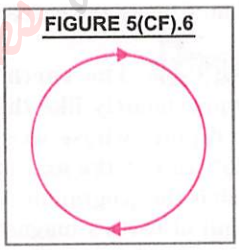
(*a*) an electric field

(*b*) a magnetic field

(*c*) a gravitational field of a mass at rest

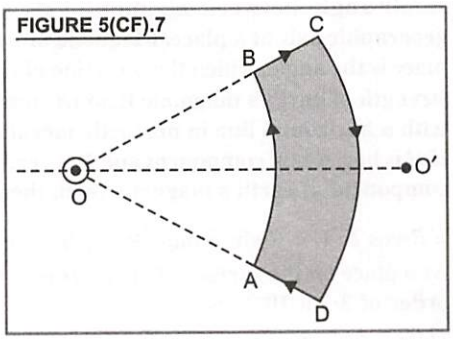
(*d*) an induced electric field

FIGURE 5(CF).6



39. An infinite current carrying wire passes through point O and is perpendicular to the plane containing a current carrying loop ABCD as shown in Fig. 5(CF).7. Choose the correct option (s).

FIGURE 5(CF).7



ANSWERS

32. (*a*) **33.** (*b*) **34.** (*c*) **35.** (*b*) **36.** (*a,b*) **37.** (*b,c*) **38.** (*a,c*)

(*a*) Net force on the loop is zero.

(*b*) Net torque on the loop is zero.

(*c*) As seen from O, the loop rotates clockwise.

(*d*) As seen from O, the loop rotates anticlockwise.

40. A particle of mass M and positive charge Q moving with a constant velocity $\vec{u_1} = 4\hat{i}$ ms^{-1}, enters in a region of uniform static magnetic field, normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the otherside after 10 milliseconds with a velocity $\vec{u_2} = 2(\sqrt{3}\hat{i} + \hat{j})$ ms^{-1}. The correct statement(s) is (are)

(*a*) the direction of magnetic field is $-z$ direction

(*b*) the direction of magnetic field is $+z$ direction

(*c*) the magnitude of magnetic field is $\frac{50\pi M}{3Q}$ units

(*d*) the magnitude of magnetic field is $\frac{100\pi M}{3Q}$ units

(JEE Advanced 2013)

41. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2 R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)

(*a*) In the region $0 < r < R$, the magnetic field is non-zero

(*b*) In the region $R < r < 2R$, the magnetic field is along the common axis

(*c*) In the region $R < r < 2R$, the magnetic field is tangent to the circle of radius r, centred on the axis

(*d*) In the region $r > 2R$, the magnetic field is non-zero **(JEE Advanced 2013)**

III Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1

The earth's magnetic field is approximately like that of a giant magnetic dipole, whose axis is inclined roughly 20° west of the axis of rotation of earth which is the geographic *N*-*S* direction. The strength of earth's magnetic field is of the order of 10^{-4} tesla. Three magnetic elements of earth are : Magnetic declination (θ), magnetic inclination or magnetic dip (δ) and horizontal component (H) of earth's magnetic field. Magnetic declination is the small angle between magnetic axis and geographic axis at a place. Magnetic dip at a place is the angle, which the direction of total strength of earth's magnetic field (R) makes with a horizontal line in magnetic meridian. If H is horizontal component and V is vertical component of earth's magnetic field, then $H = R\cos\delta$, $V = R\sin\delta$ and $R = \sqrt{H^2 + V^2}$. At a place on the surface of earth. H is of the order of $3{\cdot}2 \times 10^{-5}$ tesla.

Answer the following questons

42. The intensity of earth's magnetic field at the surface of earth is of the order of

(*a*) 10^{-4} T (*b*) 10^{-4} G

(*c*) 10^{-3} T (*d*) 10^{-3} G

43. The vertical component of earth's magnetic field is zero at a place where angle of dip is

(*a*) 45° (*b*) 60°

(*c*) 90° (*d*) 0°

44. At magnetic poles, a compass needle points out

(*a*) along *N*-*S*

(*b*) in any direction

(*c*) along *E*-*W*

(*d*) at 45° to *N*-*S* direction.

45. Magnetic declination is

(*a*) 20° west (*b*) 20° south

(*c*) 20° east (*d*) 20° north

ANSWERS

39. (*a*,*c*) **40.** (*a*,*c*) **41.** (*a*,*d*) **42.** (*a*) **43.** (*d*) **44.** (*b*) **45.** (*a*)

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices.

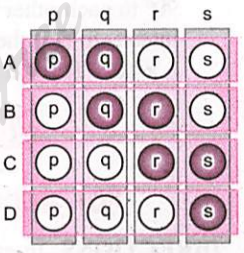
46. There are physical quantities in column I and demensional formulae in column II

	Column I		Column II
(A)	Permeability	(*p*)	$[M^0 L^0 T^0 A^0]$
(B)	Magnetic induction	(*q*)	$[M^0 L^{-1} T^0 A^1]$
(C)	Susceptibility	(*r*)	$[M L T^{-2} A^{-2}]$
(D)	Intensity of magnetisation	(*s*)	$[M L^0 T^{-2} A^{-1}]$

(*a*) A-*p* ; B-*q* ; C-*r* ; D-*s* (*b*) A-*q* ; B-*r* ; C-*s* ; D-*p* (*c*) A-*r* ; B-*s* ; C-*p* ; D-*q* (*d*) A-*s* ; B-*p* ; C-*q* ; D-*r*

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*p*, A-*q* ; B-*q*, B-*r* ; C-*r*, C-*s* and D-*s*, then the correctly bubbled matrix will look like the one shown here.



47.

	Column I		Column II
(A)	Paramagnetic material	(*p*)	They move from weaker to stronger part of the magnetic field.
(B)	Ferromagnetic material	(*q*)	The value of susceptibility is positive.
(C)	Hard magnetic material	(*r*)	Permanent magnet
(D)	Soft magnetic material	(*s*)	Core of transformer

48.

	Column I		Column II
(A)	Magnetic moment	(*p*)	scalar
(B)	Magnetic induction	(*q*)	vector
(C)	Permeability	(*r*)	*N/A-m*
(D)	Intensity of magnetization	(*s*)	Nm^3/Wb

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

49. The net magnetic moment of the system of magnetic poles shown in Fig. 5(CF).8 is *n ma* where *n* is nearly :

ANSWERS

46. (*c*) **47.** A-*p*, *q* ; B-*p*, *q*, *r*, *s* ; C-*p*, *q*, *r* ; D-*p*, *q*, *s* **48.** A-*q*, *s* ; B-*q*, *r* ; C-*p* ; D-*q*

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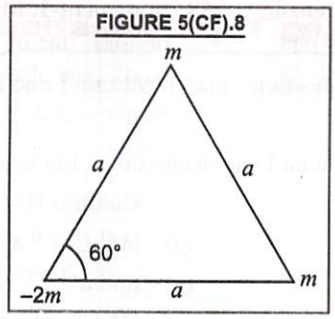
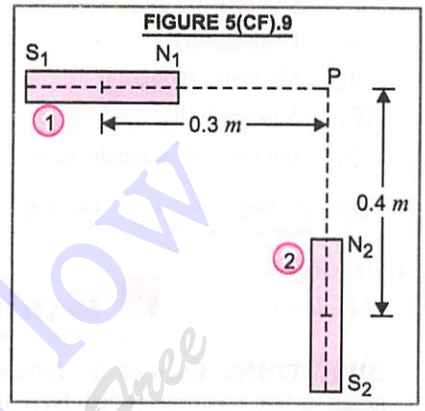
FIGURE 5(CF).8



50. The coercivity of a bar magnet is 120 A/m. It is to be demagnetised by placing it inside a solenoid of length 120 cm and number of turns 72. The current (in A) flowing through the solenoid is :

51. Two short magnets (1) and (2) of magnetic moments 2 Am^2 and 5 Am^2 respectively are placed along two lines drawn at 90° to each other as shown in Fig. 5(CF).9. At the point of intersection of their axes, the magnitude of magnetic field is $n \times 10^{-5}$ T. What is n ?

FIGURE 5(CF).9



VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if :

(A) both **Assertion** and **Reason** are true and the **Reason** is the correct explanation of the **Assertion.**

(B) both **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

(C) **Assertion** is true but the **Reason** is false.

(D) both **Assertion** and **Reason** are false.

52. Assertion. Poles of a magnet can never be separated.

Reason. Since each atom of a magnetic material is a magnet in itself.

(*a*) A (*b*) B (*c*) C (*d*) D

53. Assertion. Magnetic moment is a vector quantity, whose direction inside the magnet is, from South to North.

Reason. Magnetic lines of force emanate from *N*-pole and enter into the *S*-pole.

(*a*) A (*b*) B (*c*) C (*d*) D

54. Assertion. An atom of a magnetic material behaves as a dipole because of positive charge on the nucleus.

Reason. Charge is responsible for dipole moment.

(*a*) A (*b*) B (*c*) C (*d*) D

55. Assertion. The magnetic moment (μ) of an electron revolving around the nucleus decreases with increasing principal quantum number (n).

Reason. Magnetic moment of the revolving electron, $\mu \propto n$.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

56. Assertion. Above Curie temperature, a frerromagnetic material becomes paramagnetic.

Reason. When a magnetic material is heated to very high temperature, it loses its magnetic properties.

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

FOR ENGINEERING STUDENTS

DIRECTIONS. In each of the following questions read the two statements and choose if,

(A) Statement-1 is true ; Statement-2 is true ; Statement-2 is a correct explanation of Statement-1.

(B) Statement-1 is true ; Statement-2 is true ; Statement-2 is not a correct explanation of Statement-1.

(C) Statement-1 is true ; Statement-2 is false.

(D) Statement-1 is false ; Statement-2 is true.

ANSWERS

49. (2) **50.** (2) **51.** (2) **52.** (*a*) **53.** (*a*) **54.** (*d*) **55.** (*d*) **56.** (*a*)

57. Statement-1. A current I flows along the length of an infinitely long straight and thin walled pipe. Then the magnetic field at any point inside the pipe is zero.

Statement-2. $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$.

(*a*) A (*b*) B (*c*) C (*d*) D

58. Statement-1. Magnetic field strength at a point on axial line of a bar magnet is along South to North pole of magnet.

Statement-2. The magnetic field strength can never be along North to South pole of magnet.

(*a*) A (*b*) B (*c*) C (*d*) D

HINTS/EXPLANATIONS
For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. M – magnetic moment, m – pole strength

$2l$ – length of magnet

As $M = m \times 2l$, $m = \dfrac{M}{2l}$ ...(*i*)

When it is bent into semi-circular form, then

$$\pi r = 2l, \quad r = \frac{2l}{\pi}$$

Distance between two poles, $2l' = 2r = \dfrac{4l}{\pi}$

New magnetic moment $M' = m \times 2l' = m \times \dfrac{4l}{\pi}$

Putting value of (*i*)

$$M' = \frac{M}{2l} \times \frac{4l}{\pi} = \frac{2M}{\pi}$$

2. Charge on the element of rod of length dx at distance x from axis of rotation.

$$dq = \rho\, dx$$

Current developed on element,

$$dI = dq \times n = \rho\, dx\, n$$

Magnetic moment developed in the element due to rotation of rod, $dM = dI \times \pi x^2 = (\rho\, dx\, n) \times \pi x^2$

$$= \pi \rho\, n\, x^2\, dx$$

Total magnetic moment developed due to rotation of rod is

$$M = \int_0^l \pi \rho\, n\, x^2\, dx = \frac{\pi \rho\, n\, l^3}{3}$$

3. Here, $\theta_1 = 0°$, $\theta_2 = 60°$,

$W = \sqrt{3}\,J$, $\tau = ?$

As $W = -MB(\cos\theta_2 - \cos\theta_1)$

$$\therefore \quad \sqrt{3} = -MB(\cos 60° - \cos 0°)$$

$$= -MB\left(\frac{1}{2} - 1\right)$$

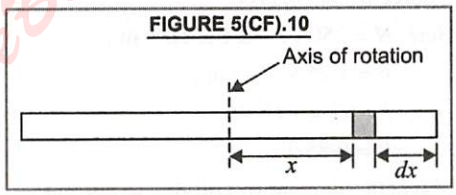
or $MB = 2\sqrt{3}$

$$\tau = MB \sin\theta = 2\sqrt{3} \sin 60°$$

$$= 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 3\,\text{J}$$

4. Refer to Fig. 5(CF).10, consider a small element of rod of length dx at distance x from the axis of rotation.

FIGURE 5(CF).10



Charge on small element dx,

$$dQ = \frac{Q}{L}dx$$

Current in the element due to rotation of rod

$$dI = v\,dQ = v\frac{Q}{L}dx$$

Magnetic moment of the small element

$$dM = (dI) \times \pi x^2 = \left(\frac{vQ}{L}dx\right)\pi x^2$$

$$= \frac{\pi v Q}{L} x^2\, dx$$

Total magnetic moment of rod system due to rotation of the whole rod is

ANSWERS

57. (*a*) 58. (*c*)

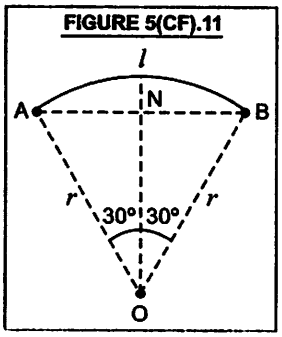
HINTS/EXPLANATIONS
For Difficult Questions

$$M = \int_{-l/2}^{+l/2} \frac{\pi v Q}{L} x^2 dx = \frac{\pi v Q L^2}{12}$$

5. Before bending, strength of each pole, $m = \frac{M}{l}$

On bending, as shown in Fig. 5(CF).11, distance between poles $= AB = 2r \sin \frac{60°}{2} = r$

FIGURE 5(CF).11



$\therefore$ New magnetic moment, $M' = m \times r$

$$M' = \frac{M}{l} \times \frac{l}{\pi/3} = \frac{3M}{\pi}$$

Choice (c) is correct.

6. Here $N = 250$; $l = 2{\cdot}1 \times 10^{-2}$ m ;
$b = 1{\cdot}25 \times 10^{-2}$ m ;
$I = 85 \times 10^{-6}$ A,
$B = 0{\cdot}85$ T

Magnetic moment of coil due to current
$M = NIA = NI\, l \times b$
$= 250 \times (85 \times 10^{-6}) \times (2{\cdot}1 \times 1{\cdot}25) \times 10^{-4}$

Work done, $W = MB\,(\cos \theta_1 - \cos \theta_2)$
$= MB\,(\cos 0° - \cos 180°) = 2\,MB$
$= 2 \times 250 \times (85 \times 10^{-6}) \times (2{\cdot}1 \times 1{\cdot}25) \times 10^{-4} \times 0{\cdot}85$
$= 9{\cdot}1 \times 10^{-6}$ J $=$ **9·1 μ J**

7. As is known, the direction of magnetic moment of a magnet is along *SN*

In case (a), From Fig. 5(CF).12(*a*) Resultant magnetic moment,

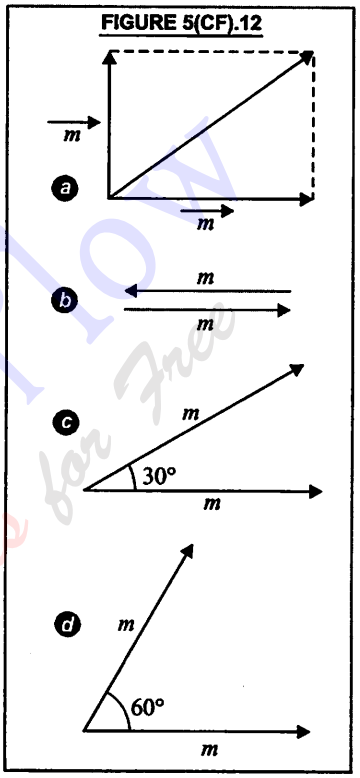
$$M_1 = \sqrt{m^2 + m^2} = m\sqrt{2} = 1{\cdot}414 \text{ m}$$

In case (b), $M_2 = m - m = 0$

In case (c),

$$M_3 = \sqrt{m^2 + m^2 + 2mm \cos 30°}$$
$$= \sqrt{2m^2 + m^2\sqrt{3}} = m\sqrt{2 + \sqrt{3}}$$
$$= m\sqrt{3{\cdot}732} = 1{\cdot}93 \text{ m}$$

FIGURE 5(CF).12



In case (d),

$$M_4 = \sqrt{m^2 + m^2 + 2m.m \cos 60°}$$
$$= \sqrt{3m^2} = 1{\cdot}732 \text{ m}$$

Choice (c) is the correct answer.

8. Here, $M = 6{\cdot}7 \times 10^{-2}$ Am^2
$I = 7{\cdot}5 \times 10^{-6}$ kg m^2
$B = 0{\cdot}01$ T ; $t = ?$

If T is time period of vibration of needle, then

$$T = 2\pi\sqrt{\frac{I}{MB}} = 2 \times \frac{22}{7}\sqrt{\frac{7{\cdot}5 \times 10^{-6}}{6{\cdot}7 \times 10^{-2} \times 0{\cdot}01}}$$

$T = 0{\cdot}665$ s

For 10 complete oscillations,
$t = 10\,T = 10 \times 0{\cdot}665$ s $=$ **6·65 s**

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9. Let r be the radius of the circle formed.

Then, $L = 2\pi r$ or $r = \dfrac{L}{2\pi}$

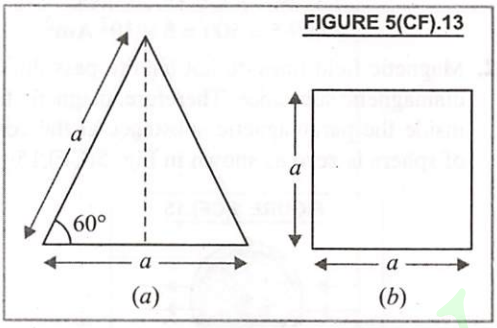
Magnetic moment $= Ia = I(\pi r^2)$

$= I\pi \times (L/2\pi)^2$

$= \dfrac{IL^2}{4\pi}$ **Am²**

10. Current $I = \dfrac{V}{R}$ is same for both the loops.

FIGURE 5(CF).13



Total length of the conducting wire = 12 a

(*i*) In case of triangle loop, of each side a,

length of wire = $3a$

Number of turns in the triangular loop of the wire

$$N = \frac{12a}{3a} = 4$$

Area of triangle, $A = \dfrac{1}{2}$ base × height

$$= \frac{1}{2} a \times a \sin 60° = \frac{a^2\sqrt{3}}{4}$$

Magnetic dipole moment of triangular loop

$$M = NIA = 4I \times \frac{a^2\sqrt{3}}{4} = \sqrt{3}\, Ia^2$$

(*ii*) Total length of four sides of a square of each side $a = 4a$. Number of turns in square loop,

$$N = \frac{12a}{4a} = 3$$

Area of square, $A' = a^2$

$\therefore$ Magnetic moment of square loop

$M' = N'IA'$

$M' = \mathbf{3}\, I a^2$

11. Here, $L = 2 \times 2\pi r$ or $r = \dfrac{L}{4\pi}$.

$$M = n I A = 2 \times I \times \pi r^2$$

$$= 2I \times \pi \times (L/4\pi)^2 = \frac{I L^2}{8\pi} \text{ Am}^2$$

12. As the axes are perpendicular, mid point lies on axial line of one magnet and on equatorial line of other magnet.

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{10^{-7} \times 2 \times 1}{1^3}$$

$$= 2 \times 10^{-7}\ \text{T}$$

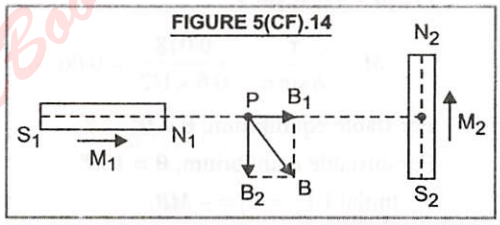
and $B_2 = \dfrac{\mu_0}{4\pi}\dfrac{M}{d^3} = \dfrac{10^{-7} \times 1}{1^3} = 10^{-7}$ T

As $B_1 \perp B_2$,

$\therefore$ Resultant field $= \sqrt{B_1^2 + B_2^2} = \sqrt{5} \times 10^{-7}$ **T**

13. The point P lies on the axial line of magnet N_1S_1 at distance r_1 (= 1 *m*) and on equatorial line of magnet N_2S_2 at distance r_2 (= 1 *m*), Fig. 5(CF).14.

FIGURE 5(CF).14



$$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{r_1^3} = 10^{-7} \times \frac{2\times 2}{1^3} = 4 \times 10^{-7}\ \text{T}$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{M_2}{r_2^3} = 10^{-7} \times \frac{2}{1^3} = 2 \times 10^{-7}\ \text{T}$$

Resultant magnetic field at P,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(4\times10^{-7})^2 + (2\times10^{-7})^2}$$

$$= 2\sqrt{5} \times 10^{-7}\ \text{T}$$

14. Here, $T_1 = 2$ s, $T_2 = ?$ $H_1 = H = 24 \times 10^{-6}$ *T*

$H_2 = H - F = 24 - 18 = 6\ \mu T = 6 \times 10^{-6}$ *T*

As $\dfrac{T_2}{T_1} = \sqrt{\dfrac{H_1}{H_2}} = \sqrt{\dfrac{24\times10^{-6}}{6\times10^{-6}}} = 2$

$\therefore \quad T_2 = 2\, T_1 = 2 \times 2 = \mathbf{4\ s}$

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15. For original magnet,

$$T_0 = 2\pi\sqrt{\frac{I}{MB}} \qquad ...(i)$$

When a magnet is cut into three equal parts, perpendicular to its length, then magnetic moment of each part is $M' = M/3$. Moment of inertia of each part is

$$I' = \frac{m}{3}\times\frac{(L/3)^2}{12} = \frac{1}{27}\times\frac{mL^2}{12} = \frac{I}{27}$$

$\therefore$ New time period, $T' = 2\pi\sqrt{\frac{I'}{M'B}}$

$$= 2\pi\sqrt{\frac{I/27}{(M/3)B}}$$

$$= \frac{1}{3}2\pi\sqrt{\frac{I}{MB}} = \frac{T_0}{3}$$

16. Here $\theta = 30°$, $B = 0{\cdot}6$ T, $\tau = 0{\cdot}018$ Nm

From, $\tau = MB\sin\theta$

$$M = \frac{\tau}{B\sin\theta} = \frac{0{\cdot}018}{0{\cdot}6\times 1/2} = 0{\cdot}06 \text{ Am}^2$$

For stable equilibrium, $\theta = 0°$

For unstable equilibrium, $\theta = 180°$

$\therefore$ Initial P.E. $= U_i = -MB$,

Final P.E. $= MB$

Work done $= MB - (-MB) = 2MB$

$= 2\times 0{\cdot}06\times 0{\cdot}6$

$= \mathbf{7{\cdot}2\times 10^{-2}\ J}$

17. Here, $\tan\theta_1 = \frac{V}{H_1}$ and $\tan\theta_2 = \frac{V}{H_2}$

and $\tan\theta = \frac{V}{H}$

$\therefore$ $H_1 = V\cot\theta_1$, $H_2 = V\cot\theta_2$, $H = V\cot\theta$

As $H^2 = H_1^2 + H_2^2$

$\therefore$ $V^2\cot^2\theta = V^2\cot^2\theta_1 + V^2\cot^2\theta_2$

Hence, $\mathbf{\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2}$

18. Angle of dip is zero at equator, **positive in northern hemisphere and negative in southern hemisphere.**

19. Here, $\chi_m = 599$, $I = 1200$ Am^{-1}, $\mu = ?$

$\mu = \mu_0(1+\chi_m) = 4\pi\times 10^{-7}(1+599)$

$= \mathbf{2{\cdot}4\pi\times 10^{-4}\ T\ mA^{-1}}$

20. The best choice is (*d*), because for permanent magnet *P*, retentivity should be as large as possible and coercivity should also be as large as possible.

21. Here, $V = AI = 10^{-3}$ m^3 ; $n = 10$ turns/cm

$= 1000$ turns/m

$N = n\times l = 1000\times l$,

$I = 0{\cdot}5$ A, $\mu_r = 1000$

$M = NIA(\mu_r - 1) = (nl)IA(\mu_r - 1)$

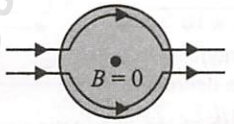
$= nIV(\mu_r - 1)$

$= 1000\times 0{\cdot}5\times 10^{-3}(1000-1)$

$= 499{\cdot}5 = 500 = \mathbf{5\times 10^2\ Am^2}$

22. Magnetic field lines do not tend to pass through diamagnetic substance. Therefore, magnetic field inside the paramagnetic substance at the centre of sphere is **zero** as shown in Fig. 5(CF).15.

FIGURE 5(CF).15



23. We know that area of the hysteresis loop tells the net energy absorbed per unit volume by the material per cycle of magnetisation.

We also know that for electromagnets and transformers, the energy loss should be small. Which is for Fig. 5(CF).4(*b*). **Thus option (*d*) is true.**

24. The relation $B = \mu H$ is valid for all Diamagnetic, paramagnetic and ferromagnetic materials.

25. As ferromagnetics tend to move towards stronger magnetic field, so, N_1 will get attracted strongly, paramagnetic also get attracted to stronger magnetic field upto some extent. So N_2 will also get weakly attracted, but diamagnetic, *i.e.*, N_3 tend to move away from magnetic field, hence N_3 will get weakly repelled.

26. According to electron theory of magnetism, an atom of a diamagnetic material has no intrinsic dipole moment, whereas atom of a para-magnetic material has some intrinsic dipole moment

i.e., $\mu_d = 0$ and $\mu_p \neq 0$

27. A ferromagnetic materials core used in transfarmer must have **high permeability** so that it may show high value of magnetic induction for

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the given magnetising field. It should have **low hysteresis loss** in order to decrease the energy losses.

28. Magnetic permeability of a ferromagnetic substance, $\mu = B/H$ or $\mu \propto \frac{1}{H}$;
where H is the magnetising field. If H increases, then μ decreases.

29. Here $N = 250$; $l = 2{\cdot}1 \times 10^{-2}$ m ;
$b = 1{\cdot}25 \times 10^{-2}$ m ; $I = 85 \times 10^{-6}$ A,
$B = 0{\cdot}85$ T
Magnetic moment of coil due to current
$M = NIA = NI\, l \times b$
$= 250 \times (85 \times 10^{-6}) \times (2{\cdot}1 \times 1{\cdot}25) \times 10^{-4}$
Work done, $W = MB\ (\cos\theta_1 - \cos\theta_2)$
$= MB\ (\cos 0° - \cos 180°)$
$= 2\,MB$
$= 2 \times 250 \times (85 \times 10^{-6}) \times (2{\cdot}1 \times 1{\cdot}25) \times 10^{-4} \times 0{\cdot}85$
$= 9{\cdot}1 \times 10^{-6}$ J $= \mathbf{9{\cdot}1\ \mu J}$

30. The magnetic susceptibility (χ) of a ferromagnetic substance decreases with increase in temperature *i.e.* $\chi \propto 1/(T - T_C)$. Hence, **option (*b*) is true.**

31. Coercivity = magnetising force, H
As, $B = \mu_0 H$ and $B = \mu_0 n I$
$\therefore\ \mu_0 n I = \mu_0 H$ or $H = n I = \frac{100}{0{\cdot}2} \times 5{\cdot}2$
$= \mathbf{2600\ A/m}$

32. Here, coercivity of given magnet
$= \frac{B}{\mu_0} = 3 \times 10^3\ \text{Am}^{-1}$
As $B = \frac{\mu_0 NI}{L}$ or $I = \frac{BL}{\mu_0 N} = \frac{(3\times10^3)\times0{\cdot}1}{100} = \mathbf{3\ A}$

33. Given, for the magnetic field 0 to 7·5 tesla, the value of critical temperature for given superconductor lies in between 100 K to 75 K. Therefore, for B = **5 tesla,** critical temperature of superconductor is more than 75 K but less than 100 K. Hence, **option (*b*) is correct.**

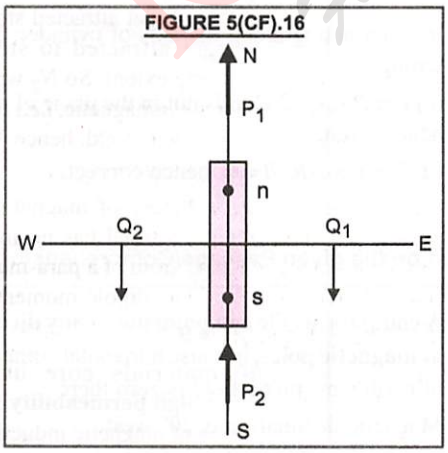
34. The gravitational potential energy gained by the diamagnetic rod comes from the energy of **current source.**

35. $T = 2\pi\sqrt{I/MH}$
where I = moment of inertia of magnet about the suspension fibre = mass $\frac{(l^2 + b^2)}{12}$
when mass is quadrupled. I becomes 4 times. Therefore, T becomes $\sqrt{4}$ times *i.e.* **2 *T*** the motion continues to **be simple harmonic.**

II. Multiple Choice Questions (with One or More than One Correct Answers)

36. As is clear from Fig. 5(CF).16, the directions of magnetic fields due to dipole are same at (P_1, P_2) and at (Q_1, Q_2).

FIGURE 5(CF).16



37. The magnetic field due to the horizontal circular loop is perpendicular to the plane of the loop and upwards. Therefore, the line NS of dipole should be perpendicular to the plane of the loop and north pole should be upwards or south pole should be downwards.

38. A current carrying circular coil does not have an electric field and a gravitational field. But it has magnetic field and induced electric field if the magnetic field due to current in the coil is changing with time.

39. There is no force on the portion of wire *AB* and *CD* as current in them is parallel to the direction of magnetic field. The magnetic force on wire *BC* is perpendicular to the loop directed outwards and on wire *OA*, the magnetic force would be

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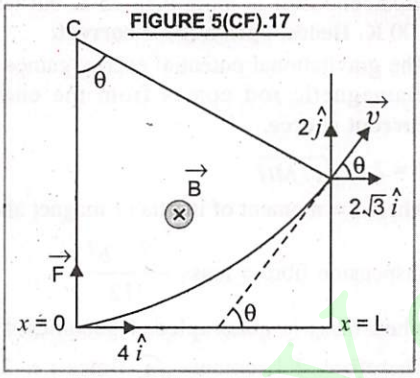
perpendicular to the loop directed inwards. Both these forces will be equal in magnitude. Therefore, the net force on the wire loop is zero and **torque on the loop would be along the clockwise direction as seen from *O*.**

40. Refer to Fig. 5(CF).17, component of final velocity of particle is in positive *y*-direction. The centre of circular path of particle in magnetic field is present on positive *y*-direction. So magnetic field is present in **negative *z*-direction.**

FIGURE 5(CF).17



If θ is the angle of deviation of the particle with *x*-axis while emerging from magnetic field, then

$$\tan\theta = \frac{v_y}{v_x} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$$

or $\theta = \dfrac{\pi}{6}$

Angular velocity of rotation of particle in magnetic field, $\omega = \dfrac{QB}{M}$

Time taken by particle to cross the magnetic field is

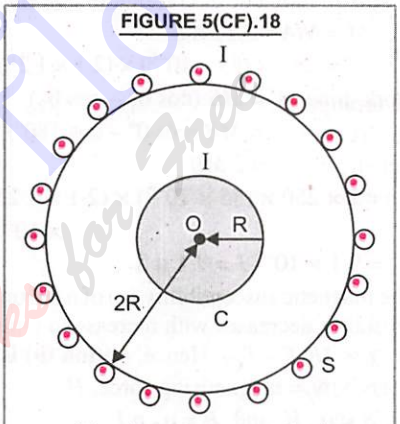
$$t = \frac{\theta}{\omega} = \frac{\pi/6}{QB/M} = \frac{M\pi}{6QB}$$

or $$B = \frac{M\pi}{6Qt} = \frac{M\pi}{6Q\times(10\times10^{-3})}$$

$$= \frac{50M\pi}{3Q}$$

41. Let B_C and B_S be the magnetic feld due to current in cylinder *C* and solenoid *S* respectively at point *P*, distance *r* from the common axis.

FIGURE 5(CF).18



(Top view of cross-section)

(*a*) For $\theta < r < R$,

$B = B_S = \mu_0 n I \neq 0$ hence **correct.**

(*b*) For $R < r < 2R$

$$B = \sqrt{B_S^2 + B_C^2}$$

It is not acting along the axis of cylinder, hence wrong.

(*c*) For $R < r < 2R$, *B* is not in the plane of circle, hence wrong.

(*d*) For $r > 2R$, $B \neq 0$, hence **correct.**

III Multiple Choice Questions (Based on the given Passage/Comprehension)

42. At the surface of earth, magnetic field intensity is of the order of **10^{-4} tesla.**

43. As $V = R\sin\delta = 0$

$\therefore \sin\delta = 0,\ \delta = 0°.$

44. A compass needle can point out in any direction at magnetic poles, because horizontal component of earth's magnetic field is zero there.

45. Magnetic declination is **20° west.**

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IV. Matching Type Questions

46. Permeability has the dimensional formula $[MLT^{-2}A^{-2}]$. The dimensional formula of magnetic induction is $[ML^0T^{-2}A^{-1}]$. Susceptibility has no dimensions and intensity of magnetisation has the dimensions $[M^0L^{-1}T^0A]$ and same units Am^{-1}.

V. Matrix-Match Type Questions

47. Knowledge based questions..

48. Magnetic moment is a vector and its units are Nm^3/Wb.

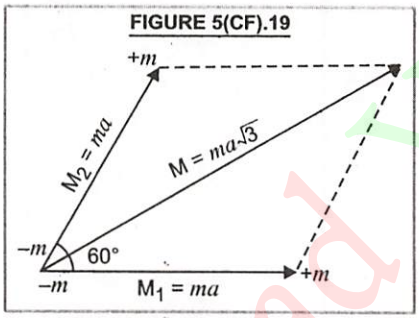
Magnetic induction is a vector, having the units N/A-m.

Permeability is scalar and intensity of magnetization is a vector.

VI. Integer Type Questions

49. The arrangement of magnetic poles as shown in Fig. 5(CF).19 is equivalent to two magnets of magnetic moments $M_1 = M_2 = (m \times a)$ inclined to each other at 60°, Fig. 5(CF).19.

FIGURE 5(CF).19



∴ Net magnetic moment is

$$M = \sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cos 60^\circ}$$

$$= \sqrt{(ma)^2 + (ma)^2 + 2(ma)(ma)\frac{1}{2}}$$

$$M = ma\sqrt{3} = 1{\cdot}732\,(ma),$$

which can be taken as **2 (*ma*)**

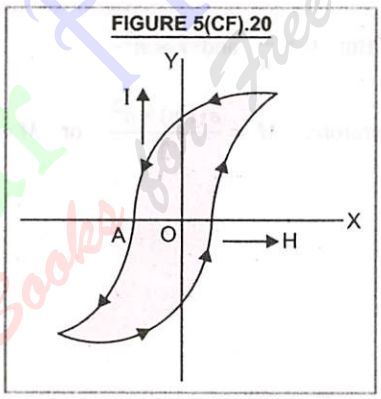
50. As is clear from Fig. 5(CF).20,

coercivity $OA = H = ni = 120$ A/m.

Here, $n = \dfrac{\text{number of turns}}{\text{length}} = \dfrac{72}{1{\cdot}2\,(m)} = 60.$

$$\therefore \quad i = \frac{H}{n} = \frac{120}{60} = \mathbf{2\,A}$$

FIGURE 5(CF).20



51. Here, $M_1 = 2$ Am^2, $d_1 = 0{\cdot}3$ m

$M_2 = 5$ Am^2, $d_2 = 0{\cdot}4$m

The point of intersection P lies on axial line of the two magnets, Fig. 5(CF).9.

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi} \times \frac{2M_1}{d_1^3} = 10^{-7} \times \frac{2 \times 2}{(0{\cdot}3)^3}$$

$$= 1{\cdot}48 \times 10^{-5} \text{ T along } S_1 N_1$$

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2M_2}{d_2^3} = 10^{-7} \times \frac{2 \times 5}{(0{\cdot}4)^3}$$

$$= 1{\cdot}56 \times 10^{-5} \text{ T along } S_2 N_2$$

As B_1 and B_2 are perpendicular to each other, therefore resultant magnetic field at P is

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{(1{\cdot}48 \times 10^{-5})^2 + (1{\cdot}56 \times 10^{-5})^2}$$

$$= 2{\cdot}15 \times 10^{-5} \text{ T} = n \times 10^{-5} \text{ T}$$

where $n = 2{\cdot}15$, which can be taken as **2**

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HINTS/EXPLANATIONS
For Difficult Questions

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

52. Both the assertion and reason are true and **latter is correct explanation of the former.**

53. Both, the assertion and reason are true and **the latter is correct explanation of the former.**

54. Both, the **assertion and reason are false.**

55. Magnetic moment of revolving electron around the nucleus is,

$$M = IA = \frac{e}{T}\pi r^2 = \frac{e\omega}{2\pi}\pi r^2 = \frac{evr}{2}$$

But $v \propto \frac{1}{n}$ and $r \propto n^2$

Therefore, $M \propto \frac{e(1/n)\times n^2}{2}$ or $M \propto n$

Thus Assertion is wrong. Here **Reason though correct but cannot explain the Assertion.**

56. Here **both Assertion and Reason are true and Reason is the correct explanation of Assertion** because when ferromagnetic material is heated above curve temperature the domains in ferromagnetic material are broken and material behaves as a paramagnetic material.

FOR ENGINEERING STUDENTS

57. Here, **both statement-1 and statement-2 are correct and statement-2 is the correct explanation of statement-1.**

58. **Statement-1 is true,** but the **statement-2 is false.** When the point lies on equatorial line of bar magnet, magnetic field strength is parallel to North to South pole of magnet.

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UNIT-4

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

CHAPTER 6.

ELECTROMAGNETIC INDUCTION

CHAPTER 7.

ALTERNATING CURRENTS

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6

ELECTROMAGNETIC INDUCTION

6.1. INTRODUCTION

In unit 3, we have seen that moving charges produce magnetic field. The question arises, as to whether moving magnets produce electric currents, *i.e.*, does nature permit such a symmetrical relation between electricity and magnetism ? The answer is a resounding Yes ! The experiments of Michael Faraday in U.K. and Joseph Henry in U.S.A. established around the year 1830 that electric currents were induced in closed coils, when subjected to changing magnetic fields.

This phenomenon of generating current/ e.m.f. in a conducting circuit by a change in strength, position or orientation of an external magnetic field is called Electromagnetic Induction (EMI). The e.m.f. so developed is called induced e.m.f. If the conductor is in the form of a closed circuit, a current flows in the circuit. This is called induced current.

This phenomenon of EMI is the basis of working of power generators, dynamos, transformers etc. We cannot imagine a world where there is no electricity, no lights, no trains, no telephones, no personal computers and nothing else that works on electricity. Therefore, the phenomenon of EMI is not merely of theoretical or academic interest. The present day civilisation owes a great deal to the discovery of the phenomenon of electromagnetic induction.

6.2. MAGNETIC FLUX

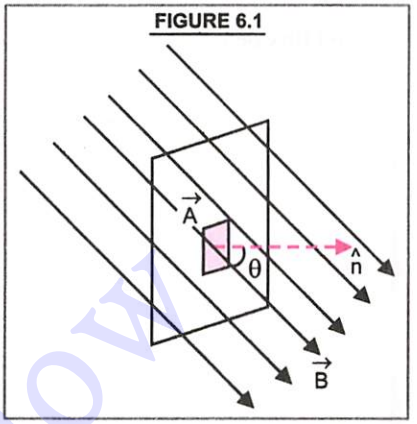
The magnetic flux ϕ through any surface of area $\vec{A}$ held in a magnetic field $\vec{B}$ is measured by the total number of magnetic lines of force crossing the surface normally. It is equal to dot product of $\vec{B}$ and $\vec{A}$.

For a uniform magnetic field $\vec{B}$ crossing the plane of area $\vec{A}$ at an angle θ with the normal to the plane, magnetic flux ϕ is given by

$$\phi = \vec{B} . \vec{A} = BA \cos\theta \quad ...(1)$$

where θ is smaller angle between $\vec{B}$ and $\vec{A}$, *i.e.*, θ is the angle, which normal to the surface area makes with $\vec{B}$. This is shown in Fig. 6.1. Here $\hat{n}$ represents unit vector along the outdrawn normal to the area element.

FIGURE 6.1



Magnetic Flux associated with variable magnetic field

If magnetic field $\vec{B}$ is variable, we divide the surface into a number of area elements. Each area element is small enough to have constant magnetic field through it.

If $\vec{B}_1, \vec{B}_2, \vec{B}_3, ..., \vec{B}_n$ be the magnetic fields at the area elements $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, ..., \Delta\vec{A}_n$ respectively, then total magnetic flux through the surface is

$$\phi = \vec{B}_1 . \Delta\vec{A}_1 + \vec{B}_2 . \Delta\vec{A}_2 + ... \vec{B}_n . \Delta\vec{A}_n = \sum_{\text{all}} \vec{B}_i . \Delta\vec{A}_i$$

If $\Delta\vec{A}$ tends to zero, we can write the summation in the form of integral.

$$\therefore \quad \phi = \int \vec{B} . d\vec{A}$$

The integration is over the entire surface area.

When the magnetic field is touching the surface tangentially, Fig. 6.2,

$$\theta = 90°$$

$$\therefore \quad \phi = BA \cos 90° = 0$$

FIGURE 6.2

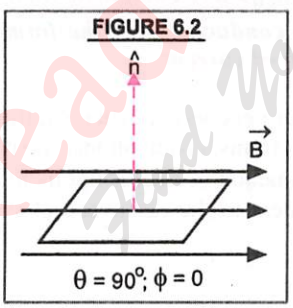
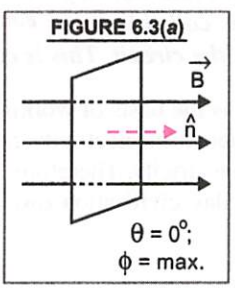


FIGURE 6.3(a)



When the magnetic field is **normal** to the surface, Fig. 6.3(*a*), $\theta = 0°$,

$$\therefore \quad \phi = BA \cos 0° = BA = \text{max. value.}$$

This means $B = \dfrac{\phi}{A}$, *i.e., magnetic field strength B is magnetic flux per unit area and is called* **magnetic flux density** or **magnetic induction.**

Thus denseness of magnetic lines of force is associated with the strength of magnetic field. For example, in Fig. 6.3(*b*), we have shown a point source, which may be a charge or a magnetic pole. As many as 16 field lines are shown to flow from this source. If Area 1 is taken as 1 sq. unit, Area 2 is 4 square units.

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$\therefore$ flux density at Area 2 $= \dfrac{16}{4} = 4$ field lines per square unit,

and flux density at Area 1 $= \dfrac{16}{1} = 16$ field lines per square unit.

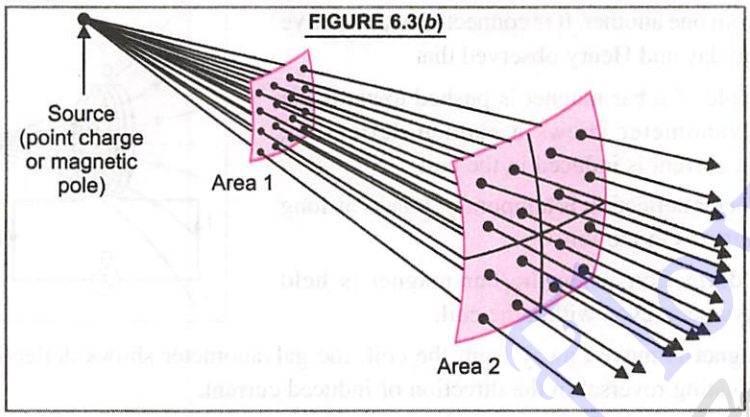

FIGURE 6.3(*b*)

Therefore, magnetic field strength or magnetic flux density at Area 1 is 4 times the magnetic field strength or magnetic flux density at Area 2.

Units of Magnetic Flux

The SI unit of magnetic flux is weber (Wb). One weber is the amount of magnetic flux over an area of 1 square metre held normal to a uniform magnetic field of one tesla. Thus

$$1 \text{ weber} = 1 \text{ tesla} \times 1 \text{ m}^2$$

The c.g.s. unit of ϕ is maxwell (Mx), where 1 weber = 10^8 maxwell.

It should be clearly understood that ***magnetic flux is a scalar quantity*** (represented by the dot product of two vectors $\vec{B}$ and $\vec{A}$).

The **dimensional formula** of ϕ can be deduced from $\phi = BA\cos\theta$

from $F = Bqv, B = F/qv$

$$\therefore \quad \phi = \frac{F}{qv} A\cos\theta = \frac{(MLT^{-2})(L^2)}{(AT)(LT^{-1})} = [ML^2T^{-2}A^{-1}] \qquad \text{...(2)}$$

In this dimensional formula, A stands for ampere, SI unit of current.

As $[ML^2T^{-2}]$ corresponds to energy, ***SI unit of magnetic flux will be***

$$\text{weber} = \frac{\text{joule}}{\text{ampere}} = \frac{\text{joule}}{\text{coulomb/sec}} = \frac{\text{joule} - \text{sec}}{\text{coulomb}} = \text{volt-sec.}$$

Sample Problem **What is the magnetic flux linked with a coil of N turns of area of cross section A held with its plane parallel to the field ?** (Kerala PET 2012)

Sol. $\phi = NBA\cos\theta$

As plane of area of coil is parallel to the field, normal to the area makes an angle of 90° with the field. Therefore, $\phi = NBA\cos 90° =$ **Zero**

6.3. THE EXPERIMENTS OF FARADAY AND HENRY

The understanding of electromagnetic induction is based on a series of experiments carried out by Faraday and Henry. Some of the important experiments are described below :

Experiment 1. Current induced by a magnet

Fig. 6.4 shows a coil or loop C of a few turns of con- ducting material insulated from one another. It is connected to a sensitive galvanometer G. Faraday and Henry observed that

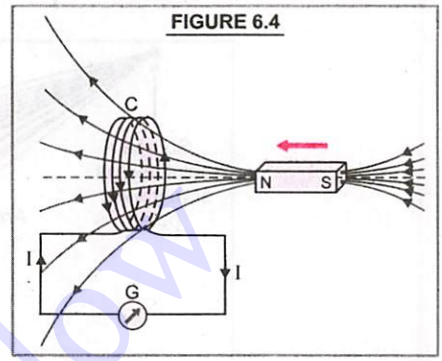

FIGURE 6.4

(*i*) When north pole of a bar magnet is pushed towards the coil, the galvanometer shows a sudden deflection, indicating that current is induced in the coil.

(*ii*) The galvanometer deflection is temporary. It lasts as long as the bar magnet is in motion.

There is no deflection when the bar magnet is held stationary, anywhere, even within the coil.

(*iii*) When the magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction, indicating reversal in the direction of induced current.

(*iv*) When south pole of bar magnet is moved towards or away from the coil, the galvanometer deflections are opposite to those observed with the north pole for similar movements.

(*v*) The galvanometer deflection (and hence induced current) is found to be ***larger*** when magnet is pushed towards or pulled away from the coil ***faster.***

(*vi*) When the bar magnet is held stationary and coil C is moved towards or away from the magnet, the same effects are observed. It shows that ***relative motion between the coil and the magnet is responsible for induction of electric current in the coil.***

Experiment 2. Current induced by current

Fig. 6.5 shows a coil or loop C of a few turns of conducting material insulated from one another connected to a sensitive galvanometer G. C' is another such coil connected to a battery. A steady current through coil C' produces a uniform magnetic field along its axis.

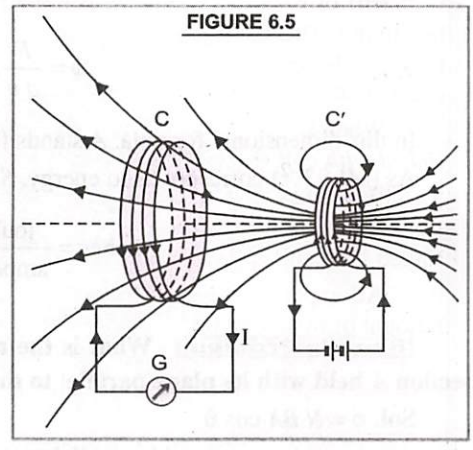

FIGURE 6.5

(*i*) When coil C' is moved towards the coil C, the galvanometer shows a sudeen deflection. This indicates that electric current is induced in coil C.

(*ii*) When coil C' is moved away from coil C, the galvanometer shows a deflection in the opposite direction. This indicates that direction of current induced in coil C is reversed.

(*iii*) The deflection is temporary. It lasts so long as there is relative motion between the two coils.

(*iv*) The galvanometer deflection (and hence induced current) is found to be larger when coils are moved faster towards/ away from each other.

We observe that coil C' carrying current in Experiment 2 behaves as a bar magnet in Experiment 1.

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Experiment 3. Current induced by changing current

In Experiment 1, *relative motion between a coil and a bar magnet* produced induced current. In Experiment 2, *relative motion between one coil and another coil carrying current produced induced current. Through the present experiment, Faraday showed that relative motion is not an absolute requirement. Current can be induced even without relative motion.* Fig. 6.6 shows two coil C and C' held stationary. Coil C is connected to a sensitive galvanometer G and coil C' is connected to a battery through a tapping key K. It is observed that

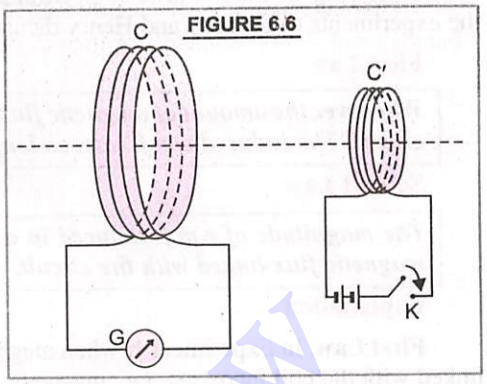

FIGURE 6.6

(*i*) When key K is pressed, galvanometer in coil C shows a momentary deflection, indicating that current is induced in coil C. The pointer in the galvanometer, however, returns to zero immediately.

(*ii*) When the key K is kept pressed continuously, there is no deflection in the galvanometer.

(*iii*) When the key K is released, the galvanometer shows again a momentary deflection, but in the opposite direction. The pointer in the galvanometer returns to zero almost instantly.

(*iv*) The galvanometer deflection increases dramatically, when an iron rod is inserted into the coils along their axis, and the key K is pressed/released.

We observe that in the absence of relative motion between coils C and C', current is induced in C by changing current in coil C'.

6.4. CAUSE OF INDUCED E.M.F.

From the foregoing experiments, we find that ***an e.m.f. is induced in a coil, when amount of magnetic flux linked with the coil changes with time.*** All observations in the three experiments can be explained in terms of this concept :

(*i*) In Experiment 1, motion of bar magnet towards or away from coil C changes the magnetic flux linked with coil C.

(*ii*) In Experiment 2, relative motion between coil C and current carrying coil C', again changes the magnetic flux linked with coil C. In either case, an e.m.f. is induced in coil C. It is this induced e.m.f. which causes electric current in the coil and galvanometer shows some deflection.

(*iii*) In Experiment 3, when the tapping key K is pressed, the current in coil C' increases from zero to maximum. Therefore, resulting magnetic field of coil C' increases from zero to maximum. Consequently, the magnetic flux linked with the neighbouring coil C also increases. It is this change in magnetic flux linked with coil C that induces e.m.f. in coil C.

When key K is kept pressed current in coil C' is constant. Therefore, there is *no change in magnetic flux linked with coil C. Hence, current induced in coil C drops to zero.*

When key K is released, current in coil C' decreases from maximum to zero. The resulting magnetic field of coil C' decreases from maximum to zero in a short time. This results in a decrease in magnetic flux linked with coil C. Hence an e.m.f. is induced in coil C in the opposite direction.

All experimental observations lead us to conclude that induced e.m.f. appears in a coil whenever the amount of magnetic flux linked with the coil changes.

Retain in Memory

Note that mere presence of magnetic flux is not enough for producing induced e.m.f. The amount of magnetic flux linked with the coil must change with time in order to produce any induced e.m.f. in the coil.

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6.5. FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Following are the two laws of electromagnetic induction as given by Faraday. Both the laws follow from the experiments of Faraday and Henry dicussed above.

First Law

Whenever the amount of magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit. The induced e.m.f. lasts so long as the change in magnetic flux continues.

Second Law

The magnitude of e.m.f. induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

Explanation

First Law. In Experiment 1, when magnet is moved towards the coil, number of magnetic lines of force linked with the coil increases, *i.e.*, magnetic flux increases. When the magnet is moved away, the magnetic flux linked with the coil decreases. In both the cases, galvanometer shows deflection indicating that e.m.f. is induced in the coil.

When there is *no relative motion* between the magnet and the coil, magnetic flux linked with the coil remains constant. That is why galvanometer shows no deflection. Thus, induced e.m.f. is produced when magnetic flux changes and induced e.m.f. continues so long as the change in magnetic flux continues. This is first law. The same results follow from Experiment 2.

Second Law. In Experiment 1, when magnet is moved ***faster***, the magnetic flux linked with the coil changes at a faster rate. Therefore, galvanometer deflection is ***more.*** However, when the magnet is moved slowly, rate of change of magnetic flux is smaller. Therefore, galvanometer deflection is smaller, *i.e.*, induced e.m.f. is smaller. Hence magnitude of e.m.f induced varies directly as the rate of change of magnetic flux linked with the coil. This is second law.

If ϕ_1 is amount of magnetic flux linked with a coil at any time and ϕ_2 is the magnetic flux linked with the coil after t sec., then

$$\text{Rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t}$$

According to Faraday's second law, induced e.m.f. $e \propto \dfrac{(\phi_2 - \phi_1)}{t}$

or $e = k\,(\phi_2 - \phi_1)/t$ ...(3)

where k is a constant of proportionality.

As $k = 1$ (in all systems of units)

$\therefore$ $\boxed{e = (\phi_2 - \phi_1)/t}$ ...(4)

If $d\phi$ is small change in magnetic flux in a small time dt, we can rewrite (5) as

$\boxed{e = -d\phi/dt}$ ...(5)

Negative sign is taken because induced e.m.f. always opposes any change in magnetic flux associated with the circuit. This follows from Lenz's law discussed in Art. 6.6.

In case of a closely wound coil of N turns, change in magnetic flux associated with each turn is the same. Therefore, total induced e.m.f. is given by

$$\boxed{e = -N\frac{d\phi}{dt}}$$

DO YOU KNOW ?

A ground fault interrupter is based on Faraday's laws. It is a device that protects us against electric shock from an appliance like a clothes dryer. This device works so fast that in less than a millisecond, it turns off the current before it reaches a dangerous level.

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By increasing number of turns N in the coil, we can increase the induced e.m.f.

Induced current in the coil is $$I = \frac{\text{induced e.m.f.}}{\text{resistance of coil}} = \frac{|e|}{R} = \frac{N d\phi/dt}{R}$$

Note that Faraday's 2nd Law gives us the magnitude of induced e.m.f./current. It does not give us the polarity of induced e.m.f. or the direction of induced current.

Sample Problem **A coil having 500 square loops each of side 10 cm is placed normal to magnetic field, which increases at the rate of 1·0 tesla/sec. What is the e.m.f. induced in the loop ?** (J & K CET 2006)

Sol. Here, $N = 500$, $A = 10 \times 10\ \text{cm}^2 = 10^2 \times 10^{-4}\ \text{m}^2 = 10^{-2}\ \text{m}^2$

$$\frac{dB}{dt} = 1{\cdot}0\ \text{T/s},\ e = ?$$

$$e = N\frac{d\phi}{dt} = NA\frac{dB}{dt} = 500 \times 10^{-2} \times 1{\cdot}0 = \textbf{5 volt}$$

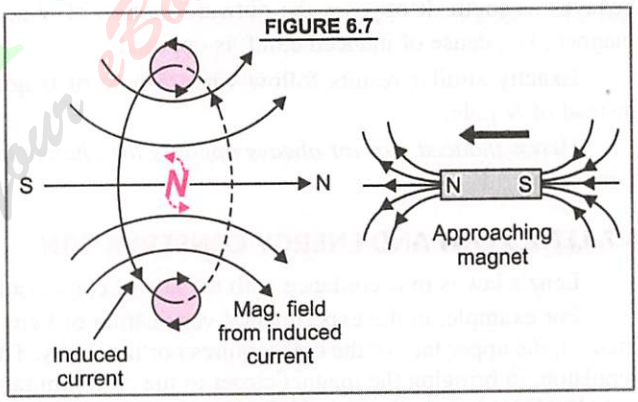
6.6. LENZ'S LAW

This law gives us the direction of current induced in a circuit.

According to Lenz's law, the direction of induced current is such that it opposes the change in magnetic flux responsible for its production.

For example, in Fig. 6.7, when north pole of a bar magnet is being pushed towards the coil, the amount of magnetic flux linked with the coil increases. Current is induced in the coil in such a direction that it *opposes the increase in flux.* This is possible only when current induced in the coil is in *anticlockwise direction* with respect to an observer on the side of the bar magnet. The magnetic moment $\vec{M}$ associated with this induced current has north polarity towards the north pole of the approaching bar magnet, as shown in Fig. 6.7. This would result in a repulsive force, which opposes the motion of approaching magnet and the corresponding increase in magnetic flux.

FIGURE 6.7



Similarly, when north pole of the bar magnet is moved away from the coil, Fig. 6.8, the magnetic flux linked with the coil decreases. To counter this decrease in magnetic flux, current induced in the coil is in clockwise direction so that its south pole faces the receding north pole of the bar magnet.

This would result in an attractive force, which opposes the motion of the magnet and the corresponding decrease in magnetic flux.

From Fig. 6.7, we find that when magnetic flux linked with a closed loop **increases,** current induced in the loop is **anticlockwise.**

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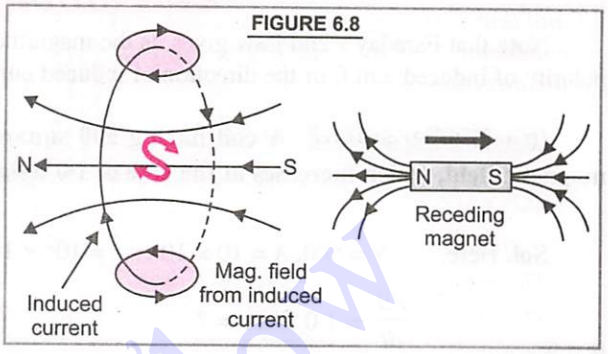
Again, Fig. 6.8 shows that when magnetic flux linked with a closed loop **decreases,** current induced in the loop is **clockwise.**

Experimental Verification of Lenz's Law

Figure 6.9 shows the experimental set up for verifying Lenz's law. A coil of a few turns is connected to a cell *C* and a sensitive galvanometer *G* through a two way key 1, 2, 3.

Put in the plug of key between 1 and 2. Cell sends current through the coil. At the upper face of the coil, the current is *anticlockwise*, which would produce *north pole* on this face. Suppose the galvanometer deflection is to the *right*. Obviously, if galvanometer deflection were to the left, current would be clockwise at the upper face, which would behave as south pole.

FIGURE 6.8

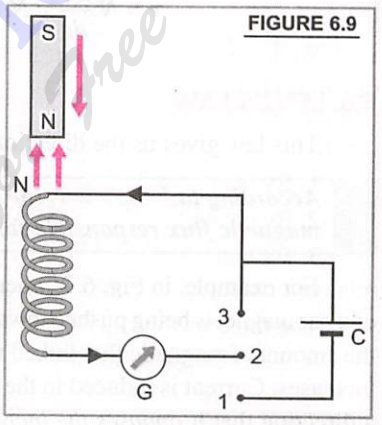


Remove the plug of key from 1 and 2. Insert the plug of key between 2 and 3. Now, move *N*-pole of a bar magnet towards the coil. The galvanometer shows a sudden deflection to the right indicating that current induced in the coil is anticlockwise and upper end of the coil behaves as north. It opposes the inward motion of *N*-pole of the bar magnet, which is the cause of induced current.

Similarly, when *N*-pole of the bar magnet is moved away from the coil, the galvanometer shows a sudden deflection to the *left*, indicating that current induced in the coil is *clockwise* and upper end of the coil behaves as south. It opposes the outward motion of *N*-pole of the bar magnet., i.e., cause of induced e.m.f. is opposed.

Exactly similar results follow when *S*-pole of magnet is moved instead of *N*-pole.

FIGURE 6.9



Hence *induced current always opposes the change in magnetic flux which produces it.* This verifies Lenz's law.

6.7. LENZ'S LAW AND ENERGY CONSERVATION

Lenz's law is in accordance with the law of conservation of energy.

For example, in the experimental verification of Lenz's law, when *N*-pole of magnet is moved towads the coil, the upper face of the coil acquires north polarity. Therefore, work has to be done **against** the force of repulsion, in bringing the magnet closer to the coil. Similarly, when *N* pole of magnet is moved away, south polarity developes on the upper face of the coil. Therefore, work has to be done against the force of attraction, in taking the magnet away from the coil.

It is this mechanical work done in moving the magnet w.r.t. the coil that changes into electrical energy producing induced current. Thus, energy is being transformed only.

When we do not move the magnet, work done is zero. Therefore, induced current is zero.

Hence Lenz's law obeys the principle of energy conservation.

Conversely, Lenz's law can be treated as a consequence of the principle of energy conservation.

6.8. FLEMING'S RIGHT HAND RULE

Fleming's right hand rule also gives us the direction of induced e.m.f./current, in a conductor moving in a magnetic field. According to this rule,

***If we stretch the first finger, central finger and thumb of our right hand in mutually perpendicular directions such that first finger points along the direction of the magnetic field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current,* Fig 6.10.**

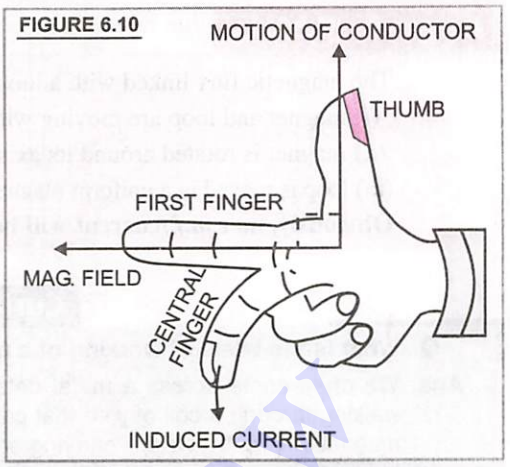


FIGURE 6.10

In a nutshell, the direction of

Magnetic Field → along First Finger
Motion → along the Thumb, then
induced current → along central finger

This right hand rule is also known as **generator rule.**

The direction of induced current given by **Lenz's law** and **Fleming's right hand rule** will obviously be the same.

6.9. VARIOUS METHODS OF PRODUCING INDUCED E.M.F.

We have learnt that e.m.f. is induced in a circuit, whenever the amount of magnetic flux linked with the circuit is changed. As $\phi = BA\cos\theta$, the magnetic flux ϕ can be changed by changing B, A or θ individually, or any two of them or changing all the three together.. Hence there are *three methods* of producing induced e.m.f.

1. By changing the magnitude of magnetic field B,

2. By changing the area A, *i.e.*, by shrinking or stretching or changing the shape of the coil.

3. By changing angle θ between the direction of B and normal to the surface area A *i.e.*, changing the relative orientation of the surface area and the magnetic field.

The three methods are discussed here briefly:

(a) Induced e.m.f. by changing the magnetic field.

In Faraday and Henry experiment 1, Fig. 6.4, motion of the magnet towards the coil increases the magnetic field B at any point on the wire loop and *vice-versa*. In either case, galvanometer shows deflection. Thus, e.m.f. is induced by changing B.

In Faraday and Henry experiment 3, Fig. 6.6, changing current in coil C' (on pressing or releasing key K), changes the magnetic field at any point on coil C. This results in the production of induced e.m.f. in coil C.

(b) Induced e.m.f. by changing the area A

Fig. 6.11 shows a rectangular conducting loop $PQRS$ in the plane of paper. The conductor PQ is free to move without any loss of energy due to friction etc. The crosses represent a uniform magnetic field B which is perpendicular to the plane of the paper and directed inwards.

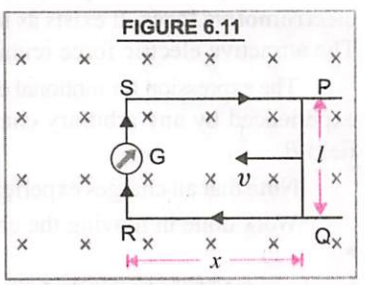


FIGURE 6.11

When PQ is moved to the left or right, the galvanometer G shows a sudden deflection. This must be due to e.m.f. induced in the loop $PQRS$. Infact, on moving PQ, area of the loop intercepted by magnetic field changes. Therefore, magnetic flux linked with the loop changes, resulting in induced e.m.f. in the loop. The galvanometer deflection lasts so long as motion of PQ continues.

(c) Induced e.m.f. by changing relative orientation of coil and magnetic field.

When a coil is rotated in a magnetic field, the angle θ between normal to the coil and direction of magnetic field changes. Therefore, magnetic flux linked with the coil changes and hence an e.m.f. is induced in the coil. This is the basis of an a.c. generator. For details, see Art. 6.19.

Retain in Memory

The magnetic flux linked with a loop does not change with time when
(*i*) magnet and loop are moving with the same velocity in the same direction,
(*ii*) magnet is rotated around its axis without changing its distance from the loop,
(*iii*) loop is moved in a uniform magnetic field and the whole of the loop remains in the field.
Obviously, no *e.m.f.*/current will be induced in the loop under these conditions.

Curiosity Question

Q. What is the basis of working of a metal detector ?

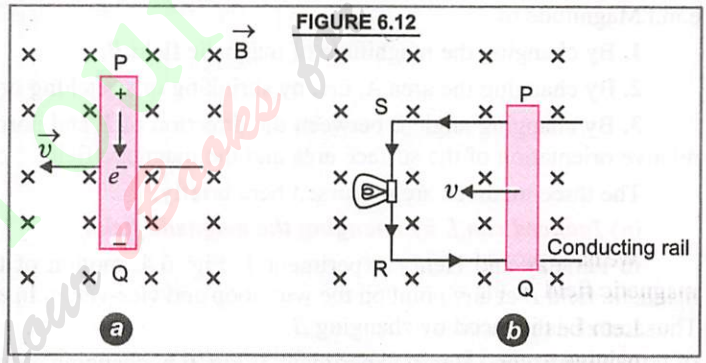
Ans. We often come across a metal detector at an airport. Walking through a metal detector is walking through a coil of wire that carries a small electric current and is associated with some magnetic field. If you are carrying some metal like iron, you change the magnetic field. This induces change in current in the coil of the metal detector. The detection of this change sets up an alarm.

6.10. MOTIONAL ELECTROMOTIVE FORCE

When a conducting rod is moved through a constant magnetic field, an e.m.f. is induced in the rod. This is called motional electromotive force, because it arises due to motion of charges through a magnetic field.

Fig. 6.12(*a*) shows a metal rod PQ of length, l being moved with a velocity $\vec{v}$ to the left in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. Each charge q within the rod moves with velocity $\vec{v}$ and experiences a magnetic force, $F = |q| v B$. The mobile free electrons in the rod are driven in the rod from P to Q, according to Fleming's left hand rule.

FIGURE 6.12



An equal amount of positive charge appears at the end P of the rod. This process goes on till the attractive electric force between the accumulated charges is balanced by the magnetic force that separates the charges.

The separated charges on the ends of rod PQ give rise to an induced e.m.f. which is called **motional electromotive force.** It exists as long as the rod moves. When the rod stops moving, magnetic force vanishes. The attractive electric force reunites the separated charges at the ends and motional e.m.f. disappears.

The expression for motional e.m.f. can be obtained from **Lorentz force equation,** $F = q v B$. This is the force experienced by any arbitrary charge + q in the conductor, moving with speed v in perpendicular magnetic field B.

Note that all charges experience the same Lorentz force irrespective of their position in the conductor PQ.

Work done in moving the charge + q from Q to P, $W = F \times QP = (q v B) \times l$

e.m.f. induced = work done per unit charge. $\therefore \quad e = \dfrac{W}{q} = \dfrac{q v B l}{q} = B l v$

$\therefore \quad \boxed{e = B l v} \quad ...(6)$

If the rod lies on a conducting rail as shown in Fig. 6.12(*b*), then due to the induced e.m.f. electrons flow in a clockwise direction around the circuit. Positive charge would flow in a direction opposite to the electron flow. Therefore, conventional current is in anticlockwise direction, as shown in Fig. 6.12(*b*).

From eqn. (6), we find that when $v = 0$, $e = 0$, because no motional e.m.f. is induced in a stationary rod.

If R is resistance of the loop $PQRS$ at a given instant, the induced current at that instant would be given by

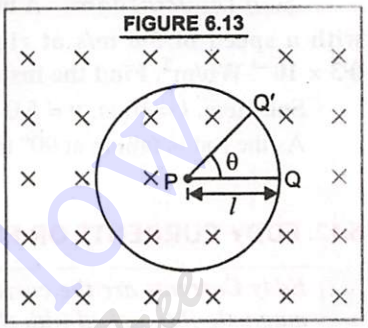
$$I = \frac{e}{R} = \frac{B l v}{R}$$

The direction of induced current is given by Fleming's right hand rule or Lenz's Law, and it is shown in Fig. 6.12(b).

Motional induced e.m.f. in a conducting rod rotated in a uniform magnetic field

In Fig. 6.13, crosses represent a uniform magnetic field of strength $\vec{B}$ directed into the paper, perpendicular to its plane. PQ is a conducting rod of length l rotating with a uniform angular velocity ω in the plane of the paper. In a small time t, let the rod turn through small angle $\theta = \angle QPQ'$

FIGURE 6.13



Area swept by the rod, $A = \frac{1}{2} \times l \times l\,\theta = \frac{1}{2} l^2 \theta$

Magnetic flux intercepted, $\phi = BA \cos 0° = B\left(\frac{1}{2} l^2 \theta\right) \times 1$

Magnitude of e.m.f. induced, $e = \frac{d\phi}{dt} = \frac{d}{dt}\left(B \frac{1}{2} l^2 \theta\right) = \frac{1}{2} B l^2 \frac{d\theta}{dt}$

$$e = \frac{1}{2} B l^2 . \omega \quad , \text{ where } \omega = \frac{d\theta}{dt} = \text{angular velocity of rod.}$$

6.11. POWER DISSIPATED IN A CONDUCTOR MOVED IN MAGNETIC FIELD

We have proved in Art. 6.10 that when a conductor of length l is moved with a velocity v in a perpendicular magnetic field B, the motional e.m.f. produced in the conductor is $e = B l v$

Let r be the resistance of movable arm PQ of the rectangular conductor. If we assume that the resistance of remaining arms QR, RS and SP is negligible compared to r, then overall resistance of the rectangular loop would remain r even when PQ is moved, Fig. 6.12(b).

$\therefore$ Current induced in the loop $I = \frac{e}{r} = \frac{B l v}{r}$

The magnitude of force on the conductor PQ moving in the magnetic field is

$$F = BI l = B\left(\frac{B l v}{r}\right) l = \frac{B^2 l^2 v}{r}$$

The direction of this force is opposite to the velocity of the conductor.

Power required to push the conductor with velocity v,

$$P = F \times v = \frac{B^2 l^2 v}{r} \times v = \frac{B^2 l^2 v^2}{r}$$

As the conductor is pushed mechanically, the mechanical energy dissipated per second is given by

$$P = I^2 r = \left(\frac{B l v}{r}\right)^2 r = \frac{B^2 l^2 v^2}{r},$$

which is the same as the power required to push the conductor.

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Hence, mechanical energy required to move the conductor PQ is converted into electrical energy first, (*i.e.*, the induced e.m.f.) and then to thermal energy.

$$\therefore \quad \text{Heat energy produced/sec} = \frac{B^2 l^2 v^2}{r} \qquad ...(7)$$

Note that the magnetic forces do not do any work. All they do is transfer energy from mechanical to electrical.

Sample Problem **A horizontal conducting rod 10 m long extending from east to west is falling with a speed of 5·0 m/s at right angles to the horizontal component of earth's magnetic field, $0{\cdot}3 \times 10^{-4}$ Wb/m^2. Find the instantaneous value of e.m.f. induced in the rod.** **(CBSE 2017)**

Sol. Here, $l = 10$ m, $v = 5{\cdot}0$ m/s, $H = 0{\cdot}3 \times 10^{-4}$ Wb/m^2, $e = ?$

As the rod is falling at 90° to horizontal field, therefore, $e = Bl\,v = H\,l\,v$

$$= 0{\cdot}3 \times 10^{-4} \times 10 \times 5{\cdot}0 = \mathbf{1{\cdot}5 \times 10^{-3}\ volt}$$

6.12. EDDY CURRENTS OR FOUCAULT CURRENTS

***Eddy Currents** are the currents induced in the bulk pieces of conductors when the amount of magnetic flux linked with the conductor changes.*

However, their flow patterns resemble swirling eddies in water. That is why they are called eddy currents. These were discovered by Foucault in the year 1895 and hence they are also called **Foucault currents.**

For example, when a metallic block is placed in a non-uniform magnetic field as shown in Fig. 6.14, eddy currents are produced in the block.

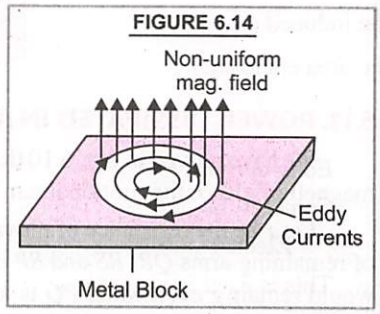

FIGURE 6.14

The magnitude of eddy current is $i = \dfrac{\text{induced e.m.f.}}{\text{resistance}} = \dfrac{e}{R}$.

As $e = -\dfrac{d\phi}{dt}$

$$\therefore \quad i = -\frac{d\phi / dt}{R} \qquad ...(8)$$

The direction of eddy currents is given by Lenz's law, or Fleming's right hand rule.

Experimental Demonstration of Eddy Currents

Experiment 1 : Jumping Disc Experiment. Hold a light metallic disc D at top of the cross-section of an electromagnet connected to a cell and a tap key K, Fig. 6.15. When key K is pressed, the disc is thrown up into the air. This is due to eddy currents developed in the disc.

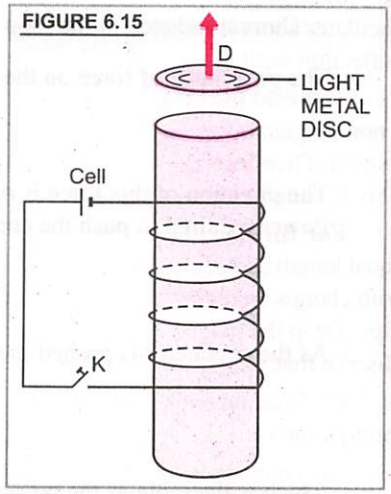

FIGURE 6.15

As current through the solenoid increases from zero to maximum, the magnetic flux along the axis of the solenoid increases. Therefore, magnetic flux linked with the disc increases. Induced currents or eddy currents develop in the disc and magnetise it. If upper end of solenoid initially acquires north polarity, the lower face of disc D also acquires north polarity in accordance with Lenz's law. The force of repulsion between the two throws the disc up in the air. Later, the disc falls down due to gravity. Things are repeated when key K is released.

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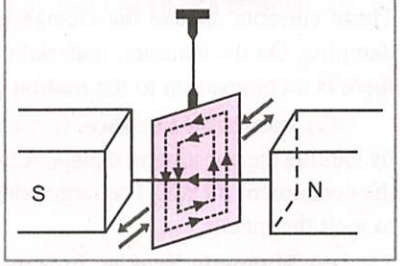
Experiment 2. Suspend a flat metallic plate between pole pieces N and S of an electromagnet. Let it come to rest with its plane perpendicular to magnetic field, Fig. 6.16.

FIGURE 6.16



When the magnetic field is off, the metallic plate disturbed once from its equilibrium position and left, oscillates freely for a longer time. But when the electromganet is switched on, the vibrations of the plate are damped and the plate stops vibrating sooner. This is because of eddy currents developed in the vibrating plate.

In the normal position of rest of the plate, magnetic flux linked with the plate is maximum. When it is displaced towards any one extreme position, area of plate in the field decreases. Therefore, magnetic flux linked with the plate decreases. Eddy currents develop in the plate which, according to Lenz's law, oppose the decrease in flux and hence the motion of the plate towards extreme position. Similarly, when plate returns from extreme position to mean position, area of plate in the field increases, magnetic flux linked with the plate increases. Eddy currents are developed which oppose the increase in flux and hence the motion of the plate towards the mean position.

In either case, vibrations of the plate are opposed and hence damped.

FIGURE 6.17

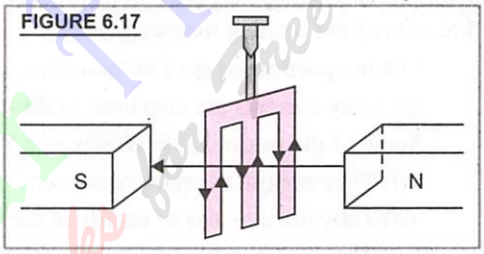


Figure 6.17 shows the same metallic plate with slots cut in it. When such a plate is made to oscillate in the magnetic field, the damping effect is there, but it is much smaller compared to the case when no slots were cut. This means eddy currents are reduced. This is because magnetic moments of the induced currents (which oppose the motion) depend upon the area enclosed by the currents $(\vec{M} = i\vec{A})$.

6.13. APPLICATIONS OF EDDY CURRENTS

Eddy currents are useful in many ways though they have some disadvantages too.

Some of the important applications of eddy currents are :

(*a*) Electro-magnetic damping

This is used in designing *dead beat galvanometers.*

When a steady current is passed through the coil of a galvanometer, it is deflected. Normally, the coil oscillates about its equilibrium position for some time before coming to rest. We cannot read the galvanometer deflection until the coil comes to rest.

To avoid the delay in reading galvanometer deflection due to these oscillations, the coil is wound over a non magnetic *metallic frame*. As the coil is deflected, eddy currents set up in the metallic frame oppose its motion. Therefore, the coil attains its equilibrium position almost instantly. Thus, the motion of coil is damped. This is called electromagnetic damping.

For further illustration of electromagnetic damping, we take two hollow thin cylindrical pipes of equal length and equal internal diameter, one made of aluminium and other of *PVC*. Fix the two pipes vertically with clamps on retort stands. Take a small cylindrical magnet whose diameter is less than the diameter of each pipe. Drop the magnet through each pipe such that it does not touch the sides of the pipe during the fall. We observe that :

(*i*) Time taken by the magnet to fall through PVC pipe is equal to time taken by the magnet to fall through the same height without the pipe.

(*ii*) Time taken by the magnet to fall through aluminium pipe is comparatively longer.

This is because as the magnet falls through the aluminium pipe, eddy currents are generated in the pipe. These currents oppose the change in magnetic flux (*i.e.*, motion of the magnet). This is electromagnetic damping. On the contrary, material of *PVC* pipe is insulating. Therefore, no eddy currents are generated and there is no opposition to the motion of the falling magnet through the *PVC* pipe.

(*b*) **Induction Furnace.** It is used to produce high temperatures which are utilised in preparing alloys by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the constituent metals. The large eddy currents generated in the metals produce high temperatures sufficient to melt the metals.

(*c*) **Magnetic Brakes.** In some electrically powered trains, strong electromagnets are situated in the train, just above the rails. When electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, therefore, the braking action is smooth.

(*d*) **Electric Power Meters.** In the power meter of your house, you must have observed a rotating shiny disc. This is a metal disc which rotates due to eddy currents developed in the disc, by magnetic fields produced by alternating current.

(*e*) **Induction Motor.** An induction motor or a.c. motor is yet another important application of eddy currents. A rotating magnetic field produces strong eddy currents in a rotor, which starts rotating in the direction of the rotating magnetic field.

(*f*) In **speedometres** of automobiles and **energy metres.**

(*g*) Eddy currents are also used in **dia-thermy, *i.e.*, in deep heat treatment of the human body.**

Some of the **undesirable effects** of eddy currents are :

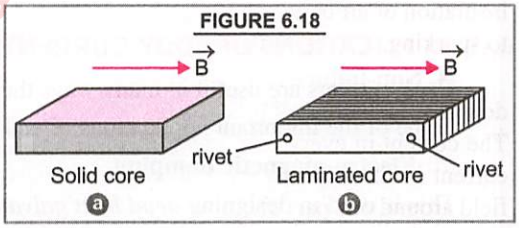
(*i*) They oppose the relative motion.

(*ii*) They involve loss of energy in the form of heat.

(*iii*) The excessive heating may break the insulation in the appliances and reduce their life.

To minimise the eddy currents, the metal core to be used in an appliance like dynamo, transformer, choke coil, motor etc. is taken in the form of ***thin sheets.*** Each sheet is electrically insulated from the other by insulating varnish like lacquer. Such a core is called ***a laminated core*** as shown in Fig. 6.18. The planes of these sheets are arranged parallel to the magnetic field so that they cut across the eddy current paths.

FIGURE 6.18



Large resistance between the thin sheets confines the eddy currents to the individual sheets. Hence the eddy currents are reduced to a large extent.

As dissipation of electric energy into heat varies directly as the square of the strength of electric current, therefore, heat loss is greatly reduced.

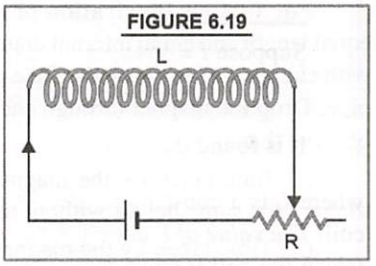
6.14. SELF INDUCTION

***Self Induction* is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an e.m.f. in itself.**

For this reason, self induction is also called the **inertia of electricity.**

In Fig. 6.19, if current in a coil *L* is changed by varying the contact position on a variable resistor *R*, a self induced *e.m.f.* appears in the coil– while the current is changing.

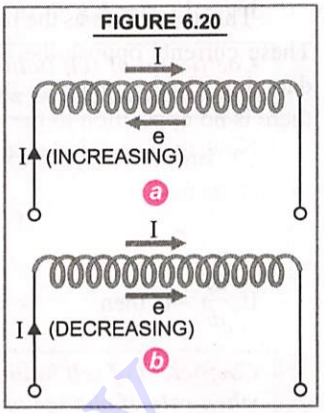
FIGURE 6.19



When current (I) is increasing, the self induced *e.m.f.* (e) appears across the coil in a direction such that *it opposes the increase*. Therefore, it would be in a direction opposite to I, Fig. 6.20(a).

When current (I) is decreasing, the self induced *e.m.f.* (e) appears across the coil in a direction, such that it *opposes the decrease*. Therefore, it would be in the direction of (I), Fig. 6.20(b).

FIGURE 6.20



Experimental Demonstration of Self-Induction

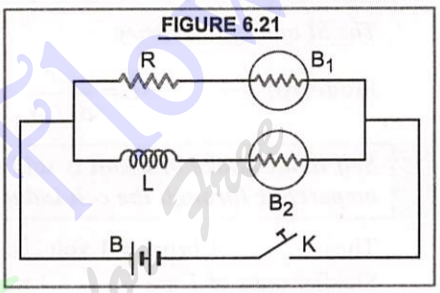
1. Simple Experiment. In Fig. 6.21, B_1, B_2 are two bulbs connected in parallel, one through resistance R and other through an inductor coil L. This combination is connected to a battery B through a key K.

When key K is pressed, bulb B_1 glows instantly with brilliance, but bulb B_2 glows slowly. This is because when current grows, no e.m.f. is induced in resistor R and bulb B_1 glows instantly. However, when current grows through L, an e.m.f. is induced across L which opposes the growth of current. Therefore, the bulb B_2 glows slowly.

When key K is opened, bulb B_1 stops glowing at once, but bulb B_2 takes some time to stop glowing. This is because when current decreases through coil L from maximum to zero, an e.m.f. is induced in L, which opposes the decay of current through L.

Current takes longer time to reduce to zero. Therefore, bulb B_2 takes some time to stop glowing.

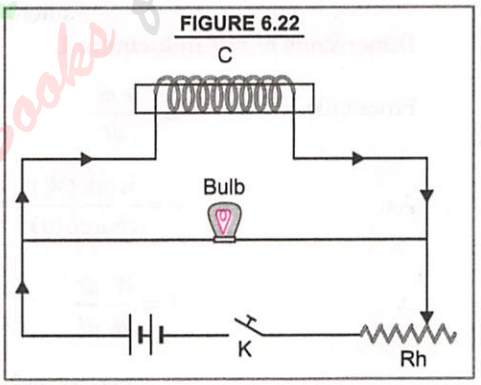
FIGURE 6.21



2. Sparking in the electric switch, when light is switched off. As time of break is very small, self induced *e.m.f.* during break may become large enough to break down the insulation of air between the switch contacts. This would lead to sparking.

3. Non-inductive coil. Every coil of a resistance box is double folded on itself to minimise the effect of self induction. The current in every part of coil is equal and opposite to the current in the opposite part of the coil. Therefore, magnetic field around one part of coil is cancelled by the magnetic field around the opposite part. Such coils are called non-inductive coils.

FIGURE 6.22



4. In Fig. 6.22, C is a coil of a few turns wound on a soft iron core. It is connected to a battery, a key K, a rheostat and a bulb as shown. Press K and adjust the rheostat so that the bulb just glows. Now, release the key. We find that the bulb **overglows** before extinguishing. Overglowing indicates that current through bulb increases on releasing the key. This must be due to self-induced current in the direction of main current, during break.

Coefficient of Self Induction or self inductance of a coil

Suppose I = strength of current flowing through a coil at any time.

ϕ = amount of magnetic flux linked with all the turns of the coil at that time.

It is found that $\phi \propto I$ or $\boxed{\phi = LI}$ ...(9)

where L is a constant of proportionality and is called *coefficient of self induction or self inductance* of the coil. *The value of L depends on number of turns, area of cross-section and nature of material of the core on which the coil is wound.*

If $I = 1$, $\phi = L \times 1$ or $L = \phi$. Therefore,

***Coefficient of self induction or self inductance of a coil** is numerically equal to the amount of magnetic flux linked with the coil when unit current flows through the coil.*

Now, the e.m.f. induced in the coil is given by

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) \quad \text{or} \quad \boxed{e = -L \times \frac{dI}{dt}} \quad \text{...(10)}$$

If $\frac{dI}{dt} = 1$, then $e = -L \times 1$ or $L = -e$.

***Coefficient of self induction or self inductance of a coil** is equal to the e.m.f. induced in the coil when rate of change of current through the coil is unity. Inductance is a scalar quantity.*

The SI unit of L is henry.

From (10), $L = \frac{-e}{dI/dt}$ $\therefore$ $1\,\text{henry} = \frac{1\,\text{volt}}{1\,\text{ampere/sec}} = \frac{1\,\text{volt-sec}}{\text{ampere}}$

Self inducatance of a coil is said to be one henry (H) when a current change at the rate of 1 ampere/sec through the coil induces an e.m.f. of 1 volt in the coil.

Thus, 1 henry = 1 volt-sec/ampere = 1 weber/ampere

Smaller units of L are 1 millihenry (1 mH) = 10^{-3} henry,

1 microhenry (1 μH) = 10^{-6} henry.

Dimensions of self inductance L

From (13), $L = \frac{e\,dt}{dI}$

As $e = \frac{\text{work}\,(W)}{\text{charge}\,(q)}$

$\therefore$ $L = \frac{W}{q}\frac{dt}{dI}$

$$L = \frac{[M^1 L^2 T^{-2}][T]}{[AT][A]}$$

$$\boxed{L = [M^1 L^2 T^{-2} A^{-2}]}$$

Note that in the dimensional formula on RHS ; L represents length, and on LHS. L stands for coefficient of self induction.

Sample Problem 1 What e.m.f. will be induced in a 10 H inductor in which current changes from 10 A to 7 A in 9×10^{-2} s ?

Sol. Here, $e = ?$, $L = 10$ H, $I_1 = 10$ A, $I_2 = 7$ A, $dt = 9 \times 10^{-2}$ s

As $e = \frac{-L\,dI}{dt} = \frac{-L(I_2 - I_1)}{dt} = \frac{-10(7-10)}{9 \times 10^{-2}}$

$= 333{\cdot}3$ volt

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Sample Problem 2 **The current passing through a choke coil of 5 H is decreasing at the rate of 2 A/s. What is the e.m.f. developing across the coil ?** **(DPMT 2002)**

Sol. Here, $L = 5$ H, $\frac{dI}{dt} = 2$ A/s, $e = ?$

$$e = L\frac{dI}{dt} = 5 \times 2 = \mathbf{10\ V}$$

6.15. SELF INDUCTANCE OF A LONG SOLENOID

A solenoid is basically a coil wound into a tightly packed helix.

A long solenoid is that whose length is very large as compared to its radius of cross section. The magnetic field B at any point inside such a solenoid is practically constant and is given by

$$B = \frac{\mu_0 N I}{l} \qquad ...(11)$$

where μ_0 is absolute magnetic permeability of free space/air, which forms the core of the solenoid, l is length of the solenoid, N is total number of turns in the solenoid.

$\therefore$ Magnetic flux through each turn of the solenoid coil = $B \times$ area of each turn $= \left(\mu_0 \frac{N}{l} I\right) A$

where A is area of each turn of the solenoid.

Total magnetic flux linked with the solenoid = flux through each turn $\times$ total number of turns

i.e., $$\phi = \mu_0 \frac{N}{l} I A \times N \qquad ...(12)$$

If L is coefficient of self inductance of the solenoid, then

$$\phi = LI \qquad ...(13)$$

From (12) and (13), $LI = \mu_0 \frac{N}{l} I A \times N$ or $\boxed{L = \frac{\mu_0 N^2 A}{l}}$ ...(14)

If core is of any other magnetic material, μ_0 is replaced by μ, where $\boxed{\mu = \mu_0 \mu_r}$

$\therefore$ $$\boxed{L' = \frac{\mu_0 \mu_r N^2 A}{l} = \mu L}$$

Clearly, coefficient of self-induction of a solenoid varies (*i*) inversely as length (l) of the solenoid (*ii*) directly as area A of cross-section of the solenoid, and (*iii*) directly as square of number of turns (N) in solenoid.

Obviously, self inductance of a solenoid increases if air core of the solenoid is replaced by core of some magnetic material.

Sample Problem **What is the self inductance of a solenoid of length 40 cm, area of cross section 20 cm^2 and total number of turns = 800 ?**

Sol. Here, $L = ?$ $l = 40$ cm $= 0{\cdot}4$ m, $A = 20$ cm$^2 = 20 \times 10^{-4}$ m^2, $N = 800$

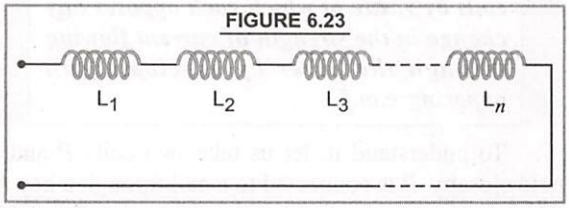
$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} (800)^2 \times 20 \times 10^{-4}}{0{\cdot}4} = \mathbf{4{\cdot}02 \times 10^{-3}\ H}$$

6.16. GROUPING OF COILS

(a) Coils in Series

When any number of coils of inductances $L_1, L_2, L_3 \ldots\ldots L_n$ are connected in series, as shown in Fig. 6.23, and coefficient of coupling* $K = 0$, then as in series, current through each coil is same and potential divides, therefore,

FIGURE 6.23



*For details, refer to coefficient of coupling on Page 6/20.

$$e = e_1 + e_2 + ... e_n$$

$$L_s \frac{dI}{dt} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + ... L_n \frac{dI}{dt} \quad \text{or} \quad \boxed{L_s = L_1 + L_2 + ...L_n} \quad ...(15)$$

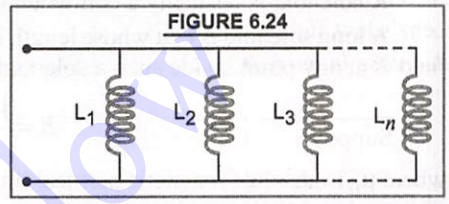
Hence combined inductance of coils in series is equal to sum of the inductances of individual coils. Therefore, in series, inductance increases.

(b) Coils in Parallel

When any number of coils of inductances L_1, L_2, L_3 .....L_n are connected in parallel as shown in Fig. 6.24, and the coefficient of coupling $K = 0$, then as in parallel, potential remains the same and current divides, therefore, $I = I_1 + I_2 + ... I_n$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + ... \frac{dI_n}{dt}$$

FIGURE 6.24



Using $e = L\frac{dI}{dt}$, $\frac{e}{L_p} = \frac{e_1}{L_1} + \frac{e_2}{L_2} + ... \frac{e_n}{L_n}$

As $e_1 = e_2 = e_3 = ... = e_n$ $\therefore$ $\boxed{\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + ... \frac{1}{L_n}}$ ...(16)

Hence reciprocal of combined inductance of any number of coils in parallel is equal to sum of the reciprocals of inductance of individual coils. Thus in parallel, combined inductance is less than the minimum value of inductance of any coil.

Eqns. (15) and (16) show that grouping of inductances is governed by formulae similar to that of grouping of resistances *i.e.*

$$R_s = R_1 + R_2 + ... R_n \quad \text{and} \quad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + ... \frac{1}{R_n}$$

Sample Problem **Three coils of self inductance 5, 10 and 15 mH are connected first in series and then in parallel. What is the ratio of the net inductance in the two cases ?**

Sol. Here, $L_1 = 5$ mH, $L_2 = 10$ mH, $L_3 = 15$ mH

$$L_s = L_1 + L_2 + L_3 = 5 + 10 + 15 = 30 \text{ mH}$$

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{5} + \frac{1}{10} + \frac{1}{15} = \frac{6+3+2}{30} = \frac{11}{30}$$

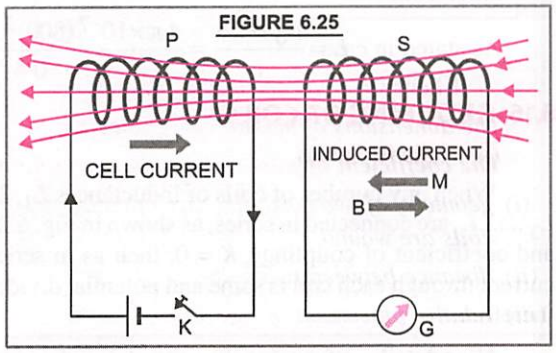
$$L_p = \frac{30}{11} \text{ mH} \qquad \frac{L_s}{L_p} = \frac{30}{30/11} = \mathbf{11}$$

6.17. MUTUAL INDUCTION

> ***Mutual induction** is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an opposing e.m.f.*

To understand it, let us take two coils P and S held closeby. P is connected to a cell through a key K. S is connected to a sensitive galvanometer G, Fig. 6.25.

FIGURE 6.25



On pressing or releasing K, galvanometer shows a sudden temporary deflection. This is due to mutual induction of two coils as detailed below :

On pressing K, current in P increases from zero to maximum value. It takes some time. During this time (of make M), current in P is increasing. Therefore, magnetic flux linked with P is increasing. As S is closeby, magnetic flux associated with S also increases. An e.m.f. is induced in S. According to Lenz's law, the induced current in S would *oppose increase in current in P* by flowing in a direction opposite to the cell current in P.

On releasing K, current in P decreases from maximum to zero value. It takes some time. During this time (of break B), current in P is decreasing. Therefore, magnetic flux linked with P is decreasing. As S is held closeby ; magnetic flux associated with S also decreases. An e.m.f. is induced in S. According to Lenz's law, induced current in S during break flows in the direction of the cell current in P so as to *oppose the decrease in current* in P, *i.e.*, it prolongs the decay of current.

Coefficient of Mutual Induction or Mutual Inductance of two coils

Suppose, I = strength of current in one coil

ϕ = total amount of magnetic flux linked with all the turns of the neighbouring coil.

It is found that $\phi \propto I$ or $\boxed{\phi = MI}$ ...(17)

where M is a constant of proportionality and is called *coefficient of mutual induction or mutual inductance* of the two coils.

If $I = 1$, $\phi = M \times 1$ or $M = \phi$. Thus,

***Coefficient of mutual induction or mutual inductance of two coils* is numerically equal to the amount of magnetic flux linked with one coil when unit current flows through the neighbouring coil.**

The e.m.f. induced in the neighbouring coil is given by $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI)$

$$\boxed{e = -M\frac{dI}{dt}} \quad \text{...(18)}$$

If $\frac{dI}{dt} = 1$, then $e = -M \times 1$ or $M = -e$. Thus,

***Coefficient of mutual induction or mutual inductance of two coils* is equal to the e.m.f. induced in one coil when rate of change of current through the other coil is unity.**

The SI unit of M is henry.

Coefficient of mutual induction or mutual inductance of two coils is said to be one henry, when a current change at the rate of one ampere/sec in one coil induces an e.m.f. of one volt in the other coil.

As stated in case of self inductance,

1 henry = 1 weber/ampere = 1 volt sec/ampere

The dimensions of mutual inductance are the same as the dimensions of self-inductance

The coefficient of mutual induction or mutual inductance of two coils depends on :

(*i*) *geometry of two coils, i.e., size of coils, their shape, number of turns, nature of material on which two coils are wound,*

(*ii*) *distance between two coils,*

(*iii*) *relative placement of two coils (i.e. orientation of the two coils).*

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Sample Problem **A current of 10 A in primary of a circuit is reduced to zero at a uniform rate in 10^{-3} s . If coeff. of mutual inductance is 3 H, what is the induced e.m.f. in the secondary ?**

Sol. Here, $I_1 = 10$ A, $I_2 = 0$, $dt = 10^{-3}$ s, $M = 3$ H, $e = ?$

As $$e = -M\frac{dI}{dt} = \frac{-M(I_2 - I_1)}{dt} \quad \therefore \quad e = \frac{-3(0-10)}{10^{-3}} = 3 \times 10^4 \text{ V}$$

DO YOU KNOW ?

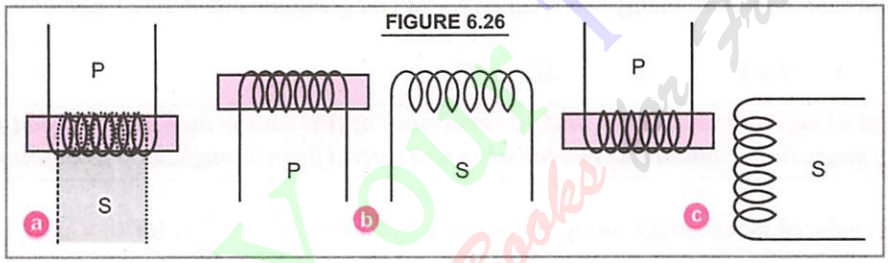
1. **Coefficient of coupling (K)** of two coils is a measure of the strength of coupling between the two coils. It is given by

$$K = \frac{M}{\sqrt{L_1 L_2}} \qquad \text{...(19)}$$

where L_1 and L_2 are coefficients of self inductance of the two coils and M is coefficient of mutual inductance of the two coils.

The value of K lies between 0 and 1. Three cases are shown in Fig. 6.26.

FIGURE 6.26



In Fig. 6.26(a), value of K is maximum

In Fig. 6.26(b), K is less than that in (a)

In Fig. 6.26(c), K is minimum.

2. If two coils are so connected in series that their coefficient of coupling $K = 1$, then we can show that $L = L_1 + L_2 + 2M$, when current in two coils is in the same direction, and $L = L_1 + L_2 - 2M$, when current in two coils is in opposite directions.

6.18. MUTUAL INDUCTANCE OF TWO LONG COAXIAL SOLENOIDS

FIGURE 6.27

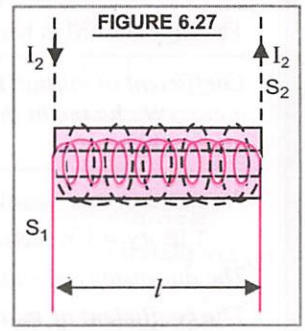


Fig. 6.27 shows two long co-axial solenoids, each of length (l). Let n_1 be the number of turns per unit length of inner solenoid S_1 of radius r_1, n_2 is number of turns per unit length of outer solenoid S_2 of radius r_2.

Imagine a time varying current I_2 through S_2, which sets up a time varying magnetic flux ϕ_1 through S_1.

$$\therefore \qquad \phi_1 = M_{12}(I_2) \qquad \text{...(20)}$$

where M_{12} is coefficient of mutual inductance of solenoid S_1 w.r.t. solenoid S_2. The magnetic field due to current I_2 in S_2 is $B_2 = \mu_0 n_2 I_2$

$\therefore$ The magnetic flux through S_1 is $\phi_1 = B_2 A_1 N_1$

where $N_1 = n_1 l$ = total no. of turns in S_1

$$\phi_1 = (\mu_0 n_2 I_2)(\pi r_1^2)(n_1 l)$$

$$\phi_1 = \mu_0 n_1 n_2 \pi r_1^2 l I_2 \qquad \text{...(21)}$$

From (20) and (21), $$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l \qquad \text{...(22)}$$

If a material of relative magnetic permeability μ_r fills the space inside S_2, we get

$$M_{12} = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$$

Note that we have neglected the **edge effects ;** and considered the magnetic field to be uniform throughout the length and width of the solenoid S_2. This aproximation is valid for $l >> r_2$.

Let us consider the reverse case :

A time varying current I_1 passed through solenoid S_1 results in the development of magnetic flux ϕ_2 through solenoid S_2, where

$$\phi_2 = M_{21} I_1 \quad ...(23)$$

where M_{21} is coefficient of mutual inductance of solenoid S_2 w.r.t. solenoid S_1.

The magnetic flux due to S_1 is confined solely inside S_1, as the solenoids are assumed to be very long. There is no magnetic field outside S_1 due to current in S_1. Therefore, magnetic flux linked with S_2 is

$$\phi_2 = B_1 A_1 N_2 = (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 l)$$

$$\phi_2 = \mu_0 n_1 n_2 \pi r_1^2 l I_1 \quad ...(24)$$

From (23) and (24), $$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l \quad ...(25)$$

From (22) and (25), we find that

$$M_{12} = M_{21} = M = \mu_0 n_1 n_2 \pi r_1^2 l \quad ...(26)$$

Thus coefficient of mutual inductance of two coils depends on their geometry, their separation and relative orientation.

We can rewrite Eqn. (22) as $$M = \mu_0 \left(\frac{N_1}{l}\right)\left(\frac{N_2}{l}\right)\pi r_1^2 \times l$$

$$M = \frac{\mu_0 N_1 N_2 A}{l} \quad ...(27)$$

Retain in Memory

Remember that in this equation (27).

N_1 = total number of turns in solenoid S_1 ; N_2 = total number of turns in solenoid S_2

l = **length of longer solenoid** (when two solenoids happen to be of different lengths)

$A = \pi r_1^2$ = **area of cross section of inner solenoid.**

We are not to take the area of cross section of outer solenoid because there is no magnetic field in the region between the two solenoids.

Sample Problem **Two circular coils, one of small radius r_1 and the other of very large radius r_2, are placed co-axially with centres coinciding. Obtain the mutual inductance of the arrangement.** **(NCERT Solved Example)**

Sol. Suppose a time varying current I_2 is made to flow through the outer circular coil, Fig. 6.28.

$\therefore$ Magnetic field at the centre of the coil is $B_2 = \frac{\mu_0 I_2}{2 r_2}$

As the inner coil placed co-axially has very small radius, therefore, B_2 may be taken as constant over its cross sectional area. Hence, flux associated with inner coil is

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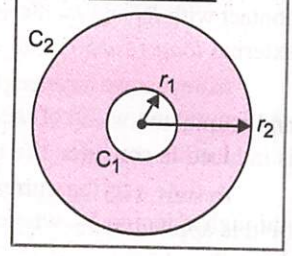
$$\phi_1 = \pi r_1^2 B_2 = \pi r_1^2 \frac{\mu_0 I_2}{2 r_2} \quad ; \quad \phi_1 = \left(\frac{\mu_0 \pi r_1^2}{2 r_2}\right) I_2 = M_{12} I_2$$

$$\therefore \quad M_{12} = \frac{\mu_0 \pi r_1^2}{2 r_2}$$

Hence, $$\boxed{M_{21} = M_{12} = \frac{\mu_0 \pi r_1^2}{2 r_2}}$$

FIGURE 6.28



6.19. A.C. GENERATOR OR A.C. DYNAMO

An a.c. generator/dynamo is a machine which produces alternating current energy from mechanical energy. It is one of the most important applications of the phenomenon of electromagnetic induction. The generator was designed originally by a Yugoslav scientist, Nikola Tesla. The word generator is a misnomer, because nothing is generated by the machine. Infact, it is an alternator converting one form of energy into another.

Principle. An a.c. generator/dynamo is based on the phenomenon of electromagnetic induction, *i.e.*, whenever amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the coil. It lasts so long as the change in magnetic flux through the coil continues. The direction of current induced is given by Fleming's right hand rule.

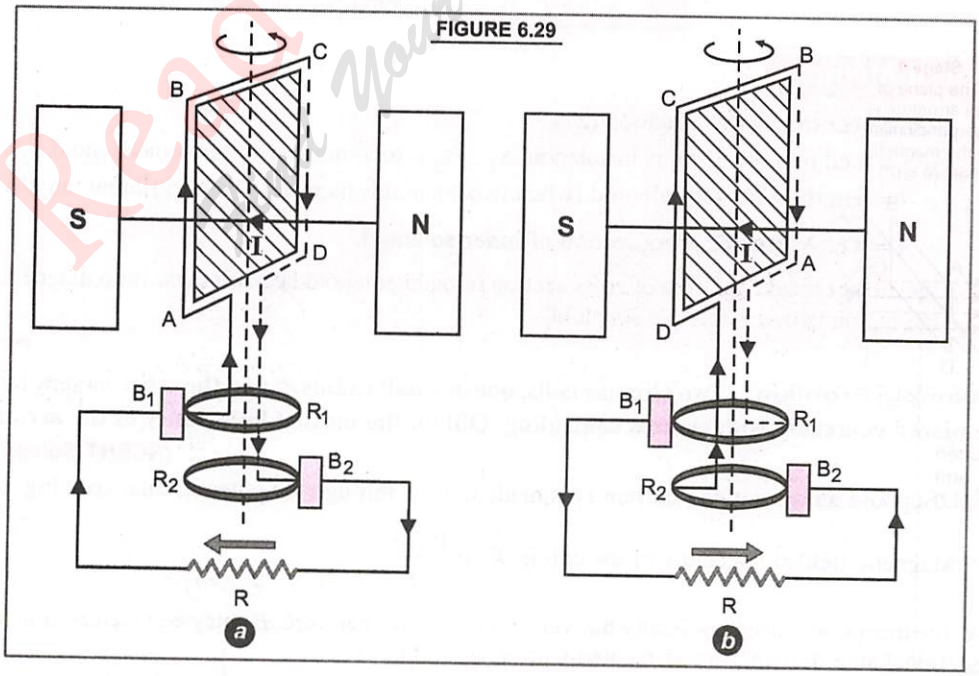
Construction. The essential parts of an a.c. dynamo are shown in Fig. 6.29.

1. *Armature. ABCD* is a rectangular armature coil. It consists of a large number of turns of insulated copper wire wound over a *laminated soft iron* core, *I*. The coil can be rotated about the central axis.

2. *Field Magnets. N* and *S* are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines. The magnetic field is of the order of 1 to 2 tesla.

3. *Slip Rings.* R_1 and R_2 are two hollow metallic rings, to which two ends of armature coil are connected. These rings rotate with the rotation of the coil.

FIGURE 6.29



4. *Brushes.* B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in light contact with R_1 and R_2 respectively. The purpose of brushes is to pass on current from the armature coil to the external *load resistance* R.

Theory and Working. As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Therefore, magnetic flux linked with the coil changes. An e.m.f. is induced in the coil.

To start with, suppose the plane of the coil is perpendicular to the plane of the paper in which magnetic field is applied, with AB at front and CD at the back, Fig. 6.29(a). The amount of magnetic flux linked with the coil in this position is maximum. As the coil is rotated anticlockwise (or clockwise), AB moves inwards and CD moves outwards. The amount of magnetic flux linked with the coil changes. According to **Fleming's right hand rule,** current induced in AB is from A to B and in CD, it is from C to D. In the external circuit, current flows from B_2 to B_1, Fig. 6.29(a).

After half the rotation of the coil, AB is at the back and CD is at the front, Fig. 6.29(b). Therefore, on rotating further, AB moves outwards and CD moves inwards. The current induced in AB is from B to A and in CD, it is from D to C. Through external circuit, current flows from B_1 to B_2 ; Fig. 6.29(b). This is repeated. Induced current in the external circuit changes direction after every half rotation of the coil. Hence the current induced is *alternating* in nature.

Fig. 6.30 depicts a clearer picture of how the current induced is alternating in nature. The configurations of the armature coil at $t = 0$, $T/4$, $2T/4$, $3T/4$ and T are shown. The current induced in the coil is plotted and it is represented by a sine curve as shown in the same figure.

To calculate the **magnitude of e.m.f. induced,** suppose

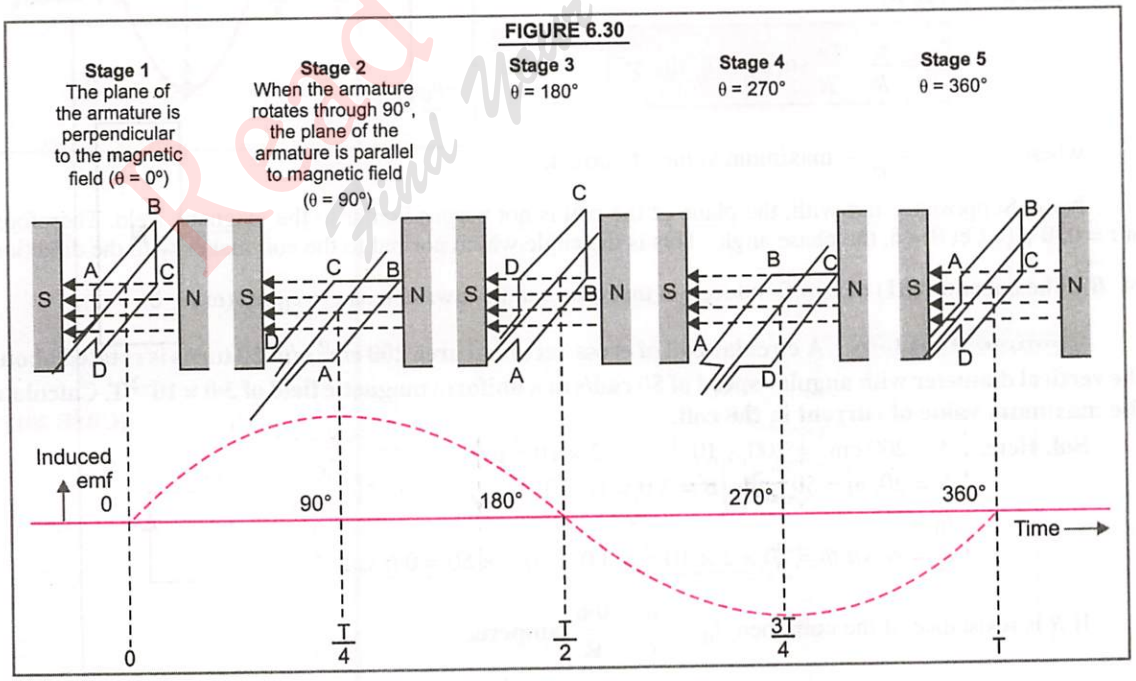
N = number of turns in the coil,

A = area enclosed by each turn of the coil

$\vec{B}$ = strength of magnetic field

θ = angle which normal to the coil makes with $\vec{B}$ at any instant t, Fig. 6.31.

FIGURE 6.30



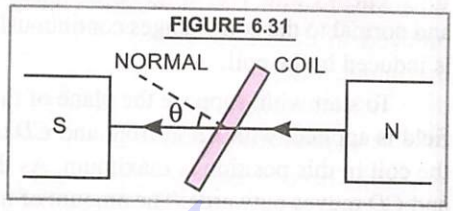
∴ Magnetic flux linked with the coil in this position

$$\phi = N\,(\vec{B}\,.\,\vec{A}) = NBA\cos\theta = NBA\cos\omega t \qquad ...(28)$$

where ω is angular velocity of the coil.

The variation of magnetic flux (ϕ) versus time (t) is shown in Fig. 6.32.

FIGURE 6.31



As the coil is rotated, θ changes ; therefore, magnetic flux ϕ linked with the coil changes and hence an e.m.f. is induced in the coil.

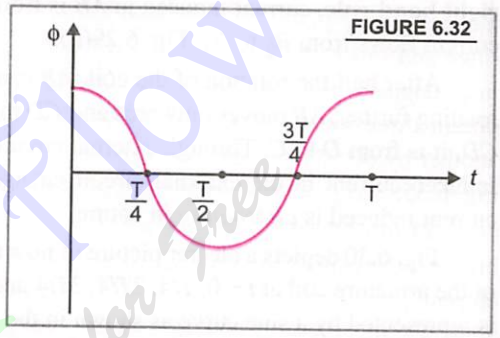
At the instant t, if e is the e.m.f. induced in the coil, then

$$e = \frac{-d\phi}{dt} = -\frac{d}{dt}\,(NAB\cos\omega t)$$

$$= -NAB\frac{d}{dt}(\cos\omega t) = -NAB\,(-\sin\omega t)\,\omega$$

$$\boxed{e = NAB\,\omega\sin\omega t} \qquad ...(29)$$

FIGURE 6.32



The induced e.m.f. will be maximum, when

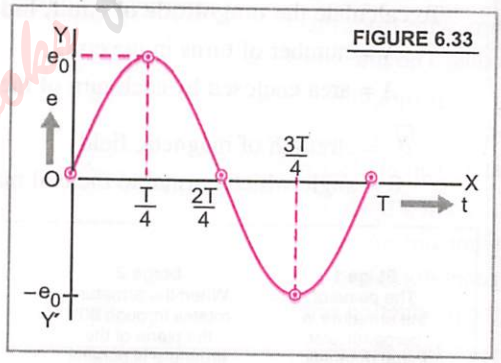
$$\sin\omega t = \text{maximum} = 1$$

$$\therefore \quad \boxed{e_{max} = e_0 = NAB\,\omega \times 1} \qquad ...(30)$$

Put in (29), $\boxed{e = e_0\sin\omega t}$ ...(31)

The variation of induced e.m.f. with time (*i.e.* with position of the coil) is shown in Fig. 6.33.

FIGURE 6.33



The current supplied by the a.c. generator is also sinusoidal. It is given by

$$\boxed{i = \frac{e}{R} = \frac{e_0}{R}\sin\omega t = i_0\sin\omega t}$$

where $i_0 = \dfrac{e_0}{R}$ = maximum value of current.

Note. Suppose to start with, the plane of the coil is not perpendicular to the magnetic field. Therefore, at $t = 0$, $\theta \neq 0$. Let $\theta = \delta$, the phase angle. This is the angle which normal to the coil makes with the direction of $\vec{B}$. The equation (31) of e.m.f. induced in that case can be rewritten as $e = e_0\sin(\omega t + \delta)$.

Sample Problem **A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad/s in a uniform magnetic field of $3{\cdot}0 \times 10^{-2}$ T. Calculate the maximum value of current in the coil.** (CBSE 2017)

Sol. Here, $A = 200\text{ cm}^2 = 200 \times 10^{-4}\text{ m}^2 = 2 \times 10^{-2}\text{ m}^2$

$N = 20$, $\omega = 50$ rad/s, $B = 3{\cdot}0 \times 10^{-2}$ T

$I_0 = ?$

$e_0 = NAB\,\omega = 20 \times 2 \times 10^{-2} \times 3{\cdot}0 \times 10^{-2} \times 50 = 0{\cdot}6$ volt

If R is resistance of the coil, then $I_0 = \dfrac{e_0}{R} = \dfrac{\mathbf{0{\cdot}6}}{\mathbf{R}}$ **ampere.**

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6.20. MULTI PHASE A.C. GENERATOR

(*a*) **Two phase a.c. generator**

In this generator, there are two armature coils held at 90° to each other. Each coil has its own pair of slip rings and brushes. When this pair of coils is rotated in the magnetic field, e.m.f. is induced in each coil. When e.m.f. induced in one coil is maximum, it is minimum in the other coil and vice-versa. Thus, the e.m.fs induced in the two coils differ in phase by 90°. This is called two phase a.c. and is shown in Fig. 6.34.

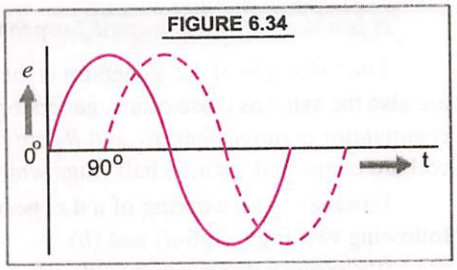


FIGURE 6.34

(*b*) **Three phase a.c. generator**

In this generator, there are three armature coils equally inclined to one another at 60°. Each coil has its own pair of slip rings and brushes. When this arrangement of coils is rotated in a magnetic field, e.m.f. is induced in each coil. Thus, we obtain three alternating e.m.fs differing in phase from one another by 60°. This is called three phase a.c. and is shown in Fig. 6.35.

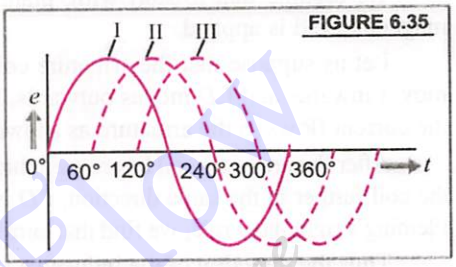


FIGURE 6.35

(*c*) *In general*, when there are a number of separate coils, each having its own pair of slip rings and brushes, the generator is called **polyphase generator**. The current produced is called polyphase alternating current.

In actual practice, one end of each coil is brought to a common point through shaft of the generator. The line wire from this point is called *Neutral line*. Separate slip rings are provided for other ends of different coils. The line wires from these rings (through their respective brushes) are called *phase lines*.

It should be clearly understood that the principle of generator discussed here applies to all the practical devices for the purpose ranging from portable generator to giant hydroelectric and thermal power generators and even nuclear power generators.

In a **hydroelectric power station,** water is stored to a great height in a dam, from where it falls on to giant turbines (popularly known as water wheels). These turbines are connected to loops of wires in a.c. generator. Thus, kinetic energy of falling water is converted into rotational energy of turbines, which leads to the production of electrical energy by the generator.

In a **thermal power station,** superheated steam is produced by boiling water using coal or oil as fuel. The superheated steam pushes past the turbines and rotates them. This leads to the production of electrical energy by the generator.

In a **Nuclear Power Generator,** nuclear energy is used to produce super heated steam that rotates the armature in the magnetic field to produce electrical energy.

IMPORTANT NOTES

1. Form factor of an *a.c.* generator is defined as the ratio of virtual value and average value of *a.c.* produced by it, *i.e.*, form factor $= \frac{I_v}{I_{av}} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}1$

2. In India, we use **three phase a.c.,** the advantages of which are : (*i*) Output of three phase system is constant. (*ii*) Output of three phase system is greater than the output of one & two phase systems. (*iii*) This system is more economical.

6.21. D.C. GENERATOR OR D.C. DYNAMO

It is a device which is used for producing direct current energy from mechanical energy.

The principle of d.c. generator is the same as that of a.c. generator. The **essential parts** of d.c. generator are also the same as those of a.c. generator *except* the *slip ring* arrangement. This is replaced by *split rings* or *commutator arrangement*. R_1 and R_2 are the two halves of the same metallic ring. The ends of the armature coil are connected to these half rings which rotate along with the armature coil, Fig. 6.36.

Working. The working of a d.c. generator is similar to that of a.c. generator and will be clear from the following two Figs. 6.36(*a*) and (*b*).

We assume that to start with, plane of the coil is perpendicular to the plane of the paper in which magnetic field is applied.

Let us suppose that the armature coil *ABCD* is rotated clockwise or anticlockwise so that the arm *AB* moves inwards and *CD* moves outwards, Fig. 6.36(*a*). Then applying Fleming's right hand rule, we see that the current flows in the armature as shown in Fig. 6.36(*a*).

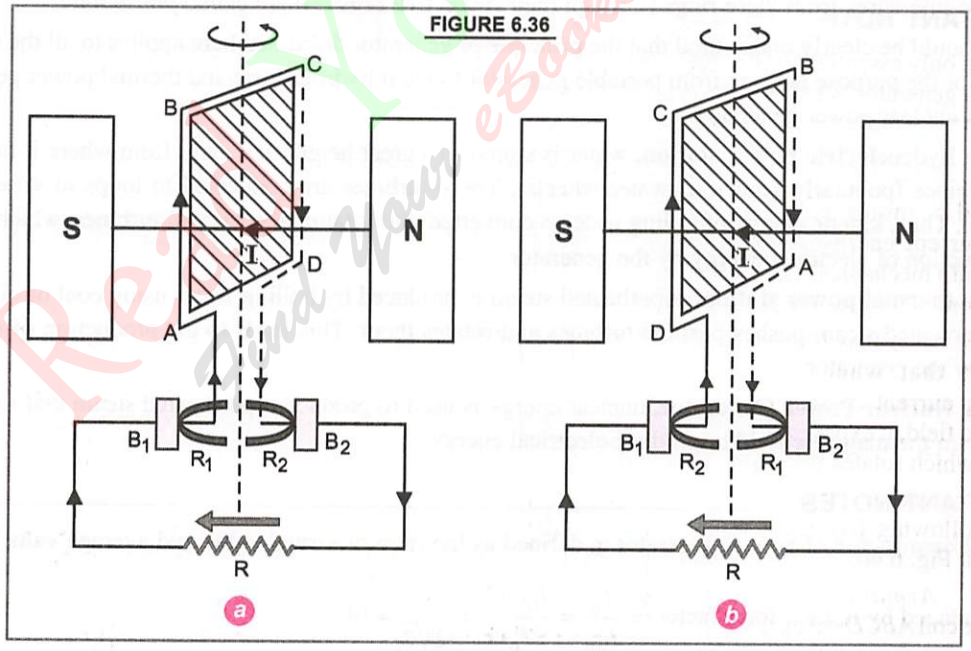
After the armature coil has rotated through 180°, it occupies the position shown in Fig. 6.36(*b*). On rotating the coil further in the same direction, *CD* is moving inwards and *AB* is moving outwards. Then again applying Fleming's right hand rule, we find that current flows in the armature as shown in Fig. 6.36(*b*).

Thus the direction of the induced e.m.f. and the induced current does not change in the external circuit during one complete rotation of the armature coil, *i.e.*, the induced current in the external circuit always flows in the same direction.

As in the case of a.c. generator, we can show that the magnitude of e.m.f. induced in d.c. generator is also

$$e = e_0 \sin \omega t$$

FIGURE 6.36



The variation of e with time in d.c. generator is shown in Fig. 6.37.

The direction of 'e' is not reversed in the second half cycle as explained in the working. It happens because after half the rotation of the coil, R_1 goes in contact with B_2 and R_2 goes in contact with B_1.

The current produced in a simple d.c. generator described above is as shown in Fig. 6.37. This current is unidirectional but its value varies considerably and falls even to zero value twice during each rotation of the coil.

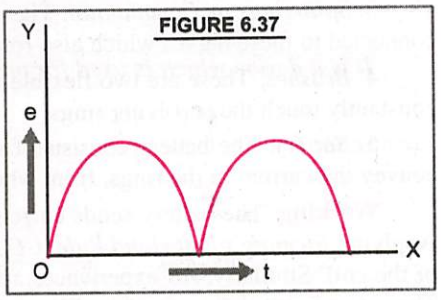


FIGURE 6.37

When we use two coils held perpendicular to each other with commutator arrangement divided into four segments connected to four ends of the two coils, and the arrangement is rotated in the magnetic field, e.m.f. of same type is induced in both the coils differing in phase by $\pi/2$. The resultant e.m.f. (e) obtained by superimposition of the two e.m.fs e_1 and e_2 is shown in Fig. 6.38.

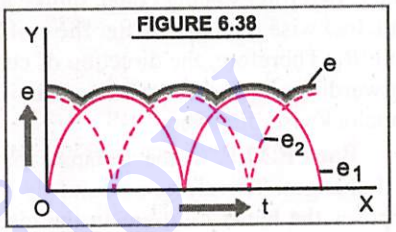


FIGURE 6.38

Obviously, the fluctuations in magnitude of *e.m.f./current* are greatly reduced.

When we use a number of such coils equally inclined to one another, with the commutator ring divided into as many segments as the total number of ends of the coils, then each coil works independently sending its own current into the outer circuit. The resultant current/e.m.f. so obtained is shown in Fig. 6.39. The magnitude of resultant e.m.f./ current is almost constant and direction, of course is the same.

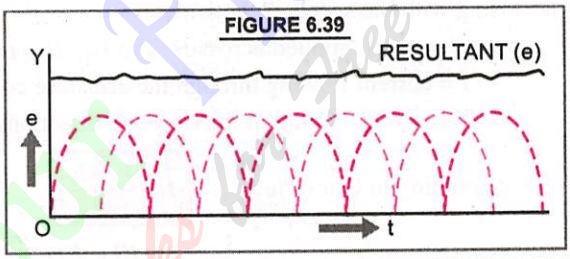


FIGURE 6.39

IMPORTANT NOTE

The only essential difference between *d.c.* generator and *a.c.* generator is that slip ring arrangement of *a.c.* generator is replaced by split ring arrangement or commutator arrangement in *d.c.* generator.

6.22. D.C. MOTOR

A d.c. motor converts direct current energy from a battery into mechanical energy of rotation.

Principle. It is based on the fact that when a coil carrying current is held in a magnetic field, it experiences a torque, which rotates the coil.

Construction. It consists of the following five parts as shown in Fig. 6.40.

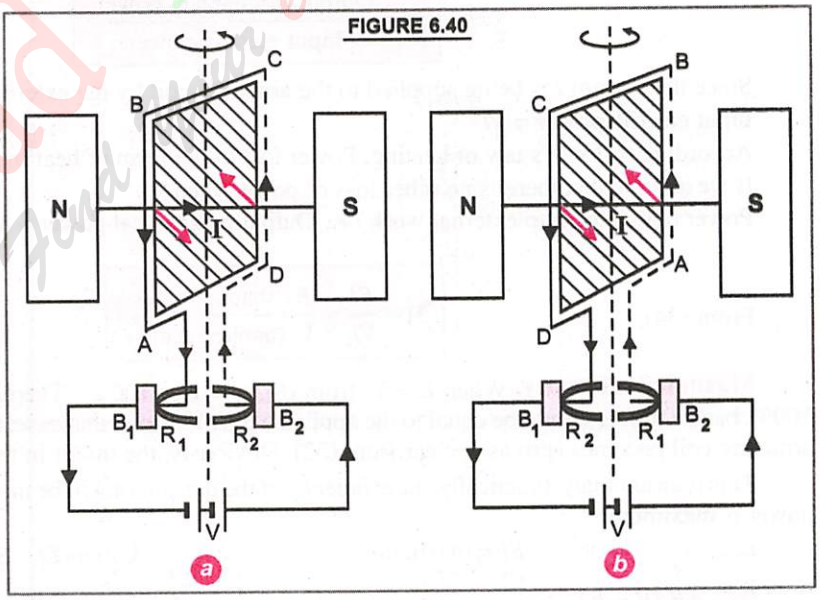


FIGURE 6.40

1. *Armature.* The armature coil *ABCD* consists of a large number of turns of insulated copper wire wound over a soft iron core.

2. *Field Magnet.* The magnetic field is supplied by a permanent magnet *NS*.

3. *Split-rings or Commutator.* These are two halves of the same ring. The ends of the armature coil are connected to these halves which also rotate with the armature.

4. *Brushes.* These are two flexible metal plates or carbon rods B_1 and B_2, which are so fixed that they constantly touch the revolving rings.

5. *Battery.* The battery consists of a few cells of voltage V connected across the brushes. The brushes convey the current to the rings, from where it is carried to the armature.

Working. The battery sends current through the armature coil in the direction shown in Fig. 6.40(*a*). Applying *Fleming's Left Hand Rule*, *CD* experiences a force directed *inwards* and perpendicular to the plane of the coil. Similarly, *AB* experiences a force directed *outwards* and perpendicular to the plane of the coil. These two forces being equal, unlike and parallel form a couple. The couple rotates the armature coil in the anticlockwise direction. After the coil has rotated through 180°, R_1 is in contact with B_2 and R_2 is in contact with B_1. Therefore, the direction of current in *AB* and *CD* is reversed, Fig. 6.40(*b*). Now *CD* experiences an outward force and *AB* experiences an inward force. The armature coil thus continues rotating in the same, *i.e.*, anticlockwise direction.

Back E.M.F. As the armature rotates in the magnetic field, the amount of magnetic flux linked with the coil changes. Therefore, an e.m.f. is induced in the coil. The direction of the induced e.m.f. is such that it opposes the battery current in the circuit. This e.m.f. is called the back e.m.f. and its magnitude goes on increasing with the speed of the armature.

Let V = e.m.f. applied across B_1 and B_2, R = resistance of the armature coil,

I = current flowing through the armature coil, at any instant t, E = back e.m.f. at that instant.

As V and E are acting in the opposite directions, $\therefore$ effective e.m.f. across B_1 and $B_2 = V - E$

According to Ohm's law, $$I = \frac{V - E}{R} \quad ...(32)$$

or $$V - IR = E \quad ...(33)$$

Efficiency of d.c. motor. It is defined as the ratio of output mechanical power to the input electrical power *i.e.*

$$\eta = \frac{\text{Output mechanical power}}{\text{Input electric power}} \quad ...(34)$$

Since the current I is being supplied to the armature coil by the external source of e.m.f. V, therefore,

Input electric power = VI

According to Joule's law of heating, Power lost in the form of heat in the coil = $I^2 R$

If we assume that there is no other loss of power, then

Power converted into external work *i.e.*, Output mechanical power = $VI - I^2 R = (V - IR)\, I = EI$ [from (33)]

From (34), $$\eta = \frac{EI}{VI} = \frac{E}{V}\frac{\text{(back e.m.f.)}}{\text{(applied e.m.f)}} \quad ...(35)$$

Maximum efficiency. When $E = V$, from (8), $\eta = 1$ or 100% . Therefore, for η to be maximum, *i.e.*, 100%, back e.m.f. E should be equal to the applied e.m.f. V. But in that case, the current I flowing through the armature coil becomes zero as is clear from (32). Obviously, the motor in this case will just cease to work.

This is an anomaly. Practically, the efficiency of the d.c. motor will be maximum, when output mechanical power is maximum.

i.e., EI = maximum or Using (5), $E.\frac{(V - E)}{R}$ = maximum

Now, $\frac{E\,(V - E)}{R}$ will be maximum (*i.e.* constant), when its differential coefficient is zero.

i.e., $\frac{d}{dE}\left[\frac{E(V-E)}{R}\right]=0$ or $\frac{d}{dE}\left[\frac{1}{R}(VE-E^2)\right]=0$ or $\frac{1}{R}(V-2E)=0$ $(\because R\neq 0)$

or $V-2E=0$ or $E=\frac{V}{2}$

From (8), $\eta=\frac{E}{V}$

When $E=\frac{V}{2}$, $\eta=\frac{V/2}{V}=\frac{1}{2}=\frac{1}{2}\times 100\%=50\%$

Hence a *d.c.* motor delivering maximum output has an efficiency of only 50%.

Further, when $E = V/2$, then from (32), Current I in the coil may become too large as R is low. Hence in practice, we do not try to get maximum output mechanical power.

Uses. 1. The d.c. motors are used in d.c. fans (exhaust, ceiling or table) for cooling and ventilation.

2. They are used for pumping water.

3. Big d.c. motors are used for running tram-cars and even trains.

Sample Problem **A motor having an armature of resistance 2·0 ohm operates on 220 V mains. At its full speed, it developes a back e.m.f. of 210 V. Find the currents in the armature when motor is switched on and when running at full speed. What is the efficiency of the motor at full speed ?**

Sol. Here, $R = 2{\cdot}0$ ohm, $V = 220$ V, $E = 210$ V,

When motor is switched on, back e.m.f. ($E = 0$).

Therefore, $I=\frac{V}{R}=\frac{220}{2{\cdot}0}=\mathbf{110\ A}$

At full speed, $I'=\frac{V-E}{R}=\frac{220-210}{2{\cdot}0}=\mathbf{5\ A}$ $\eta=\frac{E}{V}\times 100=\frac{210}{220}\times 100=\mathbf{95{\cdot}5\%}$

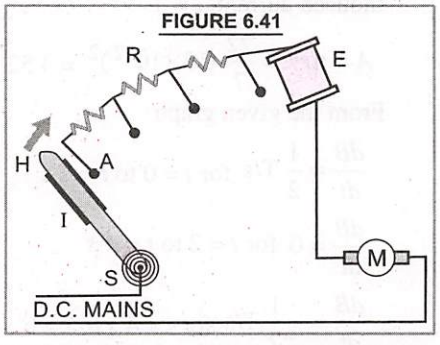
6.23. MOTOR STARTER

A starter is a device which is used for starting a d.c. motor safely. Its function is to introduce a suitable resistance in the circuit at the time of starting of the motor. This resistance decreases gradually and reduces to zero when the motor runs at full speed.

Infact, resistance of armature of d.c. motor is kept low (to reduce the copper losses) and when armature is stationary, there is no back e.m.f. Therefore, when full operating voltage is applied, the current through armature coil may become so large ($I = V/R$) that the motor may burn. A starter is needed to avoid this.

The essential parts of a starter are shown in Fig. 6.41. To start the motor, the handle H is brought in contact with the point A, so that its entire resistance R is in series with the motor M. Therefore, the initial current becomes small. The electromagnet E gets magnetised due to the passage of current. It attracts the iron piece I attached to the handle H. As the handle moves towards E, the series resistance of the starter decreases gradually. The current through the motor increases accordingly. When H just touches E, resistance R of the starter is out of circuit. The current becomes maximum and the armature rotates at full speed.

FIGURE 6.41



When the main supply is switched off, electromagnet loses its magnetism and it can no longer hold H. By the action of the spring S, the handle comes to the off position.

SOLVED EXAMPLES

TYPE I. MAGNETIC FLUX AND FARADAY'S LAWS OF EMI

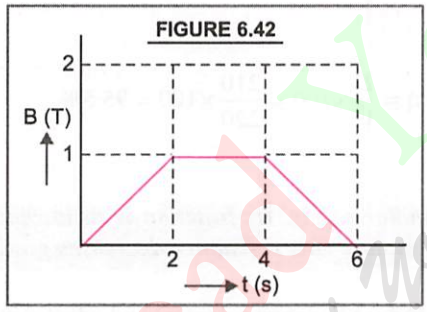
Formulae used. 1. Magnetic flux, $\phi = N(\vec{B}.\vec{A}) = NBA\cos\theta$. Here, θ is the angle which normal to the area makes with B.

2. Induced e.m.f., $e = -\dfrac{d\phi}{dt}$

Units used. ϕ in weber, e in volt, number of turns N has no units, area A in m^2, time t in second, Magnetic field B in tesla.

Example 1 **The magnetic field through a circular loop of wire 12 cm in radius and 8·5 ohm resistance, changes with time as shown in Fig. 6.42. The magnetic field is perpendicular to the plane of the loop. Calculate the induced current in the loop and plot it as a function of time.** (CBSE 2017)

FIGURE 6.42



Solution. Here, $r = 12$ cm $= 12 \times 10^{-2}$ m,

$R = 8{\cdot}5$ ohm

Induced current, $i = ?$

$$A = \pi r^2 = \frac{22}{7}(12\times10^{-2})^2 = 4{\cdot}52 \times 10^{-2}\ \text{m}^2$$

From the given graph

$$\frac{dB}{dt} = \frac{1}{2}\ \text{T/s} \quad \text{for } t = 0 \text{ to } t = 2\ s$$

$$\frac{dB}{dt} = 0 \quad \text{for } t = 2 \text{ to } t = 4\ s$$

$$\frac{dB}{dt} = -\frac{1}{2}\ \text{T/s} \quad \text{for } t = 4\ s \text{ to } t = 6\ s$$

$$i = \frac{e}{R} = \frac{d\phi/dt}{R} = \frac{A(dB/dt)}{R}$$

For $t = 0$ to $t = 2$ s

$$i = -\frac{4{\cdot}52\times10^{-2}}{8{\cdot}5}\times\frac{1}{2} = -0{\cdot}266\times10^{-2}\ \text{A}$$

$$= -2{\cdot}66\ \text{mA}$$

For $t = 2s$ to $t = 4s$

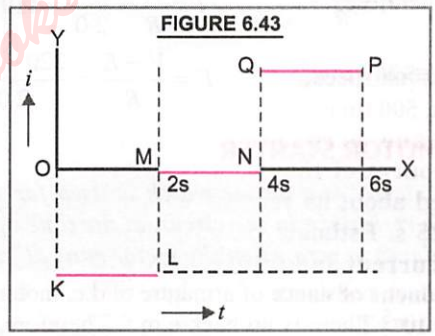
$$i = 0 \qquad \left(\because \frac{dB}{dt} = 0\right)$$

For $t = 4s$ to $t = 6$ s

$$i = -\frac{4{\cdot}52\times10^{-2}\left(-\frac{1}{2}\right)}{8{\cdot}5} = +2{\cdot}66\ \text{mA}$$

The graphical variation of induced current with time is shown in Fig. 6.43. From $t = 0$ to $t = 2s$, magnetic field is increasing. Therefore, induced current opposes the increase. From $t = 2s$ to $t = 4s$ induced current is zero. From $t = 4s$ to $t = 6s$ magnetic field is decreasing. Therefore, induced current opposes the decrease and flows in the same direction.

FIGURE 6.43



Example 2 **A conducting circular loop is placed in a uniform transverse magnetic field of 0·02 T. Somehow, the radius of the loop begins to shrink at a constant rate of 1·0 mm/s. Find the emf induced in the loop at the instant when radius is 2 cm.**

Solution. Here, $B = 0{\cdot}02\ T$,

$$\frac{dr}{dt} = 1{\cdot}0\ \text{mm/s} = 10^{-3}\ \text{m/s}$$

$$e = ? \quad r = 2\ \text{cm} = 2\times10^{-2}\ \text{m}$$

As $\phi = BA = B(\pi r^2)$

$$\therefore\ |e| = \frac{d\phi}{dt} = B(2\pi r)\frac{dr}{dt}$$

$= 0{\cdot}02 \times 2 \times 3{\cdot}14 \times 2 \times 10^{-2} \times 10^{-3}$

$= \mathbf{2{\cdot}5 \times 10^{-6}\ V}$

Example 3 **A square loop of side 10 cm and resistance 0·70 ohm is placed vertically in the east-west plane. A uniform magnetic field of 0·10 T is set up across the plane in north-east direction. The magnetic field is decreased to zero in 0·7 sec. at a steady rate. Determine the magnitudes of induced e.m.f. and current during this time interval.** (NCERT Solved Example)

Solution. Here,

$A = (10\ \text{cm})^2 = 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2$

$R = 0{\cdot}70$ ohm, $B_1 = 0{\cdot}10$ T, $\theta = 45°$

$B_2 = 0$, $dt = 0{\cdot}7$ s, $e = ?$, $I = ?$

Initial flux, $\phi_1 = B_1 A \cos\theta$

$= 0{\cdot}10 \times 10^{-2} \cos 45° = \dfrac{10^{-3}}{\sqrt{2}}$ Wb

Final flux, $\phi_2 = 0$ $(\because B_2 = 0)$

$$e = \frac{|d\phi|}{dt} = \frac{|\phi_2 - \phi_1|}{dt} = \frac{10^{-3}}{\sqrt{2} \times 0{\cdot}7} = \mathbf{10^{-3}\ V}$$

$$I = \frac{e}{R} = \frac{10^{-3}}{0{\cdot}7} = \mathbf{1{\cdot}4 \times 10^{-3}\ A}$$

Example 4 **A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0·25 s. Estimate the magnitude of the e.m.f. and current induced in the coil. Horizontal component of earth's magnetic field at the place is 3×10^{-5} T.** (NCERT Solved Example)

Solution. Here, $r = 10\ \text{cm} = 10^{-1}$ m,

$N = 500$, $R = 2\ \Omega$

$\theta_1 = 0°$, $\theta_2 = 180°$, $dt = 0{\cdot}25$ s, $e = ?$, $I = ?$

$B = 3 \times 10^{-5}$ T ; $A = \pi r^2 = 3{\cdot}14\ (10^{-1})^2$

$= 3{\cdot}14 \times 10^{-2}\ \text{m}^2$

$$e = \frac{-N|d\phi|}{dt} = \frac{-N|\phi_2 - \phi_1|}{dt}$$

$$= \frac{-NBA\,|\cos\theta_2 - \cos\theta_1|}{dt}$$

$$e = \frac{-500 \times 3 \times 10^{-5} \times 3{\cdot}14 \times 10^{-2}\,|\cos 180° - \cos 0°|}{0{\cdot}25}$$

$= \mathbf{3{\cdot}8 \times 10^{-3}\ V}$

$$I = \frac{e}{R} = \frac{3{\cdot}8 \times 10^{-3}}{2} = \mathbf{1{\cdot}9 \times 10^{-3}\ A}$$

Example 5 **The magnetic flux through a coil is varying according to the relation $\phi = (5t^3 + 4t^2 + 2t - 5)$ Wb. Calculate the induced current through the coil at $t = 2$ s, if resistance of coil is 5 ohm.** (HP Board 2007, Pb. Board 2008)

Solution. Here, $\phi = (5t^3 + 4t^2 + 2t - 5)$

$i = ?$ $t = 2$ s, $R = 5$ ohm

$$e = \left|\frac{d\phi}{dt}\right| = \frac{d}{dt}(5t^3 + 4t^2 + 2t - 5)$$

$= 15t^2 + 8t + 2 = 15\,(2)^2 + 8\,(2) + 2$

$= 78$ V

$$i = \frac{e}{R} = \frac{78}{5} = \mathbf{15{\cdot}6\ A}$$

Example 6 **An air cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500 carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back e.m.f. induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.**

Solution. Here, $l = 30\ \text{cm} = 30 \times 10^{-2}$ m,

$A = 25\ \text{cm}^2 = 25 \times 10^{-4}\ \text{m}^2$

total no. of turns, $N = 500$, $I_1 = 2.5$ ampere,

$I_2 = 0$, $dt = 10^{-3}$ sec., $e = ?$

As $e = \dfrac{d\phi}{dt} = \dfrac{(dB)\,A.(N)}{dt}$ $\left(\text{As } B = \mu_0 \dfrac{NI}{l}\right)$

$$\therefore \quad e = \frac{(\mu_0\, N\, dI)\, A(N)}{l.dt} \qquad (dI = I_1 - I_2 = 2{\cdot}5\ \text{A})$$

$$e = 4\pi \times 10^{-7} \times \frac{(500)^2}{30 \times 10^{-2}} \times \frac{2.5 \times 25 \times 10^{-4}}{10^{-3}}$$

$= \mathbf{6{\cdot}5\ volt}$

TYPE II. MOTIONAL e.m.f.

Formulae used

1. When a conductor of length l moves in a magnetic field B with a velocity v in a direction perpendicular to B, the induced emf set up across the ends of conductor is $e = B\,l\,v$.

2. When the same conductor rotates about one of its ends in the perpendicular magnetic field,

$$e = \frac{1}{2} B\omega l^2,$$

where ω is angular velocity of the rod.

3. When a disc of radius r rotates with a uniform angular speed ω in a perpendicular magnetic field B, the emf induced between the axis of disc and its rim is $e = \frac{1}{2} B\omega r^2$

Units used. l, r in metre ; v in m/s ; ω in rad/s ; B in tesla and e in volt.

Example 7 **A railway track running north south has two parallel rails 1·0 m apart. Calculate the e.m.f. induced between the rails when a train passes at a speed of 90 km h^{-1}. Horizontal component of earth's magnetic field at that place is $0{\cdot}3 \times 10^{-4}$ T and angle of dip is 60°.** **(Hr. Board 2001)**

Solution. Here, $l = 1{\cdot}0$ m, e = ?

$$v = 90 \text{ km h}^{-1} = \frac{90\times1000}{60\times60} = 25 \text{ ms}^{-1}$$

$H = 0{\cdot}3 \times 10^{-4}$ T, $\delta = 60°$

Magnetic field intercepted is

$$V = H \tan\delta = 0{\cdot}3 \times 10^{-4} \sqrt{3}\, T.$$

$$e = V l v = 0{\cdot}3 \times 10^{-4} \sqrt{3} \times 1{\cdot}0 \times 25$$

$$= \mathbf{1{\cdot}3 \times 10^{-3}\ V}$$

Example 8 **A wheel with 15 metallic spokes each 60 cm long is rotated at 360 rpm in a plane normal to the horizontal component of earth's magnetic field. The angle of dip at that place is 60°. If the e.m.f. induced between the rim of the wheel and the axle is 400 mV, calculate horizontal component of earth's magnetic field at the place. How will the induced e.m.f. change, if number of spokes is increased ?**

[CBSE 2017 (C)]

Solution. Here, $N = 15$, $r = 60$ cm $= 0{\cdot}6$ m

$$n = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ rps}$$

$\delta = 60°$, $e = 400$ mV $= 400 \times 10^{-3}$ V $= 0{\cdot}4$ V

H = ?

As the wheel rotates, linear velocity of spoke end at the rim = $r\omega$, and linear velocity of spoke end at the axle is zero.

$\therefore$ Average linear velocity $v = \frac{0 + r\omega}{2} = \frac{r\omega}{2}$.

e.m.f. induced across ends of each spoke.

$$e = Blv = H r \left(\frac{r\omega}{2}\right) = \frac{H}{2} r^2\omega$$

$$0{\cdot}4 = \frac{H}{2} (0{\cdot}6)^2 (2\pi\times6)$$

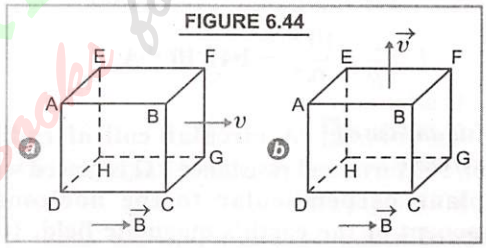
$$H = \frac{0{\cdot}8}{12\pi\ (0{\cdot}36)} = \mathbf{0{\cdot}059\ T}$$

As the spokes are in parallel, the induced *e.m.f.* is not affected by changing their number.

Example 9 **Twelve wires of equal lengths (each 10 cm) are connected in the form of a skeleton cube. If the cube is moving with a velocity of 5 m/s in the direction of a magnetic field of 0·05 Wb/m^2, find the emf induced in each arm of the cube. If the cube moves perpendicular to the field, what will be the induced *e.m.f.* in each arm ?**

Solution.

FIGURE 6.44



Here, $l = 10$ cm $= 0{\cdot}1$ m, $v = 5$ m/s,

$B = 0{\cdot}05$ Wb/m^2, e = ?

Recollect that for generation of motional *e.m.f.*, B, l and v must be along mutually perpendicular directions. If any of the two quantities are parallel, $e = 0$.

In Fig. 6.44(a), velocity of all the 12 conductors is parallel to magnetic field B, therefore, no e.m.f. is induced in any conductor.

In Fig. 6.44(b), arms AD, BC, EH and FG are parallel to velocity (v). So induced *e.m.f.* in these arms is zero. Further, arms AB, CD, EF and GH are parallel to $\vec{B}$. Therefore, no e.m.f. is induced in these arms.

The rest of the four arms AE, BF, CG and DH are perpendicular to both $\vec{v}$ and $\vec{B}$. Therefore, *e.m.f.* induced in each of these arms is

$$e = B l v = 0{\cdot}05 \times 10^{-1} \times 5$$

$$= \mathbf{2{\cdot}5 \times 10^{-2}\ V}$$

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Example 10 **A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rpm, in a plane normal to earth's magnetic field at the place. If the magnitude of the field is 0.40 gauss, what is the induced e.m.f. between the axle and rim of the wheel.**

Solution. Here, no. of spokes, $N = 10$

length of the spoke, l = radius, $r = 0.5$ m.

Frequency, $n = 120$ rpm $= 2$ rps.

$B = 0{\cdot}4$ gauss $= 0{\cdot}4 \times 10^{-4}$ tesla, $e = ?$

As the wheel rotates, linear velocity of spoke end at the rim $= r\omega$ and linear velocity of spoke end at the axle $= 0$

$\therefore$ Average linear velocity, $v = \dfrac{0 + r\omega}{2} = \dfrac{1}{2} r\omega$

As e.m.f. induced across the ends of each spoke

$$e = Bvl = B.\left(\frac{1}{2} r\omega\right).l$$

$$= B\frac{1}{2} r\, 2\pi nl = B\, r\, \pi nl$$

$$e = 0{\cdot}4 \times 10^{-4} \times 0{\cdot}5 \times \frac{22}{7} \times 2 \times 0{\cdot}5$$

$$= 6{\cdot}28 \times 10^{-5} \text{ volt}$$

As all spokes are connected in parallel between the axle and the rim, therefore net e.m.f. induced is the same as that induced across the ends of each spoke.

Example 11 **A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field of 1 T parallel to the axis is present everywhere. What is the e.m.f. between the centre and the metallic ring ?**

(CBSE 2013) (NCERT Solved Example)

Solution Here, radius of circle, $R = 1$ m. Frequency, $\nu = 50$ rev/s $B = 1$ T

In Fig. 6.45, the rotating metallic rod goes from OP to OQ in t sec tracing $\angle POQ = \theta$

The potential difference across the rod

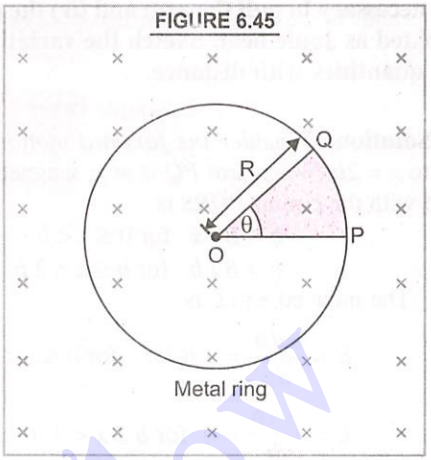
= *e.m.f.* between centre and metallic ring

$e = B \times$ rate of change of area of loop

$$= B \times \frac{d}{dt} \text{ (area of loop } POQ)$$

$$= B \times \frac{d}{dt}\left(\pi R^2 \times \frac{\theta}{2\pi}\right)$$

FIGURE 6.45



$$e = \frac{1}{2} BR^2 \frac{d\theta}{dt} = \frac{1}{2} BR^2\omega = \frac{1}{2} BR^2 (2\pi\nu)$$

$$= \pi\, \nu\, BR^2$$

$$e = \frac{22}{7} \times 50 \times 1\,(1)^2 = \mathbf{157{\cdot}1\ V}$$

Example 12 **An aeroplane is flying horizontally from west to east with a velocity of 900 km/h. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of earth's magnetic field is 5×10^{-4} T and angle of dip is 30°.**

(CBSE 2018)

Solution. Here, $v = 900$ km/h

$$= \frac{900 \times 1000}{60 \times 60} \text{ m/s} = 250 \text{ m/s}$$

$e = ?$ $l = 20$ m, $H = 5 \times 10^{-4}$ T and $\delta = 30°$

The aeroplane intercepts vertical component (V) of earth's magnetic field.

$\therefore$ $e = Vlv = (H \tan\delta)\, lv$

$$= 5 \times 10^{-4} (\tan 30°) \times 20 \times 250$$

$$= 2{\cdot}5 \times \frac{1}{\sqrt{3}} = \mathbf{1{\cdot}44\ volt}$$

Example 13 **In Fig. 6.46, the arm PQ of the rectangular conductor is moved from $X = 0$ outwards. The uniform magnetic field is perpendicular to the plane and extends from $x = 0$ to $x = b$ and is zero for $X > b$. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from $x = 0$ to $x = 2b$, and is then moved back to $x = 0$ with constant speed v. Obtain expressions for the (*i*) electric flux, (*ii*) the induced *e.m.f.*, (*iii*) the**

force necessary to pull the arm and (*iv*) the power dissipated as Joule heat. Sketch the variations of these quantities with distance.

(CBSE Sample Paper 2019-20)

Solution. *Consider the forward motion* from $x = 0$ to $x = 2b$. When arm PQ is at x, magnetic flux linked with the circuit $PQRS$ is

$$\phi = B\,l\,x \quad \text{for } 0 \le x < b$$
$$\phi = B\,l\,b \quad \text{for } b \le x < 2\,b$$

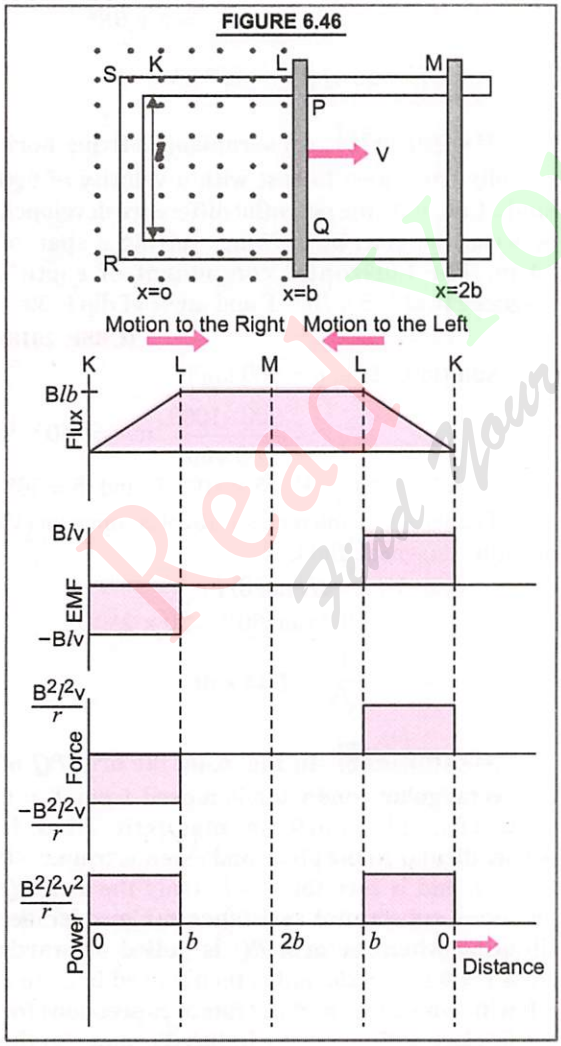
The induced e.m.f. is

$$E = \frac{-d\phi}{dt} = -B\,l\,v \quad \text{for } 0 \le x < b$$

$$E = \frac{-d\phi}{dt} = 0 \quad \text{for } b \le x < 2\,b$$

When the induced e.m.f. is non-zero, the magnitude of current is

FIGURE 6.46



$$I = \frac{E}{r} = \frac{B\,l\,v}{r}$$

Force required to keep the arm PQ in constant motion $= BI\,l = B\left(\frac{B\,l\,v}{r}\right) l$

$$= \frac{B^2\,l^2\,v}{r}, \quad \text{for } 0 \le x < b,$$

$$= \text{zero}, \quad \text{for } b \le x < 2\,b$$

The direction of the force on PQ is to the left. The joule heating loss is

$$P = I^2 r = \frac{B^2\,l^2\,v^2}{r}, \quad \text{for } 0 \le x < b$$

$$= 0, \quad \text{for } b \le x < 2\,b$$

Similar expressions are obtained for inward motion of PQ from $x = 2\,b$ to $x = 0$. The various quantities have been plotted in Fig. 6.46.

TYPE III. SELF INDUCTION

Formulae used. 1. $\phi = LI$. **2.** $e = L\frac{dI}{dt}$

3. In case of a long solenoid, $L = \frac{\mu_0 N^2 A}{l}$

Note that N is total number of turns of the solenoid.

Units used. ϕ in weber, e in volt, I in ampere, L in henry, l in meter, A in metre^2.

Standard value. $\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}

Example 14 **A solenoid of length 40 cm, area of cross section 20 cm^2 and total number of turns 800 is connected to a source that supplies current changing at the rate of 0·2 A/s. What is the emf induced across the solenoid ?**

Solution. Here, $l = 40$ cm $= 0{\cdot}4$ m,

$A = 20$ cm^2 $= 20 \times 10^{-4}$ m^2, $N = 800$

$dI/dt = 0{\cdot}2$ A/s, $e = ?$

$$e = \frac{L\,dI}{dt} = \frac{\mu_0 N^2 A}{l}\,\frac{dI}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times (800)^2 \times 20 \times 10^{-4}}{0.4} \times 0.2$$

$$= 8{\cdot}03 \times 10^{-4}\ \text{V}$$

Example 15 **A solenoid of radius 3 cm and length 1 m has 600 turns per metre. What is its**

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self inductance ? Will the value of self inductance change if it is wound on an iron piece ?

Solution. Here, $r = 3$ cm $= 3 \times 10^{-2}$ m, $l = 1$ m, $N = 600$, $L = ?$

$$A = \pi r^2 = \pi (3 \times 10^{-2})^2 = 9\pi \times 10^{-4} \text{ m}^2$$

$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} (600)^2 \, 9\pi \times 10^{-4}}{1}$$

$$= \mathbf{1{\cdot}28 \times 10^{-3} \text{ H}}$$

If the solenoid is wound on an iron core, value of its self inductance will change

TYPE IV. MUTUAL INDUCTION

Formulae used. 1. $\phi = MI$

2. $e = \frac{d\phi}{dt} = M \frac{dI}{dt}$

3. $M = \frac{\mu_0 N_1 N_2 A}{l}$,

where N_1, N_2 are total no. of turns in the two solenoids, l is length of longer solenoid and $A = \pi r^2$ = area of cross section of inner solenoid.

Units used. I in ampere, $\frac{dI}{dt}$ in ampere/sec, M in henry, ϕ in weber, e in volt, l in metre, A in metre^2

Standard Value. $\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}

Example 16 **A large circular coil of radius R and a small circular coil of radius r are put in vicinity of each other. If coefficient of mutual inductance of this pair equals 1 mH, what would be the flux linked with larger coil when a current of 0·5 A flows through the smaller coil ? When the current in the smaller coil falls to zero, what would be its effect in the larger coil ?** [CBSE 2008 (C)]

Solution. Here, $M = 1$ mH $= 10^{-3}$ H, $\phi = ?$,

$I = 0{\cdot}5$ A, $\phi = MI = 10^{-3} \times 0{\cdot}5 = \mathbf{5 \times 10^{-4}}$ **Wb**

When current in smaller coil falls to zero, an *e.m.f.* is induced in the larger coil due to mutual induction.

Example 17 **A solenoid of length 50 cm with 20 turns per cm and area of cross section 40 cm^2 completely surrounds another co-axial solenoid of the same length, area of cross section 25 cm^2 with 25 turns per cm. Calculate the mutual inductance of the system.**

Solution. Here, $l = 50$ cm $= \frac{1}{2}$ m

Total no. of turns in one solenoid

$$N_1 = 20 \times 50 = 1000$$

area of cross section of outer solenoid,

$$A_1 = 40 \text{ cm}^2 = 40 \times 10^{-4} \text{ m}^2$$

Total no. of turns in inner solenoid,

$$N_2 = 25 \times 50 = 1250$$

area of cross section of inner solenoid,

$$A_2 = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$$

$$M = \frac{\mu_0 N_1 N_2}{l} A_2^*$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times 1250 \times 25 \times 10^{-4}}{1/2}$$

$$= \mathbf{7{\cdot}85 \times 10^{-3} \text{ henry}}$$

TYPE V. ELECTRIC GENERATOR

Formulae used. 1. $e = e_0 \sin \omega t$

2. $e_0 = NAB\omega$ 3. $e = -\frac{d\phi}{dt}$

Units used. e.m.f. (e), maximum e.m.f. (e_0) in volt ; angular velocity (ω) in radian/sec. ; magnetic field B in tesla ; area A in m^2 ; magnetic flux ϕ in Wb.

Example 18 **An armature coil consists of 20 turns of wire each of area A = 0·09 m^2 and total resistance 15·0 ohm. It rotates in a magnetic field of 0·5 T at a constant frequency of $\frac{150}{\pi}$ Hz. Calculate the value of (*i*) maximum and (*ii*) average induced e.m.f. produced in the coil.** (CBSE Sample Paper 2008)

Solution. Here, $N = 20$, $A = 0{\cdot}09$ m^2,

$R = 15{\cdot}0$ ohm, $B = 0{\cdot}5$ T

$$\nu = \frac{150}{\pi} \text{ Hz}, \quad e_0 = ?$$

$$e_0 = NAB\omega = NAB(2\pi\nu)$$

$$= 20 \times 0{\cdot}09 \times 0{\cdot}5 \left(2\pi \times \frac{150}{\pi}\right) = \mathbf{270 \text{ V}}$$

As *e.m.f.* produced is alternating, average induced *e.m.f.* produced in the coil is zero.

*In all such questions, we have to take area of cross section of inner solenoid and length of longer solenoid as explained in the text.

Example 19 **An athlete peddles a stationary tricycle whose pedals are attached to a coil having 100 turns each of area 0·1 m^2. The coil lying in XY plane is rotated in this plane at the rate of 50 rpm about the Y-axis in a region where a uniform magnetic field $\vec{B} = (0{\cdot}01)\hat{k}$ tesla is present. Find the (*i*) max. e.m.f. (*ii*) average *e.m.f.* generated in the coil over one complete rotation.**

(CBSE Sample Paper 2013)

Solution. Here, $N = 100$, $A = 0{\cdot}1$ m^2,

$$\nu = 50 \text{ rpm} = \frac{5}{6} \text{ rps}$$

$\vec{B} = (0{\cdot}01)\hat{k}$ T, *i.e.*, $B = 0{\cdot}01$ T along z-axis.

$$e_0 = NAB\,\omega = 100 \times 0{\cdot}1 \times 0{\cdot}01 \times 2\pi \times \frac{5}{6}$$

$$= \mathbf{0{\cdot}52\ V}$$

(*ii*) As the *e.m.f.* generated varies sinusoidally with time, so the average *e.m.f.* generated in the coil over one complete revolution is **zero.**

Example 20 **Kamla peddles a stationary bicycle, the pedals of which are attached to a 100 turn coil of area 0·10 m^2. The coil rotates at half a revolution in one second and it is placed in a uniform magnetic field of 0·01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil ?**

(NCERT Solved Example)

Solution. Here, $N = 100$, $A = 0{\cdot}10$ m^2,

$\nu = 0{\cdot}5$ Hz, $B = 0{\cdot}01$ T, $E_0 = ?$

$$E_0 = NAB\,\omega = NAB\,(2\pi\nu)$$

$$= 100 \times 0{\cdot}10 \times 0{\cdot}01 \times 2 \times \frac{22}{7} \times 0{\cdot}5$$

$$= \mathbf{0{\cdot}314 \text{ volt}}$$

TYPE VI. ELECTRIC MOTOR

Formulae used.

1. Efficiency of d.c. motor

$$\eta = \frac{\text{Output mechanical power}}{\text{Input electric power}}$$

$$= \frac{EI}{VI} = \frac{E}{V} = \frac{\text{back emf}}{\text{e.m.f. of battery}}$$

2. For maximum efficiency, $E = \frac{V}{2}$

3. Current through armature coil,

$$I = \frac{V - E}{R}$$

Units used. E, V are in volt ; R in ohm and I in ampere ; η has no units.

Example 21 **A small d.c. motor operating at 200 V draws a current of 5.0 amp. at its full speed of 3000 r.p.m. The resistance of the armature of the motor is 8.5 Ω. Determine the back e.m.f. of the motor. Obtain the power input, power output and efficiency of motor.**

Solution. Here, $V = 200$ V, $I = 5.0$ amp.,

$n = 3000/60 = 50$ r.p.s., $R = 8.5\ \Omega$

If E is the back e.m.f., then

$$I = \frac{V - E}{R} \quad \text{or} \quad V - E = IR$$

$$\therefore E = V - IR = 200 - 5 \times 8.5 = \mathbf{157.5 \text{ volt}}$$

Input power $= VI = 200 \times 5 = \mathbf{1000 \text{ watt}}$

Output power $= VI - I^2R = I(V - IR) = IE$

$$= 5 \times 157.5$$

$$= \mathbf{787.5\ W}$$

Efficiency = Output power/Input power

$$\eta = \frac{787.5}{1000} = 0.7875 = \mathbf{78{\cdot}75\%}$$

Example 22 **(a) Suppose the windings of the armature in the d.c. motor of Example 27 cannot tolerate a current of more than 20 amp. What do you think will happen if the armature gets jammed and cannot rotate when the motor is connected to the supply ?**

(b) If the supply connection of the d.c. motor in Example 8 are removed and the motor is used as a generator by connecting the shaft of its armature to an external mechanical rotor of speed 3000 r.p.m., how much e.m.f. will be generated ?

Solution. (*a*) When the armature gets jammed and cannot rotate, back e.m.f. $E = 0$.

$\therefore$ When connected to the supply,

$$\text{current } I = \frac{V}{R} = \frac{200}{8.5} = 23{\cdot}53 \text{ amp.}$$

which exceeds the safe limit of 20 amp. Therefore, armature will burn out.

(*b*) The e.m.f. generated is obviously equal to the back e.m.f. E = **157·5 volt,** as calculated in Example 27.

TYPE VII. TYPICAL EXAMPLES

Example 23 **(a) A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Obtain the self inductance of the toroid. Ignore field variations across the cross-section of the toroid.**

(b) A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2.0 A in 0.05 s, obtain the induced e.m.f. in the second coil.

Solution. (*a*) Here, $a = 15 \text{ cm} = 0.15 \text{ m}$

$A = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$

Total no. of turns, $N_1 = 1200$

Length of toroidal solenoid,

$$l = 2\pi a = 2\pi \times 0.15 \text{ m} = 0.3\pi \text{ m}$$

As $L = \mu_0 \dfrac{N_1^2}{l} A$

$$\therefore \quad L = 4\pi \times 10^{-7} \times \frac{(1200)^2 \times 12 \times 10^{-4}}{0{\cdot}3\pi}$$

$$= \mathbf{2.304 \times 10^{-3} \text{ henry}}$$

(*b*) Here $N_2 = 300$,

$$\frac{dI}{dt} = \frac{2-0}{0.05} = 40 \text{ amp/sec.}, \quad e = ?$$

As $e = M \dfrac{dI}{dt} = \dfrac{(\mu_0 N_1 N_2 A)}{l} \dfrac{dI}{dt}$

$$\therefore e = 4\pi \times 10^{-7} \times 1200 \times \frac{300}{0.3\pi} \times 12 \times 10^{-4} \times 40$$

$$= \mathbf{0.023 \text{ volt}}$$

Example 24 **Two co-axial circular coils of radii 50 cm and 5 cm are separated by a distance of 50 cm and carry currents 3 A and 2 A respectively. Calculate the mutual inductance of the two coils.**

Solution. Here, $R = 50 \text{ cm} = 0{\cdot}5 \text{ m}$,

$r = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$, $x = 50 \text{ cm} = 0{\cdot}5 \text{ m}$,

$I_1 = 3 \text{ A}$, $I_2 = 2 \text{ A}$, $M = ?$

$$M^* = \frac{\mu_0}{4\pi} \times \frac{2\pi^2 R^2 r^2}{(R^2 + x^2)^{3/2}}$$

$$= \frac{10^{-7} \times 2(3{\cdot}14)^2 (0{\cdot}5)^2 (5 \times 10^{-2})^2}{[0{\cdot}5^2 + 0{\cdot}5^2]^{3/2}}$$

$$M = \mathbf{3{\cdot}48 \times 10^{-9} \text{ H}}$$

Example 25 **A town situated 20 km away from a power plant generating power at 440 V requires 600 kW of electric power at 200 V. The resistance of two wire lines carrying power is 0·4 Ω per km. The town gets power from the line through a 3000-220 V step down transformer at a substation in the town. Find line power losses in the form of heat. How much power must the plant supply assuming that there is negligible power loss due to leakage ?** **(CBSE 2019)**

Solution. Resistance of two line wires,

$$R = 20 \times 2 \times 0{\cdot}4 = 16 \ \Omega$$

For the transformer, $E_p = 3000$ V, $E_s = 220$ V

and $P = 600 \text{ kW} = 6 \times 10^5 \text{ W}$

$\therefore$ Current carried by line wires,

$$I_v = \frac{P}{E_p} = \frac{6 \times 10^5}{3000} = 200 \text{ A}.$$

Power loss in the form of heat $= I_v^2 \times R$

$= (200)^2 \times 16 = 640000 \text{ W} = 640 \text{ kW}$

Power supplied by plant = power demand + power loss

$= 600 + 640 = \mathbf{1240 \text{ kW}}$

REVISION AT A GLANCE

1. **Electromagnetic Induction** (EMI) is the phenomenon of generating an *e.m.f.* in a circuit by changing the number of magnetic lines of force associated with the circuit. The *e.m.f.* so generated is called *induced e.m.f.* and the corresponding current is called *induced current.*

*The derivation of this formula is ommitted.

The number of magnetic lines of force crossing a surface is called *magnetic flux* linked with the surface. It is represented by $\phi = \vec{B} \cdot \vec{A} = BA \cos\theta$

where B is strength of magnetic field, A is area of the surface and θ is the angle which *normal to the area* makes with the direction of magnetic field.

The SI unit of magnetic flux is **weber** which is the amount of magnetic flux over an area of 1 m^2 held normal to a uniform magnetic field of one tesla. The c.g.s. unit of ϕ is **maxwell.** 1 weber = 10^8 maxwell.

2. **Faraday** and **Henry** discovered the phenomenon of EMI and established the following two laws of EMI.

First Law. Whenever the amount of magnetic flux linked with a circuit changes, an *e.m.f.* is induced in the circuit. This induced e.m.f. lasts so long as the change in magnetic flux continues.

Second Law. The magnitude of *e.m.f.* induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit, *i.e.*, $e = \frac{\phi_2 - \phi_1}{t}$ or $e = \frac{-d\phi}{dt}$

3. The direction of induced *e.m.f.* is given by **Lenz's law**. According to this law, the direction of induced *e.m.f.* in a circuit is always such as to oppose the change in magnetic flux responsible for it.

Lenz's law is in accordance with the principle of conservation of energy. Infact, work done in moving the magnet w.r.t. the coil changes into electric energy producing induced current.

There is also another law for finding the direction of induced current. This is **Fleming's right hand rule.** According to this rule, if we stretch the first finger, the central finger and thumb of our right hand in mutually perpendicular directions, such that first finger points along the magnetic field and thumb points along the direction of motion of the conductor, then central finger would give us the direction of induced current.

4. As is known, *e.m.f.* is induced in a circuit only when amount of *magnetic flux* linked with the circuit *changes*. As $\phi = BA \cos\theta$, therefore, three methods of producing induced e.m.f. are (*i*), by changing B, (*ii*) by changing A and, (*iii*) by changing θ. When a conductor of length l moves with a velocity v in a perpendicular magnetic field of strength B so that magnetic flux linked with the circuit changes, the *e.m.f.* induced (e) is given by $e = Blv$.

5. **Eddy currents** are the currents induced in the body of a conductor when the amount of magnetic flux linked with the conductor changes. These currents are also called *Foucault currents*. The magnitude of eddy currents is given by $i = \frac{-e}{R} = \frac{d\phi/dt}{R}$ where R is resistance of the conductor. The direction of eddy currents is given by Lenz's law or Fleming's right hand rule. Some of the *important applications* of eddy currents are : Electromagnetic damping, induction furnace, electromagnetic brakes, induction motor, speedometers and in diathermy, *i.e.*, deep heat treatment of parts of human body.

Some of the *undesirable effects* of eddy currents are that they oppose the relative motion, involve loss of energy in the form of heat and reduce the life of electrical devices. To minimise eddy currents, we use *laminated cores.*

6. **Self induction** is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an *e.m.f.* in itself.

When a current I flows through a coil, the magnetic flux ϕ linked with the coil is $\phi = LI$, where L is *coefficient of self induction* or *self inductance* of the coil. On differentiating , we get

$$\frac{d\phi}{dt} = L\frac{dI}{dl} = -e$$

If $dI/dt = 1$; $L = -e$. Hence self inductance of a coil is equal to *e.m.f.* induced in the coil when rate of change of current through the coil is unity. The value of L depends on *geometry* of the coil and is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

where l is length of the coil (solenoid), N is total number of turns of solenoid and A is area of cross section of the solenoid.

The SI unit of L is *henry*. Coefficient of self induction of a coil is said to be one henry when a current change at the rate of 1 ampere/sec. in the coil induces an *e.m.f. of one volt in the coil.*

7. **Mutual Induction** is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced *e.m.f.*

If ϕ is the amount of magnetic flux linked with one coil when a current I flows through the other coil, then $\phi = MI$, where M is coefficient of mutual induction or mutual inductance of two coils.

As $e = \frac{-d\phi}{dt} = \frac{-d}{dt}(MI) = -M\frac{dI}{dt}$ $\quad\therefore\quad$ If $\frac{dI}{dt} = 1$, then $e = -M \times 1$ or $M = -e$

Hence coefficient of mutual induction of two coils is equal to the *e.m.f.* induced in one coil, when rate of change of current through the other coil is unity. The SI unit of M is *henry*. Coefficient of mutual induction/ mutual inductance of two coils is said to be one henry, when a current change at the rate of 1 ampere/sec. in one coil induces an *e.m.f.* of one volt in the other coil. The value of M depends on *geometry* of two coils, distance between two coils, relative placement of two coils etc.

The coefficient of mutual inductance of two long co-axial solenoids, each of length l, area of cross section A, wound on an air core is $M = \frac{\mu_0 N_1 N_2 A}{l}$, where N_1, N_2 are total number of turns of the two solenoids.

8. **An a.c. generator/dynamo** produces alternating current energy from mechanical energy of rotation of a coil. It is based on the phenomenon of *EMI*. The form of *e.m.f.* induced is $e = e_0 \sin \omega t$, where $e_0 = NAB\,\omega$, max. e.m.f. induced. Here, N is total number of turns in the coil, A is face area of the coil, B is strength of magnetic field applied and ω is angular velocity of the armature coil.

The a.c. supply in our country is 220 volt, 50 c/s.

9. **A d.c. generator/dynamo** produces direct current energy from mechanical energy of rotation of a coil. Its principle and working are same as those of a.c. generator. There is only a little change in the design of the generator. *Slip Ring* arrangement used in a.c. generator is replaced by *Split Ring* arrangement in *d.c.* generator.

10. **A d.c. motor** converts direct current energy from a battery into mechanical energy of rotation. It is based on the fact that when a coil carrying current is held in a magnetic field, it experiences a torque, which rotates the coil. The efficiency of d.c. motor is given by

$$\eta = \frac{E}{V} = \frac{\text{back e.m.f.}}{\text{e.m.f. of battery}}$$

The efficiency becomes maximum, when $E = V/2$

A motor starter is a variable resistance, which protects the motor from damage, when it is switched on.

OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. A coil of area 100 cm^2 is kept at an angle of 30° with a magnetic field of 10^{-1} T. The magnetic field is reduced to zero in 10^{-4} s. The induced e.m.f. in the coil is

(*a*) $5\sqrt{3}$ V (*b*) $50\sqrt{3}$ V

(*c*) 5·0 V (*d*) 50·0 V **(CBSE 2022)**

2. The self inductance of a solenoid of 600 turn is 108 mH. The self inductance of a coil having 500 turns with the same length, the same radius and the same medium will be

(*a*) 95 mH (*b*) 90 mH

(*c*) 85 mH (*d*) 75 mH **(CBSE 2022)**

3. The electric potential V at any point (x, y, z) is given by $V = 3x^2$, where x is in meter and V is in volt. The electric field at the point (1 m, 0, 2 m) is

(*a*) 6 V/m along $-x$ axis

(*b*) 6 V/m along $+x$ axis

(*c*) 1·5 V/m along $-x$ axis

(*d*) 1·5 V/m along $-x$ axis **(CBSE 2022)**

4. The current in the primary coil of a pair of coils changes from 7 *A* to 3 *A* in 0·04 s. The mutual

inductance between the two coils is 0·5 H. The induced e.m.f. in the secondary coil is

(*a*) 50 V (*b*) 75 V

(*c*) 100 V (*d*) 220 V **(CBSE 2022)**

5. A constant current is flowing through a solenoid. An iron rod is inserted in the solenoid along its axis. Which of the following quantities will not increase?

(*a*) The magnetic field at the centre

(*b*) The magnetic flux linked with solenoid

(*c*) Rate of heating

(*d*) The self inductance of the solenoid

(CBSE 2022)

6. A capacitor and an inductor are connected to two different a.c. circuits. The bulb glows more brightly when

(*a*) number of turns in the inductor is increased

(*b*) the separation between the plates of capacitor is increased

(*c*) an iron rod is introduced into the inductor

(*d*) a dielectric is introduced into the gap between the plates of the capacitor **(CBSE 2022)**

7. Lenz's Law is the consequence of the law of conservation of

(*a*) energy (*b*) charge

(*c*) mass (*d*) momentum

(CBSE 2022)

8. The e.m.f. induced in a 10 H inductor in which current changes from 11 A to 2 A in 9×10^{-1} sec. is

(*a*) 10^4 V (*b*) 10^3 V

(*c*) 10^2 V (*d*) 10 V **(CBSE 2022)**

9. The magnetic flux linked with a coil is given by $\phi = 5t^2 + 3t + 16$. When ϕ is in weber and t is in seconds, the induced emf in the coil at $t = 5$ s is

(*a*) 53 V (*b*) 43 V

(*c*) 10 V (*d*) 6 V **(CBSE 2022)**

10. When number of turns in a solenoid is doubled, its self inductance becomes k times, where k =

(*a*) 2 (*b*) 1

(*c*) 8 (*d*) 4 **(CBSE 2022)**

11. An e.m.f. of 12 volt is produced in a coil when the current in it changes at the rate of 45 ampere/minute. The inductance of the coil is :

(*a*) 0·25 henry (*b*) 1·5 henry

(*c*) 9·6 henry (*d*) 16·0 henry

12. A current of 1 ampere is passed through a coil at frequency n_1, subsequently the frequency is made n_2. If ε_1 and ε_2 be the induced peak emfs in the coil, then $\varepsilon_1/\varepsilon_2$ will be equal to :

(*a*) $n_1 : n_2$ (*b*) $n_2 : n_1$

(*c*) $n_1^2 : n_2^2$ (*d*) $n_2^2 : n_1^2$

13. The peak value of alternating e.m.f. in a generator is given by e_0 =

(*a*) NAB (*b*) $NAB\omega$

(*c*) $NAB\upsilon$ (*d*) none of these

Symbols have their usual meaning.

14. The energy stored in a coil carrying current is u. If current is halved, the energy stored will be

(*a*) $\frac{u}{2}$ (*b*) $\frac{u}{4}$

(*c*) $2u$ (*d*) $4u$ **(CBSE 2022)**

15. The direction of induced current is such that it opposes the very cause that has produced it. This is the law of

(*a*) Lenz (*b*) Faraday

(*c*) Kirchhoff (*d*) Flaming **(CBSE 2022)**

16. When a magnet is moved with its north pole towards a coil placed in a closed circuit, then the nearest face of the coil shows

(*a*) south polarity (*b*) north polarity

(*c*) no polarity

(*d*) sometimes north and sometimes south polarity

17. An air cored solenoid has 1000 turns and is one metre long. Its cross-sectional area is 10 cm^2, its self inductance is :

(*a*) 0·1256 mH (*b*) 12·56 mH

(*c*) 1·256 mH (*d*) 125·6 mH

18. Choose the wrong statement :

(*a*) Whenever the amount of magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit.

(*b*) The induced e.m.f. lasts so long as the change in magnetic flux continues.

(*c*) Larger the amount of magnetic flux linked with a circuit, greater is the e.m.f. induced in it.

(*d*) The direction of induced e.m.f. is given by Lenz's Law.

19. Amount of charge induced in a circuit of resistance R is given by

(*a*) $dQ = (d\phi) \times R$ (*b*) $dQ = \frac{d\phi}{R}$

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(c) $dQ = R^2\, d\phi$ (d) $dQ = \dfrac{d\phi}{R^2}$

20. Out of the following, choose the correct relation

(a) $1\,\text{henry} = \dfrac{1\,\text{volt}}{1\,\text{ampere}}$

(b) $1\,\text{henry} = \dfrac{1\,\text{amp}}{1\,\text{volt}}$

(c) $1\,\text{henry} = \dfrac{1\,\text{volt}}{1\,\text{amp/sec}}$

(d) $1\,\text{henry} = \dfrac{1\,\text{volt}}{1\,\text{amp.sec}}$

21. Two coils of self inductance L_1 and L_2 are placed so close together that effective flux in one coil is completely linked with other. The mutual inductance between them is :

(a) $L_1 L_2$ (b) L_1/L_2

(c) $(L_1/L_2)^2$ (d) $\sqrt{L_1 L_2}$

22. The magnetic flux linked with a coil is $\phi = (3t^2 - 2t + 1)$ milliweber. The e.m.f. induced in the coil at $t = 1$ sec is

(a) 4 V (b) 4×10^{-3} V

(c) 6 V (d) 4×10^3 V

23. A wire of length 2 m moves with a speed of 5 m/s perpendicular to a magnetic field of induction 0·1 Wb/m^2. The e.m.f. induced in the wire is

(a) 1 V (b) 10 V

(c) 5 V (d) 2 V

24. A circular ring of diameter 20 cm has a resistance of 0·01 Ω. The ring is turned from a position perpendicular to a uniform magnetic field of 2 T to a position parallel to the field The amount of charge flowing through the ring in this process is :

(a) 2 C (b) π C

(c) 2π C (d) 8π C

25. The frequency of a.c. generated depends on

(a) speed of rotation of coil

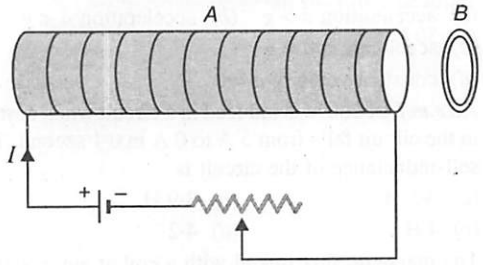
(b) amplitude of a.c.

(c) size of coil

(d) all the above

26. An aluminium ring B faces electromagnet A. The current I through A can be altered. Then which of the following statements is correct ?

FIGURE 6(Q).1



(a) If I decreases, A will attract B

(b) If I increases, A will repel B

(c) If I increases, A will attract B

(d) Whether I increases or decreases, B will not experience any force.

27. Two identical induction coils each of inductance L joined in series are placed very close to each other such that the winding direction of one is exactly opposite to that of the other, what is the net inductance ?

(a) L^2 (b) 2 L

(c) $L/2$ (d) zero

28. Due to relative motion of a magnet with respect to a coil, an *e.m.f.* is induced in the coil, identify the principle involved

(a) Ampere's circuital law

(b) Faraday's law

(c) Gauss law

(d) Biot-Savart law

29. In Faraday's experiment on electromagnetic induction, more deflection will be shown by galvanometer, when

(a) Magnet is in uniform motion towards the coil

(b) Magnet is in uniform motion away from the coil

(c) Magnet is in accelerated motion towards the coil

(d) Magnet is at rest near the coil

30. If both, the number of turns and core length of an inductor are doubled keeping other factors constant, then its self-inductance will be

(a) unaffected (b) doubled

(c) halved (d) quadrupled

31. Oscillating metallic pendulum in a uniform magnetic field directed perpendicular to the plane of oscillation

(a) slows down

(b) becomes faster

(c) remains unaffected

(d) oscillates with changing frequency

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32. A metallic cylinder is held vertically and then a small magnet is dropped along its axis. It will fall with
(*a*) acceleration $a > g$ (*b*) acceleration $a < g$
(*c*) acceleration $a = g$
(*d*) constant velocity $a = 0$

33. An *e.m.f.* of 200 V is induced in a circuit when current in the circuit falls from 5 A to 0 A in 0·1 second. The self-inductance of the circuit is
(*a*) 3·5 H (*b*) 3·9 H
(*c*) 4 H (*d*) 4·2

34. The magnetic flux linked with a coil at any instant t is $\phi = (6t^2 - 8t + 5)$ Wb, the *e.m.f.* induced in the coil at $t = 2$ second is
(*a*) − 16 V (*b*) − 24 V
(*c*) + 24 V (*d*) + 16 V

35. A conducting circular ring is placed in a uniform magnetic field B with its plane perpendicular to the field. The radius of the ring starts shrinking at the rate (da/dt). Then induced-*e.m.f.* at the instant when the radius is a is
(*a*) $(\pi a^2/2)^2 B\,(dt/dt)$ (*b*) $\pi a B\,(da/dt)$
(*c*) $\pi a^2\,(dB/dt)$ (*d*) $2\pi a B\,(da/dt)$

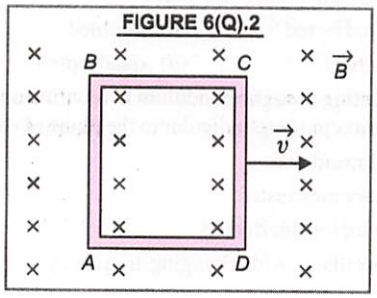
36. A small piece of metal wire is dragged across the gap between the poles of a magnet in 0·4 s. If change in magnetic flux in the wire is 8×10^{-4} Wb, then *e.m.f.* induced in the wire is
(*a*) 8×10^{-3} V (*b*) 6×10^{-3} V
(*c*) 4×10^{-3} V (*d*) 2×10^{-3} V

37. If the no. of turns per unit length of the coil of a solenoid is doubled keeping other dimensions same, then its self-inductance will be
(*a*) halved (*b*) doubled
(*c*) four times (*d*) eight times

38. Two inductors each of inductance L are joined in parallel. Their equivalent inductance will be
(*a*) zero (*b*) $L/2$
(*c*) L (*d*) 2 L

39. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere. The current induced in the loop is

FIGURE 6(Q).2



(*a*) $\frac{Blv}{R}$ clockwise

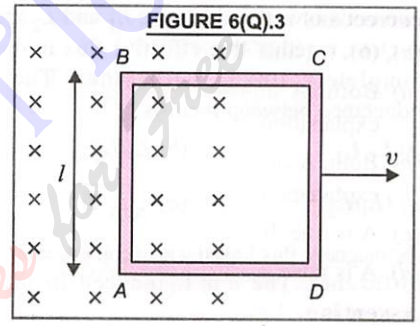
(*b*) $\frac{Blv}{R}$ anticlockwise

(*c*) $\frac{2Blv}{R}$ anticlockwise

(*d*) zero

40. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in Fig. 6(Q).3. The induced *e.m.f.* is

FIGURE 6(Q).3



(*a*) Zero (*b*) $R\,v\,B$
(*c*) $vB\ l/R$ (*d*) $vB\,l$

41. A wheel with ten metallic spokes each 0·50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0·4 Gauss, the induced *e.m.f.* between the axle and the rim of the wheel is equal to
(*a*) $1{\cdot}256 \times 10^{-3}$ V (*b*) $6{\cdot}28 \times 10^{-4}$ V
(*c*) $1{\cdot}256 \times 10^{-4}$ V (*d*) $6{\cdot}28 \times 10^{-5}$ V

42. In a circuit with a coil of resistance 2 ohms, the magnetic flux changes from 2·0 Wb to 10·0 Wb in 0·2 second. The charge that flows in the coil during this time is
(*a*) 5·0 coulomb (*b*) 4·0 coulomb
(*c*) 1·0 coulomb (*d*) 0·8 coulomb

43. The direction of induced current is such that it opposes the very cause that has produced it. This is the law of
(*a*) Lenz (*b*) Faraday
(*c*) Kirchhoff (*d*) Fleming

44. The magnetic flux through a circuit of resistance 10 Ω changes by an amount 100 Wb in time 0·1 s. Then the total quantity of electric charge, which passes during this time through any point of the circuit is given by

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(*a*) 100 *C* (*b*) 50 *C*

(*c*) 20 *C* (*d*) 10 *C*

45. A coil having an area A_0 is placed in a magnetic field which changes from B_0 to 4 B_0 in a time interval *t*. The e.m.f. induced in the coil will be

(*a*) $\frac{3A_0B_0}{t}$ (*b*) $\frac{4A_0B_0}{t}$

(*c*) $\frac{3B_0}{A_0t}$ (*d*) $\frac{4B_0}{A_0t}$

II. Assertion-Reason Type Questions

Direction. For question numbers 46 to 54, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (*a*), (*b*), (*c*) and (*d*) as given below :

(*a*) Both, A and R are true, and R is correct explanation of A.

(*b*) Both, A and R are true, but R is not the correct explanation of A.

(*c*) A is true, but R is false.

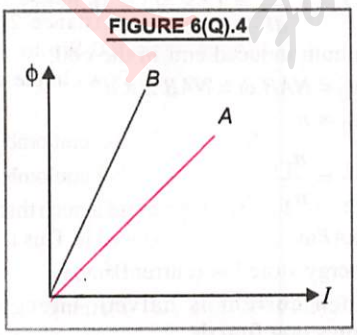
(*d*) A is false, and R is also false.

46. Assertion. Lenz's law supports the law of conservation of energy.

Reason. Change in magnetic flux linked with a coil induces an emf in it, which lasts so long as the changes last.

47. Assertion. A plot of magnetic flux (ϕ) versus current (*I*) is shown in Fig. 6(Q).4, for two inductors *A* and *B*. The inductor *B* has longer value of inductance than *A*.

FIGURE 6(Q).4



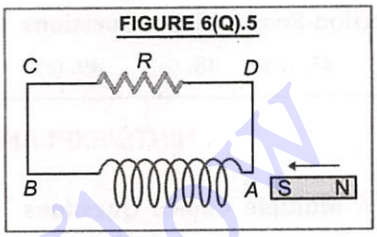
Reason. Greater the slope of $\phi - I$ graph, the larger is the value of self inductance.

48. Assertion. An ideal inductor when connected in *a.c.* circuit does not produce heating effect, though it reduces the current in the circuit.

Reason. An ideal inductor has some inductance but no ohmic resistance.

49. Assertion. A bar magnet is moved towards a coil as shown in Fig. 6(Q).5, the direction of induced current is *C* to *D*.

FIGURE 6(Q).5



Reason. According to Lenz's law, the end *A* of the coil will have north polarity.

50. Assertion. If a vertical metallic pole falls down through the plane of magnetic meridian, no emf is induced between its ends.

Reason. During the vertical falling down of metallic pole through the plane of magnetic meridian, the magnetic pole does not interact with horizontal or vertical component of earth's magnetic field.

51. Assertion. The dimensional formula of magnetic flux is $[ML^2 T^{-2} A^{-1}]$.

Reason. Magnetic flux, $\phi = B/A$, where *B* is magnetic field strength and *A* is the area of the surface.

52. Assertion. The form factor of an *a.c.* generator is 1·1.

Reason. Form factor $= \frac{I_v}{I_{av}}$.

53. Assertion. The direction of induced current is always such that it supports the change in magnetic flux responsible for its production.

Reason. This is as per Ampere's law.

54. Assertion. Magnetic flux linked with a coil of area *A* held with its plane parallel to magnetic field *B* is maximum.

Reason. Magnetic flux $\phi = |\vec{B} \times \vec{A}|$.

ANSWERS

I. Multiple Choice Questions

1. (c)	2. (d)	3. (a)	4. (a)	5. (c)	6. (c)	7. (a)	8. (c)	9. (a)	10. (d)
11. (d)	12. (a)	13. (b)	14. (b)	15. (a)	16. (b)	17. (c)	18. (c)	19. (b)	20. (c)
21. (d)	22. (b)	23. (a)	24. (a)	25. (a)	26. (b)	27. (d)	28. (b)	29. (c)	30. (b)
31. (a)	32. (b)	33. (c)	34. (a)	35. (d)	36. (d)	37. (c)	38. (b)	39. (d)	40. (d)
41. (d)	42. (b)	43. (a)	44. (d)	45. (a)					

II. Assertion-Reason Type Questions

46. (b)	47. (a)	48. (a)	49. (c)	50. (a)	51. (c)	52. (a)	53. (d)	54. (d)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. Here, $A = 100\ cm^2 = 10^{-2}\ m^2$, $B = 10^{-1}$ T,
$\theta = 90° - 30° = 60°$
$dt = 10^{-4}$ s, $e = ?$
$\phi_1 = BA\cos\theta$, $\phi_2 = 0$ or $d\phi = \phi_2 - \phi_1$

$$e = -\frac{d\phi}{dt} = -\frac{(0 - BA\cos\theta)}{dt} = \frac{BA\cos\theta}{dt}$$

$$= \frac{10^{-1} \times 10^{-2} \times \cos 60°}{10^{-4}} = \mathbf{5\ V}$$

2. Here, $N = 600$, $L = 108$ mH
As area A and length l are same for both coils,
$\therefore \quad L \propto N^2$

$$\text{or} \quad \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} = \left(\frac{600}{500}\right)^2 = \frac{36}{25}$$

$$L_2 = \frac{25}{36} \quad L_1 = \frac{25}{36} \times 108 = \mathbf{75\ mH}$$

3. Electric field, $E = \frac{-dV}{dx} = \frac{-d}{dx}(3x^2) = -6x$

$\therefore \quad E_{(1,0,2)} = -6 \times 1 = -6$ V/m $=$ **6 V/m along $-x$ axis**

4. $e = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$
where M is the mutual inductance of two coils

$$e = -0{\cdot}5\,\frac{(3-7)}{0{\cdot}04} = \mathbf{50\ V}$$

5. When an iron rod is inserted in the current carrying solenoid along its axis, the mag. field and then magnetic flux linked with solenoid will increase, self inductance of the solenoid will increase. The **rate of heating will not increase** along its axis, because there is no change in current when iron rod is inverted.

6. When an iron rod is introduced into the inductor, the self inductance of inductor increases. Due to it, for the given rate of change of flux, more induced emf is induced and hence more induced current is set up. As a result of it the bulb will glow more brightly.

7. Lenz's law is the consequence of the **law of conservation of energy.**

8. Here, $L = 10$ H, $dI = (11 - 2)$ A $= 9$ A,
$dt = 9 \times 10^{-1}$ s

$$-e = L\frac{dI}{dt} = 10 \times \frac{9}{9 \times 10^{-1}} = \mathbf{10^2\ volt}$$

9. $\phi = 5t^2 + 3t + 16$

$$-e = \frac{d\phi}{dt} = \frac{d}{dt}(5t^2 + 3t + 16)$$

$$= 10t + 3 = 10 \times 5 + 3 = \mathbf{53\ V}$$

10. As $L \propto n^2 \quad \therefore \quad L$ becomes **4 times.**

11. Here, $\varepsilon = 12$ V, $dI/dt = 45/60 = (3/4)$ A/s.
As $\quad \varepsilon = -L\,dI/dt$

$$\therefore \quad L = \varepsilon\frac{dt}{dI} = 12 \times 4/3 = \mathbf{16\ henry}$$

12. Maximum induced emf in the coil,

$$\varepsilon_0 = NAB\,\omega = NAB\,2\pi n$$

i.e., $\varepsilon_0 \propto n$

$$\therefore \quad \frac{\varepsilon_1}{\varepsilon_2} = \frac{n_1}{n_2}$$

13. $e_0 = nAB\omega$.

14. As energy stored $\propto$ (current)^2
$\therefore$ when current is halved, **energy stored becomes one fourth.**

15. This is the statement of Lenz's law.

16. When a magnet is moved with its north pole towards a coil placed in a closed circuit, then the

nearest face of the coil shows **north polarity**, in accordance with Lenz's law.

17. Here, $N = 1000$, $l = 1$ m ;

$A = 10\ cm^2 = 10 \times 10^{-4}\ m^2$

As, $L = \dfrac{\mu_0 N^2 A}{l}$

$\therefore \quad L = \dfrac{(4\pi \times 10^{-7}) \times (1000)^2 \times (10 \times 10^{-4})}{1}$

$= 1{\cdot}256 \times 10^{-3}\ H = \mathbf{1{\cdot}256\ mH}$

18. The *e.m.f.* induced does not depend upon the amount of magnetic flux.

19. Induced emf $e = \dfrac{d\phi}{dt}$;

Induced current, $di = \dfrac{e}{R} = \dfrac{d\phi/dt}{R}$

$\therefore$ Induced charge, $dQ = di \times dt = d\phi/R$.

20. From $e = L\dfrac{dI}{dt}$

$L = \dfrac{e}{dI/dt} = \dfrac{\text{volt}}{\text{amp/sec}}$

21. We know that coefficient of coupling (K) between two coils having self inductance L_1 and L_2 and mutual inductance M is given by

$K = \dfrac{M}{\sqrt{L_1 L_2}}$

When two coils are tight or completely linked, $K = 1$.

Therefore, $1 = \dfrac{M}{\sqrt{L_1 L_2}}$ or $M = \sqrt{L_1 L_2}$

22. $e = \left|\dfrac{d\phi}{dt}\right| = \dfrac{d}{dt}(3t^2 - 2t + 1)$

$= 6t - 2 = 6 \times 1 - 2 = 4\ mV = \mathbf{4 \times 10^{-3}\ V}$

23. $e = B l v = 0{\cdot}1 \times 2 \times 5 = \mathbf{1\ V}$

24. Here, $r = 10\ cm = 0{\cdot}1\ m$; $A = \pi r^2 = \pi \times (0{\cdot}1)^2$

$= 0{\cdot}01\ \pi\ m^2$, $\Delta B = (2 - 0)\ T = 2\ T$, $R = 0{\cdot}01\ \Omega$.

Induced emf due to change in magnetic flux is

$\varepsilon = \dfrac{\Delta\phi}{\Delta t} = \dfrac{\Delta B \times A}{\Delta t} = \dfrac{2 \times (0{\cdot}01\ \pi)}{\Delta t}$

Induced current, $I = \dfrac{\varepsilon}{R} = \dfrac{2 \times 0{\cdot}01\ \pi}{R\ \Delta t}$

Charge flowing through ring, $q = I\ \Delta t$

$= \dfrac{2 \times 0{\cdot}01\ \pi}{R} = \dfrac{2 \times 0{\cdot}01\ \pi}{0{\cdot}01} = \mathbf{2\pi\ C}$

25. The frequency of a.c. generated depends **on speed of rotation of coil** and is independent of amplitude of a.c. or size of coil.

26. When current I is altered, a changing magnetic flux is set up in electromagnet. Suppose, right face of the core acquires north polarity. According to Lenz's law, the nearer side of disc B acquires north polarity. As a result of it, the disc B is repelled.

27. When two induction coils are joined in series such that the winding of one coil is opposite to the other, then emf produced in the first coil is 180° out of phase of the emf produced in the second coil *i.e.*, the emf produced in the first coil is opposite to the emf produced in the second coil.

If $L_1 = \dfrac{\phi}{I}$, $\quad L_2 = \dfrac{-\phi}{I}$

In series combination, net inductance is given by

$L = L_1 + L_2 = \dfrac{\phi}{I} + \left(\dfrac{-\phi}{I}\right) = \mathbf{0}$

28. Faraday's law governs the *emf* induced in the coil due to relative motion of a magnet w.r.t. a coil.

29. The more deflection in the galvanometer is related with the more current which inturn depends upon more induced emf. The same is due to more rate of change of flux. It is possible if **magnet is in accelerated motion towards the coil.**

30. As $L = \dfrac{\mu_0 N^2 A}{l}$, therefore, when N and l are doubled, then, L is **doubled.**

31. The oscillating metallic pendulum in a uniform magnetic field directed perpendicular to the plane of oscillation, then there will be change in magnetic flux with time linked with metallic bob. Due to it, eddy current is set up in bob which opposes the relative motion of metallic bob of pendulum. As a result of it, the metallic pendulum **slows down.**

32. Acceleration, $a < g$, because motion of magnet is opposed by the current induced in the cylinder.

33. From $e = L\dfrac{dI}{dt}$

$L = \dfrac{e\,dt}{dI} = \dfrac{200 \times 0{\cdot}1}{5 - 0} = \mathbf{4\ H}$

34. $\phi = 6t^2 - 8t + 5$

Induced emf, $e = -\dfrac{d\phi}{dt} = \dfrac{-d}{dt}(6t^2 - 8t + 5)$

$= (-12t - 8)$

At $t = 2\ s$, $e = -(12 \times 2 - 8) = \mathbf{-16\ V}$

35. $\phi = B\,.\,A = B(\pi a^2)$

$e = \dfrac{d\phi}{dt} = B(\pi\, 2a)\dfrac{da}{dt} = \mathbf{2\pi a B\left(\dfrac{da}{dt}\right)}$

36. $e = \dfrac{d\phi}{dt} = \dfrac{8 \times 10^{-4}}{0{\cdot}4} = 2 \times 10^{-3}$ V

37. As $L \propto N^2$, therefore, when N is doubled, L becomes **4 times.**

38. When two conductors of inductances L_1 and L_2 are joined in parallel, then equivalent inductance L_{eq} is given by

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

As $L_1 = L = L_2$,

so $\dfrac{1}{L_{eq}} = \dfrac{1}{L} + \dfrac{1}{L} = \dfrac{2}{L}$ or $L_{eq} = \dfrac{L}{2}$

39. **No current** is induced because there is no change in magnetic flux linked with the loop.

40. Knowledge based question.

41. Here, $n = 10$, $l = r = 0{\cdot}50$ m ; $\nu = 120/60 = 2$ *rps*, $B = 0{\cdot}4 \times 10^{/4}$ T

As the wheel rotates, linear velocity of the spoke at the rim $= r\,\omega$, and linear velocity of spoke end at the axle is zero.

$\therefore$ Average linear velocity,

$$v = \frac{0 + r\,\omega}{2} = \frac{r\,\omega}{2} = \frac{r \times 2\pi\nu}{2} = \pi r \nu$$

emf induced across the end of each spoke.

$= B l v = B r \times \pi r \nu = \pi B r^2 \nu$

$= 3{\cdot}14 \times (0{\cdot}4 \times 10^{-4}) \times (0{\cdot}5)^2 \times 2$

$= 6{\cdot}28 \times 10^{-5}$ V

As the spokes are in parallel, the induced emf is not affected by changing their number.

42. From $Q = \dfrac{d\phi}{R} = \dfrac{10{\cdot}0 - 2{\cdot}0}{2} =$ **4·0 Coulomb**

43. This statement is as per Lenz's law.

44. Here, $R = 10\ \Omega$, $\Delta\phi = 100$ Wb, $\Delta t = 0{\cdot}1$ s,

$$\Delta Q = \frac{\Delta\phi}{R} = \frac{100}{10} = 10\ \text{C}$$

45. $e = \dfrac{d\phi}{dt} = \dfrac{A_0(4B_0 - B_0)}{t} = \dfrac{3A_0B_0}{t}$

II. Assertion-Reason Type Questions

46. **Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.** Here, Reason is the statement of Faraday's first law of electromagnetic induction where as Lenz's law tells the direction of induced emf.

47. As, $\phi = LI$ or $L = \dfrac{\phi}{I}$ = slope of $\phi - I$ graph. Since slope of graph for inductor B is greater than that of A, so B has larger value of self inductance than A. Hence **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

48. In ohmic resistance R, the amount of heat produced in time t due to current I, $H = I^2 R t$. As $R = 0$, $H = 0$. However the inductor offers inductive reactance, $X_L = \omega L = 2\pi\nu L$ to the *a.c.*, which helps in reducing the *a.c.* current. Thus **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

49. **Here Assertion is true but Reason is false** as the end A of the coil will have south polarity and the direction of induced current in resistance R will be from C to D.

50. **Both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

51. Here **Reason is false** as $\phi = BA\cos\theta$.

As $F = Bqv$ or $B = F/qv$

$$\therefore \quad \phi = \frac{F}{qv} \times A \times \cos\theta.$$

Dimensions of $\phi = \dfrac{(MLT^{-2}) \times L^2 \times 1}{(AT) \times (LT^{-1})}$

$= [ML^2T^{-2}A^{-1}]$

Thus Assertion is true.

52. Here, **Reason is true.**

$$I_{av} = I_0/\sqrt{2} \text{ and } I_{av} = 2I_0/\pi$$

$$\therefore \text{ Form factor } = \frac{I_v}{I_{av}} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}1$$

Thus **Assertion is true.** So the **Reason is the correct explanation of Assertion.**

53. **Both Assertion and Reason are false,** as the direction of induced current is such that it opposes the change in magnetic flux responsible for its production and is given by Lenz's law.

54. Here **Reason is false** as $\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$ when a coil is held with its plane parallel to magnetic field, $\theta = 90°$. Therefore,

$\phi = BA\cos 90° = 0$. Thus **Assertion is also false.**

CONCEPTUAL PROBLEMS

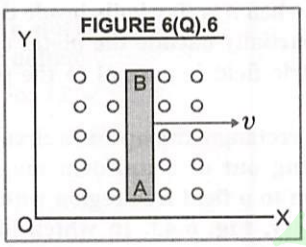
I. Faraday's Laws and Motional e.m.f.

1. A metallic rod held horizontally along east west direction is allowed to fall under gravity. Will there be an e.m.f. induced across its ends ? (CBSE 2013)

Sol. Yes. This is because horizontal component of earth's magnetic field along N-S, is intercepted by the falling rod.

2. As shown in Fig. 6(Q).6 a conducting rod *AB* moves parallel to X-axis in a uniform magnetic field pointing in the positive Z-direction. What charges are acquired by the ends *A* and *B* of the rod ?

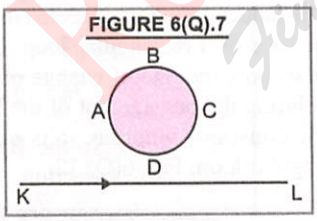
FIGURE 6(Q).6



Sol. According to Fleming's left hand rule, free electrons in the rod experience force along *AB*. Therefore, end *A* acquires positive charge and end *B* acquires negative charge.

3. What is the magnitude of induced current in the circular loop *ABCD* of radius *r*, when a steady current of *i* ampere is passed through wire *KL* ? (CBSE 2002)

FIGURE 6(Q).7



Sol. As current through wire *KL* is steady, magnetic flux linked with the loop *ABCD* is constant. As there is no change in magnetic flux, induced current = 0.

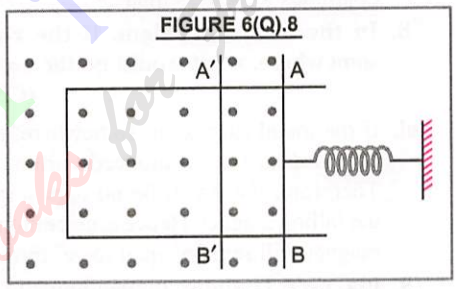
4. Two spherical bobs, one metallic and other of glass, of the same size are allowed to fall freely from the same height above the ground. Which of the two would reach the ground earlier and why ? (CBSE 2014)

Sol. Glass bob will reach the ground earlier. This is because due to presence of earth's magnetic field, there will be no effect of electromagnetic induction on glass bob, unlike in case of metallic bob.

5. A rectangular frame of wire is placed in a uniform magnetic field directed outwards, normal to the paper *AB* is connected to a spring which is stretched to *A′ B′* and then released at time *t* = 0. Explain qualitatively how induced e.m.f. in the coil would vary with time. (Neglect damping of oscillations of spring). [CBSE 2018 (C)]

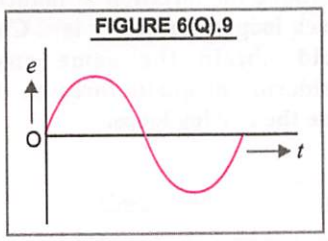
Sol. Refer to Fig. 6(Q).8. When the spring is stretched and released, it executes SHM about

FIGURE 6(Q).8



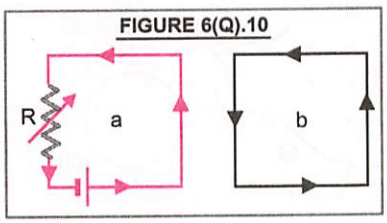
the equilibrium position *AB* of the wire. As a result, the induced e.m.f. developed in the frame will vary simple harmonically with time as shown in Fig. 6(Q).9.

FIGURE 6(Q).9



6. If resistance *R* in circuit '*a*' of Fig. 6(Q).10 be decreased, what will be the direction of induced current in the circuit '*b*'? (CBSE 2011)

FIGURE 6(Q).10



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Sol. When R is decreased, current in 'a' increases. An e.m.f. is induced in 'b' which opposes the increase of current in 'a'. The induced current must, therefore, be *anticlockwise*.

7. A bar magnet falls through a metal ring. Will its acceleration be equal to 'g'?

(Manipur Board 2012, CBSE 2008)

Sol. No, acceleration of the magnet will not be equal to g. It will be less than 'g'. This is because as the magnet falls, amount of magnetic flux linked with the ring changes. An induced current is developed in the ring which opposes the downward motion of the magnet. After the magnet has crossed the metal ring, amount of magnetic flux linked with the ring goes on decreasing. An induced current developes in the ring and opposes the fall of the magnet. Therefore, downward acceleration of the magnet continues to be less than 'g'.

8. In the above problem, if the ring is cut somewhere, what would be the answer?

(CBSE 2008)

Sol. If the metal ring is cut somewhere, e.m.f. will be induced, but no induced current can flow. Therefore, there will be no opposing force on the falling magnet. Hence acceleration of falling magnet will remain equal to 'g' throughout.

9. Fig. 6(Q).11 shows planer loops of different shapes moving out of or into a region of magnetic field which is directed normal to the plane of the loops away from the reader. Determine the direction of induced current in each loop using Lenz's law. Check if you would obtain the same answers by considering magnetic force on the charge inside the moving loops.

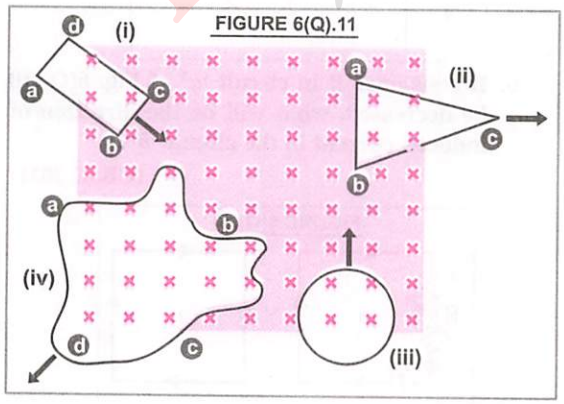


FIGURE 6(Q).11

Sol. (*i*) Due to motion, the magnetic flux linked with the loop *abcd* increases. According to Lenz's law, this increase in flux is opposed by the induced current. Therefore, the induced current must flow along *bcdab*.

(*ii*) Due to motion, magnetic flux linked with *abc* decreases. Induced current is along *bacb*.

(*iii*) The magnetic flux increases in this case. Therefore, induced current is anticlockwise.

(*iv*) As magnetic flux decreases due to motion, the induced current is along *cdabc*.

Yes, we would obtain the same answers by considering magnetic force on the charges inside the loop.

10. Answer the following questions :

(a) A closed conducting loop moves normal to the electric field between the plates of a large capacitor. Is a current induced in the loop when it is (i) wholly inside the capacitor (ii) partially outside the plates of capacitor. Electric field is normal to the plane of the loop.

(b) A rectangular loop and a circular loop are moving out of a uniform magnetic field region to a field free region with a constant velocity, Fig. 6.43. In which loop do you expect the induced e.m.f. to be constant during the passage out of the field region, the field is normal to the loops ? **(CBSE 2010)**

(c) Predict the polarity of the capacitor in the situation described by Fig. 6.44 :

(Raj. Board 2012, CBSE 2017, 2011)

Sol. (*a*) No, current is not induced by change in electric flux.

(*b*) The induced e.m.f. is expected to be constant in the case of a rectangular loop. In case of a circular loop, the rate of change of area of the loop during the passage out of the field region is not constant, whereas it is constant for rectangular loop, Fig. 6(Q).12.

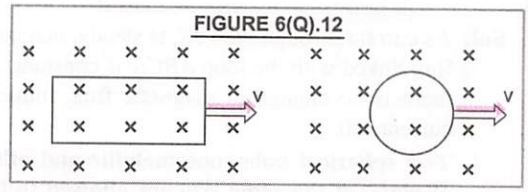
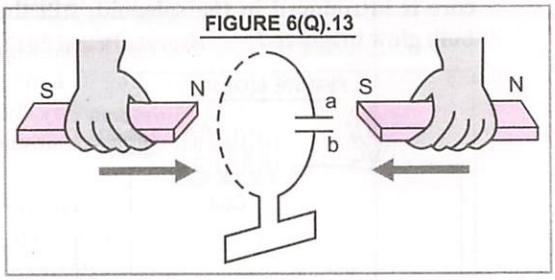


FIGURE 6(Q).12

(*c*) In the situation shown in Fig. 6(Q).13, *a* will become positive with respect to *b*, as current induced is in clockwise direction.

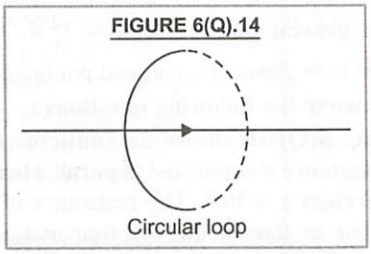
FIGURE 6(Q).13



11. A long straight current carrying wire passes normally through the centre of a circular loop. If current through the wire increases, will there be an induced e.m.f. in the loop ? Justify. (CBSE 2017)

Ans. In Fig. 6(Q).14, we have shown a long straight current carrying wire passing normally through

FIGURE 6(Q).14

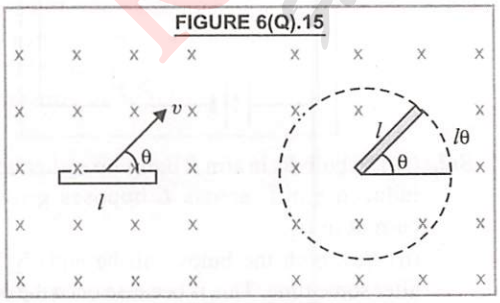


the centre of a circular loop. When current through the wire increases, the amount of magnetic flux linked with the loop increases. Therefore, an induced e.m.f. developes in the loop.

12. Calculate the rate at which the flux linked with the generated area changes with time when a rod of length l is (a) translated (b) rotated in a uniform field of induction B as shown in Fig. 6(Q).15.

Sol. (*a*) Component of velocity perpendicular to the rod $= v \sin\theta$. Therefore, in time t, area traversed $= l \times v \sin\theta \times t$.

FIGURE 6(Q).15



$$\therefore \quad \phi = B\,(l \times v \sin\theta \times t)\cos 0° = B\,l\,v\,t \sin\theta$$

$$\frac{d\phi}{dt} = B\,l\,v \sin\theta$$

(*b*) In time t, if θ is the angle traced by the free end, then

$$\therefore \quad \text{Area swept, } A = \pi l^2 \times \left(\frac{\theta}{2\pi}\right) = \frac{1}{2} l^2 \theta$$

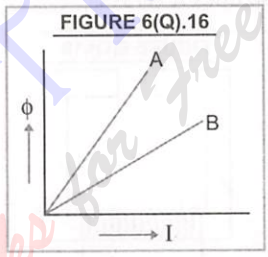
$$\phi = B\left(\frac{1}{2} l^2 \theta\right)\cos 0° = \frac{1}{2} B l^2 \theta$$

$$\frac{d\phi}{dt} = \frac{1}{2} B l^2 \frac{d\theta}{dt} = \frac{1}{2} B l^2 \omega$$

II. Self Induction and Mutual Induction

13. A plot of magnetic flux (ϕ) versus current (I) is shown for two inductors A and B. Which of the two has larger value of self inductance ? (CBSE 2010)

FIGURE 6(Q).16



Sol. As $\phi = LI$; $L = \dfrac{\phi}{I}$ = slope of $\phi - I$ graph.

Since slope of ϕ-I graph for inductor A is greater than slope of ϕ-I graph for inductor B, therefore, inductor A has larger value of self inductance than B.

14. Self inductance of an air core inductor increases from 0·01 mH to 10 mH when an iron core is introduced in it. What is the relative permeability of the core ? (CBSE 2002)

Sol. $L_0 = \dfrac{\mu_0 N^2 A}{l}$ and $L = \dfrac{\mu N^2 A}{l}$

$$\therefore \quad \frac{L}{L_0} = \frac{\mu}{\mu_0} = \mu_r$$

$$\therefore \quad \mu_r = \frac{L}{L_0} = \frac{10\,\text{mH}}{0{\cdot}01\,\text{mH}} = \mathbf{1000}$$

15. In Fig. 6(Q).17, P and S are two coils. What shall be the direction of induced momentary current in S immediately after the switch is closed, (ii) if the switch is opened, after it has been closed for some time.

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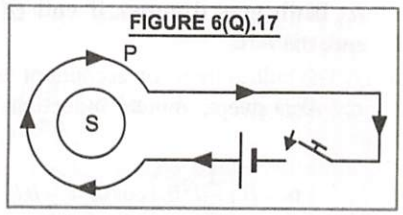
FIGURE 6(Q).17

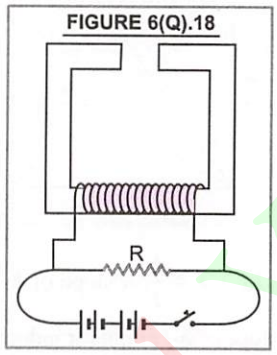


Sol. When switch is closed, current in P grows. Induced current in S will oppose this growth. Hence the current in S must be *anticlockwise*.

When switch is opened, current in P decays from maximum to zero. Induced current in S will oppose the decay. Hence current in S must be *clockwise*.

16. A small resistor R (say, a lamp) is usually put in parallel with the current carrying coil of an electromagnet, [Fig. 6(Q).18] . What purpose does it serve?

FIGURE 6(Q).18



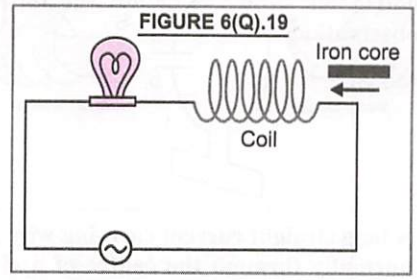
Sol. When current in the coil of a large electromagnet is switched off, magnetic flux changes at a very high rate. Therefore, induced e.m.f. is very high and may cause sparking which would damage the insulation. The small resistor R placed in parallel provides a conducting path to the e.m.f. induced. In this way, risks of high voltages and sparking etc. are reduced.

17. An ideal inductor when connected in a.c. circuit does not produce heating effect though it reduces the current in the circuit. Explain why ?

Sol. An ideal inductor is a coil having some inductance (L) but no ohmic resistance R. Amount of heat produced in time $t = I^2 R\, t$. As $R = 0$, therefore, heat produced = 0. However, the inductor offers inductive reactance $X_L = \omega L = 2\pi \nu L$ to the a.c. Therefore, the current is reduced.

18. A bulb connected in series with a solenoid is lit by a.c. source, Fig. 6(Q).19. If the soft iron core is introduced in the solenoid, will the bulb glow brighter ? (Kerala Board 2012)

FIGURE 6(Q).19



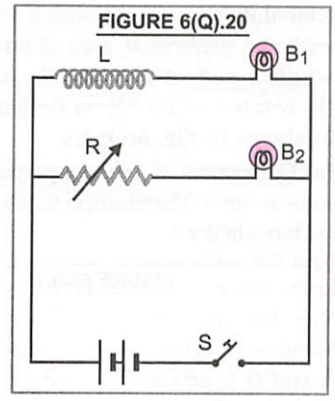
Sol. We know that the inductive reactance offered by the solenoid is $X_L = \omega L = 2\pi \nu L$. On introducing soft iron core in the solenoid, its inductance L increases. Therefore, X_L increases. As $I_v = E_v/X_L$, therefore, I_v decreases.

As glowing power of bulb = $I_v^2 R$, therefore, the bulb glows dimmer, and not brighter.

19. Answer the following questions :

Fig. 6(Q).20 shows an inductor L and a resistance R connected in parallel to a battery through a switch. The resistance of R is the same as that of the coil that makes L. Two identical bulbs are put in each arm of the circuit. (i) Which of the bulb lights up earlier when S is closed ? (ii) Will the bulb be equally bright after some time ?

FIGURE 6(Q).20

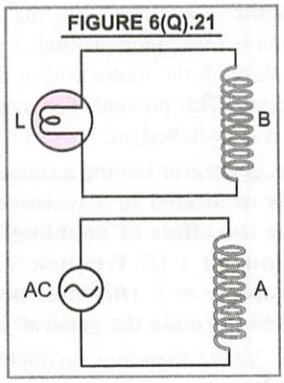


Sol. (*i*) The bulb B_2 in arm R lights up earlier because induced e.m.f. across L opposes growth of current in B_1.

(*ii*) Yes, both the bulbs will be equally bright after some time. This is because once the current has reached its maximum value, self inductance plays no role. Resistance of L and R is same ; and both the bulbs are identical and are connected to same source of potential difference.

20. In Fig. 6(Q).21, a coil *B* is connected to a low voltage bulb *L* and placed parallel to another coil *A*, as shown. Explain the following observations :

FIGURE 6(Q).21

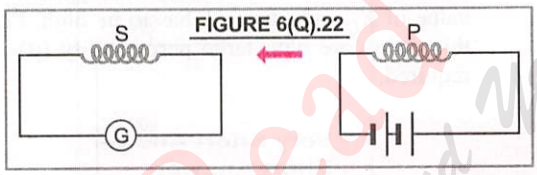


(*i*) bulb lights (*ii*) bulb gets dimmer if the coil *B* is moved upwards.

Sol. (*i*) Bulb lights because of emf induced in coil *B* due to mutual induction between *A* and *B*.
(*ii*) On moving coil *B* upwards, coeff. of mutual inductance (*M*) decreases. Induced emf (*e*) decreases. Bulb gets dimmer.

21. When primary coil P is moved towards secondary coil *S*, Fig. 6(Q).22, the galvanometer shows momentary deflection. What can be done to have larger deflection in the galvanometer with same battery ? State the related law ? **(CBSE 2011, 2010)**

FIGURE 6(Q).22

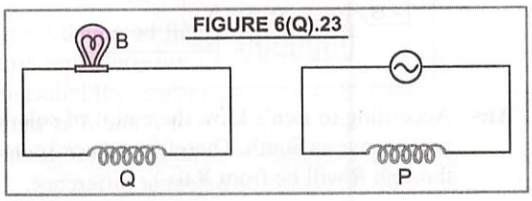


Sol. To have larger deflection in the galvanometer with the same battery, coil *P* has to be moved faster towards *S*.

The law governing this phenomenon is Faraday's 2nd law of electromagnetic induction.

22. A coil *Q* is connected to low voltage bulb *B* and placed near another coil *P* as shown in Fig. 6(Q).23. Give reasons to explain the following observations : (*a*) The bulb *B* lights

FIGURE 6(Q).23

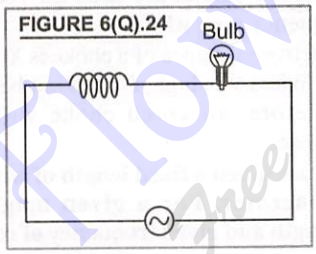


(*b*) Bulb gets dimmer if coil *Q* is moved towards left. **(CBSE 2010)**

Sol. (*i*) The bulb *B* lights on account of emf induced in coil *Q* due to mutual induction between *P* and *Q*.
(*ii*) When coil *Q* is moved towards left, magnetic flux linked with *Q* decreases, and may even reduce to zero at some distance. The emf induced may decrease and the bulb *B* gets dimmer.

23. An iron cored coil is connected in series with an electric bulb with an a.c. source as shown in Fig. 6(Q).24. When iron piece is taken out of the coil the brightness of the bulb will

FIGURE 6(Q).24



(*a*) decrease (*b*) increase
(*c*) remain unaffected (*d*) fluctuate
(CBSE Sample Paper 2022-23)

Sol. When iron piece is taken out of the coil, *L* decreases, $X_L = \omega L$ drecrease, current increases. Thus, brightness of bulb will increase.

24. A rectangular, a square, a circular and an elliptical loop, all in *XY* plane, are moving out of a uniform magnetic field with a constant velocity, $\vec{v} = v\hat{i}$. The magnetic field is directed along the negative *Z*-axis direction. The induced e.m.f., during the passage of these loops, out of the field region will not remain constant for

(*a*) any of the four loops (*b*) the circular and elliptical loops (*c*) the rectangular, circular and elliptical loops (*d*) only the elliptical loops
(CBSE Sample Paper 2022-23)

Sol. Out of the four given shapes of loops, area crossing the magnetic filed will not remain constant in case of circular and elliptical loops. Therefore, induced e.m.f. during their passage will not remain constant. Thus **option (*b*) is true.**

III. A.C./D.C. Generator and D.C. Motor

25. The word generator is a misnomer. Why ?

Sol. In a generator, mechanical energy of rotation of an armature coil is being transformed into

electrical energy. It is thus only a converter, rather than a generator.

26. Why do we prefer a choke coil to a rheostat in controlling a.c. ? (Kerala 2010)

Sol. This is because no energy/power is dissipated in a choke coil. In a rheostat, some energy is dissipated in the form of heat.

27. Iron cored chokes are used for reducing low frequency a.c., why ?

Sol. With iron core, coefficient of self inductance (L) increases. Therefore, $X_L = \omega L = 2\pi\nu L$ becomes large, even when ν is low. Hence the current $I_v = E_v/X_L$ reduces.

28. Air cored chokes are used for reducing high frequency a.c., why ?

Sol. Inductive reactance of a choke is $X_L = \omega L = 2\pi\nu L$. When ν is high, L need not be made high. Therefore, air cored choke will serve the purpose.

29. You are given a fixed length of wire to design a generator. For a given magnetic field strength and given frequency of rotation, will you use one turn or two turn square coil to generate maximum peak e.m.f. ?

Sol. As is known, the peak e.m.f. is given by

$$e_0 = NAB\omega.$$

As B and ω are fixed, e_0 will be maximum, when $N \times A$ = maximum. Let L be the length of given wire.

If $N = 1$, $A = (L/4)^2 = \dfrac{L^2}{16}$; $NA = 1 \times \dfrac{L^2}{16} = \dfrac{L^2}{16}$

If $N = 2$, $A = (L/8)^2 = \dfrac{L^2}{64}$; $NA = 2 \times \dfrac{L^2}{64} = \dfrac{L^2}{32}$

Hence one turn square coil can be used to generate maximum peak e.m.f.

30. What is the use of a motor starter ?

Sol. A motor starter is a variable resistance. When the motor is switched on, the starter offers maximum resistance so that a small current flows through the motor coil in the absence of back e.m.f. This prevents damage to the motor, when it is switched on.

31. An a.c. generator having a constant magnetic field is connected to a resistive load. What will be the effect of doubling the speed of rotation on : (*i*) frequency of a.c. (*ii*) generated *e.m.f.* (*iii*) mechanical power required to rotate the generator ?

Sol. As $e_0 = NAB\omega$, therefore, on doubling the speed of rotation : frequency $\omega = 2\pi\nu$ of a.c. will be doubled ; generated *e.m.f.* e_0 will be doubled and mechanical power required to rotate the generator will be doubled.

32. A radio frequency choke is air-cored whereas an audio frequency choke is iron cored. Why ?

Sol. Inductive reactance of a coil of inductance L is

$$X_L = \omega L = 2\pi\nu L$$

Radio frequency is high. To maintain X_L low, inductance L should have a small value. This is obtained using air cored choke.

Audio frequency is low. To maintain particular value of X_L, inductance L has to be high. For this, iron core with large permeability (μ) is required.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Faraday's Laws and Motional e.m.f.

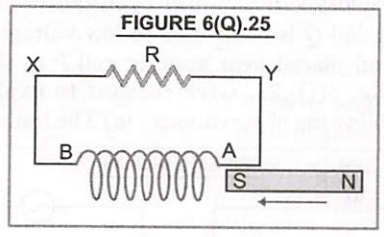
1. What is the dimensional formula of magnetic flux ?

Ans. $\phi = BA\cos\phi = \left(\dfrac{F}{Il}\right)A = \dfrac{MLT^{-2}}{AL} \cdot L^2$

$= [ML^2T^{-2}A^{-1}]$

2. A bar magnet is moved towards a solenoid as shown in Fig. 6(Q).25. What is the direction of induced current in R ? (CBSE 2013)

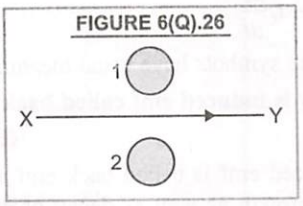
FIGURE 6(Q).25



Ans. According to Len's Law, the end A of solenoid will behave as South. Therefore induced current through R will be from X to Y.

3. Current I in wire XY is increasing steadily. What will be the direction of induced currents in metal rings 1 and 2 ? (CBSE 2012)

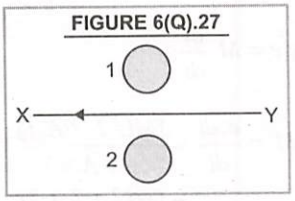
FIGURE 6(Q).26



Ans. According to Lenz's law, induced current in metal ring 1 will be *clockwise* and in metal ring 2, induced current will be *anticlockwise*.

4. Current in wire XY is decreasing steadily. Predict the direction of induced current in metal rings 1 and 2. (CBSE 2012)

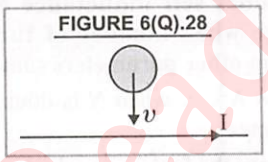
FIGURE 6(Q).27



Ans. According to Lenz's law, the induced current in ring 1 will be clockwise and in ring 2, induced current will be anticlockwise.

5. A metal ring moves towards a straight wire carrying current. What is the direction of induced current in the ring ?

FIGURE 6(Q).28



Ans. The induced current will oppose the motion of metal ring. It must be clockwise.

6. A vertical metallic pole falls down through the plane of magnetic meridian. Will any *e.m.f.* be induced between its ends ? (CBSE 2011)

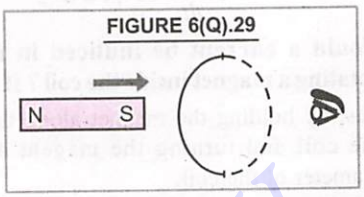
Ans. No, because the pole intercepts neither H nor V.

7. Two straight and parallel wires A and B are being brought towards each other. If current in A be i, what will be the direction of induced current in B? If A and B are being taken away from each other, then ?

Ans. In the first case, induced current in B will be opposite to i, so that A and B repel eachother and the operation of bringing them closer is opposed. Similarly, in the second case, induced current in B will be in the direction of i.

8. The south pole of a magnet is brought near a conducting loop. What is the direction of induced current as seen by a person on the other side of the loop ? [Fig. 6(Q).29]

FIGURE 6(Q).29



Ans. It is anitclockwise because this side of the loop will develop north pole and the side facing the magnet develops a south pole.

9. A train is moving with uniform speed from north to south. Will any induced e.m.f. appear across the ends of its axle ? Will the answer be affected if train moves from east to west ?

Ans. Yes, it will appear as the train is intercepting vertical component of earth's magnetic field. No, answer is not affected.

10. Name the S.I. units of magnetic flux and magnetic induction.

Ans. Weber, Tesla.

11. When is magnetic flux linked with a coil held in a magnetic field zero ?

Ans. When plane of coil is along the direction of field.

12. The induced e.m.f. is sometimes called back e.m.f. Why ?

Ans. This is because induced e.m.f. opposes the current due to the actual source of e.m.f.

13. Why are oscillations of a copper sheet in a magnetic field highly damped ?

Ans. This is because of eddy currents developed in the copper sheet.

14. What causes sparking in the switches when light is put off ?

Ans. Large induced e.m.f. at break causes the sparking.

15. What is the basic cause of induced e.m.f. ?

Ans. Change in magnetic flux linked with the circuit.

16. Does Lenz's law violate the principle of energy conservation ?

Ans. No, Lenz's law does not violate this principle.

17. A cylindrical bar magnet is kept along the axis of a circular coil. Will there be a current induced in the coil if the magnet is rotated about its axis ? Give reasons.

Ans. No, because $\phi = NBA$ = constant

$$\therefore \quad e = \frac{d\phi}{dt} = 0 \; ; i = 0$$

18. Could a current be induced in a coil by rotating a magnet inside the coil ? If so, how ?

Ans. Yes, by holding the magnet along the axis of the coil and turning the magent about the diameter of the coil.

19. Is induced electric field conservative or non conservative ?

Ans. It is non conservative, as $\oint \vec{E}.\vec{dl} = \frac{d\phi}{dt} \neq 0$

20. A glass rod of length l moves with velocity v in a uniform magnetic field B. What is the *e.m.f.* induced in the rod ?

Ans. No induced *e.m.f.* is set up because glass rod is insulating.

21. A wheel with a certain number of spokes is rotated in a plane normal to earth's magnetic field so that an *e.m.f.* is induced between the axle and rim of the wheel. Keeping all other things same, number of spokes is changed. How is the *e.m.f.* affected ?

Ans. The *e.m.f.* induced across the spokes is in parallel. Therefore, number of spokes does not affect the net *e.m.f.*

22. Why does a metallic piece become very hot when it is surrounded by a coil carrying high frequency alternating current ? **(CBSE Sample Paper 2010)**

Ans. High frequency alternating current passed through the coil surrounding the metal piece produces eddy currents in the metal piece. The eddy currents produce joule heating in the metal piece on account of its resistance.

23. What are the dimensions of inductance ? **(Hr. Board 2015)**

Ans. From $e = L\frac{dI}{dt}$, $L = \frac{e.dt}{dI} = \frac{W}{q}.\frac{dt}{dI}$

$$L = \frac{(ML^2T^{-2})T}{(AT)A} = [ML^2T^{-2}A^{-2}]$$

24. The self induced emf in a coil when current changes in it is given by : **(Hr. Board 2015)**

Ans. $e = -L\frac{dI}{dt}$

where symbols have usual meaning.

25. Why is induced emf called back e.m.f. ? **(Imphal 2015)**

Ans. Induced emf is called back emf as it opposes the growth as well as decay of current in the circuit.

II. Self Induction and Mutual Induction

26. What are the dimensions of mutual inductance ?

Ans. From $e = M\frac{di}{dt}$;

$$M = \frac{e.dt}{di} = \frac{[ML^2T^{-3}A^{-1}]}{A}$$

$$= [ML^2T^{-2}A^{-2}]$$

27. The inductance of a coil is 0·25 H. Calculate its inductive reactance in a.c. of frequency 50 Hz. **(Chhatisgarh Board 2012)**

Ans. $X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times 0{\cdot}25$

$$= 25\pi \text{ ohm}$$

28. How does self inductance of a solenoid change when number of turns is double keeping other parameters same ?

Ans. As $L \propto N^2$, $\therefore$ when N is double, L becomes four times.

29. What are the factors on which self inductance of a coil depend ?

Ans. (*i*) Number of turns

(*ii*) area of cross section

(*iii*) permeability of core material.

30. Name the physical quantity which is measured in weber amp^{-1}. **(Pb. Board 2011)**

Ans. Self inductance and Mutual inductance.

31. When current in a coil changes with time, how is the back e.m.f. induced in the coil related to it ? **(Pb. Board 2011, CBSE 2008)**

Ans. Back e.m.f. induced in the coil,

$$e = -L\frac{dI}{dt}$$

where L is coefficient of self-inductance of the coil and dI/dt is the rate of change of current through the coil.

32. Two inductors L_1 and L_2 sufficient distance apart are connected (*i*) in series (*ii*) in parallel. What is their equivalent inductance ?

Ans. In series combination, $L_s = L_1 + L_2$

In parallel combination, $L_p = \dfrac{L_1 L_2}{L_1 + L_2}$

As the two inductors are sufficient distance apart, mutual inductance between them is ignored.

33. What is one henry ?

Ans. One henry is the self inductance of a coil in which a current change at the rate of one ampere/sec. induces an e.m.f. of one volt in the coil.

34. Write an expression for self inductance of a long solenoid.

Ans. $L = \mu_0 \dfrac{N^2 A}{l}$

where symbols have usual meaning.

35. Write an expression for mutual inductance of two co-axial solenoids.

Ans. $M = \dfrac{\mu_0 N_1 N_2 A}{l}$,

Here, A is area of cross section of inner solenoid and l is length of longer solenoid.

III. A.C./D.C. Generator and D.C. Motor

36. In a.c. generator, which rule determines the direction of induced *e.m.f.* ?

Ans. Fleming's right hand rule.

37. In d.c. motor, is the supplied voltage alternating ?

Ans. No. The voltage is supplied by a d.c. battery.

38. In d.c. motor, which rule is applied to determine the direction of rotation of coil ?

Ans. Fleming's left hand rule.

39. Name the main component which changes an *a.c.* generator into *d.c.* generator.

Ans. Slip ring arrangement in an *a.c.* generator is replaced by split ring arrangement or commutator arrangement in the *d.c.* generator.

40. Which is the best method of reducing a.c. in a circuit ?

Ans. Using a choke coil.

41. Is a motor starter a variable R or L or C ?

Ans. It is a variable resistance (R).

42. A small d.c. motor operates at 110 V d.c. What is back e.m.f. when its efficiency is maximum ?

Ans. $E = V/2 = 110/2 = 55$ V

SHORT ANSWER QUESTIONS

Carrying 2 marks

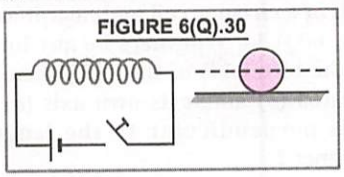
I. Faraday's Laws and Motional e.m.f.

1. An artificial satellite with a metal surface is orbiting the earth around the equator : Will the earth's magnetism induce some current in it ? (CBSE 2010)

Ans. No current is induced. This is because orbiting satellite intercepts only the vertical component of earth's magnetic field, which is zero at the equator.

2. A horizontal solenoid is connected to a cell and a switch, Fig. 6(Q).30. A copper ring with its axis along the axis of the solenoid is placed on a frictionless track. What happens to the ring, as the switch is closed ? (CBSE 2001)

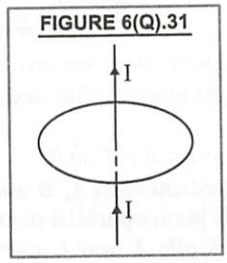
FIGURE 6(Q).30



Ans. When switch is closed current in the solenoid grows from zero to maximum. The induced current in the copper ring opposes the growth of current and hence it moves away from the coil.

3. Current is passed through a wire passing normally through the centre of a conducting loop, Fig. 6(Q).31. If current is increased gradually, what will be the direction of induced current in the loop ?

FIGURE 6(Q).31

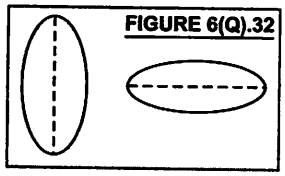


Ans. The magnetic lines of force due to current passed are parallel to the plane of the loop. Therefore, magnetic flux linked with the loop is zero. Hence *no current is induced in the loop.*

4. An artificial satellite with a metal surface is orbiting the earth around the poles. Will there be any induced current due to earth's magnetic field ?

Ans. Yes, induced current will be there. This is because, at poles, earth's magnetic field is totally vertical. The orbiting satellite will intercept this field and *emf* will be induced.

5. The planes of two circular conductors are perpendicular to each other, as shown in Fig. 6(Q).32. If current in any one is changed, will there be induced current in the other ?



Ans. No. This is because magnetic flux due to changing current in any one conductor is parallel to the plane of other conductor. Therefore, magnetic flux linked with the other conductor is zero. Hence there will be no induced current.

6. State whether the following statements are true or false giving reason in brief :

(*a*) The dimension of (*h*/*e*) is the same as that of magnetic flux ϕ.

(*b*) The dimensions of electric and magnetic flux are same.

(*c*) A coil of a metal wire is kept stationary in a non-uniform magnetic field. An *e.m.f.* is induced in the coil.

(*d*) An *e.m.f.* can be induced between the two ends of a straight copper wire when it is moved through a magnetic field.

Ans. (*a*) True.

$$\frac{h}{e} = \frac{\text{joule} - \text{sec}}{\text{coulomb}} = \frac{\text{joule}}{\text{ampere}}$$

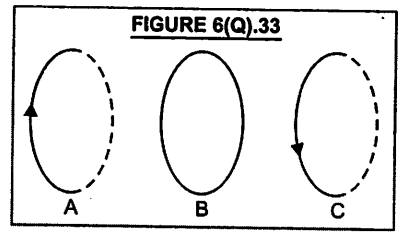
$$= \text{magnetic flux}$$

(*b*) False, for reason, see text.

(*c*) False. As magnetic flux does not change with time.

(*d*) True, as $e = B v \ l \sin\theta$.

7. Three identical coils *A*, *B* and *C* are placed with their planes parallel to one another, Fig. 6(Q).33. Coils *A* and *C* carry currents as shown. Coils *B* and *C* are fixed. The coil *A* is moved towards *B* with uniform speed. Is there any induced current in *B*



Ans. As coils *B* and *C* are fixed, there is no induced current in *B* due to *C*. However, when *A* is moved towards *B*, an *e.m.f.* is induced in *B*, which would oppose relative motion of *A* towards *B*. As current in *A* is clockwise, induced current in *B* must be anticlockwise.

8. Show that the rate of change of magnetic flux has the same units as induced e.m.f.

Ans. Induced e.m.f., $e = V = \dfrac{\text{work}}{\text{charge}} = \dfrac{M L^2 T^{-2}}{AT}$

$$= [M^1 L^2 T^{-3} A^{-1}]$$

Rate of change of magnetic flux $= \dfrac{d\phi}{dt} = \dfrac{BA}{t}$

$$= \frac{F A}{q v t} = \frac{(MLT^{-2})(L^2)}{(AT)(LT^{-1})(T)} \qquad (\because F = B q v)$$

$$= [M^1 L^2 T^{-3} A^{-1}]$$

$\therefore$ Both have the same units/dimensions.

9. A circular brass loop of radius *a* and resistance *R* is placed with its plane perpendicular to a magnetic field, which varies with time as $B = B_0 \sin \omega t$. Obtain the expression for the induced current in the loop.

Ans. Induced current ,

$$I = \frac{\text{induced e.m.f.}}{\text{resistance}} = \frac{e}{R}$$

$$= -\frac{d\phi/dt}{R} = \frac{-1}{R}\frac{d}{dt}(BA\cos 0°)$$

$$I = -\frac{A}{R}\frac{d}{dt}(B_0 \sin \omega t) = -\frac{AB_0}{R}\cos \omega t\,(\omega)$$

$$= -\frac{A\omega B_0}{R}\cos \omega t$$

10. A cylindrical bar magnet is kept along the axis of a circular coil and near it as shown in Fig. 6(Q).34. Will there be any induced emf at the terminals of the coil, when magnet is rotated (*a*) about its own axis (*b*) about an axis perpendicular to the length of the magnet ?

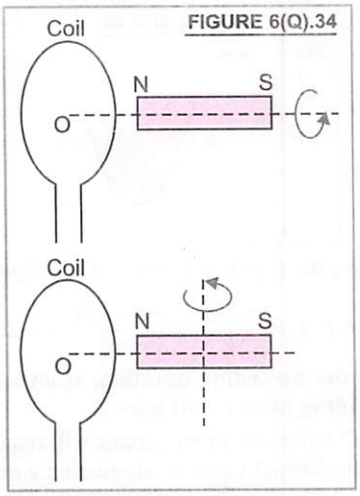


FIGURE 6(Q).34

Ans. (*a*) When the magnet is rotated about its own axis, there is no change in magnetic flux linked with the coil. Therefore, induced *emf* = 0.

(*b*) When the magnet is rotated about an axis perpendicular to its length, orientation of the magnet *wrt* the coil changes. Therefore, magnetic flux linked with the coil changes. Hence an *emf* is induced in the coil.

11. Consider Experiment 2, Art. 6.3,

(a) What would you do to obtain a large deflection of the galvanometer ?

(b) How would you demonstrate the presence of an induced current in the absence of a galvanometer ? (CBSE 2011)

(NCERT Solved Example)

Ans. (*a*) To obtain a large deflection, one or more of the following steps can be taken :

(*i*) Use a soft iron rod inside the coil *C*′.

(*ii*) Connect the coil *C*′ to a powerful battery.

(*iii*) Move coil *C*′ rapidly towards or away from coil *C*.

(*b*) Replace the galvanometer by a small bulb of torch light. The relative motion between the two coils will cause the bulb to glow indicating the production of induced current in coil *C*.

II. Self Induction and Mutual Induction

12. A solenoid with an iron core and a bulb are connected to a d.c source. How does the brightness of bulb change when iron core is removed from the solenoid ?

Ans. In a d.c. circuit, reactance of inductor/solenoid is zero. Therefore, removal of iron core does not affect the brightness of bulb.

13. How does the mutual inductance of a pair of coils change when

(*i*) distance between the coils is increased

(*ii*) an iron sheet is placed between the two coils ? (CBSE 2013, 2006)

Ans. (*i*) On increasing distance between the two coils, magnetic flux passing from one coil to the other decreases. Therefore, mutual inductance decreases.

(*ii*) When an iron sheet is placed between the two coils, mutual inductance increases because $M \propto \mu$ (the permeability).

14. Can one have an inductance without a resistance ? How about a resistance with an inductance ?

Ans. No, as every material has some resistance. Yes, we can coil a wire to have resistance with inductance.

15. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely wound wires in series carrying current in opposite senses. How is its self inductance affected ?

Ans. As two sets of closely wound wires carry currents in opposite senses, therefore, their induced effects cancel. The self inductance reduces. In a special case, we may have

$$L_{equiv.} = L_1 + L_2 - 2M = L + L - 2L = 0$$

16. The current flowing through inductor of self inductance *L* is continuously increasing. Plot a graph showing the variation of

(*i*) Magnetic flux versus the current

(*ii*) Induced emf versus *dI/dt*

(*iii*) Magnetic potential energy stored versus the current. (CBSE 2014, 2012)

Ans. (*i*) Magnetic flux $\phi = LI$ or $\phi \propto I$

Thus, graph between ϕ and *I* is a st. line as shown in Fig. 6(Q).35(*a*).

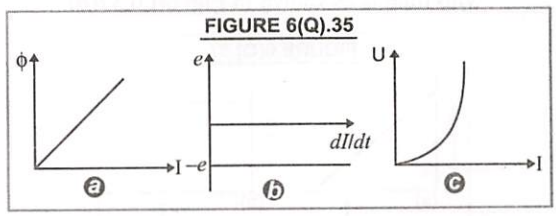


FIGURE 6(Q).35

(*ii*) Induced emf $e = -L\dfrac{dI}{dt}$

If *I* is increasing at constant rate then *dI/dt* is constant and *L dI/dt* = a constant. Then graph between *e* and *dI/dt* is a st. line as shown in Fig. 6(Q).35(*b*).

(*iii*) Magnetic potential energy stored in inductor, $U = \frac{1}{2}LI^2$ or $U \propto I^2$

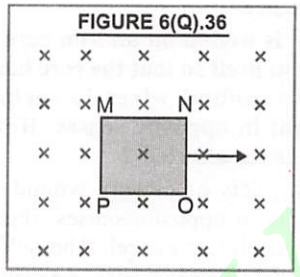
Thus, the graph between U and I is a parabolic curve as shown in Fig. 6(Q).35(*c*).

17. A square loop MNOP of side 20 cm is placed horizontally in a uniform magnetic field acting vertically downwards as shown in Fig. 6(Q).36. The loop is pulled with a constant velocity of 20 cm/s till it goes out of the field.

(*i*) Depict the direction of induced current in the loop as it goes out of the field. For how long will the current in loop persist ?

(*ii*) Plot a graph showing the variation of magnetic flux and induced emf as function of time. **(CBSE 2015)**

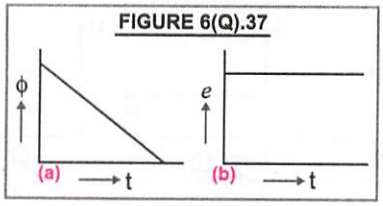
FIGURE 6(Q).36



Ans. (*i*) As loop goes out of field, magnetic flux through the loop decreases. Applying Lenz's law, the induced current must be along MNOP so as to oppose the decrease in magnetic flux. Current in loop will persist till the entire loop goes out of the magnetic field.

(*ii*) As the loop goes out at a constant speed, $\phi = BA\cos 0° = BA$ decreases at a constant rate. The variation in magnetic flux as a function of time is shown in Fig. 6(Q).37(*a*). As $|e| = d\phi/dt$ = constant, therefore, variation of induced emf with time is as shown in Fig. 6(Q).37(*b*).

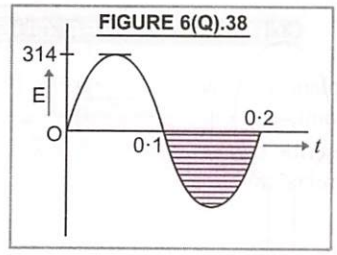
FIGURE 6(Q).37



III. A.C. Generator and D.C. Motor

18. What is the average value of shaded portion of the graph of E vs t ? **(CBSE Sample Paper 2004)**

FIGURE 6(Q).38



Ans. Over the negative half cycle, average

$$e.m.f.\ E = \frac{-2E_0}{\pi} = \frac{-2\times 314}{3{\cdot}14} = -200\ \text{V}$$

19. In the preceding question, what will be the reading of a.c. voltmeter ?

Ans. A.C voltmeter in the circuit will read effective value/virtual value of alternating *e.m.f.*

$$E_v = \frac{E_0}{\sqrt{2}} = \frac{314}{1{\cdot}414} = 222\ \text{volt}$$

20. What is the basic difference in the design of an a.c. generator and d.c. generator ?

Ans. The **slip ring** arrangement in an a.c. generator is replaced by **split ring** arrangement or commutator arrangement in d.c. generator.

21. A d.c. motor acts as generator too. Is it true ?

Ans. Yes. A d.c. motor generates induced *e.m.f.* which opposes the supplied *e.m.f.* It is called back *e.m.f.*

22. What are the advantages of three phase a.c.?

Ans. (*i*) Output of three phase a.c. is almost constant.
(*ii*) Output of three phase a.c. is greater than the output of one phase and two phase systems.
(*iii*) Three phase a.c. system is more economical.

23. What is meant by form factor of an a.c. generator?

Ans. Form factor $= \frac{I_v}{I_{av}} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}1$

24. What are phase lines and neutral line in respect of a generator?

Ans. In a polyphase generator, one end of each coil is brought to a common point through shaft of the generator. The line wire from this point is called **neutral line.** The line wires from other ends of different coils through their slip rings and brushes are called **phase lines.**

25. In a d.c. motor, do we use slip ring or split ring arrangement and why?

Ans. In d.c. motor, we use split rings. This helps us in keeping same, the direction of rotational torque on the coil.

26. Which rule do you use for direction of forces in d.c. motor?

Ans. Fleming's left hand rule is used for finding direction of forces on the armature coil of d.c. motor.

SHORT ANSWER QUESTIONS

Carrying 3 marks

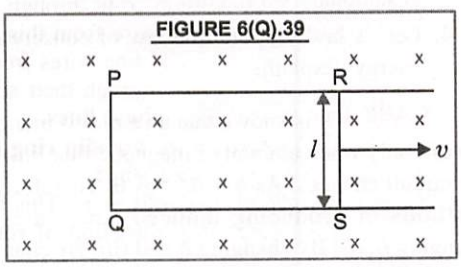
1. Explain the concept of magnetic flux. Discuss its units and dimensions. **[Art. 6.2]**
2. Describe two simple experiments to show that currents are induced by currents. **(CBSE 2011)** **[Art. 6.3]**
3. State and Explain Faraday's laws of electromagnetic induction. **(CBSE 2017, 2009, Karnataka Board 2012, Jharkhand Board 2013)** **[Art. 6.5]**
4. Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it. **(CBSE 2014)** **[Art. 6.6]**
5. What are eddy currents ? Discuss briefly any two applications of eddy currents. **(Raj. Board 2012, Uttarakhand Board 2012, CBSE 2011, 2009, Jharkhand Board 2011)** **[Art. 6.11 and 6.12]**
6. Distinguish between Self induction and Self inductance. Give two factors on which self inductance of an air cored coil depends ? **[Art. 6.14]**
7. Define self inductance. Find the relation for self inductance of two coils (*i*) in series (*ii*) in parallel. **(CBSE 2015)** **[Art. 6.16]**
8. What do you understand by Mutual Induction and Mutual Inductance. Derive an expression for the mutual inductance of two long coaxial solenoids having different radii and different number of turns. **(CBSE 2017, 2009)** **[Art. 6.17]**
9. State the principle of d.c. motor. Give a brief account of a motor starter. **[Arts. 6.22 and 6.23]**

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. State and explain Lenz's law. Give one example to illustrate the law. How will you verify it experimentally ? Does it obey the principle of energy conservation ? **(CBSE 2017, 2013, 2009, Jharkhand Board 2012, Pb. Board 2011, MP Board 2013)** **[Art. 6.6]**
2. Fig. 6(Q).39 shows a rectangular conducting loop *PQSR* in which arm *RS* of length *l* is movable. The loop is kept in a uniform magnetic field *B* directed downwards perpendicular to the plane of the loop. The arm *RS* is moved with a uniform speed *v*. Deduce an expression for

FIGURE 6(Q).39



(*i*) the emf induced across the arm *RS*.

(*ii*) the external force required to move the arm, and

(*iii*) the power dissipated as heat. **(CBSE 2009)** **[Arts. 6.9 and 6.10]**

3. What are eddy currents ? Give some applications of eddy currents. How can the eddy currents be minimised ? **(HP Board 2013, Kerala Board 2013)** **[Art. 6.12 and 6.13]**
4. What do you understand by motional electromotive force ? Find the relation for induced emf and induced current when a conducting rod is moved through a constant magnetic field. Also, find out the relation for mechanical energy dissipated per second. **[Arts. 6.10 and 6.11]**
5. Explain the phenomenon of self induction. Define coefficient of self inductance. What are its units ? Calculate self inductance of a long solenoid. **(Chhatisgarh Board 2011)** **[Arts. 6.13 and 6.14]**
6. Explain the phenomenon of mutual induction. Define coefficient of mutual induction. What are its units ? Calculate coefficient of mutual inductance between two long solenoids. **(CBSE 2019, 2017, 2016, 2015, Manipur Board 2012, MP Board 2011)** **[Arts. 6.16 and 6.17]**

7. A metallic rod of length l is moved perpendi- cular to its length with velocity v in a magnetic field $\vec{B}$ acting perpendicular to the plane in which the rod moves. Derive an expression for the induced e.m.f. **[CBSE 2017 (C)] [Art. 6.10]**

8. Give the principle, construction, theory and working of an a.c. generator. What is the source of energy generation in this device ?
(Chhatisgarh Board 2012, West Bengal Board 2012, Manipur Board 2011, CBSE 2011, 2010)
[Art. 6.19]

9. Draw a schematic sketch of an ac generator describing its basic elements. State briefly its working principle. Show a plot of variation of (*i*) Magnetic flux and (*ii*) Alternating emf versus time generated by a loop of wire rotating in a magnetic field. **(CBSE 2017, 2014) [Art. 6.19]**

10. State the principle of an a.c. generator and explain its working with the help of a labelled diagram. Obtain an expression for the e.m.f. induced in a coil having N turns each of cross sectional area A rotating with a constant angular speed ω in a magnetic field $\vec{B}$ directed perpendicular to the axis of rotation.
(CBSE 2018) [Art. 6.19]

11. Discuss briefly the principle, construction and working of a d.c. generator.
(Jharkhand 2015) [Art. 6.21]

12. What is a d.c. motor ? Discuss its construction and working. Derive an expression for efficiency of d.c. motor.
(Pb. Board 2011, 2010) [Art. 6.22]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. Electromagnetic induction (EMI) is the phenomenon of generating an e.m.f. in a circuit by changing the number of magnetic lines of force associated with the circuit. The e.m.f. so generated is called **induced e.m.f.** and the corresponding current is called **induced current**. The number of magnetic lines of force crossing a surface is called **magnetic flux (ϕ)** linked with the circuit.

$$\phi = BA\cos\theta = \vec{B}\cdot\vec{A}$$

Here, θ is the angle, which normal to the area makes with the direction of magnetic field. SI unit of magnetic flux is **weber** and CGS unit of magnetic flux is **Maxwell.**

$$1 \text{ weber} = 10^8 \text{ Maxwell}$$

Faraday and **Henry** establisted the following **two laws** of EMI :–

First Law. Wherever the amount of magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit. The induced e.m.f. lasts so long as change in magnetic flux continues.

Second Law. The magnitude of e.m.f. induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

$$e = \frac{\phi_2 - \phi_1}{t}$$

or $$e = -\frac{d\phi}{dt}$$

The direction of induced e.m.f. is given by **Lenz's Law.** According to this law, the direction of e.m.f. induced in a circuit is always such as to oppose the change in magnetic flux responsible for it.

Based on the above paragraph, answer questions no. 1 to 4 :

1. What are the units and dimensions of magnetic flux ?
2. What led Faraday to establish the laws of electromagnetic Induction ?
3. What is the direction of induced current due to relative motion of magnet and coil when N-pole of magnet (*a*) is moving towards the coil and (*b*) is stationary w.r.t., coil.
4. Lenz's law supports the law of conservation of energy. Explain.

CASE 2. It is known that an e.m.f. is induced in a circuit, only when amount of magnetic flux linked with the circuit chnages. As $\phi = BA\cos\theta$. Therefore, **three methods** of producing induced e.m.f. are (*i*) By changing B, (*ii*) By changing A and (*iii*) By changing θ.

When a conductor of length l moves with a velocity v in a perpendicular magnetic field B, the e.m.f. is given by $e = Blv$. Currents induced in the body of a

conductor, when amount of magnetic flux linked with the conductor changes are called **eddy currents** or **Foucault currents**

$$i = \frac{-e}{R} = \frac{d\phi/dt}{R},$$

where R = resistance of the conductor. The direction of eddy currents is given by Lenz's law or Fleming's right hand rule.

Some of the important applications of eddy currents are : electromagnetic damping, induction furnace, e.m. brakes, speed meters etc.

As eddy currents oppose the relative motion, they involve loss of energy. To minimise eddy currents, we use laminated cores.

Based on the above paragraph, answer questions no. 5 to 8 :

5. When is the magnetic flux crossing a given surface area held in a magnetic field maximum ?
6. What are the three methods of inducing e.m.f. ?
7. What are eddy currents or Foucault currents ?
8. Give one plus point and one minus point of eddy currents. How can they be minimised ?

CASE 3. Mutual Induction is the property of two coils by virtue of which each coil opposes any change in the strength of current flowing through the other by developing an induced e.m.f. .

If ϕ is the amount of magnetic flux linked with one coil, when a current I flows through the other coil, then

$\phi = MI$, where M is coefficient of mutual induction of the two coils or **Mutual Inductance** of the two coils.

$$\text{As} \quad e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$$

$$\text{If} \quad \frac{dI}{dt} = 1, \text{ Then, } e = -M \times I \text{ or } M = -e$$

$\therefore$ Coeff. of mutual induction of two coils = e.m.f. induced in one coil, when rate of change of current through the other coil is unity.

The SI unit of M is also **Henry.** The value of M depends on geometry of two coils, distance between two coils and relative placement of the two coils.

Based on the above paragraph, answer questions no. 9 to 12 :

9. What is meant by mutual induction and mutual inductance ?
10. What are the units and dimensions of mutual inductance ?
11. How do you define one henry of mutual inductance ?
12. On what factors coefficient of mutual inductance depend ?

ANSWERS

1. S.I unit of magnetic flux is weber and dimensional formula of magnetic flux is $[ML^2 T^{-2} A^{-1}]$.
2. The experimental observations of Faraday and Henry on the experiments related to (*i*) current induced by a magnet (*ii*) current induced by current and (*iii*) current induced by changing current, led them to conclude Faraday's laws of electromagnetic induction.
3. (*a*) When N pole of magnet is moving towards the coil, then N-pole is induced on the face of coil towards N pole of magnet. Due to it, the direction of induced current on that face of coil is anticlockwise direction.

   (*b*) No induced current is set up in the coil as there is no change in magnetic flux linked with the coil.
4. According to Lenz's law, the direction of induced emf or current is such that it opposes the change in magnetic flux responsible for its production. It is the mechanical workdone in moving the magnet w.r.t.coil, changes into electrical energy producing induced emf. This means that the energy is being transformed only. Hence Lenz's law supports the law of conservation of energy.
5. The magnetic flux is maximum when area is held perpendicular to the direction of magnetic field *i.e.* $\theta = 0°$.

   As, $\phi = BA \cos 0° = BA$ (maximum)
6. Three methods of inducing e.m.f. are :

   (*i*) By changing mag. field B

   (*ii*) By changing area A

   (*iii*) By changing θ (*i.e.*, relative orientation)
7. When amount of magnetic flux linked with a conductor changes, currents induced in the body of conductor are called eddy currents or Foucault currents.

8. Eddy currents are used in e.m. brakes. Eddy currents oppose relative motion involving loss of energy. To minimise eddy currents, we use laminated cores.
9. Mutual induction is the property of two coils by virtue of which each coil opposes any change in strength of current flowing through the other coil. Mutual inductance also called coefficient of mutual induction of two coils is equal to e.m.f. induced in one coil, when the rate of change of current through the other coil is unity.
10. SI unit of mutual inductance of two coils is **henry** and dimensions of mutual inductance is $[ML^2 T^{-2} A^{-2}]$.
11. Coefficient of mutual indirectance of two coils is said to be one henry, when a current changing at the rate of 1 ampere/sec in one coil induces an e.m.f. of one volt in the other coil.
12. The coefficient of mutual inductance of two coils depends on geometry of two coils, distance between two coils and relative placement of two coils.

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. An a.c. generator/dynamo produces alternating current energy for mechanical energy of rotation of a coil in a magnetic field. It is based on the phenomenon of electromagentic induction (EMI).

The form of e.m.f. induced is, $e = e_0 \sin \omega t$

where, $e_0 = NA B\omega$ = max. e.m.f. induced. Here N is total number of truns in the coil, A is face area of the coil, B is strength of magnetic field applied and ω is angular velocity of the armature coil. The direction of induced current is given by **Fleming's right hand rule.**

The essential parts of an A.C. generator are armature coil, field magnets ; slip rings, brushes, load resistance. The magnitude of e.m.f. generated is constant. The direction of e.m.f./current is reversed after every half rotation of the coil.

In our country, the a.c. supply is three phase : 220 volt and 50 Hz.

Based on the above paragraph, answer questions no. 1 to 4 :

1. What is the basis of an a.c. generator?
   (*a*) magnetic effect of current
   (*b*) heating effect of current
   (*c*) Chemical effect of current
   (*d*) electromagnetic induction (EMI)
2. If N is total number of turns A is face area of the coil, B is strength of mag. field applied, ω is angular velocity of the coil, then max. value of induced e.m.f. is
   (*a*) $NAB\omega^2$ (*b*) $NA^2 B\omega$
   (*c*) $NAB\omega$ (*d*) $N^2 AB\omega$

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is correct explanation of A.
(*b*) Both A and R are true and R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) Both A and R are false.

3. **Assertion.** In induction brakes use is made of eddy currents.
   Reason. Eddy currents always oppose the relative motion.
4. **Assertion.** 1 henry = 1 volt/1 amp/s.
   Reason. $M = e \times dI/dt$.

CASE 2. **Self induction** is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an e.m.f. in itself.

When current I flows through a coil, magnetic flux linked with the coil is, $\phi = LI$, where L is the **coefficient of self induction** or **self inductance** of the coil. Therefore,

$$\frac{d\phi}{dt} = L\frac{dI}{dt} = -e$$

If, $\frac{dI}{dt} = 1$ then $L \times 1 = -e$ or $L = -e$

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Value of L depends on geometry of the coil and is given by, $L = \frac{\mu_0 N^2 A}{l}$, where l is the length of the coil (solenoid), N is the total number of turns in the solenoid and A is the area of cross-section of the solenoid. SI unit of L is henry.

Based on the above paragraph, answer questions no. 5 to 8 :

5. Self inductance of a coil is directly proportional
(*a*) to number of turns and inversely proportional to length of coil
(*b*) to square of number of turns and inversely proportional to square of length of coil
(*c*) to square of number of turns and inversely proportional to length of coil
(*d*) to the product of number of turns and length of the coil

6. The dimensional formula of self inductance is :
(*a*) $[M^1 L^2 T^{-2} A^{-1}]$
(*b*) $[M^1 L^2 T^{-1} A^{-2}]$
(*c*) $[M^1 L^2 T^{-2} A^{-2}]$
(*d*) $[M^1 L^1 T^{-2} A^{-2}]$

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is correct explanation of A.
(*b*) Both A and R are true but R is not a correct explanation of A.
(*c*) A is true, but R is false.
(*d*) Both A and R are false.

7. Assertion. When the current in a coil of self inductance 10 henry is changing from 10 A to 7 A in time 9×10^{-2} s, then the e.m.f. induced is 333·3 V.
Reason. e.m.f. induced, $e = -L\, dI/dt$.

8. Assertion. Self induction is the inertia of current.
Reason. Solid friction is the inertia of mass.

ANSWERS

1. (*d*) **2.** (*c*) **3.** (*a*) **4.** (*a*) **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** (*b*)

HINTS/EXPLANATIONS For Difficult Questions

1. AC generator is based on EMI.

2. $e_{max} = e_0 = NAB\omega$

3. Both, A and R are true and R is correct explanation of A.

4. Both, A and R are true and R is the correct explanation of A.

5. As, $L = \frac{\mu_0 N^2 A}{l}$ *i.e.*, $L \propto \frac{N^2}{l}$

6. Induced emf, $e = L\frac{dI}{dt}$ or $L = \frac{e\,dt}{dI} = \frac{(W/q)\,dt}{dI}$ $\left[e = \frac{\text{work } (W)}{\text{charge } (q)}\right]$

$\therefore$ Dimensions of $L = \frac{[M^1 L^2 T^{-2}][T]}{[AT][A]} = [M^1 L^2 T^{-2} A^{-2}]$

7. Here **Reason is true.** Now $e = -10 \times \frac{(7-10)}{9\times10^{-2}} = 333{\cdot}3$ V

Thus **Assertion is also true.** Hence **Reason is a correct explanation of Assertion.**

8. Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

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PROBLEMS FOR PRACTICE

TYPE I. MAGNETIC FLUX AND FARADAY'S LAWS OF EMI

1. A coil with an average diameter of 0·02 m is placed perpendicular to a magnetic field of 6000 T. If the induced *e.m.f.* is 11 V when magnetic field is changed to 1000 T in 4 s, what is the number of turns in the coil ? **[Ans. 28]**

2. An air cored solenoid of length 50 cm and area of cross section 28 cm^2 has 200 turns and carries a current of 5·0 A. On switching off, the current decreases to zero in 0·1 ms. Find the average e.m.f. induced across the ends of the solenoid. **[Ans. 1·4 V]**

3. The area of a coil of 25 turns is 1.6 cm^2. This coil is inserted in 0.3 sec in a magnetic field of 1.8 Wb/m^2 such that its plane is perpendicular to the flux lines of the field. Calculate the e.m.f. induced in the coil. Also, calculate the total charge that passes through the wire, if its resistance is 10 Ω. **[Ans. 2.4×10^{-2} V, 7.2×10^{-4} C]**

4. A uniform magnetic field *B* exists in a direction perpendicular to the plane of a square frame made of copper wire.The wire has a diameter of 2 mm and a total length of 40 cm. The magnetic field changes with time at a steady rate dB/dt = 0·02 T/s. Find the current induced in the frame. Resistivity of copper = $1·7 \times 10^{-8}$ ohm-m. **[Ans. $9·3 \times 10^{-2}$ A]**

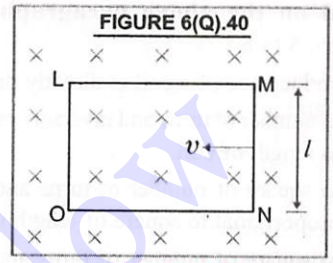
5. A wire 88 cm long bent into a circular loop is placed perpendicular to the magnetic field of flux density 2.5 Wb m^{-2}. Within 0.5 sec, the loop is changed into a square of each side 22 cm and flux density is increased to 3.0 Wb m^{-2}. Calculate the value of e.m.f. induced. **[Ans. 0.018 V]**

6. A coil of 100 loops is pulled in 0·01 *s* between the poles of a magnet, where its area intercepts the flux of 300 μ Wb, to a place where it intercepts the flux of 100 μ Wb. Calculate the e.m.f. induced in the coil. **[Ans. 2 V]**

TYPE II. MOTIONAL e.m.f.

7. A rectangular conductor LMNO is placed in a uniform magnetic field of 0·5 T, Fig. 6(Q).40. The field is directed perpendicular to the plane of the conductor. When the arm *MN* of length 20 cm is moved with a velocity of 10 m/s, calculate the *emf* induced in the arm. Find the value of induced current if resistance of arm *MN* is 5 Ω and other arms have negligible resistance.

FIGURE 6(Q).40



(CBSE 2013) **[Ans. 1 V, 0·2 A]**

8. A circular copper disc of 10 cm radius rotates at 20 π radian/sec about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0·2 *T* acts perpendicular to the disc. Calculate the potential difference developed between axis of the disc and the rim. What is the induced current if resistance of disc is 2 ohm ? (CBSE 2001)

[Ans. 0·0629 V ; 0·0314 A]

9. A metre gauge train is running due north with a constant speed of 90 km h^{-1} on a horizontal track. If the vertical component of earth's magnetic field is 3×10^{-5} Wb m^{-2}, calculate the *e.m.f.* induced across the axle of the train of length 1·25 m.

(West Bengal 2012) **[Ans. $9·38 \times 10^{-4}$ V]**

10. A jet plane is travelling west at 450 m/s. If the horizontal component of earth's magnetic field at that place is 4×10^{-4} T and the angle of dip is 30°, find the e.m.f. induced between the ends of wings having a span of 30 m.

(CBSE 2008) **[Ans. 3.12 V]**

11. An aeroplane is travelling west at the speed of 500 m/s. What is the voltage difference between the ends of the wings 25 m long, if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and dip angle is 30° ?

(Pb. Board 2009) **[Ans. 3·125 V]**

12. When a wheel with metal spokes 1·2 m long rotates in a magnetic field of flux density 5×10^{-5} T normal to the plane of the wheel, an e.m.f. of 10^{-2} V is induced between the rim and the axle. Find the rate of rotation of the wheel. **[Ans. 44·2 rps]**

TYPE III. SELF INDUCTION

13. What is the self inductance of a coil, in which magnetic flux of 40 milliweber is produced when 2 *A* current flows through it ?

(CBSE 2002) **[Ans. 2×10^{-2} Wb]**

14. An inductor of 5 H inductance carries a steady current of 2 A. How can a 50 V self induced *e.m.f.* be made to appear in the inductor ?

(Pb. Board 2001)

[Ans. In 0·2 s, current is reduced to zero]

15. The current in a solenoid of 240 turns, having a length of 12 cm and radius of 2 cm, changes at the rate of 0·8 As^{-1}. Find the emf induced in it.

[Ans. 6×10^{-4} V]

16. Magnetic flux of 5 microweber is linked with a coil when a current of 1 mA flows through it. What is self inductance of the coil ?

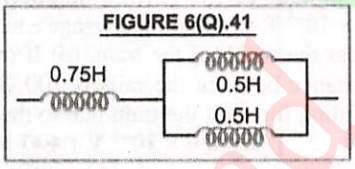
(Pb. Board 2011) **[Ans. 5 mH]**

17. The self inductance of an inductance coil having 100 turns is 20 mH. Calculate the magnetic flux through the cross section of the coil corresponding to a current of 4 milliampere. Also, find the total flux. (CBSE 2000)

[Ans. 8×10^{-5} Wb ; 8×10^{-3} Wb]

18. Three inductances are connected as shown in Fig. 6(Q).41. Calculate the resultant inductance.

FIGURE 6(Q).41



[Ans. 1·0 H]

19. What is the magnetic flux through each turn of a solenoid of self inductance $8{\cdot}0 \times 10^{-5}$ H, when a current of 3·0 A flows through it ? Assume that the solenoid has 1000 turns and is wound from wire of diameter 1·0 mm. What is the cross sectional area of the solenoid ?

[Ans. $2{\cdot}4 \times 10^{-7}$ Wb ; $6{\cdot}37 \times 10^{-5}$ m^2]

TYPE IV. MUTUAL INDUCTION

20. Over a solenoid of 50 cm length and 2 cm radius having 500 turns, is wound another wire of 50 turns near the centre. Calculate mutual inductance of the two coils. If current in primary changes from 0 to 5 A in 0·02 s, what is the *emf* induced in secondary coil ?

[Ans. $7{\cdot}896 \times 10^{-5}$ H ; 19·74 mV]

21. An air cored solenoid is of length 0·3 m, area of cross section $1{\cdot}2 \times 10^{-3}$ m^2 and has 2500 turns. Around its central section, a coil of 350 turns is wound. The solenoid and the coil are electrically insulated from eachother. Calculate the *e.m.f.* induced in the coil if the initial current of 3 A in the solenoid is reversed in 0·25 s.

[Ans. 0·106 V]

22. Find the mutual inductance between the two coils if a current of 10 ampere in primary coil changes the flux by 500 Wb per turn in the secondary coil of 200 turns. Also, find the induced e.m.f. across the ends of the secondary coil if this change occurs in 0.5 sec. **[Ans. 10^4 H, 2×10^5 volt]**

23. Two solenoids *A* and *B* spaced close to eachother and sharing the same cylindrical axis have 400 and 700 turns respectively. A current of 3·5 A in coil A produced an average flux of 300 μ T – m^2 through each turns of *A* and a flux of 90 μ T – m^2 through each turns of *B*. Calculate.

(*a*) mutual inductance of two solenoids.

(*b*) the self inductance of *A*.

What emf is induced in *B* when the current in *A* increases at the rate of 0·5 A/s ?

[Ans. $1{\cdot}8 \times 10^{-2}$ H ; $3{\cdot}43 \times 10^{-2}$ H ; 9×10^{-3} V]

TYPE V. ELECTRIC GENERATOR

24. An a.c. generator consists of a coil of 100 turns and cross sectional area of 3 m^2, rotating at a constant angular speed of 60 rad/sec in a uniform magnetic field of 0·04 T. The resistance of the coil is 500 Ω. Calculate (*i*) maximum current drawn from the generator and (*ii*) max. power dissipation in the coil. (CBSE 2002)

[Ans. 1·44 A ; 518·4 W]

25. An a.c. generator consists of a coil of 2000 turns each of area 80 cm^2 and rotating at an angular speed of 200 rpm in a uniform magnetic field of $4{\cdot}8 \times 10^{-2}$ T. Calculate the peak and *rms* values of *e.m.f.* induced in the coil. (Pb. Board 2002)

[Ans. 16·1 V, 11·4 V]

26. A flat coil of 500 turns each of area 50 cm^2 rotates in a uniform magnetic field of 0·14 Wb/m^2 at an angular speed of 150 rad/sec. The coil has a resistance of 5 Ω. The induced e.m.f. is applied to an external resistance of 10 ohm. Calculate the peak current through the resistance.

[Ans. 3·5 A]

27. A coil has 50 turns and its area is 500 cm^2. It is rotating at the rate of 50 r.p.s. at right angles to a magnetic field of 0·5 Wb/m^2. Calculate the

maximum value of electromotive force developed across the ends of the coil. **[Ans. 392·8 volt]**

28. An a.c. generator consists of a coil of 50 turns and area 2·5 m^2 rotating at an angular speed of 60 rad s^{-1} in a uniform magnetic field $B = 0·3$ T between two fixed pole pieces. The resistance of the circuit including that of the coil is 500 Ω. Find

(*i*) the max. current drawn from the generator.

(*ii*) What will be the orientation of the coil w.r.t. the magnetic field to have (*a*) maximum (*b*) zero magnetic flux ?

(*iii*) Would the generator work if the coil were stationary and instead, the pole pieces rotated together with the same speed as above ?

(CBSE 2003)

[Ans. (*i*) 4·5 A (*ii*) Vertical, Horizontal (*iii*) Yes]

29. A circular coil having 20 turns, each of radius 8 cm, is rotating about its vertical diameter with an angular speed of 50 rad/s in a uniform horizontal magnetic field of 30 mT. Obtain the maximum, average and *rms* value of *e.m.f.* induced in the coil. If the coil forms a closed loop of resistance 10 ohm, how much power is dissipated as heat ? **(CBSE Sample Paper 2012)**

[Ans. 0·603 V ; Zero ; 0·4265 V ; 0·0182 W]

TYPE VI. ELECTRIC MOTOR

30. The resistance of the armature of an electric motor is 55 ohm. The motor draws a current of 2 A when running at 220 V. What is the back *e.m.f.* ? **[Ans. 110 V]**

31. A motor has an armature resistance of 4 ohm. On a 240 V supply and a light load, the motor speed is 200 rpm and the armature current is 5 A. Calculate the motor speed at a full load, when armature current is 20 A. **[Ans. 145·45 rpm]**

32. A motor runs at 220 V. The resistance of the armature is 11 ohm. If the back *emf* produced is 198 V, when at full speed, then calculate the current through armature, when (*i*) motor is just switched on and (*ii*) motor is at full speed.

[Ans. 20 A ; 2 A]

TYPE VII. TYPICAL PROBLEMS

33. A rectangular loop of sides 8 cm and 2 cm with a small cut is stationary in a uniform magnetic field directed normal to the loop. The magnetic field is reduced from its initial value of 0·3 T at the rate of 0·02 T s^{-1}. If the cut is joined and the loop has a resistance of 1·6 Ω, how much power is dissipated by the loop as heat ?

[Ans. $6·4 \times 10^{-10}$ W]

34. A conducting circular loop is placed in a uniform magnetic field $B = 0·020$ T with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of 1·0 mm/s. Find the induced *e.m.f.* in the loop at an instant when the radius is 2 cm ?

[Ans. 2·5 μ V]

35. Magadh express takes 16 hours to cover the distance of 960 km between Patna and Ghaziabad. The rails are separated by 130 cm and the vertical component of the earth's magnetic field is $4·0 \times 10^{-5}$ T. (*a*) Find the average *e.m.f.* induced across the width of the train. (*b*) If the leakage resistance between the rails is 100 Ω, find the retarding force on the train due to the magnetic field. **[Ans. $8·6 \times 10^{-4}$ V ; $4·47 \times 10^{-10}$ N]**

36. A metallic wire bent in the form of a semi-circle of radius 0·1 m is moved in a direction parallel to its plane, but perpendicular to a magnetic field $B = 20$ mT with a vel. of 10 m/s. Find the *e.m.f.* induced in the wire. **[Ans. 0·04 V]**

HINTS . . . .
For Difficult Problems

1. Here, $r = \frac{0·02}{2} = 0·01$ m, $\theta = 0°$, $B_1 = 6000$ T

$e = 11$ V, $B_2 = 1000$ T, $t = 4$ s, $N = ?$

$$|e| = \frac{\phi_2 - \phi_1}{t} = NA\,\frac{(B_2 - B_1)}{t} = \frac{N\pi r^2 (B_2 - B_1)}{t}$$

$$11 = \frac{N \times 3·14\,(0·01)^2\,(1000 - 6000)}{4}$$

$$N = \frac{11 \times 4}{3·14 \times 10^{-4} \times 5000} = \mathbf{28}\ \text{(leaving – sign)}$$

2. Here, $l = 50$ cm $= \frac{1}{2}$ m,

$A = 28$ cm$^2 = 28 \times 10^{-4}$ m^2

$N = 200$, $I = 5·0$ A,

$t = 0·1$ ms $= 10^{-4}$ s, $e = ?$

$$B = \frac{\mu_0 NI}{l}\ ;\ \phi_1 = NBA = \frac{N(\mu_0 NI)}{l} A$$

$$= \frac{\mu_0 N^2 I A}{l}$$

$$\phi_1 = \frac{4\pi\times10^{-7}(200)^2\times5{\cdot}0\times28\times10^{-4}}{1/2}$$

$= 1{\cdot}406 \times 10^{-3}$ Now, $\phi_2 = 0$,

$$|e| = \frac{\phi_2 - \phi_1}{t} = \frac{0-1{\cdot}406\times10^{-3}}{10^{-3}} = \mathbf{1{\cdot}4\ V}$$

3. Use $e = -N\frac{d\phi}{dt}$; $q = i\times dt = \frac{e}{R}\times dt$

4. Area of frame, $A = \left(\frac{40}{4}\times10^{-2}\text{ m}\right)^2 = 10^{-2}\text{ m}^2$

$$e = \frac{d\phi}{dt} = A\frac{dB}{dt} = 10^{-2}\times0{\cdot}02 = 2\times10^{-4}\text{ V}$$

Resistance of loop, $R = \frac{\rho l}{a} = \frac{\rho l}{\pi r^2}$

$$R = \frac{1{\cdot}7\times10^{-8}\times40\times10^{-2}}{\frac{22}{7}\times(10^{-3})^2} = 2{\cdot}16\times10^{-3}\text{ ohm}$$

Induced current, $i = \frac{e}{R} = \frac{2\times10^{-4}}{2{\cdot}16\times10^{-3}}$

$= \mathbf{9{\cdot}3\times10^{-2}\ A}$

6. Here, $N = 100$, $dt = 0{\cdot}01\ s$

$\phi_1 = 300\ \mu\text{ Wb} = 300\times10^{-6}$ Wb;

$\phi_2 = 100\times10^{-6}$ Wb

$$e = -N\frac{(\phi_2 - \phi_1)}{dt}$$

$$= -\frac{100(100-300)10^{-6}}{0{\cdot}01}$$

$$= \frac{2\times10^{-2}}{0{\cdot}01} = \mathbf{2\ V}$$

7. Here, $B = 0{\cdot}5$ T, $l = 20$ cm $= \frac{1}{5}$ m,

$v = 10$ m/s, $e = ?$, $I = ?$, $R = 5\ \Omega$

$$|e| = Blv = 0{\cdot}5\times\frac{1}{5}\times10 = \mathbf{1\ V}$$

$$i = \frac{|e|}{R} = \frac{1}{5} = \mathbf{0{\cdot}2\ A}$$

8. Here, $r = 10$ cm $= 10^{-1}$ m, $\omega = 20\ \pi$ rad/s

$B = 0{\cdot}2$ T, $e = ?$

Potential difference developed between the axis of the disc and its rim, $e = \frac{1}{2}Br^2\omega$

$$e = \frac{1}{2}\times0{\cdot}2\ (10^{-1})^2\times20\pi = \mathbf{0{\cdot}0629\ V}$$

Induced current, $i = \frac{e}{R} = \frac{0{\cdot}0628}{2} = \mathbf{0{\cdot}0314\ A}$

9. Here, $v = 90$ km h^{-1} $= 90\times\frac{5}{18}$ ms^{-1} $= 25$ ms^{-1}

$B = V = 3\times10^{-5}$ Wb m^{-2}, $e = ?$, $l = 1{\cdot}25$ m

As $e = Blv$

$\therefore\ e = 3\times10^{-5}\times1{\cdot}25\times25 = \mathbf{9{\cdot}38\times10^{-4}\ V}$

10. Here, $v = 450$ m/s, $H = 4\times10^{-4}$ T,

$\delta = 30°$, $l = 30$ m

A jet plane flying horizontally intercepts vertical component (V) of earth's magnetic field.

$\therefore\ B = V = H\tan\delta = 4\times10^{-4}\tan30°$

$$= \frac{4\times10^{-4}}{\sqrt{3}}\text{ T}$$

$e = Blv$

$$e = \frac{4}{\sqrt{3}}\times10^{-4}\times30\times450\text{ V} = \mathbf{3{\cdot}12\ volt}$$

11. Here, $v = 500$ m/s $\quad e = ?\quad l = 25$ m

$R = 5\times10^{-4}$ T, $\delta = 30°$

$e = Blv = Vlv = (R\sin\delta)lv$

$= 5\times10^{-4}\sin30°\times25\times500 = \mathbf{3{\cdot}125\ V}$

12. Here, $l = 1{\cdot}2$ m, $B = 5\times10^{-5}$ T

$e = 10^{-2}$ V frequency, $v = ?$

From $e = BAv = B(\pi l^2)v$

$$v = \frac{e}{B\pi l^2} = \frac{10^{-2}}{5\times10^{-5}\times3{\cdot}14\ (1{\cdot}2)^2}$$

$\mathbf{v = 44{\cdot}2\ rotations/sec.}$

13. Here, $L = ?$, $\phi = 40$ milli weber $= 40\times10^{-3}$ Wb,

$I = 2$ A

From $\phi = LI$, $L = \frac{\phi}{I} = \frac{40\times10^{-3}}{2} = \mathbf{2\times10^{-2}\ Wb}$

14. Here, $L = 5$ H, $I = 2$ A, $e = 50$ V

To induce this e.m.f. let the current be reduced to zero in time dt

As $e = L\frac{dI}{dt}$ $\therefore$ $50 = \frac{5(2-0)}{dt}$

$$dt = \frac{5\times2}{50} = \mathbf{0{\cdot}2\ sec.}$$

15. Here, $N = 240$, $l = 12$ cm $= 12\times10^{-2}$ m,

$r = 2$ cm $= 2\times10^{-2}$ m, $A = \pi r^2$

$dI/dt = 0{\cdot}8$ As^{-1}, $e = ?$

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$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi\times10^{-7}\,(240)^2\times\pi\,(0{\cdot}02)^2}{0{\cdot}12}$$

$$|e| = \frac{L\,dI}{dt} = \frac{4\pi\times10^{-7}\,(240)^2\times3{\cdot}14\times4\times10^{-4}\times0{\cdot}8}{0{\cdot}12}$$

$$= \mathbf{6\times10^{-4}\ V}$$

17. Here, $n = 100$, $L = 20$ mH $= 20\times10^{-3}$ H ; $i = 4$ mA $= 4\times10^{-3}$ A

Magnetic flux through the cross section of coil $= Li = 20\times10^{-3}\times4\times10^{-3} = \mathbf{8\times10^{-5}\ Wb}$

Total magnetic flux $= n\,Li = 100\times8\times10^{-5}$ $= \mathbf{8\times10^{-3}\ Wb}$

18. $$\frac{1}{L_p} = \frac{1}{0{\cdot}5} + \frac{1}{0{\cdot}5} = \frac{2}{1} + \frac{2}{1} = 4$$

$$L_p = \frac{1}{4} = 0{\cdot}25\,\text{H}$$

$\therefore\ L_s = 0{\cdot}75 + 0{\cdot}25 = \mathbf{1{\cdot}0\ H}$

19. Here, $L = 8{\cdot}0\times10^{-5}$ H, $I = 3{\cdot}0$ A

$N = 1000$; diameter, $d = 1{\cdot}0$ mm $= 10^{-3}$ m.

If ϕ is magnetic flux linked with each turn of solenoid, then $N\phi = LI$

$$\phi = \frac{LI}{N} = \frac{8\times10^{-5}\times3{\cdot}0}{1000} = \mathbf{2{\cdot}4\times10^{-7}\ Wb}$$

Now, $\phi = BA = \left(\mu_0\times\frac{Ni}{l}\right)A$ where $l = N\times d$

$$\therefore\quad \phi = \mu_0\frac{Ai}{d} \text{ or } A = \frac{\phi d}{\mu_0 i}$$

$$\therefore\quad A = \frac{2{\cdot}4\times10^{-7}\times10^{-3}}{(4\pi\times10^{-7})\times3} = \mathbf{6{\cdot}37\times10^{-5}\ m^2}$$

20. Here, $l = 50$ cm $= \frac{1}{2}$ m, $r = 2$ cm $= 2\times10^{-2}$ m,

$N_1 = 500$; $N_2 = 50$, $M = ?$, $dI = 5 - 0 = 5$ A

$dt = 0{\cdot}02$ s, $e = ?$

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

$$= \frac{4\pi\times10^{-7}\times500\times50\times3{\cdot}14\,(2\times10^{-2})^2}{1/2}$$

$$= \mathbf{7{\cdot}896\times10^{-5}\ H}$$

$$e = \frac{M\,dI}{dt} = 7{\cdot}896\times10^{-5}\times\frac{5}{0{\cdot}02}$$

$$= \mathbf{19{\cdot}74\times10^{-3}\ V = 19{\cdot}74\ mV}$$

21. Here, $l = 0{\cdot}3$ m, $A = 1{\cdot}2\times10^{-3}$ m^2, $N_1 = 2500$

$N_2 = 350$, $e = ?$, $dI = -3 - (3) = -6$ A,

$dt = 0{\cdot}25$ s.

$$e = -M\frac{dI}{dt} = -\frac{\mu_0 N_1 N_2 A}{l}\left(\frac{dI}{dt}\right)$$

$$= \frac{-4\pi\times10^{-7}\times2500\times350\times1{\cdot}2\times10^{-3}\,(-6)}{0{\cdot}3\times0{\cdot}25}$$

$$e = \mathbf{0{\cdot}106\ V}$$

23. Here, $N_1 = 400$, $N_2 = 700$

$I_1 = 3{\cdot}5$ A $\quad \phi_1 = 300\times10^{-6}$ Tm^2

$\phi_2 = 900\times10^{-6}$ Tm^2

$$M = \frac{N_2\phi_2}{I_1} = \frac{700\times90\times10^{-6}}{3{\cdot}5} = \mathbf{1{\cdot}8\times10^{-2}\ H}$$

$$L_1 = \frac{N_1\phi_1}{I_1} = \frac{400\times300\times10^{-6}}{3{\cdot}5} = \mathbf{3{\cdot}43\times10^{-2}\ H}$$

$$e_2 = M\left(\frac{dI_1}{dt}\right) = 1{\cdot}8\times10^{-2}\times(0{\cdot}5)$$

$$= \mathbf{9\times10^{-3}\ V}$$

24. Here, $N = 100$, $A = 3$ m^2, $\omega = 60$ rad/sec.

$B = 0{\cdot}04$ T, $R = 500\ \Omega$,

$I_0 = ?$, $P_0 = ?$

$$I_0 = \frac{E_0}{R} = \frac{NAB\omega}{R} = \frac{100\times3\times0{\cdot}04\times60}{500}$$

$$= \mathbf{1{\cdot}44\ A}$$

Max. power dissipation $= E_v\,I_v$

$$= \frac{E_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}} = \frac{I_0^2 R}{2} = \frac{(1{\cdot}44)^2\times500}{2}$$

$$= \mathbf{518{\cdot}4\ W}$$

26. Here, $N = 500$, $A = 50$ cm$^2 = 50\times10^{-4}$ m^2

$B = 0{\cdot}14$ Wb/m^2, $\omega = 150$ rad/s,

$R = 5\ \Omega$ $\quad R' = 10\ \Omega$, $i_0 = ?$

$e_0 = NAB\,\omega = 500\times50\times10^{-4}\times0{\cdot}14\times150$

$= 52{\cdot}5$ volt

As total resistance $= R + R' = 5 + 10 = 15\ \Omega$

$$\therefore\ i_0 = \frac{e_0}{15} = \frac{52{\cdot}5}{15} = \mathbf{3{\cdot}5\ A}$$

28. (*i*) Here, $N = 50$, $A = 2{\cdot}5$ m^2, $\omega = 60$ rad s^{-1}

$B = 0{\cdot}3$ T, $R = 500\ \Omega$, $I_0 = ?$

$$I_0 = \frac{e_0}{R} = \frac{NAB\omega}{R} = \frac{50\times2{\cdot}5\times0{\cdot}3\times60}{500}$$

$$= \mathbf{4{\cdot}5\ A}$$

(*ii*) Magnetic flux will be max. if the coil is in the **vertical position.** On the other hand, magnetic flux will be zero if the coil is in the **horizontal position.**

(*iii*) **Yes,** the generator will work if the coil is stationary and instead, the pole pieces are rotated together.

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29. Here, $N = 20$, $r = 8$ cm $= 8 \times 10^{-2}$ m,
$\omega = 50$ rad/s, $B = 30$ mT $= 30 \times 10^{-3}$ T,
$A = \pi r^2 = \pi (8 \times 10^{-2})^2$ m^2
$e_0 = NA B \omega$
$= 20 \times \pi (8 \times 10^{-2})^2 \times 30 \times 10^{-3} \times 50$
$= \mathbf{0{\cdot}603\ V}$
The average value of *e.m.f.*
$e_{av} =$ Zero

$$e_{rms} = e_v = \frac{e_0}{\sqrt{2}} = \frac{0{\cdot}603}{1{\cdot}414} = \mathbf{0{\cdot}4265\ V}$$

$R = 10$ ohm
Power dissipated as heat

$$= I_v^2 R = \left(\frac{e_v}{R}\right)^2 R = \frac{e_v^2}{R} = \frac{(0{\cdot}4265)^2}{10}$$
$$= \mathbf{0{\cdot}0182\ W}$$

30. From $I = \dfrac{V - E}{R}$; $2 = \dfrac{220 - E}{55}$
$E = \mathbf{110\ V}$

31. Here, $R = 4$ ohm, $V = 240$ V, $I_1 = 5$ A,
$\nu_1 = 200$ rpm, $I_2 = 20$ A, $\nu_2 = ?$
Let E_1, E_2 be the back *emfs* in the two cases.
As $I = \dfrac{V - E}{R}$

$\therefore \quad 5 = \dfrac{240 - E_1}{4}$ or $E_1 = 220$ V

and $20 = \dfrac{240 - E_2}{4}$ or $E_2 = 160$ V

As speed of motor is proportional to back *e.m.f.*, therefore, full load speed

$$= \frac{E_2}{E_1} \times \nu_1 = \frac{160}{220} \times 200 = \mathbf{145{\cdot}45\ rpm}$$

32. Here, $V = 220$ V, $R = 11\ \Omega$, $E = 198$ V
Current at full speed $I = \dfrac{V - E}{R}$

$$= \frac{220 - 198}{11} = \frac{22}{11} = \mathbf{2\ A}$$

When motor is just switched on, $E = 0$

$$\therefore \quad I = \frac{V}{R} = \frac{220}{11} = \mathbf{20\ A}$$

33. Here, $A = 8 \times 2 = 16$ sq cm $= 16 \times 10^{-4}$ m^2

$$\frac{dB}{dt} = -0{\cdot}02\ \mathrm{T\,s^{-1}}$$

Induced *e.m.f.*, $e = \dfrac{-d\phi}{dt} = -\dfrac{A\,dB}{dt}$
$= -16 \times 10^{-4} (-0{\cdot}02) = 32 \times 10^{-6}$ V

Power loss $= \dfrac{e^2}{R} = \dfrac{(32 \times 10^{-6})^2}{1{\cdot}6} = \mathbf{6{\cdot}4 \times 10^{-10}\ W}$

34. At instant t, let the radius of the loop be r. Therefore, magnetic flux linked with the loop at this instant, $\phi = \pi r^2 B$

$$\frac{d\phi}{dt} = 2\pi r B \frac{dr}{dt}$$

When $r = 2 \times 10^{-2}$ m and $\dfrac{dr}{dt} = 10^{-3}$ m/s,

$$e = \frac{d\phi}{dt} = 2 \times \frac{22}{7} (2 \times 10^{-2}) (0{\cdot}02) \times 10^{-3}$$
$$= 2{\cdot}5 \times 10^{-6}\ \mathrm{V} = \mathbf{2{\cdot}5\ \mu V}$$

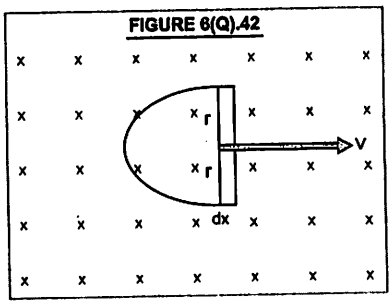
35. Here, $\theta = \dfrac{960}{16} = 60$ km/h $= 16{\cdot}67$ m/s.
Field intercepted, $B = 4 \times 10^{-5}$ T,
$l = 130$ cm $= 1{\cdot}3$ m
Average *e.m.f.* induced, $e = B v l$
$= 4 \times 10^{-5} \times 16{\cdot}67 \times 1{\cdot}3$
$= \mathbf{8{\cdot}6 \times 10^{-4}\ V}$

Leakage current, $i = \dfrac{e}{R} = \dfrac{8{\cdot}6 \times 10^{-4}}{100}$
$= 8{\cdot}6 \times 10^{-6}$ A
$\therefore$ Retarding force, $F = B i l$
$= (4 \times 10^{-5}) \times (8{\cdot}6 \times 10^{-6}) \times 1{\cdot}3$
$F = \mathbf{4{\cdot}47 \times 10^{-10}\ N}$

36. Here, $r = 0{\cdot}10$ m,
$B = 20$ mT $= 20 \times 10^{-3}$ T
$v = 10$ m/s ; $e = ?$
In a small time dt, let the semi-circular wire move through a small distance dx (in or out of the field), Fig. 6(Q).42.
As $e = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(B \times A) = B \dfrac{dA}{dt}$

$$\therefore \quad e = \frac{B(2r\,dx)}{dt} = 2Brv$$
$$= 2 \times 20 \times 10^{-3} \times 0{\cdot}1 \times 10 = \mathbf{0{\cdot}04\ V}$$

FIGURE 6(Q).42



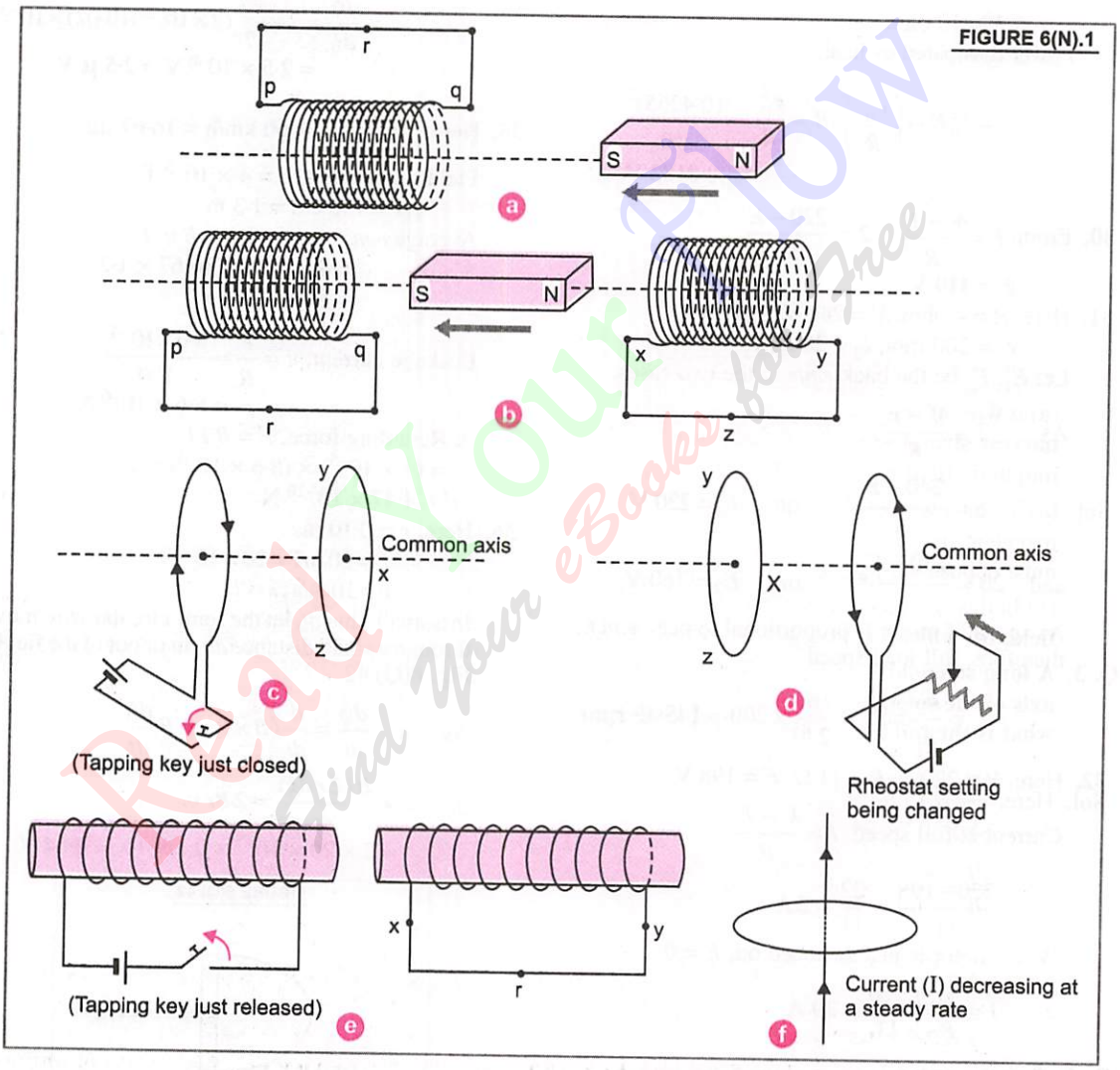
Note. If the entire loop remains inside the field, despite moving, $e = 0$, as flux would remain constant.

NCERT EXERCISES

WITH SOLUTIONS

Q. 1. Predict the direction of induced current in the situations described by the following Figs. 6(N.1 (a) to (f). (CBSE 2012)

FIGURE 6(N).1



Sol. (*a*) South pole developes at *q*, current induced must be clockwise at *q*.

∴ In the coil, induced current is from *p* to *q*.

(*b*) Coil *pq* in this case would develop *S*-pole at *q* and coil *XY* would also develop *S* pole at *X*. Therefore, induced current in coil *pq* will be from *q* to *p* and induced current in coil *XY* will be from *X* to *Y*.

(*c*) Induced current in the right loop will be along *XYZ*.

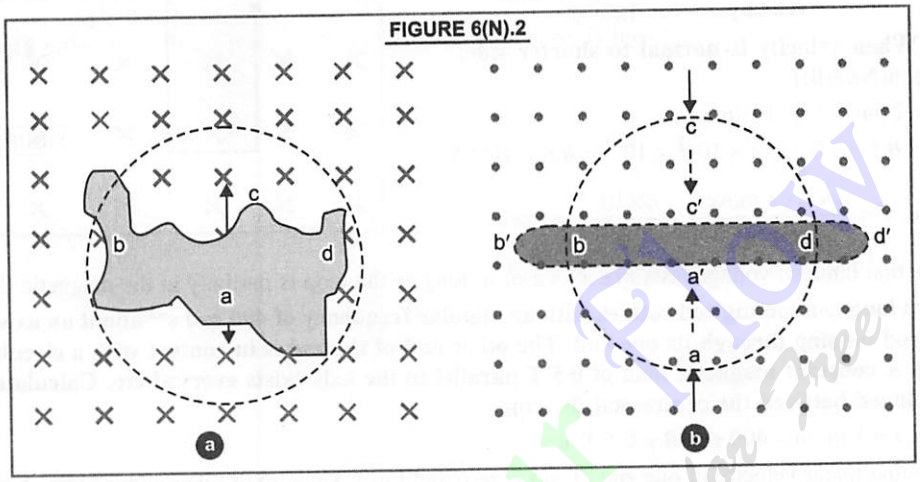
(*d*) Induced current in the left loop will be along *ZYX* as seen from front.

(*e*) Induced current in the right coil is from *X* to *Y*.

(*f*) No current is induced because magnetic lines of force lie in the plane of the loop.

Q. 2. Use Lenz's law to determine the direction of induced current in the situation described by Fig. 6(N).2.

FIGURE 6(N).2



(a) a wire of irregular shape turning into a circular shape (b) a circular loop being deformed into a narrow straight wire. The crosses indicate the magnetic field into the paper and the dots indicate magnetic field out of the paper.

Sol. (*a*) When a wire of irregular shape turns into a circular loop, area of the loop tends to increase. Therefore, magnetic flux linked with the loop increases. According to Lenz's law, the direction of induced current must oppose the magnetic field, for which induced current should flow along *adcba*.

(*b*) In this case, the magnetic flux tends to decrease. Therefore, induced current must support the magnetic field, for which induced current should flow along *adcba*.

Q. 3. A long solenoid with 15 turns per cm has a small loop of area 2·0 cm^2 placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0·1 s, what is the induced voltage in the loop, while the current is changing ?

Sol. Here, $\frac{N}{l}$ = 15 turns/cm = 1500 turns/metre., $A = 2{\cdot}0\ \text{cm}^2 = 2 \times 10^{-4}\ \text{m}^2$

$$\frac{di}{dt} = \frac{4-2}{0{\cdot}1} = 20\,A\,s^{-1}$$

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{d}{dt}\left(\mu_0 \frac{N}{l} i\right) = A\mu_0\left(\frac{N}{l}\right)\frac{di}{dt} = (2 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 1500 \times 20\ \text{V}$$

$$e = \mathbf{7{\cdot}5 \times 10^{-6}\ V}$$

Q. 4. A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 tesla directed normal to the loop. What is the voltage developed across the cut if velocity of loop is 1 cm s^{-1} in a direction normal to the (i) longer side (ii) shorter side of the loop ? For how long does the induced voltage last in each case ?

Sol. Here, $A = 8 \times 2 = 16\ \text{cm}^2 = 16 \times 10^{-4}\ \text{m}^2$, $B = 0.3$ tesla, $v = 1$ cm/sec $= 10^{-2}\ \text{ms}^{-1}$

induced e.m.f., e = ?

(*i*) When velocity is normal to longer side, [Fig. 6(N).3(i)]

$l = 8\text{ cm} = 8 \times 10^{-2}\text{ m}$

$e = B l v = 0.3 \times 8 \times 10^{-2} \times 10^{-2}$

$= \mathbf{2.4 \times 10^{-4}\text{ V}}$

Time, $t = \dfrac{\text{distance moved}}{\text{velocity}} = \dfrac{2 \times 10^{-2}}{10^{-2}} = 2\text{ sec}.$

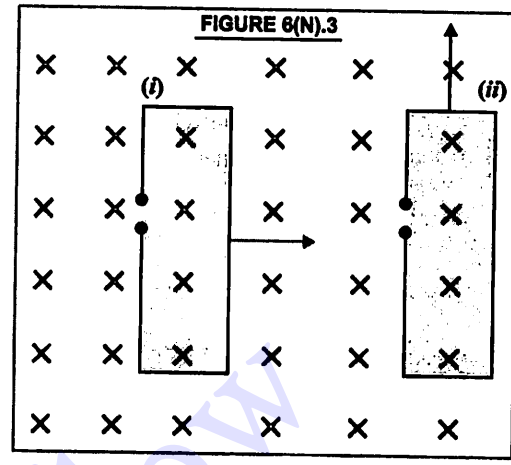
(*ii*) When velocity is normal to shorter side, [Fig. 6(N).3(ii)]

$l = 2\text{ cm} = 2 \times 10^{-2}\text{ m}$

$e = B l v = 0.3 \times 2 \times 10^{-2} \times 10^{-2} = \mathbf{0.6 \times 10^{-4}\text{ V}}$

Time, $t = \dfrac{\text{distance moved}}{\text{velocity}} = \dfrac{8 \times 10^{-2}}{10^{-2}} = 8\text{ sec}$

FIGURE 6(N).3



Note that induced voltage lasts in each case, so long as the loop is partially in the magnetic field.

Q. 5. A 1m long conducting rod rotates with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0·5 T parallel to the axis exists everywhere. Calculate the e.m.f. developed between the centre and the ring.

Sol. Here, $l = 1$ m, $\omega = 400\text{ s}^{-1}$, $B = 0{\cdot}5$ T, $e = ?$

Note that linear velocity of one end of rod is zero and linear velocity of other end is ($l\,\omega$). Average linear velocity

$$v = \frac{0 + l\omega}{2} = l\omega/2 \quad \therefore \quad e = Blv = Bl\frac{(l\omega)}{2} = \frac{Bl^2\omega}{2} = \frac{0{\cdot}5 \times 1^2 \times 400}{2} = \mathbf{100\text{ V}}$$

Q. 6. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5·0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field $0{\cdot}30 \times 10^{-4}$ Wb m^{-2}.

(a) What is the instantaneous value of the *e.m.f.* induced in the wire ?

(b) What is the direction of the *e.m.f.* ?

(c) Which end of the wire is at higher electric potential ?

Sol. Here, $l = 10$ m, $v = 5{\cdot}0\text{ ms}^{-1}$, $B = 0{\cdot}30 \times 10^{-4}$ T

(*a*) $e = B l v = 0{\cdot}30 \times 10^{-4} \times 10 \times 5{\cdot}0 = \mathbf{1{\cdot}5 \times 10^{-3}\text{ V}}$

(*b*) According to Fleming's right hand rule, the direction of induced e.m.f. is **from west to east.**

(*c*) **West end** of the wire must be at higher electric potential.

Q. 7. Current in a circuit falls from 5·0 A to 0·0 A in 0·1 s. If an average e.m.f. of 200 V is induced, give an estimate of the self inductance of the circuit ?

Sol. Here, $\dfrac{dI}{dt} = \dfrac{(I_2 - I_1)}{t} = \dfrac{0{\cdot}0 - 5{\cdot}0}{0{\cdot}1} = -50\text{ As}^{-1}$, $e = 200$ V, $L = ?$

As $|e| = L\left|\dfrac{dI}{dt}\right| \quad \therefore \quad L = \dfrac{|e|}{|dI/dt|} = \dfrac{200}{50} = \mathbf{4\text{ H}}$

Q. 8. A pair of adjacent coils has a mutual inductance of 1·5 H. If the current in one coil changes from 0 to 20 A in 0·5 s, what is the change in flux linkage with the other coil ?

Sol. Here, $M = 1{\cdot}5$ H, $\dfrac{dI}{dt} = \dfrac{20 - 0}{0{\cdot}5} = 40\text{ As}^{-1}$

From $e = M\dfrac{dI}{dt} = \dfrac{d\phi}{dt}$; $d\phi = M\,dI = 1{\cdot}5\,(20 - 0) = \mathbf{30\text{ Wb}}$

NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. A square of side L meters lies in the x-y plane in a region, where the magnetic field is given by $B = B_0 (2\hat{i} + 3\hat{j} + 4\hat{k})$ T, where B_0 is constant. The magnitude of flux passing through the square is

(a) $2 B_0 L^2$ Wb (b) $3 B_0 L^2$ Wb

(c) $4 B_0 L^2$ Wb (d) $\sqrt{29}\ B_0 L^2$ Wb

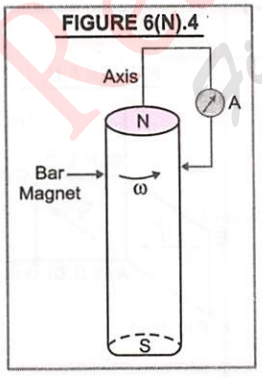
2. A loop, made of straight edges has six corners at A (0, 0, 0), B (L, O, 0), C (L, L, 0), D (0, L, 0), E (0, L, L) and F (0, 0, L). A magnetic field $B = B_0 (\hat{i} + \hat{k})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is

(a) $B_0 L^2$ Wb (b) $2 B_0 L^2$ Wb

(c) $\sqrt{2}\ B_0 L^2$ Wb (d) $4 B_0 L^2$ Wb

3. A cylindrical bar magnet is rotated about its axis, Fig. 6(N).4. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then

FIGURE 6(N).4



(a) a direct current flows in the ammeter A

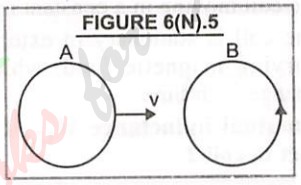
(b) no current flows through the ammeter A

(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = 2\pi/\omega$

(d) a time varying non-sinosoidal current flows through the ammeter A

4. There are two coils A and B as shown in Fig. 6(N).5. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter-clockwise. B is kept stationary when A moves. We can infer that

FIGURE 6(N).5



(a) there is a constant current in the clockwise direction in A

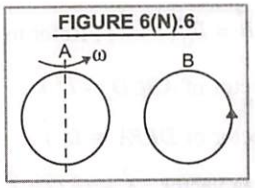
(b) there is a varying current in A

(c) there is no current in A

(d) there is a constant current in the counter-clockwise direction in A

5. Same as problem 4 except the coil A is made to rotate about a vertical axis, Fig. 6(N).6. No current flows in B if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counterclockwise and the coil A is as shown at this istant, $t = 0$, is

FIGURE 6(N).6



(a) constant current clockwise

(b) varying current clockwise

(c) varying current counterclockwise

(d) constant current counterclockwise

6. The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(a) l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) both l and A decrease

MULTIPLE CHOICE QUESTIONS-II

7. A metal plate is getting heated. It can be because

(a) a direct current is passing through the plate
(b) it is placed in a time varying magnetic field
(c) it is placed in a space varying magnetic field, but does not vary with time
(d) a current (either direct or alternating) is passing through the plate

8. An e.m.f. is produced in a coil, which is not connected to an external voltage source. This can be due to

(a) the coil being in a time varying magnetic field
(b) the coil moving in a time varying magnetic field
(c) the coil moving in a constant magnetic field
(d) the coil is stationary in external spatially varying magnetic field, which does not change with time

9. The mutual inductance M_{12} of coil 1 with respect to coil 2

(a) increases when they are brought nearer
(b) depends on the current passing through the coils
(c) increases when one of them is rotated about an axis
(d) is the same as M_{21} of coil 2 with respect to coil 1

10. A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because

(a) the magnetic field is constant
(b) the magnetic field is in the same plane as the circular coil and it may or may not vary
(c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably
(d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction

ANSWERS

1. (c) **2.** (b) **3.** (b) **4.** (d) **5.** (a) **6.** (b) **7.** (a, b, d) **8.** (a, b, c) **9.** (a, d) **10.** (b, c)

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

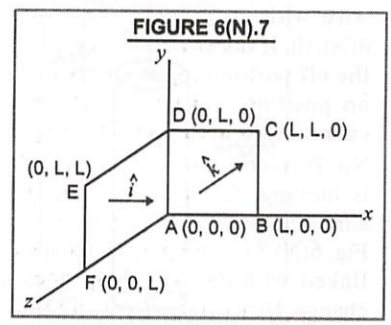
Multiple Choice Questions – I

1. Here, $\vec{A} = L^2 \hat{k}$ and $\vec{B} = B_0 (2\hat{i} + 3\hat{j} + 4\hat{k})$ tesla, $\phi = ?$

Refer to Fig. 6(N).7

As $\phi = \vec{B} . \vec{A} = B_0 (2\hat{i} + 3\hat{j} + 4\hat{k}) . L^2 \hat{k}$

$\therefore$ $\phi = 4 B_0 L^2$ **weber**



2. Here, $\vec{B} = B_0 (\hat{i} + \hat{k})$, Refer to Fig. 6(N).7.

Area vector of $ABCD$ $= L^2 \hat{k}$

Area vector of $DEFA$ $= L^2 \hat{i}$

Total area vector, $\vec{A} = L^2 (\hat{i} + \hat{k})$

Total magnetic flux, $\phi = \vec{B} . \vec{A} = B_0 (\hat{i} + \hat{k}) . L^2 (\hat{i} + \hat{k}) = B_0 L^2 (1 + 1) = 2 B_0 L^2$ **Wb.**

3. As there is no change in magnetic flux associated with the circuit, no current is induced in the circuit. The ammeter A shows no deflection. **Choice (b) is correct.**

4. Coil A must be carrying a constant current in counter clockwise direction. That is why when A moves towards B, current induced in B is in counter clockwise direction, as per Lenz's law. The current in B would stop when A stops moving. **Choice (d) is correct.**
5. When current in coil B (at $t = 0$) is counterclockwise, constant current in A (at $t = 0$) must be clockwise. Therefore, when coil A is made to rotate about a vertical axis, it would induce current in coil B in clockwise direction, as per Lenz's law. **Choice (a) is correct.**
6. The self-inductance L of a solenoid of length l and area of cross section A with fixed number of turns is

$$L = \frac{\mu_0 N^2 A}{l}$$

Obviously, L increases when l decreases and A increases. **Choice (b) is correct.**

Multiple Choice Questions – II

7. When a metal plate is getting heated, it may be due to the passage of direct current ; alternating current or even induced current through the plate. As time varying magnetic field produces induced current in the plate, therefore, **choices (a), (b), (d) are correct.**
8. An e.m.f. is produced in a coil without connecting it to an external voltage source only when amount of magnetic flux linked with the coil is changing with time. **Choices (a) and (b) are correct.** Choice (c) will be correct when the coil is entering or leaving the constant magnetic field partially.
9. The mutual inductance M_{12} of coil 1 with respect to coil 2 increases when they are brought nearer. Further, $M_{12} = M_{21}$. **The choices (a) and (d) are correct.**
10. When a circular coil expands radially in a region of magnetic field, induced e.m.f. developed is

$$e = B \times l \times v = B \times \text{rate of change of area}$$

Here, magnetic field B is in a plane perpendicular to the plane of circular coil. As $e = 0$, magnetic field must be in the plane of circular coil so that its component perpendicular to plane of coil is zero.

Further, if the magnetic field has a component perpendicular to the plane of the coil, whose magnitude is decreasing suitably so that magnetic flux linked with the coil stays constant then $e = \frac{d\phi}{dt} = 0$.

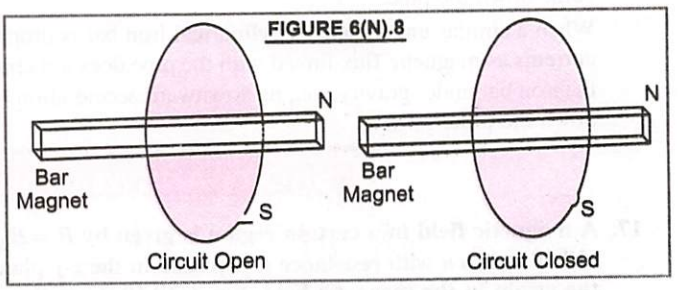
Choices (b) and (c) are correct.

VERY SHORT ANSWER QUESTIONS

11. **Consider a magnet surrounded by a wire with an on/off switch S, Fig. 6(N).10. If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit ? Explain.**

Ans. No. This is because no part of wire loop is moving. So there is no motional e.m.f. Further, as the magnet enclosed, Fig. 6(N).8 is stationary, magnetic flux linked with the wire loop does not change. Hence, no current will flow in the circuit.

FIGURE 6(N).8



12. **A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease ? Explain.**

Ans. The current drawn from d.c. source will increase. This is because when the coil is stretched and air gaps are created between successive elements of the spiral coil, magnetic flux will leak through the gaps. According to Lenz's law, emf induced must resist the decrease in flux. This can be done when more current is drawn from d.c. source.

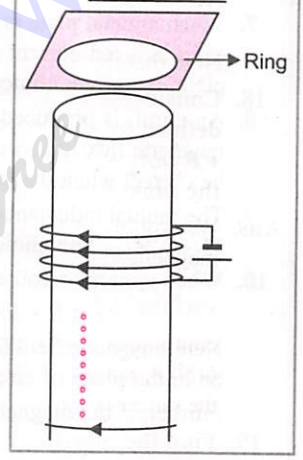
13. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease ? Explain.

Ans. The current will decrease. This is because on inserting an iron core in the solenoid, magnetic field and hence magnetic flux linked with the solenoid increases. As per Lenz's law, the emf induced in the solenoid will oppose this increase, which can be achieved by a decrease in current.

14. Consider a metal ring kept on top of a fixed solenoid (say on a cardboard), Fig. 6(N).9. The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.

Ans. In Fig. 6(N).9, when current is suddenly switched on, magnetic flux (downwards) linked with the solenoid and hence with metal ring increases. Current is induced in the metal ring in counterclockwise direction (as seen from top of the ring). As the flow of current in the ring is in a direction opposite to the current in the solenoid, they will repel each other. Therefore, the ring jumps up.

FIGURE 6(N).9



15. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current *I*, Fig. 6(N).11. The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring ?

Ans. In the same Fig. 6(N).11, when current in the solenoid is suddenly switched off, magnetic flux linked with the solenoid and hence with metal ring decreases. Current induced in the metal ring is clockwise, *i.e.*, in the same direction as the solenoid current. They will attract each other. The metal ring, therefore, will remain on the cardboard. However, reaction of cardboard on the ring will increase.

16. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0·8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

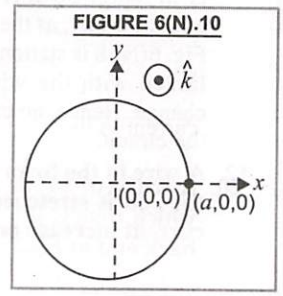
Ans. When a cylindrical bar magnet is dropped through the metallic pipe, amount of magnetic flux linked with the pipe changes. Eddy currents are induced in the pipe, which oppose the free fall of bar magnet. The acceleration of bar magnet will be less than 'g', acceleration due to gravity. Therefore, it takes more time to come down the pipe.

When a similar unmagnetised cylindrical iron bar is dropped through the metallic pipe, there are no eddy currents as magnetic flux linked with the pipe does not change. Hence, there is no opposition to free fall of the iron bar under gravity, *i.e.*, its downward acceleration = g. Hence, the iron bar takes lesser time to come down the pipe.

SHORT ANSWER QUESTIONS

17. A magnetic field in a certain region is given by $B = B_0 \cos(\omega t)\ \hat{k}$ and a coil of radius a with resistance R is placed in the x-y plane with its centre at the origin in the magnetic field, Fig. 6(N).10. Find the magnitude and the direction of the current at $(a, 0, 0)$ at $t = \pi/2\omega$, $t = \pi/\omega$ and $t = 3\pi/2\omega$.

FIGURE 6(N).10



Ans. Here, $\vec{B} = B_0 \cos(\omega t)\ \hat{k}$

i.e., magnetic field is along positive z-axis, Fig. 6(N).10

Magnetic flux linked with the ring

$$\phi = BA \cos 0^\circ = B_0 \cos \omega t\ (\pi a^2)$$

emf induced in the ring, $e = -\frac{d\phi}{dt} = -\frac{d}{dt}[B_0 \pi a^2 . \cos \omega t] = B_0 \pi a^2 \omega . \sin \omega t$

current induced, $I = \frac{e}{R} = \frac{B_0 \pi a^2 \omega . \sin \omega t}{R}$

At $t = \frac{\pi}{2\omega}$, $I = \frac{B_0 \pi a^2 \omega}{R} \sin \frac{\pi}{2} = \frac{B \pi a^2 \omega}{R}$, along $\hat{j}$

At $t = \frac{\pi}{\omega}$, $I = \frac{B_0 \pi a^2 \omega}{R} \sin \pi = \text{zero}$

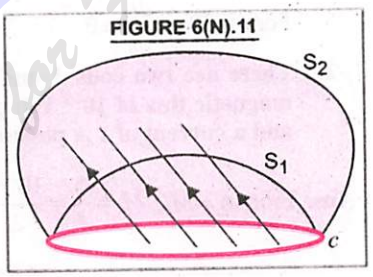
At $t = \frac{3\pi}{2\omega}$, $I = \frac{B_0 \pi a^2 \omega}{R} \sin \frac{3\pi}{2} = -\frac{B_0 \pi a^2 \omega}{R}$, along $-\hat{j}$

Note that the direction of current at any instant is as per Lenz's law.

18. Consider a closed loop C in a magnetic field, Fig. 6(N).11. The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = B_1.dA_1 + B_2.dA_2 + ...$ Now if we chose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.

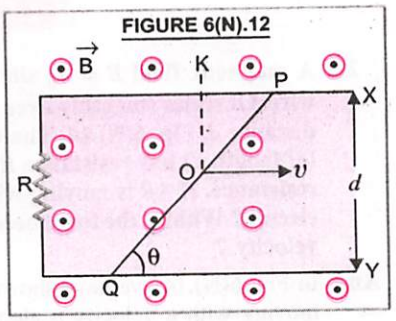
Ans. We would get the same answer for magnetic flux. This is because magnetic flux = number of magnetic field lines passing through the surface $(d\phi = \vec{B}.\Delta\vec{S})$. As magnetic field lines form closed continuous loops and cannot end or start in space, therefore, in Fig. 6(N).8, number of magnetic lines passing through surface S_1 must be the same as the number of magnetic lines passing through the surface S_2.

FIGURE 6(N).11



19. Find the current in the wire PQ for the configuration shown in Fig. 6(N).12. Wire PQ has negligible resistance. B, the magnetic field is coming out of the paper. θ is a fixed angle made by PQ travelling smoothly over two conducting parallel wires seperated by a distance d.

FIGURE 6(N).12



Ans. In Fig. 6(N).12, PQ is a wire of negligible resistance travelling smoothly over two conducting parallel wires X and Y separated by distance (d). θ is fixed angle made by PQ and B is strength of magnetic field perpendicular to the plane of paper and outwards. Motional electric field = $v\,B$.

Its direction is along 0 K, perpendicular to $\vec{v}$ and $\vec{B}$.

∴ e.m.f. induced in wire PQ

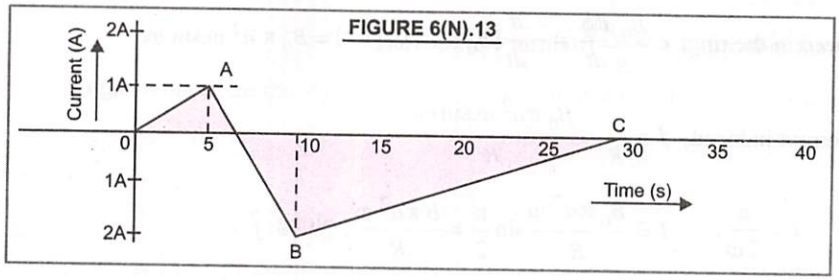
e = length PQ × motional electric field along QP

$$= \left(\frac{d}{\sin\theta}\right) \times v\,B \sin\theta = d\,v\,B$$

current in the wire, $I = \frac{e}{R} = \frac{d\,v\,B}{R}$

20. A (current vs time) graph of the current passing through a solenoid is shown in Fig. 6(N).13. For which time is the back electromotive force (u) a maximum ? If the back emf at $t = 3$ s is e, find the back emf at $t = 7$ s,15 s and 40 s. OA, AB and BC are straight line segments.

FIGURE 6(N).13



Ans. As back e.m.f., $e = L\frac{dI}{dt}$; e will be maximum when dI/dt = max. The graph in Fig. 6(N).13 shows that dI/dt is maximum for **5 s < t < 10 s.**

For 0 s < t < 5 s, $\frac{dI}{dt} = \frac{1}{5}$ ∴ At t = 3 sec ; $e = L\frac{dI}{dt} = \frac{L}{5}$

For 5 s < t < 10 s, $\frac{dI}{dt} = \frac{-2-1}{5} = \frac{-3}{5}$ ∴ At t = 7 s, $e = L\frac{dI}{dt} = L\left(-\frac{3}{5}\right) = -3e$

For 10 s < t < 30 s, $\frac{dI}{dt} = \frac{2}{20} = \frac{1}{10}$ ∴ At t = 15 s, $e = L\frac{dI}{dt} = \frac{L}{10} = \frac{e}{2}$

For t > 30 s, $\frac{dI}{dt} = 0$, ∴ At t = 40 s, $e = 0$

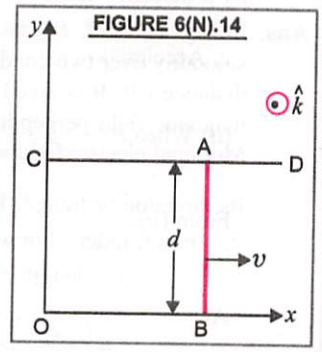
21. There are two coils A and B seperated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A ?

Ans. From $\phi = MI$, $M = \frac{\phi}{I} = \frac{10^{-2}}{2}$ henry. Again, from $\phi = MI = \frac{10^{-2}}{2} \times 1 = 5 \times 10^{-3}$ **Wb.**

LONG ANSWER QUESTIONS

22. A magnetic field $B = B_0 \sin(\omega t)\,\hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d, Fig. 6(N).14. The wires are in the x-y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in the circuit ? What is the force needed to keep the wire moving at constant velocity ?

FIGURE 6(N).14



Ans. In Fig. 6(N).14, we have shown a wire AB of length $l = d$, resistance R moving with a velocity v along positive direction of X-axis over two parallel wires OX and CD at $y = 0$ and $y = d$ respectively.

The magnetic field covering the region is $\vec{B} = B_0 \sin(\omega t)\,\hat{k}$.

At $t = 0$, AB is at $x = 0$ and velocity $= v\,\hat{i}$; At $t = t$, AB is at $x = v\,t$

Motional emf in $AB = (B_0 \sin \omega t)\, d.v\ (-\hat{j})$.

Its direction, as per Fleming's right hand rule is along negative y-axis.

e.m.f. due to change in magnetic field $= -B_0\,\omega \cos \omega t\,(x)\,(d)$

It is also along negative y-axis.

∴ total e.m.f. induced $= -B_0 \sin \omega t.d.v - B_0\,\omega \cos(\omega t)\,x.d = -B_0\,d\,[v \sin \omega t + \omega x \cos \omega t]$

Current induced, $I = \frac{B_0 d}{R}[v \sin \omega t + \omega x \cos \omega t]$

The direction of current is clockwise, *i.e.*, along *ABOC* (absorbing negative sign).

$\therefore$ Force needed along $\hat{i}$; $F = B\,I\,l$

$$F = B_0 \sin \omega t . \frac{B_0 d}{R}[v \sin \omega t + \omega x \cos \omega t] d$$

$$F = \frac{B_0^2 d^2}{R}[v \sin \omega t + \omega x \cos \omega t] \sin \omega t$$

23. A conducting wire *XY* of mass *m* and negligible resistance slides smoothly on two parallel conducting wires as shown in Fig. 6(N).15. The closed circuit has a resistance *R* due to *AC*. *AB* and *CD* are perfect conductors. There is a magnetic field $B = B(t)\,\hat{k}$. (i) Write down equation for the acceleration of the wire *XY*. (ii) If *B* is independent of time, obtain $v(t)$, assuming $v(0) = u_0$. (iii) For (b), show that the decrease in kinetic energy of *XY* equals the heat lost in *R*.

Ans. Fig. 6(N).15 shows a conducting wire *XY* of mass *m* and negligible resistance sliding smoothly over two parallel conducting wires *AB* and *CD*. The magnetic field applied is $\vec{B} = B(t)\hat{k}$

(*i*) At time *t*, suppose *XY* is at $x = x(t)$

Magnetic flux associated with area *ACYX*

$$\phi = B(t).l.x(t)$$

emf induced, $e = -\frac{d\phi}{dt} = -\left[\frac{dB}{dt} l x(t) + B(t).l.v(t)\right]$ ...(*i*)

(due to change in mag. field) (due to motional emf)

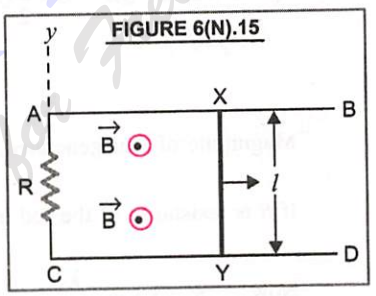


FIGURE 6(N).15

current induced, $i = \frac{e}{R}$

Force on *XY*, $F = Bil = B\left(\frac{e}{R}\right)l$ or $m\frac{d^2x}{dt} = \frac{B(t).l}{R}\left[-\frac{dB}{dt} l x(t) - B(t).l.v(t)\right]$ ...using (*i*)

$\therefore$ Acceleration of wire ; $\frac{d^2x}{dt^2} = -\frac{l^2 B}{mR}\frac{dB}{dt} x(t) - \frac{l^2 B^2}{mR}\left(\frac{dx}{dt}\right)$ ...(*ii*)

(*ii*) When *B* is independent of time, it is constant w.r.t. time. Therefore, $\frac{dB}{dt} = 0$

From (*ii*), $\frac{d^2x}{dt^2} + \frac{l^2 B^2}{mR}\frac{dx}{dt} = 0$ or $\frac{dv}{dt} + \frac{l^2 B^2}{mR} v = 0$. $\therefore$ $v = A \exp\left(-\frac{l^2 B^2 t}{mR}\right)$

At $t = 0, v = u_0 = A$ $\therefore$ $v(t) = u_0 \exp\left(-\frac{l^2 B^2 t}{mR}\right)$ ...(*iii*)

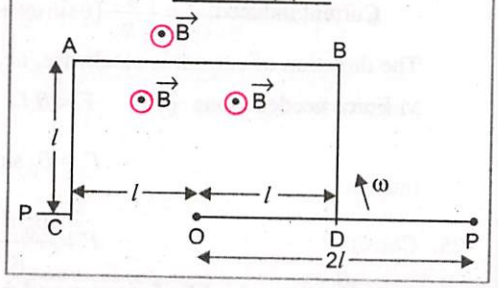
(*iii*) Now, $I^2 R = \frac{B^2 l^2 v^2(t)}{R^2} \times R$ using (*iii*), $I^2 R = \frac{B^2 l^2}{R} u_0^2 \exp\left(-\frac{2l^2 B^2 t}{mR}\right)$

$$\text{Power lost} = \int_0^t I^2 R\, dt = \frac{B^2 l^2}{R} u_0^2 \frac{mR}{2l^2 B^2}[1 - e^{-l^2 B^2 t/mR}] = \frac{m}{2}u^2 - \frac{m}{2}v^2(t)$$

= decrease in KE of wire *XY*, which was to be proved.

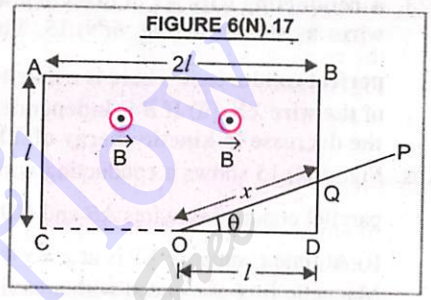
24. *ODBAC* is a fixed rectangular conductor of negligible resistance (CO is not connected) and *OP* is a conductor which rotates clockwise with an angular velocity ω, Fig. 6(N).16. The entire system is in a uniform magnetic field *B* whose direction is along the normal to the surface of the rectangular conductor *ABDC*. The conductor *OP* is in electric contact with *ABDC*. The rotating conductor has a resistance of λ per unit length. Find the current in the rotating conductor, as it rotates by 180°.

FIGURE 6(N).16



Ans. The set up is shown in Fig. 6(N).17. Between time $t = 0$ and $t = \frac{T}{8} = \frac{\pi}{4\omega}$, the rod *OP* will make contact with side *BD*. At any time t: $0 < t < \frac{\pi}{4\omega}$, let the length *OQ* of contact be x.

FIGURE 6(N).17



∴ Magnetic flux through the area *ODQ* is

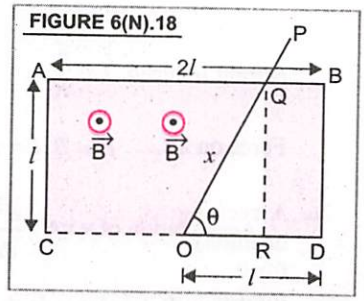
$$\phi = B \times \text{area } OQD = B \times \frac{1}{2} QD \times OD = B \times \frac{1}{2} l \tan\theta \times l$$

$$\phi = \frac{1}{2} Bl^2 \tan\theta, \text{ where } \theta = \omega t.$$

Magnitude of emf generated, $e = \frac{d\phi}{dt} = \frac{1}{2} Bl^2 \omega \sec^2 \omega t$

If *R* is resistance of the rod in contact, then induced current $I = \frac{e}{R}$.

FIGURE 6(N).18



Now, $R = \lambda x = \frac{\lambda l}{\cos\omega t}$ ∴ $I = \frac{1}{2} \frac{Bl^2 \omega \sec^2 \omega t}{\lambda l / \cos\omega t} = \frac{Bl\omega}{2\lambda \cos\omega t}$

For $t = \frac{T}{8} = \frac{\pi}{4\omega}$ to $t = \frac{3T}{8} = \frac{3\pi}{4\omega}$, the rod is in contact with the side *AB*, as shown in Fig. 6(N).18. Let the length of the rod in contact, $OQ = x$.

Magnetic flux linked with area *ODBQ*

$$\phi = B \times \text{area of trapezium } ODBQ = B\left(l^2 - \frac{l^2}{2\tan\theta}\right), \text{ where } \theta = \omega t.$$

Magnitude of emf generated, $e = \frac{d\phi}{dt} = \frac{1}{2} Bl^2 \omega \operatorname{cosec}^2 \omega t = \frac{1}{2} Bl^2 \omega \frac{\sec^2 \omega t}{\tan^2 \omega t}$

Induced current, $I = \frac{e}{R} = \frac{e}{\lambda x} = \frac{e \sin\omega t}{\lambda l} = \frac{1}{2\lambda} \times \frac{Bl\omega}{\sin\omega t}$

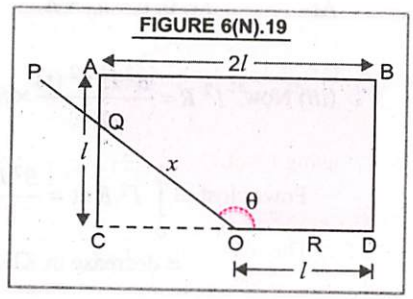
For $t = \frac{3T}{8} = \frac{3\pi}{4\omega}$ to $t = \frac{4T}{8} = \frac{\pi}{\omega}$, the rod is in contact with the side *AC* as shown in Fig. 6(N).19.

Let the length of the rod in contact $OQ = x$.

Magnetic flux linked with area *ODBAQ*

$\phi = B \times$ [area of rectangle *ABDC* – area of ΔOQC]

FIGURE 6(N).19



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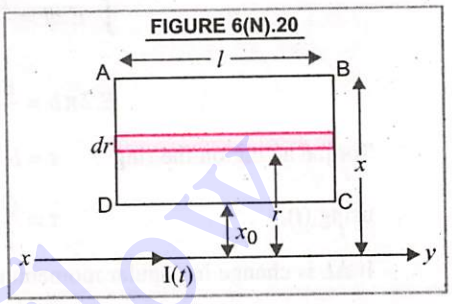
$$\phi = B\left[2l^2 - \frac{l^2}{2\tan \omega t}\right]$$

Magnitude of induced emf, $e = \frac{d\phi}{dt} = \frac{d}{dt}B\left(2l^2 - \frac{l^2}{2\tan \omega t}\right) = \frac{B\omega l^2 \sec^2 \omega t}{2\tan^2 \omega t}$

Induced current, $I = \frac{e}{R} = \frac{e}{\lambda x} = \frac{e \sin \omega t}{\lambda l} = \frac{Bl\omega}{2\lambda \sin \omega t}$.

25. Consider an infinitely long wire carrying a current I (t), with $\frac{dI}{dt} = \lambda$ = constant. Find the current produced in the rectangular loop of wire $ABCD$ if its resistance is R, Fig. 6(N).20.

FIGURE 6(N).20



Ans. In Fig. 6(N).20, XY is infinitely long wire carrying current I (t), such that $\frac{dI}{dt} = \lambda$ = constant.

At any distance r from the wire, strength of magnetic field, $B(r) = \frac{\mu_0 I}{2\pi r}$. It is directed out of the paper.

Total magnetic flux linked with the rectangular loop

$$\phi = \int_{x_0}^{x} B(r).dA = \int_{x_0}^{x} \frac{\mu_0 I}{2\pi r} l\, dr \qquad ...(i)$$

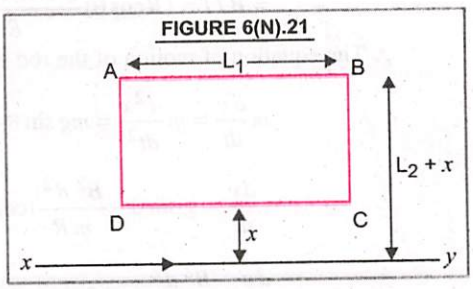
$$\phi = \frac{\mu_0 I l}{2\pi} \log_e \frac{x}{x_0} = LI \text{ , where } L \text{ is self-inductance of the loop}$$

$$L = \frac{\mu_0 l}{2\pi} \log_e \frac{x}{x_0}$$

Induced current, $I = \frac{e}{R} = \frac{L}{R}\frac{dI}{dt} = \frac{L}{R}\lambda = \frac{\mu_0 l \lambda}{2\pi R} \log_e \frac{x}{x_0}$

26. A rectangular loop of wire $ABCD$ is kept close to an infinitely long wire carrying a current $I(t) = I_0(1 - t/T)$ for $0 \le t \le T$ and $I(0) = 0$ for $t > T$, Fig. 6(N).21. Find the total charge passing through a given point in the loop, in time T. The resistance of the loop is R.

FIGURE 6(N).21



Ans. In Fig. 6(N).21, XY is an infinitely long wire carrying current

$I(t) = I_0(1 - t/T)$ for $0 \le t \le T$

If $I(t)$ is current induced in the loop, then

$$I(t) = \frac{d\phi/dt}{R} = \frac{dQ}{dt}$$

Integrating both sides, we get $Q(t_1) - Q(t_2) = \frac{1}{R}[\phi(t_1) - \phi(t_2)]$

using (i) of Q. 25 ; $\phi(t_1) = L_1 \frac{\mu_0}{2\pi} \int_{x}^{L_2+x} \frac{dx'}{x'} I(t_1) = \frac{\mu_0 L_1}{2\pi} I(t_1) \log_e \left(\frac{L_2 + x}{x}\right)$

$\therefore$ The magnitude of charge would be $Q = \frac{\mu_0 L_1}{2\pi} \log_e \left(\frac{L_2 + x}{x}\right)[I_1 - 0] = \frac{\mu_0 L_1 I_1}{2\pi} \log_e \left(\frac{L_2 + x}{x}\right)$

27. A magnetic field B is confined to a region $r \le a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge = Q) of radius b, $b > a$ and mass m lies in the x-y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

Ans. If E is the electric field generated around the charged ring of radius b, then as $e = \dfrac{d\phi}{dt}$

$$\oint E.dl = \frac{B.\pi a^2}{\Delta t}$$

$$E.2\pi b = \frac{B\pi a^2}{\Delta t} \quad \text{or} \quad Eb = \frac{Ba^2}{2(\Delta t)} \qquad ...(i)$$

Torque acting on the ring $\tau = b \times \text{force} = b \times Q \times E$

using (i), $\tau = \dfrac{QBa^2}{2(\Delta t)}$

If ΔL is change in angular momentum of the charged ring, then as

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L_2 - L_1}{\Delta t} \quad \therefore \quad L_2 - L_1 = \tau(\Delta t) = \frac{QBa^2(\Delta t)}{2(\Delta t)} = \frac{QBa^2}{2}$$

As initial angular momentum, $L_1 = 0$ $\therefore$ $L_2 = \dfrac{QBa^2}{2} = I\omega = (mb^2)\omega$

$$\omega = \frac{QBa^2}{2mb^2}$$

This is the angular velocity of the ring after the field vanishes.

28. A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to the horizontal, Fig. 6(N).22. The circuit is closed through a perfect conductor at the top. There is a constant magnetic field B along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.

Ans. The set up is shown in Fig. 6(N).22. Component of weight of rod down the incline = $mg \sin\theta$. Component of magnetic field up the incline = $B\cos\theta$. Therefore, force on the rod due to magnetic field

$$= BIl = (B\cos\theta)\frac{(B\cos\theta.d.v)}{R}d = \frac{B^2d^2(\cos\theta)^2v}{R}$$

$\therefore$ The equation of motion of the rod can be written as

$$m\frac{dv}{dt} = m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{B^2d^2}{R}(\cos\theta)^2v$$

$$\therefore \quad \frac{dv}{dt} = g\sin\theta - \frac{B^2d^2}{mR}(\cos\theta)^2v$$

or $$\frac{dv}{dt} + \frac{B^2d^2}{mR}(\cos\theta)^2v = g\sin\theta$$

$$\therefore \quad v = \frac{g\sin\theta}{B^2d^2\cos^2\theta/mR} + A\exp\left[-\frac{B^2d^2}{mR}(\cos^2\theta)t\right]$$

where A is a constant that can be determined from initial conditions ($A = 1$).

$$v = \frac{mRg\sin\theta}{B^2d^2\cos^2\theta}\left[1 - \exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)\right].$$

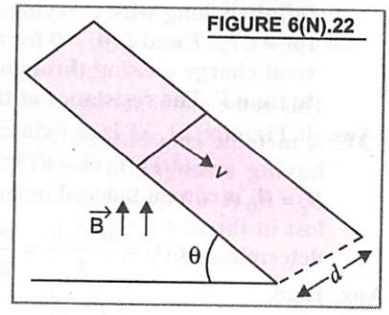
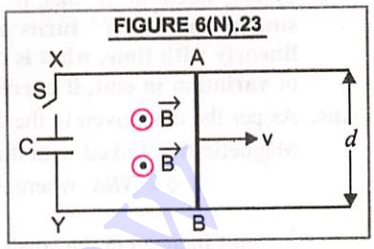


FIGURE 6(N).22

29. Find the current in the sliding rod AB (resistance = R) for the arrangement shown in Fig. 6(N).23. B is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch S is closed at time $t = 0$.

Ans. In Fig. 6(N).23, magnetic field $\vec{B}$ is constant and is out of paper. When the sliding rod AB moves with a velocity v in the direction shown, induced current in AB is from A to B. As switch S is closed at time $t = 0$, the capacitor gets charged. If $Q\,(t)$ is charge on the capacitor, then



$$I = \frac{dQ}{dt} = \frac{B\,d\,v}{R} - \frac{Q}{RC} \quad \text{or} \quad \frac{Q}{RC} + \frac{dQ}{dt} = \frac{B\,d\,v}{R}$$

$$Q = v\,B\,d\,C + A\,e^{-t/RC} \qquad ...(i)$$

where A is an arbitrary constant.

At $t = 0, Q = 0$

$\therefore$ $A = -v\,B\,d\,C$

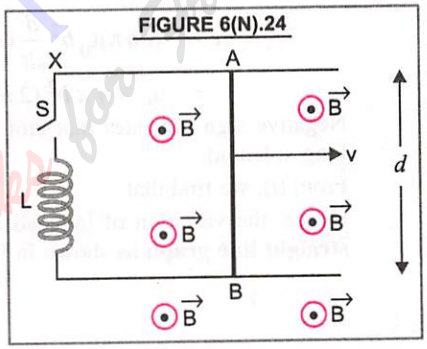
Put in (i), $Q = v\,B\,d\,C\,[1 - e^{-t/RC}]$

Differentiating w.r.t. t, we get

$$I = \frac{dQ}{dt} = v\,B\,d\,C \times \frac{1}{RC} e^{-t/RC} = \frac{v\,B\,d}{R} e^{-t/RC}$$

This is the current in the sliding rod.

30. Find the current in the sliding rod AB (resistance = R) for the arrangement shown in Fig. 6(N).24. B is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch S is closed at time $t = 0$.



Ans. Proceeding as in Q. 29 above,

$$-L\frac{dI}{dt} + B\,v\,d = IR \quad \text{or} \quad L\frac{dI}{dt} + IR = B\,v\,d$$

$$\therefore \quad I = \frac{B\,v\,d}{R} + A\,e^{-Rt/L} \qquad ...(i)$$

where A is an arbitrary constant.

At $t = 0$, $I = 0$ $\quad \therefore$ From (i), $A = -\dfrac{B\,v\,d}{R}$

Putting this value of A in (i), we get

$$I = \frac{B\,v\,d}{R} - \frac{B\,v\,d}{R} e^{-Rt/L} = \frac{B\,v\,d}{R}(1 - e^{-Rt/L})$$

This is the required current in sliding rod AB.

31. A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z-component of magnetic field is $B_z = B_0\,(1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with a velocity v, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m, B, λ and acceleration due to gravity g.

Ans. Here, $B_z = B_0\,(1 + \lambda z)$

As the metallic ring falls under gravity in this magnetic field, magnetic flux linked with the ring changes. Therefore, an emf is induced in the ring. If I is induced current in the ring at any instant, then

$$\text{induced emf} = IR = \frac{d\phi}{dt} = A \times \frac{dB_z}{dt} = (\pi l^2)\,B_0\,\lambda\,\frac{dz}{dt} \quad \therefore \quad I = \frac{\pi l^2\,B_0\,\lambda\,v}{R}, \text{ where } \frac{dz}{dt} = v$$

Energy lost/sec in the form of heat produced in the ring $= I^2\,R = \dfrac{(\pi l^2\,\lambda)^2\,B_0^2\,v^2}{R}$.

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This comes from rate of change of potential energy of ring $= mg\dfrac{dz}{dt} = mg\,v$

(Note that as v = constant, KE of ring is constant)

$$\therefore \quad \frac{(\pi l^2 \lambda)^2 B_0^2 v^2}{R} = mg\,v \quad \text{or} \quad v = \frac{mg\,R}{(\pi l^2 \lambda B_0)^2}.$$

32. A long solenoid 'S' has 'n' turns per meter, with diameter 'a'. At the centre of this coil, we place a smaller coil of 'N' turns and diameter 'b' (where $b < a$). If the current in the solenoid increases linearly with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in emf, if current varies as a function of $mt^2 + C$.

Ans. As per the data given in the question, magnetic field due to current in solenoid S, $B = \mu_0 n I$

Magnetic flux linked with the smaller coil due to this field is

$\phi = NBA$, where A = area of smaller coil $= \pi b^2$

$$\therefore \quad \text{emf induced in the smaller coil, } e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NB\pi b^2) = -N\pi b^2 \frac{d}{dt}(\mu_0 n I)$$

$$= -N\pi b^2 \mu_0 n \frac{dI}{dt}$$

As current I varies as a function of $(mt^2 + C)$, therefore,

$$e = -N n \pi \mu_0 b^2 \frac{d}{dt}(mt^2 + C) = -N n \pi \mu_0 b^2 (2mt)$$

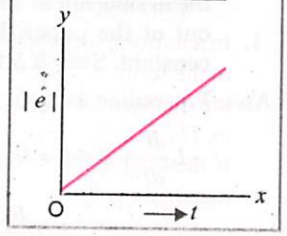
$$= -\mu_0 N n \pi b^2 (2mt) \quad \ldots(i)$$

Negative sign indicates that emf induced opposes the main current in the long solenoid.

From (i), we find that $|e| \propto t$.

Hence, the variation of induced emf $|e|$ with time t is represented by a **straight line graph** as shown in Fig. 6(N).25.

FIGURE 6(N).25



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I. Multiple Choice Questions (with One Correct Answer)

I. Electromagnetic Induction

1. In a uniform magnetic field of induction B, a wire in the form of semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, then the mean power generated per period of rotation is

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$

(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r^2\omega)^2}{8R}$

(AIEEE 2004)

2. A long solenoid of diameter 0·1 m has 2×10^4 turns per meter. At the centre of the solenoid, a coil of 100 turns and radius 0·01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduces at a constant rate to 0A from 4 A in 0·05 s. If the resistance of the coil is $10\ \pi^2\ \Omega$, the total charge flowing through the coil during this time is

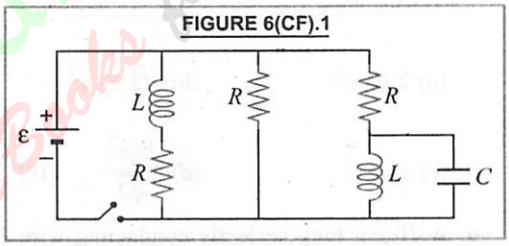
(a) $32\ \pi\ \mu C$ (b) $16\ \mu C$

(c) $32\ \mu C$ (d) $16\ \pi\ \mu C$

(NEET 2017)

3. Fig. 6(CF).1, shows a circuit that contains three identical resistors with resistance $R = 9{\cdot}0\ \Omega$ each, two identical inductors with inductance $L = 2{\cdot}0$ mA each, and an ideal battery with emf $\varepsilon = 18$ V, current 'i' through each resistance just after the switch is closed is

FIGURE 6(CF).1



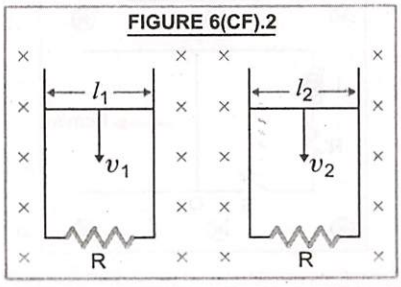
(a) 2 mA (b) 0·2 A

(c) 2 A (d) 0 ampere

(NEET 2017)

4. Two horizontal wires of same mass and lengths l_1 and l_2 are free to slide on different vertical rails with velocities v_1 and v_2 respectively in a magnetic field. [Fig. 6(CF).2]. If a_1 and a_2 are the accelerations of the two horizontal wires respectively, the condition for $a_1 > a_2$ is :

FIGURE 6(CF).2



ANSWERS

1. (d) 2. (c) 3. (c)

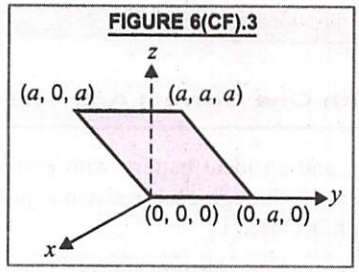
[Take the resistances of the two circuits to be equal]

(a) $\frac{l_1}{l_2} = \frac{v_2}{v_1}$ (b) $\frac{l_1}{l_2} = \left(\frac{v_2}{v_1}\right)^{1/2}$

(c) $\frac{l_1}{l_2} > \left(\frac{v_1}{v_2}\right)^{1/2}$ (d) $\frac{l_1}{l_2} > \left(\frac{v_1}{v_2}\right)$

5. Consider an electric field $\vec{E} = E_0 \hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the Fig. 6(CF).3 due to this field is

FIGURE 6(CF).3

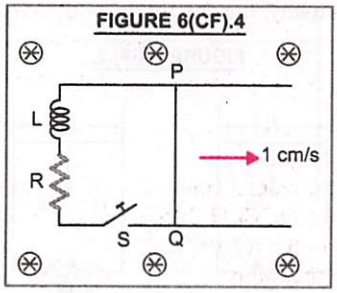


(a) $2 E_0 a^2$ (b) $\sqrt{2} E_0 a^2$

(c) $E_0 a^2$ (d) $\frac{E_0 a^2}{\sqrt{2}}$ **(IIT 2011)**

6. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor $L = 1$ mH and a resistance $R = 1$ ohm as shown in Fig. 6(CF).4. The horizontal rails lie in the same plane with a uniform magnetic field $B = 1$ T perpendicular to the plane. If the key S is closed at certain instant,

FIGURE 6(CF).4



current in the circuit after 1 millisec is $x \times 10^{-3}$ A. What is the value of x ? Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given $e^{-1} = 0.37$, where e is base of the natural log.

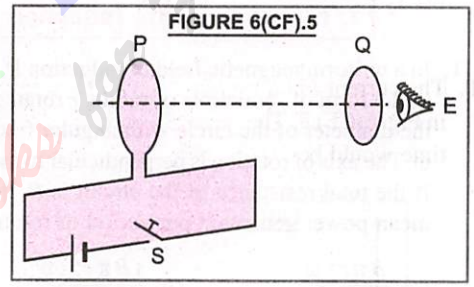
(a) 0·63 (b) 0·4

(c) 0·5 (d) 0·6

(JEE Advanced 2019)

7. As shown in Fig. 6(CF).5, P and Q are two co-axial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E), and an induced current I_{Q_1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q_2} flows in Q. Then the directions of I_{Q_1} and I_{Q_2} (as seen by E) are

FIGURE 6(CF).5



(a) respectively clockwise and anticlockwise

(b) both clockwise

(c) both anticlockwise

(d) respectively anticlockwise and clockwise

8. A short circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved keeping area of coil fixed, the electrical power dissipated would be

(a) halved (b) the same

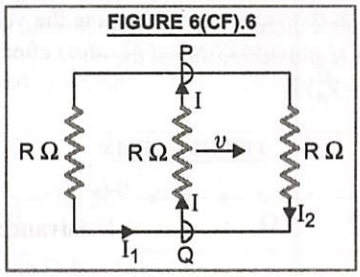
(c) doubled (d) quadrupled

9. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown in Fig. 6(CF).6. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are

ANSWERS

4. (b) **5.** (c) **6.** (a) **7.** (d) **8.** (d)

FIGURE 6(CF).6



(*a*) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

(*b*) $I_1 = I_2 = I = \frac{Blv}{R}$

(*c*) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{Blv}{3R}$

(*d*) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$

(AIEEE 2010)

10. The current *i* in a coil varies with time as shown in Fig. 6(CF).7. The variation of induced emf with time would be

FIGURE 6(CF).7

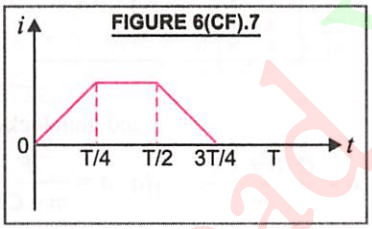
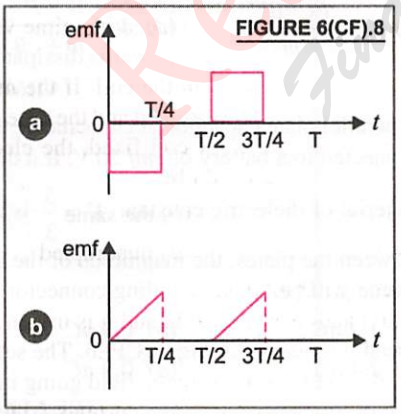
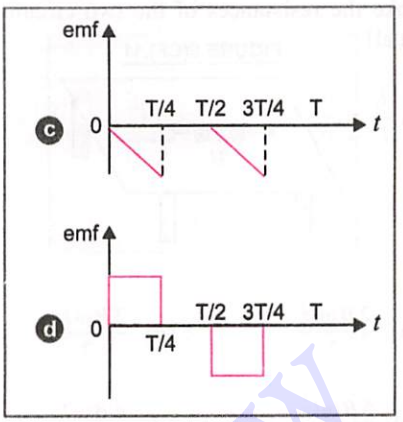


FIGURE 6(CF).8

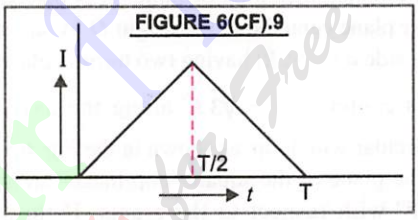




[AIPMT (Prelim) 2011]

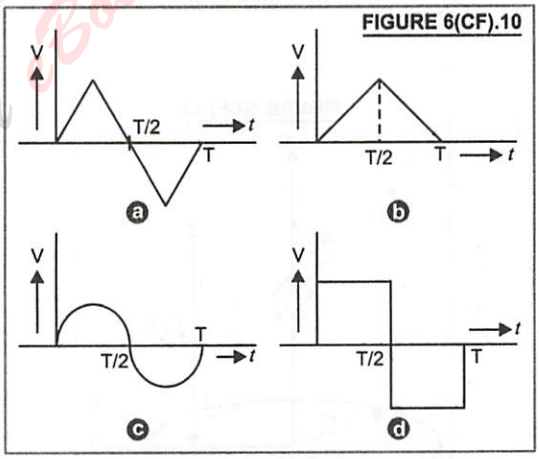
11. The current (I) in the inductance is varying with time according to the plot shown in Fig. 6(CF).9.

FIGURE 6(CF).9



Which one of the following is the correct variation of voltage with time in the coil ?

FIGURE 6(CF).10



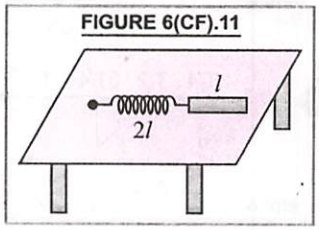
[AIPMT (Prelim) 2012]

12. A metallic rod of length *l* is tied to a string of length 2 *l* and made to rotate with angular speed ω on a horizontal table with one end of the string fixed, Fig. 6(CF).11. If there is a vertical magnetic field *B* in the region, the *emf* induced across the ends of the rod is

ANSWERS

9. (*a*) 10. (*a*) 11. (*d*)

FIGURE 6(CF).11



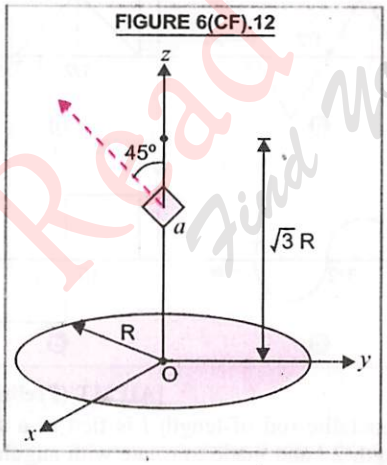
(a) $\frac{2B\omega l^2}{2}$ (b) $\frac{3B\omega L^2}{2}$

(c) $\frac{5B\omega l^2}{2}$ (d) $\frac{4B\omega L^2}{2}$

(JEE Main 2013)

13. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in the Fig. 6(CF).12. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is

FIGURE 6(CF).12

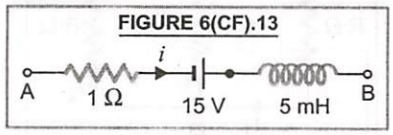


(a) 2 (b) 4

(c) 7 (d) 9 **(IIT 2012)**

14. The network shown in Fig. 6(CF).13 is a part of a complete circuit. If at a certain instant, the current i is 5 A and decreasing at the rate of 10^3 A/s, then $(V_B - V_A)$ is

FIGURE 6(CF).13

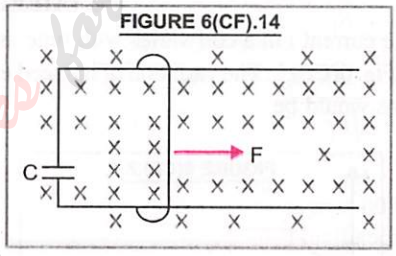


(a) 5 V (b) 10 V

(c) 15 V (d) 20 V

15. A wire having mass m and length l can freely slide on a pair of parallel smooth horizontal rails placed in vertical magnetic field B. The rails are connected by a capacitor of capacitance C. The electric resistance of the rails and the wire is zero. If a constant force F acts on the wire as shown in the Fig. 6(CF).14. Then, the acceleration of the wire can be given as

FIGURE 6(CF).14



(a) $a = \frac{C^2B^2l - F}{m}$ (b) $a = \frac{F}{m + CBl}$

(c) $a = \frac{FC^2B^2l}{m}$ (d) $a = \frac{F}{m + cB^2l^2}$

(AIIMS 2015)

16. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(a) 1.2 nC (b) 0.3 nC

(c) 2.4 nC (d) 0.9 nC

(JEE Main 2018)

ANSWERS

12. (c) 13. (c) 14. (c) 15. (d) 16. (a)

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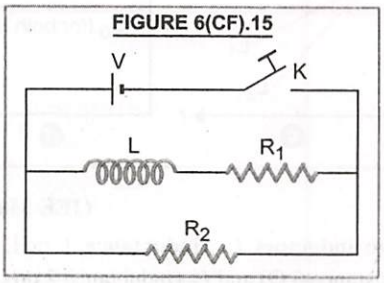
17. In the circuit shown in Fig. 6(CF).15, the key K is closed at $t = 0$. The current through the battery is

FIGURE 6(CF).15



(*a*) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(*b*) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(*c*) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(*d*) $\frac{V R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(AIEEE 2010)

18. A 800 turn coil of effective area 0·05 m^2 is kept perpendicular to a magnetic field 5×10^{-5} T. When the plane of the coil is rotated by 90° around any of its co-planar axis in 0·1 s, the *e.m.f.* induced in the coil will be

(*a*) 2 V (*b*) 0·2 V

(*c*) 2×10^{-3} V (*d*) 0·02 V

(NEET 2019)

19. A wheel with 20 metallic spokes each 1 m long is rotated with a speed of 120 rpm in a plane perpendicular to magnetic field of 0·4 G. The induced e.m.f. between the axle and rim of the wheel will be (1 G = 10^{-4} T)

(*a*) 2·51 V (*b*) 2.51×10^{-4} V

(*c*) 2.51×10^{-5} V (*d*) 4.0×10^{-5} V

(NEET 2020)

20. The magnetic flux linked with a coil (in wb.) is given by the equation $\phi = 5t^2 + 3t + 16$.

The magnitude of induced e.m.f in the coil at the fourth second will be

(*a*) 10 V (*b*) 33 V

(*c*) 43 V (*d*) 108 V

(NEET 2020)

21. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_1 >> R_2$, the mutual inductance M between them will be directly proportional to

(*a*) $\frac{R_2^2}{R_1}$ (*b*) $\frac{R_1}{R_2}$

(*c*) $\frac{R_2}{R_1}$ (*d*) $\frac{R_1^2}{R_2}$ **(NEET 2021)**

22. Consider a circular coil of wire carrying constant current I forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of circular coil is given by ϕ_0. Which of the following options is correct ?

(*a*) $\phi_i = -\phi_0$ (*b*) $\phi_i = \phi_0$

(*c*) $\phi_i < \phi_0$ (*d*) $\phi_i > \phi_0$

(JEE Main 2020)

23. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3×10^{-5} T, with its plane perpendicular to the field intensity. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0·2 s. The maximum value of emf induced (in μV) in the coil will be close to the integer.

(*a*) 25 μV (*b*) 15 μV

(*c*) 10 μV (*d*) 20 μV

(JEE Main 2020)

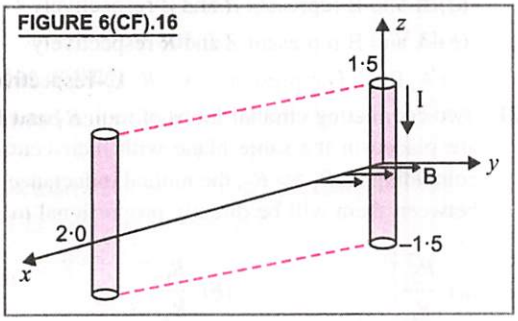
II. Electrical devices

24. A conductor lies along the z-axis at $-1{\cdot}5 \le z < 1{\cdot}5$ m and carries a fixed current of 10·0 A in $-\hat{a}_z$ direction, Fig. 6(CF).16. For a field $\vec{B} = 3{\cdot}0 \times 10^{-4} e^{-0{\cdot}2x} \hat{a}_y$ T, find the power required to move the conductor at constant speed to x = 2·0 m, y = 0 m in 5×10^{-3} s. Assume parallel motion along the x-axis.

ANSWERS

17. (*a*) 18. (*d*) 19. (*b*) 20. (*c*) 21. (*a*) 22. (*a*) 23. (*b*)

FIGURE 6(CF).16

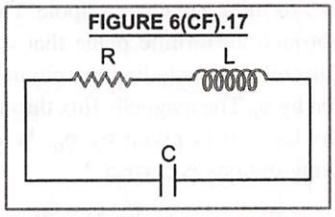


(a) 14·85 W (b) 29·7 W
(c) 1·57 W (d) 2·97 W

(JEE Main 2014)

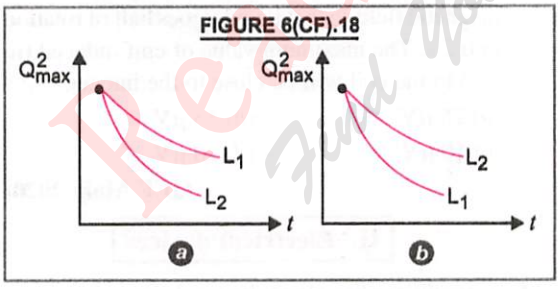
25. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit, the capacitor is

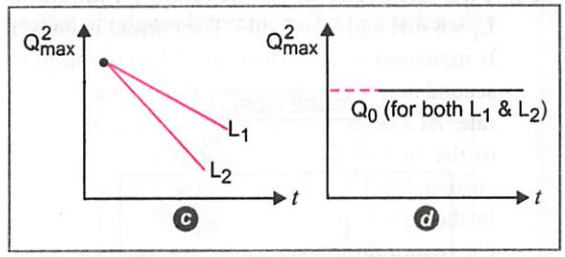
FIGURE 6(CF).17



charged to Q_0 and then connected to the L and R as shown in Fig. 6(CF).17. If a student plots graphs of the square of maximum charge (Q^2_{max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (Plots are schematic and not drawn to scale)

FIGURE 6(CF).18





(JEE Main 2015)

26. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω) and a resistor R of 12 Ω are all connected in parallel across a 5 V battery. The circuit is switched on at $t = 0$. The ratio of maximum to minimum current drawn from the battery is

(a) 8
(b) 7
(c) 6
(b) 5

(JEE Advance 2016)

27. In which of the following devices, the eddy current effect is not used ?

(a) Induction furnace
(b) Magnetic braking in train
(c) Electromagnet
(d) Electric heater

(NEET 2019)

28. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0·25 A to zero in a duration of 0·025 ms. The self inductance of the choke in mH is

(a) 20 (b) 10
(c) 25 (d) 30

(JEE Main 2020)

II. Multiple Choice Questions (with One or More than One Correct Answers)

29. L, C, R represent the physical quantities ; inductance, capacitance and reactance respectively. Which of the following combinations have dimensions of frequency ?

(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$
(c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

ANSWERS

24. (d) 25. (a) 26. (a) 27. (d) 28. (b) 29. (a,b,c)

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30. Two different coils have self-inductance $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then :

(*a*) $\frac{i_1}{i_2} = \frac{1}{4}$ (*b*) $\frac{i_1}{i_2} = 4$

(*c*) $\frac{W_1}{W_2} = \frac{1}{4}$ (*d*) $\frac{V_1}{V_2} = 4$

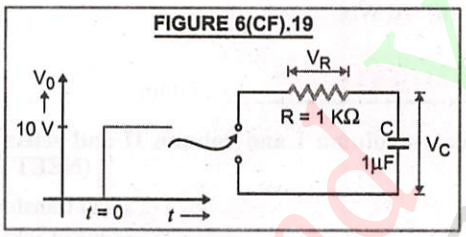
31. The SI unit of inductance, the henry, can be written as :

(*a*) Weber/Ampere (*b*) Volt-Second/ampere

(*c*) Joule/(ampere)^2 (*d*) ohm-second

32. A step voltage V_0 is applied to a series combination of R and C as shown in Fig. 6(CF).19. Then

FIGURE 6(CF).19



(*a*) after sufficiently long time $V_R = 0$

(*b*) as time passes, V_R decreases as $(1/t)$

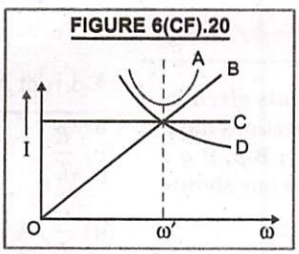
(*c*) after 1 ms, $V_C = 6{\cdot}3$ volt (approximately)

(*d*) initially, current through R is 10 mA.

(National Standard Exam in Physics 2010)

33. For an LCR circuit,

FIGURE 6(CF).20



(*a*) A and B represent R and Z respectively

(*b*) A and B represent Z and R respectively

(*c*) A, B, C, D represent Z, X_L, R, X_C respectively

(*d*) for $\omega = \omega'$, the phase difference between current and voltage is zero

(National Standard Exam in Physics 2011)

34. A current carrying infinitely long wire is kept along the diameter of circular wire loop, without touching it. The correct statement (s) is (are)

(*a*) The emf induced in the loop is zero if the current is constant

(*b*) The emf induced in the loop is finite if the current is constant

(*c*) The emf induced in the loop is zero if the current decreases at a steady rate

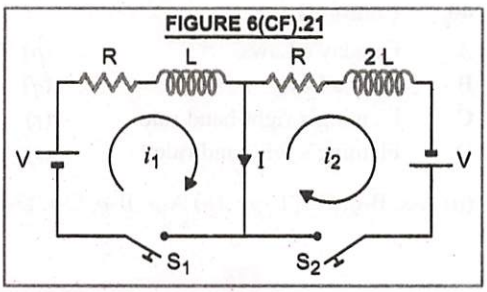
(*d*) The emf induced in the loop is finite if the current decreases at a steady rate

(IIT JEE 2012)

35. In Fig. 6(CF).21, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of e.m.f. and polarities are as indicated in the figure.

Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statements are true ?

FIGURE 6(CF).21



(*a*) $I_{max} = \frac{V}{2R}$ (*b*) $i_{max} = \frac{V}{4R}$

(*c*) $\tau = \frac{L}{R} \ln 2$ (*d*) $\tau = \frac{2L}{R} \ln 2$

(JEE Advanced 2018)

ANSWERS

30. (*a,c,d*) **31.** (*a,b,c,d*) **32.** (*a,c,d*) **33.** (*c,d*) **34.** (*a,c*) **35.** (*b,d*)

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension **Whenever the amount of magnetic flux linked with a circuit changes, an *e.m.f.* is induced in the circuit. The magnitude of *e.m.f.* induced is equal to the rate of change of magnetic flux. The induced *e.m.f.* lasts so long as change in magnetic flux continues. The direction of *e.m.f.* induced is given by Lenz's Law. For a uniform magnetic field $\vec{B}$ crossing the plane of area $\vec{A}$ at an angle θ with the normal to the plane, magnetic flux ϕ is given by**

$$\phi = \vec{B} . \vec{A} = BA \cos \theta$$

36. I_s magnetic flux a scalar or a vector ?

(*a*) scalar (*b*) vector

(*c*) cannot say

37. When plane of area is along the magnetic field, magnetic flux linked with it is

(*a*) positive

(*b*) negative

(*c*) zero

(*d*) infinite

38. The units of magnetic flux are

(*a*) ampere

(*b*) tesla

(*c*) weber

(*d*) tesla/ampere

39. A square coil of area 10^{-2} m^2 is held perpendicular to a uniform magnetic field of 10^3 Wb/m^2. The magnetic flux through the coil is

(*a*) 100 wb

(*b*) 50 Wb

(*c*) Zero

(*d*) 10 Wb

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices

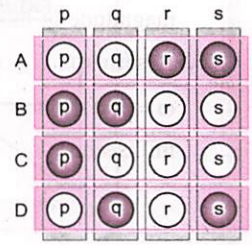
40.

	Column I		Column II
A.	Faraday's Laws	(*p*)	Direction of force on current carrying conductor in mag. field.
B.	Lenz's Law	(*q*)	Obeys principle of energy conservation
C.	Fleming's right hand rule	(*r*)	Direction of induced current
D.	Fleming's left hand rule	(*s*)	EMI

(*a*) A-*s*, B-*q*, C-*r*, D-*p* (*b*) A-*p*, B-*q*, C-*r*, D-*s* (*c*) A-*s*, B-*r*, C-*q*, D-*p* (*d*) A-*q*, B-*r*, C-*p*, D-*s*

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*r*, A-*s* ; B-*p*, B-*q* ; C-*p* ; D-*q*, D-*s*, then the correctly bubbled matrix will look like the one shown here :



ANSWERS

36. (*a*) 37. (*c*) 38. (*c*) 39. (*d*) 40. (*a*)

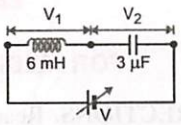
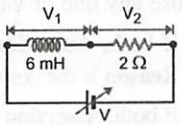
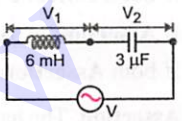
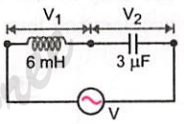
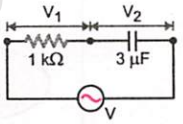
41.

Column I		Column II
A.	Resistance	*p.* can reduce a.c.
B.	Inductance	*q.* average power/cycle = 0
C.	Capacitor	*r.* phase diff. between alt. current and alt. e.m.f. = 0°
D.	Transformer	*s.* phase diff. between alt. current and alt. e.m.f. = 90°

42.

Column I		Column II
A.	Choke coil	*p.* can reduce both d.c. and a.c.
B.	Condenser	*q.* cannot reduce d.c.
C.	Inductor	*r.* power loss ≠ 0
D.	Resistance	*s.* power loss = 0

43. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or *rms* for AC) flows through the circuit, the corresponding voltage V_1 and V_2, (indicated in circuits) are related as shown in Column I. Match the two.

Column I	Column II
(A) $I \neq 0$, V_1 is proportional to I	(*p*) 
(B) $I \neq 0$, $V_2 > V_1$	(*q*) 
(C) $V_1 = 0$, $V_2 = V$	(*r*) 
(D) $I \neq 0$, V_2 is proportional to I	(*s*) 
	(*t*) 

(IIT 2010)

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

44. The sum and difference of self inductances of two coils are 13 mH and 5 mH respectively. What is the maximum value of mutual inductance (in milli henry) of the two coils ?

45. The magnetic flux of ϕ (in weber) in a closed circuit of resistance 3 ohm varies with time t (in second) according to the equation $\phi = 2t^2 - 10t + 3$. What will be the magnitude of induced current at $t = 0{\cdot}25$ s ?

46. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **(IIT 2011)**

ANSWERS

41. A-*p*, *r* ; B-*p*, *q*, *s* ; C-*q*, *s* ; D-*p* **42.** A-*q*, *s* ; B-*p*, *s* ; C-*q*, *s* ; D-*p*, *r*
43. A-*r*, *s*, *t* ; B-*q*, *r*, *s*, *t* ; C-*p*, *q* ; D-*q*, *r*, *s*, *t* **44.** (6) **45.** (3) **46.** (4)

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VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. Read the following questions and choose any one of the following four responses.

A. If both, **Assertion** and **Reason** are true and the **Reason** is the correct explanation of the **Assertion.**

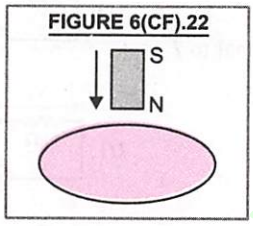
B. If both, **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

C. If **Assertion** is true but the **Reason** is false.

D. If both **Assertion** and **Reason** are false.

47. Assertion. The bar magnet falling vertically along the axis of the horizontal coil will be having acceleration less than g.

FIGURE 6(CF).22



Reason. Clockwise current induced in the coil.

(AIIMS 2015)

48. Assertion. Only a change in magnetic flux will maintain an induced current in the coil.

Reason. The presence of large magnetic flux through a coil maintains a current in the coil if the circuit is continuous.

(*a*) A (*b*) B (*c*) C (*d*) D

49. Assertion. The magnetic flux through a closed surface containing a bar magnet is zero.

Reason. Gauss's law applies in the case of electric flux only.

(*a*) A (*b*) B (*c*) C (*d*) D

50. Assertion. A glowing bulb becomes dim when an iron bar is put in the inductor in the a.c. circuit.

Reason. Resistance of the circuit increases.

(*a*) A (*b*) B (*c*) C (*d*) D

51. Assertion. Use is made of eddy currents in induction brakes.

Reason. As eddy currents always oppose the relative motion.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. Read the following two statements. Of the four choices given, choose the one that best describes the two statements :

A. Statement-1 is true ; Statement-2 is true and Statement-2 is correct explanation of Statement-1.

B. Statement-1 is true ; Statement-2 is true, but Statement-2 is not a correct explanation of Statement-1.

C. Statement-1 is true ; Statement-2 is false.

D. Statement-1 is false ; Statement-2 is true.

52. Statement-1. When a magnet is made to fall freely through a closed coil, its acceleration is always less than acceleration due to gravity.

Statement-2. Current induced in the coil opposes the motion of the magnet, as per Lenz's law.

(*a*) A (*b*) B (*c*) C (*d*) D

53. Statement-1. When we consider mutual induction of two coils, their self induction is taken as zero.

Statement-2. This is because mutual induction of two coils is over and above self induction of each coil.

(*a*) A (*b*) B (*c*) C (*d*) D

54. Statement-1. In a given situation, the direction of induced current given by Lenz's Law and Fleming's right hand rule is the same.

Statement-2. Statement of both the laws is the same.

(*a*) A (*b*) B (*c*) C (*d*) D

ANSWERS

47. (*c*) **48.** (*c*) **49.** (*c*) **50.** (*a*) **51.** (*a*) **52.** (*a*) **53.** (*d*) **54.** (*c*)

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For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. As $\phi = BA\cos\theta = B\left(\frac{\pi r^2}{2}\right)\cos\omega t$

and $e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{B\pi r^2\cos\omega t}{2}\right)$

$= -\frac{B\pi r^2\omega\sin\omega t}{2}$

Also, $P = \frac{e^2}{R} = \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$

But $<\sin^2\omega t> = \frac{1}{2}$

$\therefore \quad \boldsymbol{P = \frac{(B\pi r^2\omega)^2}{8R}}$

2. Here, $r_1 = \frac{1}{20}$ m, $N_1 = 2\times10^4$, $l = 1$ m

$r_2 = \frac{1}{100}$ m, $N_2 = 100$, $\frac{dI}{dt} = \frac{4-0}{0{\cdot}05} = 80$ A/s

$R = 10\,\pi^2$

$e = M\frac{dI}{dt} = \frac{\mu_0 N_1 N_2 A_2}{l}\times\frac{dI}{dt}$

$= 4\pi\times10^{-7}(2\times10^4)\,10^2\,\pi\left(\frac{1}{100}\right)^2\times80$

$= 640\,\pi^2\times10^{-5}$

$Q = I\times dt = \frac{e}{R}\times dt = \frac{640\pi^2\times10^{-5}}{10\pi^2}\times0{\cdot}05$

$= 32\times10^{-6}$ C $= \mathbf{32}\ \mu$C

3. As the supply is from *d.c.* battery,

$X_L = \omega L = 2\pi n L = 0$, and

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C} = \infty$

$\therefore$ C draws no current, and both inductors L have no role. Three resistances of 9 Ω each are in parallel.

$\frac{1}{R_p} = \frac{1}{R}+\frac{1}{R}+\frac{1}{R} = \frac{1}{9}+\frac{1}{9}+\frac{1}{9} = \frac{3}{9}$

$R_p = 9/3\ \Omega = 3\ \Omega.$

Total current from the battery $= \frac{E}{R_p} = \frac{18}{3} = 6\,A$.

Its divides equally among three resistances.

Current through each resistance $= \frac{6}{3}$ A $= \mathbf{2\,A}$

4. Since $F = \frac{B^2 l^2 v}{R}$,

so $\frac{B^2 l_1^2 v_1}{R} - mg = ma_1$ ...(*i*)

and $\frac{B^2 l_2^2 v_2}{R} - mg = ma_2$ ...(*ii*)

Subtracting (*ii*) from (*i*)

$ma_1 - ma_2 = \frac{B^2}{R}(l_1^2 v_1 - l_2^2 v_2)$

$a_1 - a_2 = \frac{B^2}{mR}(l_1^2 v_1 - l_2^2 v_2)$,

for $a_1 > a_2$

$l_1^2 v_1 - l_2^2 v_2 > 0$ or $\frac{l_1}{l_2} = \left(\frac{v_2}{v_1}\right)^{1/2}$

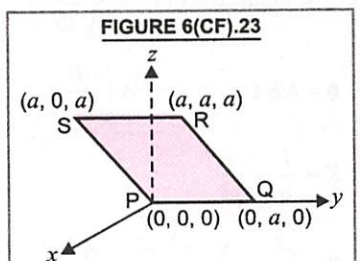
5. Here, $\vec{E} = E_0\,\hat{x}$

Let $\hat{x}$, $\hat{y}$ and $\hat{z}$ be the unit vectors along x, y and z axis respectively.

In Fig. 6(CF).23, the shaded area,

$\vec{A} = \overrightarrow{PQ}\times\overrightarrow{PS}$

FIGURE 6(CF).23



$\therefore \quad \vec{A} = (0\hat{x} + a\hat{y} + 0\hat{z})\times(a\hat{x} + 0\hat{y} + a\hat{z})$

$= (a^2\hat{x} - a^2\hat{z})$

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Electric flux through the shaded area,

$$\phi = \vec{E}.\vec{A} = (E_0 \hat{x}) \cdot (a^2 \hat{x} - a^2 \hat{z}) = E_0 a^2$$

6. As velocity of PQ is constant, *e.m.f.* developed across. It is constant $E = B\,l\,v$

Current at any time t is

$$i = \frac{E}{R}(1 - e^{-\frac{R}{L}t}) = \frac{Blv}{R}(1 - e^{-\frac{R}{L}t})$$

$$= 1 \times \left(\frac{10}{100}\right) \times \left(\frac{1}{100}\right) \times \frac{1}{1}\left(1 - e^{\frac{-1\times10^{-3}}{1\times10^{-3}}}\right)$$

$$= \frac{1}{1000}(1 - e^{-1})$$

$$i = \frac{1}{1000}(1 - 0{\cdot}37)$$

$$i = \frac{1}{1000} \times 0{\cdot}63 = 0{\cdot}63 \times 10^{-3}\ \text{A}$$

Hence $x = \mathbf{0{\cdot}63}$

7. When switch S is closed, Fig. 6(CF).5, current in coil P increases in clockwise direction. Induced current in Q must oppose this increase, as per Lenz's law. Therefore, induced current in Q *i.e.* I_{Q1} must be **anticlockwise.**

When switch S is opened, current in coil P decreases. Induced current in Q opposes this decrease, as per Lenz's law. Therefore, induced current in Q, *i.e.*, I_{Q2} must be **clockwise.**

8. Power dissipated, $P = \frac{e^2}{R}$

where e = induced e.m.f. $= -\left(\frac{d\phi}{dt}\right)$

and $\phi = NBA \therefore e = -NA\left(\frac{dB}{dt}\right)$

Also $R \propto \frac{l}{a} \propto \frac{l}{r^2}$

$$\therefore \quad P = \frac{e^2}{R} \propto \frac{N^2 A^2 . r^2}{l} \propto \frac{N^2 r^2}{l}$$

As r is halved, and N is quadrupled.

$\therefore$ P is quadrupled.

9. Induced emf produced in the arm PQ is $e = Blv$. It means there is a source of emf e $(= Blv)$ in arm QP with resistance R in the arm and other two resistances each of value R in parallel to it.

Therefore, effective resistance

$$R' = \frac{R \times R}{R + R} + R = \frac{3R}{2}$$

Current, $$I = \frac{e}{R'} = \frac{Blv}{3R/2} = \frac{\mathbf{2}Blv}{\mathbf{3}R}$$

In Fig. 6(CF).6, $I_1 = I_2 = \frac{I}{2} = \frac{Blv}{3R}$

10. Induced emf is $e = -L\frac{di}{dt}$

In Fig. 6(CF).7,

During 0 to $\frac{T}{4}$, $\frac{di}{dt}$ = a constant ; as i–t graph is a straight line with positive constant slope. Therefore, e is negative and constant.

For time, $\frac{T}{4} \le t \le \frac{T}{2}$; $\frac{di}{dt} = 0$, as i–t graph is a straight line parallel to t-axis. Therefore, $e = 0$.

For time, $\frac{T}{2} \le t \le \frac{3T}{4}$, $\frac{di}{dt}$ a negative constant, as i–t graph is a straight line with negative slope. Therefore, e is +ve and constant.

For time $\frac{3T}{4} \le t \le T$, $\frac{di}{dt} = 0$, therefore, $e = 0$.

Hence, the variation of induced emf (e) with time (t) will be as shown in **option (*a*) of Fig. 6(CF).8.**

11. As is clear from Fig. 6(CF).9,

current increases uniformly from $t = 0$ to $t = T/2$ and decreases uniformly at the same rate from $t = T/2$ to $t = T$

$\therefore$ $\frac{dI}{dt}$ is positive constant in the first half $(0 \to T/2)$ and $\frac{dI}{dt}$ is negative constant of same value in the second half $(T/2 \to T)$.

As $V = L\frac{dI}{dt}$ = positive constant from $t = 0$ to $t = T/2$ and $V = L\frac{dI}{dt}$ = negative constant from $t = T/2$ to $t = T$

Choice (*d*) is correct in Fig. 6(CF).10.

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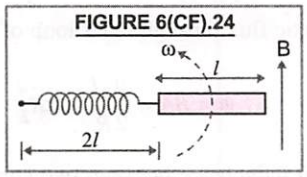
12. As shown in Fig. 6(CF).24,

$$e = \frac{d\phi}{dt} = \frac{B\,dA}{dt} = \frac{B\pi[(3l)^2 - (2l)^2]}{1/n} = B\pi n \times 5\,l^2$$

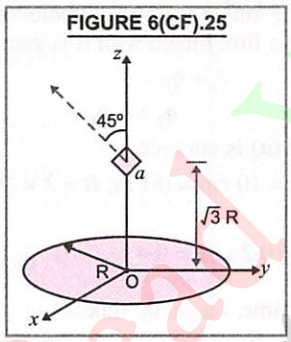
$$e = B(2\pi n) \times \frac{5}{2} l^2 = \frac{5}{2} B\omega l^2$$

FIGURE 6(CF).24



13. In Fig. 6(CF).25, we have shown a circular wire loop of radius R centred at the origin O. A square loop of side $a << R$ having two turns is held at $z = \sqrt{3}R$ along the axis of circular loop. Plane of square loop makes an angle of 45° w.r.t. z-axis. Due to current I in the circular loop, magnetic field at the centre of square loop is

FIGURE 6(CF).25



$$B = \frac{\mu_0 IR^2}{2[R^2 + (\sqrt{3}R)^2]^{3/2}} = \frac{\mu_0 IR^2}{2 \times 4R^2 \times 2R}$$

$$= \frac{\mu_0 I}{16R}$$

Magnetic flux linked with the square loop

$$\phi = 2\,BA\cos 45° = 2\left(\frac{\mu_0 I}{16R}\right)(a^2)\frac{1}{\sqrt{2}}$$

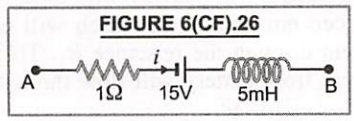
$$\phi = \frac{\mu_0 I a^2}{(2)^{7/2} R}$$

As $M = \frac{\phi}{I} = \frac{\mu_0 a^2}{(2)^{7/2} R} = \frac{\mu_0 a^2}{2^{p/2} R}$ $\therefore$ $p = 7$

14. Here, $i = 5$ A

$$\frac{di}{dt} = -10^3 \text{ A/s}$$ (minus sign for decrease)

FIGURE 6(CF).26



Applying Kirchhoff's 2nd Law to the given circuit in Fig. 6(CF).26, we get

$$V_A - iR + 15 - L\frac{di}{dt} = V_B$$

$$V_A - 5 \times 1 + 15 - (5 \times 10^{-3})(-10^3) = V_B$$

$$V_A + 15 = V_B$$

or $V_B - V_A = \mathbf{15\ volt}$

15. Let at time t, v be the velocity of wire.

EMF induced, $\varepsilon = B\,l\,v$

Charge (in magnitude) appearing on the plates of capacitor is

$$q = C\,\varepsilon = CB\,l\,v$$

Now current, $I = \frac{dq}{dt} = CB\,l\frac{dv}{dt} = CB\,la$

$$\left[\because \frac{dv}{dt} = a, i.e., \text{ acceleration}\right]$$

Force on wire due to the current is,

$$F' = IlB = (CB\,la)\,lB = CB^2 l^2 a$$

The net force in the direction of motion of conductor is $F - F' = ma$ or $F = ma + CB^2 l^2 a$

or $a = \frac{F}{m + CB^2 l^2}$

16. Here, $C_0 = 90$ pF $= 90 \times 10^{-12}$ F ;

$V = 20$ V, $K = 5/3$,

Capacitance of capacitor with dielectric

$$C = K\,C_0$$

Charge on capacitor, $Q = CV = K\,C_0\,V$

When dielectric is inserted inbetween the plates of capacitor then induced charge,

$$Q_{\text{induced}} = Q\left(1 - \frac{1}{K}\right) = K\,C_0\,V\left(1 - \frac{1}{K}\right)$$

$$= \frac{5}{3} \times 90 \times 10^{-12} \times 20\left(1 - \frac{3}{5}\right)$$

$$= 1{\cdot}2 \times 10^{-9}\text{ C} = \mathbf{1{\cdot}2\ nC}$$

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17. When key K is closed at $t = 0$, the current through inductance L will increase with time, resulting in induced emf across L, which will oppose the current through the reistance R_1. Therefore, the current from battery will flow through R_2. This current is $I = \dfrac{V}{R_2}$ **at $t = 0$**

When $t = \infty$, the current will reach to steady state in the circuit. The presence of L will become ineffective. Now effective resistance of circuit

$$= \frac{R_1 R_2}{R_1 + R_2}.$$

Current through battery **at $t = \infty$,**

$$I' = \frac{V}{R_1 R_2/(R_1 + R_2)} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

18. Here, $n = 800$, $A = 0{\cdot}05\ \text{m}^2$, $\theta_1 = 0°$, $B = 5 \times 10^{-5}$ T, $\theta_2 = 90°$, $dt = 0{\cdot}1$ s, $e = ?$

$$e = \frac{-d\phi}{dt} = -\frac{nBA(\cos\theta_2 - \cos\theta_1)}{dt}$$

$$= -\frac{800 \times 5 \times 10^{-5} \times 0{\cdot}05(\cos 90° - \cos 0°)}{0{\cdot}1}$$

$$e = \frac{-(-)2 \times 10^{-3}}{0{\cdot}1} = \mathbf{0{\cdot}02\ V}$$

19. Here, $n = 20$, $l = 1$ m, $n = \dfrac{120}{60} = 2$ rps

$$B = 0{\cdot}4\ \text{G} = 0{\cdot}4 \times 10^{-4}\ \text{T}$$

$$e = \frac{1}{2}Bl^2\omega = \frac{1}{2}(0{\cdot}4 \times 10^{-4}) \times 1^2 \times 2\pi(2)$$

$$= 0{\cdot}8\,\pi \times 10^{-4}\ \text{volt} = \mathbf{2{\cdot}51 \times 10^{-4}\ V}$$

20. Here, $\phi = 5t^2 + 3t + 16$

$$|e| = \frac{d}{dt}(\phi) = \frac{d}{dt}(5t^2 + 3t + 16) = 10t + 3$$

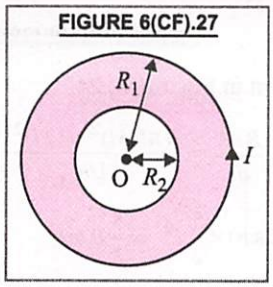
At $t = 4$ sec., $e = 10 \times 4 + 3 = \mathbf{43\ V}$

21. Let I be the current in the outer loop, Fig. 6(CF).27.

Magnetic field induction at the centre O is

$$B = \frac{\mu_0 I}{2R_1}$$

FIGURE 6(CF).27



Magnetic flux through inner loop of area πR_2^2 is

$$\phi = BA = \frac{\mu_0 I}{2R_1} \times \pi R_2^2$$

Mutual inductance between two current loops

$$M = \frac{\phi}{I} = \frac{\mu_0 \pi R_2^2}{2R_1}$$

Clearly, $M \propto \dfrac{R_2^2}{R_1}$

22. As the circular coil of wire carrying constant current forms a magnetic dipole, therefore, total magnetic flux linked with it is zero, *i.e.*,

$$\phi_i + \phi_0 = 0$$

$$\phi_i = -\phi_0$$

Choice (*a*) is correct.

23. Here, $r = 10$ cm $= 0{\cdot}1$ m, $B = 3 \times 10^{-5}$ T

$$\frac{T}{2} = 0{\cdot}2\ \text{s},\ T = 0{\cdot}4\ \text{s}$$

At any time, magnetic flux

$$\phi = BA\cos\omega t$$

$$e = \left(\frac{d\phi}{dt}\right) = BA\omega\sin\omega t$$

$$e_{max} = BA\omega = BA\left(\frac{2\pi}{T}\right)$$

$$= \frac{3 \times 10^{-5} \times \pi(0{\cdot}1)^2 \times 2\pi}{0{\cdot}4}$$

$$e_{max} = \frac{6\pi^2}{4} \times 10^{-6} = 15 \times 10^{-6}\ \text{V} = \mathbf{15\ \mu V}$$

24. Here, $l = 1{\cdot}5 + 1{\cdot}5 = 3{\cdot}0$ m ; $I = 10{\cdot}0$ A,

$$\text{Average Power} = \frac{\text{work}}{\text{time}}$$

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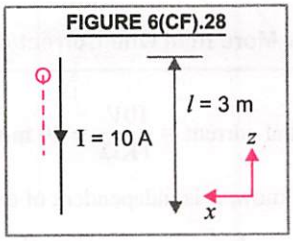
FIGURE 6(CF).28



$$W = \int_0^2 F\,dx = \int_0^2 BIl\,dx$$

$$= \int_0^2 3{\cdot}0\times10^{-4}\,e^{-0{\cdot}2x}\times10\times3\,dx$$

$$= 9\times10^{-3}\times\int_0^2 e^{-0{\cdot}2x}\,dx$$

$$= 9\times10^{-3}\left[\frac{e^{-0{\cdot}2x}}{-0{\cdot}2}\right]_0^2 = 9\times10^{-3}\left[\frac{e^{-0{\cdot}2\times2}-1}{-0{\cdot}2}\right]$$

$$= \frac{9\times10^{-3}}{0{\cdot}2}\times[1-e^{-0{\cdot}4}] = \frac{9\times10^{-3}}{0{\cdot}2}\times0{\cdot}33$$

$$W = \frac{2{\cdot}97\times10^{-3}}{0{\cdot}2}\text{ J}$$

$$P = \frac{W}{t} = \frac{2{\cdot}97\times10^{-3}}{(0{\cdot}2)\times5\times10^{-3}} = \mathbf{2{\cdot}97\ W}$$

25. Let at an instant of time t, during discharging, q be the charge on capacitor and I be the current in the circuit and $\frac{dI}{dt}$ be the rate of change of current.

The emf equation of the circuit is

$$\frac{q}{C} = IR + L\frac{dI}{dt} = \frac{dq}{dt}R + L\frac{d}{dt}\left(\frac{dq}{dt}\right)$$

or $$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} - \frac{q}{LC} = 0 \qquad ...(i)$$

For damped harmonic oscillator, the general equation of damped oscillation is

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} - \frac{k}{m}x = 0 \qquad ...(ii)$$

The amplitude of damped oscillation,

$$A = A_0\,e^{-\frac{bt}{2m}}$$

Comparing equations (*i*) and (*ii*), we note that the equation (*i*) represents damped oscillatory discharge of capacitor in the circuit whose amplitude of oscillation,

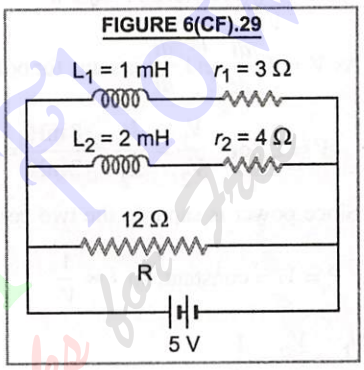
$$Q_{max}(t) = Q_0\,e^{-\frac{R}{2L}t}$$

or $$Q_{max}^2 = Q_0^2\,e^{-\frac{Rt}{L}}$$

It shows that the lesser self-inductance L, faster will be the damping force. Thus, **option (*a*) is true.**

26. Refer to Fig. 6(CF).29,

FIGURE 6(CF).29



At $t = 0$, $I_{min} = \frac{E}{R} = \frac{5}{12}$ A

Finally, in the steady state

$$I_{max} = \frac{E}{R_{eq.}} = E\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{R}\right)$$

$$= 5\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{12}\right) = \frac{10}{3}\text{ A}$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \mathbf{8}$$

27. Electric heater does not involve eddy currents. It is based on Joule heating effect.

28. Here, $V = 100$ volt, $dI = 0{\cdot}25 - 0 = 0{\cdot}25$ A

$dt = 0{\cdot}025$ ms $= 0{\cdot}025\times10^{-3}$ s

If L is self inductance of the choke coil, then

from $V = L\frac{dI}{dt}$

$$L = \frac{V}{dI/dt} = \frac{100}{0{\cdot}25/0{\cdot}025\times10^{-3}}$$

$$= \frac{100\times0{\cdot}025\times10^{-3}}{0{\cdot}25} = 10^{-2}\text{ H}$$

$$L = 10^3\times10^{-2}\text{ mH} = \mathbf{10\ mH}$$

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II. Multiple Choice Questions (with One or More than One Correct Answers)

29. (*a*) $\frac{1}{RC} = \frac{1}{R(Q/V)} = \frac{V/R}{Q} = \frac{I}{Q} = \frac{I}{It} = s^{-1}$

(*b*) $\frac{R}{L} = \frac{\Omega}{\Omega s} = s^{-1}$

(*c*) $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(L/R)(RC)}} = \frac{1}{\sqrt{s \cdot s}} = s^{-1}$

30. (*d*) As $V = L\frac{di}{dt}$ and $\frac{di}{dt}$ is same for both the coils

$$V \propto L \quad \text{or} \quad \frac{V_1}{V_2} = \frac{L_1}{L_2} = \frac{8\,\text{mH}}{2\,\text{mH}} = 4$$

(*a*) Since power is same in the two coils,

So $P = Vi$ = constant, so $i \propto \frac{1}{V}$

$$\frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

(*c*) $W = \frac{1}{2}Li^2$,

$$\frac{W_1}{W_2} = \left(\frac{L_1}{L_2}\right)\left(\frac{i_1}{i_2}\right)^2 = 4\left(\frac{1}{4}\right)^2 = \frac{1}{4}$$

31. (*a*) As $L = \frac{\phi}{I}$ $\therefore$ 1 H = 1 Wb/A

(*b*) As $L = \frac{e}{dI/dt}$; $1H = \frac{1V}{1A/s} - 1V\text{-}s/A$

(*c*) As $1H = \frac{1V}{1A/s} = \frac{1J/1C}{1A/s} = \frac{1J/1As}{1A/s} = 1J/A^2$

(*d*) $1H = \frac{1V}{1A/s} = 1\Omega s$

32. (*a*) After sufficiently long time, since the charging current drops to zero, **voltage drop across resistance is zero.**

(*c*) Time constant = RC = 1 KΩ × 1 μF = 1 ms, So, according to the definition of time constant, voltage across capacitor would be about 63% of maximum voltage, *i.e.*, **6·3 V after 1 ms.**

(*d*) Initial current $= \frac{10V}{1K\Omega} = \mathbf{10\ mA}$

33. As we know, R is independent of ω, $X_L \propto \omega$,

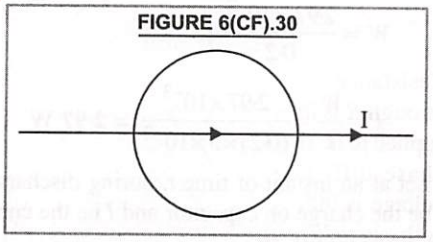
$$X_C \propto \frac{1}{\omega}$$

So **A, B, C, D represent** Z, X_L, R, X_C **respectively.**

Now Z is minimum when $\omega = \omega'$, for which **current and voltage are in phase.**

34. When an infinitely long wire carrying current is kept along the diameter of a circular wire loop, without touching it, as shown in Fig. 6(CF).30, the amount of magnetic flux entering the loop is equal to amount of magnetic flux leaving the loop, *i.e.*, net magnetic flux through the loop is always zero, whether the current is constant or current is decreasing at a steady rate. Hence e.m.f. induced in the loop is zero in either case. **Choices (*a*) and (*c*) are correct.**

FIGURE 6(CF).30



35. If i_1 and i_2 are the currents in the middle wire due to two batteries, as shown in Fig. 6(CF).34, then

$i_{max} = (i_2 - i_1)_{max}$

or $\Delta i = (i_2 - i_1)$

$$= \frac{V}{R}\left[1 - e^{-\left(\frac{R}{2L}\right)t}\right] - \frac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

$$= \frac{V}{R}\left[e^{-\frac{-R}{L}t} - e^{\frac{-R}{2L}t}\right]$$

For $(\Delta i)_{max}$, $\frac{d}{dt}(\Delta i) = 0$

$$\frac{V}{R}\left[-\frac{R}{L}e^{-\frac{R}{L}t} - \left(\frac{-R}{2L}\right)e^{-\frac{R}{2L}t}\right] = 0$$

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or $e^{\frac{-R}{L}t} = \frac{1}{2} e^{\frac{-R}{2L}t}$

or $e^{\frac{R}{2L}t} = \frac{1}{2}$

or $\frac{R}{2L} t = \ln 2$

$t = \frac{2L}{R} \ln 2$

This is the time when net current in the middle wire is maximum.

$\therefore \quad i_{max} = \frac{V}{R}\left[e^{\frac{-R}{L}\left(\frac{2L}{R}\ln 2\right)} - e^{\frac{-R}{2L}\left(\frac{2L}{R}\ln 2\right)}\right]$

$i_{max} = \frac{V}{R}\left[\frac{1}{4} - \frac{1}{2}\right] = \frac{V}{4R}$

Choices (*b*) and (*d*) are correct.

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

36. Magnetic flux is scalar.

37. Here, $\theta = 90°$, $\phi = BA \cos 90° =$ Zero.

38. **weber** is the SI unit of magnetic flux.

39. $\phi = BA \cos \theta = 10^3 \times 10^{-2} \cos 0^0 = 10$ weber

IV. Matching Type Questions

40. Faraday's laws are of EMI (electromagnetic induction). Lenz's law obeys the law of energy conservation. Fleming's right hand rule gives us the direction of induced current, and Fleming's left hand rule gives us the direction of force on current carrying conductor in magnetic field. **Choice (*a*) is correct.**

V. Matrix-Match Type Questions

41. Resistance can reduce a.c. Alternating current through R is in phase with the alternating voltage applied to it. Through inductor and capacitor both, phase diff. between alternating current and voltage is 90°. Average power/cycle is zero and both can reduce a.c. A transformer can reduce a.c. by stepping up the alternating voltage.

42. A choke coil cannot reduce d.c. However, power loss in a choke coil is zero. A condenser can reduce both d.c. and a.c. without power loss. An inductor cannot reduce d.c. However, power loss in an inductor is zero. A resistance can reduce both, d.c. and a.c., but power loss $\neq 0$.

43. **In circuit (*p*),** when steady state is reached, the capacitor is fully charged current $I = 0$ and $\frac{dI}{dt} = 0$. Then $V_1 = 0$ and $V_2 = V$ ($\because X_L = \omega L = 0$)

In circuit (*q*), when steady state is reached I is constant, $\frac{dI}{dt} = 0$; so $V_1 = -L\frac{dI}{dt} = 0$ and $V_2 = IR = V$.

In circuit (*r*),

$X_L = 2\pi \nu L = (2\pi \times 50 \times 6 \times 10^{-3}) = 1.884\ \Omega$

$V_1 = IX_L$ and $V_2 = IR = (I \times 2)$ volt.

As, $R > X_L$, so $V_2 > V_1$

In circuit (*s*), $X_L = 1.884\ \Omega$

$$X_C = \frac{1}{2\pi \nu C} = \frac{1}{2\pi \times 50 \times (3 \times 10^{-6})} = \frac{1}{3\pi} \times 10^4 = 1.06 \times 10^3\ \Omega$$

$V_1 = IX_L = I \times 1.884$ volt.

$V_2 = IX_C = I \times 1.06 \times 10^3$ volt.

$\therefore \quad V_2 > V_1$

In circuit (*t*),

$R = 1\ \text{k}\Omega = 10^3\ \Omega$;

$X_C = 1.06 \times 10^3\ \Omega$

$V_1 = IR = I \times 10^3$ volt ;

$V_2 = IX_C = I \times 1.06 \times 10^3$ volt

$\therefore \quad V_2 > V_1$

Thus options are **(*a*) → (*r, s, t*) ; (*b*) → (*q, r, s, t*) (*c*) → (*p, q*) ; (*d*) → (*q, r, s, t*)**

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VI. Integer Type Questions

44. Here, $L_1 + L_2 = 13$ mH,

$L_1 - L_2 = 5$ mH ; $M = ?$

Adding, we get $2L_1 = 18$, $L_1 = 9$ mH

$\therefore\ L_2 = 13 - L_1 = 13 - 9 = 4$ mH

Max. value of mutual inductance of two coils,

$$M = \sqrt{L_1 L_2} = \sqrt{9 \times 4} = \mathbf{6\,mH}$$

45. Here, $R = 3\ \Omega$, $\phi = 2t^2 - 10t + 3$, $I = ?$, $t = 0{\cdot}25$ s

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(2t^2 - 10t + 3)$$

$$e = -4t + 10$$

At $t = 0{\cdot}25$ s, $e = -4 \times 0{\cdot}25 + 10 = 9$ volt

$$I = \frac{e}{R} = \frac{9}{3} = \mathbf{3\,A}$$

46. Impedance of R – C circuit is

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\therefore\ R\sqrt{1{\cdot}25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\text{or}\quad R^2 \times \frac{5}{4} = R^2 + \left(\frac{1}{\omega C}\right)^2$$

$$\text{or}\quad \frac{R^2}{4} = \left(\frac{1}{\omega C}\right)^2 \quad \text{or} \quad \frac{R}{2} = \frac{1}{\omega C}$$

$$\text{or}\quad RC = \frac{2}{\omega} = \frac{2}{500} = 4 \times 10^{-3}\ \text{s}$$

Time constant of RC circuit,

$$\tau = RC = 4 \times 10^{-3}\ \text{s} = \mathbf{4\,ms}$$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

47. Here, **Assertion is true but Reason is false** because when magnet is falling towards the ring, the induced current is set up in the ring due to change in magnetic flux linked with it, which will oppose the magnetic flux linked with ring. It will be so if the direction of induced current is **anticlockwise** in the ring. This will set up an induced north polarity on the face of ring towards N pole of falling magnet. A force of repulsion will act on falling magnet. Due to it, the magnet falls with acceleration less than g.

Choice (*c*) is correct.

48. Induced current is due to *emf* induced which is directly proportional to the rate of change of magnetic flux linked with the coil.

Mere presence of magnetic flux does not maintain current in the coil.

Assertion is true, but reason is false.

49. **Assertion is true, but the reason is false** as Gauss's law applies to both in the case of electric flux and magnetic flux.

50. On introducing an iron bar in the inductor, inductive reactance $X_L = \omega L$ increases. Therefore, impedance of the circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$; increases, current through the bulb decreases and it becomes dim.

Thus, **both Assertion and Reason are true and Reason is the correct explanation of Assertion.**

51. **Both, the assertion and reason are true and latter is correct explanation of the former.**

FOR ENGINEERING STUDENTS

52. **Both the statements are true and statement-2 is correct explanation of statement-1.**

53. **Statement-1 is false, mutual induction** of two coils is in addition to their self induction. **Statement-2 is however true.**

54. **Statement-1 is true,** but the **statement-2 is false.** The two laws have different statements, though the result of their application is the same.

7

ALTERNATING CURRENTS

7.1. INTRODUCTION : THE ALTERNATING CURRENT

So far we have studied d.c. circuits. The primary source of e.m.f. in such circuits is a battery and the basic circuit element is an ohmic resistance (R) which controls the current (I) as per Ohm's law

$$V = IR \qquad \text{...(1)}$$

But most of the electric power generated and used in the world is in the form of a.c., *i.e.*, **alternating current.** This is because

(i) alternating voltages can be easily and efficiently converted from one value to the other by means of transformers.

(ii) The alternating current energy can be transmitted and distributed over long distances economically without much loss of energy.

***Alternating current** is the current whose magnitude changes continuously with time and whose direction is reversed periodically.* It is represented by

$$I = I_0 \sin \omega t$$

or

$$I = I_0 \cos \omega t$$

Here, I is **instantaneous value** of current, *i.e.*, magnitude of current at any instant of time t and I_0 is the **peak value** or **maximum value** of a.c. It is also called **amplitude** of a.c., ω is called **angular frequency** of a.c.

Also, $$\omega = \frac{2\pi}{T} = 2\pi\nu$$

where T is the **time period** or **period** of a.c. *It is equal to the time taken by the a.c. to go through one complete cycle of variation (zero to maximum, maximum to zero ; zero to maximum in opposite direction and finally max. to zero).* Again, ν is the **frequency** of a.c. *It is equal to the number of complete cycles of variation gone through by the a.c. in one second.*

FIGURE 7.1

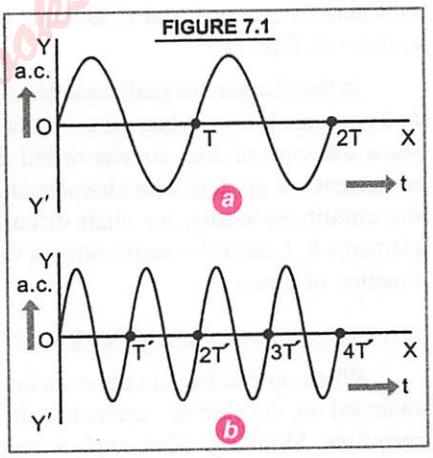
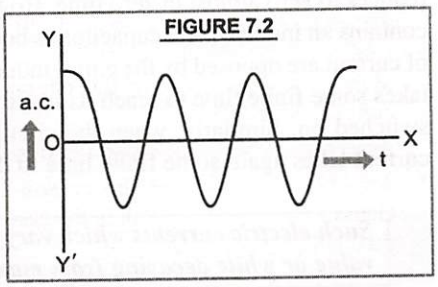


FIGURE 7.2



In Fig. 7.1(a) and Fig. 7. 1(b), we have shown **a.c. as sine function of t.** The time periods of the two wave forms are indicated as T and T', and their frequencies are ν and ν' respectively. From these figures, we find

$$\nu' = 2\nu \qquad \text{and} \qquad T = 2T'$$

The terms used for alternating current hold equally for alternating e.m.f. which may be represented by

$$E = E_0 \sin \omega t \quad \text{or} \quad E = E_0 \cos \omega t$$

Fig. 7.2 represents a.c. as **cosine function of time t.**

In a.c. circuits, two additional circuit elements are used. These are inductor (L) and capacitor (C). Thus current and voltage in a.c. circuits are controlled by three circuit elements R, L and C.

The voltage V across a pure inductor L is given by

$$V = L\frac{dI}{dt} \qquad ...(2)$$

where dI/dt is rate of change of current.

For an ideal capacitor of capacitance C, $Q = CV$.

As $$I = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} \qquad ...(3)$$

where dV/dt is rate of change of potential.

Equation (2) shows that an inductor affects the voltage only when current I changes with time. Eqn. (3) shows that a capacitor affects the current only when V changes with time, *i.e.*, V and I must be time depedent in case of L and C. But this is not the requirement in case of resistor R, Eqn. (1).

In this chapter, we shall consider two kinds of time varying voltages and currents. Firstly, when a d.c. circuit is switched on and off, we shall show that current does not rise or fall instantly. The growth and decay of current are gradual with characteristic time, called *time constant* of the circuit. Secondly, we shall discuss the behaviour of the circuit elements R, L and C towards voltages varying as sine function or cosine function of time t.

7.2. TRANSIENT CURRENTS

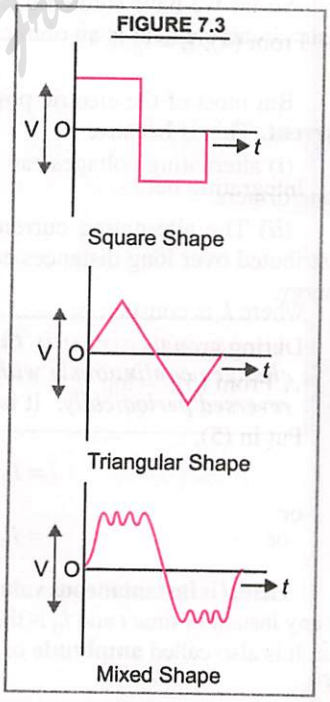
When an electric circuit containing an ohmic resistance only is switched on, the electric current acquires its maximum value almost in zero time. Similarly, when such a circuit is switched off, the current reduces to zero almost in zero time. However, when the electric circuit contains an inductor or a capacitor or both, the growth as well as decay of current are opposed by the e.m.f. induced. Therefore, electric current takes some finite time to reach its maximum value, when the circuit is switched on. Similarly, when the circuit is switched off, the electric current takes again some finite time to decay to zero value.

Such electric currents which vary for a small finite time, while growing from zero to maximum value or while decaying from maximum value to zero value are called transient currents.

DO YOU KNOW ?

1. Alternating current or voltage or e.m.f. can be of many forms. Some of them are shown in Fig. 7.3.

FIGURE 7.3



2. Polarity is never marked in the symbol of A.C. source as the same keeps on reversing itself.

7.3. GROWTH AND DECAY OF CURRENT IN AN INDUCTOR

(a) Growth of current.

Consider an ohmic resistance R and a coil of inductance L connected to a battery E through a morse key K as shown in Fig. 7.4.

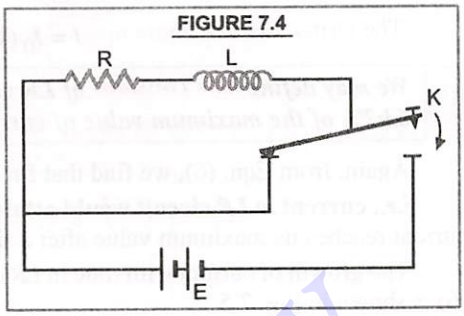


FIGURE 7.4

On pressing K, the battery is connected. Current grows in the R–L circuit. Due to self induction, an induced e.m.f. is set up across L. By Lenz's law, this induced e.m.f. opposes the growth of current.

If I is strength of current at any instant t and dI/dt is rate of growth of current at this instant, then

Potential difference across $R = IR$

Potential difference across $L = L\dfrac{dI}{dt}$

As E is the e.m.f. of the battery, $\therefore\ E = IR + L\dfrac{dI}{dt}$ ...(4)

When current reaches its maximum value, *i.e.*, $I = I_0$; $\dfrac{dI}{dt} = 0$

From (4), $E = I_0 R$. Using this value in (4), we get, $I_0 R = IR + L\dfrac{dI}{dt}$

$$R(I_0 - I) = L\frac{dI}{dt} \quad \text{or} \quad \frac{dI}{I_0 - I} = \frac{R}{L}dt$$

Integrating both sides, we get, $\int \dfrac{dI}{I_0 - I} = \int \dfrac{R}{L}dt$

$$-\log_e (I_0 - I) = \frac{R}{L}t + k \qquad \text{...(5)}$$

where k is constant of integration.

During growth, $I = 0$, at $t = 0$

$\therefore$ From (5), $-\log_e I_0 = 0 + k$ or $k = -\log_e I_0$

Put in (5), $-\log_e (I_0 - I) = \dfrac{R}{L}t - \log_e I_0$ or $\log_e (I_0 - I) - \log_e I_0 = -\dfrac{R}{L}t$

or $\log_e \dfrac{I_0 - I}{I_0} = -\dfrac{R}{L}t$ or $\dfrac{I_0 - I}{I_0} = e^{-\frac{R}{L}t}$

$1 - \dfrac{I}{I_0} = e^{-\frac{R}{L}t}$ or $\dfrac{I}{I_0} = 1 - e^{-\frac{R}{L}t}$

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) = I_0\left(1 - e^{-t/\tau}\right) \qquad \text{...(6)}$$

where $\tau = L/R$

This is **Helmholtz equation for growth of current in *LR* circuit.**

It shows that **growth of current in an inductor is exponential.**

Time constant. The quantity $\tau = L/R$ is called **time constant or inductive time constant** of LR circuit. This is because dimensions of $\tau = L/R$ are those of time* and for a given LR circuit, its value is constant.

*From $e = L\,dI/dt$, $L = \dfrac{e\,dt}{dI}$. As $e = IR$, $R = e/I$ $\therefore$ $\dfrac{L}{R} = \dfrac{e\,dt\,I}{dI\,.\,e} \to dt$

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At $t = L/R$, from (6), $I = I_0(1 - e^{-1}) = I_0\left(1 - \frac{1}{e}\right) = I_0\left(1 - \frac{1}{2{\cdot}718}\right)$

$I = I_0(1 - 0{\cdot}368) = 0{\cdot}632\, I_0 = 63{\cdot}2\%\ I_0$

We may define time constant of LR circuit as the time in which current in the circuit grows to 63·2% of the maximum value of current.

Again, from Eqn. (6), we find that for $I = I_0$, $e^{-t/\tau} = 0$ or $t = \infty$

***i.e.,* current in *LR* circuit would attain maximum value only after infinite time.** However, practically, current reaches its maximum value after a time which is roughly *five* times the value of time constant.

The growth of current with time in *LR* circuit represented by (6) is shown in Fig. 7.5.

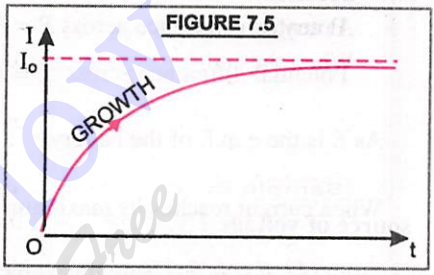

FIGURE 7.5

(b) Decay of current.

In Fig. 7.3, when morse key *K* is released, the battery is cut off. The current in the *LR* circuit decays. If *I* is the current at any time *t* during break*, then as battery is disconnected, putting $E = 0$ in Eq. (4), we get,

$$IR + L\frac{dI}{dt} = 0 \quad ; \quad L\frac{dI}{dt} = -IR \quad \text{or} \quad \frac{dI}{I} = -\frac{R}{L}dt$$

Integrating both sides, we get, $\int \frac{dI}{I} = -\int \frac{R}{L}dt$

$$\log_e I = -\frac{R}{L}t + k \qquad \ldots(7)$$

where k is a constant of integration. At $t = 0$, $I = I_0$

$\therefore$ from (7), $\log_e I_0 = 0 + k$ or $k = \log_e I_0$

Put in (7), $\log_e I = -\frac{R}{L}t + \log_e I_0$; $\log_e I - \log_e I_0 = -\frac{R}{L}t$

$$\log_e \frac{I}{I_0} = -\frac{R}{L}t \quad \text{or} \quad \frac{I}{I_0} = e^{-\frac{R}{L}t} \quad \text{or} \quad \boxed{I = I_0\, e^{-\frac{R}{L}t} = I_0\, e^{-t/\tau}} \qquad \ldots(8)$$

where $\boxed{\tau = L/R}$

This is the **Helmholtz equation for decay of current in *LR* circuit.**

Time constant. The quantity $\tau = L/R$ is the time constant or inductance time constant of *LR* circuit, as said already.

At $t = L/R$, from (8), $I = I_0\, e^{-1} = \frac{I_0}{e} = \frac{I_0}{2{\cdot}718}$

$= 0{\cdot}368\ I_0 = 36{\cdot}8\%\ I_0$

Hence we may define time constant or inductive time constant of LR circuit as the time in which current decays to 36·8% of the maximum value.

Again, from (8), for $I = 0$

$e^{-t/\tau} = 0$

or $t = \infty$

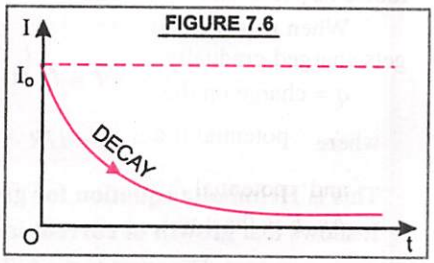

FIGURE 7.6

i.e., **current reduces to zero only after infinite time.** The decay of current with time represented by (8) has been shown in Fig. 7.6.

*Infact, during make, some magnetic flux is linked up with the inductance. During break, this flux reduces to zero and gives rise to the induced current *I*.

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Physical significance of time constant

If $\tau = \frac{L}{R}$ is small, then from (6), I grows to its final value I_0 more rapidly ; and from (8), current decays to zero also more rapidly. If $\frac{L}{R}$ is large, both the growth and decay of current in LR circuit are slow. Thus, time constant of LR circuit is a measure of the rate at which current grows or decays in the LR circuit. ***Smaller the value of time constant, faster are the growth as well as decay of current in the circuit. The reverse is also true.***

Sample Problem 1 **A cell of 1·5 V is connected across an inductor of 2 mH in series with a 2 Ω resistor. What is the rate of growth of current immediately after the cell is switched on ?**

Sol. Here, $E = 1{\cdot}5$ V, $L = 2$ mH $= 2 \times 10^{-3}$ H, $R = 2\ \Omega$

At any time, $E = RI + L\,dI/dt$

When the cell is switched on, current starts growing from zero value, $I = 0$,

$$\therefore \quad E = \frac{L\,dI}{dt} \quad \text{or} \quad \frac{dI}{dt} = \frac{E}{L} = \frac{1{\cdot}5}{2\times10^{-3}} = \mathbf{750\ A\,s^{-1}}$$

Sample Problem 2 **A coil of inductance 300 mH and resistance 2 ohm is connected to a source of voltage 2 V. After how much time does the current reach half of its steady value ?**

(AIEEE 2005)

Sol. Here, $L = 300$ mH $= 0{\cdot}3$ H, $R = 2$ ohm, $t = ?$ $I = I_0/2$

From $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$$\frac{1}{2} = 1 - e^{-\frac{2}{0{\cdot}3}t}$$

$$e^{-\frac{20}{3}t} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$-\frac{20}{3}t = \log_e^1 - \log_e^2 = 0 - 0{\cdot}693$$

$$t = \frac{0{\cdot}693\times3}{20} = \mathbf{0{\cdot}1\ s}$$

7.4. CHARGING AND DISCHARGING OF A CAPACITOR THROUGH RESISTOR

(a) Charging of Capacitor through Resistor

Let a battery of e.m.f. E be connected to a capacitor of capacitance C and a resistor of resistance R in series through a morse key K as shown in Fig. 7.7.

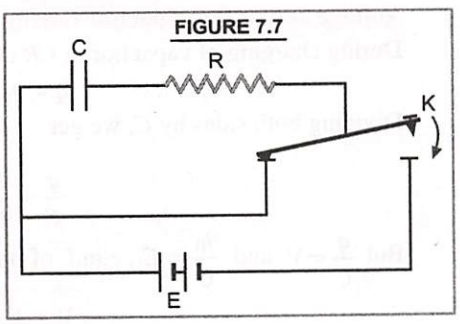

FIGURE 7.7

When K is pressed, battery is connected and the capacitor gets charged gradually. At any instant t, suppose,

q = charge on the capacitor, I = current in the circuit.

$\therefore$ potential difference across $C = \frac{q}{C}$

and potential difference across $R = IR$

As E is the e.m.f. of the source (battery)

$$\therefore \quad E = \frac{q}{C} + IR \qquad \text{...(9)}$$

When the condenser is fully charged, $q = q_0$ (maximum), then $I = \frac{dq}{dt} = 0$

From (9), $E = \frac{q_0}{C} + 0 = \frac{q_0}{C}$

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Put in (9), $\frac{q_0}{C} = \frac{q}{C} + \frac{dq}{dt} R$

$$\frac{q_0 - q}{C} = \frac{dq}{dt} R \qquad \text{or} \qquad \frac{dq}{q_0 - q} = \frac{dt}{RC}$$

Integrating both sides, we get, $\int \frac{dq}{q_0 - q} = \int \frac{dq}{RC}$

$$-\log_e (q_0 - q) = \frac{t}{RC} + k \qquad \text{...(10)}$$

where k is constant of integration. At $t = 0, q = 0$;
From (10), $-\log_e q_0 = k$

Put in (10) $-\log_e (q_0 - q) = \frac{t}{RC} - \log q_0$ or $\log_e (q_0 - q) - \log_e q_0 = -\frac{t}{RC}$

$$\log_e \left(\frac{q_0 - q}{q_0} \right) = -\frac{t}{RC} \quad ; \quad \frac{q_0 - q}{q_0} = e^{-t/RC}$$

$$1 - \frac{q}{q_0} = e^{-t/RC} \qquad \text{or} \qquad \frac{q}{q_0} = 1 - e^{-t/RC}$$

$$\boxed{q = q_0 (1 - e^{-t/RC}) = q_0 (1 - e^{-t/\tau})} \qquad \text{...(11)}$$

where $\boxed{\tau = RC}$

This equation governs charging of capacitor through resistor R.

Time constant. The quantity $\tau = RC$ is called **time constant** or **capacitative time constant** *of RC* circuit. This is because demensions of RC are those of time* and for a given RC circuit, its value is constant.

At $t = RC$, from (11),

$$q = q_0 (1 - e^{-1}) = q_0 \left(1 - \frac{1}{e}\right) = q_0 \left(1 - \frac{1}{2{\cdot}718}\right) \quad ; \quad q = q_0 (1 - 0{\cdot}368) = 0{\cdot}632 = 63{\cdot}2\%\ q_0$$

We may define time constant or capacitative time consant of CR circuit as the time in which charge on the capacitor grows to 63·2% of the maximum value of charge.

Again, from (11), we find that for $q = q_0$, $e^{-t/\tau} = 0$ or $t = \infty$, *i.e.*, **charge on the capacitor will attain its maximum value only after infinite time.** The charging of capacitor with time is shown in Fig. 7.8.

Voltage across the capacitor during charging in *CR* circuit

During charging of capacitor in CR circuit, charge on the plates of capacitor at any time t is

$$q = q_0 (1 - e^{-t/RC})$$

Dividing both sides by C, we get

$$\frac{q}{C} = \frac{q_0}{C} (1 - e^{-t/RC})$$

But $\frac{q}{C} = V$ and $\frac{q_0}{C} = E$, e.m.f. of battery,

$\therefore$ $\boxed{V = E (1 - e^{-t/RC})}$

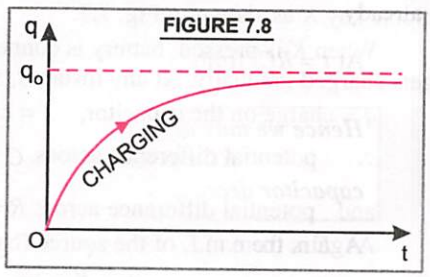


FIGURE 7.8

This is how voltage across the capacitor varies with time, during the charging process.

*As $R = \frac{V}{I}$ and $C = \frac{q}{V}$ $\therefore$ $RC = \frac{V}{1} \cdot \frac{q}{V} = \frac{q}{I} = \frac{I \times t}{I} = t$

Current in *RC* circuit at any instant during charging of capacitor.

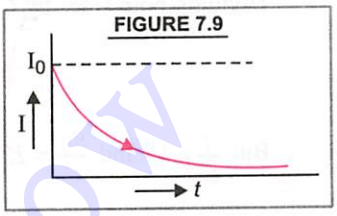
During charging of capacitor, through resistance R, charge on capacitor at any time t is

$$q = q_0 (1 - e^{-t/RC})$$

current at this instant, $$I = \frac{dq}{dt} = \frac{d}{dt}[q_0 (1 - e^{-t/RC})]$$

$$I = \frac{q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

where $I_0 = \frac{q_0}{RC}$ = max. value of current. Therefore, $\boxed{I = I_0 e^{-t/RC}}$



The variation of current in *RC* circuit with time is shown in Fig. 7.9. When we compare this figure with Fig. 7.8, we find **that as charge on capacitor increases, current in the circuit decreases. When capacitor is fully charged at $t = \infty$, current in the *RC* circuit reduces to zero.**

(b) Discharging of capacitor through a resistor

In the experimental set up shown in Fig. 7.7, when key K is released, the battery is cut off. Therefore, fully charged capacitor starts discharging through resistor.

Suppose q is the charge left on the condenser at instant t, and I is the current in the circuit at this instant. As battery is disconnected, putting $E = 0$ in (9), we get

$$0 = \frac{q}{C} + IR \quad \text{or} \quad IR = -\frac{q}{C} \quad \text{or} \quad \frac{dq}{dt} R = -\frac{q}{C} \quad \text{or} \quad \frac{dq}{q} = -\frac{dt}{RC}$$

Integrating both sides, we get, $$\int \frac{dq}{q} = \int -\frac{dt}{RC} \quad \text{or} \quad \log_e q = -\frac{t}{RC} + k \qquad \text{...(12)}$$

where k is a constant of integration

At $t = 0$, $q = q_0$ (as condenser is fully charged) $\therefore \log q_0 = 0 + k$, *i.e.*, $k = \log_e q_0$

From (12), $$\log_e q = -\frac{t}{RC} + \log_e q_0$$

$$\log_e q - \log_e q_0 = -\frac{t}{RC} \quad \text{or} \quad \log_e \frac{q}{q_0} = -\frac{t}{RC} \quad \text{or} \quad \frac{q}{q_0} = e^{-t/RC}$$

or $$\boxed{q = q_0 e^{-t/RC} = q_0 e^{-t/\tau}} \qquad \text{...(13)}$$

where $\tau = RC$

This equation governs the discharging of condenser through R.

Time constant. The quantity $\tau = RC$ is time constant or capacitative time constant of *RC* circuit, as said already.

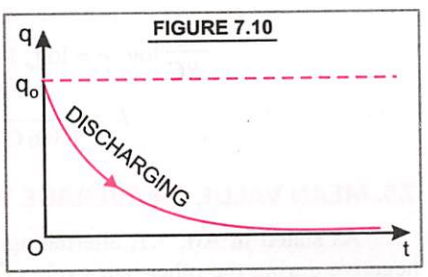
At $t = RC$, from (13), $$q = q_0 e^{-1} = \frac{q_0}{e} = \frac{q_0}{2{\cdot}718} = 0{\cdot}368 = 36{\cdot}8\%\ q_0$$

Hence we may define time constant or capacitative time constant of RC circuit as the time in which charge on the capacitor decreases to 36·8% of the maximum value.

Again, from (13), for $q = 0$

$$e^{-t/\tau} = 0 \quad \text{or} \quad t = \infty$$

i.e., condenser is discharged completely only after infinite time. The discharging of condenser with time through R is shown in Fig. 7.10.



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Physical significance of time constant

If $\tau = RC$ is small, then from (11), q grows to its maximum value q_0 more rapidly and from (13), q decays to zero also more rapidly. If RC is large, both, the charaging and discharging of condenser in RC circuit are slow. Thus, time constant of RC circuit is a measure of the rate at which charging and discharging of condenser occurs through R.

Voltage across the capacitor during discharging in *RC* circuit.

During discharging of capacitor in an RC circuit, charge on capacitor at any instant t is

$$q = q_0\, e^{-t/RC}$$

Dividing both sides by C, we get

$$\frac{q}{C} = \frac{q_0}{C} e^{-t/RC}$$

But $\frac{q}{C} = V$ and $\frac{q_0}{C} = E$ = emf of battery

$\therefore$ $$\boxed{V = E\, e^{-t/RC}}$$

i.e., voltage across the capacitor decreases exponentially with time.

Current at any instant during discharging of capacitor in *RC* circuit.

During discharging of capacitor in RC circuit, charge on capacitor at any time t is

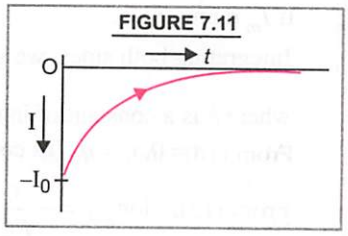
$$q = q_0\, e^{-t/RC}$$

$$I = \frac{dq}{dt} = \frac{d}{dt}[q_0\, e^{-t/RC}]$$

$$I = q_0\, e^{-t/RC}(-1/RC) = -\frac{q_0}{RC} e^{-t/RC}$$

$$\boxed{I = -I_0\, e^{-t/RC}}, \quad \text{where } I_0 = \frac{q_0}{RC}$$

FIGURE 7.11



Negative sign implies that *discharging current flows in a direction opposite to that of the charging current.* The variation of discharging current with time is shown in Fig. 7.11.

Sample Problem **A 0·18 μ *F* capacitor is first charged and then discharged through a high resistance. If it takes 0·5 sec for the charge to reduce to one fourth of its initial value, find the value of the resistance. Given $\log_e 4 = 1{\cdot}386$.**

Sol. Here, $C = 0{\cdot}18\ \mu F = 0{\cdot}18 \times 10^{-6}\ F$, $t = 0{\cdot}5$ s, $\frac{q}{q_0} = \frac{1}{4} = 0{\cdot}25$, $R = ?$

During discharging, $q = q_0\, e^{-t/RC}$ $\therefore$ $e^{-t/RC} = \frac{q}{q_0} = \frac{1}{4}$

$$\frac{-t}{RC}\log_e e = \log_e 1 - \log_e 4 \quad ; \quad \frac{-t}{RC} \times 1 = 0 - 1{\cdot}386$$

$$R = \frac{t}{1{\cdot}386\,C} = \frac{0{\cdot}5}{1{\cdot}386 \times 0{\cdot}18 \times 10^{-6}} = 2 \times 10^6\ \Omega = \mathbf{2\ M\ \Omega}$$

7.5. MEAN VALUE OR AVERAGE VALUE OF ALTERNATING CURRENT

As stated in Art. 7.1, alternating current is positive during the first half cycle (0 to $T/2$) and equally negative during the other half cycle ($T/2$ to T), therefore, the mean value or average value of a.c. over one complete cycle is zero. We can, however calculate the mean or average value of a.c. over any half cycle.

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***The mean value or average value of a.c.** over any half cycle is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle (i.e. T/2) as is sent by the a.c. through the same circuit, in the same time.*

To calculate the mean value or average value, let an alternating current be represented by

$$I = I_0 \sin \omega t \qquad ...(14)$$

If the strength of current is assumed to remain constant for a small time, dt, then small amount of charge sent in a small time dt is

$$dq = I\, dt \qquad ...(15)$$

Let q be the total charge sent by a.c. in the first half cycle (*i.e.* $0 \rightarrow T/2$).

$$\therefore \qquad q = \int_0^{T/2} I\, dt$$

Using (14), we get, $q = \int_0^{T/2} I_0 \sin \omega t \,.\, dt = I_0 \left[-\frac{\cos \omega t}{\omega} \right]_0^{T/2} = -\frac{I_0}{\omega} \left[\cos \omega \frac{T}{2} - \cos 0^\circ \right]$

$$= -\frac{I_0}{\omega} [\cos \pi - \cos 0^\circ] \qquad (\because\ \omega T = 2\pi)$$

$$q = -\frac{I_0}{\omega} [-1-1] = \frac{2 I_0}{\omega} \qquad ...(16)$$

If I_m represents the mean or average value of a.c. over the 1st half cycle, then

$$q = I_m \times \frac{T}{2} \qquad ...(17)$$

From (16), and (17), we get $I_m \times \frac{T}{2} = 2\frac{I_0}{\omega} = \frac{2 I_0 . T}{2\pi}$ ...(18)

or $$I_m = \frac{2}{\pi} I_0 = 0.637\ I_0$$

Hence, mean or average value of a.c. over positive half cycle is 0.637 times the peak value of a.c., i.e., 63.7% of the peak value.

Similarly, the mean or average value of a.c. over the negative half cycle is obtained by integrating (15) within the limits $T/2$ to T. It comes out to be $-0.637\ I_0$. Hence the mean or average value of a.c. over one complete cycle is $0.637\ I_0 - 0.637\ I_0$ = zero. This can also be derived directly by integrating (15) within the limits 0 to T.

7.6. MEAN VALUE OR AVERAGE VALUE OF ALTERNATING E.M.F.

***The mean value or average value of alternating e.m.f.** over a half cycle is that value of constant e.m.f. which would send the same amount of charge through a circuit in the time of half cycle (T/2), as is sent by alternating e.m.f. through the same circuit in the same time.*

To calculate its value, let an alternating e.m.f. be represented by $E = E_0 \sin \omega t$

If I is the value of current at instant t, then $I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$, where R is resistance of the circuit.

If this current remains constant for a small time dt, then small amount of charge sent by alternating e.m.f. in the small time dt is

$$dq = I\, dt = \frac{E_0}{R} \sin \omega t\, dt \qquad ...(19)$$

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$\therefore$ Total charge sent by alternating e.m.f. in the first half cycle ($0 \rightarrow T/2$) would be

$$q = \int_0^{T/2} \frac{E_0}{R} \sin \omega t \, dt = \frac{E_0}{R}\left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2} = \frac{-E_0}{\omega R}\left[\cos \omega \frac{T}{2} - \cos 0°\right] = -\frac{E_0}{\omega R}[\cos \pi - \cos 0°]$$

$(\because \ \omega T = 2\pi)$

$$q = -\frac{E_0}{\omega R}(-1-1) = \frac{2E_0}{\omega R} \qquad \text{...(20)}$$

If E_m is mean or average value of alternating e.m.f. over the first half cycle, then

$$q = \frac{E_m}{R} \times \frac{T}{2} \qquad \text{...(21)}$$

From (20) and (21),

$$\frac{E_m}{R}\frac{T}{2} = \frac{2E_0}{\omega R} = \frac{2E_0}{\left(\frac{2\pi}{T}\right)R} = \frac{2E_0 T}{2\pi R}$$

or $\boxed{E_m = \frac{2}{\pi}E_0 = 0.637\, E_0 = 63.7\%\, E_0}$

Hence, mean or average value of alternating e.m.f. over positive half cycle is 0.637 times the peak value of alternating e.m.f.

Similarly, over the negative half cycle ($T/2 \rightarrow T$)

$$\boxed{E_m = \frac{-2E_0}{\pi} = -0.637\, E_0}$$

Hence **average value of alternating e.m.f. over the full cycle = zero.** This can also be derived directly by integrating (19) within the limits 0 to T.

Retain in Memory

Note that ordinary *d.c.* ammeter and *d.c.* voltmeter cannot measure alternating currents/voltages. When used in *a.c.* circuits, they record zero reading because average value of alternating current/ voltage over a full cycle is zero.

7.7. ROOT MEAN SQUARE VALUE OF ALTERNATING CURRENT

The root mean square (r.m.s.) value of a.c. is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the a.c., when passed through the same resistance for the same time.

The r.m.s. value is also called **effective value** of a.c. or **virtual value** of a.c. It is represented by I_{rms} or I_{eff} or I_v. To calculate it, suppose, an alternating current is represented by

$$I = I_0 \sin \omega t \qquad \text{...(22)}$$

Let this current flow through a resistance R. In a small time dt, the amount of heat produced in resistance R is

$$dH = I^2 R \, dt \qquad \text{...(23)}$$

In one complete cycle (time $0 \rightarrow T$), the total amount of heat produced in the resistance R would be

$$H = \int_0^T I^2 R \, dt$$

Using (22), we get,

$$H = \int_0^T (I_0^2 \sin^2 \omega t)\, R \, dt = I_0^2 R \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T 1\,dt - \int_0^T \cos 2\omega t\,.\,dt\right] = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2 R}{2}\left[(T-0) - \left(\frac{\sin 2\omega T}{2\omega} - \sin 0°\right)\right] = \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\times 2\pi}{2\omega}\right] \quad (\because\ \omega T = 2\pi)$$

As $\sin 4\pi = 0$, $\therefore \quad H = \dfrac{I_0^2 RT}{2}$ ...(24)

If r.m.s. value or virtual value of a.c. is represented by I_v, then the amount of heat produced in the same resistance R, in the same time T would be

$$H = I_v^2 RT \qquad ...(25)$$

From (24) and (25), we get $I_v^2 RT = \dfrac{I_0^2 RT}{2}$

$$I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \qquad ...(26)$$

Hence, the r.m.s value or effective value or virtual value of a.c. is 0.707 times the peak value of a.c., i.e., 70.7% of the peak value of a.c.

Sample Problem **The instantaneous current from an a.c. source is $I = 6 \sin 314\,t$. What is the r.m.s. value of the current ?**

Sol. Compare the given eqn. with the standard form $I = I_0 \sin \omega t$

$$I_0 = 6\text{ A},\ I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{6}{\sqrt{2}}.\frac{\sqrt{2}}{\sqrt{2}} = \frac{6}{2}\times 1{\cdot}414 = \mathbf{4{\cdot}242\ A}$$

7.8. ROOT MEAN SQUARE VALUE OF ALTERNATING E.M.F.

The root mean square (r.m.s.) value of alternating e.m.f. is defined as that value of steady voltage ; which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating e.m.f., when applied to the same resistance for the same time.

The r.m.s. value is also called *effective value* or *virtual value* of alternating e.m.f. It is represented by E_{rms} or E_{eff} or E_v. To calculate it, suppose the alternating e.m.f. is represented by $E = E_0 \sin \omega t$

Small amount of heat produced when this alternating e.m.f. is applied to a resistance R for a small time dt is

$$dH = \frac{E^2}{R}dt = \frac{(E_0 \sin \omega t)^2}{R}dt \quad ; \quad dH = \frac{E_0^2}{R}\sin^2 \omega t\, dt$$

In one complete cycle ($0 \to T$), total amount of heat produced in resistance R is

$$H = \int_0^T \frac{E_0^2}{R}\sin^2 \omega t\, dt = \frac{E_0^2}{R}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right)dt = \frac{E_0^2}{2R}\int_0^T 1\,dt - \frac{E_0^2}{2R}\int_0^T \cos 2\omega t\, dt$$

$$H = \frac{E_0^2}{2R}[t]_0^T - \text{Zero} \qquad \left[\because \int_0^T \cos 2\omega t\, dt = 0 \text{ from Art. 7.7}\right]$$

$$H = \frac{E_0^2}{2R}(T-0) = \frac{E_0^2 T}{2R} \qquad ...(27)$$

If E_v is the rms value of alternating e.m.f., then amount of heat produced in the same resistance R in the same time T is

$$H = \frac{E_v^2}{R} T \qquad ...(28)$$

From (27), and (28), we get $\frac{E_v^2}{R} T = \frac{E_0^2}{2R}$

$$E_v^2 = \frac{E_0^2}{2} \quad \text{or} \quad \boxed{E_v = \frac{E_0}{\sqrt{2}}}$$

i.e., $$E_v = \frac{E_0}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{E_0 \times 1.414}{2} = 0.707\ E_0$$

Hence, r.m.s. value of alternating e.m.f. is 0.707 times the peak value of alternating e.m.f.

DO YOU KNOW ?

220 V *a.c.* is more dangerous than 220 V *d.c.*, because 220 V *a.c.* has peak value $E_0 = \pm\sqrt{2}\, E_v$ $= \pm 1{\cdot}414 \times 220\ \text{V} = \pm 311$ V, but 220 V *d.c.* has value fixed at 220 V only.

Retain in Memory

Alternating currents/voltages are measured by *a.c.* ammeter/voltmeter respectively. These instruments are called hot wire instruments and they measure only virtual values of alternating currents/ voltages. The values of alternating voltages/ currents quoted any where are virtual values only. For example, 220 V *a.c.* means E_v = 220 volt. An *a.c.* of 1 A means I_v = 1 A.

Sample Problem **What will be the instantaneous voltage for a.c. supply of 220 V at 50 hertz ?**

Sol. $E = ?$, $E_v = 220$ V, $\nu = 50$ hertz, $E_0 = \sqrt{2}\, E_v = 1{\cdot}414 \times 220 = 311$ volt

The instantaneous value of alternating *e.m.f.* is $E = E_0 \sin \omega t = E_0 \sin 2\pi \nu t = 311 \sin 2\pi \times 50\ t$

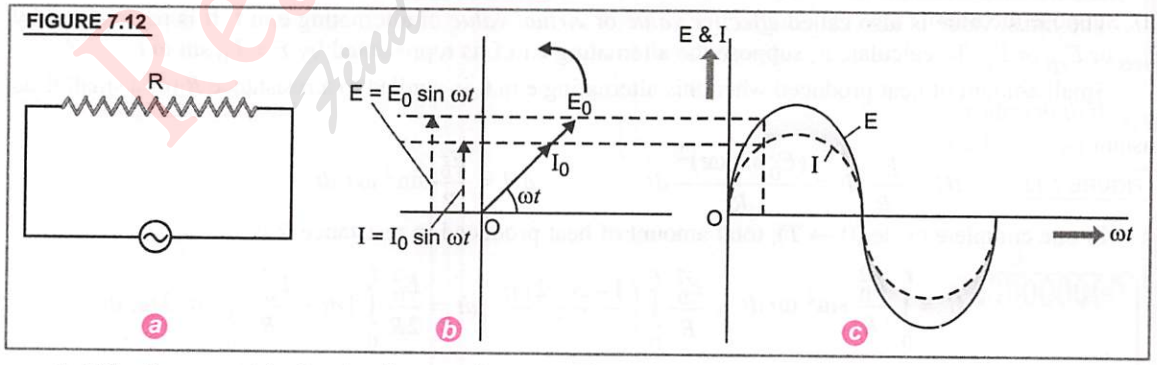
$$E = 311 \sin 100\, \pi t$$

7.9. A.C. CIRCUIT CONTAINING RESISTANCE ONLY

Let a source of alternating e.m.f. be connected to a pure resistance R, Fig. 7.12(a). Suppose the alternating e.m.f. supplied is represented by

$$\boxed{E = E_0 \sin \omega t} \qquad ...(29)$$

FIGURE 7.12



Let I be the current in the circuit at any instant t. The potential difference developed across R will be IR. This must be equal to e.m.f. applied at that instant, *i.e.*,

$$IR = E = E_0 \sin \omega t \quad \text{or} \quad \boxed{I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t} \qquad ...(30)$$

where $I_0 = E_0/R$, maximum value of current

This is the form of alternating current developed.

Comparing $I_0 = E_0/R$ with Ohm's law equation, viz. current = voltage/resistance, we find that resistance to *a.c.* is represented by R–which is the value of resistance to *d.c.*

Hence behaviour of R in d.c. and a.c. circuits is the same. R can reduce a.c. equally effectively as it can reduce d.c.

Comparing (30) and (29), we find that E and I are *in phase*. ***Therefore, in an a.c. circuit containing R only, the voltage and current are in the same phase, as shown in Fig. 7.12(c).***

It means the minima, zero and maxima of alternating voltage and alternating current in a pure resistor occur at the same respective times.

Phasor Diagram

The analysis of an a.c. circuit is facilitated by the use of a phasor diagram.

A rotating vector that represents a quantity varying sinusoidally with time is called a *phasor*. This vector is imagined to rotate with angular velocity equal to angular velocity of that quantity. In the phasor diagram, peak values of alternating current (I_0) and alternating e.m.f. (E_0) are represented by *arrows* called phasors. They are inclined to horizontal axis at $\angle \omega t$ and rotate in the anticlockwise direction. The length of the arrow represents the maximum value of the quantity, *i.e.*, $\vec{I}_0$ and $\vec{E}_0$.

The projection of the arrow on any axis represents the instantaneous value of the quantity. In the sine form, projection is taken on the vertical axis and in the cosine form, projection is taken on the horizontal axis.

The phase difference between the two alternating quantities is represented by the *angle* between the two vectors $\vec{E}_0$ and $\vec{I}_0$.

In the a.c. circuit containing R only, current and voltage are in the same phase. Therefore, in Fig. 7.12(*b*), both phasors $\vec{I}_0$ and $\vec{E}_0$ are in the same direction making an angle (ωt) with OX. This is so for all times. It means that the phase angle between alternating voltage and alternating current through R is zero. Their projections on YOY' represent the instantaneous values of alternating current and voltage *i.e.*

$$I = I_0 \sin \omega t \quad \text{and} \quad E = E_0 \sin \omega t.$$

Note that as E_0 and I_0 rotate with frequency ω, curves in Fig. 7.12(*c*) are generated.

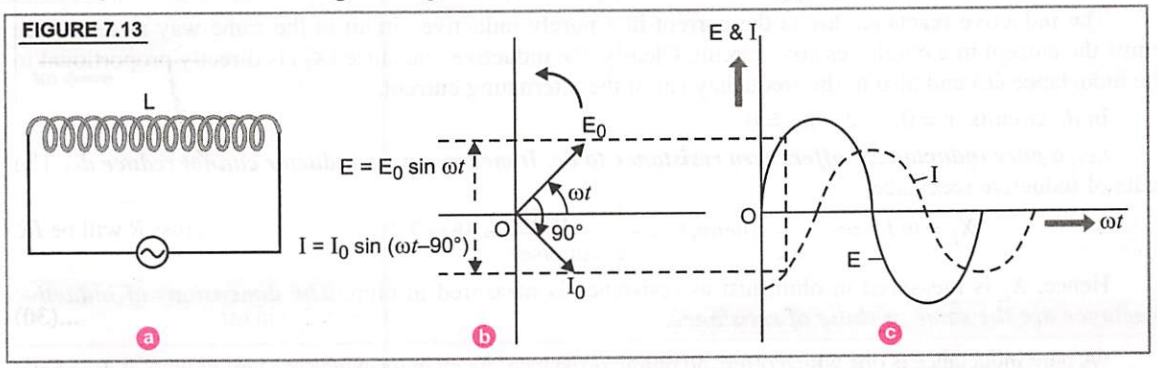
7.10. A.C. CIRCUIT CONTAINING INDUCTANCE ONLY

Let a source of alternating e.m.f. be connected to a circuit containing a pure inductance L only, Fig. 7.13 (a). Suppose the alternating e.m.f. supplied is represented by

$$E = E_0 \sin \omega t \qquad \text{...(31)}$$

If dI/dt is the rate of change of current through L at any instant, then induced e.m.f. in the inductor at the same instant is, $e = -L\, dI/dt$. The negative sign indicates that induced e.m.f. opposes the change of current.

FIGURE 7.13



Usually, inductors have some resistance in their windings. But we shall assume that resistance of this inductor is negligible. The circuit is therefore, a purely inductive circuit. To maintain the flow of current in this circuit, applied voltage must be equal and opposite to the induced voltage, e

i.e., $$E = -\left(-L\frac{dI}{dt}\right) = E_0 \sin \omega t \qquad \text{or} \qquad dI = \frac{E_0}{L}\sin \omega t\, dt$$

Integrating both sides, we get, $$I = \frac{E_0}{L}\int \sin \omega t\, dt$$

$$I = \frac{E_0}{L}\left(\frac{-\cos \omega t}{\omega}\right) + \text{constant} \qquad \text{...(32)}$$

The *integration constant* has the dimensions of current, and is time-independent. As e.m.f. of the source oscillates symmetrically about zero, the current it sustains must also oscillate symmetrically about zero. Therefore, no component of current which is time independent exists. Therefore, *integration constant is zero.*

$\therefore$ From (32), $$I = \frac{-E_0}{\omega L}\cos \omega t = \frac{-E_0}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2) \qquad \text{...(33)}$$

The current will be maximum, *i.e.,* $I = I_0$, when $\sin(\omega t - \pi/2) = \text{maximum} = 1$

From (33), $$I_0 = \frac{E_0}{\omega L} \times 1 \qquad \text{...(34)}$$

Putting in (33), we get, $$I = I_0 \sin(\omega t - \pi/2) \qquad \text{...(35)}$$

This is the form of alternating current developed.

Comparing (31) with (35), we find that ***in an a.c. circuit containing L only, alternating current I lags behind the alternating voltage E by a phase angle of 90°, i.e., by one fourth of a period.*** Conversely, voltage across L leads the current by a phase angle of 90°. This is shown in Fig. 7.13 (*c*).

Figure 7.13(*b*) represents the vector diagram or the phasor diagram of a.c. circuit containing L only. The vector representing $\vec{E_0}$ makes an angle (ωt) with OX. As current lags behind the e.m.f. by 90°, therefore, phasor representing $\vec{I_0}$ is turned **clock wise** through 90° from the direction of $\vec{E_0}$. The projections of these phasors on YOY' give instantaneous values E and I as shown in Fig. 7.13(*b*). When E_0 and I_0 rotate with frequency ω, curves in Fig. 7.13(*c*) are generated.

Comparing (34) with Ohm's law equation, viz. *Current = Voltage/Resistance, we find that* (ωL) represents the effective resistance offered by inductance L. This is called ***inductive reactance*** and is denoted by X_L.

Thus, $X_L = \omega L = 2\pi\nu L$ where ν is the frequency of a.c. supply.

The inductive reactance limits the current in a purely inductive circuit in the same way as resistance limits the current in a purely resistive circuit. Clearly, the inductive reactance (X_L) is directly proportional to the inductance (L) and also to the frequency (ν) of the alternating current.

In *dc* circuits, $\nu = 0$. $\therefore$ $X_L = 0$

i.e., a pure inductance* offers zero resistance to dc. It means a pure inductor cannot reduce dc. The units of inductive reactance

$$X_L = \omega L \Rightarrow \frac{1}{\text{sec}}\,(\text{henry}) = \frac{1}{\text{sec}}\,\frac{\text{volt}}{\text{amp/sec}} = \text{ohm}$$

Hence, X_L is measured in ohm, just as resistance is measured in ohm. ***The dimensions of inductive reactance are the same as those of resistance.***

*A pure inductance is one which offers no ohmic resistance. As an inducatance is a coil of wire, it does have some ohmic resistance (due to material of the wire). Thus a pure inductance is not possible in actual practice.

Retain in Memory

It should be clearly understood, that in a circuit containing R, opposition arises on account of obstruction to the passage of electrons through the resistor. In an inductive circuit, it is the self induced e.m.f. that opposes the growth as well as decay of current.

Sample Problem 1 **A 100 hertz a.c. is flowing in a 14 mH coil. Find its reactance.**

Sol. Here, $\nu = 100$ hz, $L = 14$ mH $= 14 \times 10^{-3}$ H

$$X_L = \omega L = 2\pi \nu L = 2\pi \times 100 \times 14 \times 10^{-3} = \mathbf{8{\cdot}8\ ohm}$$

Sample Problem 2 **An inductor of 1 H and negligible resistance is used on 220 V a.c. mains of frequency 50 Hz. Calculate the effective current.**

Sol. Here, $L = 1$ H, $E_v = 220$ V, $\nu = 50$ Hz

$$I_v = \frac{E_v}{X_L} = \frac{E_v}{\omega L} = \frac{E_v}{(2\pi\nu)L} = \frac{220}{2\times 3{\cdot}14\times 50\times 1} = \mathbf{0{\cdot}7\ A}$$

7.11. A.C. CIRCUIT CONTAINING CAPACITANCE ONLY

Let a source of alternating e.m.f. be connected to a capacitor only of capacitance C, Fig. 7.14(a). Suppose the alternating e.m.f. supplied is

$$E = E_0 \sin \omega t \qquad \text{...(36)}$$

The current flowing in the circuit transfers charge to the plates of the capacitor. This produces a potential difference between the plates. The capacitor is alternately charged and discharged as the current reverses each half cycle. At any instant t, suppose q is the charge on the capacitor. Therefore, potential difference across the plates of capacitor $V = q/C$.

At every instant, the potential difference V must be equal to the e.m.f. applied, *i.e.*,

$$V = \frac{q}{C} = E = E_0 \sin \omega t \qquad \text{or} \qquad q = CE_0 \sin \omega t$$

If I is instantaneous value of current in the circuit at instant t, then $I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega t)$

$$I = CE_0 (\cos \omega t)\, \omega$$

$$I = \frac{E_0}{1/\omega C} \sin (\omega t + \pi/2)^* \qquad \text{...(37)}$$

The current will be maximum *i.e.* $I = I_0$, when $\sin(\omega t + \pi/2) = \text{maximum} = 1$

$$\therefore \text{ From (37)}, \qquad I_0 = \frac{E_0}{1/\omega C} \times 1 \qquad \text{...(38)}$$

Put in (37), $$I = I_0 \sin (\omega t + \pi/2) \qquad \text{...(39)}$$

This is the form of alternating current developed.

Comparing (39) with (36), ***we find that in an a.c. circuit containing C only, alternating current I leads the alternating e.m.f. by a phase angle of 90°.*** This is shown in Fig. 7. 14(*c*).

The phasor diagram or vector diagram of a.c. circuit containing C only is shown in Fig. 7.14(*b*). The phasor $\vec{I}_0$ is turned **anticlockwise** through 90° from the direction of phasor $\vec{E}_0$. Their projections on YOY′

*Here, $\cos \omega t = \sin\left(\frac{\pi}{2} - \omega t\right)$ cannot be used because we have to compare I with $E = E_0 \sin \omega t$, which involves $(+\omega t)$ and not $(-\omega t)$. Hence we use $\cos \omega t = \sin\left(\frac{\pi}{2} + \omega t\right) = \sin(\omega t + \pi/2)$.

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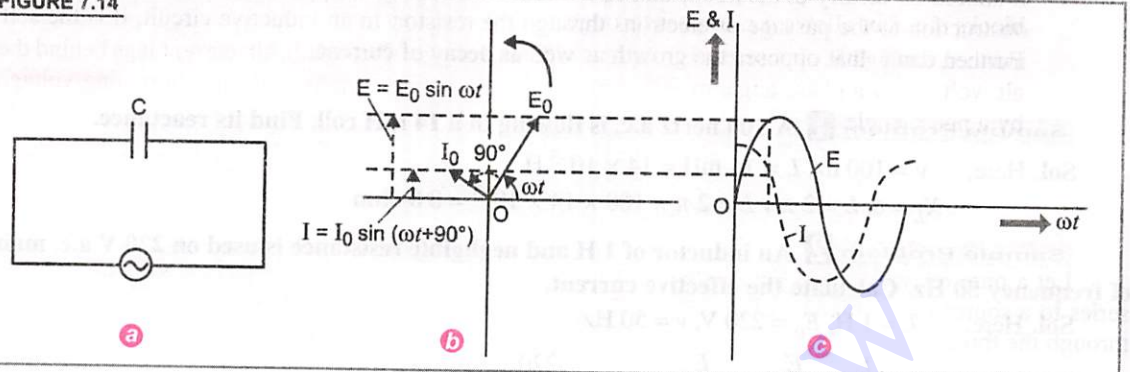
give the instantaneous values E and I as shown in Fig. 7.14(b). When E_0 and I_0 rotate with frequency ω, curves in Fig. 7.14(c) are generated.

FIGURE 7.14



Comparing (38) with Ohm's law eqn., *viz* current = voltage/resistance, we find that $(1/\omega C)$ represents effective resistance offered by the capacitor. This is called ***capacitative reactance*** and is denoted by X_C.

Thus $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C} \quad \text{...(40)}$$

where ν is frequency of a.c. supply. The capacitive reactance limits the amplitude of current in a purely capa- citive circuit in the same way as the resistance limits the current in a purely resistive circuit. Clearly, **capacitive reactance varies inversely as the frequency of a.c. and also inversely as the capacitance of the condenser.**

In a d.c. circuit, $\nu = 0$, $\therefore X_C = \infty$ *i.e.*, ***a condenser will block d.c.***

The units of X_c can be deduced as : $X_c = \frac{1}{\omega C} = \sec \frac{1}{\text{farad}} = \frac{\sec}{\text{coulomb/volt}} = \frac{\text{volt sec.}}{\text{amp. sec}} = \text{ohm}$

Hence X_c is measured in ohms, just like resistance R. The dimensions of capacitive reactance are the same as that of resistance.

Retain in Memory

It should be clearly understood that the physical processes involved in capacitive and resistive circuits are quite different. In a resistive circuit, the resistance is due to the obstruction to the passage of electrons through the resistor. But in a capacitive circuit, resistance to the flow of current is offered by the charged capacitor.

Sample Problem 1 **One microfarad capacitor is joined to 200 V, 50 hz alternator. Calculate the rms current through capacitor.**

Sol. Here, $C = 1\ \mu\text{F} = 10^{-6}\ \text{F}$, $E_v = 200\ \text{V}$, $\nu = 50\ \text{hz.}$, $I_v = ?$

$$I_v = \frac{E_v}{X_C} = \frac{E_v}{1/2\pi \nu C} = (2\pi \nu C)\, E_v \ ; \ I_v = 2 \times 3{\cdot}14 \times 50 \times 10^{-6} \times 200 = \mathbf{6{\cdot}28 \times 10^{-2}\ A}$$

Sample Problem 2 **220 V, 50 Hz a.c. is connected across a capacitor of 5 μF. Calculate the effective current.**

Sol. Here, $E_v = 220\ \text{V}$, $\nu = 50\ \text{Hz}$, $C = 5 \times 10^{-6}\ \text{F}$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C} = \frac{1}{2\pi \times 50 \times 5 \times 10^{-6}} = \frac{10^6}{500\,\pi}\ \text{ohm}$$

$$I_v = \frac{E_v}{X_C} = \frac{220 \times 500\,\pi}{10^6} = 0{\cdot}11 \times 3{\cdot}14\ \text{A} = \mathbf{0{\cdot}375\ A}$$

Retain in Memory

Note that '*R*' can reduce *a.c.* as well as *d.c.* '*L*' cannot reduce *d.c.* It can reduce only *a.c.* Again '*C*' blocks *d.c.*, but allows *a.c.* to pass through.

Further, through *R*, alt. current is in phase with alt. voltage. Through *L*, alt. current lags behind the alt. voltage by a phase angle of 90°. Through *C*, alternating current leads the alternating voltage by a phase angle of 90°.

7.12. A.C. CIRCUIT CONTAINING RESISTANCE, INDUCTANCE AND CAPACITANCE IN SERIES (RLC CIRCUIT)

(a) Phasor Treatment

Let a pure resistance *R*, a pure inductance *L* and an ideal capacitor of capacitance *C* be connected in series to a source of alternating e.m.f., Fig. 7.15. As *R*, *L*, *C* are in series, therefore, current at any instant through the three elements has the *same amplitude and phase*. Let it be represented by

$$I = I_0 \sin \omega t$$

FIGURE 7.15

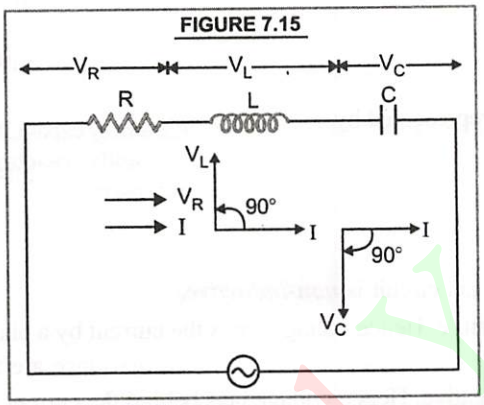
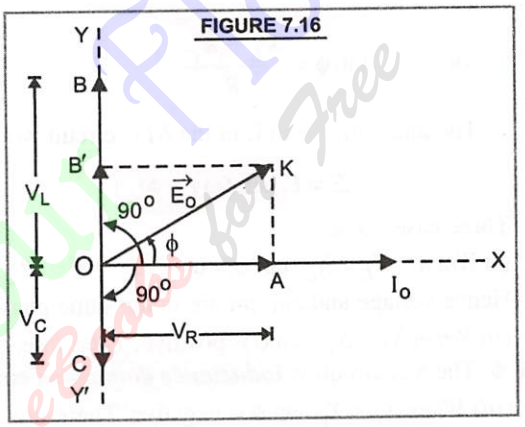


FIGURE 7.16



However, voltage across each element bears a different phase relationship with the current. Now,

(*i*) The maximum voltage across *R* is $\vec{V}_R = \vec{I}_0 R$

In Fig. 7.16, current phasor $\vec{I}_0$ is represented along *OX*.

As $\vec{V}_R$ is in phase with current, it is represented by the vector $\overrightarrow{OA}$, along *OX*.

(*ii*) The maximum voltage across *L* is $\vec{V}_L = \vec{I}_0 X_L$

As voltage across the inductor leads the current by 90°, it is represented by $\overrightarrow{OB}$ along *OY*, 90° ahead of $\vec{I}_0$.

(*iii*) The maximum voltage across *C* is $\vec{V}_C = \vec{I}_0 X_C$

As voltage across the capacitor lags behind the alternating current by 90°, it is represented by $\overrightarrow{OC}$ rotated clockwise through 90° from the direction of $\vec{I}_0$. $\overrightarrow{OC}$ is along *OY′*.

As the voltages across *L* and *C* have a phase difference of 180°, the net reactive voltage is $(\vec{V}_L - \vec{V}_C)$, assuming that $\vec{V}_L > \vec{V}_C$.

In Fig. 7.16, it is represented by $\overrightarrow{OB'}$. The resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB'}$ is the diagonal $\overrightarrow{OK}$ of the rectangle *OAKB′*. Hence the vector sum of $\vec{V}_R, \vec{V}_L$ and $\vec{V}_C$ is phasor $\vec{E}_0$ represented by $\overrightarrow{OK}$, making an angle ϕ with current phasor $\vec{I}_0$.

As $OK = \sqrt{OA^2 + OB'^2}$

$\therefore$ $E_0 = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$

$$E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

The total effective resistance of *RLC* circuit is called ***Impedance*** of the circuit. It is represented by *Z*, where

$$Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2} \quad ...(41)$$

From Fig. 7.13, it is clear that in an a.c. circuit containing *R, L, C*, the voltage leads the current by a phase angle ϕ, where

$$\tan\phi = \frac{AK}{OA} = \frac{OB'}{OA} = \frac{V_L - V_C}{V_R} = \frac{I_0 X_L - I_0 X_C}{I_0 R}$$

or $$\tan\phi = \frac{X_L - X_C}{R} \quad ...(42)$$

$\therefore$ The alternating e.m.f. in the *RLC* circuit would be represented by

$$E = E_0 \sin(\omega t + \phi) \quad ...(43)$$

Three cases arise :

(*i*) *When* $X_L = X_C$, $\tan\phi = 0$ $\therefore$ $\phi = 0°$

Hence voltage and current are in the same phase. The a.c. circuit is ***non-inductive.***

(*ii*) *When* $X_L > X_C$, $\tan\phi$ is positive. Therefore, ϕ is positive. Hence voltage *leads* the current by a phase angle ϕ. The a.c. circuit is ***inductance dominated circuit.***

(*iii*) *When* $X_L < X_c$, $\tan\phi$ is negative. Therefore, ϕ is negative. Hence voltage lags behind the current by a phase angle ϕ. The a.c. circuit is ***capacitance dominated circuit.***

(b) Analytical Treatment of RLC series circuit

Let a pure resistance *R*, a pure inductance *L* and an ideal condenser of capacity *C* be connected in series to a source of alternating e.m.f. Suppose the alternating e.m.f. supplied is

$$E = E_0 \sin\omega t \quad ...(44)$$

At any instant of time *t*, suppose q = charge on capacitor, I = current in the circuit

$\frac{dI}{dt}$ = rate of change of current in the circuit

$\therefore$ Potential difference across the condenser = $\frac{q}{C}$, potential difference across inductor = $L\frac{dI}{dt}$

potential difference across resistance = RI

$\therefore$ The voltage equation of the circuit is

$$L\frac{dI}{dt} + RI + \frac{q}{C} = E = E_0 \sin\omega t \quad ...(45)$$

As $I = \frac{dq}{dt}$, therefore, $\frac{dI}{dt} = \frac{d^2 q}{dt^2}$

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$\therefore$ The voltage equation becomes $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E_0 \sin \omega t$ ...(46)

This is like the equation of a forced, damped oscillator. Let the solution of eqn. (46) be

$$q = q_0 \sin(\omega t + \theta)$$

$\therefore$ $\frac{dq}{dt} = q_0 \omega \cos(\omega t + \theta)$; $\frac{d^2q}{dt^2} = -q_0 \omega^2 \sin(\omega t + \theta)$

Substituting these values in eqn. (46), we get

$$L[-q_0 \omega^2 \sin(\omega t + \theta)] + R q_0 \omega \cos(\omega t + \theta) + \frac{q_0}{C} \sin(\omega t + \theta) = E_0 \sin \omega t$$

$$q_0 \omega \left[R \cos(\omega t + \theta) - \omega L \sin(\omega t + \theta) + \frac{1}{\omega C} \sin(\omega t + \theta)\right] = E_0 \sin \omega t$$

As $\omega L = X_L$ and $\frac{1}{\omega C} = X_C$, therefore, $q_0 \omega [R \cos(\omega t + \theta) + (X_C - X_L) \sin(\omega t + \theta)] = E_0 \sin \omega t$

Multiplying and dividing by $Z = \sqrt{R^2 + (X_C - X_L)^2}$, we get

$$q_0 \omega Z \left[\frac{R}{Z} \cos(\omega t + \theta) + \frac{X_C - X_L}{Z} \sin(\omega t + \theta)\right] = E_0 \sin \omega t \quad \text{...(47)}$$

Let $\frac{R}{Z} = \cos\phi$ and $\frac{X_C - X_L}{Z} = \sin\phi$ ...(48)

so that $\tan\phi = \frac{X_C - X_L}{R}$ ...(49)

$\therefore$ $q_0 \omega Z [\cos(\omega t + \theta)\cos\phi + \sin(\omega t + \theta)\sin\phi] = E_0 \sin \omega t$

or $q_0 \omega Z \cos(\omega t + \theta - \phi) = E_0 \sin \omega t = E_0 \cos(\omega t - \pi/2)$ ...(50)

Comparing the two sides of this equation, we find that

$E_0 = q_0 \omega Z = I_0 Z$, where $I_0 = q_0 \omega$ ...(51)

and $\omega t + \theta - \phi = \omega t - \pi/2$

$\therefore$ $\theta - \phi = \frac{-\pi}{2}$ or $\theta = \frac{-\pi}{2} + \phi$ ...(52)

$\therefore$ Current in the circuit is $I = \frac{dq}{dt} = \frac{d}{dt}[q_0 \sin(\omega t + \theta)] = q_0 \omega \cos(\omega t + \theta)$

$I = I_0 \cos(\omega t + \theta))$ {using (51)}

Using (52), we get, $I = I_0 \cos(\omega t + \phi - \pi/2)$

$$\boxed{I = I_0 \sin(\omega t + \phi)} \quad \text{...(53)}$$

From (49), $$\boxed{\phi = \tan^{-1}\frac{(X_C - X_L)}{R}} \quad \text{...(54)}$$

As $\cos^2\phi + \sin^2\phi = 1$ $\therefore$ $\left(\frac{R}{Z}\right)^2 + \left(\frac{X_C - X_L}{Z}\right)^2 = 1$

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or $R^2 + (X_C - X_L)^2 = Z^2$ or $\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$ ...(55)

The equations (53), (54) and (55) for current, phase angle and impedance agree perfectly with the corresponding equations obtained by the technique of phasors.

Sample Problem 1 **A resistor of 12 Ω, a capacitor of reactance 14 Ω and a pure inductor of inductance 0·1 H are joined in series and placed across 200 V, 50 hz a.c. supply. Calculate (i) current in the circuit (ii) phase angle between current and voltage. Take π = 3.**

Sol. Here, $R = 12\ \Omega$, $X_C = 14\ \Omega$, $L = 0{\cdot}1$ H, $E_v = 200$ V, $v = 50$ hz, $I_v = ?$, $\phi = ?$

$$X_L = 2\pi v L = 2 \times 3 \times 50 \times 0{\cdot}1 = 30\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{12^2 + (30-14)^2} = 20\,\Omega \quad ; \quad I_v = \frac{E_v}{Z} = \frac{200}{20} = \mathbf{10\ A}$$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{30-14}{12} = \frac{16}{12} = 1{\cdot}3333 \quad ; \quad \phi = \mathbf{53°9'}\text{, voltage leads the current.}$$

Sample Problem 2 **Calculate the *rms* value of current in a circuit containing a coil of resistance 50 Ω and inductance 0·5 H, and a condenser of capacity 100 μF in series connected to a 110 V – 50 Hz a.c. source.**

Sol. Here, $I_v = ?$ $R = 50$ ohm, $L = 0{\cdot}5$ H, $C = 100\ \mu F = 100 \times 10^{-6}\ F = 10^{-4}$ F,

$E_v = 110$ V, $v = 50$ Hz, $X_L = \omega L = 2\pi v L = 2\pi \times 50 \times 0{\cdot}5 = 157$ ohm

$$X_C = 1/\omega C = 1/2\ \pi v \times 10^{-4} = \frac{10^4}{2 \times 3{\cdot}14 \times 50} = 31{\cdot}85 \text{ ohm}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (157 - 31{\cdot}85)^2} = 134{\cdot}8 \text{ ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{110}{134{\cdot}8}\ \text{A} = \mathbf{0{\cdot}816\ A}$$

7.13 IMPEDANCE TRIANGLE

As stated already, resistance (R) stands for **ohmic resistance.** It arises on account of material of the conductor ($R = \rho\, l/a$).

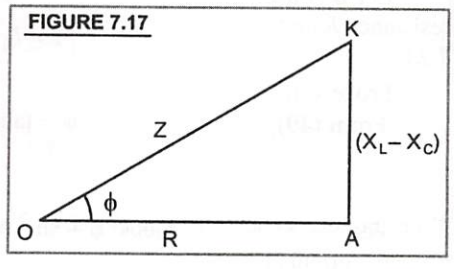
The **reactance** (X) is the resistance that arises on account of opposing e.m.f. induced due to change in the strength of current. The inductive reactance, $X_L = \omega L$ is the resistance offered by the inductor coil. The capacitive reactance $X_C = 1/\omega C$ is the resistance offered by the condenser. As voltage across the inductor *leads* the current by a phase angle of 90° and voltage across the capacitor *lags* behind the current by a phase angle of 90°, therefore X_L and X_C have opposite signs. Total reactance is taken to be $\pm (X_L - X_C)$.

The total effective resistance offered by the *RLC* circuit is called **Impedance.** It is represented by Z.

From the three phasors, $\vec{V}_R = \vec{I}_0 R$; $\vec{V}_L = \vec{I}_0 X_L$ and $\vec{V}_C = \vec{I}_0 X_C$, we obtain, what is known as **Impedance Triangle**, Fig. 7.17.

The base OA of this triangle represents ohmic resistance R, the perpendicular AK represents reactance $(X_L - X_C)$ and

FIGURE 7.17



diagonal OK represents the impedance (Z) of the a.c. circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $\angle AOK = \phi$ is the phase angle by

which voltage leads the current in the circuit, where $\tan\phi = \dfrac{X_L - X_C}{R}$.

The reciprocal of reactance is called susceptance of the a.c. circuit and reciprocal of impedance is called admittance of a.c. circuit. Both, the susceptance and admittance are measured in mho, i.e., ohm^{-1} or Siemen.

Retain in Memory

In Fig. 7.18, we have shown the variation of R,

$$X_L = \omega L;$$

$$X_C = \frac{1}{\omega C};$$

and $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

with the frequency (ν) of the applied alternating voltage.

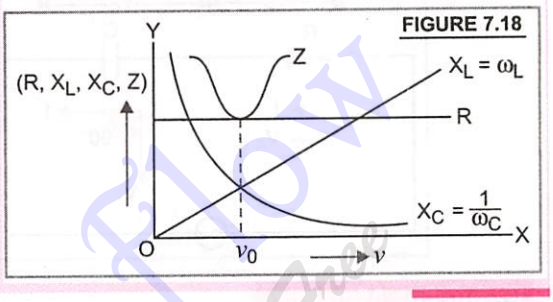


FIGURE 7.18

7.14 A.C. CIRCUIT CONTAINING RESISTANCE AND INDUCTANCE

Let a source of alternating *e.m.f.* be connected to an ohmic resistance R and a coil of inductance L, in series as shown in Fig. 7.19.

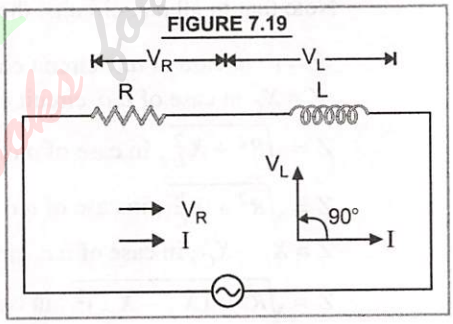


FIGURE 7.19

Proceeding as in Art. 7. 12, and ignoring $\vec{V}_C$ as there is no capacitor in the circuit, we shall obtain

$$Z = \sqrt{R^2 + X_L^2} \quad * \qquad \text{...(56)}$$

Fig. 7.20 represents phasor diagram of RL circuit.

We find that in RL circuit, voltage leads the current by a phase angle ϕ, where

$$\tan\phi = \frac{AK}{OA} = \frac{OL}{OA} = \frac{V_L}{V_R} = \frac{I_0 X_L}{I_0 R}$$

$$\tan\phi = \frac{X_L}{R} \quad ** \qquad \text{...(57)}$$

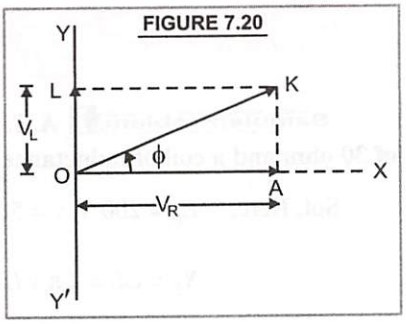


FIGURE 7.20

7.15 A.C. CIRCUIT CONTAINING RESISTANCE AND CAPACITANCE

Let a source of alternating *e.m.f.* be connected to an ohmic resistance R and a condenser of capacity C, in series as shown in Fig. 7.21.

Proceeding as in Art. 7.12 and ignoring $\vec{V}_L$ as there is no inductor in the circuit, we shall obtain

$$Z = \sqrt{R^2 + X_C^2} \quad * ** \qquad \text{...(58)}$$

*This formula is obtained by putting $X_C = 0$ in eqn. (41). **This formula is obtained by putting $X_C = 0$ in eqn. (42). ***This formula is obtained by putting $X_L = 0$ in eqn. (41).

Fig. 7.22 represents phasor diagram of RC circuit. We find that in RC circuit, *voltage lags behind the current* by a phase angle ϕ, where

$$\tan\phi = \frac{AK}{OA} = \frac{OC}{OA} = \frac{V_C}{V_R} = \frac{-I_0 X_C}{I_0 R}$$

$$\boxed{\tan\phi = \frac{-X_C}{R}*} \quad \text{...(59)}$$

FIGURE 7.21

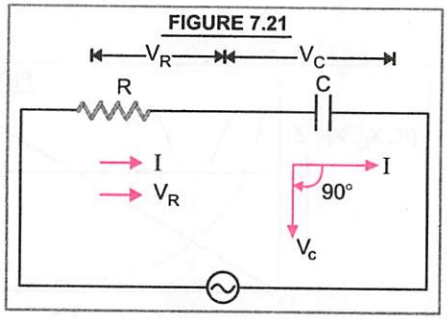
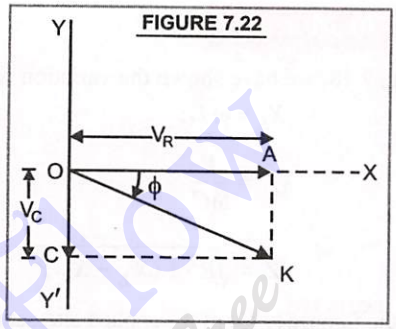


FIGURE 7.22



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Note that in all *a.c.* circuits, the relation that holds is $\frac{E_v}{I_v} = Z$, where

$Z = R$, in case of *a.c.* circuit containing R only, $Z = X_L$ in case of *a.c.* circuit containing L only,

$Z = X_C$ in case of *a.c.* circuit containing C only,

$Z = \sqrt{R^2 + X_L^2}$, in case of *a.c.* circuit containing R & L, *in series*

$Z = \sqrt{R^2 + X_C^2}$, in case of *a.c.* circuit containing R & C, *in series*

$Z = X_L - X_C$, in case of *a.c.* circuit containing L & C, *in series*

$Z = \sqrt{R^2 + (X_L - X_C)^2}$, in case of *a.c.* circuit containing R, L & C *in series.*

The phase relation between alternating voltage and current in any *a.c.* circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$

Sample Problem 1 **A.C. mains of 200 V, 50 Hz is connected to a circuit containing a resistance of 30 ohm and a coil of inductance 4/π henry in series. Calculate the virtual current in the circuit.**

Sol. Here, $E_v = 200$ V, $v = 50$ Hz, $R = 30$ ohm, $L = \frac{4}{\pi}$ henry, $I_v = ?$

$$X_L = \omega L = 2\pi v L = 2\pi \times 50 \times \frac{4}{\pi} = 400 \text{ ohm}$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 400^2} = 401{\cdot}1 \text{ ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{200}{401{\cdot}1} = \mathbf{0{\cdot}4986\ A}$$

*This formula is obtained by putting $X_L = 0$ in eqn. (42). Negative sign is for lagging of alternating voltage behind alternating current.

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Sample Problem 2 **A source of 200 V – 50 Hz a.c. is connected to a resistance of 10 ohm, a capacitor of capacity C and an ammeter in series. If reading of ammeter is 2 A, what is the capacity of condenser ?**

Sol. Here, $E_v = 200$ V, $v = 50$ Hz, $R = 10$ ohm, $C = ?$ $I = 2$ A

From $$Z = \frac{E_v}{I_v} = \frac{200}{2} = 100 \text{ ohm}$$

Again $$R^2 + X_C^2 = Z^2$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{100^2 - 10^2} = 99{\cdot}5$$

As $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C}$$

$\therefore$ $$C = \frac{1}{2\pi v X_C} = \frac{1}{2\times 3{\cdot}14\times 50\times 99{\cdot}5} = \frac{1}{314\times 99{\cdot}5} = \mathbf{3{\cdot}2\times 10^{-6}\ F = 3{\cdot}2\ \mu F}$$

7.16. ENERGY STORED IN AN INDUCTOR

When a.c. is fed to an inductor of inductance L, the current in it grows from zero to maximum steady value I_0. If I is the current at any instant t, then the magnitude of induced e.m.f. developed in the inductor at that instant is

$$E = L\frac{dI}{dt} \qquad \text{...(60)}$$

The self induced e.m.f. is also called the **back e.m.f.,** as it opposes any change in the current in the circuit.

Physically, the self inductance plays the role of inertia. It is the electromagnetic analogue of mass in mechanics. Therefore, work needs to be done against the back e.m.f. E in establishing the current. This work done is stored in the inductor as *magnetic potential energy.*

For the current I at an instant t, the rate of doing work is $\frac{dW}{dt} = EI$

If we ignore the resistive losses, and consider only inductive effect, then

Using (60), $\frac{dW}{dt} = EI = L\frac{dI}{dt}\times I$ or $dW = LI\,dI$

Total amount of work done in establishing the current I is $W = \int dW = \int_0^I LI\,dI = \frac{1}{2}LI^2$

Thus energy required to build up current in an inductor = energy stored in inductor

$$U_B = W = \frac{1}{2}LI^2 \qquad \text{...(61)}$$

For an air core solenoid, $B = \frac{\mu_0 NI}{l}$ $\therefore$ $I = \frac{Bl}{\mu_0 N}$ and $L = \frac{\mu_0 N^2 A}{l}$

$\therefore$ $$U_B = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\times\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{B^2}{2\mu_0}Al$$

Magnetic energy density, $$\overline{U}_B = \frac{U_B}{\text{volume}} = \frac{U_B}{Al}$$

$$\overline{U}_B = \frac{B^2}{2\mu_0}$$

IMPORTANT NOTE

We have already proved in unit 1 that electrostatic energy per unit volume stored in a parallel plate capacitor is $\overline{U}_E = \frac{\epsilon_0 E^2}{2}$

Clearly, in both the cases, *energy per unit volume is directly proportional to the square of the strength of the field.*

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When we compare Eqn. (61) with mechanical kinetic energy of a particle of mass m, *i.e.*, $KE = \frac{1}{2}mv^2$, we find that L corresponds to m.

As mass m is a measure of inertia of linear motion, self inductance L is a measure of electrical inertia, which opposes both, the growth and decay of current in the circuit.

Sample Problem **A 1·5 mH inductor in LC circuit stores a maximum energy of 17 μ J. What is the peak current ?**

Sol. Here, $L = 1{\cdot}5 \text{ mH} = 1{\cdot}5 \times 10^{-3}$ H, $E_0 = 17 \mu$ J $= 17 \times 10^{-6}$ J, $I_0 = ?$

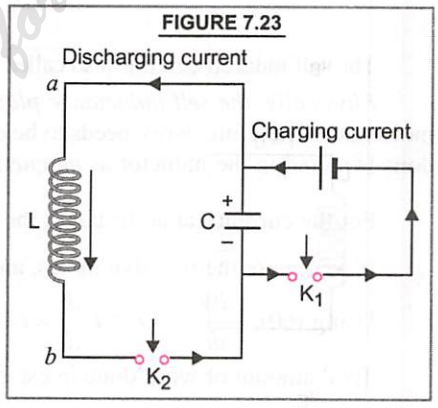
From $E_0 = \frac{1}{2}LI_0^2$; $I_0 = \sqrt{\frac{2E_0}{L}} = \sqrt{\frac{2\times17\times10^{-6}}{1{\cdot}5\times10^{-3}}} = \sqrt{\frac{34}{15}\times10^{-2}} = \mathbf{0{\cdot}15\ A}$

7.17. LC OSCILLATIONS

In Fig. 7.23, a capacitor of capacitance C is connected to an inductor of inductance L through a key K_2. A cell is connected to C through key K_1.

When plug of K_1 is put in, the cell charges the capacitor to a potential $V = q/C$, where q is the charge on capacitor plates at any instant and V is voltage across the plates at the same instant. Some energy from the cell is stored in the dielectric medium between the plates of capacitor in the form of electrostatic energy $\left[U_E = \frac{q^2}{2C}\right]$.

FIGURE 7.23



On removing plug of K_1 and putting in plug of K_2, the charged capacitor is connected to L and starts discharging through L. An induced e.m.f. developes in the circuit which opposes the growth of current in L and hence delays it. When capacitor is completely discharged, the energy stored in the capacitor appears in the form of magnetic field energy around L $\left[U_B = \frac{1}{2}LI^2\right]$.

As soon as discharge of the capacitor is complete, current stops and magnetic flux linked with L starts collapsing. Therefore, an induced e.m.f. developes which starts recharging the condenser in the opposite direction. The recharging is also opposed and hence delayed.. When condenser is recharged completely, the magnetic field energy around L reappears in the form of electric field energy between the plates. The entire process is repeated. Thus, energy taken once from the cell and given to capacitor keeps on oscillating between C and L. If the circuit has no resistance, no loss of energy would occur. The oscillations produced will be of constant amplitude, Fig. 7.24. These are called **undamped oscillations.**

FIGURE 7.24

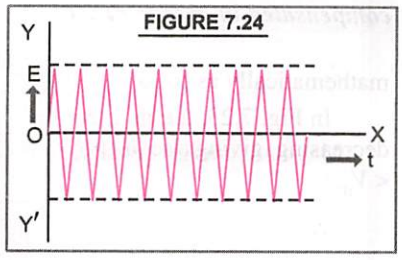


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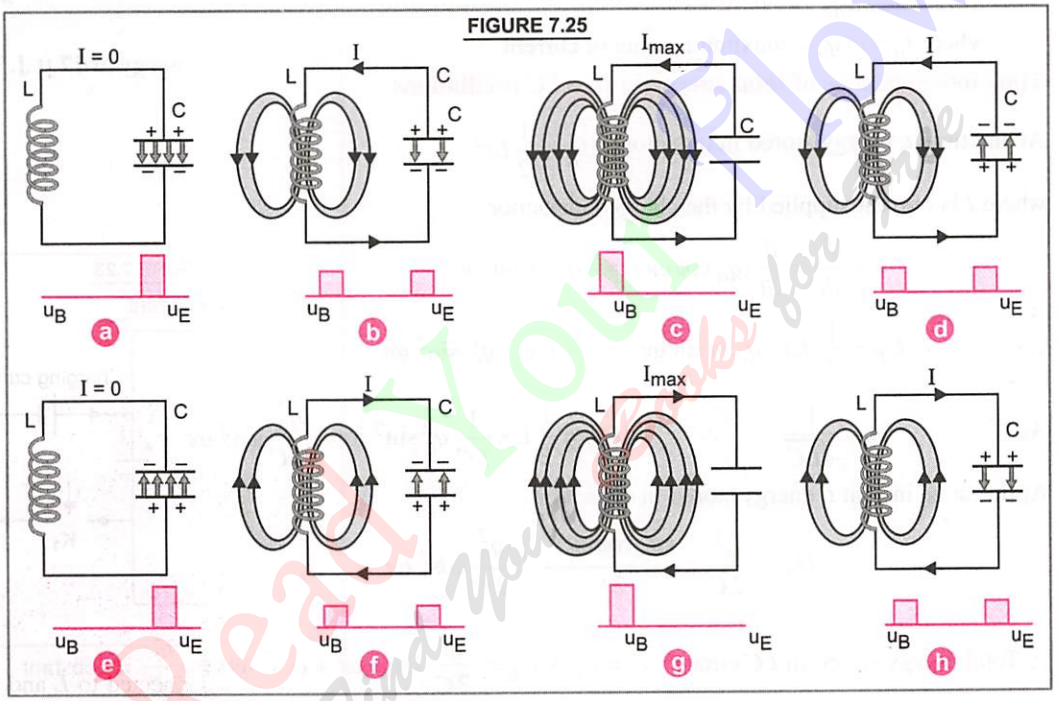
Figure 7.25 shows eight stages in a single cycle of oscillation of a resistanceless *LC* circuit. The bar graphs in each figure represent the stored electric energy $\left(U_E = \frac{q_0^2}{2C}\right)$ and stored magnetic energy $\left(U_B = \frac{1}{2}LI_0^2\right)$. The magnetic field lines of inductor and electric field lines of capacitor are also shown.

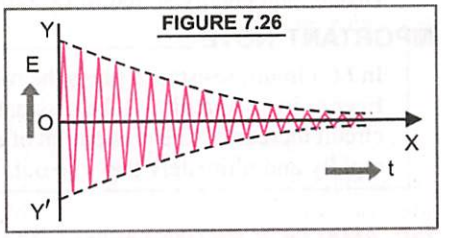
Fig. 7.25(*a*) shows capacitor with maximum charge and no current in the circuit. Fig. 7.25(*b*) shows discharging of capacitor with increasing current. Magnetic lines of force through *L* are as per right hand rule. Fig. 7.25(*c*) shows capacitor fully discharged and maximum current. Fig. 7.25(*d*) shows charging of capacitor in opposite direction and current decreasing. Fig. 7.25(*e*) shows capacitor recharged fully with polarity opposite to that of (*a*) and no current. Fig. 7.25(*f*) shows discharging of capacitor, current increasing in opposite direction. Fig. 7.25(*g*) shows fully discharged capacitor and maximum current in opposite direction. Fig. 7.25(*h*) shows charging of capacitor with current decreasing as shown. This cycle is being repeated.

FIGURE 7.25



In actual practice, however, there do occur some losses of energy. Therefore, amplitude of oscillations goes on decreasing. These are called damped oscillations as shown in Fig. 7.26.

FIGURE 7.26



To obtain undamped oscillations, the energy loss is duly compensated in proper phase.

The frequency of electrical oscillations. It can be obtained mathematically as follows :

In Fig. 7.23, the moment the circuit is completed (by putting in plug of K_2), charge on condenser starts decreasing, giving rise to current in the circuit. As dI/dt is positive, the induced e.m.f. in *L* will be such that $V_b < V_a$.

$$\therefore \quad \frac{q}{C} - L\frac{dI}{dt} = 0 \qquad \text{...(62)}$$

As q decreases ; I increases, therefore $I = -\frac{dq}{dt}$ ...(63)

Eqn. (62) becomes, $\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$ or $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$ ...(64)

This equation has the form $\frac{d^2x}{dt^2} + \omega^2 x = 0$ for a simple harmonic oscillator.

The charge, therefore, oscillates with a natural frequency,

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi\nu \quad \therefore \quad \boxed{\nu = \frac{1}{2\pi\sqrt{LC}}} \quad ...(65)$$

The variation of charge with time is represented as $q = q_0 \cos \omega t$

The current, $I = \frac{-dq}{dt} = q_0\, \omega \sin \omega t = I_0 \sin \omega t$ ...(66)

where $I_0 = \omega\, q_0$ = maximum value of current

Time independence of total energy in free *LC* oscillations

At any time t, energy stored in inductor, $U_B = \frac{1}{2}LI^2$

where I is current supplied by the charged capacitor

$$I = \frac{dq}{dt} = \frac{d}{dt}(q_0 \cos \omega t) = -\, q_0\, \omega \sin \omega t$$

$$\therefore \quad U_B = \frac{1}{2}\, L\, (-\, q_0\, \omega \sin \omega t)^2 = \frac{1}{2}\, L\, \omega^2\, q_0^2\, \sin^2 \omega t$$

As $\omega = \frac{1}{\sqrt{LC}}$ $\therefore$ $U_B = \frac{1}{2}L \times \frac{1}{LC} q_0^2 \sin^2 \omega t = \frac{q_0^2}{2C}\sin^2 \omega t$

At the same instant t, energy stored in capacitor

$$U_E = \frac{q^2}{2C} = \frac{(q_0 \cos \omega t)^2}{2C} = \frac{q_0^2}{2C}\cos^2 \omega t$$

$\therefore$ Total energy stored in *LC* circuit, $U = U_B + U_E = \frac{q_0^2}{2C}\,(\sin^2 \omega t + \cos^2 \omega t) = \frac{q_0^2}{2C}$ = constant

Hence total energy stored in *LC* circuit at every instant is constant. It does not depend upon time.

IMPORTANT NOTE

In *LC* circuit, resistance plays the role of friction, which would decrease the amplitude of oscillations. Energy in *LC* circuit will be dissipated in the form of heat. With rise in temperature, resistance of *LC* circuit increases. The dissipation of energy becomes faster. The amplitude of *LC* oscillations decreases rapidly and ultimately they die out.

7.18. ANALOGY BETWEEN MECHANICAL AND ELECTRICAL QUANTITIES

The *LC* oscillations can be compared to mechanical oscillations of a block of mass m attached to a spring of force constant k, whose equation is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}} \quad ...(67)$$

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In case of mechanical system, $F = ma = m\dfrac{d^2x}{dt^2}$

For an electrical system, $E = -L\dfrac{dI}{dt} = -L\dfrac{d^2q}{dt^2}$

Comparison shows that q corresponds to x and L corresponds to m.

In case of LC circuit, $\omega = \dfrac{1}{\sqrt{LC}}$

Comparing with (67), we find that $\dfrac{1}{C}$ is analogous to k (= F/x). Whereas k tells us the (external) force required to produce a unit displacement ; $\dfrac{1}{C} = \dfrac{V}{q}$ tells us the potential difference required to store a unit charge. Table 7.1 gives analogy between mechanical and electrical quantities.

TABLE 7.1. Analogy between mechanical and electrical quantities

S. No.	Mechanical System	S. No.	Electrical System
1.	mass m	1.	Inductance L
2.	Force constant k	2.	Reciprocal capacitance $1/C$
3.	displacement x	3.	charge q
4.	velocity, $v = \dfrac{dx}{dt}$	4.	current, $i = dq/dt$
5.	Mechanical energy, $E = \dfrac{1}{2}kx^2 + \dfrac{1}{2}mv^2$	5.	Electromagnetic energy, $U = \dfrac{q^2}{2C} + \dfrac{1}{2}LI^2$

Sample Problem 1 A charged 30 μ F capacitor is connected to a 27 mH inductor. What is the frequency of free oscillations of the circuit ?

Sol. Here, $C = 30\ \mu\text{F} = 30 \times 10^{-6}$ F, $L = 27$ mH $= 27 \times 10^{-3}$ H, $v = ?$

$$v = \frac{1}{2\pi\sqrt{LC}} = \frac{1\times 7}{2\times 22\sqrt{27\times10^{-3}\times30\times10^{-6}}} = \frac{7\times10^4}{44\times9} = \mathbf{1{\cdot}767\times10^2\ cps}$$

Sample Problem 2 A capacitor, a 101·5 mH inductor and a 5 ohm resistor are connected in series with a 50 Hz a.c. source. If current and voltage in the circuit are in phase, what is the capacity of the condenser ?

Sol. Here, $L = 101{\cdot}5$ mH $= 101{\cdot}5 \times 10^{-3}$ H, $R = 5$ ohm, $v = 50$ Hz, $C = ?$

As current and voltage are in phase, therefore $X_C = X_L$

$$\frac{1}{\omega C} = \omega L$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi v)^2 L} = \frac{1}{4\pi^2 v^2 L} = \frac{1}{4\times(3{\cdot}14)^2\times(50)^2\times101{\cdot}5\times10^{-3}}$$

$$C = 99{\cdot}8 \times 10^{-6}\ \text{F} = \mathbf{99{\cdot}8\ \mu F}$$

7.19. ELECTRIC RESONANCE

The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the **natural frequency of oscillation** of the system. If such a system is driven by an energy source, whose frequency is equal to the natural frequency of the system, the amplitude of oscillations becomes large and resonance is said to occur. For example, a child on a swing has a natural

frequency for swinging back and forth. If another child pushes the swing at regular intervals, and the frequency of the pushes is almost the same as the frequency of swinging, the amplitude of swing becomes very large. We say that resonance has occurred.

(a) Series Resonance Circuit

A circuit in which inductance L, capacitance C and resistance R are connected in series, and the circuit admits maximum current corresponding to a given frequency of a.c., is called series resonance circuit.

The impedance (Z) of an RLC circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad ...(68)$$

At very low frequencies, inductive reactance $X_L = \omega L$ is negligible, but capacitative reactance ($X_C = 1/\omega C$) is very high.

As frequency of alternating e.m.f. applied to the circuit is increased, X_L goes on increasing and X_C goes on decreasing. For a particular value of ω ($= \omega_r$, say)

$$X_L = X_C, \quad i.e., \quad \omega_r L = \frac{1}{\omega_r C} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

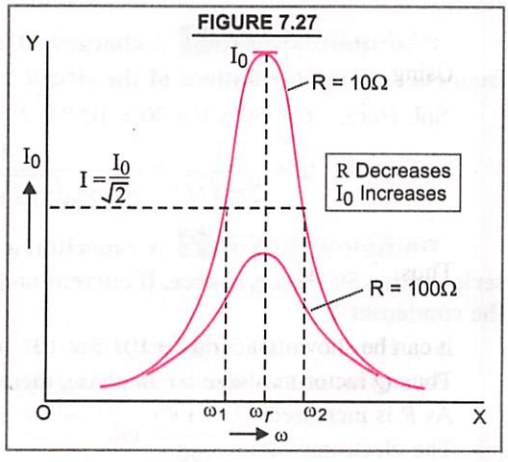
$$2\pi \nu_r = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \nu_r = \frac{1}{2\pi\sqrt{LC}}$$

At this particular frequency ν_r, as $X_L = X_C$, therefore,

from (68), $Z = \sqrt{R^2 + 0} = R =$ minimum

i.e., impedance of RLC circuit is minimum and hence the current $I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ becomes maximum. This frequency is called **series resonance frequency. Note that resonance frequency is independent of the resistance (R) of the circuit.** We have said in Art 7.17 that $\frac{1}{2\pi\sqrt{LC}}$ is the natural frequency of oscillation of an LC circuit containing no resistance.

FIGURE 7.27



The variation of circuit peak current with the changing frequency of applied voltage is shown in Fig. 7.27. It is clear that for frequencies greater than or less than ω_r the values of peak current are less than the maximum value (I_0).

Further, at $\omega = \omega_r$, value of peak current is maximum (I_0). The maximum value of peak current is inversely proportional to R. For lower R values, I_0 is large and *vice-versa*.

As series resonance circuit admits maximum peak current through it at resonance, it is called **Acceptor circuit. The practical application of series resonance circuit is in radio and T.V. receiver sets.** The antenna of a radio/T.V. intercepts signals from many broadcasting stations. To receive one particular radio station/T.V. channel, we tune our receiver set by changing the capacitance of a capacitor in the tuning circuit of the set such that resonance frequency of the circuit becomes equal to the frequency of the desired station. Therefore, resonance occurs. The amplitude of current with the frequency of the signal from the desired station becomes maximum and it is received in our set.

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In series *LCR* circuit, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At resonance, $\omega L = \frac{1}{\omega C}$ $\therefore$ $Z = R$

i.e., at resonance, series *LCR* circuit is equivalent to a purely resistive circuit. Hence current and voltage are in phase.

DO YOU KNOW ?

The phenomenon of Resonance is exhibited by a circuit, when both *L* and *C* are present in the circuit. Only then, the voltages across *L* and *C* may cancel each other, being opposite in phase. The total source voltage appears across *R* giving maximum current $I_0 = E_0/R$.

Obviously, we cannot obtain resonance in *RL* or *RC* circuit.

7.20. Q FACTOR OF RESONANCE CIRCUIT OR SHARPNESS OF RESONANCE

The characteristic of a series resonant circuit is determined by the *Q* factor or Quality factor of the circuit. It defines *sharpness of tuning at resonance*.

The Q factor of series resonant circuit is defined as the ratio of the voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the voltage applied across R.

i.e., $Q = \dfrac{\text{voltage across } L}{\text{applied voltage } (= \text{voltage across } R)}$ or $\dfrac{\text{voltage across } C}{\text{applied voltage } (= \text{voltage across } R)}$

$$Q = \frac{(\omega_r L) I}{RI} = \frac{\omega_r L}{R} \quad \text{or} \quad Q = \frac{(1/\omega_r C) I}{RI} = \frac{I}{RC\,\omega_r}$$

Using $\omega_r = \dfrac{1}{\sqrt{LC}}$, we get

$$Q = \frac{L}{R}\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{or} \quad Q = \frac{1\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \text{...(69)}$$

It can be shown that ***Q* is just a number having no dimensions.**

Thus *Q* factor may also be taken as **voltage multiplication factor of the circuit.**

As *R* is increased, *Q* factor of the circuit decreases. This is clear from Fig. 7.27.

The electronic circuits with high *Q* values would respond to a very narrow range of frequencies and *vice-versa*. The values of *Q* usually vary from 10 to 100. However, the electronic circuits dealing with very high frequencies may have $Q = 200$. It should be clearly understood that **higher the value of *Q*, the narrower and sharper is the resonance.**

Another definition of Q factor

Suppose we choose a value of ω for which the current amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value. From Fig. 7.27, we find that there are two such values of ω, symmetrical about ω_0, say ω_1 and ω_2, one smaller and the other greater than ω_r. We may write $\omega_1 = (\omega_r - \Delta\,\omega)$ and $\omega_2 = (\omega_r + \Delta\,\omega)$

The difference $(\omega_2 - \omega_1) = 2\,\Delta\,\omega$ is often called the **band width** of the circuit.

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The quantity $\left(\frac{\omega_r}{2\Delta\omega}\right)$ *is regarded as a measure of sharpness of resonance, i.e., Q factor of resonance circuit is the ratio of resonance angular frequency to band width of the circuit (which is difference in angular frequencies at which power is half the maximum power or current is* $I_0/\sqrt{2}$ *).*

IMPORTANT NOTE

If Q factor is large, the band width ($2\Delta\omega$) is small and resonance curve is sharper. It implies that the circuit is **more selective.** On the contrary, if Q factor is small, the bandwidth ($2\Delta\omega$) is large and resonance curve is less sharp. It implies that the circuit is close to resonance for large range of frequencies and hence it is less selective.

Curiosity Question

Q. What is the significance of high Q-value circuits ?

Ans. As is known, the Q factor or Quality factor of an electronic circuit defines *sharpness of tuning at resonance.* It is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Q is only a *number* having no dimensions. It represents voltage amplification factor of the circuit.

At the receiving station, many radio signals are often present over a range of frequencies. In order to eliminate unwanted signals, we need to design high Q value circuit. Higher the value of Q, narrower and sharper is the resonance. Thus the electronic circuits with high Q values would respond to a very narrow range of frequencies. The values of Q usually vary from 10 to 100. The electronic circuits dealing with very high frequencies may have $Q = 200$.

7.21. PARALLEL RESONANCE CIRCUIT

A parallel resonance circuit consists of a coil of inductance L and a condenser of capacity C, joined in parallel to a source of alternating e.m.f. as shown in Fig. 7.28.

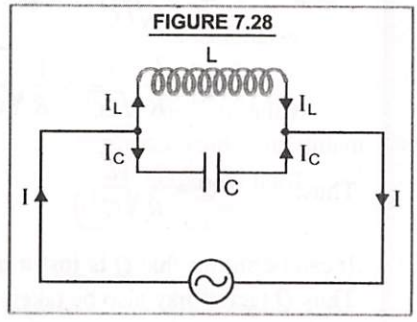


FIGURE 7.28

Let the alternating e.m.f. supplied by the source be

$$E = E_0 \sin \omega t \qquad ...(70)$$

As current through L lags behind the applied alternating e.m.f. by a phase angle $\pi/2$, therefore, we may write the value of current through L at a particular instant of time t as

$$I_L = \frac{E_0}{X_L}\sin(\omega t - \pi/2) \qquad ...(71)$$

Again, as current through C leads the applied alternating e.m.f. by a phase angle $\pi/2$, therefore, we may write the value of current through C at the same instant t as

$$I_C = \frac{E_0}{X_C}\sin(\omega t + \pi/2) \qquad ...(72)$$

The total current I in the circuit at this instant is, therefore, $I = I_L + I_C$

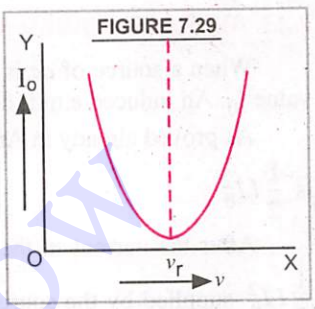
$$I = \frac{E_0}{X_L}\sin(\omega t - \pi/2) + \frac{E_0}{X_C}\sin(\omega t + \pi/2)$$

$$I = \frac{E_0}{X_L}(-\cos\omega t) + \frac{E_0}{X_C}\cos\omega t = E_0\cos\omega t\left[-\frac{1}{X_L} + \frac{1}{X_C}\right]$$

$$I = E_0 \cos \omega t \left[-\frac{1}{\omega L} + \omega C \right] \quad ...(73)$$

We find that $I = 0$, when $\omega C - \frac{1}{\omega L} = 0$, *i.e.*, $\omega C = \frac{1}{\omega L}$ or $\omega^2 = \frac{1}{LC}$

or $\omega = \frac{1}{\sqrt{LC}}$ or $2\pi\nu = \frac{1}{\sqrt{LC}}$ or $\boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$



i.e., frequency of applied alternating e.m.f. becomes equal to natural frequency of oscillation of the circuit. Hence *resonance* occurs. The circuit is called parallel resonance circuit and this frequency ν is called **parallel resonance frequency.**

Therefore, at parallel resonance frequency, $I = 0$, *i.e.*, the resonance circuit does not allow any current to flow through it, as shown in Fig. 7.29. The impedance of parallel resonance circuit at this frequency must obviously be maximum.

The parallel resonance circuits are used in the *transmitting circuits*. They reject the currents corresponding to parallel resonance frequencies, and allow other frequencies to pass through. Such circuits are, therefore, called ***filter circuits or rejector circuits or even anti resonance circuits, in a communication network.***

7.22. AVERAGE POWER ASSOCIATED WITH RESISTANCE OR NON-INDUCTIVE CIRCUIT

Power, we know, is defined as the rate of doing work. In a.d.c. circuit, power is given by the product of voltage and current. However, in an a.c. circuit, values of voltage and current change every instant. Therefore, *power in an a.c. circuit at any instant is the product of instantaneous voltage (E) and instantaneous current (I)*.*

In a pure resistance, the alternating current developed is in phase with the alternating voltage applied, *i.e.*, when

$E = E_0 \sin \omega t$, then $I = I_0 \sin \omega t$

Instantaneous power $= E I = (E_0 \sin \omega t)(I_0 \sin \omega t)$

$= E_0 I_0 \sin^2 \omega t$

If the instantaneous power remains constant for a small time dt, then small amount of work done in maintaining the current for a small time dt is $dW = E_0 I_0 \sin^2 \omega t \, dt$

Total work done or energy spent in maintaining current over one full cycle

$$W = \int_0^T E_0 I_0 \sin^2 \omega t \, dt = E_0 I_0 \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt = \frac{E_0 I_0}{2} \int_0^T dt - \frac{E_0 I_0}{2} \int_0^T \cos 2\omega t \, dt$$

$$W = \frac{E_0 I_0}{2} T - 0$$

$\therefore$ Average power supplied to R over a complete cycle

$$P = \frac{W}{T} = \frac{E_0 I_0}{2} \cdot \frac{T}{T} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = E_v \, . \, I_v \quad i.e. \quad \boxed{P = E_v I_v} \quad ...(74)$$

Hence average power over a complete cycle of a.c. through the resistor is the product of virtual voltage and virtual current.

* $E.I = \left(\frac{dW}{dq} \right) \left(\frac{dq}{dt} \right) = \frac{dW}{dt}$ = Rate of energy consumption by the given circuit at any instant t = Power at that instant.

Sample Problem **A 200 ohm electric iron is connected to 220 V, 50 Hz a.c. supply. Calculate the average power delivered to iron.**

Sol. Here, $R = 200\ \Omega$, $E_v = 220$ V, $\nu = 50$ Hz

Av. power/cycle, $P = E_v\ I_v = E_v\ (E_v/R) = \dfrac{220 \times 220}{200} = 242$ W

7.23. AVERAGE POWER ASSOCIATED WITH AN INDUCTOR

When a source of *ac* is connected to an inductor L, current in it grows from zero to maximum steady value I_0. An induced e.m.f. developes in the inductor, which opposes the growth of current.

As proved already in Art. 7.16, work done by the external source in building up current from zero to I_0 is $\frac{1}{2} LI_0^2$.

After the current in the inductor reaches its maximum value I_0, it falls from I_0 to zero. The energy $\frac{1}{2} LI_0^2$ supplied by the source during build up of current is returned back to the source during the fall of current.

Thus net power supplied by the source in a complete cycle is zero.

Mathematically, work done in one complete cycle is $W = \int_0^T EI\, dt$ ...(75)

If the alternating voltage is $E = E_0 \sin \omega t$,

then through L, as current lags behind E by a phase angle of $\frac{\pi}{2}$, therefore

$$I = I_0 \sin (\omega t - \pi/2) = -I_0 \cos \omega t$$

Put in (75), $$W = -\int_0^T E_0 \sin \omega t . I_0 \cos \omega t\, dt$$

$$W = -E_0 I_0 \int_0^T \sin \omega t \cos \omega t\, dt = \frac{-E_0 I_0}{2} \int_0^T 2 \sin \omega t \cos \omega t\, dt$$

$$W = \frac{-E_0 I_0}{2} \int_0^T \sin 2\omega t\, dt = \frac{-E_0 I_0}{2} \left[\left(-\frac{\cos 2\omega t}{2\omega} \right) \right]_0^T = \text{Zero}$$

Therefore, average power over a complete cycle of a.c. through an ideal inductor is zero. Infact, whatever energy is needed in building up current in L is returned back during the decay of current.

The magnetization and demagnetization of an inductor can be understood as follows :

Fig. 7.30(a) represents *one complete cycle* of alternating voltage E, alternating current I and the consequent magnetic flux ϕ along with power P. Note that current lags behind the voltage by 90°.

Fig. 7.30(b) shows current I entering the coil at A and increasing from zero to maximum from $t = 0$ to $t = T/4$. The magnetic flux lines are set up as shown and core gets magnetized. As voltage and current both are positive, their product P is positive. Therefore, *energy is absorbed from the source.*

From $t = T/4$ to $t = 2\ T/4$, current in the coil is still positive, but is decreasing. The core gets demagnetised, Fig. 7.30(c), and the net magnetic flux becomes zero at the end of half cycle. As dI/dt is negative, voltage E is negative. The product of voltage and current is negative and *energy is being returned to the source.*

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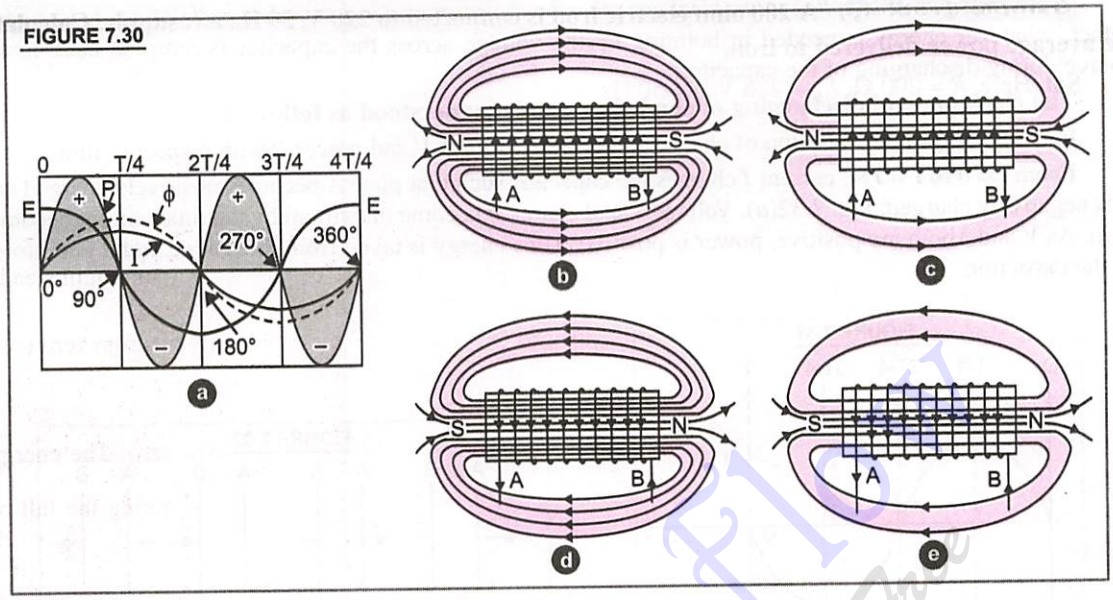
FIGURE 7.30



From $t = 2\ T/4$ to $t = 3\ T/4$, current I becomes negative, *i.e.*, it enters the coil at B and leaves at A, as shown in Fig. 7.30(d). As the direction of current is reversed, polarity of magnet is reversed. Since current and voltage, both are negative, their product P is positive. The *energy is absorbed from the source.*

From $t = 3\ T/4$ to $t = T$, I decreases and reaches its zero value at $t = T$. The core is demagnetised and magnetic flux becomes zero, Fig. 7.30(e). Voltage is positive and current is negative. Their product P is negative. *Energy absorbed during 2 T/4 to 3 T/4 is returned to the source.*

7.24. AVERAGE POWER ASSOCIATED WITH A CAPACITOR

When a source of a.c. is connected to a capacitor of capacitance C, the charge on it grows from zero to maximum steady value Q_0. A definite amount of work is done in charging the capacitor which is stored in the capacitor in the form of electrostatic energy.

As derived already in chapter 2, the energy stored in capacitor is $E = \frac{1}{2} CV_0^2$,

where V_0 is maximum potential difference across the plates of the capacitor.

We can show that energy drawn from the source during charging of capacitor is returned back to source during discharging of the capacitor so that net energy supplied by the source over a complete cycle is zero.

Mathematically, if alternating voltage applied is

$$E = E_0 \sin \omega t$$

then through C, as current leads the e.m.f. by a phase angle of $\pi/2$

$$\therefore \quad I = I_0 \sin (\omega t + \pi/2) \quad ; \quad I = I_0 \cos \omega t$$

Work done over a complete cycle is

$$W = \int_0^T EI\, dt = \int_0^T (E_0 \sin \omega t)(I_0 \cos \omega t)\, dt = \frac{E_0 I_0}{2} \int_0^T 2 \sin \omega t \cos \omega t\, dt = \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t\, dt$$

$$W = \frac{E_0 I_0}{2} \left[-\frac{\cos 2\omega t}{2\omega} \right]_0^T = \text{Zero}$$

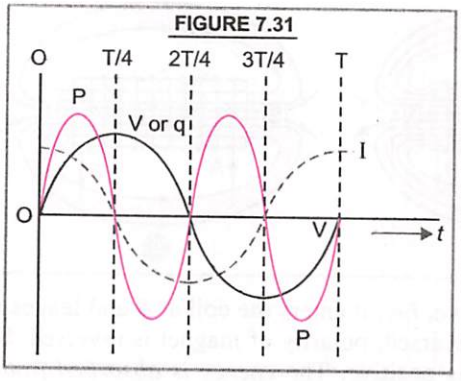
∴ Average power supplied to an ideal capacitor by the source over a complete cycle of a.c. is also zero.

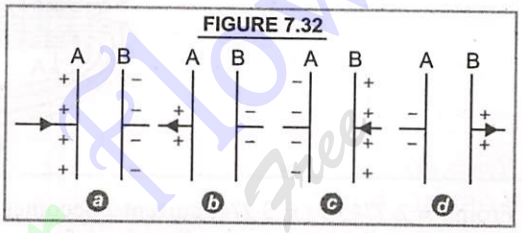
Infact, whatever energy is needed in building up the voltage across the capacitor is returned back to the source during discharging of the capacitor.

The charging and discharging of condenser can be understood as follows :

Fig. 7.31 shows the variation of charge q, current I, voltage V and power P with respect to time t.

From $t = 0$ to $t = T/4$, current I charges the capacitor such that plate A becomes positively charged and B is negatively charged, Fig. 7.32(a). Voltage V and charge q become maximum by the time current becomes zero. As V and I both are positive, power is positive. Thus energy is taken from the source and it gets stored in the capacitor.





From $t = T/4$ to $t = T/2$, current I changes its polarity. Therefore, capacitor starts getting discharged. Fig. 7.32(b), voltage is positive but getting reduced. As current is negative, power is negative. Energy is given back to the supply.

From $t = 2\ T/4$ to $t = 3\ T/4$, current I charges the capacitor such that plate B becomes positive and plate A becomes negative, Fig. 7.32(c). Since I and V, both are negative, power is positive. The capacitor takes energy from the source.

From $t = 3\ T/4$ to $t = T$, current I reverses polarity. Therefore, the capacitor starts getting discharged, Fig. 7.32(d). Voltage is reducing and current is positive. Therefore, power is negative. It implies that energy is sent back to the source. Thus, whatever energy is needed in building up the voltage across the capacitor is returned back to the source during discharging of the capacitor.

7.25. AVERAGE POWER IN RLC CIRCUIT OR INDUCTIVE CIRCUIT

Let the alternating e.m.f. applied to an RLC circuit be $E = E_0 \sin \omega t$ ...(76)

If alternating current developed lags behind the applied e.m.f. by a phase angle ϕ, then

$$I = I_0 \sin (\omega t - \phi) \quad \text{...(77)}$$

Power at instant t, $\dfrac{dW}{dt} = E\,I$

$$\frac{dW}{dt} = E_0 \sin \omega t \times I_0 \sin (\omega t - \phi) = E_0 I_0 \sin \omega t (\sin \omega t \cos \phi - \cos \omega t \sin \phi)$$

$$= E_0 I_0 \sin^2 \omega t \cos \phi - E_0 I_0 \sin \omega t \cos \omega t \sin \phi$$

$$= E_0 I_0 \sin^2 \omega t \cos \phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin \phi$$

If this instantaneous power is assumed to remain constant for a small time dt, then small amount of work done in this time is

$$dW = \left(E_0 I_0 \sin^2 \omega t \cos \phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin \phi \right) dt$$

Total work done over a complete cycle is

$$W = \int_0^T E_0 I_0 \sin^2 \omega t \cos\phi \, dt - \int_0^T \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi \, dt$$

$$W = E_0 I_0 \cos\phi \int_0^T \sin^2 \omega t \, dt - \frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t \, dt$$

As $\int_0^T \sin^2 \omega t \, dt = \frac{T}{2}$ and $\int_0^T \sin \omega t \, dt = 0$ $\therefore$ $W = E_0 I_0 \cos\phi \times \frac{T}{2}$

$\therefore$ Average power in the inductive circuit over a complete cycle

$$P = \frac{W}{T} = \frac{E_0 I_0 \cos\phi}{T} \cdot \frac{T}{2} = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$P = E_v I_v \cos\phi = E_v \left(\frac{E_v}{Z}\right) \cos\phi \qquad \text{...(78)}$$

Hence average power over a complete cycle in an inductive circuit is the product of virtual e.m.f., virtual current and cosine of the phase angle between the voltage and current.

Retain in Memory

The relation (78) is applicable to all a.c circuits. Cos ϕ and Z will have appropriate values for different circuits.

For example :

(*i*) In RL circuit, $Z = \sqrt{R^2 + X_L^2}$ and $\cos\phi = \frac{R}{Z}$

(*ii*) In RC circuit, $Z = \sqrt{R^2 + X_C^2}$ and $\cos\phi = \frac{R}{Z}$

(*iii*) In LC circuit, $Z = X_L - X_C$ and $\phi = 90°$

(*iv*) In RLC circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\cos\phi = \frac{R}{Z}$

In all a.c. circuits, $I_v = \frac{E_v}{Z}$

Sample Problem 1 **A coil of 1 H and 100 Ω resistance has a peak voltage of $5\sqrt{2}$ volt, 50 hz connected across it. Calculate the current through the coil and power absorbed.**

Sol. Here, $L = 1$ H, $R = 100\ \Omega$, $E_0 = 5\sqrt{2}$ V, $\nu = 50$ hz, $I_v = ?$ $P = ?$

$$X_L = \omega L = 2\pi \nu L = 2\pi \times 50 \times 1 = 100\pi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{100^2 + (100\pi)^2} = 3{\cdot}3 \times 10^2\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{E_0}{\sqrt{2}\,(Z)} = \frac{5\sqrt{2}}{\sqrt{2} \times 3{\cdot}3 \times 10^2} = \mathbf{1{\cdot}52 \times 10^{-2}\ A}$$

$$P = E_v I_v \cos\phi = E_v I_v (R/Z)$$

$$P = 5 \times 1{\cdot}52 \times 10^{-2} \times \frac{100}{3{\cdot}3 \times 10^2} = \mathbf{2{\cdot}29 \times 10^{-2}\ watt}$$

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Sample Problem 2 **The voltage and current in an a.c. circuit are given by $E = 300 \sin(\omega t + \pi/2)$ and $I = 5 \sin \omega t$. What is power factor of the circuit and power dissipated in each cycle ?** (CBSE 2001)

Sol. Here, $E = 300 \sin(\omega t + \pi/2)$, $I = 5 \sin \omega t$

$\therefore$ $E_v = \frac{300}{\sqrt{2}} V$ and $I_v = \frac{5}{\sqrt{2}} A$, $\phi = \pi/2$

Power factor = $\cos \phi = \cos \pi/2 =$ Zero

Average power dissipated/cycle $= E_v I_v \cos\phi = \frac{300}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \cos 90° = \mathbf{0}$

7.26. POWER FACTOR OF AN A.C. CIRCUIT

Power factor of an a.c. circuit is defined as the ratio of true power to apparent power of the circuit.

In Art. 4 (b).25, we have proved that average power/cycle in an inductive circuit is

$$P = E_v \; I_v \cos\phi \qquad ...(78)$$

Here, P is called *true power*, $(E_v \; I_v)$ is called *apparent power* or virtual power.

Thus, $$\text{Power factor} = \frac{\text{true power } (P)}{\text{apparent power } (E_v \; I_v)} = \cos\phi \qquad ...(79)$$

$$= \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$ [from impedance triangle]

$\therefore$ $$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}} \qquad ...(80)$$

Power Factor is always positive, but less than one or at the most equal to one.

(*i*) For circuit containing pure resistance R, only $\phi = 0°$.

$\therefore$ Power factor = $\cos\phi = \cos 0° = 1$.

This is the max. value of power factor.

(*ii*) For circuit containing inductor L only, power factor, $\cos\phi = \cos 90° =$ Zero.

(*iii*) For circuit containing pure capacitor only power factor, $\cos\phi = \cos 90° =$ Zero.

Hence average power consumed in a pure inductor or ideal capacitor is zero.

(*iv*) For R–L circuit, power factor $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$.

(*v*) *For RC circuit,* power factor $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$.

(*vi*) *In RLC circuit,* power factor $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

(*vii*) *In a non-inductive circuit,* $X_L = X_C$ $\therefore$ $Z = R$

Power factor = $\cos\phi = \frac{R}{Z} = 1$

This is the maximum value of power factor.

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Current through pure L or pure C, which consumes no power for its maintenance in the circuit is called Idle current or Wattless current.

In actual practice, we do not have ideal inductor or ideal capacitor. Therefore, there does occur some dissipation of energy. However, **inductance and capacitance continue to be most suitable for controlling current in a.c. circuits with minimum loss of power.**

Retain in Memory

At resonance, $X_L = X_C$ and $\phi = 0°$. $\therefore \quad \cos\phi = \cos 0° = 1$

Therefore, maximum power is dissipated in a circuit at resonance.

Further, note that whatever the circuit, power dissipation is always through resistance R.

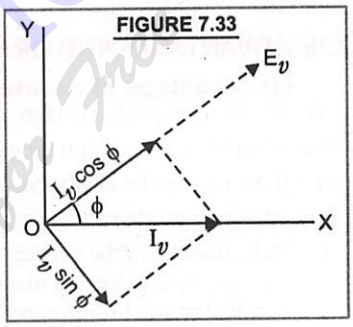
7.27. WATTLESS CURRENT OR IDLE CURRENT

The current which consumes no power for its maintenance in the circuit is called wattless current or Idle current.

We have proved that average power over a complete cycle in an inductive circuit is the product of virtual e.m.f. (E_v), virtual current (I_v) and cosine of the phase angle (ϕ) between the e.m.f. and current ; *i.e.*, $P = E_v\ I_v \cos\phi$.

In a particular circuit, suppose E_v leads I_v by phase angle ϕ, as shown in Fig. 7.33.

FIGURE 7.33



I_v can be resolved into two rectangular components :

$I_v \cos\phi$ along E_v and $I_v \sin\phi \perp E_v$

Thus I_v is the vector sum of two perpendicular components $I_v \cos\phi$ and $I_v \sin\phi$.

As phase angle between E_v and $I_v \cos\phi$ is zero.

$\therefore$ average power consumed per cycle in the circuit due to component $I_v \cos\phi$ is

$$P_{av} = E_v\ (I_v \cos\phi) \cos 0° = E_v\ I_v \cos\phi$$

Again, as phase angle between E_v and $I_v \sin\phi$ is $\pi/2$, therefore, average power consumed per cycle in the circuit due to component $I_v \sin\phi$ is

$$P_{av}' = E_v\ (I_v \sin\phi) \cos \pi/2 = 0$$

Thus, the average power consumed/cycle in the a.c. circuit is wholly due to component $I_v \cos\phi$ of the virtual current. The component $I_v \sin\phi$ makes no contribution to the consumption of power in the a.c. circuit. That is why this component $I_v \sin\phi$ is called the **idle component** or **wattless component** of a.c.

IMPORTANT NOTE

It should be noted that at the generation stage, we try to have $\cos\phi$ = maximum, *i.e.*, minimum of wattless current. Contrary to this, at the consumption stage, we try to have as much wattless current as possible, *i.e.*, $\cos\phi$ = minimum.

Sample Problem **A series LCR circuit with $R = 10$ ohm, $L = 2$ H and $C = 25\ \mu$ F is connected to a variable frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle ?**

Sol. Here, $R = 10$ ohm, $L = 2$ H, $C = 25\ \mu$ F

$E_v = 200$ V.

When the frequency of the supply equals the natural frequency of the circuit,

$$X_L = X_C \quad \text{and} \quad Z = R$$

$$I_v = \frac{E_v}{R} = \frac{200}{10} = 20\text{ A}$$

Average power transferred/cycle = $E_v\ I_v = 200 \times 20 =$ **4000 watt**

7.28. REAL RESISTORS, INDUCTORS AND CAPACITORS

The resistors, inductors and capacitors used actually are not the pure or ideal ones.

For example, a metal wire used as a *resistor* has some little inductance as a magnetic field is developed around the wire carrying current. Two wires in a circuit carrying current may also possess some little non-zero capacitance.

Similarly, an *inductor* which is a wire wound in the form of a coil does possess some Ohmic resistance. Also, there must be some capacitance between every two turns of the inductor coil.

Again, a capacitor has a very high resistance of the dielectric between the plates. Wire leads of the capacitor also possess some little inductance.

Thus in actual practice, we cannot have pure or ideal resistors, inductors or capacitors.

7.29. ADVANTAGES AND DRAWBACKS OF A.C. OVER D.C.

(*a*) Advantages of a.c. over d.c. are :

(*i*) Generation, transmission and distribution of a.c. is much more economical than d.c.

(*ii*) The a.c. voltages can be easily varied using transformers.

(*iii*) The a.c. can be easily converted into d.c.

(*iv*) The magnitude of a.c. can be reduced using a choke coil, without involving loss of energy.

(*v*) A.C. travels on the surface of the conductor. Therefore, for long distance transmission of a.c., we often save costly conducting materials using inner core of cheaper material. For example, *ACSR i.e.*, aluminium conductor steel reinforced is widely used for electric supply lines.

(*vi*) A.C. machines are often small sized, easy to use and have longer life.

(*b*) Drawbacks of a.c. are :

(*i*) It is more dangerous to work with a.c. at high voltages. The moment the insulation is faulty, one gets a severe shock.

(*ii*) The shock of a.c. is attractive, whereas that of d.c. is repulsive.

(*iii*) There are certain phenomena like electroplating, electrorefining, electrotyping etc. where a.c. cannot be used. In such cases, d.c. is needed.

(*iv*) The a.c. is transmitted more from the surface of conductor than from inside. Therefore, several fine insulated wires (and not a single thick wire) are required for the transmission of a.c.

DO YOU KNOW ?

The *a.c.* can be converted into *d.c.* with the help of a rectifier, while *d.c.* can be converted into *a.c.* with the help of an inverter.

The *a.c.* can be stepped up or down with the help of a transformer, while *d.c.* cannot be.

The *a.c.* cannot produce chemical effects of current *e.g.* electrolysis ; electroplating etc. This is due to large inertia of heavy ions which cannot follow the frequency of *a.c.*

7.30. TRANSFORMER

A transformer is an electrical device which is used for changing the a.c. voltages.

A transformer which increases the a.c. voltages is called a *step up transformer*. A transformer which decreases the a.c. voltages is called a *step down transformer*.

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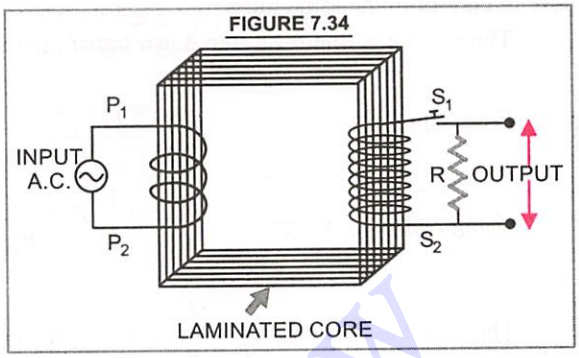
Principle. A transformer is based on the principle of mutual induction, *i.e.*, whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the neighbouring coil.

Construction. A transformer consists of a rectangular soft iron core made of *laminated sheets*, well insulated from one another, Fig. 7.34. Two coils $P_1\ P_2$ (the primary coil) and $S_1\ S_2$ (the secondary coil) are wound on the same core, but are well insulated from each other. Note that both the coils are also insulated from the core. The source of alternating e.m.f. (to be transformed) is connected to the primary coil $P_1\ P_2$ and a load resistance R is connected to the secondary coil $S_1\ S_2$ through an open switch S. Thus, there can be no current through the secondary coil so long as the switch is open.

FIGURE 7.34



For an ideal transformer, we assume that the resistances of the primary and secondary windings are negligible. Further, the energy losses due to magnetic hysterisis in the iron core is also negligible. Well designed high capacity transformers may have energy losses as low as 1%.

Theory and working

Let the alternating *e.m.f.* supplied by the *a.c.* source connected to primary be

$$E_p = E_0 \sin \omega t \qquad \text{...(81)}$$

As we have assumed the primary to be a pure inductance with zero resistance, the sinusoidal primary current I_p lags the primary voltage E_p by 90°. The primary's power factor, $\cos \phi = \cos 90° = 0$. Therefore, no power is dissipated in primary.

The alternating primary current induces an alternating magnetic flux ϕ_B in the iron core. Because the core extends through the secondary winding, the induced flux also extends through the turns of secondary.

According to Faraday's law of electromagnetic induction, the induced *e.m.f. per turn* (E_{turn}) is same for both, the primary and secondary. Also, the voltage E_p across the primary is equal to the *e.m.f.* induced in the primary, and the voltage E_s across the secondary is equal to the *e.m.f.* induced in the secondary. Thus,

$$E_{turn} = \frac{d\phi_B}{dt} = \frac{E_p}{n_p} = \frac{E_s}{n_s} \quad \text{or} \quad \boxed{E_s = E_p \frac{n_s}{n_p}} \qquad \text{...(82)}$$

Here, n_p ; n_s represent total number of turns in primary and secondary coils respectively

If $n_s > n_p$; $E_s > E_p$, the transformer is a *step up transformer.* Similarly, when $n_s < n_p$; $E_s < E_p$. The device is called a *step down transformer.* $\frac{n_s}{n_p} = K$ represents **transformation ratio.**

Note that this relation (82) is *based on three assumptions*

(*i*) the primary resistance and current are small, (*ii*) there is no leakage of magnetic flux. The same magnetic flux links both, the primary and secondary coils, (*iii*) the secondary current is small.

Now, the rate at which the generator/source transfers energy to the primary = $I_p E_p$. The rate at which the primary then transfers energy to the secondary (via the alternating magnetic field linking the two coils) is $I_s E_s$.

As we assume that no energy is lost along the way, conservation of energy requires that

$$\boxed{I_p E_p = I_s E_s} \quad \therefore \quad I_s = I_p \frac{E_p}{E_s} \qquad \text{From (82),} \quad \frac{E_p}{E_s} = \frac{n_p}{n_s} \quad \therefore \quad \boxed{I_s = I_p . \frac{n_p}{n_s} = \frac{I_p}{K}} \qquad \text{...(83)}$$

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For a step up transformer, $E_s > E_p$; $K > 1$ $\therefore$ $I_s < I_p$

i.e. secondary current is weaker when secondary voltage is higher, *i.e.*, whatever we gain in voltage, we lose in current in the same ratio.

The reverse is true for a step down transformer.

From eqn. (83), $$I_p = I_s\left(\frac{n_s}{n_p}\right) = \frac{E_s}{R}\left(\frac{n_s}{n_p}\right)$$

Using eqn. (82), we get $$I_p = \frac{1}{R}.E_p\left(\frac{n_s}{n_p}\right)\left(\frac{n_s}{n_p}\right) = \frac{1}{R}\left(\frac{n_s}{n_p}\right)^2 E_p \quad \text{...(84)}$$

This eqn. has the form $I_p = \frac{E_p}{R_{eq}}$, where the equivalent resistance R_{eq} is $$R_{eq} = \left(\frac{n_p}{n_s}\right)^2 R \quad \text{...(85)}$$

Thus R_{eq} is the value of load resistance as seen by the source/generator, *i.e.*, the source/generator produces current I_p and voltage E_p as if it were connected to a resistance R_{eq}.

Efficiency of a transformer is defined as the ratio of output power to the input power.

i.e., $$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p} \quad \text{...(86)}$$

In an ideal transformer, where there is no power loss, $\eta = 1$ (*i.e.* 100%). However, practically there are many energy losses. Hence efficiency of a transformer in practice is less than one (*i.e.* less than 100%).

Sample Problem **A power transmission line feeds input power at 2200 V to a step down transformer with its primary windings having 3000 turns. Find the number of turns in secondary to get the power output at 220 V.** **(CBSE 2017)**

Sol. Here, $E_p = 2200$ V, $n_p = 3000$, $n_s = ?$ $E_s = 220$ V

For a step down transformer,

$$\frac{n_s}{n_p} = \frac{E_s}{E_p} = \frac{220}{2200} = \frac{1}{10}$$

$$n_s = \frac{n_p}{10} = \frac{3000}{10} = \mathbf{300}$$

7.31. ENERGY LOSSES IN A TRANSFORMER

Following are the major sources of energy loss in a transformer :

1. *Copper loss* is the energy loss in the form of heat in the copper coils of a transformer. This is due to Joule heating of conducting wires. These are minimised using thick wires.
2. *Iron loss* is the energy loss in the form of heat in the iron core of the transformer. This is due to formation of eddy currents in iron core. It is minimised by taking laminated cores.
3. *Leakage of magnetic flux* occurs inspite of best insulations. Therefore, rate of change of magnetic flux linked with each turn of S_1 S_2 is less than the rate of change of magnetic flux linked with each turn of P_1 P_2. It can be reduced by winding the primary and secondary coils one over the other.
4. *Hysteresis loss.* This is the loss of energy due to repeated magnetisation and demagnetisation of the iron core when a.c. is fed to it. The loss is kept to a minimum by using a magnetic material which has a low hysteresis loss.
5. *Magnetostriction*, *i.e.*, humming noise of a transformer.

Therefore, output power in the best transformer may be roughly 90% of the input power.

7.32. USES OF TRANSFORMER

A transformer is used in almost all a.c. operations, *e.g.*,

(*i*) In voltage regulators for T.V., refrigerator, computer, air conditioner etc.

(*ii*) In the induction furnaces.

(*iii*) A step down transformer is used for welding purposes.

(*iv*) *In the transmission of a.c. over long distances.* The loss of power in the transmission lines is $I^2 R$, where I is strength of current and R is the resistance of the wires. To reduce the power loss, a.c. is transmitted over long distances at extremely high voltages. This reduces I in the same ratio. Therefore, $I^2 R$ becomes negligibly low. As I has been reduced sufficiently, $I^2 R$ remains negligible even when R is not very small. This means we can use even thin line wires of large resistance R instead of thick ones. This saves us a lot of material (copper). Therefore, cost of transmission is reduced considerably.

Normally, at the generating station, we use a step up transformer, which raises a.c. voltage to about 132000 V. Thin line wires carry the power to the receiving station. The voltage is reduced in steps using a number of step down transformers. For the domestic consumers, the voltage is 220 V, 50 Hz.

Curiosity Question

Q. A small town with a demand of 1 MW at 220 V is situated 20 km away from an electric plant generating power at 440 V. The resistance of two line wires carrying power is 0·5 Ω per km. Two step down transformers are available : 4000 V – 220 V and 100000 V – 220 V at a substation in the town for supply of power. Which one will you prefer and why ?

Ans. Here, power required = 1 MW = 10^6 W = 10^3 kW

Total resistance of two line wires $R = 2 \times 20 \times 0{\cdot}5 = 20$ ohm.

(*a*) ***If power is supplied through 4000 V – 220 V transformer***

$$E_v = 4000 \text{ V}, \quad I_v = \frac{P}{E_v} = \frac{10^6 \text{ W}}{4000 \text{ V}} = 250 \text{ A}$$

Line power loss $= I_v^2 R = (250)^2 \times 20 = 1250000$ W = 1250 kW

If there is no other power loss, then the essential plant supply = (1000 + 1250) kW = 2250 kW

voltage drop on the line = $I_v \times R = 250 \times 20 = 5000$ V $\therefore$ Voltage for transmission = 4000 + 5000 = 9000 V

As power is generated at 440 V, the step up transformer needed at the plant is **440 V – 9000 V**

$$\text{Power loss in the process} = \frac{1250}{2250} \times 100\% = \mathbf{55{\cdot}5\%}$$

(*b*) ***If power is supplied through 100,000 V – 220 V transformer***

$$E_v = 100{,}000 \text{ V}$$

$$I_v = \frac{P}{E_v} = \frac{10^6 \text{ W}}{100{,}000 \text{ V}} = 10 \text{ A}$$

Line power loss $= I_v^2 R = 10^2 \times 20 = 2000$ W = 2 kW

If there is no other power loss, then the essential plant supply = (1000 + 2) kW = 1002 kW

voltage drop on the line = $I_v \times R = 10 \times 20 = 200$ V

$\therefore$ Voltage for transmission = (100000 + 200) V = 100200 V

As power is generated at 440 V, the step up transformer needed at the plant is **440 V – 100200 V**

$$\text{Power loss in the process} = \frac{2}{1002} \times 100\% \simeq \mathbf{0{\cdot}2\%}$$

which is much less than the power loss in case (*a*). Hence, we shall prefer to use 100,000 V – 220 V transformer at the substation. That is why transmission of a.c. over long distances is preferred at high voltages.

7.33. CHOKE COIL

A choke coil is an electrical appliance used for controlling current in an a.c. circuit. If we use a resistance R for the same purpose, a lot of energy would be wasted in the form of heat, etc.

The choke coil consists of a number of turns of thick copper wire wound closely over a *laminated* soft iron core. The inductive reactance offered by the coil, $X_L = \omega L$ is large. Therefore, *magnitude of a.c. is reduced, as*

$$I_v = \frac{E_v}{X_L} \qquad \text{...(87)}$$

In *an ideal choke coil*, Ohmic resistance = 0. Therefore, there is no energy dissipation. Through an ideal inductor, current lags behind the applied voltage by a phase angle of 90°.

∴ Average power consumed by the choke coil over a complete cycle is

$$P = E_v\, I_v \cos\phi = E_v\, I_v \cos 90° = 0$$

However, in actual practice, a choke coil of inductance L is having a small resistance r. It may be treated as a *series combination of L and r*. Therefore, average power consumed over a complete cycle in the practical choke coil is

$$P = E_v\, I_v \cos\phi \qquad \text{...(88)}$$

$$P = E_v\, I_v \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \qquad \text{...(89)}$$

This comes out to be much smaller than the power loss ($I^2 R$) if a resistance R is used for reducing a.c.

For reducing low frequency alternating currents, choke coils with laminated soft iron cores are used.

This is because $I_v = \frac{E_v}{X_L} = \frac{E_v}{\omega L} = \frac{E_v}{2\pi v L}$

When v is low, L is made high using soft iron core.

These are called the ***a.f. chokes*** represented in Fig. 7.35.

For reducing high frequency alternating currents, air cored chokes are used.

This is because $I_v = \frac{E_v}{X_L} = \frac{E_v}{\omega L} = \frac{E_v}{2\pi v L}$

When v is high, L need not be made high, *i.e.*, air cored chokes are used.

These are called the ***r.f. chokes*** represented in Fig. 7.36.

We have seen the use of choke coil in series with a fluorescent tube working on a.c. mains.

This is shown in Fig. 7.37.

FIGURE 7.35

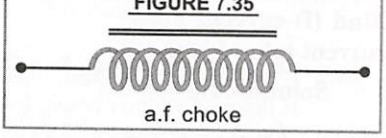


FIGURE 7.36

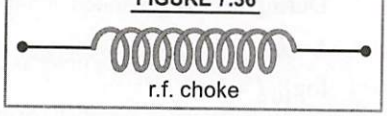
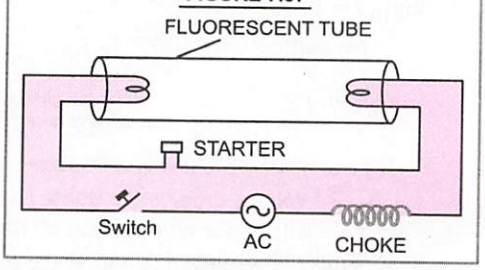


FIGURE 7.37



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Note that a choke coil can be replaced by a condenser for reducing *a.c.* equally effectively. This is because average power dissipated/cycle in case of a condenser is also zero.

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SOLVED EXAMPLES

TYPE I. TRANSIENT CURRENTS

Formulae used.

(*i*) Growth of current in *RL* circuit

$$I = I_0 [1 - e^{-(R/L)t}]$$

(*ii*) Decay of current in *RL* circuit

$$I = I_0 e^{-(R/L)t}$$

(*iii*) Time constant of *RL* circuit, $\tau = L/R$

(*iv*) Charging of capacitor through resistor,

$$q = q_0 (1 - e^{-t/RC})$$

(*v*) Discharging of capacitor through resistor,

$$q = q_0 (e^{-t/RC})$$

(*vi*) Time constant of *R–C* circuit is $\tau = RC$

Units used. I, I_0 in ampere ; R in ohm ; L in henry and t, τ in seconds.

q, q_0 in coulomb ; t, τ in sec. ;

R in ohm and C in farad.

Example 1 **When a current of 10 ampere is flowing through a resistance of 20 ohm and inductance of 10 henry, the battery is switched off. Find (i) current after 0·4 sec. (ii) the time the current takes to fall to 60% of its initial value.**

Solution. Here, $I_0 = 10$ A, $R = 20\ \Omega$, $L = 10$ H

(*i*) $I = ?, t = 0{\cdot}4$ sec.

During decay,

$$I = I_0 e^{-(R/L)t} = 10\, e^{-20 \times 0{\cdot}4/10} = 10\, e^{-0{\cdot}8}$$

$$\log_{10} I = \log_{10} 10 - 0{\cdot}8 \log_{10} e$$

$$= 1 - 0{\cdot}8 \times \log_{10} 2{\cdot}718 = 1 - 0{\cdot}8 \times 0{\cdot}4343$$

$$\log_{10} I = 1 - 0{\cdot}3474 = 0{\cdot}6526$$

$$I = \text{antilog } 0{\cdot}6526 = \mathbf{4{\cdot}49\ A}$$

(*ii*) $t = ?\ I = \dfrac{60}{100} I_0$ From $I = I_0 e^{(-R/L)t}$

$$\frac{I}{I_0} = e^{(-R/L)t} \quad ; \quad \frac{60}{100} = e^{\frac{-20}{10}t} = e^{-2t}$$

$$\log_{10} 6 - \log_{10} 10 = -2t \log_{10} e$$

$$0{\cdot}7782 - 1{\cdot}0000 = -2t \times 0{\cdot}4343$$

or $\quad -0{\cdot}2218 = -t \times 0{\cdot}8686$

$$t = \frac{0{\cdot}2218}{0{\cdot}8686} = \mathbf{0{\cdot}255\ s}$$

Example 2 **A capacitor of capacitance 0·5 μ F is discharged through a resistance. Find the value of resistance if half the charge on capacitor escapes in one minute.**

Solution. Here, $C = 0{\cdot}5\ \mu\text{F} = 0{\cdot}5 \times 10^{-6}$ F,

$R = ?$

$$q = \frac{q_0}{2}, \quad t = 1 \text{ min.} = 60 \text{ sec.}$$

As $q = q_0 e^{-t/RC}$ $\quad \therefore \quad \dfrac{q_0}{2} = q_0 e^{-60/RC}$

$$\log_e 1 - \log_e 2 = -\frac{60}{RC} \log_e e$$

$$0 - 2{\cdot}303 \log_{10} 2 = -\frac{60}{RC} \times 1$$

$$2{\cdot}303 \times 0{\cdot}3010 = \frac{60}{R(0{\cdot}5 \times 10^{-6})}$$

$$\therefore \quad R = \frac{60}{0{\cdot}5 \times 10^{-6} \times 2{\cdot}303 \times 0{\cdot}3010}$$

$$= \mathbf{1{\cdot}73 \times 10^8\ ohm}$$

Example 3 **A 20 mH coil is connected in series with a 2 kΩ resistor and a 12 V battery. Calculate**

(*i*) time constant of the circuit,

(*ii*) time during which current decays to 10% of its maximum value.

Solution. Here, $L = 20$ mH $= 20 \times 10^{-3}$ H,

$R = 2\ \text{k}\Omega = 2 \times 10^3$ ohm, $V = 12$ V

(*i*) Time constant $= \dfrac{L}{R} = \dfrac{20 \times 10^{-3}}{2 \times 10^3} = \mathbf{10^{-5}\ s}$

(*ii*) For decay of current, $I = I_0 e^{-\frac{R}{L}t}$

$$\frac{I_0}{10} = I_0 e^{-10^5 t}$$

$$\therefore \quad e^{10^5 t} = 10$$

$$10^5 t = 2{\cdot}303 \log_{10} 10$$

$$t = \frac{2{\cdot}303}{10^5} = \mathbf{2{\cdot}303 \times 10^{-5}\ s}$$

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Example 4 **A capacitor of 1·0 μF is connected to series with a resistance of 10^4 ohm ; and a battery of 2·0 V. Find the maximum value of current and current after 0·02 s.**

Solution. Here, $C = 1{\cdot}0\ \mu F = 10^{-6}$ F,

$R = 10^4$ ohm, $E_0 = 2{\cdot}0$ volt

During charging of the condenser,

$q = q_0\,(1 - e^{-t/RC})$

$I = -I_0\, e^{-t/RC}$, where

$$I_0 = \frac{E}{R} = \frac{2{\cdot}0}{10^4} = 2{\cdot}0 \times 10^{-4} \text{ amp}$$

At $t = 0{\cdot}02$ s, $I = I_0\,(e^{-0{\cdot}02/10^4 \times 10^{-6}})$

$= 2 \times 10^{-4}\, e^{-2}$

$$= \frac{2\times10^{-4}}{e^2} = \frac{2\times10^{-4}}{(2{\cdot}718)^2} = 0{\cdot}27 \times 10^{-4} \text{ A}$$

$I = 27 \times 10^{-6}$ A = **27 μ A**

TYPE II. MEAN VALUE AND R.M.S. VALUE OF A.C.

Formulae used

1. $(E_m)_{0\to T/2} = \dfrac{2E_0}{\pi} = 0{\cdot}637\,E_0$;

$(E_m)_{T/2\to T} = -\dfrac{2E_0}{\pi} = -0{\cdot}637\,E_0$;

$(E_m)_{0\to T} = 0$

2. $(I_m)_{\text{half cycle}} = \pm\dfrac{2I_0}{\pi} = \pm 0{\cdot}637\,I_0$;

$(I_m)_{\text{full cycle}} = 0$

3. $E_v = \dfrac{E_0}{\sqrt{2}} = 0{\cdot}707\,E_0$, $I_v = \dfrac{I_0}{\sqrt{2}} = 0{\cdot}707\,I_0$

Units used. E_m, E_0, E_v in volt ; I_m, I_0, I_v in ampere.

Example 5 **The instantaneous current from an a.c. source is $I = 10 \sin 314\,t$. What is the frequency of the source and virtual value of current ?** (CBSE 2000)

Solution. Here $I = 10 \sin 314\,t$

Compare with standard form :

$I = I_0 \sin 2\pi\nu t$

$I_0 = 10$ A, $2\pi\nu = 314$; $\nu = \dfrac{314}{2\times3{\cdot}14} = \mathbf{50\ Hz}$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{10}{1{\cdot}414} = \mathbf{7{\cdot}07\ A}$$

Example 6 **The instantaneous current from an a.c. source is $I = 5 \sin 100\,\pi t$. What is the frequency of a.c. ? What is the rms value of current ?** (CBSE 2001)

Solution. Here, $I = 5 \sin 100\,\pi t$

Compare with $I = I_0 \sin \omega t$

$I_0 = 5$ A, $\omega = 2\pi\nu = 100\,\pi$,

$$\nu = \frac{100\,\pi}{2\,\pi} = \mathbf{50\ Hz}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = \mathbf{3{\cdot}54\ A}$$

TYPE III. A.C. CIRCUIT CONTAINING RESISTANCE ONLY

Formulae used. 1. Effective resistance to *a.c.* $= R$

$\therefore\ I_v = \dfrac{E_v}{R}$

2. No phase change is introduced, *i.e.*, if

$E = E_0 \sin \omega t$, then $I = I_0 \sin \omega t$.

Units used. E, E_0, E_v in volt ; I, I_0, I_v in ampere ; R in ohm ; ω in radian/sec ; t in sec.

Example 7 **A light bulb is rated at 100 W for a 220 V supply. Find**

(*a*) the resistance of the bulb.

(*b*) the peak voltage of the source

(*c*) the rms current through the bulb.

(CBSE 2012) (NCERT Solved Example)

Solution. Here, $P = 100$ W, $E_v = 220$ V,

$R = ?\ E_0 = ?\ I_v = ?$

From, $P = \dfrac{E_v^2}{R}$, $R = \dfrac{E_v^2}{P} = \dfrac{220\times220}{100} = \mathbf{484\,\Omega}$

$E_0 = \sqrt{2}\ E_v = 1{\cdot}414 \times 220 = \mathbf{311\ V}$

$$I_v = \frac{P}{E_v} = \frac{100}{220} = \mathbf{0{\cdot}45\ A}$$

Example 8 **An alternating voltage given by $V = 140 \sin 314\,t$ is connected across a pure resistor of 50 ohm. Find (*i*) the frequency of the source. (*ii*) the rms current though the resistor.**

(CBSE 2012)

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Solution. Here, $V = 140 \sin 314\, t$, $R = 50\ \Omega$, $v = ?$ $I_v = ?$

Compare the given equation with the standard form, $E = E_0 \sin \omega t$, $E_0 = 140\ V$, $\omega = 2\pi v = 314$,

$$v = \frac{314}{2\pi} = \frac{314}{2 \times 3{\cdot}14} = \mathbf{50\ Hz}$$

$$I_0 = \frac{E_0}{R} = \frac{140}{50} = \frac{14}{5}\ A$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{14}{5 \times 1{\cdot}4} = \mathbf{2A}$$

Example 9 **A resistance of 40 Ω is connected to an a.c. source of 220 V, 50 Hz. Find (*i*) the rms current (*ii*) maximum instantaneous current in the resistor (*iii*) time taken by the current to change from max. value to rms value.**

Solution. Here, $R = 40\ \Omega$, $E_v = 220$ V, $v = 50$ Hz,

(*i*) $I_v = \frac{E_v}{R} = \frac{220}{40} = \mathbf{5{\cdot}5\ A}$

$$I_0 = \sqrt{2}\, I_v = 1{\cdot}414 \times 5{\cdot}5 = \mathbf{7{\cdot}8\ A}$$

If alternating current is given by

$I = I_0 \sin \omega t$, then

$I_0 = I_0 \sin \omega t_1$; $\sin \omega t_1 = 1$ or $\omega t_1 = \frac{\pi}{2}$

and $I_v = \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t_2$, which implies

$$\omega t_2 = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\therefore\ \omega(t_2 - t_1) = \frac{\pi}{2} + \frac{\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4}$$

$$t_2 - t_1 = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi v} = \frac{1}{8v} = \frac{1}{8 \times 50}$$

$$= 2{\cdot}5 \times 10^{-3}\ s = \mathbf{2{\cdot}5\ ms}$$

TYPE IV. A.C. CIRCUIT CONTAINING INDUCTANCE ONLY

Formulae used.

1. Alternating current through an inductor lags behind the alt. emf by a phase angle of 90°. If

$E = E_0 \sin \omega t$; $I = I_0 \sin (\omega t - 90°)$

2. Inductive reactance, $X_L = \omega L = 2\pi v L$

$$\therefore\ I_v = \frac{E_v}{X_L}$$

Units used. E, E_0, E_v in volt ; I, I_0, I_v in ampere, L in henry, X_L in ohm ; t in sec ; v in hertz ; ω in radian/sec.

Example 10 **Alternating e.m.f. $E = 220 \sin 100\, \pi\, t$ is applied to a circuit containing an inductance of $\left(\frac{1}{\pi}\right) H$. Write the equation for instantaneous current through the circuit. What will be the reading of a.c. ammeter in the circuit ?**

Solution. Here, $E = 220 \sin 100\, \pi\, t$, $L = \frac{1}{\pi}\ H$

$E_0 = 220$ V, $\omega = 100\, \pi$

$$I_0 = \frac{E_0}{X_L} = \frac{E_0}{\omega L} = \frac{220}{(100\,\pi)\frac{1}{\pi}} = 2{\cdot}2\ A$$

As current through L lags behind the applied *e.m.f.* through a phase angle of 90°, therefore instantaneous current through the circuit is

$I = I_0 \sin (\omega t - \pi/2) = \mathbf{2{\cdot}2 \sin (100\, \pi\, t - \pi/2)}$

Reading of a.c. ammeter $= I_v = \frac{I_0}{\sqrt{2}} = \frac{2{\cdot}2}{\sqrt{2}}$

$= \mathbf{1{\cdot}556\ A}$

Example 11 **A pure inductor of 25·0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.**

(NCERT Solved Example)

Solution. Here, $L = 25{\cdot}0$ mH $= 25 \times 10^{-3}$ H,

$E_v = 220$ V, $X_L = ?$, $I_v = ?$, $v = 50$ Hz

$$X_L = \omega L = 2\pi v L = 2 \times \frac{22}{7} \times 50 \times 25 \times 10^{-3}$$

$$= \mathbf{7{\cdot}85\ ohm}$$

$$I_v = \frac{E_v}{X_L} = \frac{220}{7{\cdot}85} = \mathbf{28\ A}$$

TYPE V. A.C. CIRCUIT CONTAINING CONDENSER ONLY

Formulae used.

1. Through capacitor, alt. current leads the applied alt. voltage by a phase angle of 90°, *i.e.*, if $E = E_0 \sin \omega t$, then $I = I_0 \sin (\omega t + 90°)$

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2. Capacitative reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

$$I_v = \frac{E_v}{X_C}$$

Units used. E, E_0, E_v in volt ; I, I_0, I_v in ampere ; C in farad ; X_C in ohm ; t in sec ; ν in hertz and ω in radian/sec.

Example 12 **A 1·5 μF capacitor has a capacitative reactance of 12 Ω. What is the frequency of the source ? If frequency of source is doubled, what will be the capacitative reactance ?**

Solution. Here, $C = 1{\cdot}5\ \mu F = 1{\cdot}5 \times 10^{-6}$ F,

$X_C = 12\ \Omega$, $\nu = ?$

As $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu c}$

$$\therefore\ \nu = \frac{1}{2\pi C X_C} = \frac{1}{2\times 3{\cdot}14\times 1{\cdot}5\times 10^{-6}\times 12}$$

$\nu = \mathbf{8846\ Hz}$

As $X_C \propto \frac{1}{\nu}$, therefore, when ν is doubled, X_C becomes half, *i.e.*, $\frac{12}{2} = \mathbf{6\,\Omega}$

Example 13 **A coil has an inductance of 1 henry. (a) At what frequency will it have a reactance of 3142 ohm ? (b) What should be the capacity of a condenser which has the same reactance at that frequency ?**

Solution. *(a)* Here, $L = 1$ henry, $\nu = ?$,

$X_L = 3142$ ohms

As $X_L = \omega L = 2\pi\nu L$

$$\therefore\ \nu = \frac{X_L}{2\pi L} = \frac{3142}{2\times 3{\cdot}142\times 1} = \mathbf{500\ hertz}$$

(b) $C = ?$ $\nu = 500$ hertz, $X_C = X_L = 3142$ ohm

As $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

$$\therefore\ C = \frac{1}{2\pi\nu X_C} = \frac{1}{2\times 3{\cdot}142\times 500\times 3142}$$

$$= \frac{10^{-6}}{(3{\cdot}142)^2}\ \text{farad} = \mathbf{0.11\,\mu F}$$

Example 14 **A 15·0 μ F capacitor is connected to a 220 V, 50 Hz source. Find the capacitative reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitative reactance and current ?** (CBSE 2009)

(NCERT Solved Example)

Solution. Here, $C = 15{\cdot}0\ \mu F = 15 \times 10^{-6}$ F

$E_v = 220$ V, $\nu = 50$ Hz, $X_C = ?$

$I_v = ?$, $I_0 = ?$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\times\frac{22}{7}\times 50\times 15\times 10^{-6}}$$

$$= \frac{7\times 10^4}{22\times 15} = \mathbf{212\ ohm}$$

$$I_v = \frac{E_v}{X_C} = \frac{220}{212} = \mathbf{1{\cdot}04\ A},$$

$$I_0 = \sqrt{2}\, I_v = 1{\cdot}414 \times 1{\cdot}04 = \mathbf{1{\cdot}47\ A}$$

When frequency is doubled, X_C **is halved and current is doubled.**

TYPE VI. A.C. CIRCUIT CONTAINING *R* AND *L*

Formulae used. 1. Alternating current in *RL* circuit lags behind the applied alternating *e.m.f.* by a phase angle ϕ, where $\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R}$

2. Impedance of *RL* circuit

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$$

3. $I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + X_L^2}}$

Units used. R, X_L, Z in ohm ; L in henry ; E_v in volt ; I_v in ampere.

Example 15 **A coil when connected across a 10 V d.c. supply draws a current of 2 A. When it is connected across 10 V–50 hz a.c. supply the same coil draws a current of 1 A. Explain why ? Hence determine self inductance of the coil.** (CBSE Sample Paper 2003)

Solution. The coil draws lesser current in 2nd case because of inductive reactance of the coil.

In *1st case*, $R = \frac{V}{I} = \frac{10}{2} = 5\,\Omega$

In second case, $Z = \frac{V}{I} = \frac{10}{1} = 10\,\Omega$

Inductive reactance,

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{10^2 - 5^2} = 5\sqrt{3}$$

$$2\pi\nu L = 5\sqrt{3}$$

$$L = \frac{5\sqrt{3}}{2\pi\nu} = \frac{5\sqrt{3}}{2\times 3{\cdot}14\times 50} = \mathbf{0{\cdot}0288\ H}$$

Example 16 **In an *RL* circuit, pot. diff. across the inductor *L* is 120 V and pot. diff. across *R* is 90 V. If *rms* value of current is 3 A, what is impedance of the circuit and what is phase angle between voltage and current ?** **(CBSE 2004)**

Solution. Here, $V_L = 120$ V, $V_R = 90$ V,

$I_v = 3$ A, $Z = ?$, $\phi = ?$

As voltages across *R* and *L* are 90° out of phase,

∴ resultant voltage,

$$V = \sqrt{V_L^2 + V_R^2} = \sqrt{120^2 + 90^2} = 150\text{ V}$$

$$Z = \frac{V}{I_v} = \frac{150}{3} = 50\,\Omega$$

$$\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{120}{90} = 1{\cdot}33$$

$$\phi = \tan^{-1}(1{\cdot}33) = \mathbf{53{\cdot}1^\circ}$$

In *RL* circuit, current lags behind the voltage.

Example 17 **Following readings were obtained in the experiment to determine self inductance of a coil. If frequency of a.c. is 50 Hz, what is the value of self inductance ?** **(Karnataka Board, 2012)**

Solution. From D.C. Part,

$$R_1 = \frac{V_1}{I_1} = \frac{0{\cdot}5}{0{\cdot}1} = 5\,\Omega,\quad R_2 = \frac{0{\cdot}6}{0{\cdot}12} = 5\,\Omega$$

$$\therefore \text{Average value of } R = \frac{R_1 + R_2}{2} = \frac{5+5}{2} = 5\,\Omega$$

D.C. Part

S.No.	V	I
1	0·5 V	0·1 A
2	0·6 V	0·12 A

A.C. Part

S.No.	V	I
1	1·0 V	0·1 A
2	1·6 V	0·16 A

From A.C. Part,

$$Z_1 = \frac{V_1}{I_1} = \frac{1{\cdot}0}{0{\cdot}1} = 10\,\Omega;\quad Z_2 = \frac{1{\cdot}6}{0{\cdot}16} = 10\,\Omega$$

$$\text{Average value of } Z = \frac{Z_1 + Z_2}{2} = \frac{10+10}{2} = 10\,\Omega$$

$$\text{As } X_L = \omega L = 2\pi\nu L = \sqrt{Z^2 - R^2}$$

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi\nu} = \frac{\sqrt{10^2 - 5^2}}{2\times 3{\cdot}14\times 50} = \frac{\sqrt{75}}{314} = \mathbf{0{\cdot}0276\ H}$$

Example 18 **An electric lamp which runs at 80 volt d.c. and consumes 10 ampere is connected to 100 volt, 50 Hz a.c. mains. Calculate the inductance of the choke required.** **(Uttarakhand Board 2012)**

Solution. Here, $V = 80$ V, $I = 10$ A,

$$R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega$$

$E_v = 100$ V, $\nu = 50$ Hz, $I_v = 10$ A, $L = ?$

If Z is impedance of lamp and choke coil,

$$\text{then}\quad Z = \frac{E_v}{I_v} = \frac{100}{10} = 10\,\Omega$$

$$\text{As}\quad R^2 + X_L^2 = Z^2$$

$$\therefore\quad X_L^2 = Z^2 - R^2 = 10^2 - 8^2 = 36,\ X_L = 6\,\Omega$$

$$\text{Now}\quad X_L = \omega L = 2\pi\nu L$$

$$\therefore\quad L = \frac{X_L}{2\pi\nu} = \frac{6\times 7}{2\times 22\times 50} = \mathbf{1{\cdot}9\times 10^{-2}\ H}$$

Retain in Memory

To run an electric lamp in an a.c. circuit, I_v is same as direct current required to run the lamp.

Example 19 **A 60 V–10 W electric lamp is to be run on 100 V – 60 Hz mains. Calculate the inductance of the choke coil required. If a resistor is to be used in place of choke coil to achieve the same result, calculate its value.**

Solution. Here, $E_v = 60$ V, $P = 10$ W

$$R = \frac{E_v^2}{P} = \frac{60\times 60}{10} = 360\,\Omega$$

Current through the lamp,

$$I_v = \frac{E_v}{R} = \frac{60}{360} = \frac{1}{6}\text{ A}$$

Let L be the inductance of the choke required.

$$\therefore \quad Z = \frac{E_v}{I_v} = \frac{100}{1/6} = 600\,\Omega$$

From $R^2 + X_L^2 = Z^2$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(600)^2 - (360)^2}$$
$$= 480\ \Omega$$
$$X_L = \omega L = 2\pi\nu L = 480$$
$$L = \frac{480}{2\pi\nu} = \frac{480}{2\times 3{\cdot}14\times 60}$$
$$L = \mathbf{1{\cdot}274\ H}$$

To achieve the same result resistance required

$$R' = Z - R = 600 - 360 = \mathbf{240\ \Omega}$$

Example 20 **When an inductor L and a resistor R are connected across 12 V, 50 Hz a.c. supply, a current of 0·5 A flows through the circuit. What is the value of R if current differs in phase from applied voltage by $\pi/3$ radian ?** (CBSE 2006)

Solution. Here, $E_v = 12$ V, $\nu = 50$ Hz,

$I_v = 0{\cdot}5$ A, $R = ?$ $\phi = \pi/3$ radian

$$Z = \frac{E_v}{I_v} = \frac{12}{0{\cdot}5} = 24$$

From $\tan\phi = \frac{X_L}{R} = \tan\pi/3 = \sqrt{3}$

$$X_L = \sqrt{3}\,R$$

As $R^2 + X_L^2 = Z^2$

$$\therefore\ R^2 + (\sqrt{3}R)^2 = Z^2 = 24\times 24$$

$$R = \sqrt{\frac{24\times 24}{4}} = \mathbf{12\ ohm}$$

Example 21 **An alternating voltage of 220 V is applied across a device X, a current of 0·25 A flows, which lags behind the applied voltage in phase by $\pi/2$ radian. If the same voltage is applied across another device Y, the same current flows, but now it is in phase with the applied voltage.**

(i) Name the devices X and Y.

(ii) Calculate the current flowing in the circuit when the same voltage is applied across the series combination of X and Y. [CBSE 2018 (C)]

Solution. (i) In device X, current lags behind the applied voltage in phase by $\pi/2$. Therefore, device X must be an **inductor.** In device Y, current is in phase with the applied voltage. Therefore, device Y must be a **resistor.**

(ii) $$X = \frac{E_v}{I_v} = \frac{220}{0{\cdot}25} = 880\text{ ohm} = X_L$$

$$Y = \frac{E_v}{I_v} = \frac{220}{0{\cdot}25} = 880\text{ ohm} = R$$

when X and Y are in series,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{880^2 + 880^2} = 880\sqrt{2}\text{ ohm}$$

$$\therefore\ I_v = \frac{E_v}{Z} = \frac{220}{880\sqrt{2}} = \frac{1}{4\sqrt{2}}\text{ A}$$

TYPE VII. A.C. CIRCUIT CONTAINING R AND C ONLY

Formulae used. **1.** Alternating current in R-C circuit leads the applied alternating *emf* by a phase angle ϕ, where $\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR}$

2. Impedance of RC circuit

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (1/\omega C)^2}$$

3. $$I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + X_C^2}}$$

Units used. R, X_C, Z in ohm ; C in farad ; E_v in volt ; I_v in ampere.

Example 22 **An a.c. circuit consists of a series combination of circuit elements X and Y. The current is ahead of voltage in phase by $\pi/4$. If element X is a pure resistor of 100 ohm, name the circuit element Y, and calculate the *rms* value of current if *rms* voltage is 141 V.** [CBSE 2004 (C)]

Solution. As current is ahead of voltage, the circuit element Y must be a **capacitor.**

As $\cos\phi = \frac{R}{Z} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$Z = R\sqrt{2} = 100\times 1{\cdot}414 = 141{\cdot}4\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{141}{141{\cdot}4} = \mathbf{1\ A}$$

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Example 23 **A resistor of 200 ohm and a capacitor of 15·0 μF are connected in series to a 220 V, 50 Hz a.c. source. (a) Calculate the current in the circuit, (b) Calculate the r.m.s. voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage ? If yes, resolve the paradox.**

(NCERT Solved Example)

Solution. Here, $R = 200$ ohm,

$$C = 15{\cdot}0\ \mu\text{F} = 15 \times 10^{-6}\ \text{F}$$

$$E_v = 220\ \text{V},\ v = 50\ \text{Hz},\ I_v = ?$$

$$V_R = ?,\ V_C = ?$$

Now $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi v C}$

$$= \frac{1}{2\times 3{\cdot}14\times 50\times 15\times 10^{-6}} = 212{\cdot}3\,\Omega$$

(*a*) Impedance of the circuit,

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{200^2 + (212{\cdot}3)^2}$$

$$= 291{\cdot}7 \text{ ohm}$$

∴ Current in the circuit,

$$I_v = \frac{E_v}{Z} = \frac{220}{291{\cdot}7} = \mathbf{0{\cdot}75\ A}$$

(*b*) $V_R = I_v \times R = 0{\cdot}75 \times 200 = \mathbf{150{\cdot}8\ V}$

$V_C = I_v\ X_C = 0{\cdot}75 \times 212{\cdot}3 = \mathbf{159{\cdot}2\ V}$

$V_R + V_C = 150{\cdot}8 + 159{\cdot}2 = 310$ V, which is more than the source voltage of 220 V.

This paradox is resolved by the fact that the two voltages are ***not in same phase.*** Therefore, they cannot be added like ordinary numbers. As V_R and V_C are out of phase by 90°, therefore,

$$V_{RC} = \sqrt{V_R^2 + V_C^2} = \sqrt{(150{\cdot}8)^2 + (159{\cdot}2)^2}$$

$= 220$ V, the source voltage.

TYPE VIII. A.C. CIRCUIT CONTAINING *R*, *L* AND *C*

Formulae used. 1. Impedance of *RLC* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$

2. $I_v = \dfrac{E_v}{Z}$

3. Phase angle ϕ by which alternating current lags behind the alternating voltage is given by,

$$\tan\phi = \frac{X_L - X_C}{R}$$

Units used. E_v in volt ; I_v in ampere ; Z, R, X_L, X_C in ohm.

Example 24 **A capacitor of unknown capacitance, a resistance of 100 ohm and an inductor of self inductance $L = 4/\pi^2$ henry are connected in series across an a.c. source of 200 V and 50 hz. Calculate the value of capacitance and the current that flows in the circuit, when the current is in phase with the voltage.** **[CBSE 2013 (C)]**

Solution. Here, $R = 100\ \Omega$, $L = \dfrac{4}{\pi^2}\ H$,

$E_v = 200$ V, $v = 50$ Hz, $C = ?$, $I_v = ?$

As current is in phase with voltage, therefore,

$$X_C = X_L\ ;\ \frac{1}{\omega C} = \omega L,$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi v)^2 L}$$

$$C = \frac{1}{4\pi^2 v^2 L} = \frac{1\times\pi^2}{4\times\pi^2\times 50^2\times 4}$$

$$= 25 \times 10^{-6}\ \text{F} = \mathbf{25\ \mu F}$$

As $X_C = X_L$,

∴ $Z = R = 100\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{200}{100} = \mathbf{2\ A}$$

Example 25 **A series LCR circuit is connected to an a.c. source (200 V, 50 Hz). The voltages across resistor capacitor and inductor are respectively 200 V, 250 V and 250 V. The algebraic sum of the voltages across the three elements is greater than the source voltage. Why ?**

If $R = 40\ \Omega$, calculate current in the circuit.

(CBSE 2013)

Solution. Here, $E_v = 200$ V, $v = 50$ Hz ;

$V_R = 200$ V, $V_C = 250$ V, $V_L = 250$ V

Now, $V_R + V_C + V_L = 200 + 250 + 250 = 700$ V, which is greater than the source voltage 200 V. This is because voltages across C and L are in opposite phase and cancel out. The sum continues to be 200 V, the source voltage.

$$I_v = \frac{E_v}{Z} = \frac{E_v}{R} = \frac{200}{40} = 5 \text{ A}$$

Example 26 **A resistor of 50 ohm, an inductor of (20/π) H and a capacitor of (5/π) μF are connected in series to an a.c. source 230 V, 50 Hz. Find the current in the circuit.** (CBSE 2006)

Solution. Here, $R = 50$ ohm,

$$L = \frac{20}{\pi}\text{H}, \; C = \frac{5}{\pi}\mu\text{F} = \frac{5\times10^{-6}}{\pi}\text{F}$$

$E_v = 230$ V, $\nu = 50$ Hz, $I_v = ?$

$$X_L = \omega L = 2\pi\nu L = 2\pi\nu\left(\frac{20}{\pi}\right)$$

$$= 40 \times 50 = 2000\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu c} = \frac{1\pi}{2\pi\nu\times5\times10^{-6}}$$

$$= \frac{10^6}{10\times50} = 2000\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{50^2 + (2000-2000)^2} = 50\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{230}{50} = 4{\cdot}6 \text{ A}$$

Example 27 **An LCR circuit has $L = 10$ mH, $R = 3\ \Omega$ and $C = 1$ μF, and is connected in series to a source of (20 sin ωt) volt. Calculate the current amplitude at a frequency 20% lower than the resonance frequency of the circuit.**

Solution. Here, $L = 10$ mH $= 10^{-2}$ H, $R = 3\ \Omega$

$C = 1\ \mu\text{F} = 10^{-6}$ F,

$E = 20 \sin \omega t = E_0 \sin \omega t$

$\therefore E_0 = 20$ V, $I_0 = ?$

Resonant frequency, $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{10^{-2}\times10^{-6}}} = \frac{10^4}{2\pi}$$

Frequency at which current amplitude is required $= \nu = 80\%\ \nu_0 = \frac{80}{100}\times\frac{10^4}{2\pi} = \frac{8\times10^3}{2\pi}$ Hz

$$X_L = \omega L = 2\pi\nu L$$

$$= 2\pi\times\frac{8\times10^3}{2\pi}\times10^{-2} = 80 \text{ ohm}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{2\pi}{2\pi\times(8\times10^3)\times10^{-6}}$$

$$= \frac{10^3}{8} = 125 \text{ ohm}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{3^2 + (80-125)^2} = 45{\cdot}1 \text{ ohm}$$

$$I_0 = \frac{E_0}{Z} = \frac{20}{45{\cdot}1} = \mathbf{0{\cdot}443\ A}$$

TYPE IX. ELECTRIC RESONANCE

Formulae used.

1. At resonance, $X_L = X_C$,

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

$\therefore$ Resonance frequency $\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

2. At series resonance frequency,

$$Z = R, \; I_v = \frac{E_v}{R} = \text{max.}$$

3. At parallel resonance frequency, $I_v = 0$

4. Q factor of resonance : $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Units used. L in henry ; C in farad ; R, Z, X_L, X_C, in ohm ; ω in radian/sec ; ν in sec.$^{-1}$; E_v in volt ; I_v in ampere and Q has no units.

Example 28 **Compute the resonant frequency and the Q factor of a series *LCR* circuit having $L = 4{\cdot}0$ H, $C = 36$ μF and $R = \frac{10}{3}\Omega$. How can sharpness of resonance of the circuit be improved by a factor of 2 by reducing its full width at half maximum ?**

Solution. Here, $\omega_r = ?$, $Q = ?$, $L = 4{\cdot}0$ H,

$C = 36\ \mu\text{F} = 36\times10^{-6}$ F, $R = \frac{10}{3}\Omega$.

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Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4{\cdot}0\times36\times10^{-6}}} = \frac{1000}{12}$$

$$= 83{\cdot}3 \text{ rad/s.}$$

$$\text{Q factor of the circuit} = \frac{\omega_r L}{R} = \frac{250}{3}\times\frac{4{\cdot}0}{10/3}$$

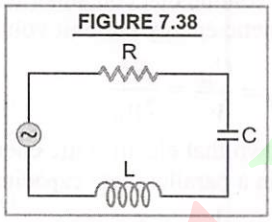
$$= 100$$

To improve sharpness of resonance by a factor of 2 ; Q should be doubled. To double Q, without changing ω_r, resistance R should be reduced to half *i.e.*, R should be **10/6 ohm.**

Example 29 **Fig. 7.38 shows series *LCR* circuit with L = 5·0 H, C = 80 μF, R = 40 Ω connected to a variable frequency 240 V source. Calculate**

(*i*) the angular frequency of the source which drives the circuit at resonance.

(*ii*) current at the resonanting frequency.



(*iii*) the rms pot. drop across the capacitor at resonance. (CBSE 2012)

Solution. Here, L = 5·0 H,

$C = 80\ \mu\text{F} = 80\times10^{-6}$ F, R = 40 ohm

E_v = 240 V, ω = ?, I = ?, V_C = ?

At resonance, $X_L = X_C$

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{5{\cdot}0\times80\times10^{-6}}} = \frac{1000}{20} = \mathbf{50 \text{ rad/s}}$$

(*ii*) At resonance, $Z = R$ = 40 Ω

$$I_v = \frac{E_v}{Z} = \frac{240}{40} = \mathbf{6\ A}$$

(*iii*) $V_C = I_v X_C$

$$= \frac{I_v}{\omega C} = \frac{6}{50\times80\times10^{-6}} = \frac{6000}{4}$$

$$= \mathbf{1500\ V}$$

Example 30 **A series *LCR* circuit with L = 80 mH, C = 50 μF and R = 60 ohm is connected to a variable frequency 220 V source. Determine (*i*) the source frequency which drives the circuit in resonance (*iii*) the quality factor Q of the circuit.** (CBSE 2014)

Solution. Here, $L = 80 \text{ mH} = 80\times10^{-3}$ H,

$C = 50\times10^{-6}$ F, R = 60 Ω

E_v = 220 V, v = ?

At resonance, $X_L = X_C$,

i.e., $\omega L = \frac{1}{\omega C}$, $\omega = \frac{1}{\sqrt{LC}}$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{80\times10^{-3}\times50\times10^{-6}}}$$

$$= 500 \text{ rad/s}$$

$$v = \frac{\omega}{2\pi} = \frac{500}{2\times3{\cdot}14} = \mathbf{79{\cdot}6\ Hz}$$

$$Q = \frac{\omega_r L}{R} = \frac{500\times80\times10^{-3}}{60} = \mathbf{0{\cdot}67}$$

Example 31 **A series *LCR* circuit is connected to an a.c. source 200 V, 50 Hz. The voltages across the resistor, inductor and capacitor are respectively 200 V, 250 V and 250 V. (*i*) The algebraic sum of the three voltages is greater than the source voltage. How is the paradox resolved ?**

(*ii*) If R = 40 Ω, calculate the current in the circuit. (CBSE F 2013)

Solution. Here, E_v = 200 V, v = 50 Hz

V_R = 200 V, V_L = 250 V, V_C = 250 V

Now $V_R + V_L + V_C = 200 + 250 + 250 = 700$ V

This is greater than source voltage 200 V.

It is because, V_L and V_C are 180° out of phase and both are 90° out of phase with V_R.

$$\therefore \quad V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{200^2 + (250-250)^2}$$

$$= 200 \text{ V, the source voltage.}$$

The paradox is resolved.

Current in the circuit,

$$I_v = \frac{E_v}{Z} = \frac{E_v}{R} = \frac{200}{40} = \mathbf{5\ A}$$

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TYPE X. ENERGY STORED IN CONDENSER, INDUCTOR AND LC OSCILLATIONS

Formulae used. 1. Energy stored in capacitor,

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

2. Energy stored in inductor, $U_L = \frac{1}{2}LI^2$

3. Average power consumed/cycle in pure inductor is zero.

4. Average power consumed/cycle in an ideal capacitor is zero.

5. The natural frequency of energy oscillations in LC circuit is $\nu = \frac{1}{2\pi\sqrt{LC}}$

Units used. Q in coulomb ; C in farad ; V in volt ; I in ampere ; U_C, U_L in joule ; power in watt ; L in henry and ν in hertz.

Example 32 A transmitter operates at 1 MHz. The oscillating circuit has a capacitance of 200 pF. What is the inductance and capacitative reactance of the resonant circuit ?

Solution. Here $\nu = 1$ M Hz $= 10^6$ Hz,

$C = 200$ pF $= 2 \times 10^{-10}$ F, $L = ?$, $X_C = ?$

As $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$\therefore L = \frac{1}{4\pi^2\nu^2 C} = \frac{1}{4\times\frac{22}{7}\times\frac{22}{7}\times(10^6)^2\times 2\times10^{-10}}$$

$$= \mathbf{13 \times 10^{-5}\ H}$$

As $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

$$\therefore X_C = \frac{1}{2\times\frac{22}{7}\times10^6\times2\times10^{-10}} = \mathbf{795{\cdot}5\ ohm}$$

Example 33 (a) Obtain an expression for magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid.

(b) How does this magnetic energy compare with the electrostatic energy stored in a capacitor ? (NCERT Solved Example)

Solution. *(a)* Refer to Art. 7.16. The mag- netic energy is $U_B = \frac{1}{2}LI^2$

As, $B = \frac{\mu_0 NI}{l}$, therefore,

$$I = \frac{Bl}{\mu_0 N} \qquad \therefore U_B = \frac{1}{2}L\left(\frac{Bl}{\mu_0 N}\right)^2$$

Using $L = \frac{\mu_0 N^2 A}{l}$, we get,

$$U_B = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{Bl}{\mu_0 N}\right)^2$$

$$U_B = \frac{B^2}{2\mu_0}Al \qquad ...(i)$$

(b) The volume that contains flux, $V = A \times l$

$\therefore$ Magnetic energy per unit volume,

$$u_B = \frac{U_B}{V} = \frac{B^2}{2\mu_0} \qquad ...(ii)$$

It is known that electrostatic energy stored per unit volume in a parallel plate capacitor is

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad ...(iii)$$

In both the cases, energy is directly proportional to the square of the field strength.

Note that Eqns. *(ii)* and *(iii)* have been derived for special cases—a solenoid and a parallel plate capacitor respectively. But they are valid for any region of space in which a magnetic field or/and an electric field exists.

Example 34 Find the natural frequency of oscillation of a circuit containing an inductance of 400 mH and a capacity of 40 μF. To which wavelength will its response be maximum ? How long will the oscillations last ?

Solution. $\nu = ?$ $L = 400$ mH $= 400 \times 10^{-3}$ H,

$C = 40\ \mu$F $= 40 \times 10^{-6}$ F, $\lambda = ?$

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{400\times10^{-3}\times40\times10^{-6}}}$$

$$= \frac{1000}{8\pi} = \mathbf{39{\cdot}8\ Hz}$$

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$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{39{\cdot}8}\ \text{m} = 7{\cdot}5 \times 10^6\ \text{m}$$

When there is no loss of energy, oscillations will last for **infinite time.**

TYPE XI. POWER ASSOCIATED WITH A.C. CIRCUITS, POWER FACTOR

Formulae used. 1. Average power dissipated/cycle in a non-inductive circuit ($X_L = X_C$)

$$P = E_v\ I_v$$

2. Average power dissipated/cycle in *RLC* circuit $P = E_v\ I_v \cos\phi$

3. Power factor,

$$\cos\phi = \frac{\text{true power}}{\text{apparent power}} = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Units used. L in henry ; C in farad ; E_v and V in *volt* ; Power in watt ; I_v in ampere and ν in s^{-1}.

Example 35 At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 $m^3\ s^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8\ ms^{-2}$).

Solution. Here, $h = 300$ m, volume/sec, $V = 100\ m^3\ s^{-1}$, $\eta = 60\%$,

Electric power = ?, $g = 9.8\ m/s^2$

$$\text{Hydroelectric power} = \frac{\text{Work}}{\text{time}} = \frac{\text{force}\times\text{dist.}}{\text{time}}$$

= Force × velocity

= (Pressure × area) × velocity

But area × velocity = Volume/sec = V

$\therefore$ Hydroelectric power = $P \times V$

As efficiency is 60%, therefore

$$\text{Power available} = \frac{60}{100} PV = \frac{3}{5}(h\rho g)\,V,$$

where $\rho = 10^3\ kg/m^3$, for water

$$\therefore \quad \text{Power} = \frac{3}{5} \times 300 \times 10^3 \times 9.8 \times 100$$

$$= 1.764 \times 10^8\ \text{W} = \mathbf{176.4\ MW}$$

Example 36 Suppose the frequency of the source in the above example can be varied (a) What is the frequency of the source at which resonance occurs ? (b) Calculate the impedance, the current and power dissipated at the resonant condition. (NCERT Solved Example)

Solution. (*a*) Resonant frequency

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\times3{\cdot}14\sqrt{25{\cdot}48\times10^{-3}\times796\times10^{-6}}}$$

$$= \mathbf{35{\cdot}4\ Hz}$$

(*b*) At resonance, $Z = R = \mathbf{3\ \Omega}$

$$I_v = \frac{E_v}{Z} = \frac{E_v}{R} = \frac{283}{\sqrt{2}\times3} = \mathbf{66{\cdot}7\ A}$$

(*c*) Power dissipated,

$$P = I_v^2\ R = (66{\cdot}7)^2 \times 3 = \mathbf{13350\ watt}$$

Example 37 A circuit containing a 80 mH inductor and a 60 μ F capacitor in series is connected to a 230 V, 50 Hz supply. The resistance in the circuit is negligible.

(a) Obtain the current amplitude and r.m.s. value.

(b) Obtain the r.m.s. values of potential drop across each element.

(c) What is the average power transferred to inductor ?

(d) What is the average power transferred to capacitor ?

(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle']. (Jharkhand Board 2012)

Solution. Here, $L = 80\ mH = 80 \times 10^{-3}$ H,

$C = 60\ \mu F = 60 \times 10^{-6}$ F, $R = 0$, $E_v = 230$ V,

$E_0 = \sqrt{2} \times E_v = \sqrt{2} \times 230$ V

$\nu = 50$ Hz, $\omega = 2\pi\nu = 100\pi$ rad/s

(*a*) $I_0 = ?$ $I_v = ?$

$$I_0 = \frac{E_0}{\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{230\sqrt{2}}{\left(100\pi\times80\times10^{-3} - \frac{1}{100\pi\times60\times10^{-6}}\right)}$$

$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = \frac{230\sqrt{2}}{-27{\cdot}91} = -11{\cdot}63\,\text{amp.}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{-11{\cdot}63}{1{\cdot}414} = -8{\cdot}23 \text{ amp}$$

Negative sign appears as $\omega L < \frac{1}{\omega C}$

$\therefore$ *e.m.f.* lags behind the current by 90°

(*b*) Across L, $V = I_v \times \omega L$

$= 8{\cdot}23 \times 100\,\pi \times 80 \times 10^{-3}$

$= \mathbf{206{\cdot}74 \text{ volt}}$

Across C, $V = I_v \times \frac{1}{\omega C}$

$$= 8{\cdot}23 \times \frac{1}{100\,\pi \times 60 \times 10^{-6}}$$

$= \mathbf{436.84 \text{ volt}}$

As voltages across L and C are 180° out of phase, therefore, they get subtracted.

That is why applied *r.m.s.* voltage

$= 436{\cdot}84 - 206{\cdot}74 = 230{\cdot}1$ volt

(*c*) Average power transferred over a complete cycle by the source to inductor is always **zero** because of phase difference of $\pi/2$ between voltage and current through L.

(*d*) Average power transferred over a complete cycle by the source to the capacitor is also **zero** because of phase difference of $\pi/2$ between voltage and current through C.

(*e*) Total average power absorbed by the circuit is also, therefore **zero.**

Example 38 **A circuit containing an 80 mH inductor and a 250 μF capacitor in series is connected to 240 V, 100 rad/s supply. The resistance of the circuit is negligible. (*i*) Obtain rms value of current. (*ii*) What is the total average power consumed in the circuit ?** (CBSE 2015)

Solution. Here, $L = 80 \text{ mH} = 80 \times 10^{-3}$ H,

$C = 250\ \mu\text{F} = 250 \times 10^{-6}$ F, $E_v = 240$ V,

$\omega = 100$ rad/s

$X_L = \omega L = 100 \times 80 \times 10^{-3}\ \text{H} = 8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 250 \times 10^{-6}} = 40\,\Omega$$

$Z = X_C - X_L = 40 - 8 = 32\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{240}{32} = \mathbf{7{\cdot}5\ A}$$

As $R = 0$, therefore power consumed is **zero.**

Example 39 **A series *LCR* circuit is made by taking $R = 100\ \Omega$, $L = \frac{2}{\pi}$ H and $C = \frac{100}{\pi}\ \mu$F. This series combination is connected to an a.c. source of 220 V, 50 Hz. Calculate the impedance of circuit and peak value of current. Also calculate power factor of the circuit and compare this value with the one at its resonant frequency.** (CBSE 2008)

Solution. Here, $R = 100\ \Omega$,

$$L = \frac{2}{\pi}\text{H},\quad C = \frac{100}{\pi} \times 10^{-6}\text{ F},$$

$E_v = 220$ V, $v = 50$ Hz, $Z = ?$ $I_0 = ?$ $\cos\phi = ?$

$$X_L = \omega L = 2\,\pi\, v\, L = 2\,\pi \times 50 \times \frac{2}{\pi} = 200\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\,\pi\, v \times \frac{100}{\pi} \times 10^{-6}} = 100\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{100^2 + (200 - 100)^2} = \mathbf{141{\cdot}4\ \Omega}$$

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\,E_v}{Z} = \frac{\sqrt{2} \times 220}{141{\cdot}4} = \mathbf{2{\cdot}2\ A}$$

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}}$$

At resonance, $Z = R$; $\cos\phi = \frac{R}{R} = \mathbf{1}$

TYPE XII. TRANSFORMER

Formulae used. 1. Efficiency of transformer,

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p}$$

2. When efficiency of transformer is 100% ,

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = \frac{I_p}{I_s} = K \text{ , transformer ratio.}$$

Units used. E_s, E_p in volt ; I_s, I_p in ampere ; K and η have no units.

Example 40 **Given the input current 15 A and input voltage of 100 V for a step up transformer having 90% efficiency. Find the output power and voltage in the secondary if output current is 3 A.** (CBSE 2017)

Solution. Here, $I_p = 15$ A, $E_p = 100$ V,

$$\eta = 90\%,\ P_o = ?\ E_s = ?\ I_s = 3 \text{ A}$$

$$\text{As } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p}$$

$$\therefore \frac{90}{100} = \frac{E_s \times 3}{100 \times 15};$$

$$E_s = \frac{90 \times 100 \times 15}{3 \times 100} = \mathbf{450\ V}$$

Output power $= P_o = E_s I_s$

$$= 450 \times 3 = \mathbf{1350\ W}$$

Example 41 **A transformer has 500 turns in the primary and 1000 turns in its secondary. The primary voltage is 200 V and load in secondary is 100 ohm. Calculate the current in primary assuming it to be ideal transformer.**

(ICSE 2002)

Solution. Here, $n_p = 500$, $n_s = 1000$

$$E_p = 200 \text{ V}, R_s = 100\ \Omega, I_p = ?$$

In an ideal transformer,

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = \frac{1000}{500} = 2,$$

$$E_s = 2\,E_p = 2 \times 200 = 400 \text{ V}$$

Also, $$\frac{I_p}{I_s} = \frac{n_s}{n_p} = 2,$$

$$I_p = 2 I_s = 2\left(\frac{E_s}{R_s}\right) = \frac{2 \times 400}{100} = \mathbf{8\ A}$$

Example 42 **A step down transformer converts transmission line voltage from 2200 V to 220 V. Primary coil is having 5000 turns. Efficiency of transformer is 90% and output power is 8 kW. Evaluate number of turns in secondary coil and input power.** (Uttarakhand Board 2012)

Solution. Here, $E_p = 2200$ V,

$$E_s = 220 \text{ V},\ n_p = 5000$$

$$\eta = 90\%,\quad P_0 = E_s I_s = 8 \text{ kW} = 8000 \text{ W}$$

$$n_s = ? \qquad P_i = E_p I_p = ?$$

$$\text{As } \eta = \frac{P_0}{P_i}$$

$$\therefore P_i = \frac{P_0}{\eta} = \frac{8 \text{ kW}}{90/100} = \mathbf{8{\cdot}88\ kW}$$

$$\text{As } \frac{E_s}{E_p} = \frac{n_s}{n_p}$$

$$\therefore n_s = \frac{E_s}{E_p} \times n_p = \frac{220}{2200} \times 5000 = \mathbf{500}$$

Example 43 **Calculate current drawn by primary coil of a transformer, which steps down 200 V to 20 V to operate a device of 20 ohm resistance. Assume efficiency of transformer 80%.**

(CBSE 2007)

Solution. Here, $I_p = ?$, $E_p = 200$ V,

$$E_s = 20 \text{ V}, R_s = 20 \text{ ohm}$$

$$\eta = 80\% \qquad \eta = \frac{E_s I_s}{E_p I_p} = \frac{E_s (E_s/R_s)}{E_p I_p}$$

$$\frac{80}{100} = \frac{20 \times 20}{20 \times 200\, I_p} = \frac{1}{10\, I_p}$$

$$I_p = \frac{100}{800} = \mathbf{0{\cdot}125\ A}$$

Example 44 **A 66% efficient transformer is working on 110 V and 2·2 kW power. If current in secondary coil is 6·0 A, calculate (*i*) current in primary coil and (*ii*) voltage across secondary coil.**

Solution. Here $\eta = 66/100$, $E_p = 110$ V,

$$P_p = 2{\cdot}2 \text{ kW} = 2200 \text{ W}$$

$$I_s = 6{\cdot}0 \text{ A},\quad I_p = ?,\quad E_s = ?$$

As $P_p = E_p I_p$

$$\therefore\ I_p = P_p/E_p = \frac{2200}{110} = \mathbf{20\ A}$$

$$\text{As } \eta = \frac{E_s I_s}{E_p I_p} \quad \therefore \frac{66}{100} = \frac{E_s \times 6}{2200}$$

$$E_s = \frac{66 \times 2200}{100 \times 6} = \mathbf{242\ V}$$

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Example 45 **A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two line wires carrying power is 0.5 Ω per km. The town gets power from the lines through a 4000–220 V step down transformer at a substation in the town.**

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterize the step up transformer at the plant.

Solution. Here, Power required,

$P = 800 \text{ kW} = 800 \times 10^3 \text{ W}$

Total resistance of two wire lines

$R = 2 \times 15 \times 0.5 = 15\ \Omega.$

As supply is through 4000 – 220 V transformer

$\therefore \quad E_v = 4000$ volt

As $\quad P = E_v\, I_v \qquad \therefore \quad 800 \times 10^3 = 4000\, I_v$

$$I_v = \frac{800 \times 10^3}{4000} = 200 \text{ amp}$$

Line power loss $= I_v^2 R = (200)^2 \times 15$

$= 60 \times 10^4$ watt

$= \mathbf{600\ kW}$

If there is no power loss due to leakage,

Then the essential plant supply = 800 + 600

$= \mathbf{1400\ kW}$

Voltage drop on the line $= I_v R$

$= 200\,(15)$

$= 3000$ volt.

$\therefore$ Voltage for transmission = 3000 + 4000

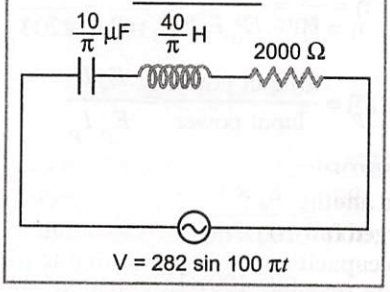
= 7000 V

As power is generated at 440 volt, the step up transformer needed at the plant is **440 V – 7000 V**

TYPE XIII. TYPICAL EXAMPLES

Example 46 **Equation of emf of a generator is $V = 282 \sin 100\,\pi t$ volt. Internal resistance of generator is 2000 Ω. It is connected as shown in Fig. 7.39. Find the frequency of generator and impedance of circuit.** (Kerala Board 2012)

FIGURE 7.39



Solution. Here, $V = 282 \sin 100\,\pi t$, $R = 2000\ \Omega$

$$L = \frac{40}{\pi} H \quad \text{and} \quad C = \frac{10}{\pi}\mu F. \quad \nu = ?, Z = ?$$

The standard form of alternating emf is

$E = E_0 \sin \omega t$. Compare it with

$V = 282 \sin 100\,\pi t$, to get

$E_0 = 282\ V$, $\omega = 2\pi\nu = 100\,\pi$,

$$\nu = \frac{100}{2} = \mathbf{50\ Hz}$$

$$X_L = \omega L = 2\pi\nu \times \frac{40}{\pi}$$

$$= 2 \times 50 \times 40 = 4000 \text{ ohm}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2\pi \times 50 \times \frac{10}{\pi} \times 10^{-6}} = 1000\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(2000)^2 + (4000 - 1000)^2}$$

$$= 10^3\sqrt{4+9} = \mathbf{3{\cdot}6 \times 10^3\ \Omega}$$

Example 47 **A step-up transformer operates on a 220 volt line and supplies to load, a current of 2A. The ratio of primary and secondary winding is 1 : 25. Calculate secondary voltage, primary current and power output, if efficiency is 80%.**

Solution. Here, $E_p = 220$ V, $I_s = 2$ A,

$$\frac{n_p}{n_s} = \frac{1}{25}\ ;\ E_s = ?, I_p = ?, E_s I_s = ?$$

As $\quad \dfrac{E_s}{E_p} = \dfrac{n_s}{n_p} \quad \therefore \quad \dfrac{E_s}{220} = 25,\ E_s = \mathbf{5500\ V}$

Output power $= E_s I_s = 5500 \times 2 = \mathbf{11000}$ **watt**

As $\eta = \frac{P_0}{P_i} = \frac{E_s I_s}{E_p I_p}$ $\therefore \frac{80}{100} = \frac{11000}{220 I_p} = \frac{50}{I_p}$

$$\therefore \quad I_p = \frac{50 \times 5}{4} = \mathbf{62{\cdot}5\ A}$$

Example 48 **A capacitor C_1 of capacitance 1 μ F and another capacitor C_2 of capacitance 2 μF are charged fully separately by a common battery. The two capacitors are then allowed to discharge separately through equal resistors at time $t = 0$. Which of the following is true?**

(a) A current in each of the two discharging circuits is zero at $t = 0$.

(b) the current in the two discharging circuits at $t = 0$ are equal but not zero.

(c) the currents in the two discharging circuits at $t = 0$ are unequal.

(d) capacitor C_1 loses 50% of its initial charge sooner then C_2 loses 50% of its initial charge.

Solution. During discharging of condenser,

$$q = q_0 e^{-t/RC} \quad \text{and} \quad i = \frac{E_0 e^{-t/RC}}{R}$$

is same in the two cases.

Also, R is same as they are discharged through equal resistors.

$$\therefore \text{At } t = 0, \quad i = \frac{E_0}{R} e^0 = \frac{E_0}{R} = \text{constant}$$

Hence, current in the two discharging circuits at $t = 0$ are equal but not zero.

Now, time constant, $\tau = RC$

As $C_2 > C_1$, therefore $\tau_2 > \tau_1$, or $\tau_1 < \tau_2$, *i.e.*, C_1 would lose charge faster than C_2. Hence **alternatives (*b*) and (*d*) are correct.**

Alternatives (*a*) and (*c*) are wrong.

Example 49 **A solenoid of resistance 50 Ω and inductance 80 H is connected to a 200 V battery. How long will it take for the current to reach 50% of its final equilibrium value ? Calculate the maximum energy stored ?**

Solution. Here, $R = 50\ \Omega$, $L = 80$ H,

$E = 200$ V, $t = ?$

$$I = \frac{50}{100} I_0$$

$$\tau = \frac{L}{R} = \frac{80}{50} = \frac{8}{5}\text{s}, \quad I_0 = \frac{E}{R} = \frac{200}{50} = 4\text{ A}$$

From $I = I_0 (1 - e^{-t/\tau})$

$$\frac{50}{100} I_0 = I_0 (1 - e^{-t/\tau});$$

$$t = \tau \log_e 2 = \frac{8}{5} \times 0{\cdot}693 = \mathbf{1{\cdot}109\ s}$$

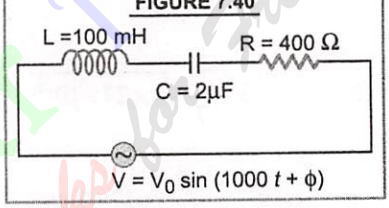
Maximum energy stored

$$= \frac{1}{2} L I_0^2 = \frac{1}{2} \times 80 \times (4)^2 = \mathbf{640\ J}$$

Example 50 **Find the value of phase lag/lead between the current and voltage in the given *CLR* circuit, Fig. 7.40. Without making any other change, find the value of additional capacitor, such that when joined suitably to $C = 2$ μF, would make the power factor of this circuit unity.**

(CBSE 2015)

FIGURE 7.40



Solution. Here, $L = 100$ mH $= 10^{-1}$ H,

$C = 2 \times 10^{-6}$ F

$R = 400\ \Omega$, $V = V_0 \sin(1000\, t + \phi_0)$

$\omega = 1000$ rad/s

$X_L = \omega L = 1000 \times 10^{-1} = 100\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 2 \times 10^{-6}} = 500\ \Omega$$

As $X_C > X_L$, current leads the applied emf by phase angle ϕ, where

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{500 - 100}{400} = 1, \quad \phi = \mathbf{45^\circ}$$

The power factor will be unity, when $\phi = 0°$.

$$\therefore \quad X_{C'} - X_L = 0$$

$$X_{C'} = X_L = 100\,\Omega$$

$$\frac{1}{\omega C'} = 100,$$

$$C' = \frac{1}{100\,\omega} = \frac{1}{100 \times 1000}\text{ F} = 10\ \mu\text{F}$$

Hence, additional capacitance, $C' - C = 10 - 2 = 8\ \mu F$ **has to be connected in parallel** with the given capacitor of 2 μF to make the power factor equal to unity.

Example 51 **Obtain the resonant frequency and Q-factor of a series LCR circuit with $L = 3$ H, $C = 27\ \mu F$, $R = 7{\cdot}4$ ohm. It is desired to improve the sharpness of resonance of circuit by reducing its full width at half maximum by a factor of 2. Suggest a suitable way.** (CBSE Sample Paper 2019-20)

Solution. Here, $L = 3$ H ;

$C = 27\ \mu F = 27 \times 10^{-6}$ F,

$R = 7{\cdot}4$ ohm

Resonant frequency $\nu_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\times\frac{22}{7}\sqrt{3\times27\times10^{-6}}}$$

$$\nu_r = \frac{7\times10^3}{44\times9} = \mathbf{17{\cdot}67\ c/s}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{7{\cdot}4}\sqrt{\frac{3}{27\times10^{-6}}} = \frac{10^3}{7{\cdot}4\times3}$$

$$= \mathbf{45{\cdot}04}$$

The sharpness of resonance of the circuit can be improved by reducing its full width at half maximum by a factor of 2. A suitable way is to decrease the resistance to $7{\cdot}4/2 = 3{\cdot}7$ ohm.

Example 52 **An inductor of 200 mH, capacitor of 400 μF and a resistor of 10 ohm are connected in series to an a.c. source of 50 V of variable frequency. Calculate (*i*) angular frequency at which maximum power dissipation occurs in the circuit and the corresponding value of effective current, and (*ii*) value of *Q* factor in the circuit.** [CBSE 2017 (C)]

Solution. Here, $L = 200$ mH $= \dfrac{2}{10}$ H

$C = 400\ \mu F = 400 \times 10^{-6}$ F $= 4 \times 10^{-4}$ F

$R = 10$ ohm, $E_v = 50$ V

(*i*) Maximum power dissipation occurs in the circuit at resonance, *i.e.*, when

$$\omega L = \frac{1}{\omega C}$$

or $$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{2}{10}\times4\times10^{-4}}}$$

$$\omega = \frac{1}{\sqrt{80\times10^{-6}}} = \frac{10^3}{8{\cdot}944} = \mathbf{111{\cdot}8\ rad/s}$$

$$I_v = \frac{E_v}{Z} = \frac{E_v}{R} = \frac{50}{10} = \mathbf{5\ A}$$

(*ii*) $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{10}\sqrt{\frac{2/10}{4\times10^{-4}}} = \frac{1}{10}\times\frac{100}{4{\cdot}47}$$

$$= \mathbf{2{\cdot}237}$$

Example 53 **Calculate the current drawn by the primary of a 90% efficient transformer which steps down 220 V to 22 V, if the output resistance is 440 ohm.** [CBSE 2018 (C)]

Solution. Here, $I_p = ?$ $\eta = 90\%$

$E_p = 220$ V, $E_s = 22$ V,

$R_s = 440$ ohm

$$I_s = \frac{E_s}{R_s} = \frac{22}{440} = \frac{1}{20}\ \text{A}$$

From $$\eta = \frac{E_s\, I_s}{E_p\, I_p}$$

$$\frac{90}{100} = \frac{22\times1/20}{220\times I_p}$$

$$I_p = 22\times\frac{1}{20}\times\frac{100}{220\times90}$$

$$I_p = \frac{1}{180}\ \text{A} = \mathbf{5{\cdot}55\times10^{-3}\ A}$$

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REVISION AT A GLANCE

1. **An alternating current** is that which changes continuously in magnitude and periodically in direction. It can be represented by a sine curve or a cosine curve, *i.e.*, $I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$
Here, I_0 is *peak value* of current and I is *instantaneous value* of current. $\omega = 2\pi/T = 2\pi\nu$ where T is *period* of a.c. and ν is *frequency* of a.c. The alternating *e.m.f.* can similarly be represented as
$$E = E_0 \sin \omega t \qquad \text{or} \qquad E = E_0 \cos \omega t$$

2. **Mean or Average value** of alternating current over any half cycle is that value of steady current, which would send the same amount of charge through a circuit in the time of half cycle (*i.e.* $T/2$) as is sent by a.c. through the same circuit in the same time.
Thus $(I_m \text{ or } I_{av})_{0 \to T/2} = \frac{2}{\pi} I_0 = 0.637\ I_0$ and $(I_m \text{ or } I_{av})_{T/2 \to T} = -\frac{2}{\pi} I_0 = -0.637\ I_0$
Over a complete cycle ($0 \to T$), I_m or $I_{av} = 0$.
Similarly, for alternating *e.m.f.*, we can write mean value as
$(E_m \text{ or } E_{av})_{0 \to T/2} = \frac{2}{\pi} E_0 = 0.637\ E_0$, $(E_m \text{ or } E_{av})_{T/2 \to T} = \frac{-2}{\pi} E_0 = -0.637\ E_0$
Over a complete cycle ($0 \to T$), E_m or $E_{av} = 0$

3. **The root mean square (r.m.s.) value** of alternating current is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating current, when passed through the same resistance for the same time. The r.m.s. value of a.c. is also called **effective value** or **virtual value** of a.c. It is represented by I_{rms}, I_{eff} or I_v. It can be shown that
$$I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \qquad \text{and} \qquad E_v = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$
All a.c. instruments measure *virtual values* of a.c.
The behaviour of an ohmic resistance R in a.c. circuit is the same as in d.c. circuit. Through R, alternating *e.m.f.* and alternating current are in *same phase*.

4. **Inductive reactance** (X_L) is the resistance offered by an inductor and is given by $X_L = \omega L = 2\pi\nu L$
Through a pure inductor, alternating current lags behind the alternating *e.m.f.* by phase angle of 90°.

5. **Capacitative reactance** (X_C) is the resistance offered by a capacitor and is given by $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$
Through a pure capacitor, alternating current leads the alternating *e.m.f.* by a phase angle of 90°.
X_L and X_C, both are measured in ohm.

6. The total effective resistance of *RLC* circuit is called **Impedance** (Z) of the circuit. It is given by
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
The angle by which alternating voltage leads the alternating current in *RLC* circuit is given by
$$\tan \phi = \frac{X_L - X_C}{R}$$

7. **Impedance Triangle** is a right angled triangle whose base represents ohmic resistance R, perpendicular represents reactance ($X_L - X_C$) and hypotenuse represents impedance Z of the *RLC* circuit. From this triangle, we can calculate Z and ϕ as stated above. The formulae for Z and ϕ are given at S. No. 13.

8. When a condenser of capacity C charged to a certain potential is connected to an inductor L, energy stored in C oscillates between L and C. The frequency of **energy oscillations** is given by $\nu = \frac{1}{2\pi\sqrt{LC}}$
If there is no loss of energy, total energy in L and C at every instant remains constant.

9. A circuit in which inductance L, capacitance C and resistance R are connected in series and the circuit admits *maximum current*, is called **series resonance circuit**. The necessary condition for resonance is

$$X_L = X_C, \quad \text{which gives} \quad \nu = \frac{1}{2\pi\sqrt{LC}}$$

i.e., frequency of a.c. fed to the circuit becomes equal to natural frequency of energy oscillations in the circuit.

Under this condition, $Z = R$, $I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ = maximum

The series resonance circuit is used as an *acceptor* circuit in radio and T.V. receiver sets.

The *sharpness of tuning* at *resonance* is measured by **Q factor** or Quality factor of the circuit. It is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

10. **Average power** associated over a complete cycle in a pure resistive circuit or a non inductive circuit (containing R only or containing R, L, C with $X_L = X_c$) is $P = E_v\ I_v$

On the contrary, average power associated over a complete cycle with a pure inductor or pure capacitor is *zero*. Hence current through L and through C is said to be *Wattless* or *Idle current*.

In general, if ϕ is the phase angle between alternating voltage and alternating current in any a.c. circuit, then magnitude of **wattless current** is I_v **sin** ϕ.

In an inductor, energy stored during growth of current is $\frac{1}{2}LI_0^2$. This is spent in maintaining the current during decay.

In a capacitor, energy stored during charging is

$$\frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This is spent during discharging of the condenser.

In an *LCR* circuit, average power associated over a complete cycle is $P = E_v\ I_v \cos\phi$,

where ϕ is phase angle between alternating voltage and alternating current in the circuit, $\cos\phi = P/E_v\ I_v$ = true power/apparent power, is called *power factor* of the a.c. circuit. The value of power factor varies from 0 to 1.

11. **A transformer** is an electrical device, which is used for changing alternating voltages.

It is based on the phenomenon of mutual induction. If n_p is number of turns in primary and n_s is number of turns in secondary, then

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{n_s}{n_p} = K$$

For a step up transformer, $K > 1$ and for a step down transformer, $K < 1$. The main use of transformer is in transmission of a.c. over long distances, at extremely high voltages. This reduces the energy losses in transmission, and also makes the transmission cost effective.

$$\text{Efficiency of transformer} = \frac{\text{Output power}}{\text{input power}}, \quad \eta = \frac{E_s\ I_s}{E_p\ I_p}$$

12. **A choke coil** is a pure inductor used for controlling current in an a.c. circuit. Through pure L, phase difference between E and I is $\theta = 90°$.

$\therefore$ Average power consumed/cycle $= E_v\ I_v \cos 90° = 0$.

As $$I_0 = \frac{E_0}{X_L} = \frac{E_0}{\omega L} = \frac{E_0}{2\pi\nu L}$$

$\therefore$ for reducing low frequency alternating current, choke coils with laminated soft iron cores are used, and for reducing high frequency alternating currents, air cored chokes are used.

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OBJECTIVE TYPE QUESTIONS

Based on NCERT Book

I. Multiple Choice Questions

1. The r.m.s. current in a circuit connected to 50 Hz a.c. source is 15 A. The value of current in the circuit $\left(\frac{1}{600}\right)s$ after the instant the current is zero is

(a) $\frac{15}{\sqrt{2}}$ A (b) $15\sqrt{2}$ A

(c) $\frac{\sqrt{2}}{15}$ A (d) 8 A **(CBSE 2022)**

2. In a circuit, the phase difference between the alternating current and the source voltage is $\pi/3$. Which of the following cannot be the elements of the circuit?

(a) only C (b) only L

(c) only R (d) All the three

3. A variable copacitor is connected to a 200 V battery. If its capacitance is changed from 2 μF to X μF, the decrease in energy of the capacitor is 2×10^{-2} J. The value of X is

(a) 1 μF (b) 2 μF

(c) 3 μF (d) 4 μF **(CBSE 2022)**

4. The impedance of a series LCR circuit is

(a) $R + X_L + X_C$ (b) $\sqrt{\frac{1}{X_C^2} + \frac{1}{X_C^2} + R^2}$

(c) $\sqrt{X_L^2 - X_C^2 + R^2}$ (d) $\sqrt{R^2 + (X_L - X_C)^2}$

(CBSE 2022)

5. The voltage across a resistor, an inductor and a capacitor connected in series to an *a.c.* source are 20 V, 15 V and 30 V respectively. The resultant voltage in the circuit is

(a) 5 V (b) 20 V

(c) 25 V (d) 65 V **(CBSE 2022)**

6. When an alternating voltage $E = E_0 \sin \omega t$ is applied to a circuit, a current $I = I_0 \sin(\omega t + \pi/2)$ flows through it. The average power dissipated in the circuit is.

(a) $E_{rms} \times I_{rms}$ (b) $E_0 I_0$

(c) $\frac{E_0 I_0}{\sqrt{2}}$ (d) Zero **(CBSE 2022)**

7. Choose the quantity whose SI unit is not ohm.

(a) Resistance (b) Reactance

(c) Capacitance (d) Impedance

8. The peak value of 220 V a.c. is

(a) 220 V (b) $\frac{220}{\sqrt{2}}$ V

(c) 440 V (d) $220\sqrt{2}$ V

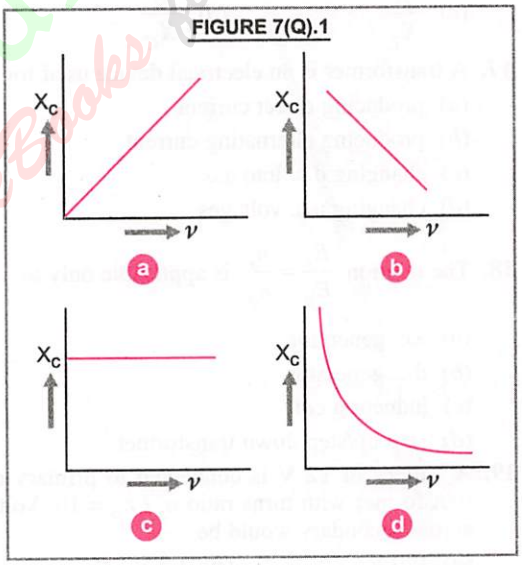
9. Which of the following does not have the dimensions of time ?

(a) RC (b) $\frac{L}{R}$

(c) $\frac{R}{L}$ (d) $\sqrt{LC}$

10. The variation of reactance of a capacitor with frequency is represented correctly by

FIGURE 7(Q).1



11. Phase difference between voltages across L and C in series is

(a) 0° (b) 90°

(c) 180° (d) 360°

12. The resistance of a coil for direct current is 10 ohm. When a.c. is sent through the same coil, its resistance would be

(a) 10 Ω (b) > 10 ohm

(c) < 10 ohm (d) cannot say

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13. The average value of a.c. voltage $E = E_0 \sin \omega t$ over the time interval $t = 0$ to $t = \pi/\omega$ is

(a) $-2E_0/\pi$ (b) E_0/π

(c) $\frac{2E_0}{\pi}$ (d) zero

14. The alternating current from a source is represented by $I = 0{\cdot}5 \sin 314\ t$. The frequency of a.c. is

(a) 314 Hz (b) 100 Hz

(c) 50 Hz (d) zero

15. Q factor of resonance is given by

(a) $\frac{1}{R}\sqrt{\frac{L}{C}}$ (b) $\frac{1}{R}\sqrt{\frac{C}{L}}$

(c) $\frac{1}{L}\sqrt{\frac{R}{C}}$ (d) $\frac{1}{C}\sqrt{\frac{L}{R}}$

16. The power factor of an a.c. circuit is given by $\cos \phi =$

(a) $\frac{R}{Z}$ (b) $\frac{Z}{R}$

(c) $\frac{R}{X_L}$ (d) $\frac{R}{X_C}$

17. A transformer is an electrical device used for

(a) producing direct current

(b) producing alternating current

(c) changing d.c. into a.c.

(d) changing a.c. voltages

18. The relation $\frac{E_s}{E_p} = \frac{n_s}{n_p}$ is applicable only to

(a) a.c. generator

(b) d.c. generator

(c) induction coil

(d) step up/step down transformer

19. A battery of 12 V is connected to primary of a transformer with turns ratio $n_s / n_p = 10$. Voltage across secondary would be

(a) 120 V (b) 1·2 V

(c) 12 V (d) Zero

20. Out of the following, choose the **wrong** statement :

(a) A transformer cannot work on d.c.

(b) A transformer cannot change the frequency of a.c.

(c) A transformer can produce a.c. power

(d) In a transformer, when a.c. voltage is raised n times, the alternating current reduces to $1/n$ time.

21. In a series LR-circuit, the inductive reactance is equal to the resistance R of the circuit. An *e.m.f.* $E = E_0 \cos(\omega t)$ is applied to the circuit. The power consumed in the circuit is

(a) $\frac{E_v^2}{R}$ (b) $\frac{E_v^2}{2R}$

(c) $\frac{E_v^2}{4R}$ (d) $\frac{E_v^2}{8R}$

22. One 60 V, 100 W bulb is to be connected to 100 V, 50 Hz ac-source. The potential drop across the inductor is ($f = 50$ Hz)

(a) 80 V (b) 40 V

(c) 10 V (d) 20 V

23. An AC voltage source of variable angular frequency ω and fixed amplitude V is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(a) The bulb glows dimmer

(b) The bulb glows brighter

(c) Net impedance of circuit is unchanged

(d) Total impedance of the circuit increases

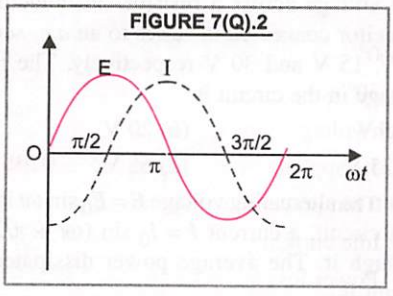
24. An alternating *e.m.f.* of angular frequency ω is applied across an inductance. The instantaneous power developed across it has an angular frequency

(a) $\omega/4$ (b) $\omega/2$

(c) ω (d) 2ω

25. The variation of the instantaneous current $I(t)$ and instantaneous *e.m.f.* $E(t)$ in a circuit is as shown in Fig. 7(Q).2. Which of the following statements is correct ?

FIGURE 7(Q).2



(a) The voltage lags behind the current by $\pi/2$

(b) The voltage leads the current by $\pi/2$

(c) The voltage and the current are in phase

(d) The voltage leads the current by π

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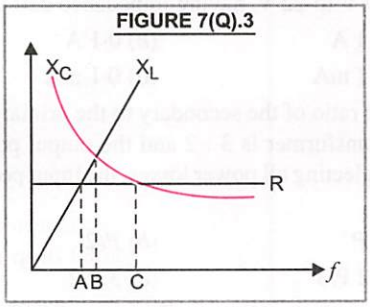
26. Fig. 7(Q).3 shows variation of R, X_L and X_C with frequency f in a series L, C, R circuit. Then for what frequency point, the circuit is inductive.

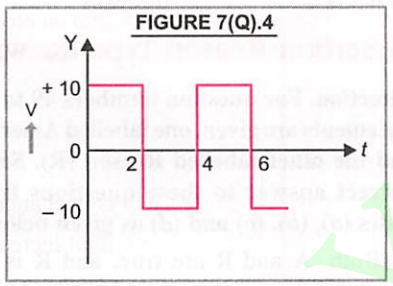
FIGURE 7(Q).3



(*a*) A (*b*) B
(*c*) C (*d*) A and B

27. The r.m.s. voltage of the wave form shown is

FIGURE 7(Q).4



(*a*) 10 V (*b*) 7 V
(*c*) 6·37 V (*d*) 12 V

28. In electric arc furnace, Cu or Iron is melted due to variation of
(*a*) current (*b*) magnetic field
(*c*) voltage (*d*) electric field

29. When AC source is connected across series R-C combination, the *ac*-current may lead *ac*-voltage by
(*a*) 0° (*b*) 180°
(*c*) 30° (*d*) 90°

30. High voltage transmission line is preferred as
(*a*) Its appliances are less costly
(*b*) Thin power cables are required
(*c*) Idle current very low
(*d*) Power loss is very less

31. In series R-L-C circuit, quality factor can be improved by
(*a*) decreasing L (*b*) increasing C
(*c*) decreasing R (*d*) decreasing R and L

32. When *ac*-source is connected across series R-L-C combination, maximum power loss will occur provided
(*a*) current and voltage are in phase
(*b*) current from source is minimum
(*c*) inductance is minimum
(*d*) capacitance is maximum

33. In R-L-C series *ac*-circuit, impedance cannot be increased by
(*a*) increasing frequency of source
(*b*) decreasing frequency of source
(*c*) increasing the resistance
(*d*) increasing the voltage of the source

34. In highly inductive load circuit, it is more dangerous when
(*a*) we close the switch
(*b*) open the switch
(*c*) increasing the resistance
(*d*) decreasing the resistance

35. In electric sub-station in township, large capacitor banks are used
(*a*) to reduce power factor
(*b*) to improve power factor
(*c*) to decrease current
(*d*) to increase current in the circuit

36. In a purely resistive *a.c.* circuit, the current
(*a*) is in phase with the *e.m.f.*
(*b*) leads the *e.m.f.* by a difference of π radians phase
(*c*) leads the *e.m.f.* by a phase difference of $\pi/2$ radians
(*d*) lags behind the *e.m.f.* by phase difference of $\pi/4$ radians

37. A capacitor of capacitance C has reactance X. If capacitance and frequency become double, then the capacitative reactance will be
(*a*) $2X$ (*b*) $4X$
(*c*) $\frac{X}{2}$ (*d*) $\frac{X}{4}$

38. Reactance of a capacitor of capacitance C for an alternating current of frequency $\frac{400}{\pi}$ Hz is 25 Ω. The value of C is
(*a*) 25 μF (*b*) 50 μF
(*c*) 75 μF (*d*) 100 μF

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39. The core of a transformer is laminated, so as to
(a) make it light weight
(b) make it robust and strong
(c) increase the secondary voltage
(d) reduce energy loss due to eddy current

40. In a transformer, the no. of turns of primary and secondary coil are 500 and 400 respectively. If 220 V a.c. is supplied to the primary coil, then ratio of currents in primary and secondary coils is
(a) 4 : 5 (b) 5 : 4
(c) 5 : 9 (d) 9 : 5

41. In an a.c. circuit, the applied voltage and resultant current are $E = E_0 \sin \omega t$ and $I = I_0 \sin (\omega t + \pi/2)$. The average power consumed in the circuit is
(a) $E_0 I_0$ (b) $E_0 I_0/2$
(c) $E_0 I_0/\sqrt{2}$ (d) Zero (CBSE 2022)

42. In a series *LCR* circuit, at resonance, the current is equal to
(a) $\frac{V}{R}$ (b) $\frac{V}{X_C}$
(c) $\frac{V}{X_L - X_C}$ (d) $\frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$
(CBSE 2022)

43. The frequency of an a.c source for which a 10 μF capacitor has a resistance of 1000 Ω is
(a) $\frac{1000}{\pi}$ Hz (b) 50 Hz
(c) $\frac{50}{\pi}$ Hz (d) $\frac{100}{\pi}$ Hz
(CBSE 2022)

44. Which one of the following statements is true ?
(a) An inductor has infinite resistance in d.c. circuit
(b) An inductor and a capacitor both cannot conduct in d.c. circuit
(c) A capacitor can conduct in a d.c. circuit but not an inductor
(d) An inductor can conduct in a d.c. circuit but not a capacitor. (CBSE 2022)

45. In a transformer having 100% efficiency, the input power is 60 W. The number of turns of primary coils is 100 and the number of turns of secondary coils is 300. Then the output power is :
(a) 60 W (b) 120 W
(c) 180 W (d) 240 W

46. The current flowing in a step down transformer 220 V to 22 V, having impedance 220 Ω is :
(a) 1 A (b) 0·1 A
(c) 1 mA (d) 0·1 mA

47. The ratio of the secondary to the primary turns in a transformer is 3 : 2 and the output power is P. Neglecting all power losses, the input power must be
(a) *P* (b) *P*/2
(c) 2 P/3 (d) 3 P/2

48. A transformer has an effeciency of 80%. It works at 4 kW, 100 V. If secondary voltage is 240 V, the current in primary coil is :
(a) 10 A (b) 4 A
(c) 0·4 A (d) 40 A

II. Assertion-Reason Type Questions

Direction. For question numbers 49 to 57, two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below :
(a) Both, A and R are true, and R is correct explanation of A.
(b) Both, A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, and R is also false.

49. **Assertion.** Time constant of *C-R* circuit depends on the emf of the source of current.
Reason. Time constant of *C-R* circuit is *CR*.

50. **Assertion.** A *d.c.* ammeter can not read alternating current.
Reason. Average value of *a.c.* over half cycle is zero.

51. **Assertion.** The impedance offered by an ideal inductor to the flow of direct current is zero.
Reason. The frequency of *d.c.* is zero and reactance inductance *L* is, of $X_L = \omega L$ where ω is the angular frequency of current.

52. **Assertion.** A series *L-C-R* circuit fed with *a.c.* will have minimum impedance when current through the circuit is maximum.
Reason. The power consumed in purely inductive or capacitative circuit is zero.

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53. Assertion. The minimum inductance that can be obtained by combining three inductors of 2 H, 3 H and 6 H is 1 H.

Reason. The minimum inductance can be achieved when the various inductors are connected in series.

54. Assertion. The power factor of inductor is zero.

Reason. When a pure inductor is used in *a.c.* circuit, then the current and emf differ in phase by 90°.

55. Assertion. Condenser serves as a good conductor for *d.c.*

Reason. Capacitative reactance, $X_C = \omega C$.

56. Assertion. The core of a transformer is made of a material of high permeability.

Reason. The leakage of magnetic flux through core increases the effeciency of transformer.

57. Assertion. A step up transformer can not be used as a step down transformer.

Reason. A transformer works only on *d.c.*

ANSWERS

I. Multiple Choice Questions

1. (*a*)	**2.** (*d*)	**3.** (*a*)	**4.** (*d*)	**5.** (*c*)	**6.** (*d*)	**7.** (*c*)	**8.** (*d*)	**9.** (*c*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**13.** (*c*)	**14.** (*c*)	**15.** (*a*)	**16.** (*a*)	**17.** (*d*)	**18.** (*d*)	**19.** (*d*)	**20.** (*c*)
21. (*b*)	**22.** (*a*)	**23.** (*b*)	**24.** (*d*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*b*)	**29.** (*c*)	**30.** (*d*)
31. (*c*)	**32.** (*a*)	**33.** (*d*)	**34.** (*b*)	**35.** (*b*)	**36.** (*a*)	**37.** (*d*)	**38.** (*b*)	**39.** (*d*)	**40.** (*a*)
41. (*d*)	**42.** (*a*)	**43.** (*c*)	**44.** (*d*)	**45.** (*a*)	**46.** (*b*)	**47.** (*c*)	**48.** (*d*)		

II. Assertion-Reason Type Questions

49. (*d*)	**50.** (*c*)	**51.** (*a*)	**52.** (*b*)	**53.** (*c*)	**54.** (*b*)	**55.** (*d*)	**56.** (*c*)	**57.** (*d*)

HINTS/EXPLANATIONS For Difficult Questions

I. Multiple Choice Questions

1. Here, $I_{rms} = 15$ A

$$I_0 = \sqrt{2}\, I_{rms} = 15\sqrt{2} \text{ A}$$

Now, $I = I_0 \sin \omega t = I_0 \sin 2\pi \nu t$

$$= 15\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{600}$$

$$= 15\sqrt{2} \sin \pi/6$$

$$= 15\sqrt{2} \times \frac{1}{2}$$

$$= \frac{15}{\sqrt{2}} \text{ amp}$$

2. Through C, current leads the source voltage by 90°. Through L, current lags behind the source voltage by 90°. Through R, current is in phase with source voltage. Phase diff. of $\pi/3$ is not possible in all the three cases. **Choice (*d*) is correct.**

3. Here, $V = 200$ volt, $C = 2\ \mu$F, $C' = X\ \mu$F

Decrease in energy $= \frac{1}{2}CV^2 - \frac{1}{2}C'V^2$

$$= \frac{1}{2}(C - C')\, V^2$$

$$2 \times 10^{-2} = \frac{1}{2}(C - C')\, (200)^2$$

$$C - C' = \frac{4 \times 10^{-2}}{4 \times 10^4} = 10^{-6} \text{ F} = 1\ \mu\text{F}$$

$$C' = C - 1 = 2 - 1 = \mathbf{1\ \mu F}$$

4. Impedance of a series LCR circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

5. Here, $V_R = 20$ V, $V_L = 15$ V and $V_C = 30$ V

V_R is in phase of *a.c.* V_C and V_L are in opposite direction to each other but perpendicular to V_R.

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Hence,

Resultant voltage, $V = \sqrt{V_R^2 + (V_C - V_L)^2}$

$= \sqrt{20^2 + (30-15)^2}$

$= \sqrt{400+225} = \sqrt{625}$

$= 25$ V

6. Here, phase diff. $\phi = \pi/2$

Average power dissipated.

$$P_{av} = E_v I_v \cos\phi = \frac{1}{2} E_0 I_0 \cos\phi$$

$$= \frac{1}{2} E_0 I_0 \cos \pi/2 = \textbf{Zero}$$

7. SI unit of capacitance is farad.

8. $E_0 = \sqrt{2}\, E_v = \sqrt{2} \times 220$ V

9. $RC = \frac{V}{I} \times \frac{Q}{V} = \frac{Q}{I} = \frac{It}{I} = t$

$\frac{L}{R} = \frac{e/(dI/dt)}{e/I} = \frac{I \times dt}{dI} \rightarrow dt$;

$\frac{R}{L}$ will be reciprocal of time

$$\sqrt{LC} = \sqrt{\frac{e}{dI/dt} \times \frac{Q}{e}} = \sqrt{\frac{Q\,dt}{dI}} = \sqrt{\frac{I\,dt \times dt}{dI}} \rightarrow dt.$$

Thus **option (c) is true.**

10. As $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$

$\therefore \quad X_C = \frac{1}{\nu}$

Hence choice (d) is correct.

11. Across L, voltage leads the current by 90°, and across C, voltage lags behind the current by 90°.

$\therefore$ Phase difference between voltage across L and C in series is $(90° + 90°) = \mathbf{180°}$

12. Here, $R = 10$ ohm.

When a.c. is sent through the coil.

$$Z = \sqrt{R^2 + X_L^2} > R \quad \text{or} \quad \mathbf{Z > 10\ ohm.}$$

13. Average value of *a.c.* voltage over the time internal $t = 0$ to $t = \pi/\omega$ is

$$= \frac{\int_0^{\pi/\omega} E_0 \sin \omega t\, dt}{\int_0^{\pi/\omega} dt} = \frac{E_0}{\omega} (-\cos \omega t)_0^{\pi/\omega} \frac{1}{\pi/\omega}$$

$$= \frac{-E_0}{\pi}\left[\cos \omega \times \frac{\pi}{\omega} - \cos 0°\right]$$

$$= \frac{-E_0}{\pi}[-1-1] = \frac{2E_0}{\pi}$$

14. As $I = 0{\cdot}5 \sin 314\, t$

$\therefore \quad \omega = 2\pi\nu = 314$

$$\nu = \frac{314}{2\pi} = \frac{314}{2 \times 314} = \mathbf{50\ Hz}$$

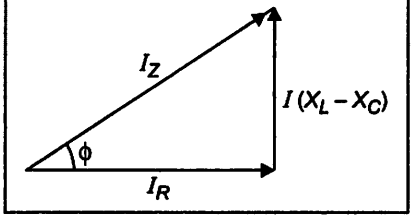
15. Q factor $= \dfrac{\text{(Voltage across } L \text{ or } C\text{) (at resonance)}}{\text{Voltage across } R}$

$$= \frac{(\omega_r L)\, I}{RI} = \frac{\omega_r L}{R}$$

$$= \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \left[\because \omega_r = \frac{1}{\sqrt{LC}}\right]$$

16. Power factor $= \cos\phi = \frac{IR}{Iz} = \frac{R}{z}$

FIGURE 7(Q).5



where z is the impedance, X_L and X_C are the rectance due to L and C in *a.c.* circuit.

19. A transformer does not work on d.c.

$\therefore \quad E_s = \mathbf{0}.$

20. A transformer **cannot produce a.c. power.**

21. Here, $X_L = R, \quad Z = \sqrt{R^2 + X_L^2} = R\sqrt{2}$

$$\tan\theta = \frac{X_L}{R} = 1, \quad \theta = 45°$$

$$P = E_v I_v \cos\theta = E_v \frac{E_v}{Z} \cos 45°$$

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$$P = \frac{E_v^2}{R\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{E_v^2}{2R}$$

22. From $V_L = \sqrt{E_v^2 - E_R^2}$

$$V_L = \sqrt{100^2 - 60^2} = \sqrt{(100+60)(100-60)}$$

$$= \sqrt{160 \times 40} = \mathbf{80\ V}$$

23. When ω is increased, $X_L = \frac{1}{\omega C}$ decreases.

Current I increases. $\therefore$ **The bulb glows brighter.**

24. As is known from theory, instantaneous power developed has an angular frequency (2 ω).

25. As is clear from Fig. 7(Q).2, voltage leads the current by $\pi/2$.

26. The circuit is inductive, when $X_L > X_C$.
This happens for point C in the given graphs.

27. $V_{rms} = \sqrt{\frac{10^2 + 10^2 + 10^2 + 10^2}{4}} = \mathbf{10\ V}$

28. In electric arc furnace, Cu or Fe is melted due to **variation of magnetic field**, causing eddy currents which produces strong heating effect.

29. In RC circuit, a.c. current may lead a.c. voltage **by 30°.**

30. To transmit the given power through transmission line if voltage is high, the current is low. Due to it, there will be less heating effect as heat produced $\propto$ (current)^2. **So, power loss is very less.**

31. As $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\therefore$ quality factor can be improved by **decreasing R.**

32. Power loss in series R-L-C circuit fed by *a.c.* is
$= E_v I_v \cos\theta$
Power loss is maximum when $\cos\theta$ = max. *i.e.*, $\theta = 0°$. It means, **current and voltage are in phase.**

33. As $Z = \sqrt{R^2 + (X_L - X_C)^2}$, therefore, Z cannot be increased by **increasing the voltage of the source.**

34. In highly inductive load circuit, it is **more dangerous when we open the switch.** This is because time of break is very small and induced *e.m.f.* becomes large.

35. In electric substation in township, large capacitor banks are used **to improve power factor.**

36. In purely resistive a.c. circuit, current is **in phase** with the *e.m.f.*

37. As $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = X$, therefore, when both frequency and capacitance become double, the reactance becomes $X/4$.

38. From $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

$$C = \frac{1}{2\pi\nu X_C} = \frac{1}{2\pi \times \frac{400}{\pi} \times 25} \text{ farad}$$

$$C = \frac{1}{800 \times 25} = \frac{1}{200,00}\ f = \mathbf{50\ \mu F}$$

39. The core of a transformer is laminated, so as to **reduce energy loss due to eddy currents.**

40. Here, $n_p = 500$, $n_s = 400$, $E_p = 220$ V.

$$\frac{I_p}{I_s} = \frac{n_s}{n_p} = \frac{400}{500} = \mathbf{4:5}$$

41. Here, $E = E_0 \sin\omega t$,
$I = I_0 \sin(\omega t + \pi/2) = I_0 \cos\omega t$
$P = E \times I$

$$P = E_0 \sin\omega t \,.\, I_0 \cos\omega t = \frac{E_0 I_0}{2} \sin 2\omega t$$

$$\text{Average power} = \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t\, dt = \mathbf{Zero.}$$

42. At resonance, $X_L = X_C$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R}$$

43. $\nu = ?$, $C = 10\ \mu F = 10 \times 10^{-6}\ F = 10^{-5}\ F$,
$X_C = 1000$ ohm.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$\nu = \frac{1}{2\pi C X_C} = \frac{1}{2\pi \times 10^{-5} \times 1000} = \frac{50}{\pi}\ \mathbf{Hz}$$

44. In a d.c circuit, $\nu = 0$.
Resistance of inductor $X_L = \omega L = 2\pi\nu X_L = 0$
$\therefore$ Inductor can conduct in a d.c. circuit.

45. Efficiency of a transformer, $\eta = \frac{\text{output power}}{\text{input power}}$
when efficiency of a transformer is 100%, then

$$\eta = 1, \text{ so } 1 = \frac{\text{output power}}{\text{input power}}$$

or output power = input power = **60 W**

46. Current in secondary circuit of transformer

$$I = \frac{V_S}{z} = \frac{22}{220} = \mathbf{0{\cdot}1\ A}$$

47. $\dfrac{\text{output power, } P_o}{\text{input power, } P_i} = \dfrac{n_s}{n_p}$

$$\therefore \quad \frac{P}{P_i} = \frac{3}{2} \quad \text{or} \quad P_i = \frac{2}{3}\mathbf{P}$$

48. Here, for primary of transformer

$$P_p = 4\text{ kW} = 4 \times 10^3\text{ W};\ V_p = 100\text{ V}$$

$\therefore$ Current in primary coil,

$$I_p = \frac{P_p}{V_p} = \frac{4\times10^3}{100} = \mathbf{40\ A}$$

II. Assertion-Reason Type Questions

49. Time constant of *C-R* circuit is $1/CR$ and it does not depend on the emf of the source. Thus **both Assertion and Reason are false.**

50. Here **Reason is false** as average value of *a.c.* over complete cycle is zero. **Assertion is true** as *d.c.* ammeter measures average value of current and average value of alternating current over complete cycle is zero.

51. $X_L = \omega L = 2\pi\nu L$.

For *d.c.*, $\nu = 0$ $\therefore$ $X_L = 2\pi \times 0$ $X_L = \mathbf{0}$

Thus both Assertion and Reason are true and Reason is the correct explanation of Assertion.

52. Here **Assertion and Reason are true** but **Reason is not the correct explanation of Assertion,** as

$$\text{Current} \quad I = \frac{\text{voltage } (\varepsilon)}{\text{impedance } (z)} = \frac{\varepsilon}{\sqrt{R^2 + (X_L - X_C)^2}}$$

where z is minimum, if $(X_L - X_C) = 0$, then I is maximum. The phase difference between emf and current due to L or C is 90°. So the power consumed in purely inductive or capacitative circuit is zero.

53. Here **Reason is false** as the minimum inductance can be achieved when the various inductors are connected in parallel. Now

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6} = 1$$

or $L = \mathbf{1\ H}$. **Thus Assertion is true.**

54. Here **Reason is true** as the phase difference between emf and current in pure inductive circuit fed with *a.c.* is 90°, *i.e.*, $\phi = 0$.

Power factor of an inductor circuit = $\cos\phi$

$= \cos 90° = 0.$

Thus Assertion is true, but Reason can not explain properly the assertion.

55. Here **Reason is false** as $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C}$

For *d.c.*, $\nu = 0$, $\therefore$ $X_C = \dfrac{1}{2\pi\times0\times C} = \infty$

It means a condenser serves as a bad conductor for *d.c.* Thus **Assertion is false.**

56. Here **Reason is false** as the leakage of magnetic flux through core of transformer decreases the effeciency of transformer. Here, **Assertion is true** because the energy loss is minimum if the core of transformer is filled with a material of high permeability, *i.e.*, soft iron.

57. **Both Assertion and Reason are false** as transformer works only on a.c. and step up transformer can be used as step down transformer if the secondary of transformer is used as input and primary of transformer as output.

CONCEPTUAL PROBLEMS

I. Transient currents and Alternating currents

1. What is the average value of a.c. over a cycle and why ?

Sol. Average value of a.c. over a cycle is zero. This is because a.c. is positive during one half cycle and equally negative in the other half cycle.

2. Find the time taken by 60 Hz a.c. to reach its peak value from zero.

Sol. As $\nu = 60$ Hz, $T = \dfrac{1}{\nu} = \dfrac{1}{60}$ s

In going from zero value to peak value,

time taken, $t = \dfrac{T}{4} = \dfrac{1}{4\times60}\text{ s} = \dfrac{1}{240}\text{ s}$

3. What is the significance of time constant of *R–L* circuit?

Sol. Time constant of *R–L* circuit tells us how fast or how slow is the growth/decay of current in the *R–L* circuit. Low value of time constant indicates that the growth and decay are fast. Large values of time constant indicate that growth and decay of current in the circuit are slow.

For a given circuit, time constant is fixed. Its value can be optimised by selecting suitable values of R and L.

4. What is the significance of time constant of *RC* circuit?

Sol. Time constant of *RC* circuit indicates how fast or how slow will be the charging and discharging of a capacitor through a resistor. A small value of $t = RC$, represents faster charging and discharging, whereas large value of $t = RC$, indicates slower charging and discharging of the capacitor.

For a given circuit, time constant is fixed. We can optimise the value of time constant by choosing suitable values of R and C.

5. Can we use 15 c/s. a.c. for lighting purposes ?

Sol. Yes, we can use 15 c/s. a.c. for lighting purposes. The fluctuations in current will be so rapid (30 times/sec) that the bulb will appear glowing continuously due to persistence of vision.

6. 220 volt a.c. is more dangerous than 220 volt d.c. Why ? (Hr. Board 2011)

Sol. 220 volt a.c. means the effective or virtual value of a.c. is 220 volt, *i.e.*, $E_v = 220$ volt.

As peak value $E_0 = \sqrt{2}\, E_v$

$\therefore \quad E_0 = 1.414 \times 220 = 311$ volt.

But 220 volt d.c. has the same peak value (*i.e.* 220 volt only).

Moreover, the shock of a.c. is attractive and that of d.c. is repulsive.

Hence 220 volt a.c. is more dangerous than 220 volt d.c.

7. The d.c. and a.c. both, can be measured by a hot wire instrument. Why?

Sol. This is because both a.c. and d.c. produce heat, which is proportional to square of the current. The reversal of direction of current in a.c. is immaterial so far as production of heat is concerned.

8. Show that the sum of instantaneous currents during growth and decay of current in LR circuit is independent of time.

Sol. During growth of current in *LR* circuit,

$$I_g = I_0 (1 - e^{-Rt/L}) \quad \text{and}$$

during decay of current in *LR* circuit,

$$I_d = I_0\, e^{-Rt/L}$$

Sum of the currents at any instant t,

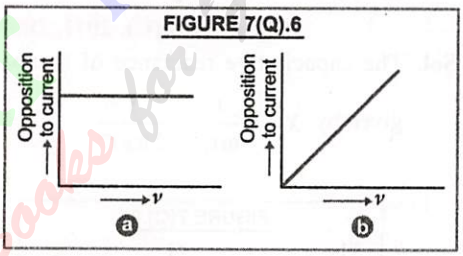
$$I_g + I_d = I_0 (1 - e^{-Rt/L} + e^{-Rt/L}) = I_0$$

which is constant, and is independent of time.

II. A.C. circuits and Electric Resonance

9. The graphs shown in Fig. 7.(Q).6 represent the variation of opposition offered by the circuit element to the flow of a.c. with frequency of applied e.m.f. Identify the circuit element. (CBSE 2004)

FIGURE 7(Q).6

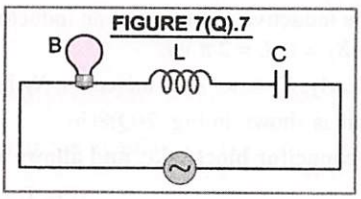


Sol. In Fig. 7(Q).6(a), opposition to current does not vary with frequency. The circuit element must be a *pure resistor.* In Fig. 7(Q).6(b), opposition to current increases directly with applied frequency. The circuit element must be a pure inductor ($X_L = \omega L = 2\pi\nu L$; $X_L \propto \nu$).

10. In the circuit shown in Fig. 7(Q).7, when frequency of supply is doubled, how should the values of *L* and *C* be changed so that glow in the bulb remains unchanged ?

(CBSE Sample Paper 2008)

FIGURE 7(Q).7



Sol. Current in *RLC* circuit is

$$I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + \left(2\pi\nu L - \dfrac{1}{2\pi\nu C}\right)^2}}$$

The glow of bulb will remain unchanged when I_v remains constant. When v is doubled, L should be halved and C should also be halved.

11. A bulb B and a capacitor C are connected in series to an a.c. source. A dielectric slab is now introduced between the plates of the capacitor. How will the brightness of bulb change ? (CBSE 2007)

Sol. Current through the bulb is

$$I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

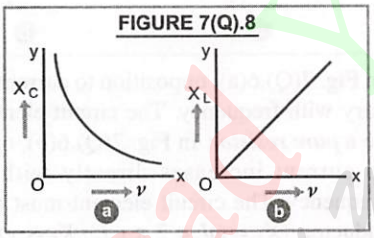
On introducing a dielectric slab between the plates of capacitor, C increases. Z decreases, I_v increases. Therefore, glow of the bulb increases.

12. Sketch a graph to show how the reactance of (*i*) a capacitor (*ii*) an inductor varies as a function of frequency ?

[CBSE 2018 (C), 2011, 2007, 2005]

Sol. The capacitative reactance of a capacitor is given by $X_c = \frac{1}{\omega C} = \frac{1}{2\pi v C}$

FIGURE 7(Q).8



Clearly, $X_c \propto \frac{1}{v}$

As frequency (v) increases, X_c decreases.

This is shown in Fig. 7(Q).8(*a*).

The inductive reactance of an inductor is given by $X_L = \omega L = 2\pi v L$

Clearly, $X_L \propto v$. As v increases, X_L increases.

This is shown in Fig. 7(Q)8(*b*).

13. A capacitor blocks d.c. and allows a.c. Why? (Pb. Board 2011)

Sol. The capacitative reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C}$$

For d.c., $v = 0$, $\therefore X_C = \infty$

i.e., capacitor offers infinite resistance to *d.c.* and hence blocks it.

For a.c., $v \neq 0$,

$\therefore X_C \neq \infty$, but it has only a finite value

Therefore, *a.c.* can pass through the capacitor.

14. The resistance of a coil for direct current is 10 Ω. An alternating current is sent through it. Will its resistance remain the same ?

Sol. No, the resistance of the coil will not remain the same. It will increase to $Z = \sqrt{R^2 + (\omega L)^2}$

15. An electric heater is heated turn by turn with d.c. and a.c. keeping pot. diff. across the ends of the heater same. Will the rate of production of heat in the two cases be same ? Explain.

Sol. An electric heater is in the form of a coil of ohmic resistance R and inductance L. On d.c. supply, resistance = R. On a.c. supply, impedance $= \sqrt{R^2 + (\omega L)^2}$, which is obviously more.

When potential difference is constant, heat produced/sec is inversely proportional to resistance. Hence heat produced/sec in case of a.c. will be less compared to heat produced/sec in case of *d.c.*

16. Explain : Voltages across L and C in series are 180° out of phase, while for L and C in parallel, current in L and C are 180° out of phase.

Sol. When L and C are in series, *for a given a.c.*, voltage across L leads by $\pi/2$ and voltage across C lags by $\pi/2$. Therefore, voltages across L and C differ in phase by π.

When L and C are in parallel, *for a given voltage*, current in L lags by $\pi/2$ and current in C leads by $\pi/2$. Therefore, currents in L and C differ in phase by π.

17. A device X is connected across an a.c. source of voltage $V = V_0 \sin \omega t$. The current through X is given as

$$I = I_0 \sin(\omega t + \pi/2).$$

(*a*) Identify the device X and write expression for its reactance.

(*b*) Draw graphs showing variation of voltage and current with time over one cycle of a.c. for X.

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(c) How does the reactance of X vary with frequency of a.c. ? Show this variation graphically.

(d) Draw the phasor diagram for device X.

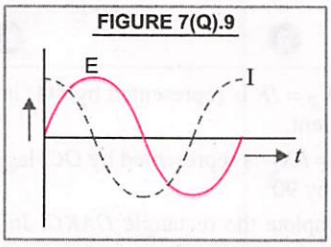
(CBSE 2018)

Sol. (*a*) Through the device X, alternating current leads the applied a.c. voltage by $\pi/2$. Therefore, the device X must be a *condenser*.

Rectance of $X = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \nu C}$

(*b*) Variation of voltage (E) and current (I) with time over one cycle of a.c. is shown in Fig. 7(Q).9.

FIGURE 7(Q).9

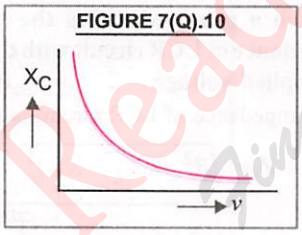


(*c*) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \nu C}$

Clearly, $X_C \propto \dfrac{1}{\nu}$

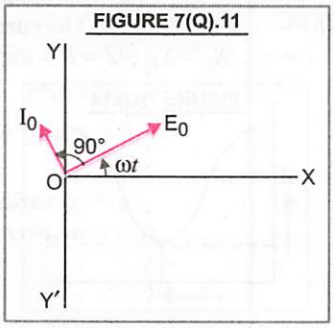
The variation is shown graphically in Fig. 7(Q).10.

FIGURE 7(Q).10



(*d*) Phasor diagram for device X is as shown in Fig. 7(Q).11.

FIGURE 7(Q).11



18. Does the current in an A.C. circuit lag, lead or remain in phase with the voltage of frequency ν applied to the circuit when (i) $\nu = \nu_r$ (ii) $\nu < \nu_r$ (iii) $\nu > \nu_r$ where ν_r is the resonance frequency.

Sol. (*i*) When $\nu = \nu_r$, $X_L = X_C$.

The circuit is purely resistive, *i.e.*, it is non-inductive. Therefore, current and voltage in the circuit are in the same phase.

(*ii*) When $\nu < \nu_r$. As $X_L = \omega L = 2\pi \nu L$

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \nu C}$

$\therefore$ when ν is small, X_L is small, X_c is large. The circuit is capacitance dominated. Therefore, current in the circuit leads the voltage by phase angle ϕ.

(*iii*) When $\nu > \nu_r$

X_L becomes large and X_C becomes small. The circuit is inductance dominated. The current lags behind the voltage, by phase angle ϕ.

19. At an airport, a person is made to walk through the door way of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work ? **(NCERT Solved Example)**

Sol. The metal detector works on the *principle of resonance* in a.c. circuits.

When we walk through a metal detector, we are infact, walking through a coil of many turns connected to a capacitor tuned suitably. If we carry some metal on our body, impedance of the circuit changes, bringing about significant changes in the current. This change in current is detected and the electronic circuitry causes an alarm.

20. Can the voltage drop across the inductor or capacitor in a series LCR circuit be greater than the applied voltage of the ac source ? Justify your answer.

Sol. Yes, the voltage drops across the inductor or the capacitor in a series circuit can be greater than the applied voltage. This is because these voltages are *not in phase* and they cannot be added like ordinary numbers.

21. A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if capacitance of the capacitor is reduced ? **(NCERT Solved Example)**

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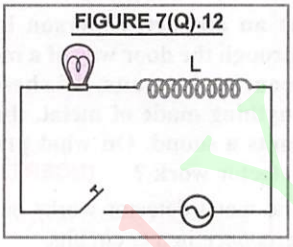
Sol. When *dc* source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When *ac* source is connected, the capacitor offers capacitative reactance $X_C = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C ; X_C increases. Therefore, glow of the bulb reduces.

22. A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig. 7(Q).12. The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increases (b) decreases (c) is unchanged as the iron is inserted. Give your answer with reasons. What will be your answer if ac source is replaced by dc source ?

(CBSE 2016) (NCERT Solved Example)

FIGURE 7(Q).12



Sol. When an iron rod is inserted into the interior of the inductor, inductance (L) of the coil increases, increasing thereby the inductive reactance $X_L = \omega L$. The current $I_v = E_v/X_L$ decreases. Therefore, glow of light bulb decreases.

If *ac* source is replaced by *dc* source of same voltage ; glow of the bulb is much more ($\because X_L = \omega L = 2\pi \nu L = 0$). However, this glow does not change on inserting iron rod.

23. A bulb and a capacitor are connected in series to an a.c. source of variable frequency. How will the brightness of the bulb change on increasing the frequency of a.c. source ?

Sol. The bulb acts as a resistor. When it is connected to an a.c. source through the capacitor, the circuit is *RC* circuit, whose impedance,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

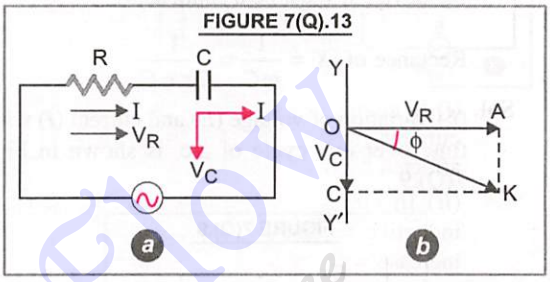
As frequency $\nu = \frac{\omega}{2\pi}$ of a.c. source is increased, Z decreases. The current through the bulb $I_v = E_v/Z$ increases. Therefore, brightness of the bulb (which corresponds to $I_v^2 R$) increases.

24. Draw the phasor diagram for a series *RC* circuit connected to an ac source.

[CBSE 2018 (C)]

Sol. In Fig. 7(Q).13(a), a series *RC* circuit is connected to an a.c. source.

FIGURE 7(Q).13



$\therefore V_R = IR$ is represented by OA, in phase with current.

$V_C = IX_C$ is represented by OC, lagging behind V_R by 90°.

Complete the rectangle $OAKC$. Join OK. This is the resultant voltage in the series *RC* circuit. Let $\angle AOK = \phi$. The voltage lags behind the current through a phase angle ϕ,

where $\tan\phi = \frac{V_C}{R}$

Fig. 7(Q).13(b) represents the phasor diagram of series *RC* circuit.

25. Sketch a graph showing the variation of impedance of LCR circuit with the frequency of applied voltage. **(CBSE 2011)**

Sol. The impedance of LCR circuit is

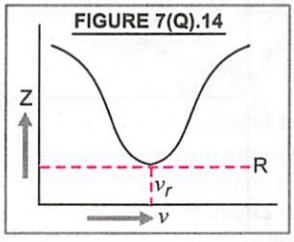
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}$$

The variation of Z with ν is as shown in Fig. 7(Q).14.

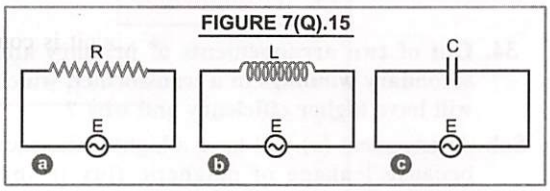
At $\nu = \nu_r$; $X_L = X_C$; $Z = R$ = minimum.

FIGURE 7(Q).14



26. Figs. 7(Q).15(a), (b), (c) show three alternating circuits with equal currents. If frequency of alt. emf be increased, what will be the effect on currents in the three cases ? Explain. (CBSE 2011)

FIGURE 7(Q).15



Sol. (*i*) In circuit (a), there will be no effect on the current flowing. This is because, R is not affected by frequency.

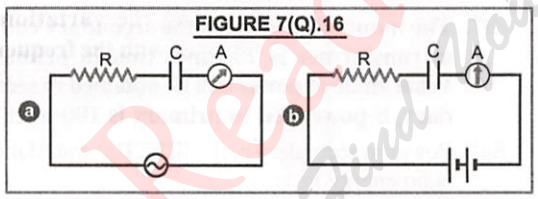
(*ii*) In circuit (b), current will decrease as inductive reactance $X_L = \omega L = 2\pi \nu L$ will increase with increasing frequency.

(*iii*) In circuit (c), current will increase as capacitative reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$ will decrease on increasing the frequency.

27. The hot wire ammeter A in Fig. 7(Q).16(a) shows some deflection, but not in Fig. 7(Q).16. Why ?

Sol. This is because in Fig. 7(Q).16(b), condenser blocks d.c. offering infinite resistance. Therefore, current in circuit of Fig. 7(Q).16(b) is zero. In case of Fig. 7(Q).16(a), a.c. is not blocked by condenser. Therefore, ammeter A shows some deflection.

FIGURE 7(Q).16



III. Energy stored and Power associated with a.c. circuits

28. State the condition under which resonance occurs in *LCR* circuit ? (CBSE 2005)

Sol. In LCR circuit, resonance occurs, when $X_L = X_C$ or $V_L = V_C$. The resonance frequency is

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}.$$

29. What is wattless current ?

Sol. Alternating current through pure L or pure C, which consumes no power for its maintenance in the circuit is called Idle current or wattless current.

30. What is expression for magnetic energy stored in an inductor. Compare it with the electrostatic energy stored in a capacitor.

Sol. Magnetic energy $= \frac{1}{2} LI^2$.

Electrostatic energy $= \frac{1}{2} CV^2$.

31. Give analogies between some important mechanical and electrical quantities.

Sol. The table below gives the analogy between some important quantities of mechanical and electrical systems :

Mechanical System	Electrical System
1. Mass (m)	1. Inductance (L)
2. Displacement (x)	2. Charge q
3. Velocity, $v = \frac{dx}{dt}$	3. Current, $I = \frac{dq}{dt}$
4. Force constant (k)	4. Reciprocal capacitance $(1/C) = \frac{V}{q}$
5. Mechanical energy, $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	5. Electromagnetic energy, $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$

32. Answer the following questions :

(a) In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit ? Is the same true for r.m.s. voltage ?

(b) A capacitor is used in the primary circuit of an induction coil.

(c) An applied voltage signal consists of a superposition of a d.c. voltage and an a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the d.c. signal will appear across C and the a.c. signal will appear across L.

(d) A choke coil in series with a lamp is connected to a d.c. line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an a.c. line.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains ? Why can we not use an ordinary resistor instead of choke coil ?

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Sol. (*a*) Yes, applied instantaneous voltage is always equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit. No, this is not true in case of r.m.s. voltages, because such voltages across different elements may not be in same phase.

(*b*) When the circuit is broken, the large induced voltage is used up in charging the capacitor. Thus sparking etc. is avoided.

(*c*) We know, $X_L = \omega L = 2\pi\nu L$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

For d.c., $\nu = 0$, $X_C = \infty$. The capacitor blocks d.c. Therefore, d.c. signal appears across C.

For a.c. of high freq., $X_L \rightarrow$ very large.

$\therefore$ Inductor blocks it. Hence a.c. signal appears across L.

(*d*) On a *d.c.* line, choke has no impedance. Therefore, lamp shines brightly and there is no effect of inserting iron core in the choke. On an *a.c.* line, choke offers impedence. So the bulb lights dimly. When an iron core is inserted in the choke, its impedance to *a.c.* increases further. Therefore, brightness of the bulb decreases.

(*e*) A choke coil is needed in the use of fluorescent tubes to reduce *ac* without loss of power. If we use an ordinary resistor, *ac* will reduce, but loss of power due to heating will be there.

Power dissipated/cycle $= E_v\ I_v \cos\phi$

In a resistor, $\phi = 0°$ $\therefore$ power dissipated/cycle $= E_v\ I_v \cos 0° = E_v\ I_v =$ **max**

In a choke coil, $\phi = 90°$

$\therefore$ power dissipated/cycle $= E_v\ I_v \cos 90° =$ **zero**

33. (a) For circuits used for transporting electric power, a low power factor implies larger power loss in transmission. Explain.

(b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

(NCERT Solved Example)

Sol. (*a*) We know that $P = E_v\ I_v \cos\phi$, where $\cos\phi$ is the power factor.

To supply/transmit a given power P at a given voltage E_v ; if $\cos\phi$ is small, I_v has to be increased accordingly. Therefore, power loss $= I_v^2 R$ in transmission would increase.

(*b*) Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

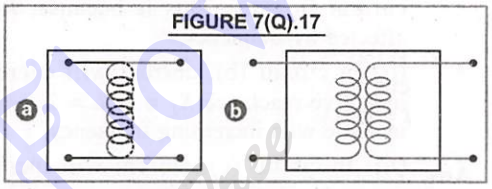
To improve power factor, $\cos\phi \rightarrow 1$, for which $Z \rightarrow R$. In a circuit with given value of L, we can use a capacitor of appropriate capacitance so that $X_C \rightarrow X_L$ and hence $Z \rightarrow R$.

IV. Transformer

34. Out of two arrangements of primary and secondary windings in a transformer, which will have higher efficiency and why ?

Sol. Arrangement (*a*) will have a higher efficiency because leakage of magnetic flux in this arrangement is almost zero.

FIGURE 7(Q).17



35. The core of a transformer is made of a material having narrow hysterisis loop. Why ?

Sol. A core material with narrow hysterisis loop involves much less energy loss (hysterisis loss) due to repeated magnetisation and demagnetisation of the core.

36. The core of a transformer is made of magnetic material of high permeability. Why ?

Sol. Due to high permeability of the core material, magnetic lines of force crowd through the core. Flux leakage decreases and efficiency of transformer increases.

37. The number of turns in the secondary coil of a transformer is 20 times that in primary. What value of power will be obtained in secondary, if power fed to primary is 100 watt ?

Sol. Power in secondary coil will be 100 watt if there is no energy loss.

38. When a current flows in the coil of a transformer, then why does its core become hot ?

Sol. When alternating current flows through the coil of a transformer, its core gets magnetised and demagnetised repeatedly. The energy spent in magnetising the core is not returned fully in demagnetisation. This energy left in the core appears in the form of heat.

39. A transformer is used to step down a.c. voltage. What appliance do you use to step down d.c. voltage ?

Sol. For stepping down d.c. voltage, we may use an ohmic resistance only such as in potential dividing arrangement.

40. Why is the core of a transformer made of a magnetic material of high permeability ?

Sol. When permeability of magnetic material of transformer core is high, almost whole of magnetic flux will be linked with the core. Therefore, magnetic flux linked with the secondary coil will almost be equal to the amount of magnetic flux linked with the primary coil. Therefore, energy loss due to leakage of magnetic flux will be reduced considerably.

QUESTIONS

- Very Short Answer
- Short Answer
- Long Answer

VERY SHORT ANSWER QUESTIONS

Carrying 1 mark

I. Transient currents and Alternating currents

1. What is the average value of alternating current, $I = I_0 \sin \omega t$ over time interval $t = \pi/\omega$ to $t = 2\pi/\omega$?

Ans. $t = \pi/\omega = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{T}{2}$ and $t = \frac{2\pi}{\omega} = T$.

Over this negative half cycle, $I_{av} = -\frac{2I_0}{\pi}$.

2. What is the average value of a.c. over a full cycle ?

Ans. Zero.

3. A 220 V, 50 Hz a.c. source is being used. What is the average e.m.f. over a full cycle ? What is the *rms* voltage ?

Ans. Average *emf* over a full cycle = 0.

rms voltage = 220 volt.

4. Show that $\left(\frac{L}{R}\right)$ has the dimensions of time.

Ans. From $e = L\frac{dI}{dt}$, $L = \frac{e.dt}{dI}$

According to Ohm's law, $R = \frac{e}{I}$

$\therefore \frac{L}{R} = \frac{e.dt}{dI} \times \frac{I}{e} \rightarrow dt$

$\therefore \left(\frac{L}{R}\right)$ has dimensions of time.

5. Show that time constant ($\tau = RC$) of R–C circuit has the dimensions of time.

Ans. From Ohm's law, $R = \frac{V}{I}$. Also, $q = CV$

$\therefore \quad C = \frac{q}{V} = \frac{I \times t}{V} \quad \therefore \quad RC = \frac{V}{I} \times \frac{I \times t}{V} = t$

Hence dimensions of (RC) are those of time.

6. We can measure d.c. by an ordinary ammeter, but not a.c. Why ?

Ans. This is because average value of a.c. over a complete cycle is zero.

7. What is peak value of 220 V a.c. ?

Ans. $E_0 = \sqrt{2}\, E_v = \sqrt{2} \times 220 \text{ volt} = 311 \text{ volt}$.

8. Can r.m.s value of current be defined in terms of chemical effect of current ?

Ans. No, because a.c. cannot be used in electrolysis.

9. Give expression for average value of a.c. voltage $V = V_0 \sin \omega t$ over time interval $t = 0$ to $t = \frac{\pi}{\omega}$

Ans. As $t = \frac{\pi}{\omega} = \frac{1}{2}.\frac{2\pi}{\omega} = \frac{1}{2}T$, therefore, we wish to know average value of a.c. voltage over first half cycle $(0 \rightarrow T/2)$. It is $E_{av} = \frac{2V_0}{\pi}$.

II. A.C. circuits and Electric Resonance

10. Alternating current through an inductor lags behind the alternating voltage by 90°. What does it imply ?

Ans. It implies that at a given time, whatever is the value of alternating *e.m.f.*, current will acquire that value after $t = T/4$.

11. How is ohmic resistance in a circuit affected when frequency of a.c. source in the circuit is doubled ?

Ans. Ohmic resistance R remains unaffected.

12. How does inductive reactance vary when frequency of a.c. source in the circuit is halved ? **(CBSE 2002)**

Ans. As $X_L = \omega L = 2\pi \nu L$, therefore, when ν is halved, X_L becomes half.

13. How is capacitative reactance affected when frequency of a.c. supply is tripled ?

(HP Board 2003)

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Ans. As $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$, therefore when ν is tripled, X_C becomes one third.

14. In a series LCR circuit, the voltages across an inductor, a capacitor and a resistor are 30 V, 30 V, 60 V respectively. What is the phase difference between the applied voltage and the current in the circuit ? (CBSE 2007)

Ans. If ϕ is phase difference between applied voltage and the current, then

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{I_v(X_L - X_C)}{I_v R}$$

$$= \frac{V_L - V_C}{V_R} = \frac{30-30}{60} = 0;$$

$\phi = 0°$.

15. Does the steady state current in *RL* circuit depend upon *L* ?

Ans. No, it depends only on *R*, and not on *L*.

16. What is the natural frequency of *LC* circuit ? What is the reactance of this circuit at this frequency ?

Ans. $\nu = \frac{1}{2\pi\sqrt{LC}}$. Reactance of the circuit at this frequency is zero.

17. What is the dimensional formula of $\sqrt{LC}$?

Ans. $[M^0 L^0 T^1]$

18. How can you improve the quality factor of a series resonance circuit ?

Ans. To improve the quality factor of series resonance circuit, ohmic resistance of the circuit should be made as small as possible.

19. An electrical element *X* when connected to an alternating voltage source has current through it leading the voltage by $\pi/2$ radian. Identify *X* and write an expression for its reactance. (CBSE 2011)

Ans. As current through element *X* leads the alternating voltage applied by $\pi/2$ radian, therefore, *X* is a pure capacitance. Its reactance is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$

20. Can a capacitor of suitable capacitance be used to control a.c. in place of the choke coil ?

Ans. Yes, this is because average power consumed/cycle in an ideal capacitor is also zero. Therefore, like a choke coil, a condenser can reduce a.c. without power dissipation.

III. Energy stored and Power associated with a.c. circuits

21. At resonance in an a.c. circuit, what is the value of power factor ?

Ans. At resonance, $Z = R$,

$\therefore$ Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$ = maximum.

22. Does it imply that power dissipated in an a.c. circuit is zero at resonance ?

Ans. No ; power dissipated at resonance is maximum.

23. At parallel resonance frequency, is current zero or maximum ?

Ans. At parallel resonance frequency, current is zero.

24. Why are parallel resonance circuits called rejector circuits/filter circuits/antiresonance circuits ?

Ans. This is because parallel resonance circuits reject the current corresponding to parallel resonance frequencies. These circuits are used in the transmitting circuits.

25. Where is the power dissipation in an alternating current circuit ? In resistance ? In inductance? In capacitance ?

Ans. Power is dissipated in an *a.c.* circuit in resistance only.

26. What is meant by wattless current ? (CBSE 2011)

Ans. Wattless current is that which involves no consumption of power per cycle for its maintenance in the circuit.

27. In a pure *LC* circuit, what is the energy stored when peak current is I_0 ?

Ans. Total energy stored in inductor $= \frac{1}{2} L I_0^2$.

28. What is the average power consumed in a circuit consisting of resistanceless inductance ?

Ans. Zero, because through resistanceless inductance, $\phi = 90°$ and $P = E_v\ I_v\ \cos 90° = 0$.

29. What is the minimum value of power factor ? When does it occur ?

Ans. Zero, in a pure inductor and in a pure capacitor.

30. What is the maximum value of power factor ? When does it occur ?

Ans. One. It occurs in a non-inductive circuit, *i.e.*, in a circuit containing *R* only.

31. In a series *LCR* circuit, $V_L = V_C \neq V_R$. What is the value of power factor ? (CBSE 2015)

Ans. When $V_L = V_C ; X_L = X_C ; Z = R ; \cos\phi = \frac{R}{Z} = 1.$

i.e., power factor is unity.

IV. Transformer

32. Why cannot a transformer be used to step up d.c. voltage ? (CBSE 2011)

Ans. This is because d.c. voltage cannot produce a changing magnetic flux required in the working of a transformer.

33. Name any appliance that can step down d.c. voltage.

Ans. A pure resistor can be used to step down d.c. voltage.

34. For transporting electric power, what is required at the generating station ?

Ans. A step up transformer to raise the voltage, reduce the current and hence reduce energy losses.

35. What is iron loss in a transformer ?

Ans. Iron loss is loss of energy in the form of heat in iron core of a transformer.

36. The number of turns in secondary coil of a transformer is 100 times the number of turns in primary coil. What is the transformer ratio ?

Ans. $K = \frac{n_s}{n_p} = 100$

37. In a transformer with transformation ratio 0·1, 220 volt *a.c.* is fed to primary. What voltage is obtained across the secondary ?

Ans. $K = \frac{n_s}{n_p} = \frac{e_s}{e_p}$

$$0{\cdot}1 = \frac{e_s}{220} \quad \therefore \quad e_s = 22 \text{ volt}$$

38. A transformer steps down 220 volt to 11 volt. What is the transformation ratio ?

Ans. $e_p = 220$ V, $e_s = 11$ V

$$K = \frac{e_s}{e_p} = \frac{11}{220} = 1/20 = 0{\cdot}05$$

39. Can a transformer work on d.c. ?

Ans. No, a transformer cannot work on d.c.

40. The primary and secondary coils of a transformer do not have any electrical contact. How is then electrical energy transferred from primary to secondary ?

Ans. Electrical energy is transferred from primary coil to secondary coil on account of mutual induction.

41. The core of a transformer is laminated, why ?

Ans. The transformer core is laminated to minimise loss of energy due to eddy currents. By doing so, the magnitude of eddy currents is reduced considerably and hence the energy loss is minimised.

42. Which of the primary and secondary of a step down transformer has a larger number of turns ?

Ans. Primary coil in a step down transformer has larger number of turns.

43. What is the function of oil in a transformer ?

Ans. It provides insulation as well as cooling.

44. Why is choke coil needed in the use of fluorescent tubes with ac mains ?

Ans. A choke coil reduces the voltage across the fluorescent tube without wastage of power.

SHORT ANSWER QUESTIONS

Carrying 2 marks

I. Transient currents and Alternating currents

1. A 110 V d.c. source replaces an a.c. source such that heat produced is same in the two cases. What is the *rms* value of alternating voltage source ?

Ans. As heat produced is the same, *rms* voltage of a.c. source = *d.c.* voltage = 110 V

2. Calculate the time taken by 60 Hz a.c. source to reach its negative peak value from zero value ?

Ans. Here, $\nu = 60$ Hz, $T = \frac{1}{\nu} = \frac{1}{60}$ s

From negative peak value to zero value, time taken $t = \frac{T}{4} = \frac{1}{50 \times 4}$ s $= \frac{1}{240}$ second.

3. Why cannot we use a.c. for electrolysis ?

Ans. For electrolysis, we require fixed cathode and fixed anode. In a.c., the direction of current is changing periodically.

4. For an *a.c.*, can ever :
(*a*) *r.m.s.* value be equal to peak value ?
(*b*) average value be equal to peak value ?
(*c*) *r.m.s.* value be equal to average value
(*d*) all the three values be equal ?

Ans. Yes, when the *a.c.* is a square wave, *r.m.s.* value, average value and peak value of *a.c.* are equal. Choice (*d*) is correct.

5. Distinguish between alternating current and direct current by giving two points.
(Manipur Board 2011)

Ans. (*i*) Alternating current is that current which shows periodic variation in its value with time. Direct current is that current whose magnitude and direction do not change.

(*ii*) The frequency of alternating current has some finite value.
The frequency of direct current is zero.

II. A.C. circuits and Electric Resonance

6. An inductor is connected in series with a bulb to an a.c. source. What happens to brightness of bulb when number of turns in the inductor is reduced ? **(CBSE 2002)**

Ans. Current through the bulb

$$I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + X_L^2}}, \text{ where } X_L = \omega L$$

As $L \propto N^2$, therefore, on decreasing number of turns, L decreases ; Z decreases, I_v increases. Hence brightness of bulb increases.

7. A light bulb and an open coil inductor are connected to an a.c. source. What happens to brightness of bulb when an iron rod is inserted into the inductor ?
(CBSE Sample Paper 2008)

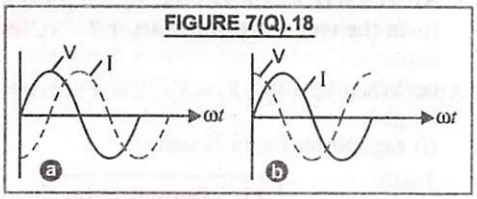
Ans. Current through the bulb

$$I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + X_L^2}}, \text{ where } X_L = \omega L$$

On introducing an iron rod into the inductor its inductance L increases. $X_L = \omega L$ increases. Therefore, impedance Z increases. Current I_v through the bulb decreases. Hence brightness of bulb decreases.

8. For a series *LCR* circuit, connected to an a.c. source, identify the graph that corresponds to $\omega > \frac{1}{\sqrt{LC}}$. Give reason.
[CBSE 2007 (C)]

FIGURE 7(Q).18



Ans. When $\omega > \frac{1}{\sqrt{LC}}$, $X_L = \omega L$ is large and $X_C = \frac{1}{\omega C}$ is small. Therefore, the circuit is inductive. Current I lags behind the applied alternating voltage as shown in Fig. 7(Q).18(a).

9. Define reactance X and impedance Z. Can these be negative ? If yes, when and what does it imply ?

Ans. For definitions, see text. As $X = X_L - X_C$ = negative, when $X_C > X_L$, *i.e.*, reactance X can be negative.

It implies, alternating current leads the applied alternating voltage in the circuit. However, Z cannot be negative.

10. An electric lamp connected in series with a capacitor and an a.c. source is glowing with certain brightness. How does the brightness of the lamp change on reducing the capacitance ?

Ans. Brightness of the lamp decreases. This is because on reducing C; X_c increases, Z increases and I decreases.

11. An inductor acts as a conductor for d.c. Why?
(Manipur Board 2012)

Ans. The inductive reactance is given by

$$X_L = \omega L = 2\pi\nu L$$

For d.c., $\nu = 0$, $\therefore X_L = 0$

i.e., inductor offers no resistance to d.c. Hence for d.c., an inductor acts as a conductor.

12. A capacitor allows a.c. to pass through. Why?
(Karnataka Board 2008)

Ans. The capacitative reactance is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

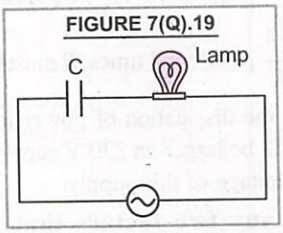
For a.c. , $\nu \neq 0$. $\therefore X_C \neq \infty$
Hence a condenser does not block a.c. It allows a.c. to pass through.

13. An electric lamp having coil of negligible inductance connected in series with a capacitor and an a.c. source is glowing with certain brightness, Fig. 7(Q).19. How does the brightness of the lamp change on reducing (*i*) capacitance (*ii*) frequency ? Justify your answer. (CBSE 2010)

Ans. Brightness of lamp $\propto I_v^2$

Assuming zero resistance and zero inductance of lamp, $I_v = \frac{E_v}{X_C} = \frac{E_v}{1/\omega C} = E_v(2\pi v)C$,

FIGURE 7(Q).19

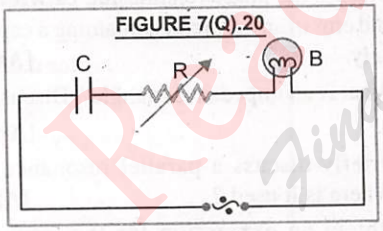


On reducing C or v ; I_v would decrease.

Therefore, brightness of the lamp will decrease.

14. A capacitor C, a variable resistor R and a bulb B are connected in series to the ac mains in circuit as shown in Fig. 7(Q).20. The bulb glows with some brightness. How will the glow of the bulb change if (*i*) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be the same, (*ii*) the resistance R is increased keeping the same capacitance. (CBSE 2014)

FIGURE 7(Q).20



Ans. (*i*) On introducing a dielectric slab between the plates of capacitor, the reactance of capacitor $\left(X_C = \frac{1}{\omega C}\right)$ will decrease resulting in the increase of current in the circuit. Due to it, the bulb will glow brighter (*ii*) On increasing resistance R, the effective resistance of circuit increases. Due to it, the current in the circuit decreases. Therefore, the glow of bulb will decrease.

III. Energy stored and Power associated with a.c. circuits

15. Under what conditions, power factor of an a.c. circuit is maximum ?

Ans. Power factor $= \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

Max. value of power factor is 1, when $Z = R$ *i.e.*, when $X_L = X_C$, *i.e.*, at resonance, when the circuit is non-inductive.

16. Alternating current through pure inductor and pure capacitor is wattless. Why ? (CBSE 2009)

Ans. Through pure inductor or pure capacitor, phase diff. between alternating voltage and alternating current is 90°. Therefore, power factor, $\cos\phi = \cos 90° = 0$.

Hence the current is said to be wattless.

17. What is power dissipation in an a.c. circuit in which $E = 230 \sin(\omega t + \pi/2)$; $I = 10 \sin \omega t$? (CBSE 2005)

Ans. As phase diff. = 90°, therefore average power dissipated $P = E_v I_v \cos 90° =$ Zero

18. In an a.c. circuit, there is no power consumption in an ideal inductor. Explain.

Ans. Average power/cycle in an a.c. circuit is

$$P = E_v I_v \cos\phi$$

In an ideal inductor, which has no ohmic resistance, phase angle $\phi = 90°$

$\therefore \quad P = E_v I_v \cos 90° =$ Zero.

Infact, power supplied during growth of current is retrieved during decay of current through ideal inductor.

19. An electromagnet has stored 648 J of magnetic energy, when a current of 9 A exists in its coils. What average e.m.f. is induced if the current is reduced to zero in 0·45 s ?

Ans. Here, $E = 648$ J, $I = 9$ A, $e = ?$,

$dI = 9 - 0 = 9$ A, $dt = 0{\cdot}45$ s

From $E = \frac{1}{2}LI^2$

$$648 = \frac{1}{2} \times L(9)^2 ; \quad L = \frac{648 \times 2}{9 \times 9} = \mathbf{16\,H}$$

As $e = \frac{L\,dI}{dt} \quad \therefore \quad e = \frac{16(9)}{0{\cdot}45} = \mathbf{320\,V}$

20. (*a*) In a series *LCR* circuit connected across an *a.c.* source of variable frequency, obtain the expression for its impedance and draw a plot showing its variation with frequency of *a.c.* source.

(*b*) What is the phase difference between the voltage across inductor and the capacitor at resonance in the *LCR* circuit. (CBSE 2019)

Ans. (*a*) Refer to Art. 7.12 and See Fig. 7.18.

(*b*) Phase difference between the voltage across inductor and the capacitor at resonance in the *LCR* circuit is 180°.

IV. Transformer

21. Why is core of a transformer laminated ?

Ans. Lamination of core is required to reduce energy losses due to eddy currents.

22. What causes the core of a transformer to get heated under operation ?

Ans. The eddy currents set up in the iron core heat up the core of transformer under operation.

23. A solenoid with an iron core and a bulb are connected to a d.c. source. How does the brightness of the bulb change when iron core is removed from the solenoid ? (CBSE 2004)

Ans. The brightness of bulb remains unchanged because the solenoid does not offer any reactance ($X_L = 2\pi\nu L$) to d.c. source ($\nu = 0$).

24. In India, domestic power supply is at 220 V, 50 hz, while in *U.S.A*, it is 110 V, 60 hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply.

(CBSE 2011)

Ans. For transfer of power ($= V \times I$) at higher voltage (220 V instead of 110 V), current carried by wires is just half. Therefore, such wires need not be very thick, saving lot of transmission material and reducing the cost of transmission. This is one advantage of 220 V supply.

But to design a device of particular wattage, $P = \frac{V^2}{R}$, as V^2 is 4 times, R must be four times.

If not, the dissipation of power in the form of heat will be larger on 220 V supply. This is one disadvantage of this supply.

25. State any two factors that reduce the efficiency of a transformer.

[CBSE 2018 (C)]

Ans. The efficiency of a transformer is reduced on account of many factors. For example : Copper loss ; Iron loss ; leakage of magnetic flux, hysterisis loss etc.

SHORT ANSWER QUESTIONS — Carrying 3 marks

1. In what situations, is it preferable to use an *a.c.* source rather than a *d.c.* source ? When is *d.c.* preferred to *a.c.* ? **[Art. 7.29]**
2. What is meant by *rms* value of a.c. ? Derive an expression for alternating emf ?
(Raj. Board 2012) **[Art. 7.8]**
3. What is a phasor diagram ? Draw a phasor diagram when a pure resistor is connected across an a.c. supply and show that current and voltage are in the same phase. **[Art. 7.9]**
4. What do you understand by peak value and *rms* value of alternating current ? Derive a relation between them.
(Bihar Board 2011, HP Board 2010, CBSE 2009) **[Art. 7.7]**
5. An ac voltage, $E = E_0 \sin \omega t$ is applied across an inductor L. Obtain an expression for the current. (CBSE 2017, 2011) **[Art. 7.10]**
6. Discuss the phase relationship between current and emf in an ac circuit containing a capacitance only. **[Art. 7.11]**
7. What is an impedance triangle ? Discuss its use. **[Art. 7.13]**
8. Briefly discuss a parallel resonance circuit. Where is it used ? **[Art 7.21]**
9. Obtain an expression for the frequency of oscillations when a capacitor discharges itself through an inductor. **[Art. 7.17]**
10. Define quality factor in electrical resonance circuit. Find quality factor of series resonance circuit. (Raj. Board 2012) **[Art. 7.20]**
11. What is wattless current ? Show mathematically that an ideal inductor does not consume any power in an a.c. circuit. **[Arts. 7.27 and 7.23]**

12. Describe briefly the choke coil. Why is a choke coil preferred for reducing a.c. over resistor ? [Art. 7.9]

13. How is a transformer used in large scale transmission and distribution of a.c. over long distances ? (Raj. Board 2012, CBSE 2012, 2010) [Art. 7.8]

14. Mention various energy losses in a transformer. (HP Board 2015, CBSE 2011) [Art. 7.7]

LONG ANSWER QUESTIONS

Carrying 5 or more marks

1. Discuss the growth and decay of current in *R.L.* circuit. [Art. 7.3]

2. Discuss charging and discharging of a condenser through resistor. [Art. 7.4]

3. What is meant by mean value of a.c. ? Derive an expression for mean value of alternating current & e.m.f. over first half cycle. (HP Board 2009) [Arts. 7.5 & 6]

4. Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltages. An *ac* voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance *L*. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\pi/2$. (Manipur Board 2012, CBSE 2007) [Art. 7.10]

5. Explain the term 'capacitive reactance'. Show graphically the variation of capacitive reactance with frequency of the applied alternating voltage.
An *ac* voltage $E = E_0 \sin \omega t$ is applied across a pure capacitor of capacitance *C*. Show mathematically that the current flowing through it leads the applied voltage by a phase angle of $\pi/2$. (CBSE 2007) [Art. 7.11]

6. An ac. supply $E = E_0 \sin \omega t$ is connected to a series combination of *L*, *C* and *R*. Calculate impedance of the circuit and discuss the phase relation between voltage and current. (CBSE 2014, HP Board 2011) [Art. 7. 12]

7. How does the term electric resistance differ from 'impedance'? With the help of a suitable phasor diagram, obtain a relation for impedance in an a.c. series *LCR* circuit. (CBSE 2012, HP Board 2011) [Art 7.12]

8. Find in an alternating *LR* circuit, (*i*) resultant voltage (*ii*) impedance of circuit, (*iii*) Phase difference, (*iv*) average power. (Chhatisgarh 2012) [Art. 7. 14]

9. Calculate the frequency of series resonance circuit. Define *Q* factor of this circuit. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or TV set. (CBSE 2011) [Arts. 7. 19 & 7. 20]

10. Define the quality factor in an a.c. circuit. Why should the quality factor have high value in receiving circuits ? Name the factors on which it depends. (CBSE 2009) [Art. 7.20]

11. State the condition for resonance to occur in a series *LCR* a.c. circuit and derive an expression for the resonant frequency. Draw a plot showing the variation of the peak current (i_0) with frequency of the a.c. source used. Define the quality factor, *Q* of the circuit. Calculate the (*i*) impedance (*ii*) wattless current of the given a.c. circuit. (CBSE 2010, 2008) [Arts. 7.19 and 7.20]

12. In a series *LCR* circuit connected to an a.c. source of variable frequency and voltage $V = V_m \sin \omega t$, draw a plot showing the variation of current (*I*) with angular frequency (ω) for two different values of resistance R_1 and R_2 ($R_1 > R_2$). Write the condition under which the phenomenon of resonance occurs. For which value of resistance out of the two curves, a sharper resonance is produced? Define *Q* factor of the circuit and give its significance. (CBSE 2013) [Arts. 7.19 & 7.20]

13. Derive an expression for the average power over a complete cycle of a.c. in a series LCR circuit connected to an a.c. source in which the phase difference between the voltage and current in the circuit is ϕ. (CBSE 2009) [Art. 7. 22]

14. Calculate energy stored in an inductor in an a.c. circuit. Show that average power consumed per cycle in an a.c. circuit containing pure inductor is zero. (CBSE 2017, 2015) [Arts. 7. 16 & 7. 23]

15. Derive an expression for the average power in *LCR* circuit connected to a.c. supply. Hence define power factor. Show that average power consumed per cycle in an a.c. circuit containing an ideal capacitor is zero. (CBSE 2016) [Arts. 7. 25 & 7. 26]

16. Prove that an ideal capacitor in an a.c. circuit does not dissipate power.

[CBSE 2017 (C)] [Arts. 7.24]

17. Draw a labelled circuit arrangement showing the windings of primary and secondary coils in a transformer. Explain the underlying principle and working of a step up transformer. Write any two major sources of energy loss in this device.

(MP Board 2011, Bihar Board 2011 ; Chhatisgarh Board 2011, CBSE 2011, 2008)

[Arts. 7.30 and 31]

18. Describe briefly with the help of a labelled diagram, working of a step up transformer. A step up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy ? Explain.

(CBSE 2017, 2016 ; HP Board 2013 ; Pb. Board 2011, CBSE 2016, 2011, 2010, 2009)

[Art. 7.30]

19. Draw the diagram of a device which is used to decrease high *a.c.* voltage into a low *a.c.* voltage and state its working principle. Write four sources of energy loss in this device.

(CBSE 2019) [Art. 7.30]

CASE-BASED VERY SHORT/SHORT QUESTIONS

CASE 1. An alternating current is that current which changes continuously in magnitude and periodically in direction. It can be represented by a sine curve or cosine curve, *i.e.*,

$$I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

Here, I_0 is peak value of current and I is the instantaneous value of current, $\omega = 2\pi/T = 2\pi\nu$, where T is the period of a.c. and ν is the frequency of a.c. The alternating emf can similarly be represented as $E = E_0 \sin \omega t$ or $E = E_0 \cos \omega t$.

Mean value or average value of alternating current over a given time is that steady value of current which would send the same amount of charge through a circuit in the given time as is sent by a.c. through the same circuit in the same time. It is represented by I_m or I_{av}.

If I is the instantaneous value of current at time t, given by $I = I_0 \sin \omega t$.

Then mean value or average value of alternating current over the time t is

$$I_m = \frac{\int_0^t I\,dt}{\int_0^t dt} = \frac{\int_0^t I_0 \sin \omega t}{t}$$

All d.c. ammeters measure mean value of a.c. over complete cycle.

Based on the above paragraph, answer questions no. 1 to 4 :

1. The value of a.c. at instant t is given by $I = I_0 \sin \omega t$, find the average value of a.c. over first half cycle of a.c.

2. In the above question, find the average value of a.c. over next half cycle of a.c.

3. In the above question, what is the average value of a.c. over complete cycle of a.c.

4. Draw a graph for the average value of a.c. over the period 0 to T.

CASE 2. The root mean square (rms) value of a.c. is that value of steady current, which would produce the same amount of heat in a given resistor in a given time, as has been done by the alternating current when passed through the same resistor for the same time. The rms value of a.c. is also called virtual value or effective value of a.c. It is represented by I_v or I_{eff} or I_{rms} : A.C. ammeter measures the rms value of a.c. over a given time.

Based on the above paragraph, answer questions no. 5 to 8 :

5. If the a.c. current at time t, given by $I = I_0 \cos \omega t$, is flowing through a resistor of resistance R, then find the virtual value of a.c. over a complete cycle of a.c.

6. Why d.c. ammeter cannot measure a.c. in a circuit ?

7. What is the ratio of virtual value of a.c. and mean value of a.c. over half cycle ?

8. Draw a graph between virtual value of a.c. and time for the given a.c., $I = I_0 \cos \omega t$.

CASE 3. The resistance offered by pure inductance L in the path of a.c. is called inductive reactance (X_L). It is given by, $X_L = \omega L = 2\pi\nu L$.

It is measured in ohm. Due to inductance in a.c. circuit the voltage leads the current by a phase angle 90°. The resistance offered by pure capacitance C in the path of a.c. is called capacitative reactance (X_C).

It is given by, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C}$.

Due to C in the a.c. circuit, the voltage lags behind the current by a phase angle 90°.

The effective resistance by R, L and C in a.c. circuit is called impedance (Z) of RLC circuit.

It is given by, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}$$

The phase angle ϕ by which alternating voltage leads the alternating current is given by

$$\tan\phi = \frac{X_L - X_C}{R}$$

Based on the above paragraph, answer questions no. 9 to 12 :

9. In series R-L-C circuit fed with a.c., for what frequency of a.c. the voltage and current will be in the same phase ?

10. What are the units and dimensions of impedance ?

11. A resistor of 12 Ω, a capacitor of reactance 14 Ω and a pure inductor of inductance 0·1 H are joined in series and placed across 200 V, 50 Hz a.c. supply. Calculate the impedance of the circuit. [Take $\pi = 3$]

12. In above question, find the value of current in the circuit.

ANSWERS

1. Mean value or average value of a.c. over first half cycle is

$$I_{av} = \frac{\int_0^{T/2} I_0 \sin\omega t\, dt}{\int_0^{T/2} dt} = \frac{I_0}{T/2}\int_0^{T/2} \sin\omega t\, dt$$

$$= \frac{2I_0}{T}\left[\frac{-\cos\omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{2I_0}{T\times 2\pi/T}\left[\cos\frac{2\pi}{T}(T/2) - \cos 0\right]$$

$$= -\frac{I_0}{\pi}[\cos\pi - \cos 0] = -\frac{I_0}{\pi}[-1-1]$$

$$= \frac{\mathbf{2I_0}}{\pi}$$

2. $I_{av} = \dfrac{\int_{T/2}^{T} I_0 \sin\left(\frac{2\pi}{T}\right) dt}{\int_{T/2}^{T} dt}$

$$= \frac{I_0}{(T - T/2)}\int_{T/2}^{T} \sin\left(\frac{2\pi}{T}\right) dt$$

$$= \frac{2I_0}{T}\left[\frac{-\cos\frac{2\pi t}{T}}{2\pi/T}\right]_{T/2}^{T}$$

$$= -\frac{I_0}{\pi}\left[\cos\frac{2\pi T}{T} - \cos\frac{2\pi}{T}\times\frac{T}{2}\right]$$

$$= -\frac{I_0}{\pi}[\cos 2\pi - \cos\pi] = -\frac{I_0}{\pi}[1+1]$$

$$= -\frac{\mathbf{2I_0}}{\pi}$$

3. Average value of a.c. over complete cycle (0 to T) is

$$I_{av} = \frac{\int_0^{T} I_0 \sin\omega t\, dt}{\int_0^{T} dt} = \frac{I_0}{T}\int_0^{T} \sin\omega t\, dt$$

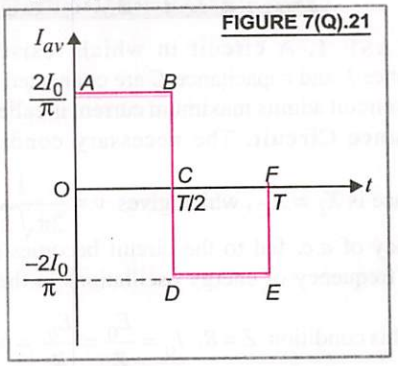
$$= \frac{I_0}{T}\left[\frac{-\cos\omega t}{\omega}\right]_0^{T}$$

$$= -\frac{I_0}{T\times 2\pi/T}\left[\cos\frac{2\pi}{T}\times T - \cos 0\right]$$

$$= -\frac{I_0}{2\pi}[\cos 2\pi - \cos 0] = -\frac{I_0}{2\pi}[1-1] = \mathbf{0}$$

4. The graph between I_{av} and t for a.c. is as shown in Fig. 7(Q).21.

FIGURE 7(Q).21



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5. From the definition of virtual value of current, we have

$$(I_v)^2 RT = \int_0^T (I_0 \cos \omega t)^2 R\, dt$$

$$= I_0^2 R \int_0^T \cos^2 \omega t\, dt$$

$$= I_0^2 R \int_0^T \frac{(1+\cos 2\omega t)}{2} dt$$

$$= \frac{I_0^2 R}{2}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2 R}{2}\left[T + \frac{\sin 2\omega T}{2\omega} - 0\right]$$

$$= \frac{I_0^2 R}{2}[T+0] = \frac{I_0^2 RT}{2}$$

$$\left[\because \sin 2\omega T = \sin 2 \times \frac{2\pi}{T} T = \sin 4\pi = 0\right]$$

$$\therefore \quad I_v^2 = \frac{I_0^2}{2} \quad \text{or} \quad I_v = \frac{I_0}{\sqrt{2}}$$

6. A d.c. ammeter measure the average value of current over a complete cycle. For a.c., average value of current over complete cycle is zero, so d.c. ammeter does not show reading in a.c. circuit.

7. $I_v = \dfrac{I_0}{\sqrt{2}}$ and $I_m = \dfrac{2I_0}{\pi}$

$$\therefore \quad \frac{I_v}{I_m} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}}$$

8. A graph between I_v and time t for the given a.c. is shown by a straight line AB parallel to time axis.

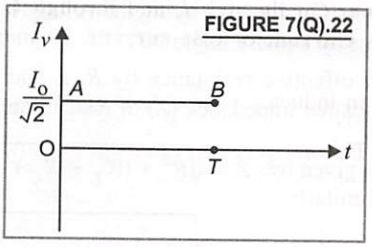

FIGURE 7(Q).22

9. In series R-L-C circuit, the phase angle ϕ by which the voltage leads the current in a.c. circuit is

$$\tan \phi = \frac{X_L - X_C}{R}.$$

If $\phi = 0°$, then $\tan 0° = \dfrac{X_L - X_C}{R_0}$

or $X_L - X_C = 0$ or $X_L = X_C$

or $\omega L = \dfrac{1}{\omega C}$ or $\omega = \dfrac{1}{\sqrt{LC}}$ or $2\pi \nu_r = \dfrac{1}{\sqrt{LC}}$

or $\nu_r = \dfrac{1}{2\pi\sqrt{LC}}$. Here ν_r is called resonant frequency of R-L-C circuit.

10. S.I. unit of impedance is **ohm.** The dimensional formula of impedance

$$Z = \frac{E_v}{I_v} = \frac{\text{workdone/charge}}{\text{current}} = \frac{(ML^2T^{-2})/AT}{A}$$

$$= [M^1 L^2 T^{-3} A^{-2}]$$

11. Here, $R = 12\ \Omega$, $X_C = 14\ \Omega$, $L = 0{\cdot}1$ H, $E_v = 200$ V, $\nu = 50$ Hz.

$X_L = \omega L = 2\pi \nu L = 2 \times 3 \times 50 \times 0{\cdot}1 = 30\ \Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{12^2 + (30-14)^2}$

$= \sqrt{144 + 256} = \mathbf{20\ \Omega}$

12. $I_v = \dfrac{E_v}{Z} = \dfrac{200}{20} = \mathbf{10\ A}$

CASE-BASED MCQs AND ASSERTION-REASON QUESTIONS

CASE 1. A circuit in which resistance R, inductance L and capacitance C are connected in series and the circuit admits maximum current is called **Series Resonance Circuit.** The necessary condition for resonance is $X_L = X_C$, which gives $\nu = \dfrac{1}{2\pi\sqrt{LC}}$, *i.e.*, frequency of *a.c.* fed to the circuit becomes equal to natural frequency of energy oscillations in the circuit.

Under this condition, $Z = R$, $I_0 = \dfrac{E_0}{Z} = \dfrac{E_0}{R}$ = max. The series resonance circuit is used as an **Acceptor** circuit in radio and T.V. receiver sets.

The sharpness of tuning at resonance is measured by **Q-factor** or **Quality** factor of the circuit

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

In a pure resistive circuit or non-inductive circuit, power, $P = E_v \times I_v$

The average power associated over a complete cycle with a pure inductor or a pure capacitor is zero.

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Hence, current through L and through C is called **Wattless Current** or **Idle current.** Its magnitude is $I_v \sin \phi$.

In an inductor 1 energy stored during growth is $\frac{1}{2} L I_0^2$. This is spent in maintaining current during decay. Similarly, energy stored in a capacitor during charging is $\frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} \frac{Q^2}{C}$.

This is spent during discharging of capacitor. In an *LCR* circuit, average power over a complete cycle is $P = E_v I_v \cos \phi$. Here, power factor = $\cos \phi$

$$= \frac{P}{E_v I_v} = \frac{\text{True power}}{\text{Apparent power}}.$$

The value of power factor varies for 0 to 1.

Based on the above paragraph, answer questions no. 1 to 4 :

1. In an electrical circuit, what is the necessary condition for resonance?

(*a*) $X_L = X_C$ (*b*) $2X_L = X_C$

(*c*) $X_L = 2X_C$ (*d*) none of these

2. What is the magnitude of wattless current or Idle current in an *a.c.* circuit?

(*a*) I_v (*b*) $I_v \sin \phi$

(*c*) $I_v \cos \phi$ (*d*) $I_v \tan \phi$

For Question No. 3 and 4, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is correct explanation of A.

(*b*) Both A and R are true and R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) Both A and R are false.

3. Assertion (A). The Q factor of an electric circuit is $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.

Reason (R). Current through L and through C is wattless.

4. Assertion (A). The power factor of an *a.c.* circuit is, $\cos \phi = \frac{\text{True power}}{\text{Apparent power}}$.

Reason (R). Current through condenser C is always wattless.

CASE 2. A transformer is an electrical device which is used for changing alternating voltages. It is based on the phenomenon of mutual induction. If n_p = number of turns in primary and n_s = number of turns in secondary, then

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = \frac{I_p}{I_s} = K$$

For a step up transformer, $K > 1$ and for a step down transformer, $K < 1$.

The main use of transformer is in transmission of *a.c.* over long distances at extremely high voltages. This reduces the energy losses.

Efficiency of transformer (η)

$$= \frac{\text{Output power } (E_s I_s)}{\text{Input power } (E_p I_p)}.$$

A choke coil is a pure inductor used for controlling current in an *a.c.* circuit.Through pure L, phase diff. between E and I is 90°.

$\therefore$ Average power comsumed/Cycle

$= E_v I_v \cos 90° =$ Zero

As $I_0 = \frac{E_0}{X_L} = \frac{E_0}{\omega L} = \frac{E_0}{2\pi \nu L}$

$\therefore$ For reducing low frequency *a.c.*, choke coils with laminated soft iron cores are used and for reducing high frequency *a.c.*, air cored chokes are used.

Based on the above paragraph, answer questions no. 5 to 8 :

5. Which appliance is used for long distance transmission of A.C.?

(*a*) *a.c.* motor (*b*) *d.c.* motor

(*c*) transformer (*d*) Choke coil

6. A choke coil is a

(*a*) pure resistor (*b*) pure inductor

(*c*) pure capacitor (*d*) a transpower

For Question No. 7 and 8, we have given two statements each, one labelled as Assertion (A) and other labelled as Reason (R).

Choose the correct option out of the four options given below :

(*a*) Both A and R are true and R is correct explanation of A.

(*b*) Both A and R are true but R is not a correct explanation of A.

(*c*) A is true, but R is false.

(*d*) Both A and R are false.

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7. **Assertion.** The transformer ratio (K) of a step up transformer is greater than one

Reason. $K = \frac{n_s}{n_p} = \frac{E_s}{E_p} > 1$

8. **Assertion.** A choke coil is used for controlling current in an *a.c.* circuit.

Reason. Through pure L, phase difference between E and I is 90°. $P = E_v I_v \cos 90° =$ Zero.

ANSWERS

1. (*a*) 2. (*b*) 3. (*b*) 4. (*b*) 5. (*c*) 6. (*b*) 7. (*a*) 8. (*a*)

HINTS/EXPLANATIONS For Difficult Questions

1. For resonance, $X_L = X_C$.
2. Wattless current consumes zero power. Its value is $I_v \sin \phi$.
3. **Both, A and R are true, but R is not a correct explanation of A.**
4. **Both, A and R are true, but R is not a correct explanation of A.**
5. A transformer is used for long distance transmission of A.C.
6. A choke coil is a pure inductor.
7. **Both, the Assertion and Reason are true, and Reason is correct explanation of Assertion.**
8. **Both, the Assertion and Reason are true, and Reason is correct explanation of Assertion.**

PROBLEMS FOR PRACTICE

TYPE I. TRANSIENT CURRENTS

1. A coil of resistance 20 ohm and inductance 0·5 H is connected to direct current supply of 200 V. Calculate the rate of increase of current at
(*i*) the instant of closing the switch,
(*ii*) at $t = L/R$ seconds after the switch is closed,
Also, calculate steady value of current in the circuit. **[Ans. 400 A s^{-1} ; 147·17 A s^{-1} ; 10 A]**
2. An inductor (L = 20 mH), a resistor (R = 100 Ω) and a battery (E = 10 V) are connected in series. Find (*a*) the time constant, (*b*) the maximum current and (*c*) the time elapsed before the current reaches 99% of the maximum value.
[Ans. 2 × 10^{-4} s ; 10^{-1} A ; 0·92 × 10^{-3} s]
3. A capacitor charged to 10 V is being discharged through a resistance R. At the end of 1 s, the voltage across the capacitor is 5 V. What will be the voltage after 2 s ? **[Ans. 2·5 V]**
4. A capacitor of 1 μF is placed in series with a resistor of 2 mega-ohm and a battery of emf 2 V. Calculate the time after which the charge will grow to 86·47% of its max. value. **[Ans. 4·0 s]**

TYPE II. MEAN VALUE AND R.M.S. VALUE OF A.C.

5. The equation of a.c. in a circuit is $I = 50 \sin 100 \pi t$. Find (*i*) frequency of a.c.
(*ii*) mean value of a.c. over positive half cycle
(*iii*) *rms* value of current
(*iv*) value of current $\frac{1}{300}$ s after it was zero.
[Ans. 50 Hz ; 31·8 A ; 35·35 A ; 43·3 A]
6. The peak value of an a.c. of frequency 50 hertz is 14.14 ampere. Find the r.m.s. value of current. How much time will the current take in reaching from zero to maximum value ?
[Ans. 10 amp ; 0.005 sec]
7. The effective value of current in a 50 cycle a.c. circuit is 5·0 A. What is the value of current (1/300) s after it is zero ? **[Ans. 6·12 A]**
8. The electric current in a circuit is given by $i = i_0 (t/\tau)$ for some time. Calculate the *r.m.s.* current for the period $t = 0$ to $t = \tau$. **[Ans. $i_0/\sqrt{3}$]**
9. The equation of alternating current for a circuit is given by $I = 50 \cos 100 \pi t$. Calculate
(*i*) frequency of *a.c.* applied, (*ii*) virtual value of current, (*iii*) value of current 1/600 *s* after it was maximum. **[Ans. 50 c/s ; 35·36 A ; $25\sqrt{3}$A]**

TYPE III. A.C. CIRCUIT CONTAINING RESISTANCE ONLY

10. An alternating voltage given by $E = 280 \sin 50 \pi t$ is connected across a pure resistor of 40 ohm. Find frequency of source and *rms* current through the bulb. **(CBSE 2012)**
[Ans. 25 Hz, 4·95 A]

11. A light bulb is rated 200 W for 220 V supply at 50 Hz. Calculate resistance of the bulb and *rms* current through the bulb. [CBSE 2012 (C)]
[Ans. 242 Ω ; 0·91 A]

12. A resistance of 20 Ω is connected to a source of alternating current rated 110 V, 50 Hz. Find (*a*) the *r.m.s.* current, (*b*) the maximum instantaneous current in the resistor and (*c*) the time taken by the current to change from its maximum value to the *r.m.s.* value.
[Ans. 5·5 A ; 7·8 A ; $2·5 \times 10^{-3}$ s]

13. A 100 ohm iron is connected to a 110 V–60 hertz wall plug. What is (*i*) peak pot. diff. (*ii*) average potential diff. over a half cycle and (*iii*) *rms* current ? **[Ans. 155·5 V ; 99·1 V ; 1·1 A]**

14. A sinusoidal voltage $V = 200 \sin 314\, t$ is applied to a resistor of 10 ohm. Calculate (*i*) rms value of voltage (*ii*) rms current and (*iii*) power dissipated as heat. (CBSE 2001)
[Ans. (*i*) 141·44 V (*ii*) 14·144 A (*iii*) 2000·5 W]

TYPE IV. A.C. CIRCUIT CONTAINING INDUCTANCE ONLY

15. An inductor of inductance 200 mH is connected to an a.c. source of peak *emf* 210 V and frequency 50 Hz. Calculate the peak current. What is the instantaneous value of voltage when current is at its peak value ? **[Ans. 3·3 A ; Zero]**

16. A pure inductor of self inductance 1 H is connected across an alternating voltage of 115 V and frequency 60 Hz. Calculate the (*i*) inductive reactance (*ii*) effective current (*iii*) peak current (*iv*) average power consumed.
[Ans. 377.1 Ω, 0.3 A, 0.43 A, Zero]

17. How much inductance should be connected to 200 V, 50 *c/s* supply so that a maximum current of 0·5 A flows through it ? **[Ans. 1·79 H]**

18. Alternating *e.m.f.* of $E = 220 \sin 100\, \pi\, t$ is applied to a circuit containing an inductance of $(1/\pi)$ henry. Write equation for instantaneous current through the circuit. What will be the reading of *a.c.* galvanometer connected in the circuit ?
[Ans. $I = 2·2 \sin (100\, \pi\, t - \pi/2)$; 1·555 A]

19. An inductance of negligible resistance, whose reactance is 22 ohm at 200 Hz is connected to a 220 V, 50 Hz power line. What is the value of inductance and reactance ? **[Ans. 0·0175 H ; 5·5 Ω]**

TYPE V. A.C. CIRCUIT CONTAINING CONDENSER ONLY

20. A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage $E = 10 \sin \omega t$ volt. If $\omega = 10$ rad/s, find the peak current and instantaneous current.
[Ans. 1·0 mA ; $I = 10^{-3} \sin (\omega t + \pi/2)$]

21. A 10 μ F capacitor is connected with 1 henry inductance in series with a 50 hertz source of a.c. Calculate the impedance of the combination.
[Ans. 4·0 Ω]

22. A 3 μ F capacitor is connected to a 220 V, 50 Hz a.c. source. Calculate the rms value of current through the circuit. Also, find the peak value of voltage across the capacitor. **[Ans. 0·207 A ; 311·1 V]**

23. A capacitor of 10 μ F is connected to an a.c. source of *e.m.f.* $E = 220 \sin 100\, \pi\, t$. Write the equation of instantaneous current through the circuit. What will be the reading of a.c. ammeter connected in the circuit ?
[Ans. $I = 0·691 \sin (100\, \pi\, t + \pi/2)$; 0·489 A]

24. A current of 30 mA is taken by a 4 μF capacitor connected across an alternating current line having frequency of 500 Hz. Calculate reactance of the capacitor and voltage across the capacitor.
[Ans. 79·6 Ω ; 2·4 V]

25. A 30 μF capacitor is connected to a 220 V, 50 Hz source. Find its capacitative reactance, *rms* current, peak current and impendance of the circuit. (Hr. Board 2010)
[Ans. 106·1 Ω ; 2·07 A ; 2·93 A ; 106·1 Ω]

TYPE VI. A.C. CIRCUIT CONTAINING *R* AND *L*

26. A 12 ohm resistance and an inductance of $0·05/\pi$ henry are connected in series. Across the ends of the circuit is connected a 130 V a.c. supply of 50 Hz. Calculate current in the circuit. What is phase diff. between current and voltage ?
(Hr. Board 2003) **[Ans. 10 A ; 22·6°]**

27. The impedance of a coil is 141.4 ohm at 50 hertz and its resistance is 100 ohm. What will be its inductance ? **[Ans. 0.318 H]**

28. When 100 volt d.c. is applied across an inductor, a current of 1 A flows through it. If the same inductor is connected to 100 V a.c. source, the current reduces to 0·5 A. Why is the current reduced in the latter case ? Calculate the value of the reactance of the inductor ?
(CBSE 2019) **[Ans. 173·2 Ω]**

29. When a series combination of inductance and resistance are connected with a 10 V, 50 Hz *a.c.* source, a current of 1 A flows in the circuit. The voltage leads the current by a phase angle of $\pi/3$ radian. Calculate the values of resistance and inductance. **[Ans. 5 Ω , 27·57 mH]**

30. A 80 V–800 W heater is to be operated on a 100 V–50 Hz a.c. supply. Calculate the inductance of the choke required.
(Hr. Board 2002) **[Ans. 0·019 H]**

31. The inductance of a resistance coil is 0.5 henry. How much potential difference will be developed across it on passing an alternating current of 0.2 amp. if the frequency of current be 50 hertz ? What will be the phase difference between the potential difference and current in the coil ?
[Ans. 31.4 volt, voltage leads current by 90°]

32. A coil of inductance 0·5 H and resistance 100 Ω is connected to 200 V, 50 Hz a.c. supply. Find the maximum current in the coil Also, find the time lag between the maximum voltage and maximum current. (CBSE 2008)
[Ans. 1·52 A ; $3·194 \times 10^{-3}$ s]

TYPE VII. A.C. CIRCUIT CONTAINING *R* AND *C* ONLY

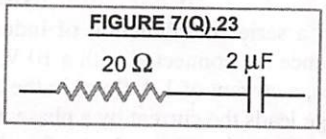
33. In a series *RC* circuit, $R = 30$ ohm, $C = 0·25$ μF, $V = 100$ V, $\omega = 10000$ rad/s. Find the current in the circuit and calculate the voltage across the resistor and capacitor. Is the algebraic sum of these voltages more than the source voltage ? If yes, resolve the paradox. (CBSE 2004)
[Ans. 0·25 A ; 7·5 V, 100 V, Yes]

34. When an electric device *X* is connected to a 220 volt, 50 hertz a.c. supply, the current is 0.5 amp, and is in same phase as the applied voltage. When another device *Y* is connected to the same supply, the electric current is again 0.5 amp, but it leads the potential difference by π/2. (*i*) What are the devices *X* and *Y* ? (*ii*) When *X* and *Y* are connected in series across the same source, what will be the current ?
[Ans. *X* is resistor, *Y* is capacitor ; 0.353 A]

35. Find the capacity of a condenser to run a 30 volt, 10 watt lamp when connected in series with an alternating e.m.f. of 220 volt and frequency 50 c/s. **[Ans. 4.86 μ F]**

36. What will be the impedance of the adjoining circuit, Fig. 7(Q).23 for (*i*) direct current source, (*ii*) an a.c. source of frequency $25/\pi$ kilohertz ?

FIGURE 7(Q).23



[Ans. (*i*) ∞ (*ii*) $10\sqrt{5}\ \Omega$]

37. A circuit consists of a 2 μ F capacitor and a resistor of 1 kΩ. An a.c. source of 12 V, 50 hz is connected across the circuit. Calculate (*i*) current flowing (*ii*) voltage across capacitor (*iii*) phase angle between voltage and current (*iv*) average power supplied.
[Ans. $6·4 \times 10^{-3}$ A ; 10·2 V ; 58°, 0·041 W]

38. When a circuit element *X* is connected across an *a.c.* source of *e.m.f.* $220\sqrt{2}$ volt, current of $\sqrt{2}$ A flows through it in phase with the voltage. When another element *Y* is connected across the same *a.c.* source, the same current flows through the circuit, but it leads the voltage by π/2 radian. Name the circuit elements *X* and *Y*.
Find the current that flows in the circuit when series combination of *X* and *Y* is connected across the same source. (CBSE Sample Paper 2010)
[Ans. *X*-Resistance, *Y*-capacitor ; 1 A]

TYPE VIII. A.C. CIRCUIT CONTAINING *R*, *L* AND *C*

39. A 50 μF capacitor, 0·05 H inductor and 48 Ω resistor are connected in series with an a.c. source of *e.m.f.* $E = 310 \sin 314\ t$. Calculate reactance of the circuit. What is its nature ?
What is phase angle between current and applied *e.m.f.* ?
[Ans. 48 Ω, capacitative ; 45°, current leads]

40. A 25·0 μ F capacitor, a 0·10 henry inductor and a 25·0 ohm resistor are connected in series with an *a.c.* source whose e.m.f. is given by $E = 310 \sin 314\ t$.
(*i*) what is the frequency of the e.m.f. ?
(*ii*) calculate
(*a*) the reactance of the circuit
(*b*) the impedance of the circuit, and
(*c*) current in the circuit.
[Ans. (*i*) 50 Hz (*ii*) (*a*) 95·84 Ω (*b*) 99·1 Ω (*c*) 2·21 A]

41. A 40 ohm resistor, 3 mH inductor and 2 μF capacitor are connected in series to a 110 V, 5000 Hz a.c. source. Calculate the value of current in the circuit. (CBSE 2005) **[Ans. 1·25 A]**

42. A series *LCR* circuit having $L = 10$ mH, $C = (400/\pi^2)$ μF and $R = 55$ ohm is connected to 220 V variable frequency *a.c.* supply.
(*i*) Find frequency of source, for which average power absorbed by the circuit is maximum
(*ii*) Calculate the amplitude of current.
(CBSE 2010 (C)) **[Ans. (*i*) 250 Hz (*ii*) $4\sqrt{2}$ A]**

TYPE IX. ELECTRIC RESONANCE

43. A variable frequency 230 V alternating voltage source is connected across a series combination of $L = 5.0$ H, $C = 80$ μ F and $R = 40$ Ω.

(*i*) Calculate the angular frequency of the source which drives the circuit in resonance.

(*ii*) Obtain the impedance of the circuit and amplitude of current at resonance frequency

(*iii*) Obtain *r.m.s.* potential drop across the three elements of the circuit at resonating frequency.

(*iv*) How do you explain the observation that the algebraic sum of voltages across the three elements obtained is greater than the supplied voltage. (CBSE 2001)

[Ans. (i) 50 rad/sec (ii) 40 Ω ; 8.13 A (iii) $V_L = V_C = 1437{\cdot}5$ V , $V_R = 230$ V]

44. An inductance of 2·0 H, a capacitance of 18 μ F and a resistance of 10 kΩ are connected in series to an *ac* source of 20 V with adjustable frequency. (*a*) What frequency should be chosen to maximise the current in the circuit ? (*b*) What is the value of this maximum current ?

[Ans. 26·5 Hz ; 2·828 mA]

45. A series *LCR* circuit with $C = 80$ μF, $L = 5{\cdot}0$ H and $R = 40$ Ω is connected to a variable frequency 240 V a.c. source. Calculate

(*i*) angular frequency of the source which drives the circuit in resonance.

(*ii*) current at the resonating frequency.

(*iii*) *rms* pot. drop across the capacitor.

(CBSE 2012) **[Ans. 50 rad/s ; 6 A ; 1500 V]**

46. In a series *LCR* circuit, the resonance frequency is 800 Hz. The half power points are obtained at frequencies 745 and 855 Hz. Calculate the Q factor of the circuit and the band width.

[Ans. 7·27 ; 110 Hz]

47. A capacitor, a resistor and a 40 mH inductor are connected in series to an *a.c.* source of frequency 60 Hz. Calculate the capacitance of the capacitor, if current is in phase with the voltage.

[Ans. 175·7 μ F]

48. A capacitor of 50 μ F, a resistor of 10 ohm and an inductor L are in series with an *a.c.* source of frequency 50 Hz. Calculate the value of L if phase angle between current and voltage is zero.

[Ans. 0·2 H]

49. A resistor of resistance 400 ohm and a capacitor of reactance 200 ohm are connected in series to a 220 V, 50 Hz source. If the current in the circuit is 0·49 A, find the voltage across the resistor and capacitor. What is the value of inductance required so that voltage and current are in phase ? (CBSE Sample Paper 2011)

[Ans. 196 V, 98 V, 0·64 H]

50. A capacitor of unknown capacitance, a resistance of 100 ohm and an inductor of inductance $L = 4/\pi^2$ henry are connected in series across an a.c. source of 200 V, 50 Hz. Calculate the value of capacitance and the current that flows in the circuit, when the current is in phase with the voltage. (CBSE 2013) **[Ans. 25 μF ; 2 A]**

51. Obtain the resonant frequency and Q factor of a series *LCR* circuit with $L = 3{\cdot}0$ H, $C = 27$ μF and $R = 7{\cdot}4$ ohm. **[Ans. 17·7 Hz ; 45·05]**

TYPE X. ENERGY STORED IN CONDENSER, INDUCTOR AND LC OSCILLATIONS

52. An e.m.f. $E = 100 \sin 314\, t$ is applied across a pure capacitor of 637 μF. Calculate the instantaneous power P and maximum energy stored in the capacitor.

[Ans. 1000 sin 628 t watt ; 3·185 J]

53. A capacitor of capacitance 54 μF is charged to a potential of 25 V and then connected to a 15 mH inductor to produce oscillations. What was the energy stored in the capacitor initially and to what wavelength will the circuit respond ?

[Ans. $1{\cdot}69 \times 10^{-2}$ J ; 176·8 Hz]

TYPE XI. POWER ASSOCIATED WITH A.C. CIRCUITS, POWER FACTOR

54. A series *LCR* circuit is made by taking $R = 100$ Ω, $L = \frac{2}{\pi} H$ and $C = \frac{100}{\pi} \mu F$. The series combination is connected across an a.c. source of 220 V, 50 Hz. Calculate impedance of the circuit and peak value of current flowing in the circuit. What is power factor of the circuit ? Compare it with one at resonance frequency.

[CBSE 2008 (C)] $\left[\text{Ans. } 141{\cdot}4\,\Omega ; 2{\cdot}2\,\text{A}, \frac{1}{\sqrt{2}}\right]$

55. A resistor R and an element X are connected in series to a.c. source of voltage. The voltage is found to lead the current in phase by $\pi/4$. If X is replaced by another element Y, the voltage lags

behind the current by $\pi/4$. Identify elements X and Y. When both, X and Y are connected in series with R, to the same source, will the power dissipated in the circuit be maximum or minimum ? Justify your answer. (CBSE 2013)

[Ans. X-inductor, Y-capacitor ; maximum]

56. An inductor of unknown value, a capacitor of 100 μF and a resistor of 10 ohm are connected in series to a 200 V, 50 Hz a.c. source. It is found that power factor of the circuit is unity. Calculate inductance of the inductor and current amplitude. (CBSE 2008) **[Ans. 101·3 mH ; 28·28 A]**

57. You are given three circuit elements X, Y and Z. When the element X is connected across an a.c. source of a given voltage, the current and voltage are in same phase. When element Y is connected in series with X across the source, voltage is ahead of the current by $\pi/4$. But the current is ahead of voltage by $\pi/4$, when Z is connected in series with X, across the source. Identify the circuit elements X, Y and Z.

When all the three elements are connected in series across the same source, determine the impedance of the circuit. Draw a plot of current versus the frequency of applied source and mention the significance of this plot.

(CBSE 2015)

[Ans. $X \rightarrow R$; $Y \rightarrow L$; $Z \rightarrow C$; $Z = R$]

58. An a.c. circuit containing 800 mH inductor and a 60 μ F capacitor is in series with 15 ohm resistance. They are connected to 230 V, 50 Hz a.c. supply. Obtain average power transferred to each element and total power absorbed.

[Ans. $P_L = 0$, $P_C = 0$, $P_R = 20{\cdot}05$ watt, $P = 20{\cdot}05$ watt]

59. A group of electric bulbs have a power rating of 300 watt. An a.c. voltage $V = 141.4 \sin (314\, t + \pi/3)$ is applied to the group. Calculate the effective current. **[Ans. 3 A]**

60. A 60 cycle AC circuit has a resistance of 200 Ω and inductance of 10 mH. What is the power factor ? What capacitance placed in the circuit will make the power factor unity ?

[Ans. 0.99, 703 μ F]

61. An *LCR* series circuit with 100 Ω resistance is connected to an a.c. source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the *LCR* circuit. **[Ans. 2 A ; 400 W]**

62. A 200 V variable frequency a.c. source is connected to a series combination of $L = 5$ H, $C = 80$ μF and $R = 40$ Ω. Calculate the angular frequency of the source to get maximum current in the circuit, the curent amplitude at resonance and power dissipated in the circuit.

(CBSE 2002)

[Ans. 50 rad/s 7·07 A ; 1000 W]

63. An inductor 200 mH, capacitor 500 μF, resistor 10 ohm are connected in series with a 100 V, variable frequency a.c. source. Calculate (*i*) frequency at which power factor of the circuit is unity (*ii*) current amplitude at this frequency (*iii*) Q factor. (CBSE 2008)

[Ans. $\frac{50}{\pi}$ Hz ; 14·14 A ; 2]

TYPE XII. TRANSFORMER

64. In an ideal transformer, number of turns in primary and secondary are 200 and 1000 respectively. If the power input to the primary is 10 kW at 200 V, calculate (*i*) output voltage (*ii*) primary current. (CBSE 2002)

[Ans. 1000 V, 50 A]

65. The primary of a transformer has 200 turns and secondary has 1000 turns. The power output from secondary at 1000 V is 9 kW. Calculate primary voltage and heat loss in primary. Take resistance of primary coil 0·2 Ω and efficiency of transformer = 90%. **[Ans. 200 V ; 500 W]**

66. A transformer has 200 primary turns and 150 secondary turns. If the operating voltage for the load connected to the secondary is measured to be 300 V, what is the voltage supplied to the primary ? **[Ans. 400 V]**

67. The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24 V, 24 W, calculate the current in the primary coil of the circuit. (CBSE 2000) **[Ans. 0·1 A]**

68. Calculate the current drawn by the primary of a transformer which steps down 200 V to 20 V to operate a device of resistance 20 ohm. Assume the efficiency of transformer to be 80%.

(CBSE 2007) **[Ans. 0·125 A]**

69. The ratio of the number of turns in primary and secondary coil of a step up transformer is 1:200. It is connected to a.c. mains of 220 volt. Calculate the voltage developed in the secondary. Determine the maximum current in secondary

coil, when a current of 2 ampere flows through the primary. **[Ans. 44 kV, 0·01 amp.]**

70. The primary coil of an ideal step up transformer has 100 turns and the transformation ratio is also 100. The input voltage and power are 220 V and 1100 W. Calculate (*i*) number of turns in secondary (*ii*) current in primary (*iii*) voltage across secondary (*iv*) current in secondary (*v*) power in secondary. (CBSE 2006)

[Ans. (*i*) 10000 (*ii*) 5 A (*iii*) 22000 V (*iv*) 0·05 A (*v*) 1100 W]

TYPE XIII. TYPICAL PROBLEMS

71. A coil of resistance 15 Ω and inductance 10 H is connected across a 90 V *d.c.* supply. Determine the value of current after 2 sec. What is the energy stored in the mag. field at that instant ?

[Ans. 5·7 A ; 162·45 J]

72. A resistor of resistance 100 Ω is connected to an *ac* source, $E = (12\text{ V})\sin(250\ \pi s^{-1})\ t$. Find the energy dissipated as heat during $t = 0$ to $t = 1{\cdot}0$ ms. **[Ans. $2{\cdot}61 \times 10^{-4}$ J]**

73. An alternating current of 1·5 mA and angular frequency $\omega = 300$ radian/sec flows through 10 kΩ resistor and a 0·50 μ F capacitor in series. Find the r.m.s. voltage across the capacitor and impedance of the circuit.

[Ans. 10 V, $1{\cdot}2 \times 10^4$ Ω]

74. A coil of inductance 1·0 H and resistance 100 Ω is connected to a battery of *e.m.f.* 12 V. Find the energy stored in the magnetic field associated with the coil at an instant 10 ms after the circuit is switched on ? **[Ans. $2{\cdot}87 \times 10^{-3}$ J]**

75. A choke of 0·5 H, a capacitor of 15 μF and resistance of 100 ohm are connected in series across 200 V, 50 Hz main. Find current in the circuit and power factor of the circuit.

[Ans. 1·752 A ; 0·876]

76. Calculate the value of inductance, which should be connected in series with a capacitance of 5 μF and resistance of 10 ohm with *a.c.* source of 50 Hz, so that power factor of the circuit is unity.

[Ans. 2·208 H]

77. A transformer has an efficiency of 80% and works at 100 volt and 4 kW. If the secondary voltage is 240 volt, calculate the primary and secondary currents. **[Ans. 40 A ; 13·3 A]**

78. A step down transformer converts a voltage of 2000 V into 200 V in the transmission line. Number of turns in primary coil is 5000, efficiency of transformer is 80% and output power is 8 kW. Calculate input power and number of turns in secondary coil. **[Ans. 10 kW ; 500]**

79. A 10 kW transformer has 20 turns in primary and 100 turns in secondary circuit. A.C. voltage $E_1 = 600 \sin 314\ t$ is applied to the primary. Find max. value of flux and max. value of secondary voltage. **[Ans. 0·0955 Wb ; 3000 V]**

80. An *a.c.* source of internal resistance 9000 Ω is to supply current to a load resistor of 10 ohm. How should the source be matched to the load and what is the ratio of the currents passing through the load and the source ?

[Ans. $\frac{n_s}{n_p} = \frac{1}{30}$; $\frac{I_s}{I_p} = 30$]

HINTS
For Difficult Problems

1. Here, $R = 20\ \Omega$, $L = 0{\cdot}5$ H, $E = 200$ V

As $I = I_0\left(1 - e^{\frac{-R}{L}t}\right)$

$$\therefore \frac{dI}{dt} = I_0\left[0 - \left(\frac{-R}{L}\right)e^{\frac{-R}{L}t}\right]$$

$$\frac{dI}{dt} = \frac{I_0 R}{L}e^{-(R/L)t} = \frac{E}{L}e^{\frac{-R}{L}t}$$

(*i*) When the switch is just closed, $t = 0$,

$$\frac{dI}{dt} = \frac{E}{L} \times 1 = \frac{200}{0{\cdot}5} \times 1 = \mathbf{400\ A\,s^{-1}}$$

(*ii*) At $t = \frac{L}{R}$,

$$\frac{dI}{dt} = \frac{E}{L}e^{-1} = \frac{200}{0{\cdot}5} \times \frac{1}{2{\cdot}718} = \mathbf{147{\cdot}17\ A\,s^{-1}}$$

(*iii*) Steady value of current

$$I_0 = \frac{E}{R} = \frac{200}{20} = \mathbf{10\ A}$$

2. Here, $L = 20$ mH $= 20 \times 10^{-3}$ H, $R = 100$ ohm. $E = 10$ V.

Time constant, $\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{100} = \mathbf{2 \times 10^{-4}\ s}$

Maximum current, $I_0 = \frac{E}{R} = \frac{10}{100} = 10^{-1}$ A

Using $I = I_0(1 - e^{-t/\tau})$

$0{\cdot}99\, I_0 = I_0(1 - e^{-t/\tau})$

$\therefore \quad e^{-t/\tau} = 0{\cdot}01.$

Solve to get $t = 0{\cdot}92 \times 10^{-3}$ s

3. The equation of discharging of capacitor is

$$q = q_0 e^{-t/RC}, \quad \frac{q_0}{q} = e^{t/RC}$$

$$\text{or} \quad t = RC \log_e\left(\frac{q_0}{q}\right) = RC \log_e\left(\frac{V_0}{V}\right) \qquad (\because q \propto V)$$

At $t = 1$ s, $V = 5$ volt from $V_0 = 10$ V

$$\therefore \quad 1 = RC \log_e\left(\frac{10}{5}\right) = RC \log_e 2 \qquad \ldots(i)$$

At $t = 2$ s, $V = ?$ $V_0 = 10$ V

$$\therefore \quad 2 = RC \log_e\left(\frac{10}{V}\right) \qquad \ldots(ii)$$

Dividing (*ii*) by (*i*), we get

$$2 = \frac{\log_e 10 - \log_e V}{\log_e 2}$$

$$2 \log_e 2 = \log_e 10 - \log_e V$$

$$\log_e V = \log_e 10 - \log_e 2^2 = \log_e\left(\frac{10}{4}\right)$$

$$V = \frac{10}{4} = 2{\cdot}5\,\text{V}$$

4. Here, $C = 1\ \mu\text{F} = 10^{-6}$ F,

$R = 2$ mega-ohm $= 2 \times 10^6\ \Omega$, $V = 2$ volt

$$t = ? \quad \frac{q}{q_0} = \frac{86{\cdot}47}{100}$$

$$\tau = RC = 2 \times 10^6 \times 10^{-6} = 2 \text{ s}$$

From $q = q_0(1 - e^{-t/\tau})$

$$\frac{q}{q_0} = 1 - e^{-t/\tau}$$

$$\text{or} \quad e^{-t/\tau} = 1 - \frac{q}{q_0} = 1 - \frac{86{\cdot}47}{100} = \frac{13{\cdot}53}{100}$$

$$e^{-t/2} = \frac{13{\cdot}53}{100} \quad \text{or} \quad e^{t/2} = \frac{100}{13{\cdot}53}$$

$$\frac{t}{2} = (\log_e 100 - \log_e 13{\cdot}53)$$

$$= 2{\cdot}303\,(2 - 1{\cdot}1310)$$

$$t = 2 \times 2{\cdot}303\,(0{\cdot}8690) = 4{\cdot}0 \text{ s}$$

5. Here, $I = 50 \sin 100\,\pi\, t$. Compare with the standard form : $I = I_0 \sin 2\,\pi\, \nu\, t$

(*i*) $2\,\pi\,\nu = 100\,\pi$; $\nu = 50$ Hz

(*ii*) Mean value of a.c. over pos. half cycle

$$= \frac{2I_0}{\pi} = \frac{2 \times 50}{3{\cdot}14} = 31{\cdot}8 \text{ A}$$

(*iii*) *rms* value of current,

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{50}{1{\cdot}414} = 35{\cdot}35 \text{ A}$$

(*iv*) $I = I_0 \sin \omega t$

$$= 50 \sin 100\,\pi \times \frac{1}{300} = \frac{50\sqrt{3}}{2} = 43{\cdot}3 \text{ A}$$

7. $I_v = 5{\cdot}0$ A, $\nu = 50$ c/s,

$I_0 = \sqrt{2}\ I_v = \sqrt{2} \times 5$ A

$I = I_0 \sin \omega t = I_0 \sin 2\,\pi\,\nu\, t$

$$= \sqrt{2} \times 5 \sin 2\,\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \pi/3 = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 6{\cdot}12\,\text{A}$$

8. Mean square current for the period $t = 0$ to $t = \tau$ is

$$\overline{i^2} = \frac{1}{\tau}\int_0^{\tau} i_0^2\,(t/\tau)^2\,dt = \frac{i_0^2}{\tau^3}\left[\frac{t^3}{3}\right]_0^{\tau} = \frac{i_0^2}{3}$$

$$\therefore \quad i_{r.m.s.} = \sqrt{\overline{i^2}} = \sqrt{\frac{i_0^2}{3}} = \frac{i_0}{\sqrt{3}}$$

9. Here, $I = 50 \cos 100\,\pi t$

Compare it with the standard form

$I = I_0 \cos \omega t$, we get $I_0 = 50$ A

$$\omega = 2\,\pi\,\nu = 100\,\pi, \quad \nu = \frac{100\pi}{2\pi} = 50 \text{ c/s}$$

Virtual value of current, $I_v = \frac{I_0}{\sqrt{2}} = \frac{50}{\sqrt{2}}$

$= 35{\cdot}36$ A

At $t = 0$, $I = 50 \cos 0 = 50$ A $=$ max.

At $t = \frac{1}{600}$ s,

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$$I = 50 \cos 100\pi \times \frac{1}{600} = 50 \cos \frac{\pi}{6}$$

$$= 50 \cos 30° = \frac{50\sqrt{3}}{2} = \mathbf{25\sqrt{3}\ A}$$

10. Here, $E = 280 \sin 50\pi t$, $R = 40\ \Omega$, $\nu = ?$ $I_v = ?$

Compare given eqn. of E with standard form $E = E_0 \sin 2\pi \nu t$

$$E_0 = 280\ \text{V}, \ 2\pi\nu = 50\pi, \ \nu = \frac{50\pi}{2\pi} = \mathbf{25\ Hz}$$

$$I_v = \frac{E_v}{R} = \frac{E_0}{\sqrt{2}\,R} = \frac{280}{\sqrt{2}\times 40} = \frac{280}{56{\cdot}56}$$

$$= \mathbf{4{\cdot}95\ A}$$

11. Here, $P = 200$ W, $E_v = 220$ V, $\nu = 50$ Hz

$R = ?$ $I_v = ?$

From $P = E_v I_v$; $I_v = \frac{P}{E_v} = \frac{200}{220} = \mathbf{0{\cdot}91\ A}$

$$R = \frac{E_v}{I_v} = \frac{220}{0{\cdot}91} = \mathbf{242\ ohm}$$

12. (*a*) $I_{r.m.s.} = \frac{E_{r.m.s.}}{R} = \frac{110}{20} = \mathbf{5{\cdot}5\ A}$

(*b*) $I_0 = \sqrt{2}\ I_{r.m.s.} = 1{\cdot}414 \times 5{\cdot}50 = \mathbf{7{\cdot}8\ A}$

Let the ac be represented by $i = i_0 \cos \omega t$

At $t = 0$, $i = i_0 \cos 0° = i_0$ (max)

At $t = t$, let $i = i_v = \frac{i_0}{\sqrt{2}} = i_0 \cos \omega t$

$$\therefore \quad \cos \omega t = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4}$$

$$\omega t = \frac{\pi}{4} \quad \text{or} \quad 2\pi\nu t = \frac{\pi}{4};$$

$$t = \frac{1}{8\nu} = \frac{1}{8\times 50} = \mathbf{2{\cdot}5 \times 10^{-3}\ s}$$

13. Here, $R = 100$ ohm, $E_v = 110$ V, $\nu = 60$ hz

$$E_0 = \sqrt{2}\ E_v = 1{\cdot}414 \times 110 = \mathbf{155{\cdot}5\ V},$$

$$E_m = \frac{2E_0}{\pi} = \frac{2\times 155{\cdot}5}{3{\cdot}14} = \mathbf{99{\cdot}1\ V}$$

$$I_v = \frac{E_v}{R} = \frac{110}{100} = \mathbf{1{\cdot}1\ A}$$

14. $V = 200 \sin 314\, t$, $R = 10\ \Omega$

Compare with $V = E_0 \sin \omega t = E_0 \sin 2\pi \nu t$

$$E_0 = 200\ \text{V}, \ E_v = \frac{E_0}{\sqrt{2}} = \frac{200}{1{\cdot}414} = \mathbf{141{\cdot}44\ V}$$

$$I_v = \frac{E_v}{R} = \frac{141{\cdot}4}{10} = \mathbf{14{\cdot}144\ A}$$

$$P = E_v I_v = 141{\cdot}144 \times 14{\cdot}144 = \mathbf{2000{\cdot}5\ W}$$

15. Here, $L = 200\ \text{mH} = 200 \times 10^{-3}$ H, $E_0 = 210$ V,

$\nu = 50$ Hz, $I_0 = ?$

$$I_0 = \frac{E_0}{X_L} = \frac{E_0}{\omega L} = \frac{E_0}{2\pi\nu L} = \frac{210}{2\times 3{\cdot}14\times 50\times 0{\cdot}2}$$

$$= \mathbf{3{\cdot}3\ A}$$

Through L, current lags behind the voltage by a phase angle of 90°. Therefore, when current is at its peak value, voltage must be **zero.**

17. Here, $L = ?$, $E_v = 200$ V, $\nu = 50$ c/s,

$I_0 = 0{\cdot}5$ A, $L = ?$

$$I_0 = \frac{E_0}{X_L} = \frac{\sqrt{2}\,E_v}{\omega L}$$

$$L = \frac{\sqrt{2}\,E_v}{\omega I_0} = \frac{\sqrt{2}\,E_v}{(2\pi\nu) I_0} = \frac{1{\cdot}414\times 200}{2\times\frac{22}{7}\times 50\times 0{\cdot}5}$$

$$= \mathbf{1{\cdot}79\ H}$$

18. From $E = 220 \sin 100\pi t$,

$E_0 = 220$ V, $\omega = 100\pi$; $L = (1/\pi)$ H

$$X_L = \omega L = 100\pi \times \frac{1}{\pi} = 100\ \text{ohm}.$$

$$\therefore \quad I_0 = \frac{E_0}{X_L} = \frac{220}{100} = 2{\cdot}2\ \text{A}$$

As current lags behind the *e.m.f.* by a phase angle $\pi/2$

$$\therefore \quad I = I_0 \sin(\omega t - \pi/2)$$

$$I = \mathbf{2{\cdot}2 \sin (100\,\pi\, t - \pi/2)}$$

Reading of *a.c.* galvanometer, $I_v = \frac{I_0}{\sqrt{2}} = \frac{2{\cdot}2}{\sqrt{2}}$

$$I_v = \frac{2{\cdot}2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = 1{\cdot}1 \times 1{\cdot}414 = \mathbf{1{\cdot}555\ A}$$

19. Here, $X_L = 22$ ohm, $\nu = 200$ Hz,

$E_v = 220$ V, $\nu' = 50$ Hz, $L = ?$, $X_L' = ?$

From $X_L = \omega L$,

$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi\nu} = \frac{22}{2\times\frac{22}{7}\times 200}$$

$$= \mathbf{0{\cdot}0175\ H}$$

$$X_L' = \omega' L = 2\pi\nu' L$$

$$= 2\pi \times 50 \times 0{\cdot}0175 = \mathbf{5{\cdot}5\ ohm}$$

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20. Here ; $C = 10\ \mu F = 10 \times 10^{-6}$ F, $E = 10 \sin \omega t$,
$\therefore \quad E_0 = 10$ V, $\omega = 10$ rad/s, $I_0 = ?$, $I = ?$

$$I_0 = \frac{E_0}{X_C} = \frac{E_0}{1/\omega C} = \omega C E_0$$
$$= 10 \times 10^{-5} \times 10 = 10^{-3} \text{ A}$$
$$= \mathbf{1{\cdot}0\ mA}$$

Through a capacitor, current leads the applied voltage by a phase angle of 90°.

$\therefore \quad I = I_0 \sin(\omega t + \pi/2) = \mathbf{10^{-3} \sin(\omega t + \pi/2)}$

22. Here, $C = 3 \times 10^{-6}$ F, $E_v = 220$ V, $\nu = 50$ hz
$I_v = ?$, $E_0 = ?$

$$I_v = \frac{E_v}{X_C} = \frac{E_v}{1/\omega C} = E_v(\omega C) = E_v(2\pi\nu C)$$

$$I_v = 220 \times 2 \times \frac{22}{7} \times 50 \times 3 \times 10^{-6} = \mathbf{0{\cdot}207\ A}$$

$$E_0 = \sqrt{2}\ E_v = 1{\cdot}414 \times 220 = \mathbf{311{\cdot}1\ V}$$

23. Here, $C = 10\ \mu F = 10 \times 10^{-6}$ F
$E = 220 \sin 100\pi t = E_0 \sin \omega t$
$\therefore E_0 = 220$ V, $\omega = 2\pi\nu = 100\pi$, $\nu = 50$ hz.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$
$$= \frac{1}{2 \times 3{\cdot}14 \times 50 \times 10^{-5}} = 318{\cdot}5\ \Omega$$

$$I_0 = \frac{E_0}{X_C} = \frac{220}{318{\cdot}5} = 0{\cdot}691\text{A}$$

Reading of a.c. ammeter,

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{0{\cdot}691}{1{\cdot}414} = \mathbf{0{\cdot}489\ A}$$

Instantaneous current in the circuit,
$I = I_0 \sin(\omega t + \pi/2)$
$\boldsymbol{I = 0{\cdot}691 \sin(100\pi t + \pi/2)}$

24. Here, $I_v = 30$ mA $= 30 \times 10^{-3}$ A,
$C = 4\ \mu F = 4 \times 10^{-6}$ F
$\nu = 500$ Hz, $X_C = ?$ $V_C = ?$

$$X_C = \frac{1}{\omega C} = \frac{2}{2\pi\nu \times C}$$
$$= \frac{1}{2 \times 3{\cdot}14 \times 500 \times 4 \times 10^{-6}}$$
$$= \frac{1000}{12{\cdot}56} = \mathbf{79{\cdot}6\ \Omega}$$
$$V_C = I_v X_C = 30 \times 10^{-3} \times 79{\cdot}6 = \mathbf{2{\cdot}4\ V}$$

25. Here, $C = 30\ \mu F = 30 \times 10^{-6}$ F, $E_v = 220$ V,
$\nu = 50$ Hz, $X_C = ?$ $I_v = ?$ $I_0 = ?$ $Z = ?$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$
$$= \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}}$$
$$= \frac{10^3}{3 \times 3{\cdot}14} = \mathbf{106{\cdot}1\ \Omega}$$

$$I_v = \frac{E_v}{X_C} = \frac{220}{106{\cdot}1} = \mathbf{2{\cdot}07\ A}$$

$$I_0 = \sqrt{2}\ I_v = 1{\cdot}414 \times 2{\cdot}07 = \mathbf{2{\cdot}93\ A}$$
$$Z = X_C = \mathbf{106{\cdot}1\ ohm}$$

26. Here, $R = 12\ \Omega$, $L = \frac{0{\cdot}05}{\pi} H$, $E_v = 130$ V,
$\nu = 50$ Hz

$$X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times \frac{0{\cdot}05}{\pi} = 5\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 5^2} = 13 \text{ ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{130}{13} = \mathbf{10\ A}$$

In *RL* circuit, current lags behind the voltage by phase angle ϕ, where

$$\tan\phi = \frac{X_L}{R} = \frac{5}{12} = 0{\cdot}4167$$
$$\phi = \tan^{-1}(0{\cdot}4167) = \mathbf{22{\cdot}6°}$$

29. $E_v = 10$ V, $\nu = 50$ Hz, $I_v = 1$ A,

$$Z = \sqrt{R^2 + X_L^2} = \frac{E_v}{I_v} = \frac{10}{1} = 10\Omega \quad \text{...(i)}$$

$$\tan\phi = \frac{X_L}{R} = \tan\pi/3 = \sqrt{3}$$

$\therefore \quad X_L = \sqrt{3}R$

Put in (*i*)

$\sqrt{R^2 + 3R^2} = 10$, *i.e.*, $2R = 10$, $\boldsymbol{R = 5\ \Omega}$

$$X_L = \sqrt{3}R = \sqrt{3} \times 5 \text{ ohm}$$

As $X_L = 2\pi\nu L$

$$\therefore \quad L = \frac{X_L}{2\pi\nu} = \frac{\sqrt{3} \times 5}{2\pi \times 50}$$
$$= 2{\cdot}757 \times 10^{-2}\ \text{H} = \mathbf{27{\cdot}57\ mH}$$

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30. Here, $V = 80$ V, $P = 800$ W

Current required, $I = \frac{P}{V} = \frac{800}{80} = 10$ A ;

$$R = \frac{V^2}{P} = \frac{80 \times 80}{800} = 8\,\Omega$$

For a.c. supply, $E_v = 100$ V, $I_v = I = 10$ A,
$\nu = 50$ hz

$$\therefore \quad Z = \frac{E_v}{I_v} = \frac{100}{10} = 10 \text{ ohm}$$

Now $R^2 + X_L^2 = Z^2$

$$\therefore \; X_L = \sqrt{Z^2 - R^2} = \sqrt{10^2 - 8^2} = 6 \text{ ohm}$$

$X_L = \omega L = 2\pi\nu L$

$$\therefore \quad L = \frac{X_L}{2\pi\nu} = \frac{6}{2 \times 3{\cdot}14 \times 50} = \mathbf{0{\cdot}019\ H}$$

32. Here, $L = 0{\cdot}5$ H, $R = 100\ \Omega$, $E_v = 200$ V, $\nu = 50$ Hz

$$X_L = \omega L = 2\pi\nu L = 2\pi \times 50 \times 0{\cdot}5 = 50\pi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{100^2 + (50\pi)^2} = 186{\cdot}1\ \Omega$$

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\,E_v}{Z} = \frac{1{\cdot}414 \times 200}{186{\cdot}1} = \mathbf{1{\cdot}52\ A}$$

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi\nu L}{R} = \frac{2\pi \times 50 \times 0{\cdot}5}{100} = 1{\cdot}57$$

$$\phi = \tan^{-1}(1{\cdot}57) = 57{\cdot}5° = \frac{57{\cdot}5\pi}{180} \text{ radian}$$

$$= 0{\cdot}3194\,\pi$$

$$\text{Time lag, } t = \frac{\phi}{\omega} = \frac{\phi}{2\pi\nu} = \frac{0{\cdot}3194\pi}{2\pi \times 50}$$

$$= \mathbf{3{\cdot}194 \times 10^{-3}\ s}$$

33. Here, $R = 30\ \Omega$, $C = 0{\cdot}25\ \mu$F, $E_v = 100$ V,
$\omega = 10000$ rad/s, $I_v = ?$

$$X_C = \frac{1}{\omega C} = \frac{1}{10000 \times 0{\cdot}25 \times 10^{-6}} = 400\ \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + (400)^2} = 401{\cdot}1\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{100}{401{\cdot}1} = \mathbf{0{\cdot}25\ A}$$

$V_R = I_v X_R = 0{\cdot}25 \times 30 = \mathbf{7{\cdot}5\ V}$
$V_C = I_v X_C = 0{\cdot}25 \times 400 = \mathbf{100\ V}$

$V_R + V_C = 7{\cdot}5 + 100 = \mathbf{107{\cdot}5\ V}$, which is more than source voltage = 100 V. This is because V_R and V_C are not in the same phase.

37. Here, $C = 2\ \mu\text{F} = 2 \times 10^{-6}$ F, $R = 1\ \text{k}\Omega = 10^3\ \Omega$
$E_v = 12$ V, $\nu = 50$ hz

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3{\cdot}14 \times 50 \times 2 \times 10^{-6}}$$

$$= 1592{\cdot}36\ \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(10^3)^2 + (1592{\cdot}36)^2}$$

$$= 1{\cdot}88 \times 10^3\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{12}{1{\cdot}88 \times 10^3} = \mathbf{6{\cdot}4 \times 10^{-3}\ A}$$

Voltage across capacitor

$$= I_v X_C = 6{\cdot}4 \times 10^{-3} \times 1592{\cdot}36 = \mathbf{10{\cdot}2\ V}$$

$$\tan\phi = \frac{X_C}{R} = \frac{1592{\cdot}36}{10^3} = 1{\cdot}592$$

$$\phi = \mathbf{58°}$$

$$\text{Power supplied} = I_v^2 R = (6{\cdot}4 \times 10^{-3})^2 \times 10^3$$

$$= 40{\cdot}96 \times 10^{-3}\ \text{W} = \mathbf{0{\cdot}041\ W}$$

38. Here, $E_v = 220\sqrt{2}$ V, $I_v = \sqrt{2}$ A

As current is in phase with the voltage, element **X must be a resistor**

$$R = \frac{E_v}{I_v} = \frac{220\sqrt{2}}{\sqrt{2}} = 220 \text{ ohm}$$

Through Y, current leads the voltage by $\pi/2$,
$\therefore$ Y must be a condenser

$$X_C = \frac{E_v}{I_v} = \frac{220\sqrt{2}}{\sqrt{2}} = 220 \text{ ohm}$$

When series combination of R and C is connected,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{220^2 + 220^2} = 220\sqrt{2} \text{ ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{220\sqrt{2}}{220\sqrt{2}} = \mathbf{1\,A}$$

39. Here, $C = 50\ \mu\text{F} = 50 \times 10^{-6}$ F, $L = 0{\cdot}05$ H
$R = 48\ \Omega$, $E = 310 \sin 314\,t$; $E_0 = 310$ V,
$\omega = 314$ rad/s
$X_L = \omega L = 314 \times 0{\cdot}05 = 15{\cdot}7\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 50 \times 10^{-6}} = 63{\cdot}7\ \Omega$$

Reactance of the circuit $= X_C - X_L$

$= 63{\cdot}7 - 15{\cdot}7 = \mathbf{48\ \Omega}$

It is **capacitative in nature.**

If ϕ is angle by which **current in *RLC* circuit leads the applied e.m.f.,** then

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{48}{48} = 1\ ;\ \phi = 45^\circ$$

41. Here, $R = 40\ \Omega$, $L = 3\ \text{mH} = 3 \times 10^{-3}$ H,

$C = 2\ \mu\text{F} = 2 \times 10^{-6}$ F, $E_v = 110$ V, $\nu = 5000$ Hz

$X_L = \omega L = 2\pi\nu L = 2 \times 3{\cdot}14 \times 5000 \times 3 \times 10^{-3}$

$= 94{\cdot}2\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2 \times 3{\cdot}14 \times 5000 \times 2 \times 10^{-6}} = 15{\cdot}92\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{40^2 + (94{\cdot}2 - 15{\cdot}92)^2} = 87{\cdot}9\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{110}{87{\cdot}9} = \mathbf{1{\cdot}25\ A}$$

42. Here, $L = 10\ \text{mH} = 10 \times 10^{-3}\ \text{H} = 10^{-2}$ H

$$C = (400/\pi^2)\ \mu\text{F} = \frac{4 \times 10^{-4}}{\pi^2}\ \text{F},$$

$R = 55$ ohm

$E_v = 220$ V

(*i*) Average power absorbed by the circuit is maximum at resonance. The resonant frequency is

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-2} \times \frac{400}{\pi^2} \times 10^{-6}}}$$

$$\nu_0 = \frac{1}{2 \times 2 \times 10^{-3}} = \frac{1000}{4} = \mathbf{250\ Hz}$$

(*ii*) Current is maximum at resonance

$$I_0 = \sqrt{2}\, I_v = \sqrt{2} \cdot \frac{E_v}{R} = \sqrt{2} \times \frac{220}{55} = \mathbf{4\sqrt{2}\ A}$$

43. (*i*) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{5{\cdot}0 \times 80 \times 10^{-6}}} = \mathbf{50\ rad\ s^{-1}}$

(*ii*) At resonance, $Z = R = \mathbf{40\ ohm}$

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\, E_v}{Z} = \frac{1{\cdot}414 \times 230}{40} = \mathbf{8{\cdot}13\ A}$$

(*iii*) $I_v = \dfrac{E_v}{Z} = \dfrac{230}{40} = \dfrac{23}{4}\ \text{A}$

$V_R = I_v \times R = 23/4 \times 40 = \mathbf{230\ volt}$

$V_L = I_v X_L = I_v\,(\omega L) = 23/4 \times 50 \times 5{\cdot}0 = \mathbf{1437{\cdot}5\ V}$

$$V_C = I_v X_C = \frac{I_v}{\omega C} = \frac{23/4}{50 \times 80 \times 10^{-6}} = \mathbf{1437{\cdot}5\ V}$$

The three voltages, $\vec{V}_R$, $\vec{V}_L$ and $\vec{V}_C$ are phasors/ vectors. Out of these, $\vec{V}_L$ and $\vec{V}_C$ are in opposite phase. Therefore, they cancel out. The vector sum of the three voltages $= \vec{V}_R = \mathbf{230\ V}$, which is the source voltage.

44. Here, $L = 2{\cdot}0$ H, $C = 18 \times 10^{-6}$ F,

$R = 10\ k\Omega = 10^4\ \Omega$, $E_v = 20$ V

To maximize current,

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times \frac{22}{7}\sqrt{2 \times 18 \times 10^{-6}}}$$

$$\nu = \frac{7 \times 10^3}{2 \times 22 \times 6} = \mathbf{26{\cdot}5\ Hz}$$

At this frequency, $X_L = X_C$; $Z = R$

$$\therefore\ I_0 = \frac{E_0}{Z} = \frac{E_v\sqrt{2}}{R}$$

$$= \frac{20 \times 1{\cdot}414}{10^4} = 2{\cdot}828 \times 10^{-3}\ \text{A}$$

$$= \mathbf{2{\cdot}828\ mA}$$

45. Here, $C = 80\ \mu\text{F} = 80 \times 10^{-6}$ F, $L = 5{\cdot}0$ H,

$R = 40\ \Omega$, $E_v = 240$ V

(*i*) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{5{\cdot}0 \times 80 \times 10^{-6}}} = \dfrac{10^3}{20}$

$= \mathbf{50\ rad/s}$

(*ii*) At resonating frequency, $Z = R = 40\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{2{\cdot}0}{40} = \mathbf{6\,A}$$

(*iii*) $V_C = I_v \cdot X_C = 6 \times \dfrac{1}{\omega C} = 6 \times \dfrac{1}{50 \times 80 \times 10^{-6}}$

$= \mathbf{1500\ V}$

46. Here, $\nu_r = 800$ Hz, $\nu_1 = 745$ Hz, $\nu_2 = 855$ Hz

$$Q = \frac{\nu_r}{\nu_2 - \nu_1} = \frac{800}{855 - 745} = \frac{800}{110} = 7{\cdot}27$$

Band width $= \nu_2 - \nu_1 = 855 - 745 = \mathbf{110\ Hz}$

47. Here, $L = 40$ mH $= 40 \times 10^{-3}$ H, $\nu = 60$ hertz

For current to be in phase with voltage,

$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi\nu)^2 L} = \frac{1}{4\pi^2 \nu^2 L}$$

$$= \frac{1 \times 7 \times 7}{4 \times 22 \times 22 (60)^2 \times 40 \times 10^{-3}}$$

$$C = \frac{49 \times 1}{4 \times 484 \times 36 \times 4} = 175{\cdot}7 \times 10^{-6} \text{ F}$$

$$C = \mathbf{175{\cdot}7\ \mu F}$$

49. Here, $R = 400\ \Omega$, $X_C = 200$ ohm,

$E_v = 220$ V, $\nu = 50$ Hz

$I_v = 0{\cdot}49$ A, $V_R = ?$, $V_C = ?$ $L = ?$

$V_R = I_v R = 0{\cdot}49 \times 400 = \mathbf{196\ V}$

$V_C = I_v X_C = 0{\cdot}49 \times 200 = \mathbf{98\ V}$

When V and I are in same phase,

$$X_L = X_C \quad \text{or} \quad \omega L = X_C$$

$$L = \frac{X_C}{\omega} = \frac{X_C}{2\pi\nu} = \frac{200}{2 \times 3{\cdot}14 \times 50} = \mathbf{0{\cdot}64\ H}$$

50. Here, $R = 100\ \Omega$, $L = \frac{4}{\pi^2} H$, $E_v = 200$ V,

$\nu = 50$ Hz $C = ?$ $I_v = ?$

When current is in phase with the voltage,

$$X_C = X_L, \text{ i.e., } \frac{1}{\omega C} = \omega L, \quad C = \frac{1}{\omega^2 L}$$

$$C = \frac{1}{4\pi^2 \nu^2 L} = \frac{1}{4\pi^2 (50)^2 \times 4/\pi^2}$$

$$= \frac{1}{16 \times 2500}$$

$$= 0{\cdot}25 \times 10^{-4} \text{ F} = 25 \times 10^{-6} \text{ F}$$

$$= \mathbf{25\ \mu F}$$

$$I_v = \frac{E_v}{Z} = \frac{E_v}{R} = \frac{200}{100} = \mathbf{2\ A}$$

51. Here, $L = 3{\cdot}0$ H, $C = 27 \times 10^{-6}$ F, $R = 7{\cdot}4\ \Omega$

$\nu_0 = ?$ $Q = ?$

Resonant frequency, $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3{\cdot}14 \sqrt{3{\cdot}0 \times 27 \times 10^{-6}}}$$

$$= \frac{1}{6{\cdot}28 \times 9 \times 10^{-3}} = \frac{1000}{6{\cdot}28 \times 9} = \mathbf{17{\cdot}7\ Hz}$$

Q factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{7{\cdot}4} \sqrt{\frac{3{\cdot}0}{27 \times 10^{-6}}} = \frac{10^3}{7{\cdot}4 \times 3} = \mathbf{45{\cdot}05}$$

52. Here, $E = 100 \sin 314\ t$. In a capacitor. Current leads the applied e.m.f. by a phase angle of 90°.

$$\therefore I = I_0 \sin (314\ t + 90°) = \frac{E_0}{1/\omega C} \cos 314\ t$$

$$= 100 \times 314 \times 637 \times 10^{-6} \cos 314\ t$$

$$= 20 \cos 314\ t$$

Instantaneous power $= EI$

$$= 100 \sin 314\ t \times 20 \cos 314\ t$$

$$= 1000\ (2 \sin 314\ t \cos 314\ t)$$

$$= \mathbf{1000\ (\sin 628\ t)\ watt}$$

Max. energy stored $= \frac{1}{2} CE_0^2$

$$= \frac{1}{2} \times 637 \times 10^{-6} (100)^2 = \mathbf{3{\cdot}185\ J}$$

53. Here, $C = 54\ \mu F = 54 \times 10^{-6}$ F

$V = 25$ V, $L = 15$ mH $= 15 \times 10^{-3}$ H

Energy stored initially is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 54 \times 10^{-6} (25)^2$$

$$= \mathbf{1{\cdot}687 \times 10^{-2}\ J}$$

Frequency of oscillations produced.

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3{\cdot}14 \sqrt{15 \times 10^{-3} \times 54 \times 10^{-6}}}$$

$$= \frac{10^4}{6{\cdot}28 \times 9} = \mathbf{176{\cdot}8\ Hz}$$

54. Here, $R = 100\ \Omega$, $L = \frac{2}{5} H$

and $C = \frac{100}{\pi} \times 10^{-6} F$

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$E_v = 220$ V, $\nu = 50$ Hz, $Z = ?$, $I_0 = ?$, $\cos\phi = ?$

$$X_L = \omega L = 2\pi\nu L = 2\pi\times 50\times\frac{2}{\pi}H = 200\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2\pi\times 50\times(100/\pi)\times 10^{-6}}\ \text{ohm} = 10^2\ \text{ohm}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{100^2 + (200-100)^2} = \mathbf{141{\cdot}4\ \Omega}$$

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\,E_v}{Z} = \frac{1{\cdot}414\times 220}{141{\cdot}4} = \mathbf{2{\cdot}2\ A}$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{100}{141{\cdot}4} = \frac{1}{\sqrt{2}}$

At resonant frequency, $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

55. As the voltage leads the current, X must be **inductor (L)**

In RL circuit, $\cos\frac{\pi}{4} = \frac{R}{Z} = \frac{1}{\sqrt{2}}$ $\quad Z = \sqrt{2}R$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{2R^2 - R^2} = R$$

Again, as voltage lags behind the current, Y must be a **capacitor.**

In RC circuit, $\cos\frac{\pi}{4} = \frac{R}{Z} = \frac{1}{\sqrt{2}}$ $\quad Z = \sqrt{2}R$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{2R^2 - R^2} = R$$

Clearly, $X_L = X_C = R$

The circuit is purely resistive. Impedance is minimum and current is maximum. Hence power dissipated in the circuit is **maximum.**

57. With X, current and voltage are in same phase, therefore, element X must be pure **resistor R.**

With X and Y in series, voltage is ahead of current, therefore, Y must be **inductor.**

$$\tan\frac{\pi}{4} = \frac{R}{\sqrt{R^2 + X_L^2}} = 1 \quad \therefore\ X_L = 0$$

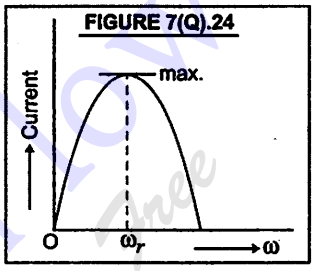
With X and Z in series, current is ahead of voltage, therefore Z must be a **capacitor.**

$$\tan\frac{\pi}{4} = \frac{R}{\sqrt{R^2 + X_C^2}} = 1 \quad \therefore\ X_C = 0$$

When all the three elements are connected in series across the same source.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \mathbf{R}$$

It is a series resonance circuit. Current is maximum at resonance frequency $\omega_r = \frac{1}{\sqrt{LC}}$ as shown in Fig. 7(Q).24.



58. Here, $L = 800$ mH $= 800\times 10^{-3}$ H

$C = 60\ \mu$F $= 60\times 10^{-6}$ F, $R = 15$ ohm

$E_v = 230$ V, $\nu = 50$ hz

$$X_L = \omega L = 2\pi\nu L$$

$$= 2\times\frac{22}{7}\times 50\times 800\times 10^{-3} = 251{\cdot}43\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2\times\frac{22}{7}\times 50\times 60\times 10^{-6}} = 53{\cdot}03\ \text{ohm}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{15^2 + (251{\cdot}43 - 53{\cdot}03)^2} = 198{\cdot}97\ \text{ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{230}{198{\cdot}97} = 1{\cdot}156\ \text{A}$$

Average power transferred to resistance

$$P_R = I_v^2 \times R = (1{\cdot}156)^2 \times 15 = \mathbf{20{\cdot}05\ watt}$$

Average power transferred to Inductor $P_L = 0$

Average power transferred to Capacitor $P_C = 0$

Total power absorbed $= E_v I_v \cos\phi$

$P = E_v I_v (R/Z)$

$$P = \frac{230\times 1{\cdot}156\times 15}{198{\cdot}97} = \mathbf{20{\cdot}05\ watt.}$$

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61. Here, $R = 100\ \Omega$, $E_v = 200$ V, $\omega = 300$ rad/s

When capacitance is removed,

$$\tan\phi = \frac{X_L}{R} = \tan 60° = \sqrt{3} \quad \therefore \quad X_L = \sqrt{3}\,R$$

When only the inductance is removed,

$$\tan\phi = \frac{X_c}{R} = \tan 60° = \sqrt{3} \quad \therefore \quad X_C = \sqrt{3}\,R$$

As $X_L = X_C$, the LCR circuit is in resonance.

$\therefore \quad Z = R = 100\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{200}{100} = \mathbf{2\,A}$$

At resonance, $P = E_v I_v \cos 0°$

$= 200 \times 2 \times 1 = \mathbf{400\ W}$

62. Here, $E_v = 200$ V, $L = 5$ H, $C = 80 \times 10^{-6}$ F

$R = 40\ \Omega$, To get maximum current,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{1000}{20} = \mathbf{50\ rad/s}$$

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\,E_v}{R} = \frac{1{\cdot}414 \times 200}{40} = \mathbf{7{\cdot}07\ A}$$

$$P = \frac{E_v^2}{R} = \frac{200 \times 200}{40} = \mathbf{1000\ W}$$

63. Here, $L = 200$ mH $= 0{\cdot}2$ H, $C = 500 \times 10^{-6}$ F,

$R = 10\ \Omega$,

$E_v = 100$ V

Power factor of the circuit is unity, when

$$X_L = X_C \quad \text{or} \quad \omega L = \frac{1}{\omega C} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0{\cdot}2 \times 500 \times 10^{-6}}} = \frac{\mathbf{50}}{\pi}\ \mathbf{Hz}$$

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\,E_v}{R} = \frac{1{\cdot}414 \times 100}{10} = \mathbf{14{\cdot}14\ A}$$

$$Q \text{ factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0{\cdot}2}{5 \times 10^{-4}}} = \mathbf{2}$$

64. Here, $n_p = 200$, $n_s = 1000$,

$P_i = 10$ kW $= 10000$ W, $E_p = 200$ V

$E_s = ?$, $I_p = ?$

As $\quad \dfrac{E_s}{E_p} = \dfrac{n_s}{n_p} = \dfrac{1000}{200} = 5,$

$E_s = 5\,E_p = 5 \times 200 = \mathbf{1000\ V}$

Also, $E_p I_p = P_i = 1000$,

$$I_p = \frac{10000}{E_p} = \frac{10000}{200} = \mathbf{50\,A}$$

65. Here, $x_p = 200$, $n_s = 1000$; $E_s = 1000$ V,

$P = 9$ kW $= 9000$ W

$E_p = ?$, $H = ?$, $R_p = 0{\cdot}2\ \Omega$, $\eta = 90\%$

As $\quad \dfrac{E_s}{E_p} = \dfrac{n_s}{n_p} = \dfrac{1000}{200} = 5,$

$$\therefore \quad E_p = \frac{E_s}{5} = \frac{1000}{5} = \mathbf{200\ V}$$

From $\quad \eta = \dfrac{P_o}{P_i}, \quad P_i = \dfrac{P_o}{n} = 10$ kW

$$\therefore \quad I_p = \frac{1000}{200} = \mathbf{50\ A}$$

Energy loss in primary

$= I_p^2 \times R_p = (50)^2 \times 0{\cdot}2$

$= \mathbf{500\ W}$

67. Here, $E_s = 24$ V, $E_p = 240$ V, $P_0 = 24$ W

$$I_s = \frac{P_0}{E_s} = \frac{24}{24} = 1\,\text{amp} \ ; \ I_p = ?$$

As $\dfrac{I_p}{I_s} = \dfrac{E_s}{E_p} \quad \therefore \quad I_p = \dfrac{E_s}{E_p} \times I_s = \dfrac{24}{240} \times 1 = \mathbf{0{\cdot}1\,A}$

68. Here, $I_p = ?$, $E_p = 200$ V, $E_s = 20$ V, $R_s = 20$ ohm,

$$I_s = \frac{E_s}{R_s} = \frac{20}{20} = 1\,\text{A}$$

As $\quad \eta = \dfrac{E_s I_s}{E_p I_p}$

$$\therefore \quad \frac{80}{100} = \frac{20 \times 1}{200\,I_p} \ ; \ I_p = \frac{1}{8} = \mathbf{0{\cdot}125\,A}$$

70. Here, $n_p = 100$, $K = 100$, $E_p = 220$ V,

$P_i = 1100$ W

(*i*) From $\quad K = \dfrac{n_s}{n_p}$

$n_s = K \times n_p = 100 \times 100 = \mathbf{10000}$

(*ii*) From $P_i = E_p I_p$

$$I_p = \frac{P_i}{E_p} = \frac{1100}{220} = \mathbf{5\,A}$$

(iii) $\frac{E_s}{E_p} = \frac{n_s}{n_p} = K$

$E_s = KE_p = 100 \times 220 = 22000$ V

(iv) $\frac{I_s}{I_p} = \frac{n_p}{n_s} = \frac{1}{K}$

$I_s = \frac{I_p}{K} = \frac{5}{100} = 0{\cdot}05$ A

(v) Power in secondary = $P_0 = P_i = 1100$ W

71. Here, $R = 15\ \Omega$, $L = 10$ H, $E = 90$ V,

$I_0 = E/R = 90/15 = 6$ A, $I = ?$, $t = 2$ s

$\tau = L/R = 10/15 = \frac{2}{3}$ s

$I = I_0\left(1 - e^{\frac{-R}{L}t}\right)$

$I = 6\left(1 - e^{-2\times\frac{3}{2}}\right) = 6\,(1 - 0{\cdot}05) = 5{\cdot}7$ A

Energy stored in mag. field $= \frac{1}{2}LI^2$

$= \frac{1}{2}\times 10(5{\cdot}7)^2 = 162{\cdot}45$ joule

72. Energy dissipated $= \int_0^t \frac{E^2}{R}dt$

where $t = 1$ ms $= 10^{-3}$ s

$= \int_0^t \frac{[(12)\sin 250\pi t]^2\,dt}{100}$

$= \frac{144}{100}\int_0^t \frac{(1 - \cos 2\times 250\pi t)\,dt}{2}$

$= \frac{144}{200}\left[t - \frac{\sin 500\pi t}{500\pi}\right]_0^t$

As $t = 1$ ms $= 10^{-3}$ s,

$\therefore$ Energy dissipated $= \frac{144}{200}\left[10^{-3} - \frac{1}{500\pi}\right]$

$= 2{\cdot}61 \times 10^{-4}$ J

73. Here, $I_v = 1{\cdot}5$ mA $= 1{\cdot}5 \times 10^{-3}$ A,

$\omega = 300$ rad/s, $R = 10$ k$\Omega = 10^4\ \Omega$,

$C = 0{\cdot}5\ \mu$F $= 0{\cdot}5 \times 10^{-6}$ F

$X_C = \frac{1}{\omega C} = \frac{1}{300\times 0{\cdot}5\times 10^{-6}} = \frac{10^6}{150}$

$= \frac{20}{3}\times 10^3\ \Omega$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^8 + \frac{4}{9}\times 10^8}$

$= 10^4\sqrt{\frac{13}{9}} = 1{\cdot}2 \times 10^4\ \Omega$

r m s voltage across the capacitor

$= I_v X_C = 1{\cdot}5\times 10^{-3}\times\frac{20}{3}\times 10^3 = 10$ volt

74. Time constant of circuit is $\tau = \frac{L}{R} = \frac{1{\cdot}0}{100} = 10^{-2}$ s

As $I = I_0\,(1 - e^{-t/\tau}) = \frac{E_0}{R}(1 - e^{-t/\tau})$

$\therefore\ I = \frac{12}{100}(1 - e^{-10^{-2}/10^{-2}})$

$= 0{\cdot}12\left(1 - \frac{1}{e}\right) = 0{\cdot}0758$ A

Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2}\times 1{\cdot}0\,(0{\cdot}0758)^2$

$= 2{\cdot}87 \times 10^{-3}$ J

75. Here, $L = 0{\cdot}5$ H, $C = 15 \times 10^{-6}$ F, $R = 100$ ohm

$E_v = 200$ V, $v = 50$ Hz, $I_v = ?$ Power factor ?

$X_L = \omega L = 2\pi\, v\, L = 2\pi \times 50 \times 0{\cdot}5 = 157\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C} = \frac{1}{2\pi\times 50\times 15\times 10^{-6}}$

$= 2{\cdot}12 \times 10^2$ ohm $= 212\ \Omega$

$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{100^2 + (212 - 157)^2}$

$= \sqrt{100^2 + 55^2} = 114{\cdot}127\ \Omega$

$I_v = \frac{E_v}{Z} = \frac{200}{114{\cdot}127} = 1{\cdot}752$ A

Power factor, $\cos\phi = \frac{R}{Z} = \frac{100}{114{\cdot}127} = 0{\cdot}876$

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76. Here, $L = ?\ C = 5\ \mu F = 5 \times 10^{-6}$ F,
$R = 10$ ohm
$\nu = 50$ Hz
Power factor is unity at resonance, where

$$\omega L = \frac{1}{\omega C}$$

or $$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 \nu^2 C}$$

$$= \frac{1}{4 \times 3{\cdot}14 \times 3{\cdot}14 \times (50)^2 \times 5 \times 10^{-6}}$$

$$L = \mathbf{2{\cdot}028\ H}$$

77. $\eta = 80\%,\ E_p = 100$ V, $E_p I_p = 4$ kW $= 4000$ W
$E_s = 240$ V, $I_p = ?,\ I_s = ?$

$$I_p = \frac{E_p I_p}{E_p} = \frac{4 \times 1000}{100} = \mathbf{40\ A}$$

To calculate I_s, use $\eta = \dfrac{E_s I_s}{E_p I_p}$

78. Here, $E_p = 2000$ V, $E_s = 200$ V,
$N_p = 5000,\ \eta = 80\%$
$P_o = 8$ kW, $P_i = ?\ N_s = ?$

From $\eta = \dfrac{P_o}{P_i};\quad P_i = \dfrac{P_o}{\eta} = \dfrac{8}{80/100} = \mathbf{10\ kW}$

From $\dfrac{N_s}{N_p} = \dfrac{E_s}{E_p}$

$$\therefore \quad N_s = \frac{E_s}{E_p} \times N_p = \frac{200}{2000} \times 5000 = \mathbf{500}$$

79. Here, $P_0 = 10$ kW $= 10000$ W, $N_p = 20$,
$N_s = 100$
As $E_1 = 600 \sin 314\ t$
$\therefore\ E_0 = 600$ V,
$\omega = 314$ rad/s.
Flux linked with each turn of primary
$\phi = BA \cos \omega t = \phi_0 \cos \omega t$

As $E_1 = -N_p \dfrac{d\phi}{dt} = -N_p \dfrac{d}{dt}(\phi_0 \cos \omega t)$

$= \omega N_p \phi_0 \sin \omega t$

Peak value of $E_1 = E_0 = \omega N_p \phi_0 \times 1$

$$\therefore \quad \phi_0 = \frac{E_0}{\omega N_p} = \frac{600}{314 \times 20} = \mathbf{0{\cdot}0955\ Wb}$$

As $\dfrac{E_s}{E_p} = \dfrac{N_s}{N_p}$ $\therefore\ E_s = \dfrac{N_s}{N_p} E_p = \dfrac{100}{20} \times 600$

$= \mathbf{3000\ V}$

80. The *a.c.* source must be connected to primary of transformer and load should be connected to secondary of the transformer.

As $I_s^2 R_s = I_p^2 R_p$

$$\frac{I_s}{I_p} = \sqrt{\frac{R_p}{R_s}} = \sqrt{\frac{9000}{10}} = \mathbf{30}$$

Also, $\dfrac{I_s}{I_p} = \dfrac{n_p}{n_s} = \mathbf{30}$ $\therefore$ $\dfrac{n_s}{n_p} = \mathbf{\dfrac{1}{30}}$

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NCERT EXERCISES WITH SOLUTIONS

Q. 1. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the rms value of current in the circuit ?

(b) What is the net power consumed over a full cycle ?

Sol. Here, $R = 100\ \Omega$, $E_v = 220$ V, $\nu = 50$ Hz

(*a*) $I_v = ?$, $I_v = \dfrac{E_v}{R} = \dfrac{220}{100} = \mathbf{2{\cdot}2\ A}$

(*b*) Net power consumed over a full cycle, $P = E_v\, I_v = 220 \times 2{\cdot}2 = \mathbf{484\ W}$

Q. 2. (a) The peak voltage of an a.c. supply is 300 V. What is its r.m.s. voltage ?

(b) The r.m.s. value of current in an ac circuit is 10 A. What is the peak current ?

Sol. (*a*) Here, $E_0 = 300$ V, $E_v = ?$

As $E_v = \dfrac{E_0}{\sqrt{2}}$ $\quad\therefore\quad$ $E_v = \dfrac{300}{\sqrt{2}} = \dfrac{300\sqrt{2}}{\sqrt{2}\sqrt{2}} = \mathbf{212{\cdot}1\ volt}$

(*b*) Here, $I_v = 10$ A, $I_0 = ?$, $\quad I_0 = \sqrt{2}\ I_v = 1{\cdot}414 \times 10 = \mathbf{14{\cdot}14\ A}$

Q. 3. A 44 m H inductor is connected to 220 V, 50 Hz a.c. supply. Determine the r.m.s. value of current in the circuit.

Sol. Here, $L = 44$ mH $= 44 \times 10^{-3}$ H, $E_v = 220$ V, $\nu = 50$ Hz, $I_v = ?$

$$X_L = \omega L = 2\pi\nu L = 2 \times \frac{22}{7} \times 50 \times 44 \times 10^{-3} = 13{\cdot}83 \text{ ohm}$$

$$I_v = \frac{E_v}{X_L} = \frac{220}{13{\cdot}83} = \mathbf{15{\cdot}9\ A}$$

Q. 4. A 60 μ F capacitor is connected to a 110 V, 60 Hz a.c. supply. Determine the r.m.s. value of current in the circuit.

Sol. Here, $C = 60\ \mu$F $= 60 \times 10^{-6}$ F, $E_v = 110$ V, $\nu = 60$ hz, $I_v = ?$

$$I_v = \frac{E_v}{X_C} = \frac{E_v}{1/2\pi\nu C} = (2\pi\nu C)\,E_v = 2 \times 3{\cdot}14 \times 60 \times (60 \times 10^{-6}) \times 110 = \mathbf{2{\cdot}49\ A}$$

Q. 5. In Questions 3 and 4, what is the net power absorbed by each circuit over a complete cycle ? Explain your answer.

Sol. In Q. 3, $P = E_v\, I_v \cos\phi$

In a pure inductor, phase difference between alternating voltage and current is $\phi = 90°$

$\therefore$ $P = E_v\, I_v \cos 90° = \mathbf{Zero}$

In Q. 4, $P = E_v\, I_v \cos\phi$

In a pure capacitor, phase difference between alternating voltage and current is $\phi = 90°$.

$\therefore$ $P = E_v\, I_v \cos 90° = \mathbf{Zero}$

Infact, in both the cases, power spent in one half cycle is retrieved in the other half cycle.

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Q. 6. A charged 30 μ F capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit ?

Sol. Here, $C = 30\ \mu F = 30 \times 10^{-6}$ F, $L = 27$ mH $= 27 \times 10^{-3}$ H, $\omega_r = ?$

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27\times10^{-3}\times30\times10^{-6}}} = \frac{10^4}{9} = 1{\cdot}1\times10^3 \text{ rad/s}$$

Q. 7. A series LCR circuit with $R = 20\ \Omega$, $L = 1{\cdot}5$ H and $C = 35\ \mu$ F is connected to a variable frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferrred to the circuit in one complete cycle ?

Sol. When the frequency of the supply equals the natural frequency of the circuit, resonance occurs.

$\therefore \quad Z = R = 20$ ohm.

$$I_v = \frac{E_v}{Z} = \frac{200}{20} = 10\text{ A}$$

Average power transferred/cycle $P = E_v\ I_v \cos 0° = 200 \times 10 \times 1 = 2000$ watt

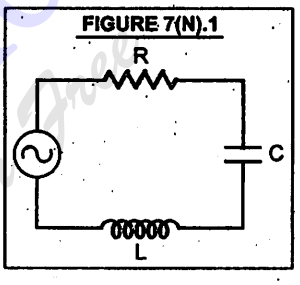
Q. 8. A series LCR circuit connected to a variable frequency 230 V source has $L = 5{\cdot}0$ H, $C = 80\ \mu$ F, $R = 40\ \Omega$, Fig. 7(N).1.

(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and amplitude of current at the resonating frequency.

(c) Determine the r.m.s. potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

FIGURE 7(N).1



Sol. Refer to Example 29, Page 7/51.

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NCERT EXEMPLAR PROBLEMS

WITH ANSWERS, HINTS AND SOLUTIONS

MULTIPLE CHOICE QUESTIONS-I

1. If the rms current in a 50 Hz ac circuit is 5 A, the value of the current 1/300 seconds after its value becomes zero is

(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A

(c) 5/6 A (d) $5/\sqrt{2}$ A

2. An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to

(a) zero (b) X_g

(c) $-X_g$ (d) R_g

3. When a voltage measuring device is connected to *AC* mains, the meter shows the steady input voltage of 220 V. This means

(a) input voltage can not be *AC* voltage, but a *DC* voltage

(b) maximum input voltage is 220 V

(c) the meter reads not v but $< v^2 >$ and is calibrated to read $\sqrt{< v^2 >}$

(d) the pointer of the meter is stuck by some mechanical defect

4. To reduce the reasonant frequency in an LCR series circuit with a generator

(a) the generator frequency should be reduced

(b) another capacitor should be added in parallel to the first

(c) the iron core of the inductor should be removed

(d) dielectric in the capacitor should be removed

5. Which of the following combinations should be selected for better tuning of an *LCR* circuit used for communication ?

(a) $R = 20\ \Omega$, $L = 1{\cdot}5$ H, $C = 35\ \mu$F

(b) $R = 25\ \Omega$, $L = 2{\cdot}5$ H, $C = 45\ \mu$F

(c) $R = 15\ \Omega$, $L = 3{\cdot}5$ H, $C = 30\ \mu$F

(d) $R = 25\ \Omega$, $L = 1{\cdot}5$ H, $C = 45\ \mu$F

6. An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6 V (rms) a.c. source. The power dissipated in the circuit is

(a) 8 W (b) 12 W

(c) 14·4 W (d) 18 W

7. The output of a step-down transformer is measured to be 24 V when connected to a 12 watt light blub. The value of the peak current is

(a) $1/\sqrt{2}$ A (b) $\sqrt{2}$ A

(c) 2 A (d) $2\sqrt{2}$ A

MULTIPLE CHOICE QUESTIONS-II

8. As the frequency of an ac circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit ?

(a) Inductor and capacitor

(b) Resistor and inductor

(c) Resistor and capacitor

(d) Resistor, inductor and capacitor.

9. In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit ?

(a) Only resistor

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(*b*) Resistor and an inductor

(*c*) Resistor and a capacitor

(*d*) Only a capacitor.

10. Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct ?

(*a*) For a given power level, there is a lower current

(*b*) Lower current implies less power loss

(*c*) Transmission lines can be made thinner

(*d*) It is easy to reduce the voltage at the receiving end using step-down transformers

11. For an *LCR* circuit, the power transferred from the driving source to the driven oscillator is $P = I^2Z \cos\phi$.

(*a*) Here, the power factor $\cos\phi \geq 0$, $P \geq 0$

(*b*) The driving force can give no energy to the oscillator ($P = 0$) in some cases

(*c*) The driving force can not syphon out ($P < 0$) the energy out of oscillator

(*d*) The driving force can take away energy out of the oscillator

12. When an *AC* voltage of 220 V is applied to the capacitor *C*

(*a*) the maximum voltage between plates is 220 V

(*b*) the current is in phase with the applied voltage

(*c*) the charge on the plates is in phase with the applied voltage

(*d*) power delivered to the capacitor is zero

13. The line that draws power supply to your house from street has

(*a*) zero average current

(*b*) 220 V average voltage

(*c*) voltage and current out of phase by 90°

(*d*) voltage and current possibly differing in phase ϕ such that $|\phi| < \frac{\pi}{2}$

ANSWERS

1. (*b*)	**2.** (*c*)	**3.** (*c*)	**4.** (*b*)	**5.** (*c*)	**6.** (*c*)	**7.** (*a*)
8. (*a*, *d*)	**9.** (*c*, *d*)	**10.** (*a*, *b*, *c*, *d*)	**11.** (*a*, *b*, *c*)	**12.** (*c*, *d*)	**13.** (*a*, *d*)	

HINTS FOR DIFFICULT MULTIPLE CHOICE QUESTIONS

Multiple Choice Questions – I

1. Here, $\nu = 50$ Hz, $I_v = 5$ A, $I = ?$, $t = \frac{1}{300}$ s, $I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 5\,\text{A}$

From $I = I_0 \sin \omega t = 5\sqrt{2} \sin 100\pi \times \frac{1}{300} = 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = \mathbf{5\sqrt{3/2}\,A}$

2. For delivering maximum power from the generator to the passive load, total reactance must vanish, *i.e.*, $X_L + X_g = 0$ or $X_L = -X_g$. **Choice (*c*) is correct.**

3. The voltmeter connected to A.C. mains is calibrated to read root mean square value or virtual value of a.c. voltage, *i.e.*, $\sqrt{< v^2 >}$. **Choice (*c*) is correct.**

4. Resonant frequency in an *LCR* series circuit is $\nu_r = \frac{1}{2\pi\sqrt{LC}}$

To reduce ν_r ; c can be increased, by adding another capacitor in parallel to the first. **Choice (*b*) is correct.**

5. The *LCR* circuit used for communication should possess high quality factor (Q factor) of resonance, which is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

To make Q high ; R should be low ; L should be high and C should be low. Therefore, **Choice (*c*) is the best suited.**

6. Here, $X_L = 1\ \Omega, R = 2\ \Omega, E_v = 6\ \text{V}, P = ?, \quad Z = \sqrt{X_L^2 + R^2} = \sqrt{1^2 + 2^2} = \sqrt{5}\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{6}{\sqrt{5}}\text{A}$$

$$P = E_v I_v \cos\theta = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \mathbf{14{\cdot}4\ W.}$$

7. Here, $E_s = 24\ \text{V}, P_s = 12\ \text{W}, I_0 = ?$

$$I_s = \frac{P_s}{E_s} = \frac{12}{24} = \frac{1}{2}\text{A} \qquad I_0 = \sqrt{2}\, I_s = \sqrt{2} \times \frac{1}{2} = 1/\sqrt{2}\ \text{A}.$$

Multiple Choice Questions – II

8. The impedance of an *LCR* circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}$$

The variation of Z with ν is as shown in Fig. 7(N).2.

As ν increases, Z decreases current increases. At $\nu = \nu_r$, Z is minimum, current is maximum. Beyond $\nu = \nu_r$, Z increases and current decreases. Hence, the circuit elements likely to comprise the circuit are L and C or R, L and C. **Choices (*a*) and (*d*) are correct.**

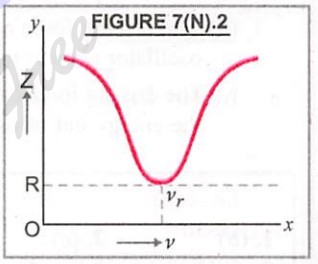

FIGURE 7(N).2

9. The reactance of a capacitor is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

On increasing the frequency ν; X_C decreases. Therefore, current increases. As R does not vary with frequency, therefore, likely elements constituting the circuit may be capacitor only or capacitor and a resistor. **Choices (*c*) and (*d*) are correct.**

10. Electrical energy is transmitted over large distances at high alternating voltages. As $P = E_v I_v$, therefore, for a given power level, current (I_v) is lower. Power loss $= I_v^2 R$ becomes low, even if transmission lines are thinner and R is a little high. At the receiving end, high voltages can be easily reduced using step down transformers. **Choices (*a*), (*b*), (*c*), (*d*), all are correct.**

11. Power transferred from the driving source to the driven oscillator is $P = I^2 Z \cos\phi$. Therefore, power factor $\cos\phi \geq 0$ and $P \geq 0$. For wattless component the driving force shall give no energy to the oscillator ($P = 0$, when $\phi = 90°$). Further, the driving force cannot syphon out energy out of the oscillator, *i.e.*, P cannot be negative. **Choices (*a*), (*b*), (*c*) are correct.**

12. When an a.c. voltage of 220 V is applied to a capacitor C, the charge on the plates is in phase with the applied voltage. As current developed leads the applied voltage by a phase angle of 90°, therefore, power delivered to the capacitor per cycle is $P = E_v I_v \cos 90° =$ Zero. **Choices (*c*) and (*d*) are correct.**

13. The line that draws power supply to our house from street supplies **alternating current,** whose average value/mean value over a cycle is zero. As the line has some resistance ($R \neq 0$), therefore, voltage and current differ in phase ϕ such that $|\phi| < \pi/2$. **Choices (*a*) and (*d*) are correct.**

VERY SHORT ANSWER QUESTIONS

14. If a *LC* circuit is considered analogous to a harmonically osciallting spring block system, which energy of the *LC* circuit would be analogous to potential energy and which one analogous to kinetic energy ?

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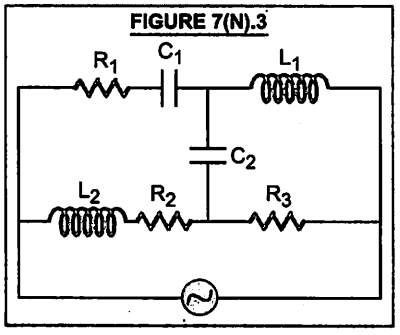
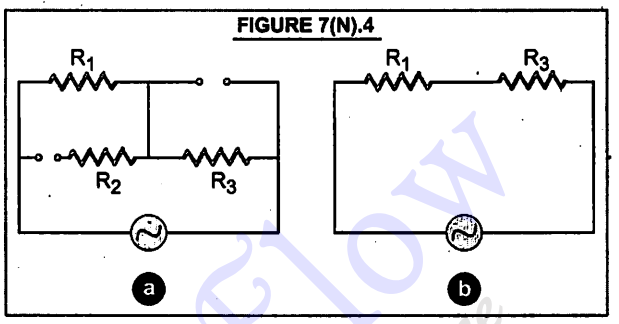
Ans. In an *LC* circuit, magnetic energy $\left(\frac{1}{2}LI^2\right)$ stored in inductor is analogous to kinetic energy and electric energy stored in condenser $\left(\frac{1}{2}CV^2\right)$ is analogous to potential energy.

15. Draw the effective equivalent circuit of the circuit shown in Fig. 7(N).3, at very high frequencies and find the effective impedance.





Ans. At very high frequencies, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} \rightarrow$ Zero

$$X_L = \omega L = 2\pi\nu L \rightarrow \text{extremely high}$$

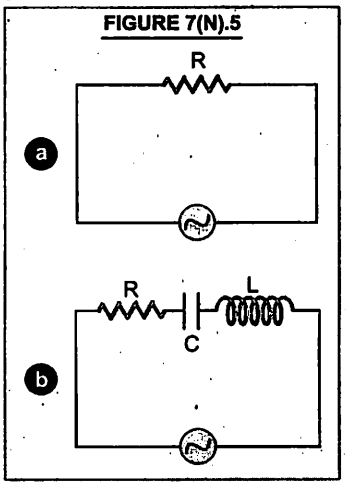
i.e., capacitor is short circuited and inductor blocks the current (serving as open key). The effective equivalent circuits are shown in Fig. 7(N).4(*a*) and (*b*)

Effective impedance, $Z = R_1 + R_3$.

16. Study the circuits (a) and (b) shown in Fig. 7(N).5 and answer the following questions.

(a) Under which conditions would the rms currents in the two circuits be the same ?

(b) Can the rms current in circuit (b) be larger than that in (a) ?



Ans. In Fig. 7(N).5(*a*), $I_v = \frac{E_v}{R}$

In Fig. 7(N).5(*b*), $I_v' = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + (X_L - X_C)^2}}$

Clearly, $I_v' = I_v$ only when $X_L = X_C$, *i.e.*, at resonance only.

(*b*) As $R \leq Z$, so $I_a \geq I_b$

Therefore, rms current in circuit (*b*) cannot be larger than that in circuit (*a*).

17. Can the instantaneous power output of an ac source ever be negative ? Can the average power output be negative ?

Ans. In an *RLC* circuit, if applied e.m.f., $E = E_0 \sin \omega t$, then current developed, $I = I_0 \sin(\omega t \pm \phi)$

Instantaneous power output of the a.c. source

$$\frac{dW}{dt} = EI = E_0 \sin \omega t \, I_0 \sin(\omega t \pm \phi)$$

It may be negative, because power might be absorbed at some instant. However, average power output over a cycle cannot be negative. It is usually positive or may be zero such as in *LC* circuit.

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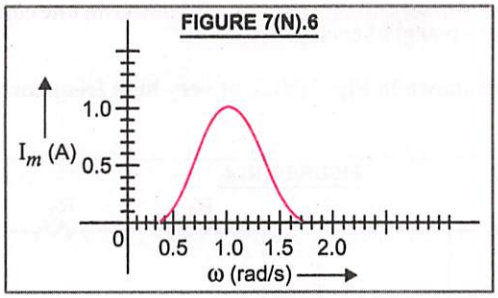
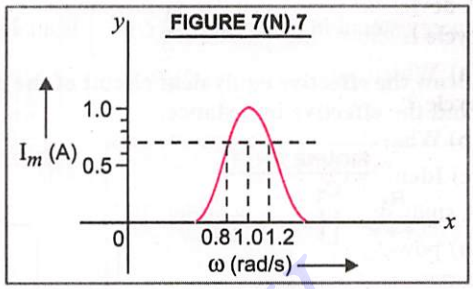
18. In series LCR circuit, the plot of I_{max} vs ω is shown in Fig. 7(N).6. Find the bandwidth and mark in the figure.



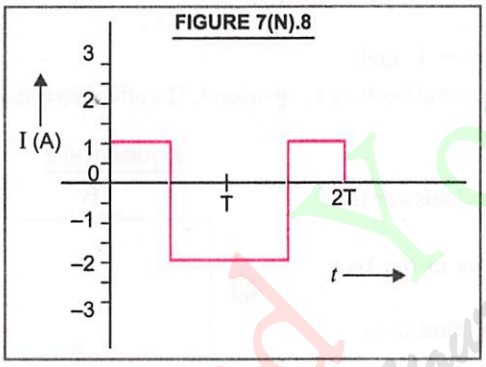


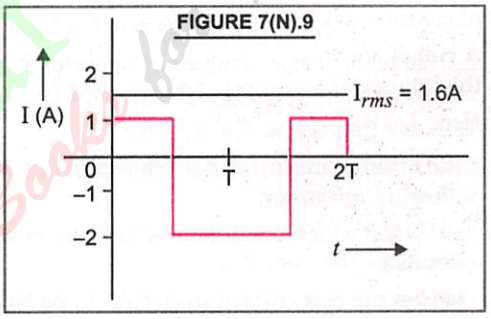
Ans. As is known from theory, bandwidth = frequency range at which

$$I = \frac{1}{\sqrt{2}} I_{max} = 0{\cdot}7\ I_{max}.$$

In Fig. 7(N).7, band width, $\Delta\omega = 1{\cdot}2 - 0{\cdot}8 = 0{\cdot}4$ rad/s.

19. The alternating current in a circuit is described by the graph shown in Fig. 7(N).8. Show rms current in this graph.





Ans. From Fig. 7(N).8, we find that

from $t = 0$ to $t = T/2$, $I = 1$ A and from $t = T/2$ to $t = T$, $I = -2$ A

$\therefore$ Mean square value of current over one full cycle $= \dfrac{1^2 + (-2)^2}{2} = 2{\cdot}5$

rms current, $I_{rms} = \sqrt{2{\cdot}5} = 1{\cdot}6$ A.

This is shown in Fig. 7(N).9.

20. How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an *LCR* series circuit, change as the supply frequency is gradually increased from very low to very high values ?

Ans. The supply voltage leads the current in *LCR* series circuit by ∠ϕ, where

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - 1/2\pi\nu C}{R}$$

When ν is low, tan ϕ is negative ; ϕ is negative.

When $\nu = \nu_r$; $2\pi\nu L = 1/2\pi\nu C$, tan ϕ = 0 ; ϕ = 0°.

When $\nu > \nu_r$; tan ϕ is positive ; ϕ is positive.

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SHORT ANSWER QUESTIONS

21. A device '*X*' is connected to an a.c. source. The variation of voltage, current and power in one complete cycle is shown in Fig. 7(N).10.

(a) Which curve shows power consumption over a full cycle ?

(b) What is the average power consumption over a cycle ?

(c) Identify the device '*X* '.

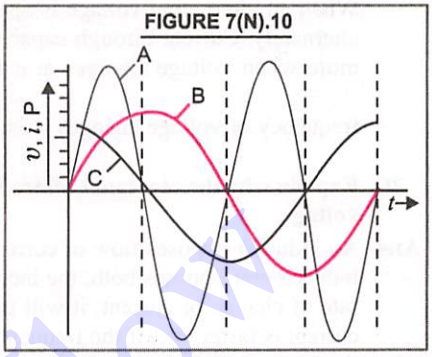

FIGURE 7(N).10

Ans. A study of given curves in Fig. 7(N).10 reveals that

(*a*) power consumption over a full cycle is represented by curve *A*.

(*b*) Average power consumption over a full cycle = Zero ; as is clear from curve *A*.

(*c*) The device *X* may be an inductor (*L*) or a capacitor (*C*) or their combination (*LC*).

22. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current ?

Ans. In direct current, 1 ampere = 1 coulomb/sec. However, in alternating current, direction of current is reversed every half cycle. Therefore, a.c. ampere is defined in terms of some property, which is independent of direction of current. As is known, a.c. ampere is defined in terms of Joule's heating, which is proportional to I^2.

23. A coil of 0·01 henry inductance and 1 ohm resistance is connected to 200 volt, 50 Hz ac supply. Find the impedance of the circuit and time lag between max. alternating voltage and current.

Ans. Here, $L = 0{\cdot}01$ H, $R = 1\ \Omega$, $E_v = 200$ V, $\nu = 50$ Hz ; $Z = ?$, $\Delta t = ?$

$$X_L = \omega L = 2\pi\nu L = 2 \times 3{\cdot}14 \times 50 \times 0{\cdot}01 = 3{\cdot}14\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{1^2 + (3{\cdot}14)^2} = \sqrt{10{\cdot}86} = 3{\cdot}3\,\text{ohm}$$

$$\tan\phi = \frac{\omega L}{R} = 3{\cdot}14$$

$$\phi = \tan^{-1}(3{\cdot}14) = 72° = 72 \times \frac{\pi}{180}\,\text{rad}$$

$$\text{Time lag, } \Delta t = \frac{\phi}{\omega} = \frac{72 \times \pi}{180 \times 2\pi \times 50} = \frac{1}{250}\,\text{sec}$$

24. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0·54 A flows in the load, what is the current in the primary coil ? Comment on the type of transformer being used.

Ans. Here, $P_s = 60$ W, $I_s = 0{\cdot}54$ A, $I_p = ?$

As line voltage is 220 V, therefore, $E_p = 220$ V

$$\text{Now, } E_s = \frac{P_s}{I_s} = \frac{60}{0{\cdot}54} = 110\,\text{V (approx.)}$$

The transformer is a **step down transformer**

$$I_p = \frac{1}{2} I_s = \frac{1}{2} \times 0{\cdot}54\,\text{A} = \mathbf{0{\cdot}27\,A}$$

25. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency.

Ans. A capacitor does not allow the flow of direct current through it as resistance across the gap between the two plates is infinite.

When an alternating voltage is applied across the capacitor plates, the plates are charged and discharged alternately. Current through capacitor is as a result of this changing voltage (or charges). Current will be more when voltage changes at a faster rate, *i.e.*, reactance offered by condenser will be smaller when frequency of voltage changes is increased. Mathematically, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$

26. Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.

Ans. An inductor opposes flow of current through it by developing a back emf in itself. As per Lenz's law, the induced emf opposes both, the increase and decrease of current. As the induced emf is proportional to the rate of change of current, it will provide greater reactance to the flow of current if the rate of change of current is faster, *i.e.*, if the frequency is higher. Hence, the reactance of inductor is directly proportional to the frequency of a.c. supplied. Mathematically, $X_L = \omega L = 2\pi \nu L$.

LONG ANSWER QUESTIONS

27. An electrical device draws 2 kW power from *AC* mains [voltage 223 V (rms) = $\sqrt{50,000}$ V]. The current differs (lags) in phase by $\phi \left(\tan\phi = \frac{-3}{4} \right)$ as compared to voltage. Find (i) R, (ii) $X_C - X_L$, and (iii) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected ?

Ans. Here, $P = 2\text{ kW} = 2000\text{ W}$. $E_v = \sqrt{50000} = 223\text{ V}$; $\tan\phi = -\frac{3}{4}$; $R = ?$; $(X_C - X_L) = ?$, $I_M = I_0 = ?$

From $P = \frac{E_v^2}{Z}$, $Z = \frac{E_v^2}{P} = \frac{50000}{2000} = 25\,\Omega$

Now, $\tan\phi = \frac{X_C - X_L}{R} = -\frac{3}{4}$ $\quad \therefore X_C - X_L = -\frac{3}{4}R$ ...(*i*)

From $Z^2 = R^2 + (X_C - X_L)^2$

$$(25)^2 = R^2 + \left(-\frac{3}{4}R\right)^2 = \frac{25}{16}R^2$$

$$R = \frac{4}{5} \times 25 = \mathbf{20\,ohm}$$

From (*i*), $X_C - X_L = -\frac{3}{4} \times 20 = -15\,\Omega$

Now, $I_v = \frac{E_v}{Z} = \frac{223}{25} \simeq 9\text{ A}$ $\qquad I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 9\text{ A} = \mathbf{12{\cdot}6\text{ A}}$

When R, X_C, X_L are all doubled, $\tan\phi$ does not change ; Z is doubled, current is halved and power drawn is halved.

28. 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0·5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if (i) power is transmitted at 220 V. Comment on the feasibility of doing this.

(ii) a step-up transformer is used to boost the voltage to 11000 V, power transmitted, then a step-down transformer is used to bring voltage to 220 V. ($\rho_{Cu} = 1{\cdot}7 \times 10^{-8}$ SI unit)

Ans. (*i*) As the town is 10 km away, length of pair of Cu-wires used, $l = 20$ km $= 20000$ m.

Resistance of wires, $R = \frac{\rho l}{a} = \frac{\rho l}{\pi r^2} = \frac{1{\cdot}7\times10^{-8}\times20000}{3{\cdot}14(0{\cdot}5\times10^{-2})^2} = 4\Omega$

When power $P = 10^6$ watt is transmitted at $V = 220$ V.

Current drawn, $I = \frac{P}{V} = \frac{10^6}{220} = 0{\cdot}45\times10^4$ A

Power loss, $I^2\ R = (0{\cdot}45\times10^4)^2\times4 = 0{\cdot}81\times10^8$ W, which is so large.

Therefore, this method of transmission of power is **not feasible.**

(*ii*) When power $P = 10^6$ watt is transmitted at 11000 V,

Current drawn, $I = \frac{P}{V} = \frac{10^6}{11000} = \frac{10^3}{11}$ A ; Power loss, $I^2R = \left(\frac{10^3}{11}\right)^2\times4 = 3{\cdot}3\times10^4$ watt

Fraction of ohmic losses to power transmitted $= \frac{3{\cdot}3\times10^4}{10^6}\times100\% = 3{\cdot}3\%$

29. Consider the *LCR* circuit shown in Fig. 7(N).11. Find the net current *i* and the phase of *i*. Show that $i = \frac{v}{Z}$. Find the impedence Z for this circuit.

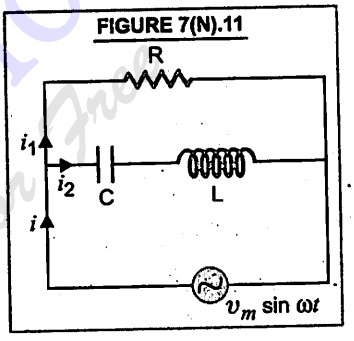

FIGURE 7(N).11

Ans. In Fig. 7(N).11, let *i* be the total current from the source. It is divided into two parts : i_1 through *R* and i_2 through series combination of *C* and *L*, such that $i = i_1 + i_2$.

As $R\,i_1 = v_m \sin \omega t$, $i_1 = \frac{v_m \sin \omega t}{R}$ ...(*i*)

If q_2 is charge on *C* at time *t*, then for series combination of *C* and *L*,

$$\frac{q_2}{C} + L\frac{dq_2^2}{dt^2} = v_m \sin \omega t \qquad ...(ii)$$

Let $q_2 = q_m \sin(\omega t + \phi)$ ...(*iii*)

$\therefore$ $\frac{dq_2}{dt} = q_m\,\omega\cos(\omega t + \phi)$ and $\frac{d^2q_2}{dt} = -q_m\,\omega^2\sin(\omega t + \phi)$

Putting in (*ii*), we get $q_m\left[\frac{1}{C} - L\omega^2\right]\sin(\omega t + \phi) = v_m \sin \omega t$

If $\phi = 0$ and $\left(\frac{1}{C} - L\omega^2\right) > 0$, then $q_m = \frac{v_m}{\left(\frac{1}{C} - L\omega^2\right)}$ ...(*iv*)

From (*iii*), $i_2 = \frac{dq_2}{dt} = \omega q_m \cos(\omega t + \phi)$; using (*iv*), $i_2 = \frac{\omega v_m \cos(\omega t + \phi)}{\frac{1}{C} - L\omega^2}$

Taking $\phi = 0$; $i_2 = \frac{v_m \cos(\omega t)}{\left(\frac{1}{\omega C} - L\omega\right)}$ ...(*v*)

From (*i*) and (*v*), we find that **i_1 and i_2 are out of phase.**

Now, $$i_1 + i_2 = \frac{v_m \sin \omega t}{R} + \frac{v_m \cos \omega t}{\left(\frac{1}{\omega C} - L\omega\right)}$$

Put $$\frac{v_m}{R} = A = C\cos\phi \quad \text{and} \quad \frac{v_m}{\left(\frac{1}{\omega C} - L\omega\right)} = B = C\sin\phi$$

$\therefore$ $$i_1 + i_2 = C\cos\phi \sin\omega t + C\sin\phi\cos\omega t = C\sin(\omega t + \phi)$$

where $$C = \sqrt{A^2 + B^2} \quad \text{and} \quad \phi = \tan^{-1}\frac{B}{A}$$

$$C = \left[\frac{v_m^2}{R^2} + \frac{v_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2} \quad \text{and} \quad \phi = \tan^{-1}\frac{R}{\left(\frac{1}{\omega C} - L\omega\right)}$$

Hence, $$i = i_1 + i_2 = \left[\frac{v_m^2}{R^2} + \frac{v_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2} \sin(\omega t + \phi)$$

or $$\frac{i}{v_m} = \frac{1}{Z} = \left[\frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2}$$

This is the expression for impedance Z of the circuit.

30. For an *LCR* circuit driven at frequency ω, the equation reads $L\frac{di}{dt} + Ri + \frac{q}{C} = v_i = v_m \sin\omega t$

(i) Multiply the equation by *i* and simplify where possible.

(ii) Interpret each term physically.

(iii) Cast the equation in the form of a conservation of energy statement.

(iv) Intergrate the equation over one cycle to find that the phase difference between *v* and *i* must be acute.

Ans. For an *LCR* circuit driven at frequency ω, the given eqn. is

$$L\frac{di}{dt} + Ri + \frac{q}{C} = v_i = v_m \sin\omega t \qquad ...(i)$$

Multiplying both sides by i, we get $$Li\frac{di}{dt} + Ri^2 + \frac{qi}{C} = vi \qquad ...(ii)$$

Now, $Li\frac{di}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$ = rate of change of energy stored in inductor, R_i^2 = rate of Joule heating loss

$\frac{q}{C}i = \frac{d}{dt}\left(\frac{q^2}{2C}\right)$ = rate of change of energy stored in capacitor

$v \times i$ = rate at which driving force pours in energy

Hence, eqn. (*ii*) is in the form of conservation of energy statement.

Integrating both sides of (*ii*) w.r.t. t over one full cycle ($0 \to T$), we may write

$$\int_0^T \frac{d}{dt}\left(\frac{1}{2}Li^2 + \frac{q^2}{2C}\right)dt + \int_0^T Ri^2\,dt = \int_0^T v\,i\,dt$$

$$\downarrow \qquad\qquad \downarrow$$

$$\text{zero} \quad + \quad \text{positive} \quad = \int_0^T v\,i\,dt$$

$\therefore \int_0^T v\,i\,dt > 0$, which is possible only when phase diff. between v and i is an acute angle.

31. In the *LCR* circuit shown in Fig. 7(N).12, the *ac* driving voltage is $v = v_m \sin \omega t$.
(i) Write down the equation of motion for q (t).
(ii) At $t = t_0$, the voltage source stops and R is short circuited. Now write down how much energy is stored in each of L and C.
(iii) Describe subsequent motion of charges.

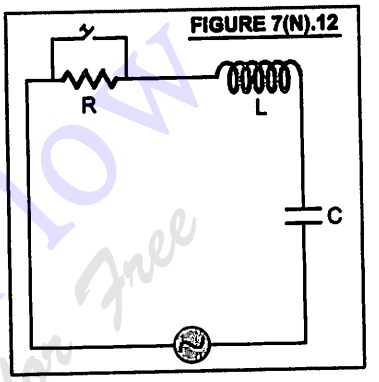

FIGURE 7(N).12

Ans. (*i*) In the circuit shown in Fig. 7(N).12.

When driving voltage $v = v_m \sin \omega t$ is applied, the valid equation for the circuit with usual notation is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = v_m \sin \omega t \qquad ...(i)$$

This is the equation of motion for q (t).

Let $\quad q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$

$\therefore \quad i = i_m \sin(\omega t + \phi) = q_m \omega \sin(\omega t + \phi)$

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}} \quad \text{and} \quad \phi = \tan^{-1}\frac{(X_C - X_L)}{R}$$

(*ii*) When source stops at $t = t_0$ and R is short circuited, energy is stored in L and C

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\left[\frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \sin^2(\omega t_0 + \phi)$$

and $$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}\left[q_m^2 \cos^2(\omega t_0 + \phi)\right] = \frac{1}{2C}\frac{i_m^2}{\omega^2}\cos^2(\omega t_0 + \phi)$$

$$U_C = \frac{1}{2C}\left[\frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \frac{1}{\omega^2}\cos^2(\omega t_0 + \phi)$$

(*iii*) When R is short circuited, the circuit becomes an LC oscillator. The capacitor will go on discharging till all energy goes to L, and back to C again. This keeps on repeating indefinitely.

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I. Multiple Choice Questions (with One Correct Answer)

I. Alternating currents

1. A resistor R and 2 μF capacitor in series are connected through a 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 sec. after the switch has been closed. ($\log_{10} 2{\cdot}5 = 0{\cdot}4$).

(*a*) $1{\cdot}7 \times 10^5\ \Omega$ (*b*) $2{\cdot}7 \times 10^6\ \Omega$

(*c*) $3{\cdot}3 \times 10^7\ \Omega$ (*d*) $1{\cdot}3 \times 10^4\ \Omega$

(AIEEE 2011)

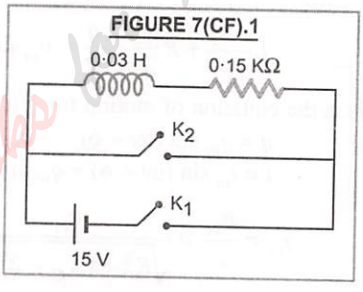
2. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. The ratio t_1/t_2 will be :

(*a*) 2 (*b*) 1

(*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$ **(AIEEE 2010)**

3. An inductor (L = 0·03 H) and a resistor (R = 0·15 KΩ) are connected in series to a battery of 15 V *emf* in a circuit shown in Fig. 7(CF).1. The key k_1 has been kept closed for a long time. Then at $t = 0$, k_1 is opened and key k_2 is closed simultaneously. At time t = 1 ms, the current in the circuit will be (Take $e^5 \simeq 150$)

FIGURE 7(CF).1



(*a*) 100 mA (*b*) 67 mA

(*c*) 6·7 mA (*d*) 0·67 mA

(JEE Main 2015)

4. A 150 ohm resistor and inductor of inductance L are connected in series to a 200 V, 50 Hz source of negligible impedence. The current comes to 1 A. When the source is changed to 400 V, 100 Hz, the current will be

(*a*) less than 1·0 A

(*b*) 1·0 A

(*c*) between 1 A and 2 A

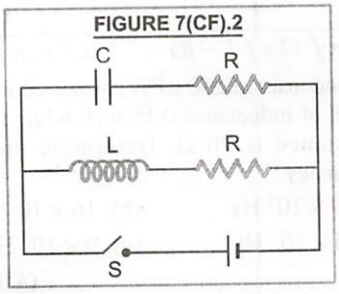
(*d*) between 4 A and 2 A

5. In the circuit shown in Fig. 7(CF).2, the switch S is closed at time $t = 0$. The current through the capacitor and inductor will be equal at time t equals (given $R = \sqrt{L/C}$)

ANSWERS

1. (*b*) 2. (*d*) 3. (*d*) 4. (*c*)

FIGURE 7(CF).2

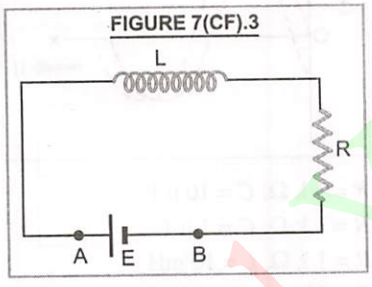


(*a*) RC (*b*) $RC \log_e 2$

(*c*) $\frac{1}{R \log_e 2}$ (*d*) LR

6. An inductor ($L = 100$ mH), a resistor ($R = 100\ \Omega$) and a battery ($E = 100$ V) are initially connected as shown in Fig. 7(CF).3. After a long time, the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is

FIGURE 7(CF).3

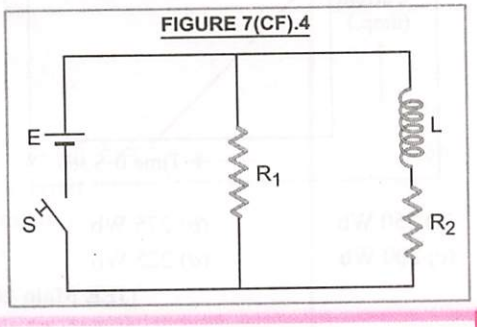


(*a*) e A (*b*) 0·1 A

(*c*) 1 A (*d*) $\frac{1}{e}$ A

7. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of e.m.f. 12 V as shown in Fig. 7(CF).4. The internal resistance of the battery is negligible. The switch is closed at $t = 0$. The potential drop across L as a function of time is

FIGURE 7(CF).4



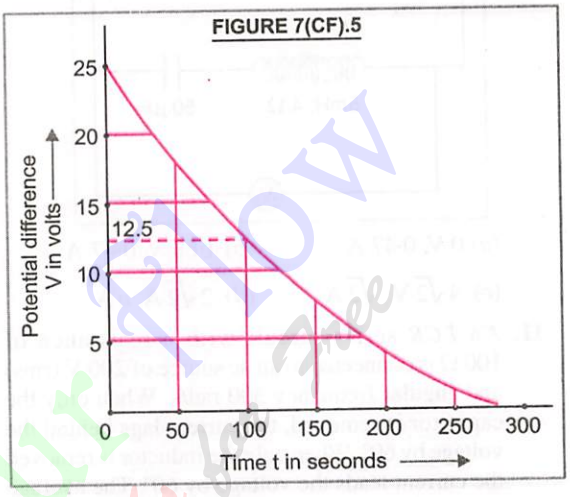
(*a*) $\frac{12}{t} e^{-3t}$ V (*b*) $6\,(1 - e^{-t/0{\cdot}2})$ V

(*c*) $12\ e^{-5t}$ V (*d*) $6\ e^{-5t}$ V

(AIEEE 2009)

8. Fig. 7(CF).5 shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between :

FIGURE 7(CF).5



(*a*) 150 sec and 200 sec

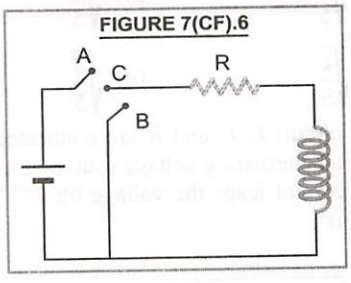
(*b*) 0 and 50 sec

(*c*) 50 sec and 100 sec

(*d*) 100 sec and 150 sec **(AIEEE 2012)**

9. In the circuit shown in Fig. 7(CF).6, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterwards, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to

FIGURE 7(CF).6



(*a*) -1 (*b*) $\frac{1-e}{e}$

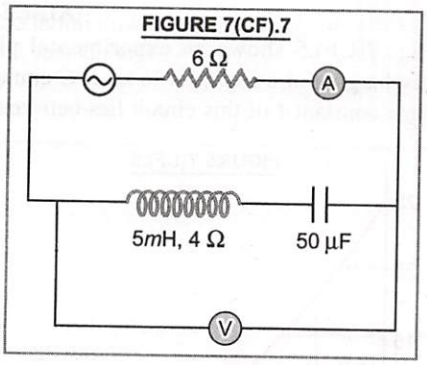
(*c*) $\frac{e}{1-e}$ (*d*) 1 **(JEE Main 2014)**

ANSWERS

5. (*b*) **6.** (*d*) **7.** (*c*) **8.** (*d*) **9.** (*d*)

10. In the circuit shown in Fig. 7(CF).7, the a.c. source gives a voltage $V = 20 \cos (2000\ t)$. Neglecting source resistance, the voltmeter and ammeter readings will be

FIGURE 7(CF).7



(*a*) 0 V, 0·47 A (*b*) 1·68 V, 0·47 A

(*c*) $4\sqrt{2}$ V, $\sqrt{2}$ A (*d*) $2\sqrt{2}$ A, *o* A

11. An *LCR* series circuit with a resistance of 100 Ω is connected to an ac source of 200 V (rms) and angular frequency 300 rad/s. When only the capacitor is removed, the current lags behind the voltage by 60°. When only the inductor is removed the current leads the voltage by 60°. The average power dissipated in original *LCR* circuit is

(*a*) 50 W (*b*) 100 W

(*c*) 200 W (*d*) 400 W

12. An ac source of angular frequency ω is fed across a resistor *R* and a capacitor *C* in series. The current registered is *I*. If now the frequency of source is changed to ω/3 (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω

(*a*) $\sqrt{\frac{3}{5}}$ (*b*) $\sqrt{\frac{2}{5}}$

(*c*) $\sqrt{\frac{1}{5}}$ (*d*) $\sqrt{\frac{4}{5}}$

13. In a circuit *L*, *C* and *R* are connected in series with an alternating voltage source of frequency *f*. The current leads the voltage by 45°. The value of *C* is :

(*a*) $\frac{1}{2\pi f\,(2\pi f L - R)}$

(*b*) $\frac{1}{2\pi f\,(2\pi f L + R)}$

(*c*) $\frac{1}{\pi f\,(2\pi f L - R)}$ (*d*) $\frac{1}{\pi f\,(2\pi f L + R)}$

14. A condenser of 250 μF is connected in parallel to a coil of inductance 0·16 mH, while its effective resistance is 20 Ω. Determine the resonant frequency

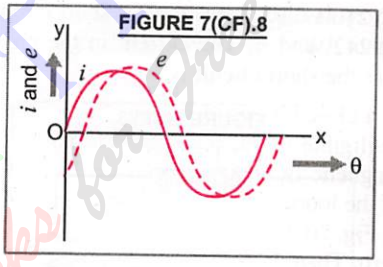
(*a*) 9×10^4 Hz (*b*) 16×10^7 Hz

(*c*) 8×10^5 Hz (*d*) 9×10^3 Hz

(AIIMS 2015)

15. When an a.c. source of emf $e = E_0 \sin 100\ t$ is connected across a circuit ; the phase difference between the emf *e* and current *i* in the circuit is observed to be π/4, as shown in Fig. 7(CF).8. If the circuit consists possibly only of *R*-*C* or *R*-*L* or *L*-*C* in series, find possible values of two elements.

FIGURE 7(CF).8



(*a*) $R = 1$ k Ω, $C = 10$ μ F

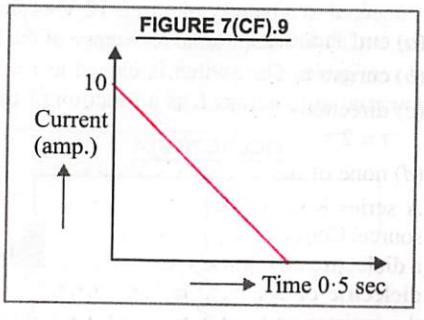
(*b*) $R = 1$ k Ω, $C = 1$ μ F

(*c*) $R = 1$ k Ω, $L = 10$ mH

(*d*) $R = 10$ k Ω, $L = 10$ μ H

16. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the 7(CF).9. The magnitude of change in flux through the coil is

FIGURE 7(CF).9



(*a*) 250 Wb (*b*) 275 Wb

(*c*) 200 Wb (*d*) 225 Wb

(JEE Main 2017)

ANSWERS

10. (*c*) **11.** (*d*) **12.** (*a*) **13.** (*b*) **14.** (*c*) **15.** (*a*) **16.** (*a*)

17. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(a) the bulb glows dimmer

(b) the bulb glows brighter

(c) total impedance of the circuit is unchanged

(d) total impedance of the circuit increases

(IIT 2010)

18. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the LCR circuit is

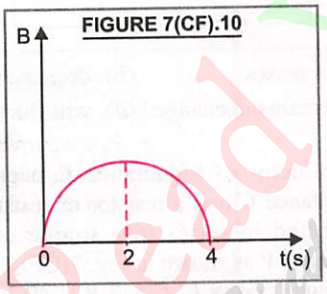
(a) 210 W (b) Zero W

(c) 242 W (d) 305 W

(AIEEE 2010)

19. In a closed loop, which has some inductance but negligible resistance, uniform but time varying magnetic field is applied directed into the plane of the loop. Variation of field with time is shown in Fig. 7(CF).10. Initially, current in the loop was zero. Then :

FIGURE 7(CF).10



(a) emf induced in the loop is zero at $t = 2$ s

(b) current in the loop will be maximum at $t = 2$ s

(c) direction of emf in the loop will not change at $t = 2$ s

(d) none of the above **(Karnataka CET 2010)**

20. A series R-C circuit is connected to AC voltage source. Consider two cases ; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true ?

(a) $I_R^A > I_R^B$ (b) $I_R^A = I_R^B$

(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

(IIT 2011)

21. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic field is

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$

(c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

(AIEEE 2011)

22. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6 V is used in an experiment on (C–R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is

(a) 20 second (b) 10 second

(c) 5 second (d) 2·5 second

(AIEEE 2011)

23. In an $a.c.$ circuit, the instantaneous emf and current are given by

$$e = 100 \sin 30\, t$$

$$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of $a.c.$, the average power consumed by the circuit and the wattless current are, respectively

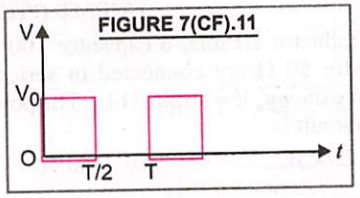
(a) 50, 10 (b) $\frac{1000}{\sqrt{2}}, 10$

(c) $\frac{50}{\sqrt{2}}, 0$ (d) 50, 0

(JEE Main 2018)

24. The r.m.s. value of potential difference V shown in Fig. 7(CF).11 is

FIGURE 7(CF).11



ANSWERS

17. (b) **18.** (c) **19.** (a) **20.** (c) **21.** (a) **22.** (d) **23.** (b)

(a) V_0 (b) $V_0/2$

(c) $V_0/\sqrt{2}$ (d) $V_0/\sqrt{3}$

[AIPMT (Main) 2011]

25. A coil has resistance 30 ohm and inductive reactance 20 ohm at 50 Hz frequency. If an ac source of 200 volt, 100 Hz, is connected across the coil, the current in the coil will be

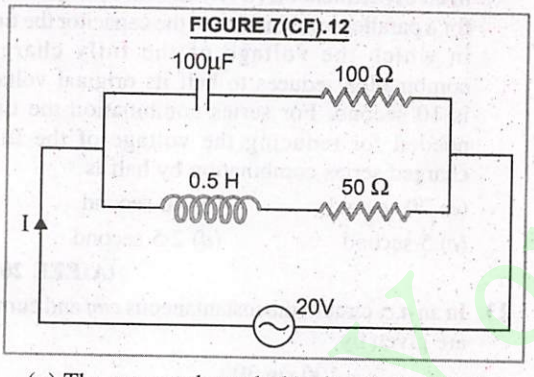
(a) 4·0 A (b) 8·0 A

(c) $\frac{20}{\sqrt{13}}$ A (d) 2·0 A

[AIPMT (Main) 2011]

26. In the given circuit, Fig. 7(CF).12, the *AC* source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice (s) is (are)

FIGURE 7(CF).12



(a) The current through the circuit, I is 0·3 A

(b) The current through the circuit, I is $0{\cdot}3\sqrt{2}$ A

(c) The voltage across 100 Ω resistor $= 15\sqrt{2}$ V

(d) The voltage across 50 Ω resistor = 10 V

(IIT 2012)

27. If an electrical circuit *R, L, C* and an a.c. voltage source are all connected in series. When *L* is removed from the circuit, the phase difference between the voltage and the current in the circuit is $\pi/3$. If instead, *C* is removed from the circuit, the phase difference is again $\pi/3$. The power factor of the circuit is :

(a) 1/2 (b) $1/\sqrt{2}$ (c) 1 (d) $\sqrt{3}/2$

[AIPMT (Prelim) 2012]

28. An inductor 20 mH, a capacitor 100 μ*F* and a resistor 50 Ω are connected in series across a source of *emf*, $V = 10 \sin 314\, t$. The power loss in the circuit is

(a) 2.74 *W* (b) 0.43 *W*

(c) 0.79 *W* (d) 1.13 *W*

(NEET 2018)

29. A 40 μF capacitor is connected to a 200 V, 50 Hz a.c. supply. The rms value of current in the circuit is nearly

(a) 2·05 A (b) 2·5 A

(c) 25·1 A (d) 1·7 A **(NEET 2020)**

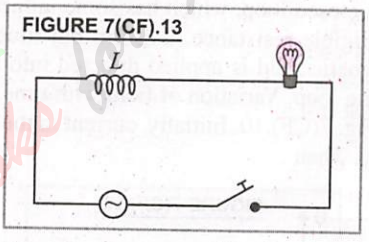
30. A series LCR circuit is connected to an a.c. voltage source. When *L* is removed from the circuit, the phase difference between current and voltage is $\pi/3$. If instead, *C* is removed from the circuit, the phase difference is again $\pi/3$ between current and voltage. The power factor of the circuit is :

(a) 0·5 (b) 1·0

(c) – 1·0 (d) zero **(NEET 2020)**

31. A light bulb and an inductor coil are connected to an a.c. source through a key as shown in Fig. 7(CF).13. The key is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb

FIGURE 7(CF).13



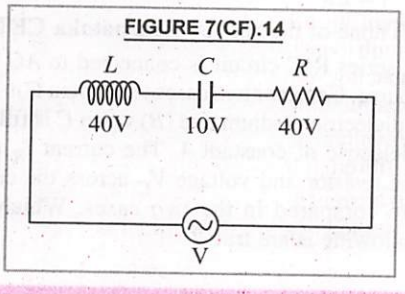
(a) increases (b) decreases

(c) remains unchanged (d) will fluctuate

(NEET 2020)

32. An inductor of inductance *L*, a capacitor of capacitance *C* and a resistor of resistance *R* are connected in series to a source of potential difference *V* as shown in Fig. 7(CF).14. Potential difference across *L*, *C* and *R* is 40 V, 10 V and 40 V, respectively. The amplitude of current flowing through *LCR* series circuit is $10\sqrt{2}$ A. The impedance of the circuit is

FIGURE 7(CF).14



ANSWERS

24. (c) 25. (a) 26. (a) 27. (c) 28. (c) 29. (b) 30. (b) 31. (b)

(a) 5 Ω (b) $4\sqrt{2}\,\Omega$

(c) $5\sqrt{2}\,\Omega$ (d) 4 Ω **(NEET 2021)**

33. A series *LCR* circuit containing 5·0 H inductor, 80 μF capacitor and 40 Ω resistor is connected to 230 V variable frequency a.c. source. The angular frequencies of the source at which power transferred to the circuit is half the power at the resonant angular frequency are likeky to be

(a) 42 rad/s and 58 rad/s

(b) 25 rad/s and 75 rad/s

(c) 50 rad/s and 25 rad/s

(d) 46 rad/s and 54 rad/s **(NEET 2021)**

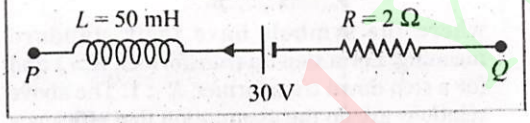
34. An a.c. circuit has R = 100 Ω, C = 2 μF and L = 80 mH connected in series. The quality factor of the circuit is

(a) 400 (b) 0·5

(c) 2 (d) 20 **(JEE Main 2020)**

35. A part of a complete circuit is shown in Fig. 7(CF).15. At some instant, the value of current I is 1A and it is decreasing at a rate of 10^2 As^{-1}. The value of potential difference $(V_P - V_Q)$ in volt at that instant is

FIGURE 7(CF).15



(a) 66 V (b) 33 V

(c) 100 V (d) 120 V **(JEE Main 2020)**

36. An electric power line, having a total resistance of 2 Ω delivers 1 kW power at 220 V. The efficiency of the transmission line is approximately

(a) 72% (b) 96%

(c) 91% (d) 85% **(JEE Main 2020)**

II. Electrical devices

37. The primary and secondary coils of a transformer have 50 and 1500 turns respectively. If the magnetic flux ϕ linked with primary coil is given by $\phi = \phi_0 + 4t$, where ϕ_0 is constant, ϕ is in weber and t is in sec., output voltage across the secondary coil is

(a) 120 V (b) 220 V

(c) 30 V (d) 90 V **(AIPMT 2007)**

38. An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of e.m.f. $V = 10 \sin 340t$. The power loss in a.c. circuit is

(a) 0·67 W (b) 0·76 W

(c) 0·89 W (d) 0·046 W **(NEET 2016)**

39. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to 220 V (rms), 50 Hz a.c. supply, the series inductor needed for it to work is close to

(a) 80 H (b) 0·08 H

(c) 0·044 H (d) 0·065 H **(JEE Mains 2016)**

40. A step down transformer connected to an a.c. mains supply of 220 V is made to operate at 11 V, 44 W lamp. Ignoring power losses in the transformer, what is the current in the primary circuit ?

(a) 4 A (b) 0·2 A

(c) 0·4 A (d) 2 A **(NEET 2021)**

II. Multiple Choice Questions (with One or More than One Correct Answers)

41. When an alternating current flows through a circuit consisting of a resistor in series with a capacitor, during the cycle at some instant, it is possible to have

(a) voltage across the circuit is zero but current through it is not zero

(b) current through the circuit is zero but the voltage across it is not zero

(c) current through the capacitor is not zero but the voltage across it is zero

(d) current through the resistor is not zero but the voltage across it is zero.

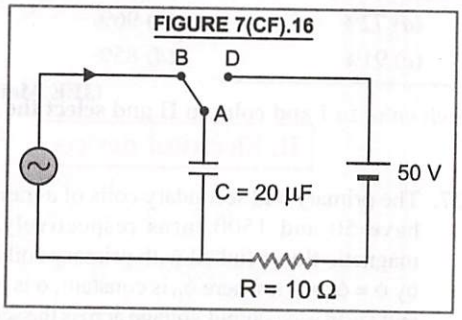
(National Standard Exam in Physics 2011)

42. At time t = 0, terminal A in the circuit shown in the Fig. 7(CF).16 is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with

ANSWERS

32. (a) 33. (d) 34. (c) 35. (b) 36. (b) 37. (a) 38. (d) 39. (d)
40. (b) 41. (a,b,c)

FIGURE 7(CF).16



$I_0 = 1$ A and $\omega = 500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20$ μF, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s).

(*a*) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C

(*b*) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(*c*) Immediately after A is connected to D, the current in R is 10 A

(*d*) $Q = 2 \times 10^{-3}$ C **(JEE Advanced 2014)**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

Each comprehension given below is followed by some multiple choice questions. Each question has one correct option. Choose the correct option.

Comprehension 1

In an a.c. circuit, values of voltage and current change every instant. Therefore, power of an a.c. circuit at any instant is the product of instantaneous voltage (E) and instantaneous current (I). The average power supplied to a pure resistance R over a complete cycle of a.c. is $P = E_v . I_v$. When the circuit is inductive, average power/cycle = $E_v\ I_v \cos\phi$, where ϕ is the phase angle between alternating voltage and alternating current in the circuit.

In an a.c. circuit, 800 mH inductor and a 60 μ F capacitor are connected in series with 15 ohm resistance. The a.c. supply to the circuit is 230 V, 50 Hz.

43. The average power transferred per cycle to resistance is

(*a*) 2·05 W (*b*) 20·05 W

(*c*) Zero (*d*) 230 W

44. The electrical energy spent in running the circuit for one hour is

(*a*) 20·05 J

(*b*) 20·05 k w h

(*c*) $7{\cdot}22 \times 10^4$ J

(*d*) Zero

Comprehension 2

A transformer is an electrical device which is used for changing the a.c. voltages. It is based on the phenomenon of mutual induction. It can be shown that

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{n_s}{n_p} = K$$

where the symbols have their standard meaning. For a step up transformer, $K > 1$ and for a step down transformer, $K < 1$. The above relations are on the assumption that efficiency of transformer is 100%. In fact, efficiency

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{E_s I_s}{E_p I_p}$$

The number of turns in the primary and secondary coils of a transformer are 2000 and 50 respectively. The primary coil is connected to main supply of 120 V and secondary to a night bulb of 0·6 ohm. The efficiency of transformer is 80%.

45. Voltage across the secondary of transformer is

(*a*) 120 V (*b*) 360 V (*c*) 40 V (*d*) 3 V

46. Current in primary coil is

(*a*) 15 A (*b*) 5/3 A (*c*) $\frac{5}{32}$ A (*d*) 0·6 A

47. Power in secondary coil is

(*a*) 15 W (*b*) 225 W (*c*) 120 W (*d*) 7 W

ANSWERS

42. (*c,d*) 43. (*b*) 44. (*c*) 45. (*d*) 46. (*c*) 47. (*a*)

IV. Matching Type Questions

DIRECTIONS. In each of the following questions, match column I and column II and select the correct match out of the four given choices

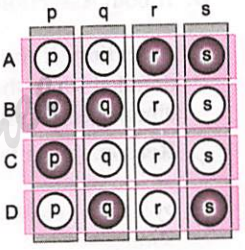
48.

	Column I		Column II
A.	A.C. Generator	(*p*)	Best way of reducing a.c.
B.	D.C. Generator	(*q*)	Works on mutual induction
C.	Choke coil	(*r*)	slip ring arrangement
D.	Transformer	(*s*)	split ring arrangement

(*a*) A-*s*, B-*r*, C-*q*, D-*p* (*b*) A-*p*, B-*q*, C-*r*, D-*s* (*c*) A-*s*, B-*r*, C-*p*, D-*q* (*d*) A-*r*, B-*s*, C-*p*, D-*q*

V. Matrix-Match Type Questions

DIRECTIONS. Each of the following questions contains statements given in two columns, which have to be matched. The answers to these questions have to be appropriately bubbled. If the correct matches are A-*r*, A-*s* ; B-*p*, B-*q* ; C-*p* ; D-*q*, D-*s*, then the correctly bubbled matrix will look like the one shown here :



49.

	Column I		Column II
	SI units of		
(*a*)	impedance	(*p*)	siemen
(*b*)	reactance	(*q*)	mho
(*c*)	susceptance	(*r*)	ohm
(*d*)	admittance	(*s*)	hertz
(*e*)	A transformer works on	(*t*)	d.c.
		(*u*)	a.c.

VI. Integer Type Questions

DIRECTIONS. The answer to each of the following questions is a single digit integer, ranging from 0 to 9. If the correct answers to the question numbers A, B, C and D (say) are 4, 0, 9 and 2 respectively, then the correct darkening of bubbles should be as shown on the side :

50. At a certain frequency ω_1, the reactance of a certain capacitor equals that of a certain inductor. If the frequency is changed to $\omega_2 = 2\omega_1$, the ratio of reactance of the inductor to that of the capacitor is :

51. A condenser of capacitance 0·144 pF is used in a transmitter to transmit at wavelength λ. If inductance of $1/\pi^2$ mH is used for resonance, what is the value of λ (in *m*) ?

52. An a.c. voltage is represented by $E = 220\sqrt{2}\cos(8\pi)t$.

How many times will the current become zero in 1 sec ?

53. An a.c. source of voltage $V = 100 \sin 100\pi t$ is connected to a resistor of $\frac{25}{\sqrt{2}}$ ohm. What is the r.m.s. value of current (in ampere) through the resistor ?

ANSWERS

48. (*d*) **49.** (*a*)–*r* ; (*b*)–*r* ; (*c*)–*p*,*q* ; (*d*)–*p*,*q* ; (*e*)–*u* **50.** (4) **51.** (7)
52. (8) **53.** (4)

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VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

DIRECTIONS. Read the following questions and choose any one of the following four responses.

A. If both, **Assertion** and **Reason** are true and the **Reason** is the correct explanation of the **Assertion.**

B. If both, **Assertion** and **Reason** are true but **Reason** is not a correct explanation of the **Assertion.**

C. If **Assertion** is true but the **Reason** is false.

D. If both **Assertion** and **Reason** are false.

54. Assertion. If the frequency of the applied *AC* is doubled, then the power factor of a series *R-L* circuit decreases.

Reason. Power factor of series *R-L* circuit is given by $\cos\theta = \frac{2R}{\sqrt{R^2 + \omega^2 L^2}}$

(*a*) A (*b*) B (*c*) C (*d*) D

(AIIMS 2015)

55. Assertion. If a varying current is flowing through a machine of iron, eddy currents are produced.

Reason. Change in the magnetic flux thro' an area causes eddy currents.

(*a*) A (*b*) B (*c*) C (*d*) D

56. Assertion. In series *LCR* resonance circuit, the impedance is equal to the ohmic resistance.

Reason. At resonance, the inductive reactance exceeds the capacitive reactance.

(*a*) A (*b*) B (*c*) C (*d*) D

FOR ENGINEERING STUDENTS

DIRECTIONS. Read the following two statements. Of the four choices given, choose the one that best describes the two statements :

A. Statement-1 is true ; Statement-2 is true and Statement-2 is correct explanation of Statement-1.

B. Statement-1 is true ; Statement-2 is true, but Statement-2 is not a correct explanation of Statement-1.

C. Statement-1 is true ; Statement-2 is false.

D. Statement-1 is false ; Statement-2 is true.

57. Statement-1. In 220 V ; 50 hz a.c. wall plug, peak value of alternating *e.m.f.* is 220 V.

Statement-2. Only rms value is specified.

(*a*) A (*b*) B (*c*) C (*d*) D

58. Statement-1. A step up transformer can also be used as a step down transformer.

Statement-2. This is because $\frac{E_s}{E_p} = \frac{n_s}{n_p}$

(*a*) A (*b*) B (*c*) C (*d*) D

VIII. Numerical Value Type Questions (In Decimal Notations)

59. In determination of frequency of a.c. mains using a sonometer, a.c. is supplied to electromagnet

(*a*) directly

(*b*) through a step up transformer

(*c*) through a step down transformer

(*d*) none of the above

60. The resonance length obtained by a student in determination of frequency of a.c. mains by sonometer is 26·7 cm. If thickness of sonometer wire were doubled without changing any other parameter, the resonance length would be

(*a*) 26·7 cm (*b*) 53·4 cm

(*c*) 13·4 cm (*d*) none of the above

ANSWERS

54. (*c*) 55. (*a*) 56. (*c*) 57. (*d*) 58. (*a*) 59. (*c*) 60. (*c*)

HINTS/EXPLANATIONS
For Difficult Questions

I. Multiple Choice Questions (with One Correct Answer)

1. Here, $V_C = 120$ V $V_0 = 200$ V

So $V_C = V_0 (1 - e^{-t/RC})$

$$1 - e^{-t/RC} = \frac{V_C}{V_0} = \frac{120\text{ V}}{200\text{ V}} = \frac{3}{5}$$

or $e^{-t/RC} = 2/5$

or $\log e^{-t/RC} = \log_e 2/5$

or $$\frac{-t}{RC} = 2{\cdot}3026 \log_{10} 0{\cdot}4 = -0{\cdot}92$$

or $$R = \frac{t}{0{\cdot}92\,C} = \frac{5}{0{\cdot}92 \times 2 \times 10^{-6}} = \mathbf{2{\cdot}7 \times 10^6\ \Omega}$$

2. As $$U = \frac{Q^2}{2C} = \frac{(Q_0 e^{-t/\tau})^2}{2C}$$

$$= \frac{Q_0^2}{2C} e^{-2t/\tau} = U_0 e^{-2t/\tau}$$

As when $t = t_1$, $U = U_0/2$, so $$\frac{U_0}{2} = U_0 e^{-2t_1/\tau}$$

$$\frac{1}{2} = e^{-\frac{2t_1}{\tau}} \quad \text{or} \quad t_1 = \frac{\tau}{2} \ln 2$$

Further, as $Q = Q_0 e^{-t/\tau}$ and when $t = t_2$, $Q = \frac{Q_0}{4}$

$$\frac{Q_0}{4} = Q_0 e^{-t_2/\tau}$$

$$t_2 = \tau \ln 4 = 2\tau \ln 2$$

$$\frac{t_1}{t_2} = \frac{1}{4}$$

3. Here $L = 0{\cdot}03$ H, $R = 0{\cdot}15$ KΩ $= 0{\cdot}15 \times 10^3\ \Omega$,

$t = 1$ ms $= 1 \times 10^{-3}$ sec.

$$I_0 = \frac{V}{R} = \frac{15}{150} = \frac{1}{10} = 0{\cdot}1$$

$$\tau = \frac{L}{R} = \frac{0{\cdot}03}{150} = 2 \times 10^{-4}\text{ s}$$

During decay, $$I = I_0 e^{-t/\tau} = 0{\cdot}1\, e^{-\frac{10^{-3}}{2\times10^{-4}}}$$

$$I = 0{\cdot}1 e^{-5} = \frac{0{\cdot}1}{e^5} = \frac{0{\cdot}1}{150}\text{ A} = \mathbf{0{\cdot}67\ mA}$$

4. Here, $R = 150\ \Omega$, $V = 200$ V, $\nu = 50$ HZ,

$I = 1$A, $V' = 400$ V, $\nu' = 100$ Hz, $I' = ?$

$$I = \frac{200}{\sqrt{R^2 + X_L^2}} = 1, \quad \sqrt{R^2 + X_L^2} = 200$$

$$(150)^2 + X_L^2 = (200)^2$$

$$X_L^2 = 17{,}500$$

$$X_L = 2\pi\nu L, \quad \frac{X_L'}{X_L} = \frac{2\pi\nu' L}{2\pi\nu L} = 2, \text{ so } X_L' = 2X_L$$

$$X_L' = 4X_L^2 = 4 \times 17500 = 70{,}000$$

$$Z' = \sqrt{R^2 + X_L'^2} = \sqrt{(150)^2 + 70{,}000} = 304\ \Omega$$

$$I' = \frac{400V}{304\,\Omega} = \mathbf{1{\cdot}3\,A}\ \textbf{(between 1 A and 2 A)}$$

5. In Fig. 7(CF).2, when switch S is closed, current through L grows and current through C decays.

As $I_C = I_L$

$$\therefore \quad \frac{V}{R} e^{-t/RC} = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \qquad \textbf{...(i)}$$

As $$R = \sqrt{\frac{L}{C}} \quad \therefore \quad R^2 C = L \quad \text{or} \quad RC = \frac{L}{R}$$

From (i), $e^{-t/RC} = 1 - e^{-t/RC}$

$$2\, e^{-t/RC} = 1$$

$$e^{-t/RC} = \frac{1}{2}$$

$$t = \boldsymbol{RC \log_e 2}$$

6. Here, $L = 100$ mH $= 100 \times 10^{-3}$ H $= 0{\cdot}1$ H

$R = 100$ ohm, $E = 100$ V

$$I_0 = \frac{E}{R} = \frac{100}{100} = 1\text{A}$$

When short circuiting is done, battery is disconnected. Current in RL circuit decays. At any time t,

$$I = I_0 e^{\frac{-R}{L}t} = 1 \times e^{\frac{-100}{0{\cdot}1}\times 10^{-3}} = e^{-1}$$

$$= \frac{1}{e}\textbf{ ampere}$$

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7. In the R-L circuit, growth of current is given by

$$I = \frac{E}{R_2}\left[1 - e^{-R_2 t/L}\right]$$

$$\therefore \quad \frac{dI}{dt} = -\frac{E}{R_2}\left(-\frac{R_2}{L}\right)e^{-R_2 t/L}$$

Potential drop across L is

$$V_L = L\frac{dI}{dt} = L\left(-\frac{E}{R_2}\right)e^{-R_2 t/L}\left(-\frac{R_2}{L}\right)$$

$$V_L = E e^{-R_2 t/L} = 12 e^{-2t/0.4}$$

$$= 12\, e^{-5t} \text{ volt}$$

8. From Fig. 7(CF).5, max. p.d., $V_0 = 25$ volt

Corresponding to $V = \frac{V_0}{2} = \frac{25}{2} = 12.5$ V,

$t = 100$ s.

The discharging of a capacitor through resistance is governed by

$$V = V_0 e^{-t/\tau} \quad \text{When } V = \frac{V_0}{2}\ ; \quad \frac{V_0}{2} = V_0 e^{-t/\tau}$$

$$\log_e 1 - \log_e 2 = -\frac{t}{\tau}\log_e e = -\frac{t}{\tau}$$

$$\tau = \frac{t}{\log_e 2} = \frac{100}{0.693} = 144.3 \text{ s}$$

Choice (*d*) is correct.

9. When C is connected to A, let I_0 be the current set up in the circuit. When C is disconnected to A and is connected to B, then current in L-R circuit decays from I_0. At time t, current is given by

$$I = I_0 e^{-(R/L)t}\ ; \quad \frac{dI}{dt} = -\frac{R}{L} I_0 e^{-\frac{R}{L}t} = -\frac{R}{L} I$$

When $t = \frac{L}{R}$, $I = I_0 e^{-(R/L)\times(L/R)} = \frac{I_0}{e}$

$$\frac{dI}{dt} = -\frac{R}{L}\times\frac{I_0}{e}$$

Voltage across L,

$$V_L = -L\frac{dI}{dt} = -L\left[-\frac{R}{L}\frac{I_0}{e}\right] = \frac{RI_0}{e}$$

Voltage across R, $V_R = IR = \frac{I_0}{e}R \quad \therefore \quad \frac{V_L}{V_R} = 1$

10. Here, $R = 6 + 4 = 10\ \Omega$; $\omega = 2000$ rad s^{-1}

$$X_L = \omega L = 2000 \times 5 \times 10^{-3}\ \Omega = 10\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000\times 50\times 10^{-6}} = 10\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{10^2 + (10-10)^2} = 10\ \Omega$$

$$I_0 = \frac{E_0}{Z} = \frac{20}{10} = 2\text{ A}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\text{ A}$$

This is the ammeter reading. Combined resistance of L and $C = \sqrt{r^2 + (X_L - X_C)^2}$

$$Z' = \sqrt{4^2 + (10-10)^2} = 4 \text{ ohm}$$

$\therefore$ Voltmeter reading $= I_v \times Z'$

$$= \sqrt{2}\times 4 = 4\sqrt{2}\text{ V}$$

11. Here, $R = 100\ \Omega$, $E_v = 200$ volt, $\omega = 300$ rad/s

When capacitor is removed,

$$\tan\phi = \frac{X_L}{R} = \tan 60° = \sqrt{3}$$

$$X_L = \sqrt{3}R$$

When inductor is removed,

$$\tan\phi = \frac{X_C}{R} = \tan 60° = \sqrt{3}$$

$$X_C = \sqrt{3}R$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{100^2 + 0} = 100\ \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{200}{100} = 2\text{ A}$$

$$P = E_v I_v \cos\phi = E_v I_v\left(\frac{R}{Z}\right)$$

$$= (200)(2)\times\frac{100}{100} = 400\text{ W}$$

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12. In RC circuit, $I_v = \dfrac{E_v}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$

and $\dfrac{I_v}{2} = \dfrac{E_v}{\sqrt{R^2 + \left(\dfrac{1}{\frac{\omega C}{3}}\right)^2}}$

$\therefore \quad 2\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2} = \sqrt{R^2 + \dfrac{9}{\omega^2 C^2}}$

or $3R^2 = \dfrac{5}{\omega^2 C^2} = 5X_C^2$

$\therefore \quad \dfrac{X_C}{R} = \sqrt{\dfrac{3}{5}}$

13. As current leads the voltage by 45°, therefore

$$\tan\phi = \frac{X_C - X_L}{R} = \tan 45° = 1$$

$\therefore X_C - X_L = R$ or $X_C = X_L + R$

or $1/\omega C = \omega L + R$

$$C = \frac{1}{\omega(\omega L + R)}, \quad C = \frac{1}{2\pi f\,(2\pi f L + R)}$$

14. Here, $C = 250 \times 10^{-6}$ F, $L = 0{\cdot}16 \times 10^{-3}$ H, $R = 20\ \Omega$

Resonant frequency of L, C, R circuit is

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{CL} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\times 3{\cdot}14}\sqrt{\frac{1}{(250\times 10^{-6})(0{\cdot}16\times 10^{-3})} - \frac{20\times 20}{(0{\cdot}16\times 10^{-3})^2}}$$

$\simeq$ **8×10^5 Hz**

15. Fig. 7(CF).8 shows that current i leads the emf e by a phase angle $\pi/4$. Therefore, the circuit can be R- C circuit alone.

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} = \tan\frac{\pi}{4} = 1$$

or $\quad CR = \dfrac{1}{\omega}$

From $e = E_0 \sin 100\,t$, $\omega = 100$ rad/s

$$\therefore \quad CR = \frac{1}{\omega} = \frac{1}{100}$$

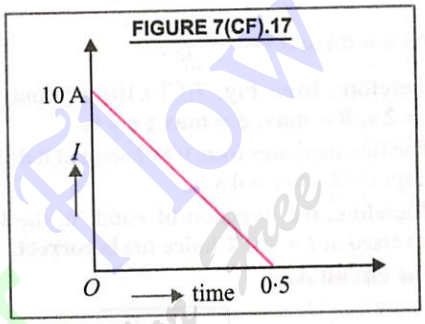
When $R = 1\ \text{k}\Omega = \mathbf{10^3\ \Omega}$

$$C = \frac{1}{\omega R} = \frac{1}{10^5} = 10^{-5}\,F = \mathbf{10\ \mu F}$$

16. Here, $R = 100\ \Omega$

As $q = \int I\,dt$ = area of current time graph

$$q = \frac{1}{2}\times 10\times 0{\cdot}5 = 2{\cdot}5\ \text{C}$$



From $q = \dfrac{\Delta\phi}{R}$

$\Delta\phi = q \times R = 2{\cdot}5 \times 100 =$ **250 Wb**

17. In R–C circuit, the impedance is

$Z = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$; As ω increases, Z decreases.

Since, Power $\propto \dfrac{1}{\text{impedance}}$,

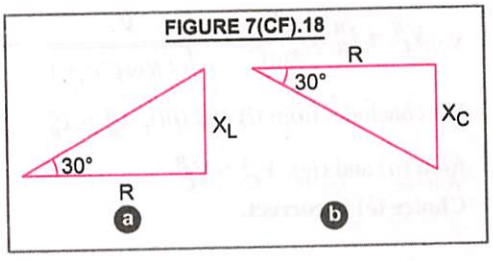
therefore, **the bulb glows brighter.**

18. Here, $R = 200\ \Omega$, $E_v = 220$ V, $\nu = 50$ Hz

In L–R circuit, Fig. 7(CF).18(a), $\tan 30° = \dfrac{X_L}{R}$

In C–R circuit, Fig. 7(CF).18(b), $\tan 30° = \dfrac{X_C}{R}$

$\therefore \quad \dfrac{X_L}{R} = \dfrac{X_C}{R}$ or $X_L = X_C$.



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In LCR circuit, if θ is the phase difference between voltage and current, then

$$\tan\theta = \frac{X_L - X_C}{R} = \frac{0}{200} = 0 \quad \text{or} \quad \theta = 0°$$

i.e., current and voltage are in the same phase.

∴ Average power $= E_v I_v \cos\theta$

$$= \frac{E_v^2}{R}\cos 0° = \frac{(220)^2}{200} = 242\,\text{W}$$

19. As $\phi = BA$ and $|e| = \frac{d\phi}{dt}$,

therefore, from Fig. 7(CF).10, we find that at $t = 2$ s, $B = \max$, $\phi = \max$; $e = 0$.

The flux increases from 0 to 2 sec and it decreases from $t = 2$ s to $t = 4$ s.

Therefore, the direction of e.m.f. in the loop is reversed at $t = 2$ s. **Choice (*a*) is correct.**

20. For circuit A,

Impedance, $Z_A = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Current in circuit, $I_R^A = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$ ...(*i*)

Pot. diff. across C,

$$V_C^A = I_R^A \times \frac{1}{\omega C} = \frac{V}{\sqrt{(R\omega C)^2 + 1}} \quad ...(ii)$$

For circuit B, impedance,

$$Z_B = \sqrt{R^2 + \frac{1}{(4\omega)^2 C^2}}$$

Current in circuit,

$$I_R^B = \frac{V}{\sqrt{R^2 + \frac{1}{(4\omega C)^2}}} \quad ...(iii)$$

Pot. diff. across C,

$$V_C^B = I_R^B \times \frac{1}{4\omega C} = \frac{V}{\sqrt{(4R\omega C)^2 + 1}} \quad ...(iv)$$

We conclude, from (*i*) and (*iii*), $I_R^B > I_R^A$

from (*ii*) and (*iv*), $V_C^A > V_C^B$

Choice (*c*) is correct.

21. During discharging of capacitor C through inductance L, let at any instant, charge in capacitor be q and current in inductance be changing at the rate of dI/dt.

The emf equation of the circuit is

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \quad \text{or} \quad \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

or $$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \quad ...(i)$$

It is a differential equation of second order which shows that the charge is oscillatory in L-C circuit, *i.e.*, $Q = Q_0 \sin\omega t$.

Comparing (*i*) with the relation

$$\frac{d^2Q}{dt^2} + \omega^2 Q = 0$$

We have $\omega^2 = \frac{1}{LC}$ or $\omega = \frac{1}{\sqrt{LC}}$.

Max. energy stored in capacitor $= \frac{1}{2}\frac{Q_0^2}{C}$

Let at an instant t, the energy be stored equally between electric and magnetic field. Then energy stored in electric field at instant t is

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\left[\frac{1}{2}\frac{Q_0^2}{C}\right] \quad \text{or} \quad Q^2 = \frac{Q_0^2}{2}$$

or $$Q = \frac{Q_0}{\sqrt{2}}$$

$$Q_0 \sin\omega t = \frac{Q_0}{\sqrt{2}} \quad \text{or} \quad \omega t = \frac{\pi}{4}$$

or $$t = \frac{\pi}{4\omega} = \frac{\pi}{4\times(1/\sqrt{LC})} = \frac{\pi\sqrt{LC}}{4}$$

22. Time constant for parallel combination of two identical capacitor and a resistor $R = 2\,RC$.

Time constant for series combination of two identical capacitor and a resistor $R = RC/2$.

As $V = V_0 e^{-t/RC}$

In first case, $\frac{V_0}{2} = V_0 e^{-t_1/2RC}$ ...(*i*)

In second case, $\frac{V_0}{2} = V_0 e^{-t_2/(RC/2)}$ ...(*ii*)

From (*i*) and (*ii*), $\frac{t_1}{2RC} = \frac{t_2}{(RC/2)}$

or $$t_2 = \frac{t_1}{4} = \frac{10}{4} = 2{\cdot}5\,\text{s}$$

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23. Here $E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{100}{\sqrt{2}}$ V

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{20}{\sqrt{2}} \text{ A};$$

$$\theta = \frac{\pi}{4} \text{ rad}$$

Average power consumed is

$$P_{av} = E_{rms} I_{rms} \cos\theta$$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos\frac{\pi}{4} = \frac{1000}{\sqrt{2}} \text{ W}$$

Wattless current, $= I_{rms} \sin\theta = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \mathbf{10\ A}$

24. From graph we note that in time interval 0 to $T/2$, the potential diff. $= V_0$ (constant) and in time interval $\frac{T}{2}$ to T, the potential difference = 0.

$$\therefore \quad V_{rms} = \sqrt{\frac{V_0^2 \times \frac{T}{2} + 0^2 \times T/2}{T}} = \frac{V_0}{\sqrt{2}}$$

25. Here, $R = 30\ \Omega$, $X_L = 20\ \Omega$, $\nu = 50$ Hz

$X_L = \omega L = 2\pi\nu L$ or $20 = 2\pi \times 50 \times L$ ...(*i*)

When $\nu' = 100$ Hz, then $X_L' = 2\pi \times 100 \times L$

$$= 2(2\pi \times 50\, L) = 2 \times 20 = 40\ \Omega$$

Current $I' = \frac{E_{rms}}{\sqrt{R^2 + X_L'}} = \frac{200}{\sqrt{30^2 + 40^2}} = \mathbf{4{\cdot}0\ A}$

26. In Fig. 7(CF).12,

$C = 100\ \mu F = 10^{-4}$ F, $R_1 = 100\ \Omega$ $L = 0{\cdot}5$ H,

$R_2 = 50\ \Omega$, $\omega = 100$ rad/s ; $E_v = 20$ V

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-4}} = 100\Omega$$

$$Z_1 = \sqrt{X_C^2 + R_1^2} = \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega$$

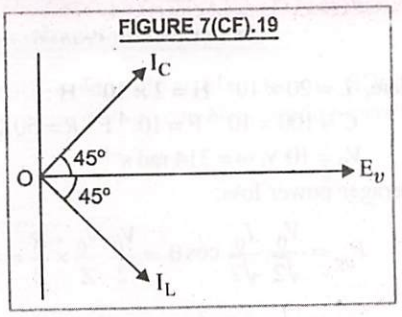
$$I_C = \frac{E_v}{Z_1} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}} \text{ A}$$

Let I_C lead E_v by $\angle\phi_1$

$$\tan\phi_1 = \frac{X_C}{R_1} = \frac{100}{100} = 1$$

$\phi_1 = 45°$, as shown in Fig. 7(CF).19.

FIGURE 7(CF).19



Again, $X_L = \omega L = 100 \times 0{\cdot}5 = 50\ \Omega$

$$Z_2 = \sqrt{X_L^2 + R_2^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}\ \Omega$$

$$I_L = \frac{E_v}{Z_2} = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}} \text{ A}$$

Let I_L lag behind E_v by $\angle\phi_2$

$$\tan\phi_2 = \frac{X_L}{R_2} = \frac{50}{50} = 1,$$

$\therefore \quad \phi_2 = 45°$

$\therefore$ Current through the circuit

$$I = \sqrt{I_C^2 + I_L^2} = \sqrt{\left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{2}{5\sqrt{2}}\right)^2} = \frac{1}{\sqrt{10}}$$

$$= \frac{\sqrt{10}}{10} \simeq \mathbf{0{\cdot}3\ A}$$

$$V_{R_1} = I_C \times R_1 = \frac{100}{5\sqrt{2}} = 10\sqrt{2} \text{ V}$$

$$V_{R_2} = I_L \times R_2 = \frac{2 \times 50}{5\sqrt{2}} = 10\sqrt{2} \text{ V}$$

Choice (*a*) is correct.

27. When L is removed from the circuit, it becomes RC circuit.

$$\tan\phi = \tan\frac{\pi}{3} = \frac{X_C}{R}$$

$\therefore \quad X_C = R \tan \pi/3 = \sqrt{3}\, R$

When C is removed from the circuit, it becomes RL circuit.

$$\therefore \quad \tan\phi = \tan\frac{\pi}{3} = \frac{X_L}{R}$$

$$X_L = R \tan \pi/3 = \sqrt{3}\, R$$

Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{R} = \mathbf{1}$

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28. Here, $L = 20 \times 10^{-3}$ H $= 2 \times 10^{-2}$ H ;
$C = 100 \times 10^{-6}$ F $= 10^{-4}$ F ; $R = 50\ \Omega$,
$V_0 = 10$ V, $\omega = 314$ rad s^{-1}

Averager power loss,

$$P_{av} = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\theta = \frac{V_0}{2} \frac{V_0}{Z} \times \frac{R}{Z} = \frac{V_0^2 R}{2Z^2}$$

where, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

Now, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$= \sqrt{50^2 + \left(314 \times 2 \times 10^{-2} - \frac{1}{314 \times 10^{-4}}\right)^2}$$

$= 56\ \Omega$

$$\therefore\ P_{av} = \frac{V_0^2 R_0}{2Z^2} = \frac{(10)^2 \times 50}{2 \times (56)^2} = 0{\cdot}79 \text{ W}$$

29. Here, $C = 40\ \mu\text{F} = 40 \times 10^{-6}$ F
$E_v = 200$ V, $\nu = 50$ Hz ; $I_v = ?$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$I_v = \frac{E_v}{X_C} = E_v (2\pi\nu C)$$

$= 200 \times 2 \times 3{\cdot}14 \times 50 \times 40 \times 10^{-6}$
$= 2{\cdot}5$ A

30. When L is removed,

$$\tan\phi = \frac{X_C}{R} = \tan \pi/3 = \sqrt{3}$$

When C is removed,

$$\tan\phi = \frac{X_L}{R} = \tan \pi/3 = \sqrt{3}$$

Clearly, $X_L = X_C = \sqrt{3}\, R \quad \therefore\ Z = R$

Power factor, $\cos\phi = \frac{R}{Z} = 1{\cdot}0$

31. On introducing an iron rod in the interior of the conductor, inductance L increases. Inductive reactance $X_L = \omega L$ increases. Current, $I_v = \frac{E_v}{X_L}$ decreases. Hence glow of light bulb $(= I_v^2 R)$ decreases.

32. Given, $V_L = 40$ V, $V_C = 10$ V, $V_R = 40$ V

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{40^2 + (40-10)^2} = 50 \text{ V}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ A}$$

Let Z be the impedance of the circuit, then

$$V_{rms} = I_{rms} \times Z$$

$$\therefore \quad Z = \frac{V_{rms}}{I_{rms}} = \frac{50}{10} = 5\ \Omega$$

33. Here, $L = 5{\cdot}0$ H, $C = 80 \times 10^{-6}$ F, $R = 40\ \Omega$
$V_{rms} = 230$ V

Resonance angular frequency of LCR circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5{\cdot}0 \times 80 \times 10^{-6}}} = \frac{10^3}{20} = 50 \text{ rad/s}$$

Half power angular frequencies are given as

$$\omega = \omega_0 \pm \frac{R}{2L}$$

$$\omega = 50 \pm \frac{40}{2 \times 5} = \mathbf{54 \text{ rad/sec. and } 46 \text{ rad/sec.}}$$

34. Here, $R = 100\ \Omega$, $C = 2\ \mu\text{F} = 2 \times 10^{-6}$ F
$L = 80$ mH $= 80 \times 10^{-3}$ H ; $Q = ?$

As, $$Q = \frac{\omega L}{R} = \frac{L}{R} \cdot \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{100}\sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}} = \frac{1}{100}\sqrt{4 \times 10^4}$$

$Q = 2$

35. Here, $L = 50$ mH $= 50 \times 10^{-3}$ H
$E = 30$ V, $R = 2\ \Omega$, $I = 1$ amp

$\frac{dI}{dt} = 10^2$ A/s, $(V_P - V_Q) = ?$

Applying Kirchhoff's second law to the circuit

$$V_P - L\frac{dI}{dt} - 30 + RI = V_Q$$

$$V_P - V_Q = L\frac{dI}{dt} + 30 - RI$$

$= (50 \times 10^{-3}) \times 10^2 + 30 - 2 \times 1$
$= 5 + 30 - 2 = 33$ volt

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36. Here, $R = 2\ \Omega$

Output power, $P = 1\ \text{kW} = 1000\ \text{W}$

Output voltage, $V = 220\ \text{V}$

As $P = VI$, $\therefore\ I = \frac{P}{V} = \frac{1000}{220} = \frac{50}{11}\ \text{A}$

$$\text{Power loss} = I^2R = \left(\frac{50}{11}\right)^2 \times 2 = \frac{5000}{121}$$

$$\text{Input power} = 1000 + \frac{5000}{121} = \frac{126000}{121}\ \text{watt}$$

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$= \frac{1000 \times 100}{126000\%} = \frac{121}{126} = \frac{121}{126} \times 100\% = \mathbf{96\%}$$

37. Here, $n_p = 50$, $n_s = 1500$

$\phi_p = \phi_0 + 4t$

$$E_p = \frac{d\phi_p}{dt} = \frac{d}{dt}(\phi_0 + 4t) = 4$$

As $\frac{E_s}{E_p} = \frac{n_s}{n_p}$

$$\therefore\quad E_s = \frac{n_s}{n_p} \times E_p = \frac{1500}{50} \times 4 = \mathbf{120\ V}$$

38. $L = 20\ \text{mH} = 20 \times 10^{-3}\ \text{H}$, $C = 50 \times 10^{-6}\ \text{F}$,

$R = 40\ \Omega$, $V = 10 \sin 340t$, Power loss = ?

$\therefore E_0 = 10\ \text{V}$, $\omega = 340\ \text{rad/s}$

$X_L = \omega L = 340 \times 20 \times 10^{-3} = 6{\cdot}8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{340 \times 50 \times 10^{-6}} = \frac{1000}{17}\ \Omega$$

$$= 58{\cdot}8\ \Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{40^2 + (58{\cdot}8 - 6{\cdot}8)^2} = 65{\cdot}6\ \text{ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{E_0}{\sqrt{2}\,Z} = \frac{10}{1{\cdot}414 \times 65{\cdot}6} = 0{\cdot}1078\ \text{A}$$

The power loss $= I_v^2 \times R = (0{\cdot}1078)^2 \times 40$

$= \mathbf{0{\cdot}046\ W}$

Choice (*d*) is correct.

39. Here, $I = 10\ \text{A}$, $V = 80\ \text{V}$

$$R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega$$

$E_v = 220\ \text{V}$, $\nu = 50\ \text{Hz}$, $L = ?$

$$Z = \frac{E_v}{I_v} = \frac{220}{10} = 22\,\Omega$$

As $R^2 + X_L^2 = Z^2$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{22^2 - 8^2} = \sqrt{420}$$

$$= 20{\cdot}49$$

$$X_L = \omega L = 2\pi\nu L = 20{\cdot}49$$

$$L = \frac{20{\cdot}49}{2 \times 3{\cdot}14 \times 50} = \mathbf{0{\cdot}065\ H}$$

40. Here, $V_P = 220\ \text{V}$, $P = 44\ \text{W}$

In ideal transformer,

Input power = Output power

$V_P I_P = P$

$$I_P = \frac{P}{V_P} = \frac{44}{220} = \frac{1}{5} = \mathbf{0{\cdot}2\ A}$$

II. Multiple Choice Questions (with One or More than One Correct Answers)

41. As current and voltage are in phase in resistor, so option (*d*) is not possible, but due to the capacitor, all other options are possible as there is phase difference between current and voltage.

42. Due to capacitor in ac circuit, the voltage lags behind the current by phase $\pi/2$.

As $I = I_0 \cos \omega t$

So $V = V_0 \cos(\omega t - \pi/2) = V_0 \sin \omega t$

Charge in circuit, at a given instant t

$$Q = \int_0^t I\,dt = \int_0^t I_0 \cos \omega t\,dt = \frac{I_0}{\omega} \sin \omega t$$

Max. charge on capacitor,

$$Q_0 = \frac{I_0}{\omega} = \frac{1}{500} = 2 \times 10^{-3}\ \text{C}$$

The current in the left part of the circuit at time $t = \frac{7\pi}{6\omega}$

$$I = I_0 \cos \omega \times \frac{7\pi}{6\omega} = I_0 \cos \frac{7\pi}{6}$$

$$= I_0 \cos\left(\pi + \frac{\pi}{6}\right)$$

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$$= I_0\left[\cos\pi\cos\frac{\pi}{6} - \sin\pi\sin\frac{\pi}{6}\right]$$

$$= I_0\left[-1\times\frac{\sqrt{3}}{2} - 0\right] = -\frac{\sqrt{3}}{2}I_0$$

Thus current in the left part of the circuit is anticlockwise as value of current is negative.

At time $t = \frac{7\pi}{6\omega}$, charge on capacitor is

$$Q = 2\times10^{-3}\sin\omega\times\frac{7\pi}{6\omega}$$

$$= 2\times10^{-3}\sin\frac{7\pi}{6} = 2\times10^{-3}\left[\sin\left(\pi+\frac{\pi}{6}\right)\right]$$

$$= 2\times10^{-3}\left[\sin\pi\cos\frac{\pi}{6} + \cos\pi\sin\frac{\pi}{6}\right]$$

$$= 2\times10^{-3}\left[0 + (-1)\times\frac{1}{2}\right] = -10^{-3}\,\text{C}$$

Potential difference across capacitor due to charge Q $V_C = \frac{Q}{C} = \frac{10^{-3}}{20\times10^{-6}} = 50\,\text{V}$

When A is connected to D, the current in resistance circuit, $I = \frac{V_C + 50}{R} = \frac{50+50}{10} = \mathbf{5\ A}$

Final charge on capacitor after shifting the switch $Q' = CV_C = (20\times10^{-6})\times50 = \mathbf{10^{-3}\ C}$

Charge flown from battery $= 10^{-3} + 10^{-3}$
$= \mathbf{2\times10^{-3}\ C}$

Thus options **(c) and (d) are true.**

III. Multiple Choice Questions (Based on the given Passage/Comprehension)

43. Here, $L = 800\ \text{mH} = 800\times10^{-3}\ \text{H} = 0{\cdot}8\ \text{H}$
$C = 60\ \mu\text{F} = 60\times10^{-6}\ \text{F}$
$R = 15$ ohm, $E_v = 230$ V and $v = 50$ Hz.

$X_L = \omega L = 2\pi v L$
$= 2\times22/7\times50\times0{\cdot}8 = 251{\cdot}43\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C} = \frac{1}{2\times22/7\times50\times60\times10^{-6}}$$

$= 53{\cdot}03$ ohm

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{15^2 + (251{\cdot}43 - 52{\cdot}03)^2}$$

$= 198{\cdot}97$ ohm

$$I_v = \frac{E_v}{Z} = \frac{230}{198{\cdot}97} = 1{\cdot}156\,\text{A}$$

$\therefore$ Average power transferred/cycle to resistance

$P_R = I_v^2\times R = (1{\cdot}156)^2\times15$ **= 20·05 watt**

44. As energy spent = power × time
$\therefore$ energy spent $= 20{\cdot}05\times60\times60$
$= \mathbf{7{\cdot}22\times10^4\ joule}$

45. Here, $n_p = 2000$, $n_s = 50$, $E_p = 120$ V,
$R_s = 0{\cdot}6$ ohm $\eta = 80\%$

As $\frac{E_s}{E_p} = \frac{n_s}{n_p}$

$$\therefore\quad E_s = \frac{n_s}{n_p}\times E_p = \frac{50}{2000}\times120 = \mathbf{3\ V}$$

46. As $\eta = \frac{E_s I_s}{E_p I_p}$

$$\therefore\quad \frac{80}{100} = \frac{3\times5}{120 I_p}$$

or $I_p = \frac{1500}{120\times80} = \frac{5}{32}\,\text{A}$

47. Power in secondary coil is $= E_s I_s$
$= 3\times5 = \mathbf{15\ W}$

IV. Matching Type Questions

48. In a.c. generator, slip ring arrangement is used. In d.c. generator, split ring arrangement is used. Choke coil is the best way of reducing a.c. A transformer works on the principle of mutual induction. **Choice (d) is correct.**

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V. Matrix-Match Type Questions

49. Impedance and reactance are sort of resistance a measured in ohm.
Susceptance is reciprocal of reactance
Admittance is reciprocal of impedance
∴ Susceptance and Admittance are measured in mho or siemen.
A transformer works on a.c. and not on d.c.

VI. Integer Type Questions

50. As $X_{L_1} = X_{C_1}$, So $\omega_1 L = \dfrac{1}{\omega_1 C}$

Thus, $X_{L_2} = \omega_2 L = 2\omega_1 L = 2\left(\dfrac{1}{\omega_1 C}\right) = 2\left(\dfrac{2}{\omega_2 C}\right)$

$= 4X_{C_2}$

i.e., $\dfrac{X_{L_2}}{X_{C_2}} = 4$

51. Here, $C = 0{\cdot}144\ \text{pF} = 0{\cdot}144 \times 10^{-12}\ \text{F}$

$L = \dfrac{1}{\pi^2}\text{mH} = \dfrac{10^{-3}}{\pi^2}\text{H}$,

$v = \dfrac{1}{2\pi\sqrt{LC}}$

$\lambda = \dfrac{v}{v} = v.2\pi\sqrt{LC}$

$= 3\times10^8 \times 2\pi\sqrt{\dfrac{10^{-3}}{\pi^2}\times 0{\cdot}144\times10^{-12}}$

$\lambda = 3\times10^8 \times 2\pi\times\dfrac{1{\cdot}2\times10^{-8}}{\pi} = \mathbf{7{\cdot}2\ m}$

The correct answer is **7 m**

52. Compare $E = 220\sqrt{2}\cos(8\pi)t$ with the standard form :

$E = E_0 \cos\omega t$

$\omega = 8\pi = \dfrac{2\pi}{T}$

$\therefore\ T = \dfrac{2\pi}{8\pi} = \dfrac{1}{4}\text{sec.}$

In $\dfrac{1}{4}$ sec., current becomes zero, twice.

∴ In 1 sec., current will become zero **8 times**

53. Here, $V = 100\sin 100\pi t$, $R = \dfrac{25}{\sqrt{2}}$ ohm.

Compare with the standard form $V = E_0 \sin\omega t$

$E_0 = 100$, volt ; $E_v = \dfrac{E_0}{\sqrt{2}} = \dfrac{100}{\sqrt{2}}$ volt

$I_v = \dfrac{E_v}{R} = \dfrac{100/\sqrt{2}}{25/\sqrt{2}} = \mathbf{4\ A}$

VII. Assertion-Reason Type Questions

FOR MEDICAL STUDENTS

54. Power factor of series *R-L* circuit is given by

$\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2+\omega^2 L^2}}$

When frequency becomes double, then

$\cos\phi' = \dfrac{R}{\sqrt{R^2+4\omega^2 L^2}}$

So $\cos\phi' < \cos\phi$

Thus **Assertion is true.** Here, **Reason is false.**

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55. Both, the assertion and reason are true and latter is correct explanation of the former.

56. In series resonance circuit, $\omega L = 1/\omega C$

$$\therefore \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R$$

Assertion is true. But reason is false as inductive reactance is only equal to capacitive reactance.

FOR ENGINEERING STUDENTS

57. Statement-1 false. 220 V is the *rms* value of alternating *e.m.f.* and not the peak value. **Statement-2 is true.**

58. This is possible when primary coil of transformer is used as secondary and vice-versa. **Both the statements are true and statement-2 can be understood as correct explanation of statement-1.**

VIII. Numerical Value Type Questions (In Decimal Notations)

59. Electromagnet needs lower voltage. So, the **supply is through a step down transformer.**

60. From $n = \frac{1}{lD}\sqrt{\frac{T}{\pi d}}$

As n is constant, $l \times D$ = constant. When D is doubled, l must become half. Therefore, new resonance length $= \frac{26{\cdot}7}{2} = 13{\cdot}35$ cm = **13·4 cm**

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